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β-testing Cosmological Holography

Based on works with Alishahiha, Dong, Horn, Karch, Matsuura, Polchinski, Tong, Torroba

(2004 - present)

May 22, 2012 KITP BitBranes12 Conference

Motivations

- Accelerated expansion + exit in early U (inflation: Structure seeded by quantum fluctuations) and late U – now precision science, limited by cosmic variance

- Gibbons/Hawking + Covariant entropy bound suggest a holographic framework for dS + exit FRW
  – Goal: precision holographic Cosmology, limited by entropy bounds

cf Bousso Harlow Susskind

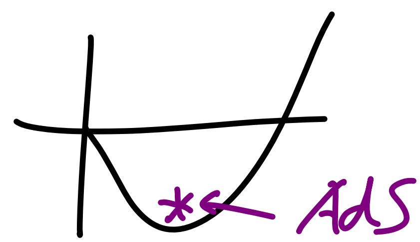
Anninos '12 review

Plan

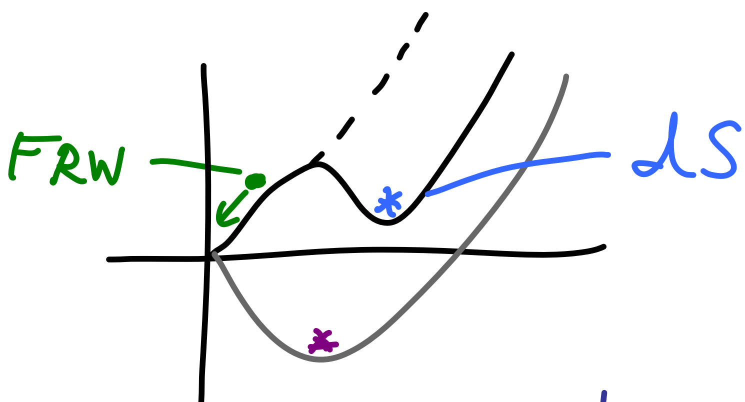
- Concrete Uplifts of AdS/CFT to cosmology
  - Alishahia, Karch, ES, Tong '04
  - Dong Horn ES Torroba '05
  - ..., Matsuura '10, '11
- Role of basic ingredients (β-testing):
  - Flavor content + unitarity
    Dong Horn ES Torroba '12
  - UV structure of dS/dS:
    Wilsonian action & (meta-) stability of the moduli

(In progress, a bits & branes production)

Start by asking what happens to AdS/CFT



when we uplift to cosmology

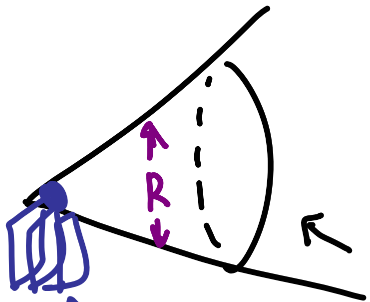


in concrete examples.

- p-brane, D-brane technology → BH entropy, AdS/CFT.
- structure of cosmo solutions in string theory provides useful guidance as well, as we'll see.

AdS/CFT brane construction

$$\frac{\left(\frac{dR}{dr}\right)^2}{R^2} = \frac{1}{R^2} \rightarrow R = r$$
(cone)

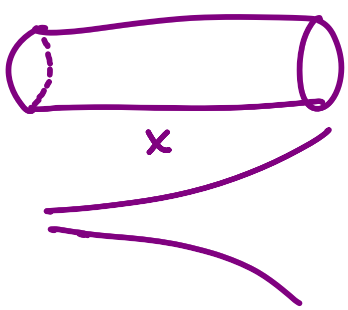


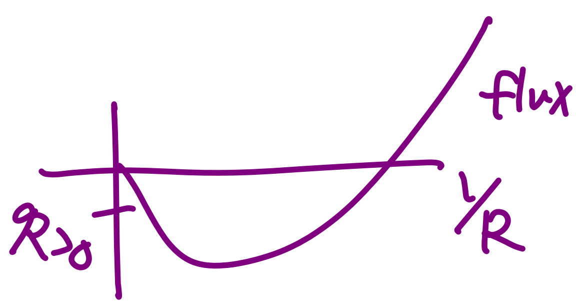
Gravitational Redshift $g_{00} \sim 1 - \frac{R^4}{r^4}$

