The black hole possibility tree

Santa Barbara 2012

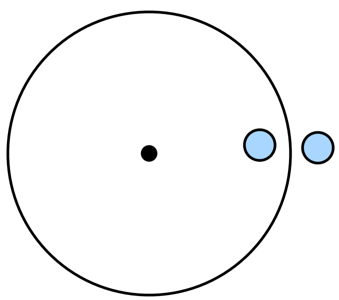
Black holes and beyond (arXiv:1205.0776)

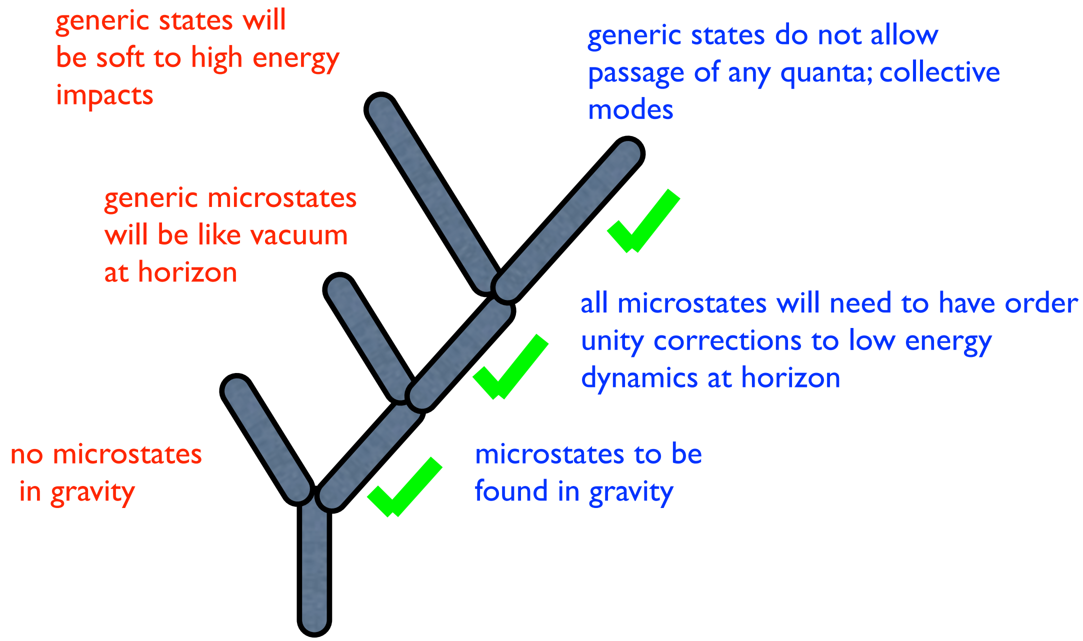
Some common questions (and my responses)

http://www.physics.ohio-state.edu/~mathur/

The black hole information paradox has been puzzling for a long time

But now there are several definite computations that guide us through the possible paths towards a resolution ...





Can address basic questions:

What happens to a collapsing shell ?

What is de Sitter entropy counting ?

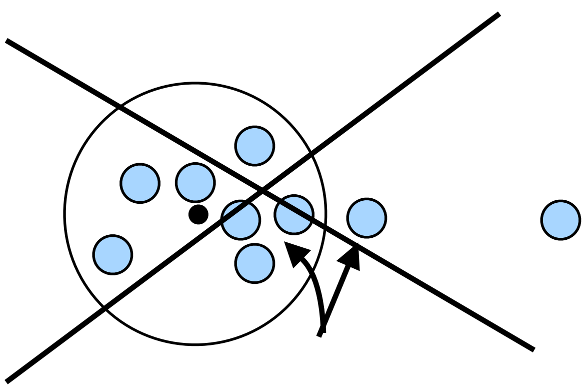
Is there a notion of ‘complementarity’ ? etc ...

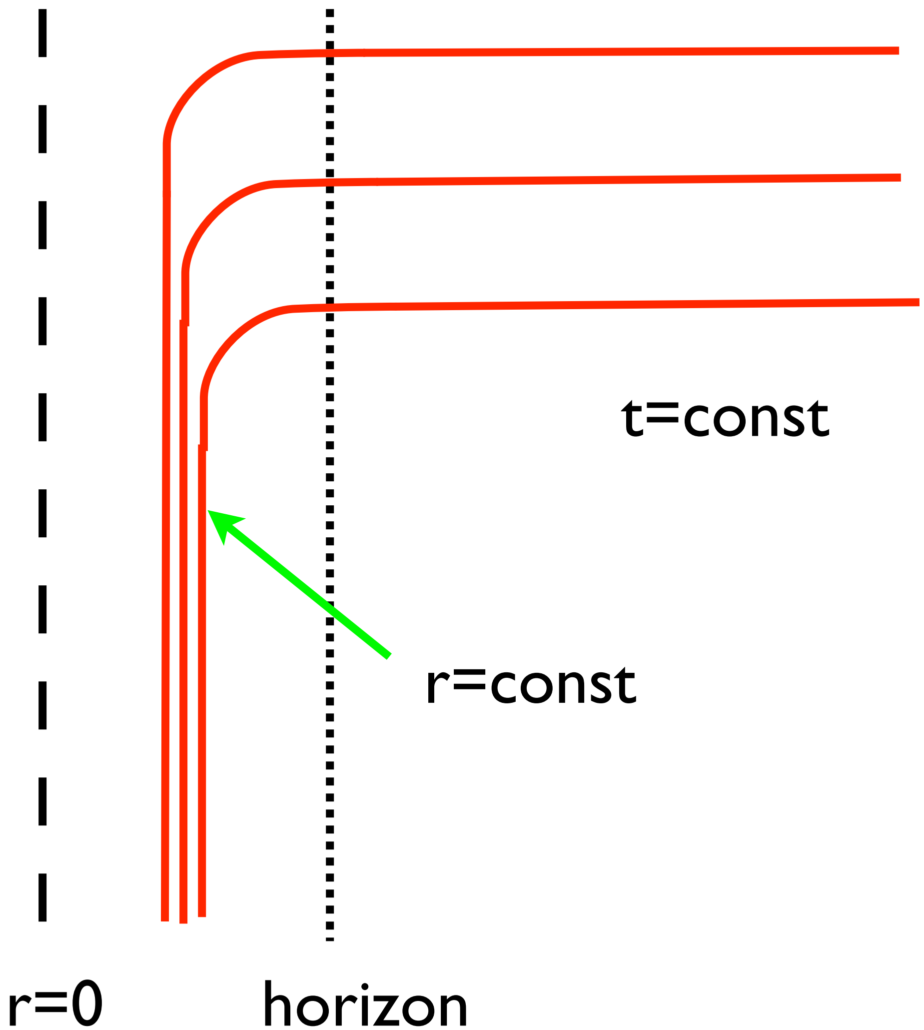
(A) The information paradox: Leading order Hawking computation

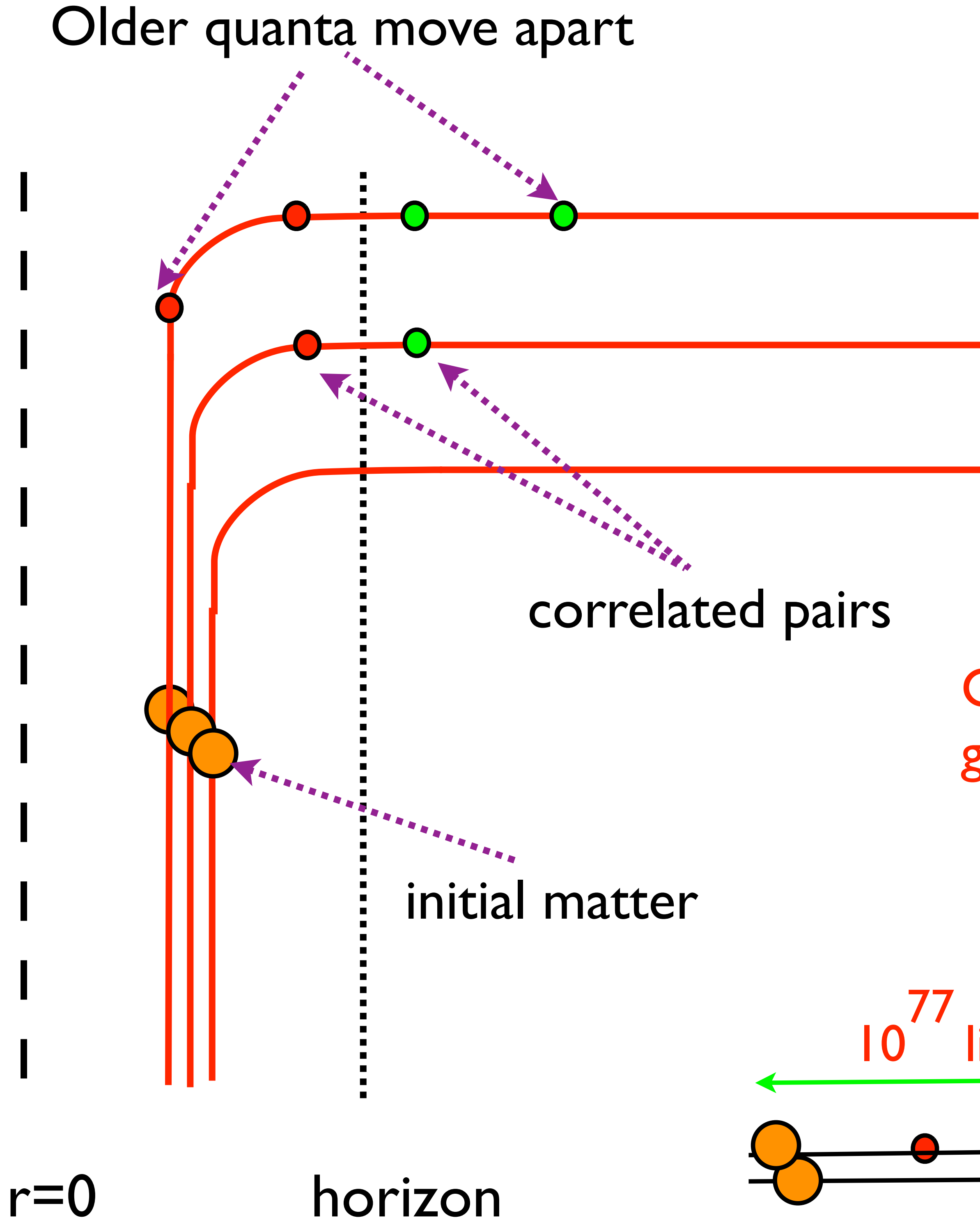
$$ds^2 \;=\; -(1-\frac{2M}{r})dt^2 + \frac{dr^2}{(1-\frac{2M}{r})} + r^2(d\theta^2+\sin^2\theta d\phi^2)$$

If we have a horizon, then we do not have a time-independent slicing of the geometry

Not like a 'ball' ...







