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Toy Models and Fast Scrambling (I)

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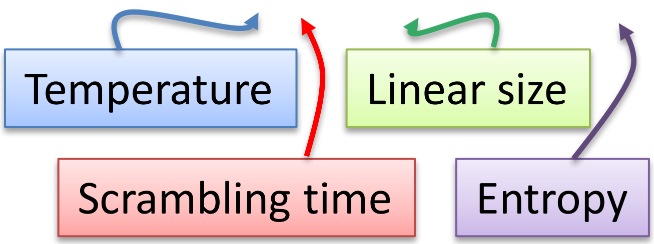
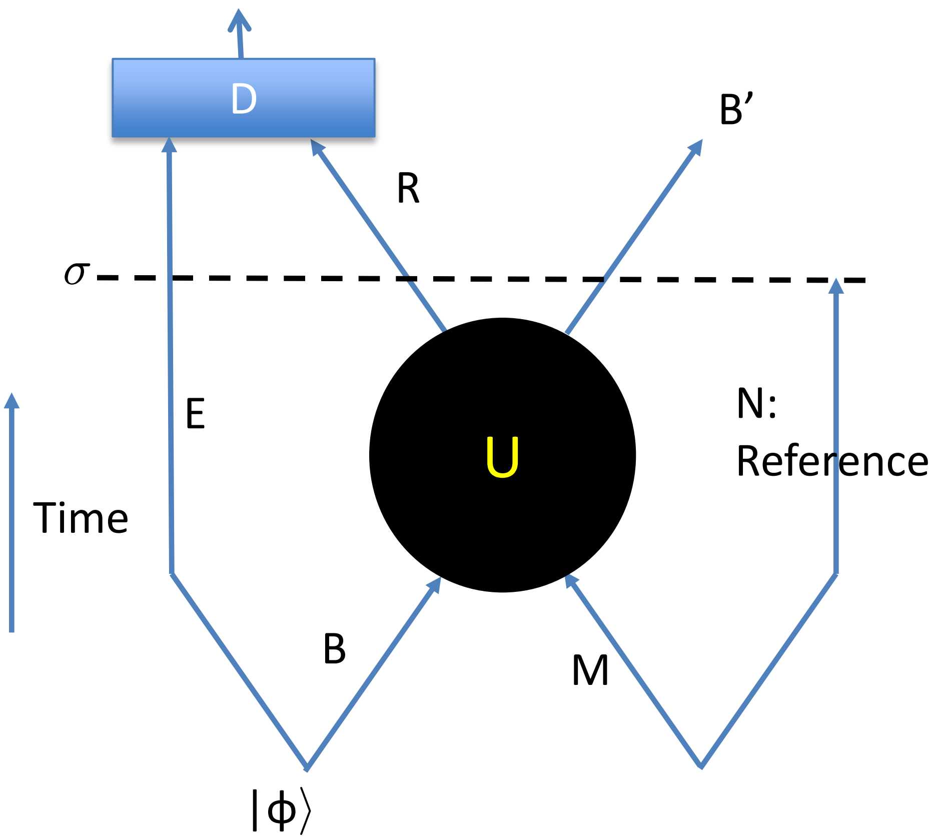
Scrambling

- Minimum time for “localized” information to become inaccessible without measuring fraction O(1) of the whole system
- Normal systems: geometrical locality
  - $Tt^* \sim L^{const} \sim S^{const/d}$



- Schwarzschild black holes:
  - $Tt^* \sim \log S$ (estimate based on charge spreading)
- Conjecture: this is correct, and no system can scramble its degrees of freedom faster [Sekino-Susskind’08, Susskind’11]
  - Motivation: black hole complementarity principle

Scrambling and quantum error correction



Sending arbitrary states from M to R is *equivalent* to establishing entanglement between N and R

Establishing entanglement between N and R is *equivalent* to eliminating all correlations between N and B’