$\Rightarrow$ Low-energy region

$$E = \sqrt{-g_{00}}\, E_{pr} << E_{pr}$$

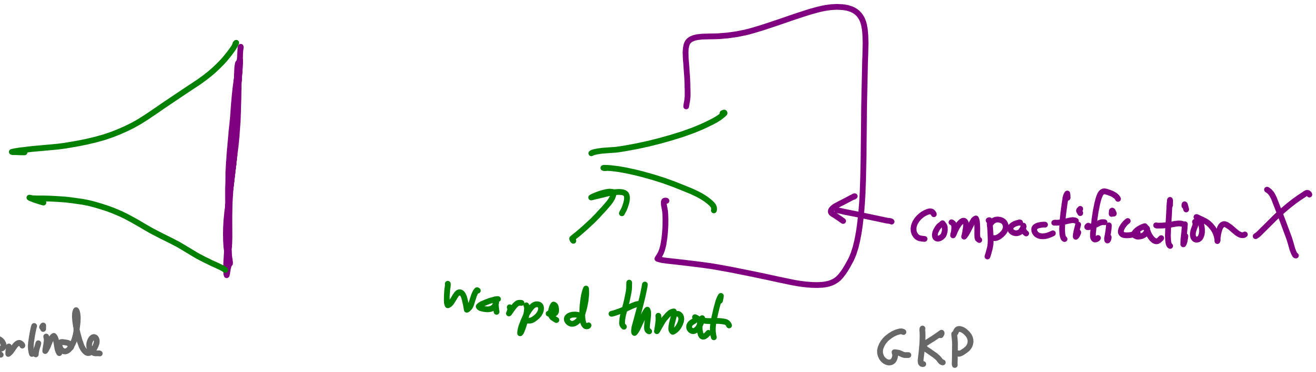
$\rightarrow$ Effective field theory (EFT) dual, complete QFT in strict near-horizon limit





RS/warped compactifications

$$ds^2 \simeq \frac{r^2}{R^2}\,dx^2 + \frac{R^2}{r^2}\,dr^2 + \text{internal}$$



$$\simeq \underbrace{CFT}_{D-1} + \underbrace{GR_{D-1}} + \cdots$$

$$E < \Lambda_c = \frac{r_{uv}}{R^2} \;\Big|\; M_p^2 \sim \frac{r_{uv}^2}{R^4} N^2 + \frac{Vol(X)}{g_s^2}$$

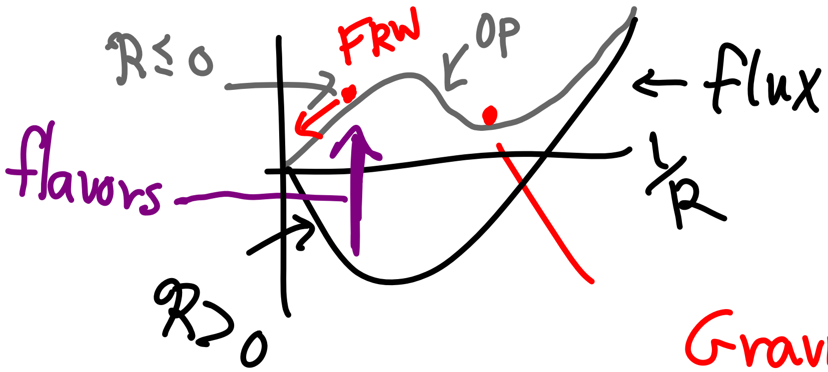
$\rightarrow$ Complete QFT in limit

$$r_{uv} \rightarrow \infty \quad \text{with} \quad \frac{r_{uv}^2}{R^4} N^2 > \frac{Vol(X)}{g_s^2}$$