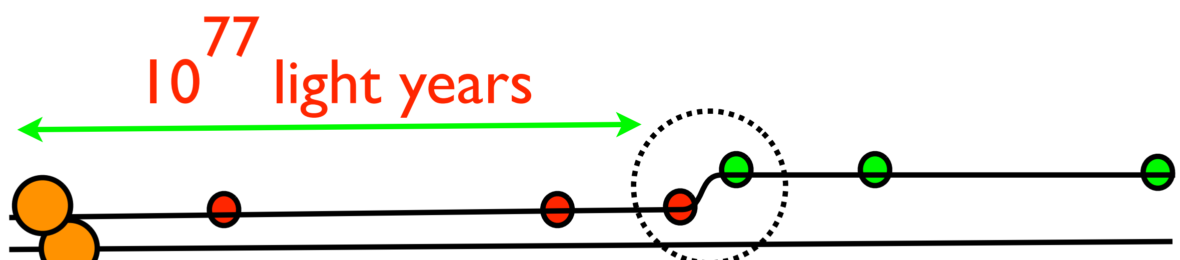
Hawking state

$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle|0\rangle + |1\rangle|1\rangle\Big)$$

(We will use a discretized picture for simplicity; for full state see e.g. Giddings-Nelson) 1

Outside and inside of hole get more and more entangled ....

$$S_{ent} = N \ln 2$$



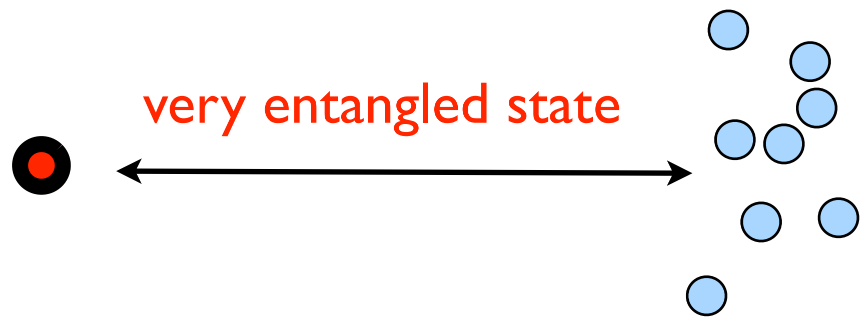
Possibilities:

(a) Complete evaporation

Radiation is entangled, but there is nothing that it is entangled ***with***

.... not a quantum state

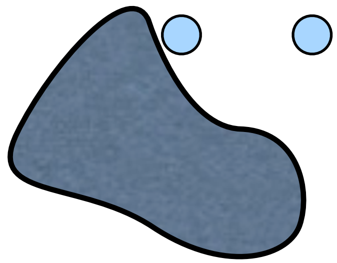
(b) Remnant



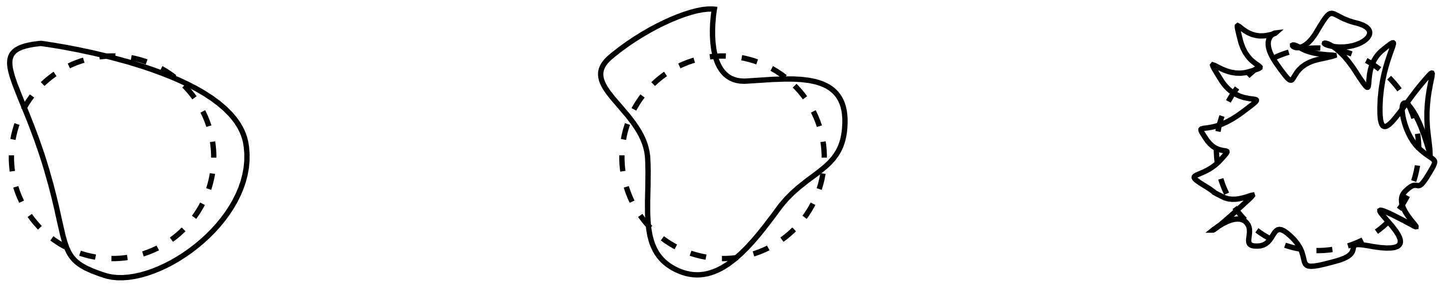
Remnant needs to have unbounded degeneracy while having bounded mass and volume ... problematic

(B) What do we need to do to resolve the paradox ?

In one sense very little, in another sense a lot !

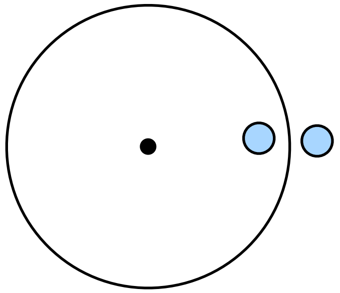


People looked for deformations of the horizon ...

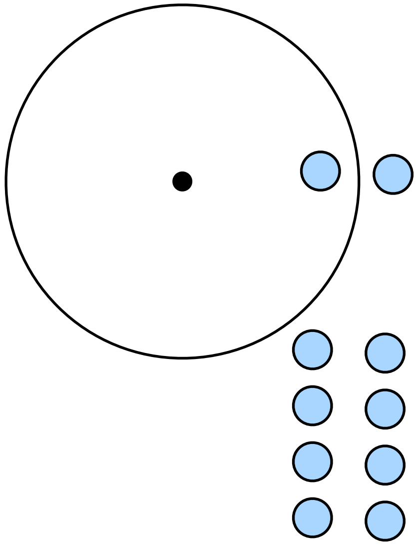


If we get such 'hair', then the horizon structure is altered, and the Hawking argument is invalidated

The problem is that people could not find the 'hair' ... this is what makes the problem difficult



(C) Many string theorists did not worry too much about the paradox, assuming that 'small corrections' would make the entanglement go away



'Nothing happens at horizon' :
Can only have a small correction at each step of pair creation

But number of pairs N is large

State may be non-entangled overall ??

But we know now that this is not true ... if correction at each step is $O(\epsilon)$ then

$$\frac{\delta S_{ent}}{S_{ent}} < 2\epsilon$$

(SDM arXiv: 09091038) ②

(see also recent work of Giddings et al ...)

Resolving the puzzle :

Work of many people has contributed to this picture ...

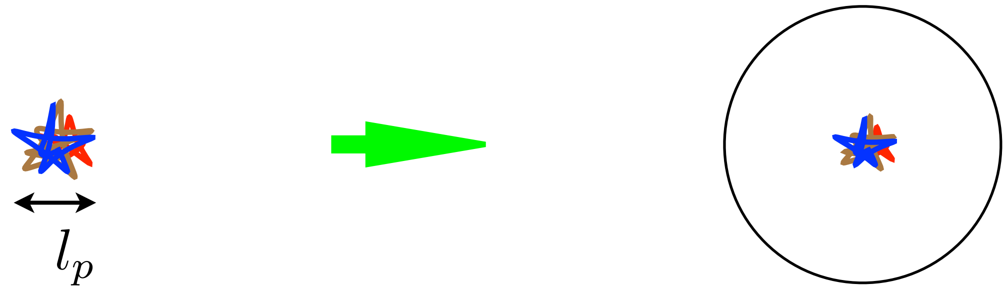
Avery, Balasubramanian, Bena, Chowdhury, de Boer, Denef, Gimon, Giusto, Keski-Vakkuri, Levi, Lunin, Maldacena, Maoz, Park, Peet, Potvin, Ross, Ruef, Saxena, Simon, Skenderis, Srivastava, Taylor, Turton, Warner ...

What do the microstates of black holes look like in string theory ?

That is, what is the nature of brane bound states when the number of branes is large ?

The traditional expectation ...

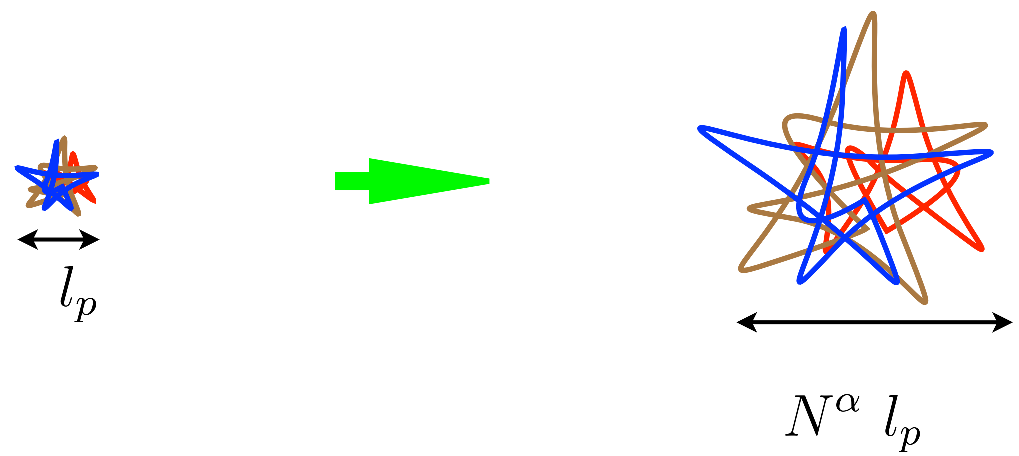
weak coupling



strong coupling

But it seems in string theory the opposite happens ...

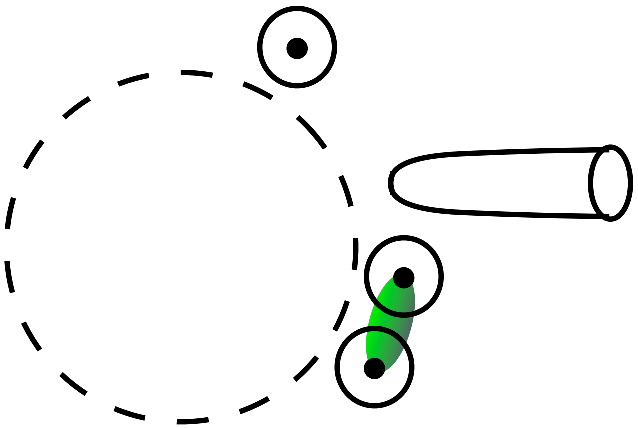
weak coupling



strong coupling

$$R \sim \left[\frac{g^2 \alpha'^3 \sqrt{n_1 n_5 n_p}}{RV}\right]^{\frac{1}{3}} \sim R_s$$

(SDM 97)