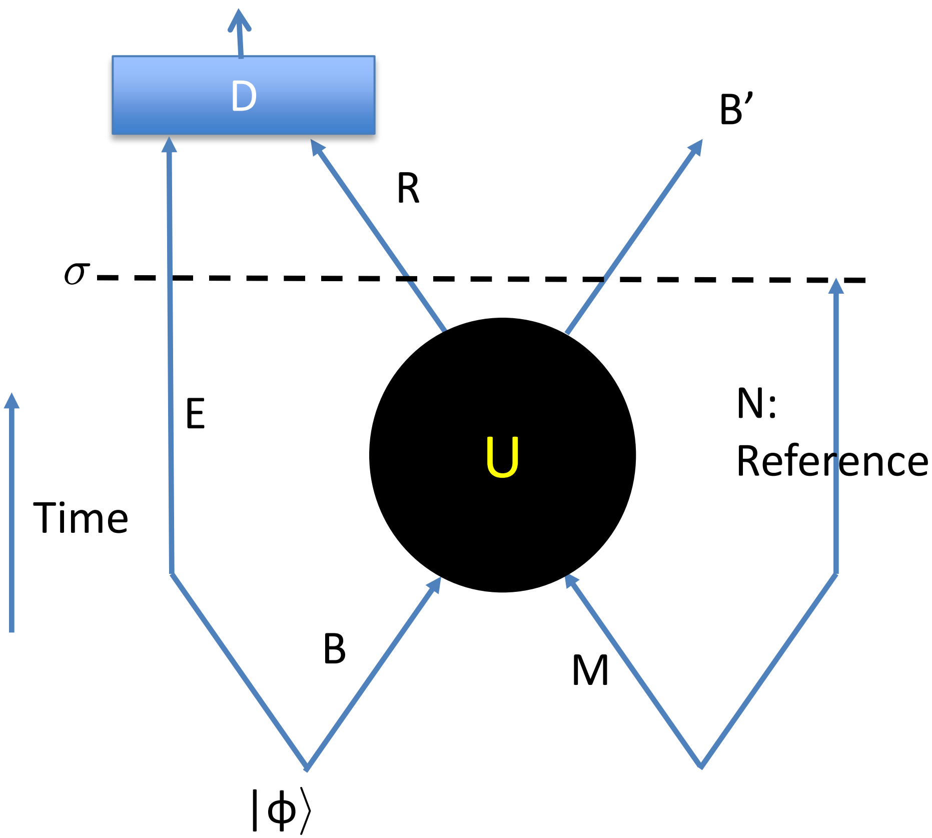
$$\mathrm{Tr}_R\, \sigma_{NB'R} = \tau_N \quad \xi_{B'}$$

$$\Rightarrow$$

$$|\sigma_{NB'R}\rangle = (\mathrm{id}_{NB'} \quad U_R)\,|\phi_{NR1}\rangle\,|\psi_{B'R2}\rangle$$

If U scrambles systems of size |B’|, then the message M can be decoded from R. (No-cloning requires |R| > |B’|.)

Scrambling and quantum error correction



Sending arbitrary states from M to ER is *equivalent* to establishing entanglement between N and ER

Establishing entanglement between N and R is *equivalent* to eliminating all correlations between N and B'

$$\mathrm{Tr}_{ER}\,\sigma_{NB'R} = \tau_N \quad \xi_{B'}$$

$$\Rightarrow$$

$$|\sigma_{NB'ER}\rangle = (\mathrm{id}_{NB'} \quad U_{ER})\,|\phi_{NR1}\rangle\,|\psi_{B'R2}\rangle$$

Scrambling time controls information release. Faster than log S leads to problems for black hole complementarity.