Turns out to be a good analogy for

$dS \rightarrow FRW$

Uplifting AdS/CFT: Brane Constructions

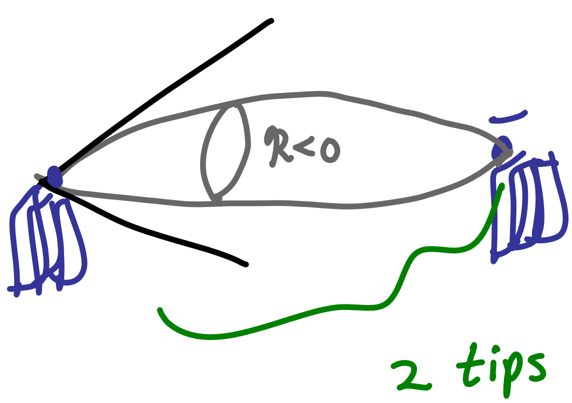


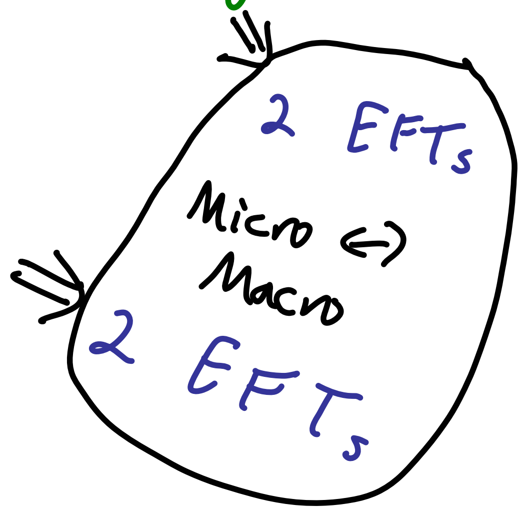
(+ other ingredients to fully meta-stabilize)

Gravity Solution:

$$ds^2_{dS_D} = \underbrace{\sin^2\frac{w}{L}\, ds^2_{dS_{D-1}}}_{\text{2 redshifted regions}} + dw^2$$

Brane Construction:





$$\frac{\left(\frac{dR}{dr}\right)^2}{R^2} = \frac{-1}{R^2} + \frac{\text{const}}{R^{n>2}}$$

- GR_{D-1} (e.g. Liouville) at finite times, but large matter sector dual to large throat

More specifically

dS) Uplifted AdS_3/CFT_2 D1-D5 system with elliptic fibration, O-planes, D-branes → $R_{dS} \gg \sqrt{\alpha'}$

→ parametric entropy count $g_s \ll 1$

FRW) Uplift $AdS_5 \times S^5$ with $n > 36$ (p,q) 7-branes → GR solution

$$ds^2_{5dE} = c^2\left(\hat{t}^{2/c} - w^2\right)^{c-1} dw^2 + \left(1 - \frac{w^2}{\hat{t}^{2/c}}\right)^{c-1}\left(-d\hat{t}^2 + c^2\hat{t}^2 d\mathbb{H}_3^2\right)$$

$c^2 = \frac{7}{3} > 1$ ⇒ 2 warped throats

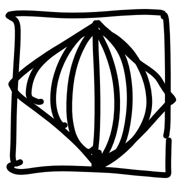
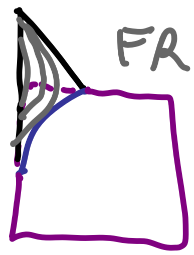
★ GR_{D-1} decouples at late times; S bound → ∞

· QFT side: t-dependent theory, with $n > 36$ magnetic flavors

→ parametric count of d.o.f. on junctions

Comparative holography:

Recall in AdS/CFT that p-brane construction lands on Poincaré slicing. In dS + FRW, above brane constructions → slicings

dS/dS  &  FRW/t-dependent QFT

- inside a causal region
- a spatial direction (↔ scale) emerges.