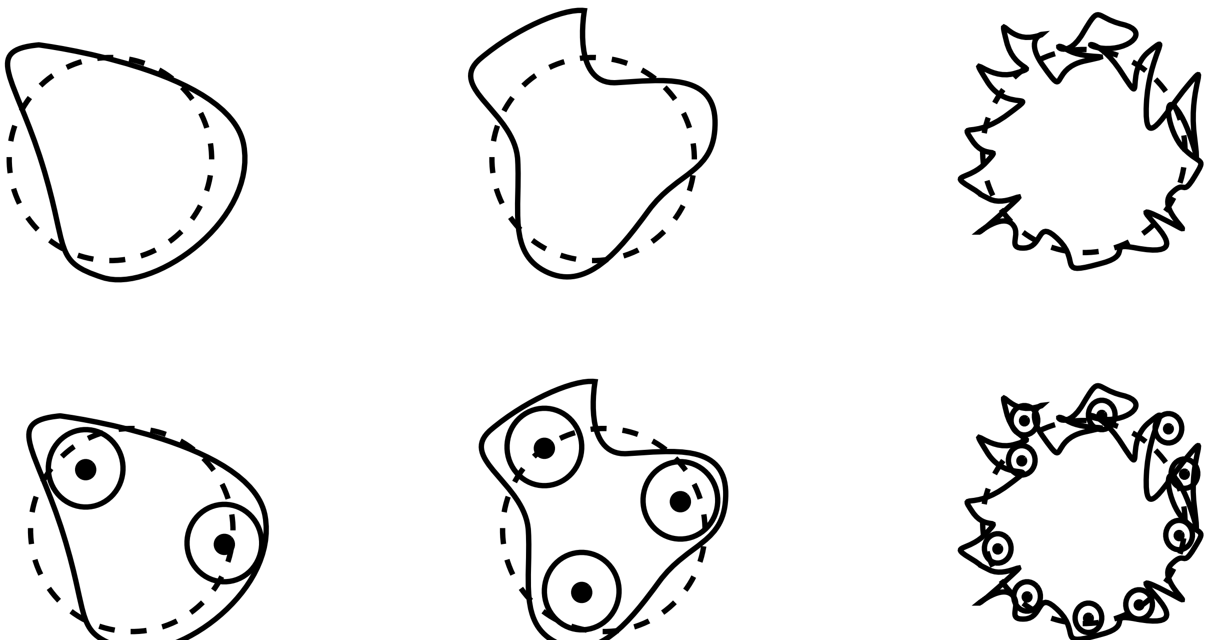


Compact directions pinch off

KK Monopoles and antimonoples

Fluxes etc. on cycles

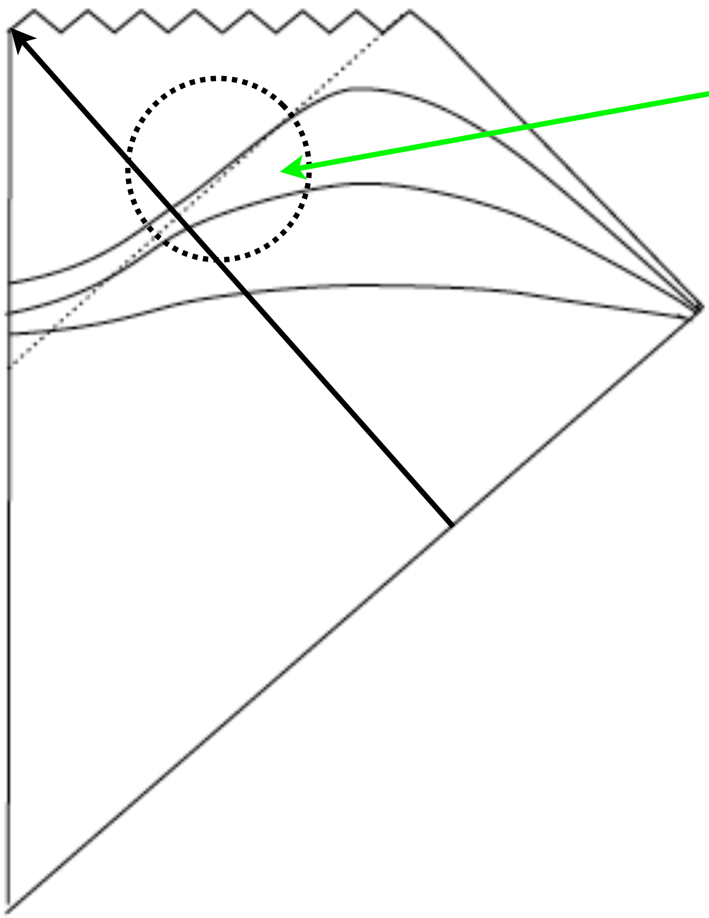
General picture: Manifold ends in allowed sources of string theory, Horizon does not form



??

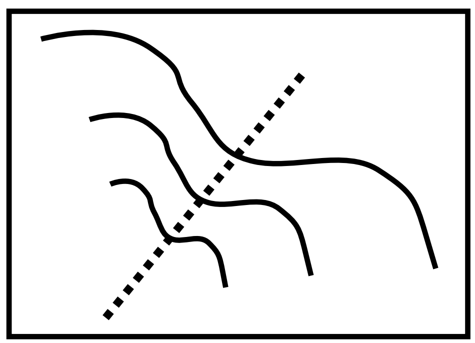
So finally we do find the hair, but it is non-perturbative ...

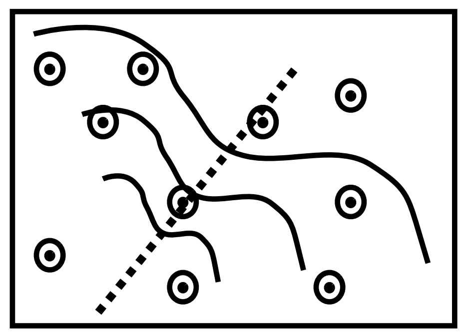
In the beginning some string theorists found this picture strange, because there was a general belief that ‘nothing should happen at the horizon’



Should we get ‘lab physics’ in a good slicing ?

Maybe quantum fluctuations at planck scale do not affect low energy physics ?



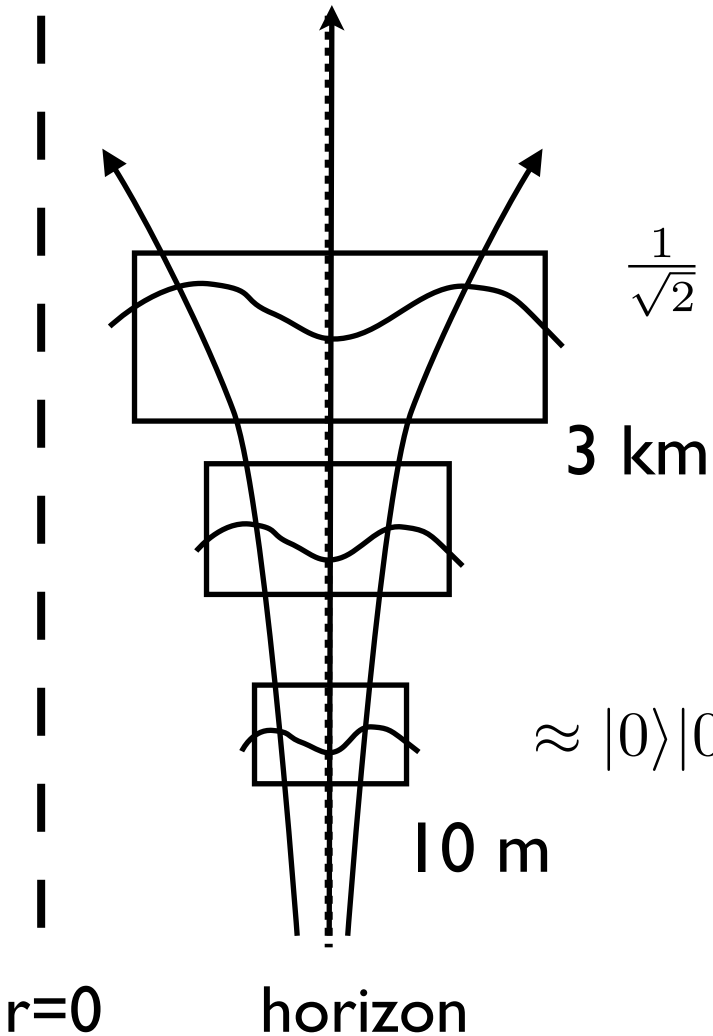


Actually one cannot hide information at planck scale this way ...

But more importantly, this is NOT what we want ...

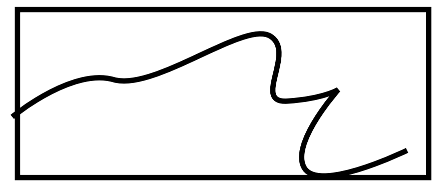
Small corrections will not help (Traditional horizon)

Need order unity correction (hair)



$$\frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |1\rangle|1\rangle\right) + \epsilon_k \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle - |1\rangle|1\rangle\right)$$

$$\approx |0\rangle|0\rangle$$

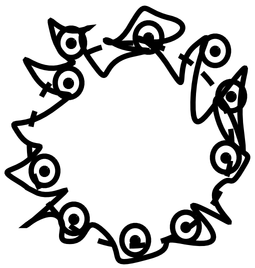


$$\frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |1\rangle|1\rangle\right)$$

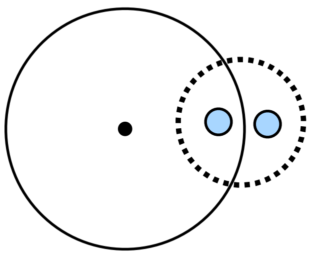
$$+\alpha_k \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle - |1\rangle|1\rangle\right)$$

$$\alpha_k = O(1)$$

Evolution at horizon in good slicing of of low energy modes cannot be close to the evolution in this room

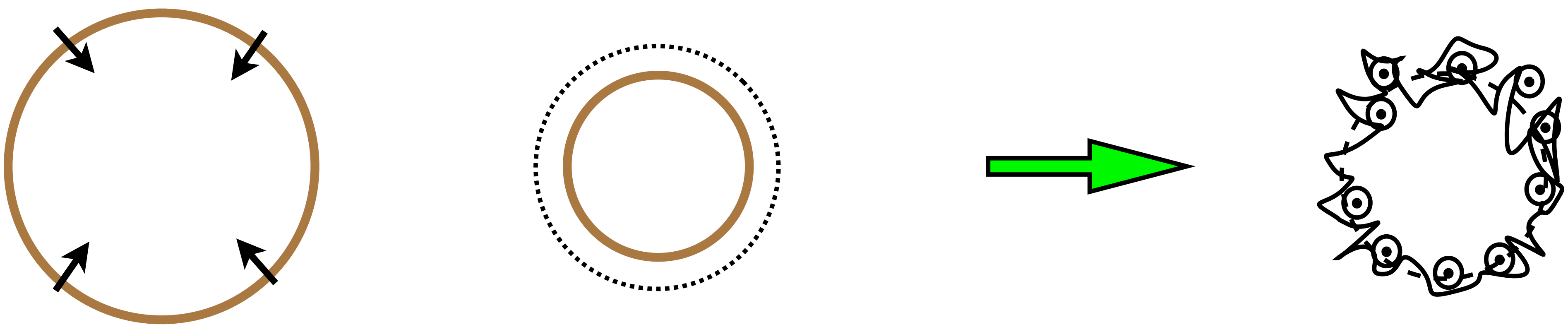


Fuzzballs give a realization of ‘hair’: Energy eigenstates of the black hole do not have a traditional horizon.