If U scrambles systems of size |B'|, then the message M can be decoded from ER. (~~No cloning requires |R| > |B'|.~~)

Big picture versus toy examples

String theory descriptions of black holes couple degrees of freedom nonlocally.

e.g. BFSS Matrix theory: $$L = \sum_a \mathrm{tr}\, \dot{M}^a \dot{M}^a - \sum_{ab} \mathrm{tr}[M^a, M^b]^2$$

Every pair of matrix entries appears together in at least one term

Would like to show by direct analysi… e system that it is a fast scrambler

HARD

Goal of this hour is more modest:

1) Find examples of toy systems that scramble quickly
    - Want time independent, 2-body interactions, unengineered
    - Should scramble a whole subspace of initial states
2) Prove general lower bounds on scrambling times

Related work

- Asplund, Berenstein, Trancanelli
  - Numerical simulation of BMN matrix model (classical)
- Bar
  - (yperbolic
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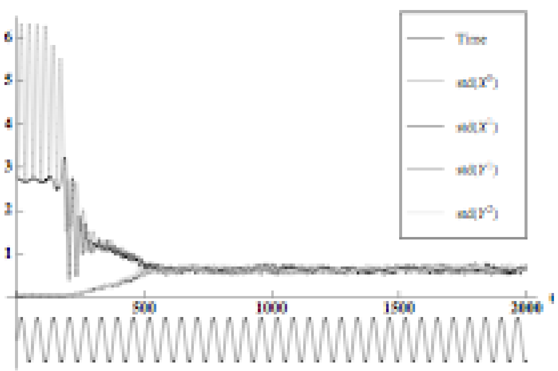
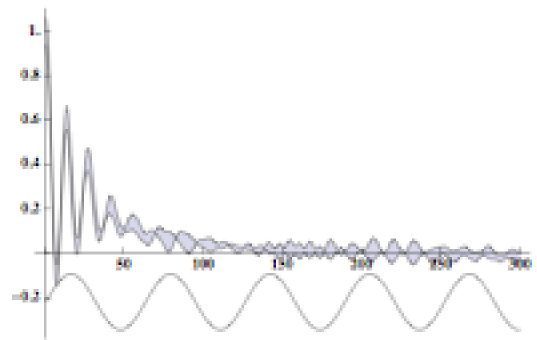
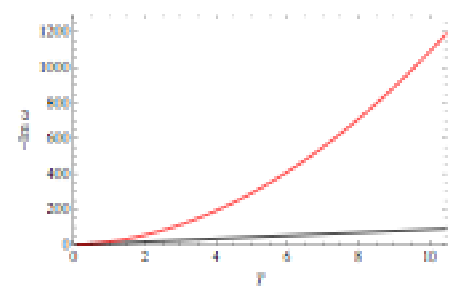




- Eda…, …, …aza, Garcia
  - Use AdS/CFT to study thermalization in strongly coupled noncommutative gauge theories