⇒ • # of degrees of freedom real, >0 and unitarity more transparent

• symmetries (such as they are) less manifest

As in AdS → Global, this may connect to other slicings (dS/CFT, FRW/CFT)

also have dynamical gravity

Anninos/Hartman/Strominger
Harlow/Shenker/Stanford/Susskind

We would like to extract the essential features of the dual theories, given the concrete brane constructions

Plan:

(1) Flavor content of FRW duals, unitarity, & time-dependent QFT couplings

(2) dS_D duals and holographic RG

Magnetic flavors & uplifting

e.g. IIB (p,q) 7Bs wrapping $\Sigma_3 \subset S^5$

Tension $\propto \Lambda \times \frac{1}{R^2}\frac{1}{g_s^2}$

$$S^1 \to S^{5 \text{ or } 3}$$
$$\downarrow$$
$$\mathbb{CP}^{2 \text{ or } 1}$$

Competes with curvature

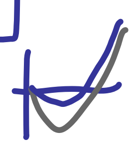
on $\mathbb{CP}^1$: 24 7Bs $\to \mathcal{R}=0$

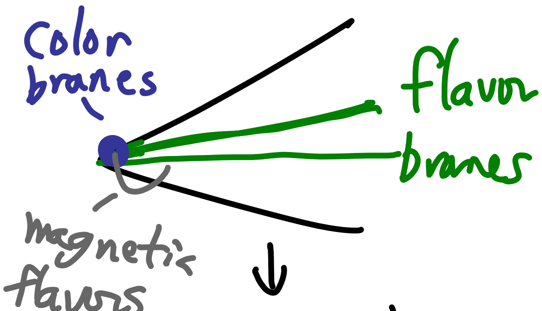
* $\mathbb{CP}^2$: 36 7Bs $\to \mathcal{R}=0$

Banks Douglas Seiberg, Aharony Maldacena Fayyazuddin Joe P ES

$$\Delta n \equiv n - n_{\mathcal{R}=0}$$

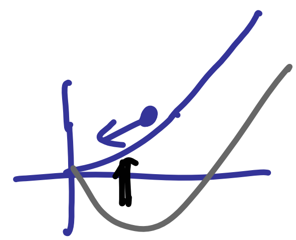
$\boxed{\Delta n < 0}$

AdS 



$\downarrow$

Static soln

$\boxed{\Delta n \geq 0}$ 

Cosmo

no static solution but $\exists$ simple, t-dep't solutions

cf Kleban + Redi

$\Delta n \gtrless 0$ distinction in dual QFT?

Unitarity bounds & t-dependent QFT.

Given well-defined QFT at some scale, unitarity bounds (e.g. $\Delta_{scalar} > \frac{d-2}{2}$ in CFT) help constrain IR physics.

(e.g. SQCD conformal window ends at $N_f = \frac{3}{2} N_c$, ...)

In D3 – (p,q)7 system,

$\Delta n > 0 \Rightarrow$ no static solution.

☆ In the simplest case (with parallel 7-branes $\Leftrightarrow \mathcal{N}=2$ susy) this follows from unitarity.

Seiberg-Witten curve

$$y^2 = x^3 - \overset{\sim u^r}{f(u)}\,x - \overset{\sim u^s}{g(u)}$$

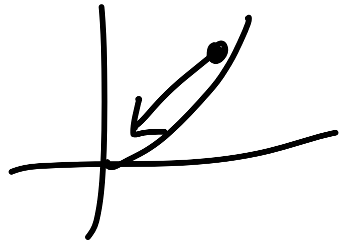
$(s < \tfrac{3}{2}r)$ $\boxed{2\Delta_y = 3\Delta_x = s\Delta_u}$

$$\frac{d\lambda_{SW}}{du} = \frac{dx}{y} \qquad a \sim \int \lambda_{SW} \sim \frac{u\,dx}{y}$$

BPS masses dim 1 $\Rightarrow$ $\boxed{\Delta_u = 1 + \Delta_y - \Delta_x}$

$$\boxed{\Rightarrow \Delta_u = \frac{12}{12 - N_f}, \quad N_f > 12 \text{ would violate unitarity}}$$

But there do exist t-dependent solutions



with the required properties (redshift, N_{dof}, $M_p \to \infty$) for a dual EFT $\to$ QFT.