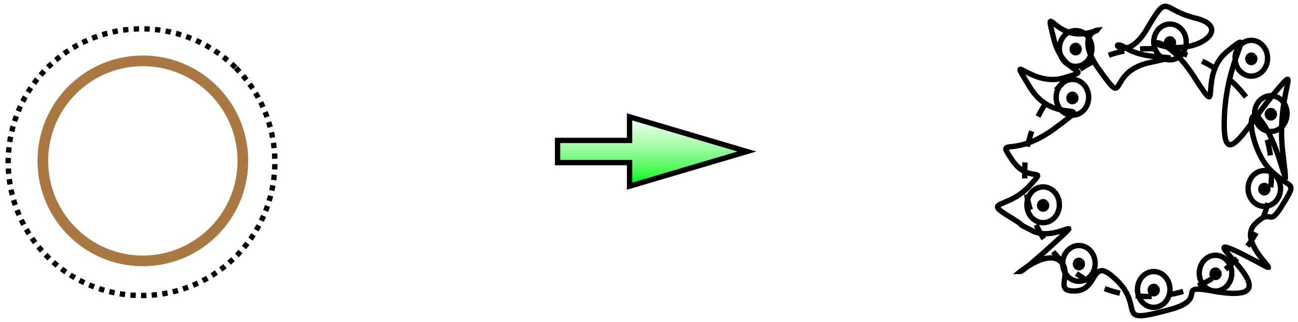


That is, there is no region around a ‘horizon’ where we can make a good slicing and get laboratory evolution for low energy modes

We can now conjecture what happens to a collapsing shell ...



There is a small amplitude for the shell to tunnel into a fuzzball state ...



$$S_{tunnel} \sim \frac{1}{G}\int R d^4x \sim \frac{1}{G}\frac{1}{(GM)^2}(GM)^4 \sim GM^2$$

$$\mathcal{A} \sim e^{-S_{tunnel}}$$

Text
Amplitude to tunnel is very small

$$\mathcal{N} \sim e^{S_{bek}} \sim e^{GM^2}$$

But the number of states that one can tunnel to is very large !

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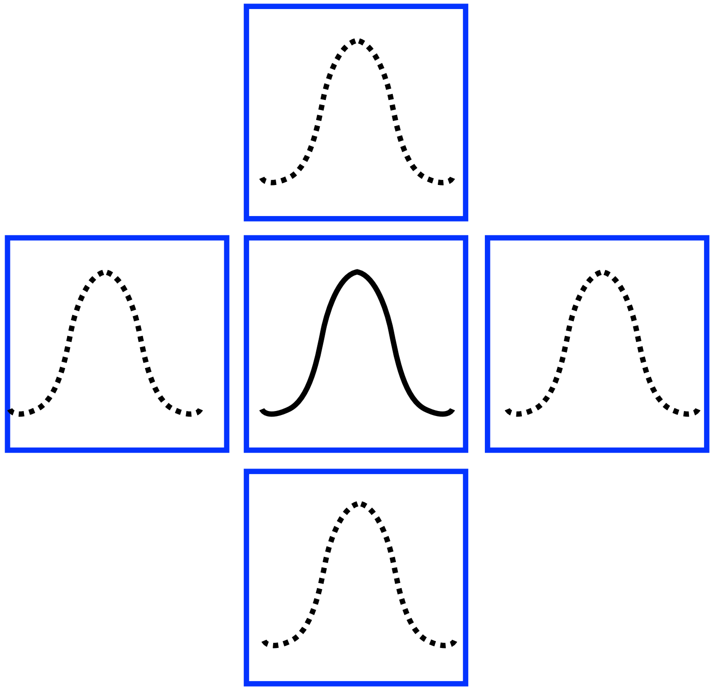
Smallness of amplitude can be cancelled by largeness of degeneracy ...

(SDM 07)

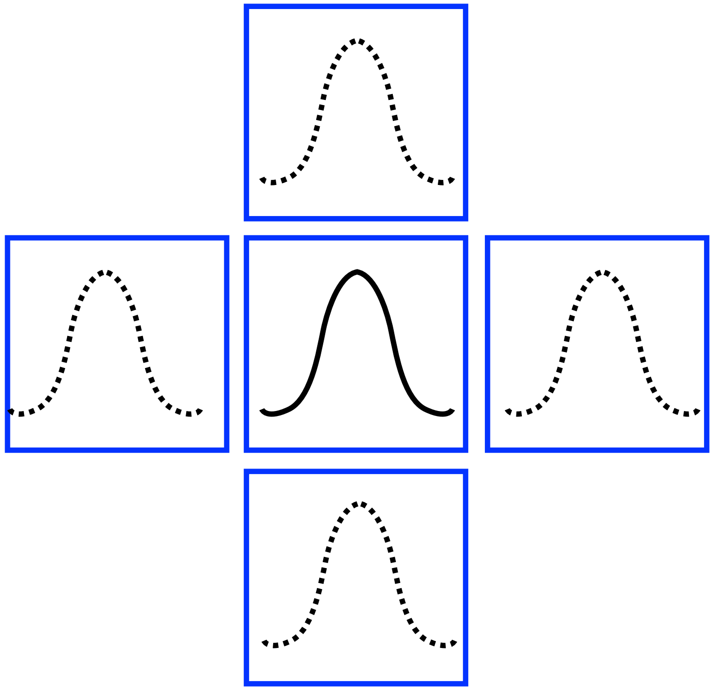
Toy model: Small amplitude to tunnel to a neighboring well, but there are a correspondingly large number of adjacent wells



In a time of order unity, the wavefunction in the central well becomes a linear combination of states in all wells

A crude estimate shows that this tunneling happens in a time much shorter than Hawking evaporation time (SDM 08)

Toy model: Small amplitude to tunnel to a neighboring well, but there are a correspondingly large number of adjacent wells



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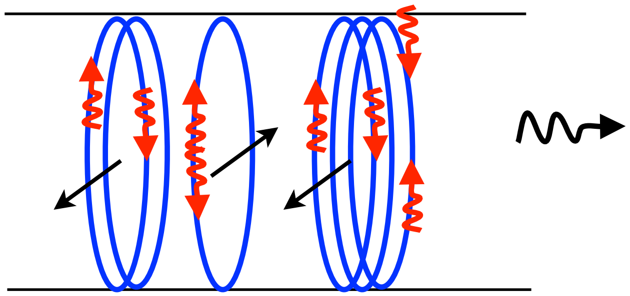
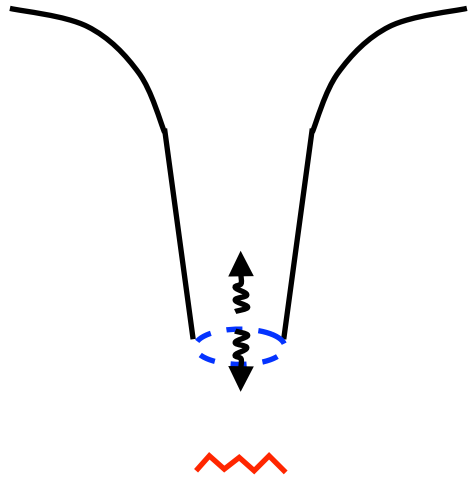
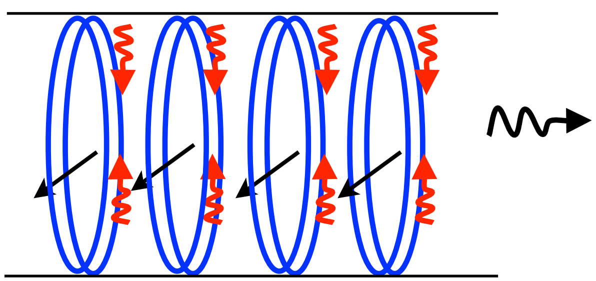
Hawking radiation from fuzzballs



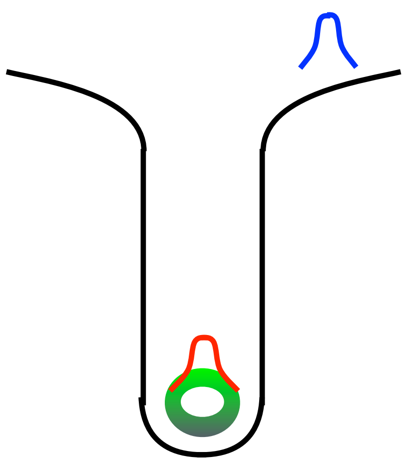
$$\Gamma = V \rho_L \rho_R$$



Unitary CFT emission rate agrees with nonunitary Hawking emission rate



$$\Gamma_{microstate} = V \bar{\rho}_L \bar{\rho}_R$$



Simple microstates

Make gravity dual

No horizon, but ergoregion emission gives exactly same radiation rate

$$\boxed{\Gamma_{microstate} = \Gamma_{gravity}}$$

(Chowdhury+SDM 07, 08)

4

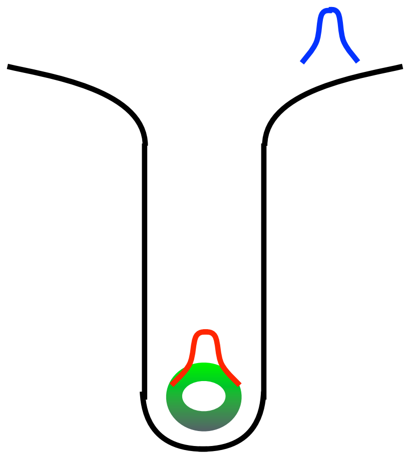
$$
\begin{aligned}
\mathrm{d}s^2 \;=\; & -\frac{f}{\sqrt{\tilde{H}_1\tilde{H}_5}}(\mathrm{d}t^2-\mathrm{d}y^2)+\frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(s_p\mathrm{d}y-c_p\mathrm{d}t)^2\\
&+\sqrt{\tilde{H}_1\tilde{H}_5}\left(\frac{r^2\mathrm{d}r^2}{(r^2+a_1^2)(r^2+a_2^2)-Mr^2}+\mathrm{d}\theta^2\right)\\
&+\left(\sqrt{\tilde{H}_1\tilde{H}_5}-(a_2^2-a_1^2)\frac{(\tilde{H}_1+\tilde{H}_5-f)\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right)\cos^2\theta\mathrm{d}\psi^2\\
&+\left(\sqrt{\tilde{H}_1\tilde{H}_5}+(a_2^2-a_1^2)\frac{(\tilde{H}_1+\tilde{H}_5-f)\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right)\sin^2\theta\mathrm{d}\phi^2\\
&+\frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(a_1\cos^2\theta\mathrm{d}\psi+a_2\sin^2\theta\mathrm{d}\phi)^2\\
&+\frac{2M\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_1c_1c_5c_p-a_2s_1s_5s_p)\mathrm{d}t+(a_2s_1s_5c_p-a_1c_1c_5s_p)\mathrm{d}y]\mathrm{d}\psi\\
&+\frac{2M\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_2c_1c_5c_p-a_1s_1s_5s_p)\mathrm{d}t+(a_1s_1s_5c_p-a_2c_1c_5s_p)\mathrm{d}y]\mathrm{d}\phi\\
&+\sqrt{\frac{\tilde{H}_1}{\tilde{H}_5}}\sum_{i=1}^{4}\mathrm{d}z_i^2
\end{aligned}
$$