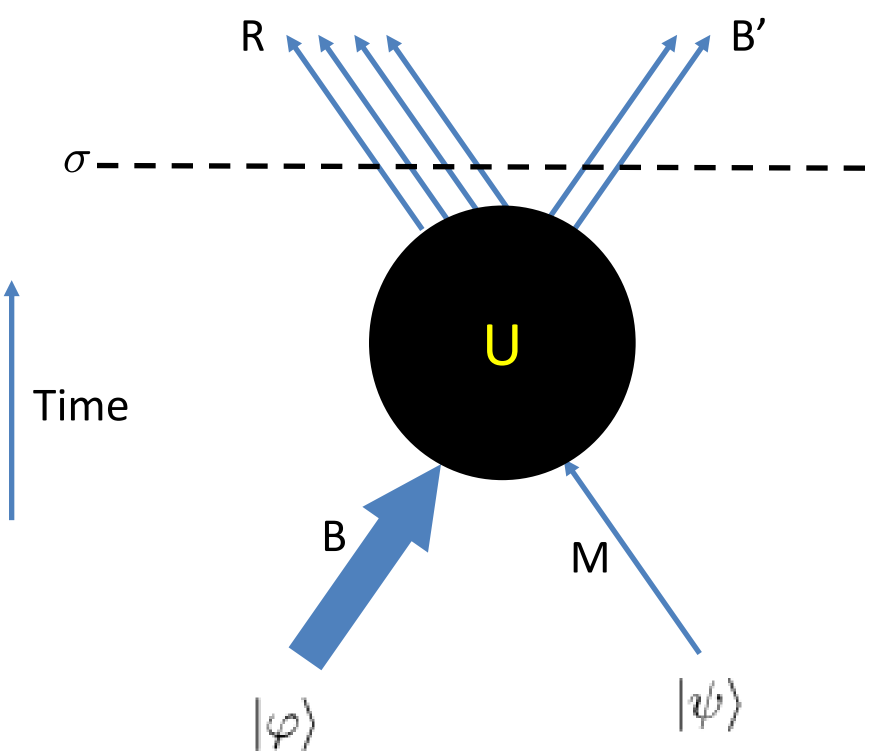


Outline

- Part I
  - Scrambling and quantum error correction ✓
  - Definitions and calibration
  - Brownian quantum circuits
- Part II (Douglas Stanford)
  - Ising interaction on random graphs
  - Lieb-Robinson bounds for nonlocal interactions
  - Comments on AdS/CFT

Some formality

Scrambling *n* subsystems:
any *n/3* should be independent of ψ



There should exist a single ψ_0 such that for all valid ψ

$$\| \mathrm{tr}_R\, \sigma(\psi) - \mathrm{tr}_R\, \sigma(\psi_0) \|_1 < \epsilon$$

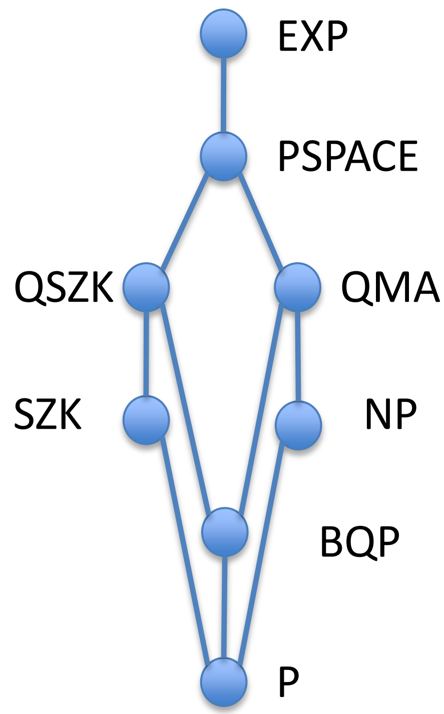
Schatten-l_1 norm measures statistical distinguishability

In our toy models, we will simply compare $\mathrm{tr}_R\sigma(\psi)$ to the unique maximum entropy state on B'.

The computer scientist's cop-out

How hard is it to determine if an efficiently specified U(t) scrambles a specific B'?

Good news! (maybe)