^ t-dependent

$\to$ More general question: how do t-dependent couplings affect unitarity bounds on IR behavior $\leftrightarrow$ field content

Consider

$$\int \Delta\mathcal{L} = \int dt\, d^{d-1}\vec{x}\; g(t,\vec{x})\, \mathcal{O}$$

$$g = g_0 t^\alpha \quad \text{or} \quad g_0 (t^2 - \vec{x}^2)^{\frac{\alpha}{2}}$$

$$\text{as } t \to \infty$$

- α can change whether $\Delta\mathcal{L}$ dominates at late times (IR)
- If $\Delta\mathcal{L}$ marginal in IR under $x^\mu \to \lambda x^\mu$, then

$$[\mathcal{O}] = d + \alpha$$

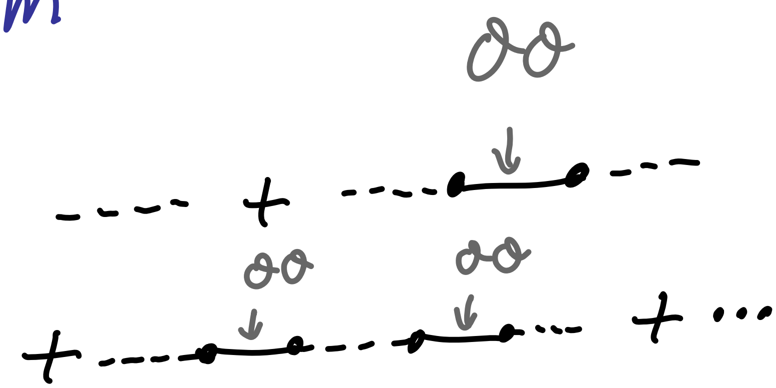
$\to$ Expect α can shift relevance condition & unitarity bounds

We can analyze this explicitly in large-N double trace flows.

$$\Delta S = \int \tfrac{1}{2}\phi\, m^2 \phi + g(t,\vec{x})\, \mathcal{O}\phi$$

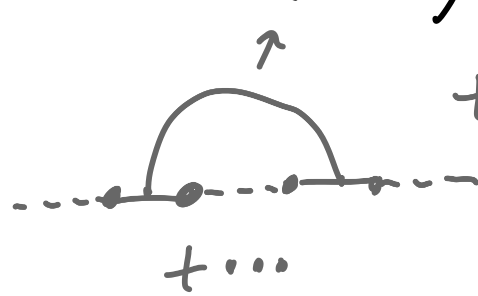
$$\simeq \int \frac{g^2}{2m^2}\, \mathcal{O}\mathcal{O}$$

$\langle \phi \phi \rangle =$



Effectively Gaussian

$+ \mathcal{O}\left(\frac{1}{N}\right)$



+ non-adiabatic effects

Static Limit:

For analysis in holographic case see Andrade, Faulkner, Marolf ...

$$\Delta_\pm = \frac{d}{2} \pm \nu$$

$$\langle \mathcal{O}_\pm(p)\, \mathcal{O}_\pm(-p) \rangle = -i\, C_{\pm\nu} (p^2 - i\varepsilon)^{\pm\nu}$$

$$S^{(+)}_{CFT} + \int \frac{g^2}{4m^2} \underbrace{\mathcal{O}_+ \mathcal{O}_+}_{\substack{\Delta = d+2\nu \\ \text{irrelevant}}}$$

$$\langle \phi(p)\, \phi(-q) \rangle = \frac{-i\, \delta(p-q)}{m^2 - g^2 C_\nu (p^2)^\nu}$$

$$C_\nu = \boxed{\frac{2^{-2\nu} \pi^{\frac{d}{2}} \Gamma(-\nu)}{\Gamma(\frac{d}{2}+\nu)}}$$

$\rightarrow \langle \mathcal{O}_- \mathcal{O}_- \rangle$ as $p^2 \rightarrow \infty$

UV $\mathcal{O}_-$

nonunitary for $\nu > 1$

$\Lambda_g^n \quad \frac{1}{g^{\frac{1}{\nu-1}}}$

ok as cut off QFT

IR $\mathcal{O}_+$ unitary

$\boxed{\text{t-dependent case}}$

- $\int \lambda_0 t^{2\gamma} \mathcal{O}_+^2$ is <u>relevant</u>

(dominates 2 pt ftns at large Δx)

when $[\lambda_0] = 2(\gamma - \nu) > 0$

- Unitarity maintained, including $\nu > 1$

- Can UV complete, e.g. SUSY models

(effects of $\dot{\lambda}$ lost for $\Delta t < \lambda / \dot{\lambda}$)