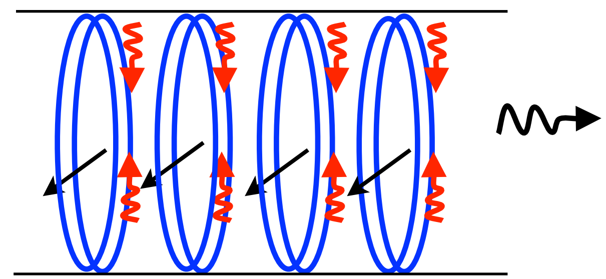
$$
Q_1 = \frac{g\alpha'^3}{V}n_1 \qquad Q_5 = g\alpha' n_5 \qquad Q_p = \frac{g^2\alpha'^4}{VR^2}n_p
$$

(Jejalla, Madden, Ross Titchener '05)

$$
\tilde{H}_i = f + M\sinh^2\delta_i, \quad f = r^2 + a_1^2\sin^2\theta + a_2^2\cos^2\theta
$$

$$
Q_1 = M\sinh\delta_1\cosh\delta_1, \quad Q_5 = M\sinh\delta_5\cosh\delta_5, \quad Q_p = M\sinh\delta_p\cosh\delta_p
$$





$$\Gamma_{microstate} = V \; \bar{\rho}_L \; \bar{\rho}_R$$

(Cardoso, Dias, Jordan, Hovdebo, Myers, '06)

(Chowdhury+SDM 07, 08, 08)

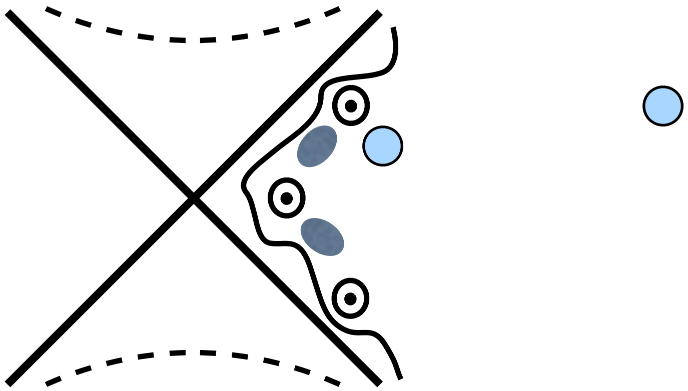
$$\boxed{\omega \simeq \omega_R = \frac{1}{R}\left(-l - m_\psi m + m_\phi n - |-\lambda - m_\psi n + m_\phi m| - 2(N+1)\right)}$$

$$\boxed{\omega_I = \frac{1}{R}\left(\frac{2\pi}{[l!]^2}\left[(\omega^2 - \frac{\lambda^2}{R^2})\frac{Q_1 Q_5}{4R^2}\right]^{l+1} {}^{l+1+N}C_{l+1} \; {}^{l+1+N+|\zeta|}C_{l+1}\right)}$$

$$\zeta \equiv -\lambda - m_\psi n + m_\phi m$$

$$N \geq 0$$

$$\boxed{\Gamma_{microstate} = \Gamma_{gravity}}$$



Real degrees of freedom at horizon
in the 'membrane paradigm'

(SDM 10)

Low energy modes (E ~ kT) involved in Hawking radiation
needed order unity correction ...

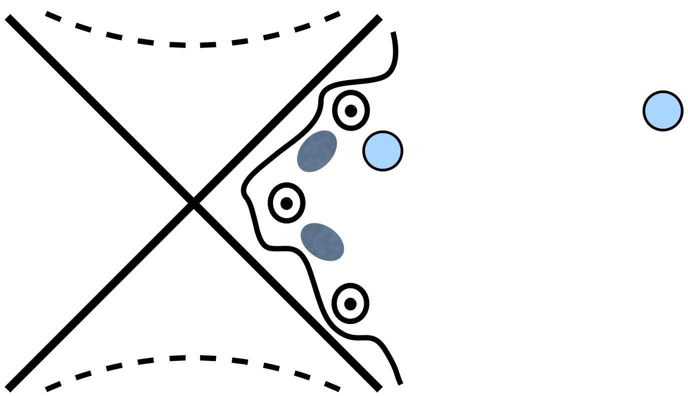
$$\frac{\delta S_{ent}}{S_{ent}} < 2\epsilon$$

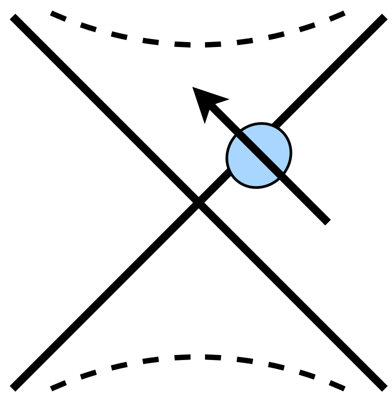
We indeed find a complete change in the evolution of these modes ...

Is the traditional back hole geometry of any use at all ?
If so, it can only be for high energy modes (E >> kT) ...

Information paradox: How can different microstates radiate low energy radiation (E~kT) *differently* ?

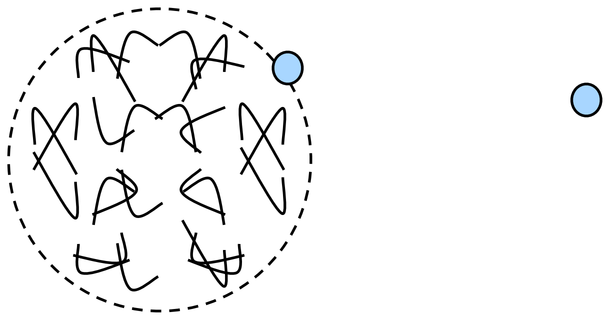
Infall problem: Is there some approximation in which high energy infalling objects (E>>kT) behave the *same* (perhaps seeing the traditional black hole geometry)

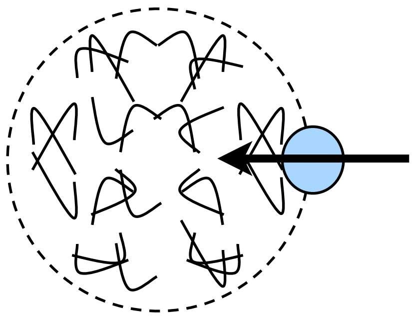




Thus the information paradox and the infall problem are, in a sense, opposite problems ...

A possibility





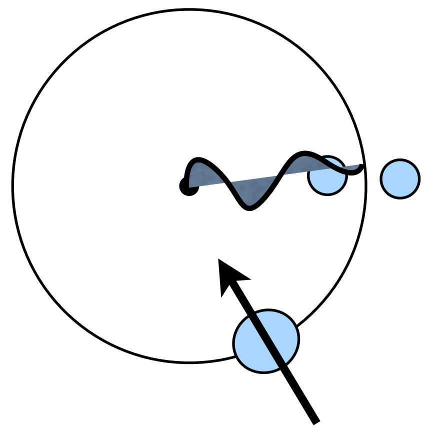
Low energy quanta suffer order unity correction

High energy quanta see only a light fuzz and sail through ...

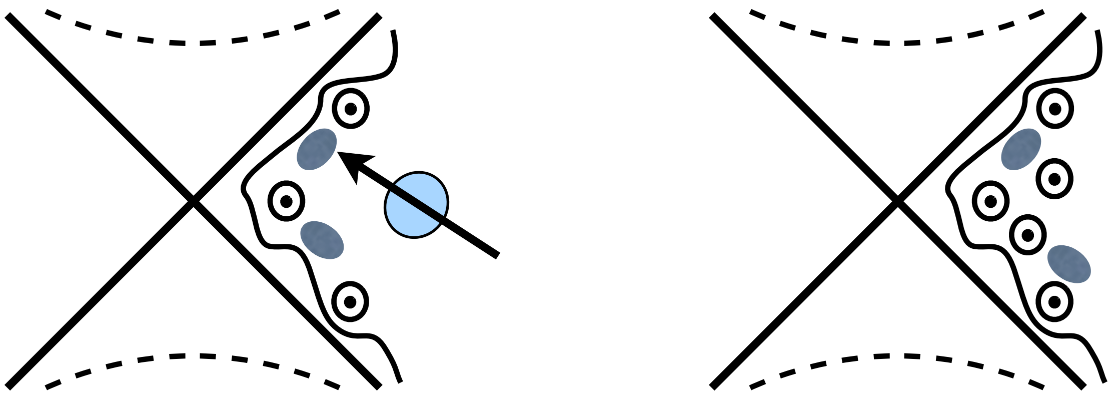
(see also Balasubramanian, de Boer, Jejjala, Simon 05)

Somewhat similar to non-locality ...

(some gentle nonlocal effect changes low energy quanta by order unity, but high energy quanta travel on the traditional metric of the hole)



But the situation appears to be more interesting ...



Impact of E>>kT quantum excites collective modes of fuzzball

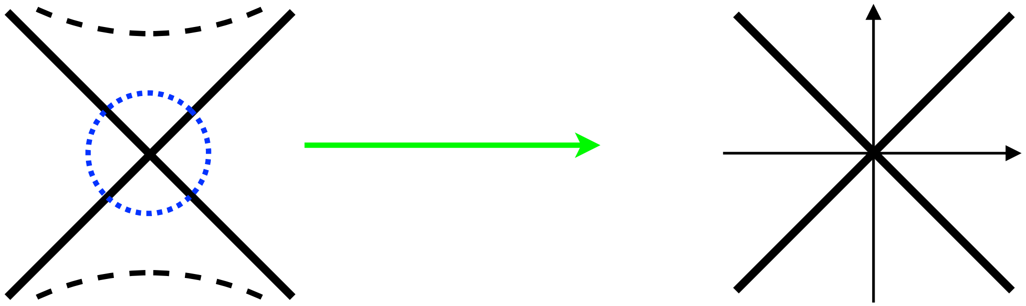
(SDM + Plumberg: arXiv: 1101.4899)

Spectrum of these modes is same as spectrum of quanta in traditional black hole geometry

Hilbert space of infalling quanta maps into Hilbert space of collective excitations

If one Hilbert space maps faithfully into another one, we cannot tell the difference ... so we cannot 'know' that the quantum has been converted into collective modes ...

We will use some ideas of Israel - Maldacena - Van Raamsdonk in making sense of this picture ...



Central part of eternal black hole diagram looks like a piece of Minkowski spacetime,
Horizons look like Rindler horizons

Scalar field ϕ: Can write Minkowski vacuum in terms of left and right Rindler states

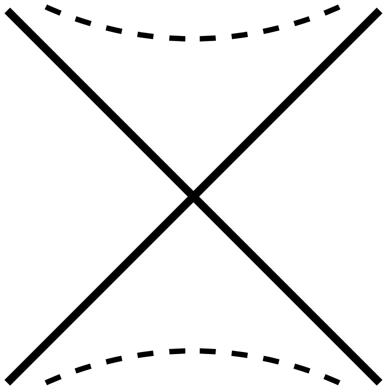
$$|0\rangle_M = \sum_i e^{-\frac{E_i}{4\pi}} |E_i\rangle_L |E_i\rangle_R$$

What is the corresponding decomposition for the gravitational field $h_{\mu\nu}$??