Graph isomorphism

Entropy of time-averaged quantum state

Gnd state energy of 2-body quantum spin systems

Gnd state energy of classical spin glass

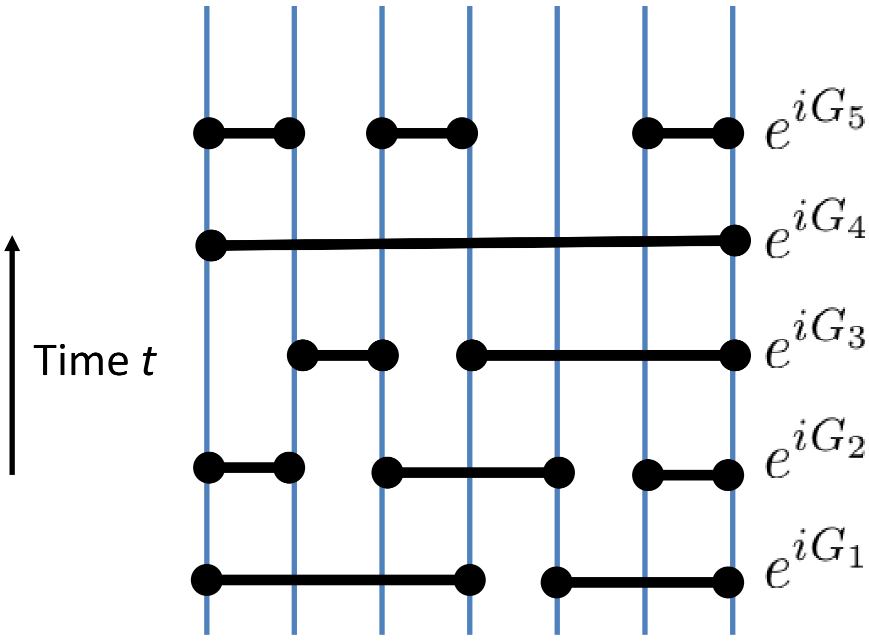
Scattering amplitudes in ϕ^4 theory

Gnd state energy of 1-d Ising model, arb couplings

Determining whether a noisy quantum evolution is correctable is QSZK-complete

[with Brian Swingle]

Brownian circuits



G_i random pairwise interaction

$$G_i = \sum_{<j,k>} \sum_{\alpha_j,\alpha_k} \sigma_{\alpha_j}^{(j)} \otimes \sigma_{\alpha_k}^{(k)} g_{ijk\alpha_j\alpha_k}$$

Location

Operator

i.i.d. Gaussian N(0,ε)

Limit of infinitesimal ε:

$$t) = \sum_{<j,k>} \sum_{\alpha_j,\alpha_k} \sigma_{\alpha_j}^{(j)} \otimes \sigma_{\alpha_k}^{(k)} U(t) dW_{jk\alpha_j\alpha_k}(t) - \frac{1}{2} U(t) dt$$

Weiner process

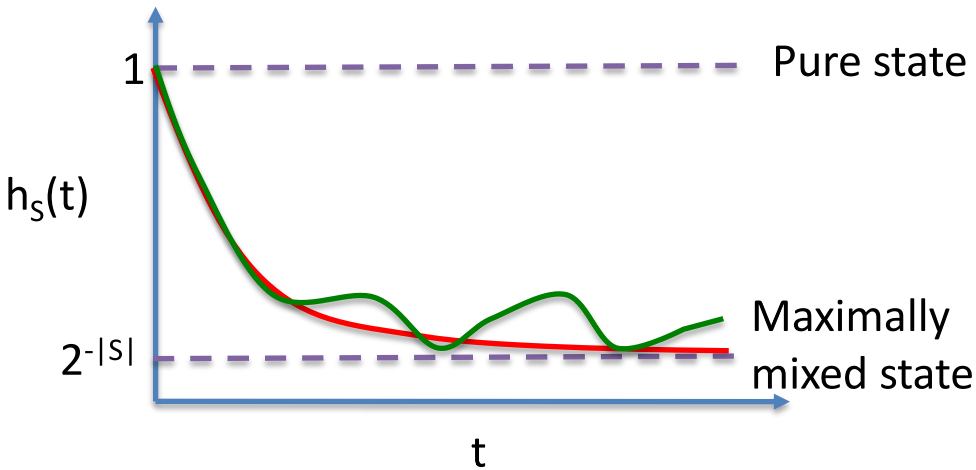
Dankert et al.: Construction of circuit scrambling in time O(log n)

$\sigma_0 = I \; \sigma_1 = \sigma_x \; \sigma_2 = \sigma_y \; \sigma_3 = \sigma_z$

Subsystem entropies

State $\Psi(t)$. Density operator for S subset of $\{1,2,\ldots,n\}$: $\Psi_S(t) = \mathrm{tr}_{[n]\setminus S}\, \psi(t)$.