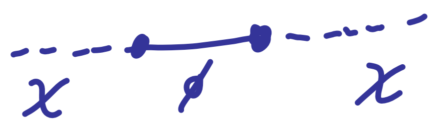
$$\Delta S = \int \tfrac{1}{2}\phi\, m^2 \phi + \underbrace{g(t,\vec{x})\,\phi}_{\equiv\ \tilde{\phi}}\ \mathcal{O}_+$$

$$\langle \tilde{\phi}(p)\,\tilde{\phi}(-q)\rangle \xrightarrow[IR]{} i\,\delta(p-q)\,\frac{(p^2-i\varepsilon)^{-\nu}}{C_\nu}$$

* Not norm of a state

$$\Rightarrow \langle \phi(x)\,\phi(x')\rangle = \frac{-1}{C_\nu C_{-\nu}\, g(x)\, g(x')\,\left[(x-x')^2\right]^{\Delta_-}}$$

*

Despite the Δ_- here, forward scattering* 

is unitary

$$\mathrm{Im}\, A(\chi\to\chi) \propto -\sin\pi\nu / C_\nu > 0$$

(Intriligator/Grinstein: $\mathrm{Im}\, A_f \propto C_{\mathcal{O}}\left(\Delta-\left(\frac{d-2}{2}\right)\right)$ in CFTs)

Scales in t-dependent case:

$\Rightarrow$ UV complete...

- $\lambda(t)$ dynamical field
- SUSY gauge theory e.g. in which ϕ, θ composite below Λ_λ

t-dependence of λ doesn't matter

$\lambda \to$ strong $\}$ $\to \Lambda_\lambda$

would get ghosts at

... or cut off

$$\frac{\dot{\lambda}}{\lambda} \sim \frac{\gamma}{t}$$

$\int \lambda_0 t^{2\gamma} \theta_+^2$ relevant & unitary

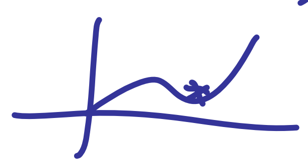
Note: can be more relevant than mass term, maintain long-range correlations!

② UV structure of dS_D duals

Maximal symmetry & RG

$$ds^2_D = \sin(h)^2 \frac{w}{L}\, ds^2_{dS_{D-1}} + dw^2$$

(1) In AdS_D, Moduli fixing $(\partial_{\Phi_I} V = 0)$ is dual to $\beta_{\{\lambda\}} = 0$.

(2) In dS_D, we also (meta-)stabilize the moduli $(\partial_{\Phi_I} V = 0)$  $M^2 \geq 0$

→ How are (1) & (2) reflected in holographic RG? What is the (D-1) dual of (2)?

Wilsonian holographic RG (in progress)

... Heemskerk
Polchinski

$$\begin{cases} ds^2 = L_D^2 \dfrac{dz^2}{z^2} + a(z,x)^2 ds^2_{dS_{D-1}} \\ \text{scalar } \phi \text{ with potential } V(\phi) \end{cases}$$

$a(z,x) \leftrightarrow T$

$\phi \leftrightarrow \mathcal{O}$

integrate out

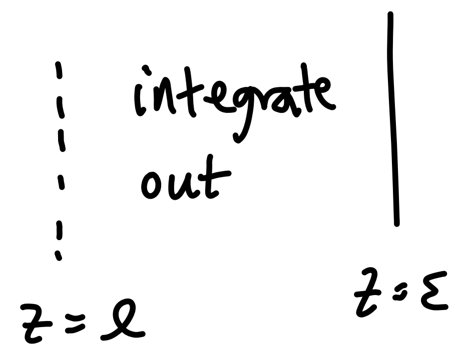
$z = \ell$ $\quad z=\varepsilon$

$\rightarrow$ wilson action $S(\ell) = \int \sigma_{nm}(\ell)\, T^n \mathcal{O}^m$