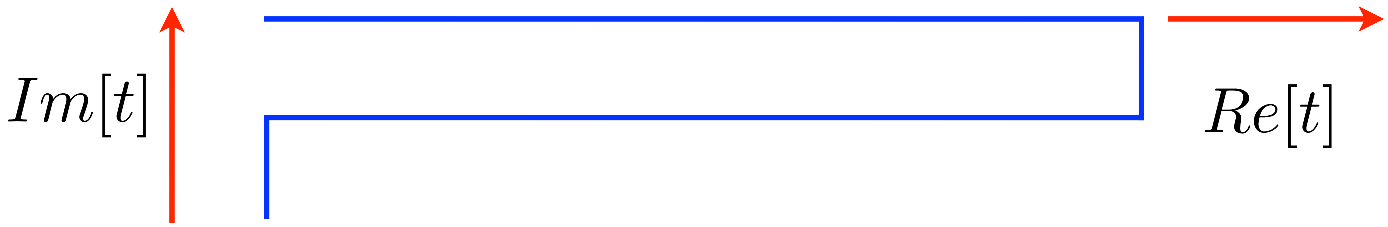
(see also Jacobson 2012)

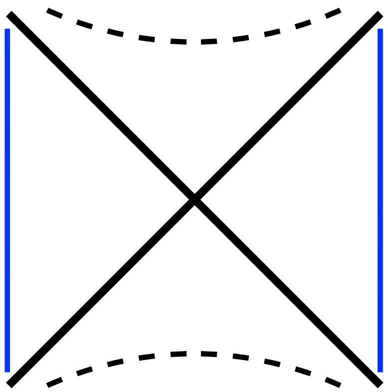
Let us recall some earlier results ...

Black Holes :

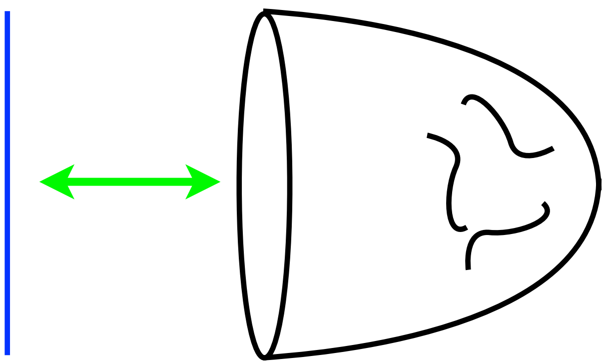


Israel (1976): The two sides of the eternal black hole are the two entangled copies of a thermal system in thermo-field-dynamics

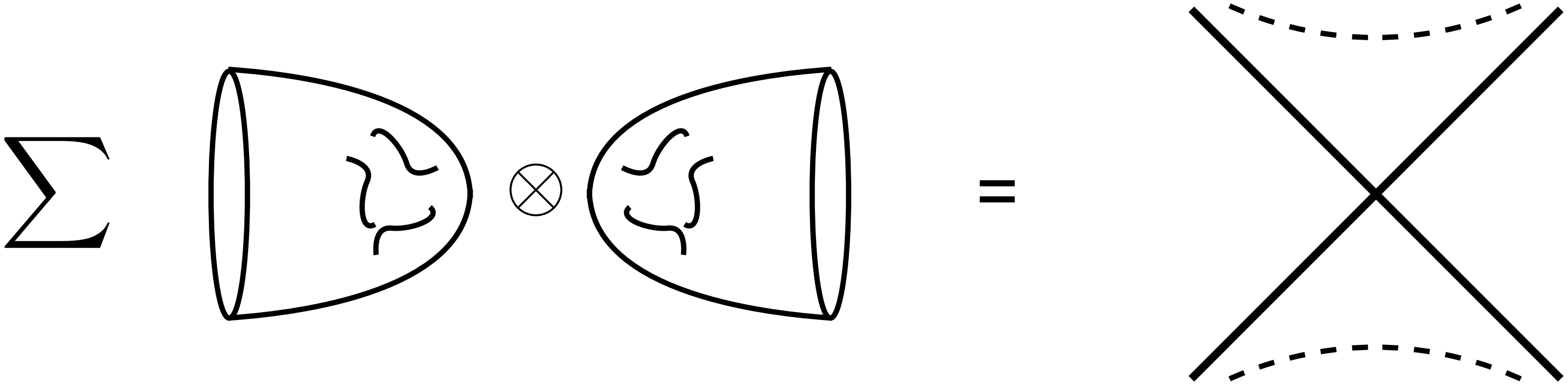




Maldacena (2001): This implies that the dual to the eternal black hole is two entangled copies of a CFT

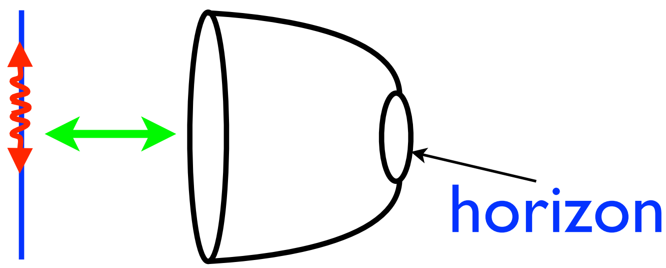


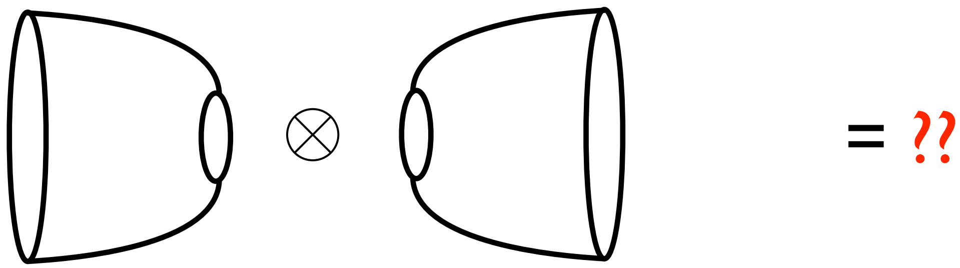
Van Raamsdonk (2009): CFT states are dual to gravity solutions ... so we should be able to write an entangled sum of CFT states as an entangled sum of gravity states ...



(Van Raamsdonk 2009)

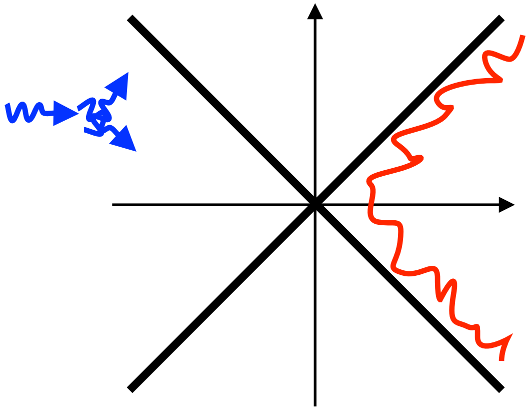
But what do we do with CFT states which are dual to black holes with a horizon ?





But the lesson from fuzzballs is that there are no microstates with horizons !! Thus there is only one 'class' of microstates, they just vary in their complexity

We can write down the properties we expect from the Rindler states ...



(a) The state of the right wedge should 'end' in the right wedge

(b) The states are eigenstates the full Hamiltonian, so interactions are included

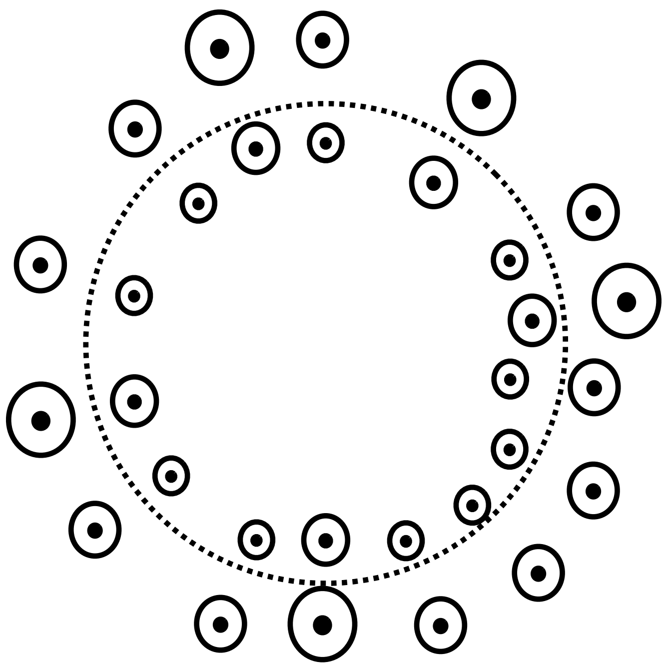
(c) Near the horizon, energies are high, so interactions are large ... very nonlinear

(d) The number of states should give S=A/4

These are just the properties of the fuzzball states ....Suggest a Van Raamsdonk type relation

Fuzzballs are the Rindler states for gravity ...

This picture gives a conjecture for what is being counted in de Sitter entropy ...



Write Vacuum as entangled sum (following Van Raamsdonk idea)

Count all compact manifolds with negative cosmological constant ending before reaching the horizon

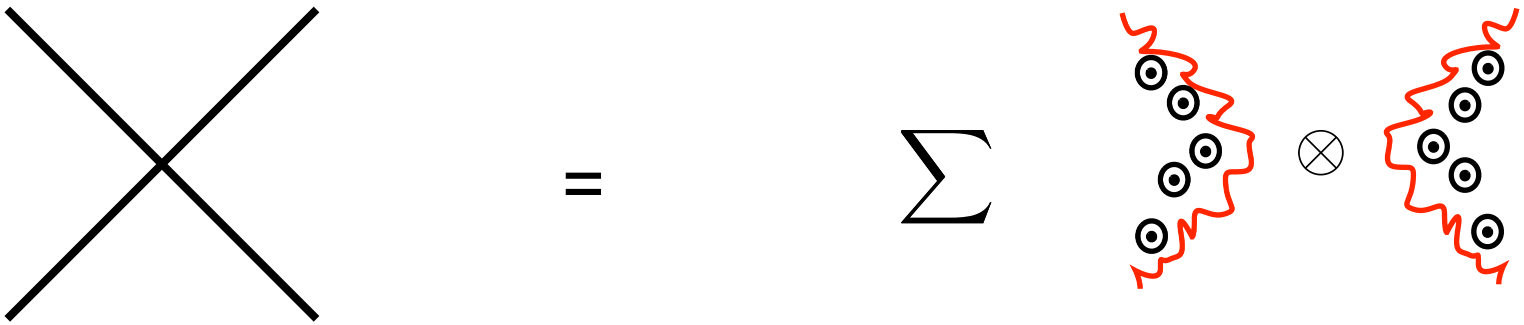
(SDM 12)

What happens when a quantum with E>> kT falls onto the fuzzball ??