Interested in *purity* $h_S(t) = \mathrm{tr}\, \psi_S(t)^2$



Smooth out fluctuations by averaging over trajectories: $\langle h_S(t)\rangle$

Simplify by choosing product pure input state $|\psi(0)\rangle = |\psi_1\rangle|\psi_2\rangle\ldots|\psi_n\rangle$

Gives $\langle h_S(t)\rangle = \langle h_{|S|}(t)\rangle = \langle h_k(t)\rangle$

Small miracle: system of linear ODE closes and is (almost) solvable

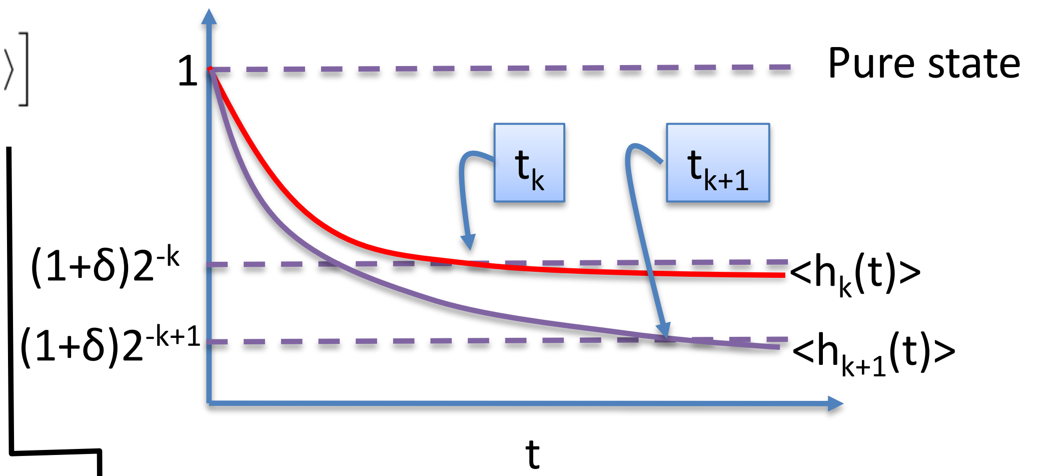
$$\frac{d\langle h_k\rangle}{dt} = k(n-k)\left[2\langle h_{k-1}\rangle - 5\langle h_k\rangle + 2\langle h_{k+1}\rangle\right]$$

Analysis of ODE

$|\psi(0)\rangle = |\psi_1\rangle|\psi_2\rangle\ldots|\psi_n\rangle$

Purity $h_k(t) = \mathrm{tr}\, \psi_{|S|=k}(t)^2$

$$\frac{d\langle h_k\rangle}{dt} = k(n-k)\Big[2\langle h_{k-1}\rangle - 5\langle h_k\rangle + 2\langle h_{k+1}\rangle\Big]$$



Quick and dirty analysis

Let t_k be time at which $\langle h_k(t)\rangle = (1+\delta)2^{-k}$

For $t > t_{k-1}$

$$\frac{d\langle h_k\rangle}{dt} \sim\leq kn\Big[2\frac{1+\delta}{2^{k-1}} - 5\langle h_k\rangle + 2\langle h_{k+1}\rangle\Big]$$
$$\leq kn\Big[2\frac{1+\delta}{2^{k-1}} - 3\langle h_k\rangle\Big]$$

Exponential decay with rate proportional to k.

So $t_k - t_{k-1} \leq O(1/k)$

$$t_k \sim \sum_{j=1}^{k} \frac{1}{j} \sim \log(k)$$

Careful analysis

Solve using Gauss hypergeometric functions

$$\langle h_k(t)\rangle \sim \sum_{j=1}^{n} \alpha_{jk} e^{-3jt} n 2^{-n-1+2j}\, {}_2F_1\left(n+1, 1-m; 2; \frac{3}{4}\right)$$

Brownian circuits: take-home

- Scramble very effectively: subsystems of size smaller than half become almost maximally mixed
- Scramble quickly: $t^*/t_1 = O(\log n)$
- But:
  - Time-dependent
  - Not very physical
  - Lots of randomness