- For $V'(\phi_*) = 0$, find solution $\partial_\ell \sigma_{01}^{(\vec{L}=0)} = 0$ regardless of sign of $V_* = V(\phi_*)$

$\hookrightarrow$ single-trace coupling ℓ-independent for both dS_D & AdS_D

Metastability & RG



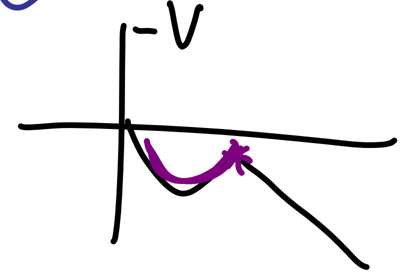
radial Hamiltonian

$$\psi_{UV}[\hat{\phi}, \hat{a}; \ell] = \langle \hat{\phi}\, \hat{a} | e^{-H \log \ell} | \phi_0\, a_0 \rangle$$

(related to Wilson action by integral transform)

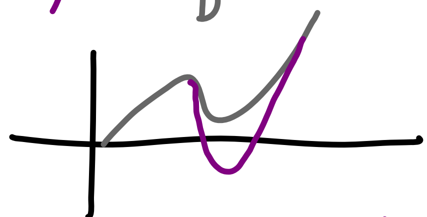
Meta-stability adds bounce solution,

$\Rightarrow$ non-perturbative correction to Wilson action



• **Metastability** & Microscopics

Our explicit (A)dS_D vacua are all metastable



• $D3$-$\overline{D3}$ annihilation • decay to $R \to \infty$ ...

• Reminiscent of

$AdS \times S^5/\mathbb{Z}_k$ (susy with free action) / Quiver $U(N)^k$ QFT's

Not complete CFT, but sensible (exponentially long-lived) as warped throat in compact geometry.

Kachru, ES ; Horowitz, Orgera, Polchinski ; Kachru, Simic, Trivedi

Can compute UV behavior of Correlators

$$ds^2_{dS_d} = \sin^2\frac{w}{L_{dS}}\, ds^2_{dS_{D-1}} + dw^2$$

$$Z_{bulk} = \int D\hat{\Phi} \underbrace{\int [D\Phi]\Big|_{w<\frac{\pi}{2}L_{dS}} e^{iS_<}\Big|_{\Phi(\frac{\pi}{2}L)=\hat{\Phi}}}_{Z^{(1)}_{QFT}[\hat{\Phi}]} \underbrace{\int [D\Phi]\Big|_{w>\frac{\pi}{2}L} e^{iS_>}\Big|_{\Phi(\frac{\pi}{2}L)=\hat{\Phi}}}_{Z^{(2)}_{QFT}[\hat{\Phi}]}$$

e.g. at Gaussian level,

$$\langle \mathcal{O}\mathcal{O}\rangle^{(1)}_L = \frac{\delta^2 Z_1}{\delta\hat{\Phi}_1^{\,2}} = \frac{\Gamma\left[\frac{1}{2}(1+d-\hat{\Delta}+L)\right]\Gamma\left[\frac{1}{2}(1+\hat{\Delta}+L)\right]}{\Gamma\left[\frac{1}{2}(d-\hat{\Delta}+L)\right]\Gamma\left[\frac{\hat{\Delta}+L}{2}\right]}$$

- $\hat{\Delta} = \frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 - M^2L^2}$ (but $\langle\mathcal{O}\mathcal{O}\rangle_\ell$ real)

cf Bousso Maloney Strominger

- Δ flows to $\frac{d}{2}+\frac{1}{2}$ in UV (double trace)
- encodes (max. symmetric) shape of dS_D warp factor

Summary

- Can uplift AdS/CFT to cosmology and derive some features of the resulting holographic duals:

  — 2 EFT's + GR_{D-1}

  Concrete brane constructions ⟷ macro warped metric

  decouples at late times

  — magnetic flavor content $N_f > N_f^{CFT}$ fits with unitarity → more general lesson for spacetime-dependent QFT

  — Can go beyond low energy description, e.g. capturing the $dS_D \to dS_{D-1}$ warp factor in RG and correlators. $\partial_\ell \sigma_{01} = 0$