We will put together two simple observations ...

(A) Minkowski vacuum

$$|0\rangle_M = C \sum_i e^{-\frac{E_i}{2}} |E_i\rangle_L |E_i\rangle_R, \qquad C = \left(\sum_i e^{-E_i} \right)^{-\frac{1}{2}}$$



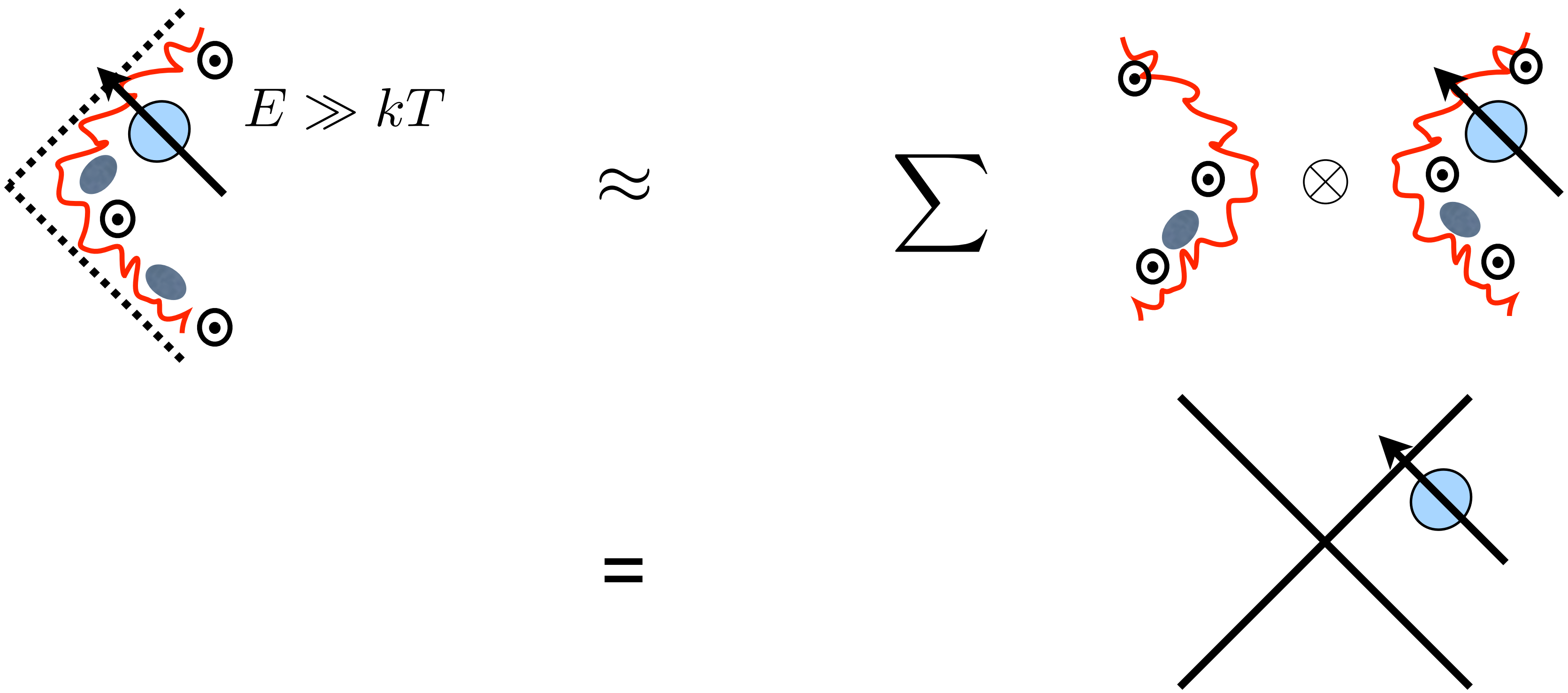
Expectation value of an operator in the right wedge is a thermal average over Rindler states ...

$$\begin{aligned} {}_M\langle 0|\hat{O}_R|0\rangle_M &= C^2 \sum_{i,j} e^{-\frac{E_i}{2}} e^{-\frac{E_j}{2}} {}_L\langle E_i|E_j\rangle_L \, {}_R\langle E_i|\hat{O}_R|E_j\rangle_R \\ &= C^2 \sum_i e^{-E_i} {}_R\langle E_i|\hat{O}_R|E_i\rangle_R \end{aligned}$$

(B) For appropriate operators, in generic states, we have

$$ {}_R\langle E_k|\hat{O}_R|E_k\rangle_R \approx \frac{1}{\sum_l e^{-E_l}} \sum_i e^{-E_i}\, {}_R\langle E_i|\hat{O}_R|E_i\rangle_R = {}_M\langle 0|\hat{O}_R|0\rangle_M $$

(Just the usual statement that measurements in one sample can be replaced by the ensemble)

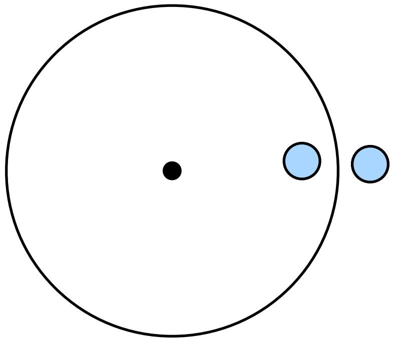


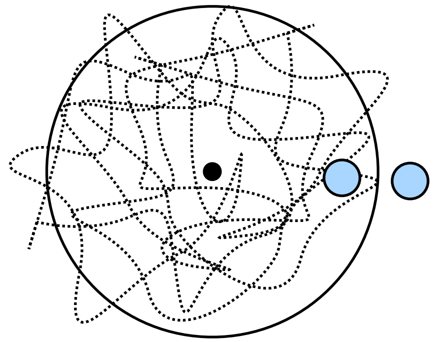
5

This appears to be a kind of 'complementarity' ...

Summary

(A) The information paradox is real

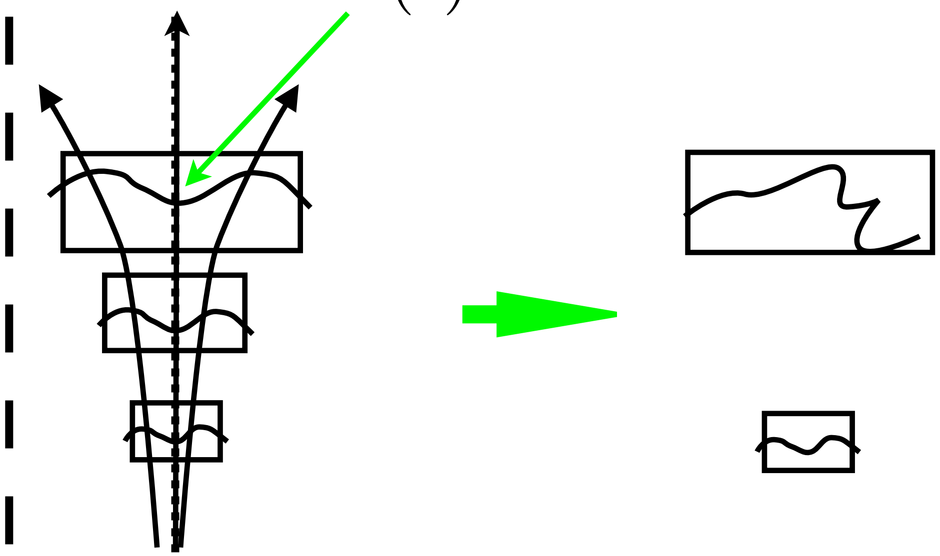




$$\frac{\delta S_{ent}}{S_{ent}} < 2\epsilon$$

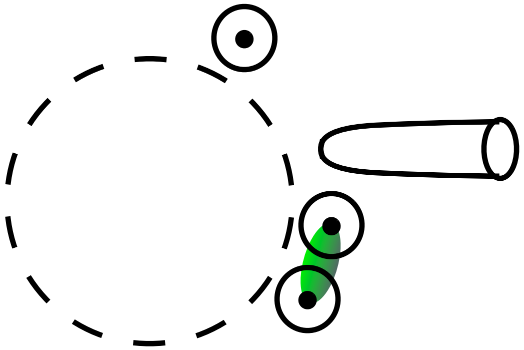
(SDM arXiv: 09091038)

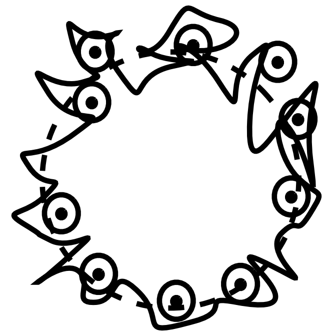
$O(\epsilon)$ corrections will not help



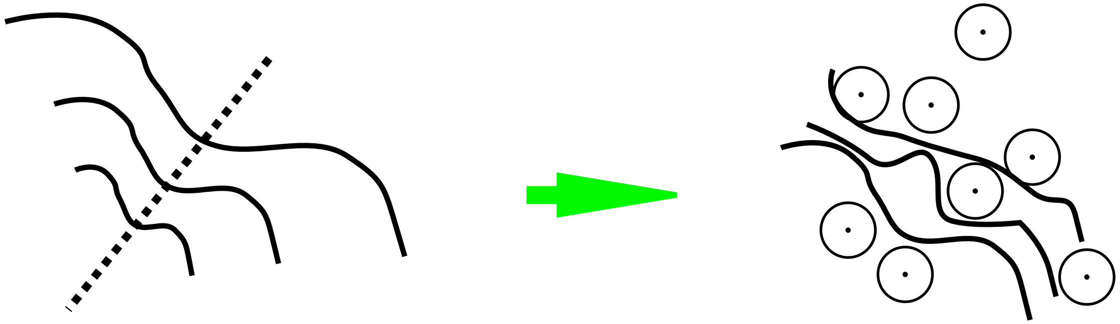
We cannot assume that there are 'subtle' quantum gravity effects hiding in the geometry, which leave the low energy evolution unaffected to leading order

(B) Microstates in string theory produce the needed structure (fuzzballs)

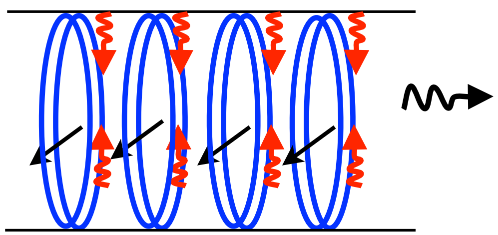




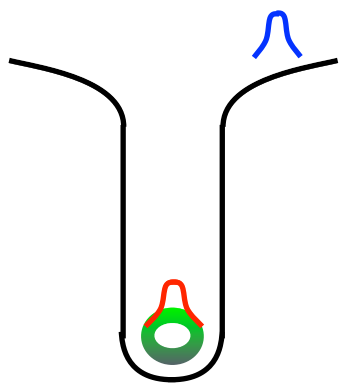
No horizon for
individual microstates



Evolution of low energy
modes at horizon
is completely altered

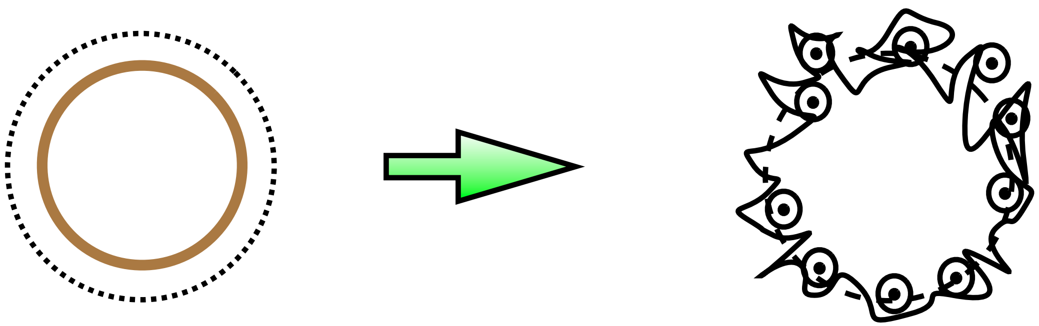


$$\Gamma_{microstate} = V \, \bar{\rho}_L \, \bar{\rho}_R$$



Radiation from simple
microstates matched
exactly to Hawking
radiation from CFT state

(C) Semiclassical evolution is violated by spreading of wavefunction over very large phase space

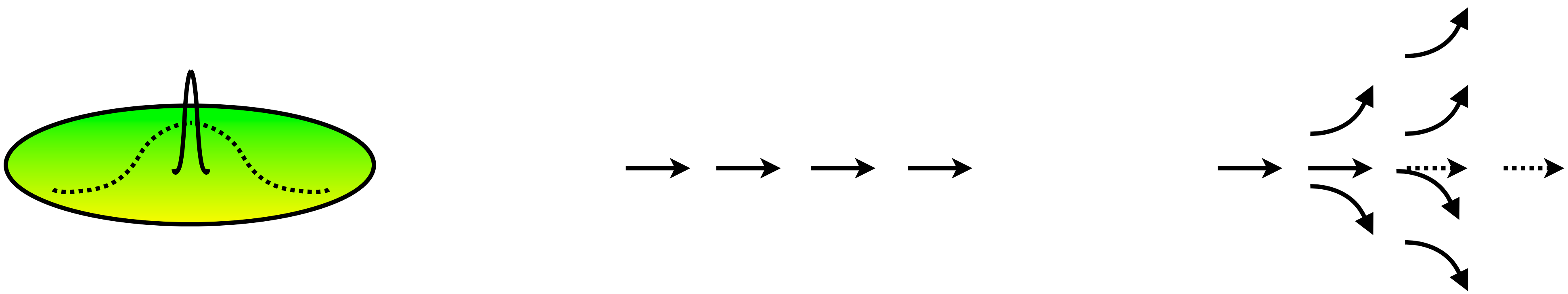


$$\mathcal{A} \sim e^{-S_{cl}}$$

can be offset by

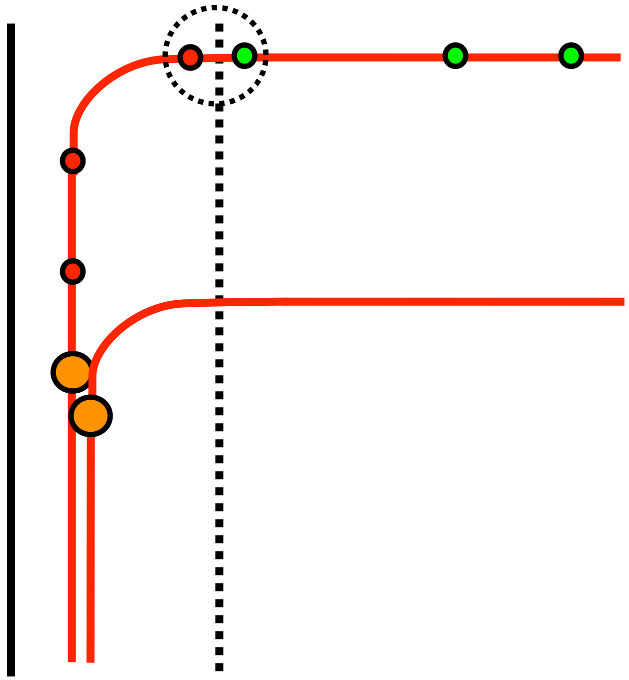
$$\mathcal{N} \sim e^{S_{bek}}$$

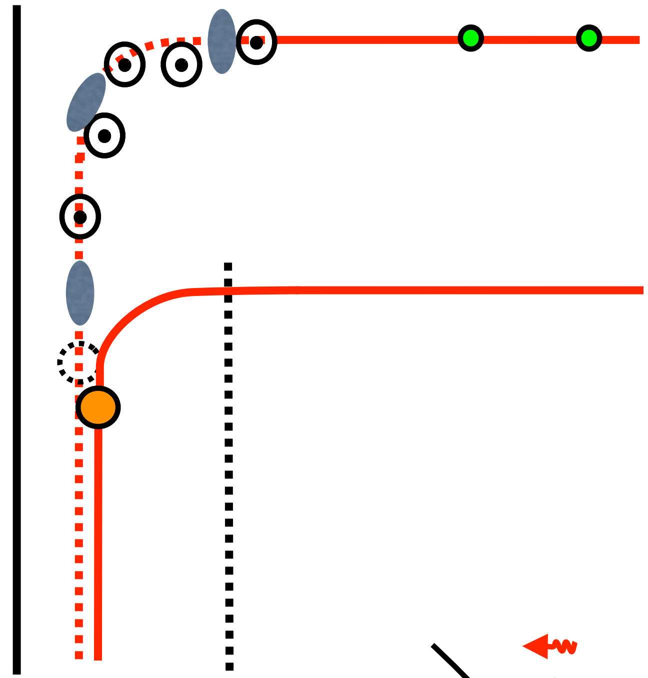
This spread of shell wavefunction over space of fuzzball states is what bypasses the Hawking problem ...



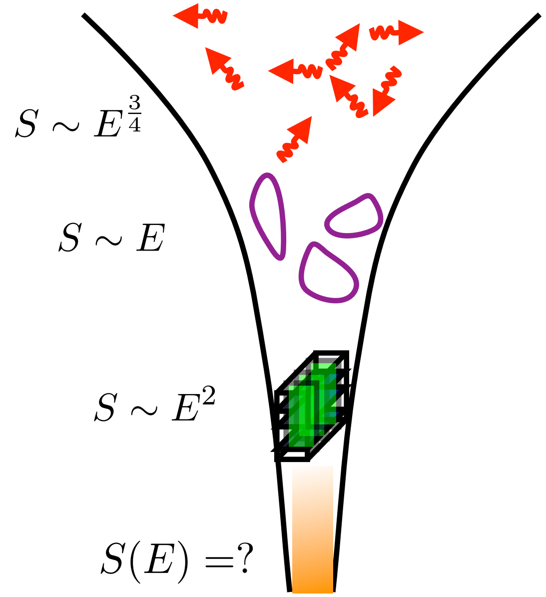
Wavefunction on initial slice spreads over vast space of fuzzball states ...

The good slicing argument breaks down ...

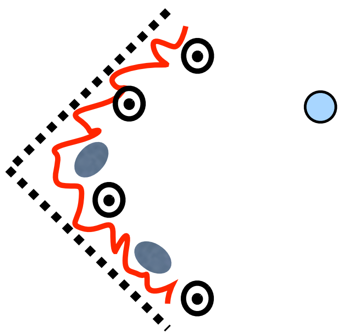




Spreading over phase space can have important consequences for Cosmology ...

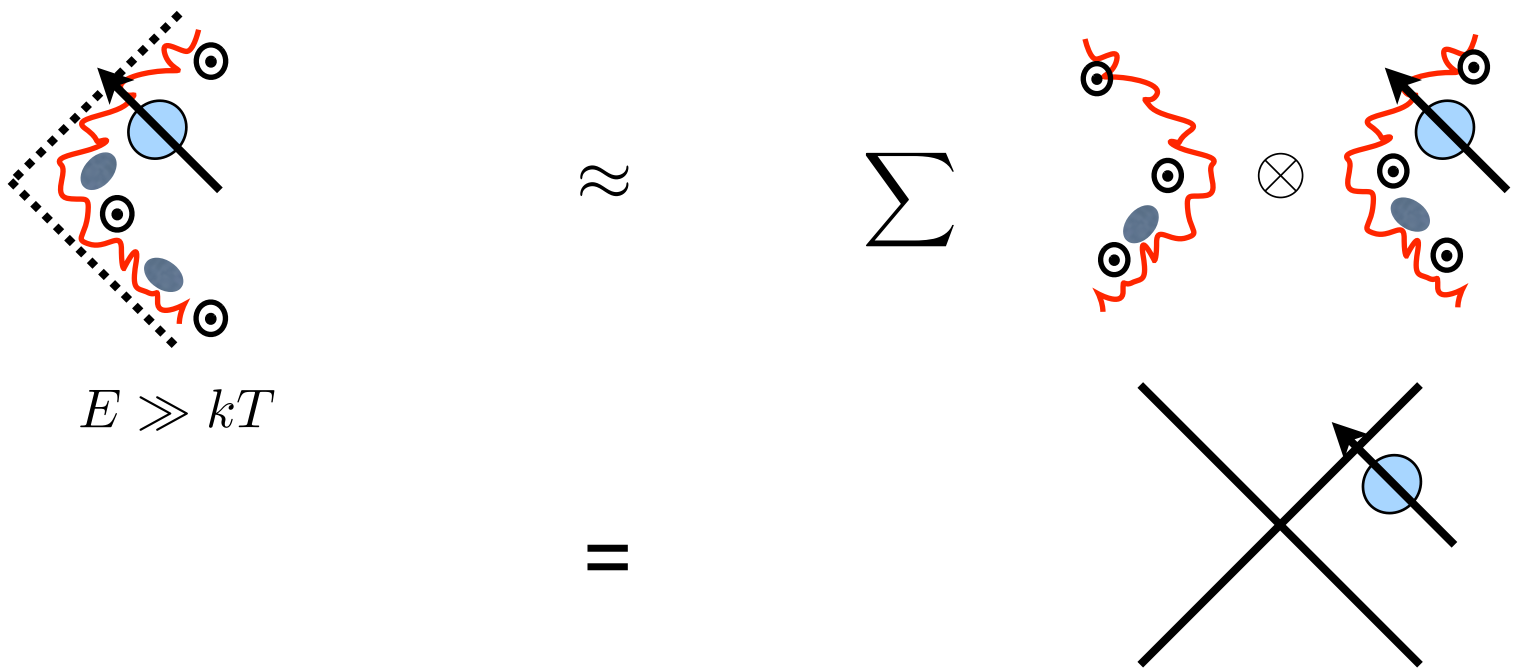


(D) The 'infall problem' is the opposite of the information paradox ...



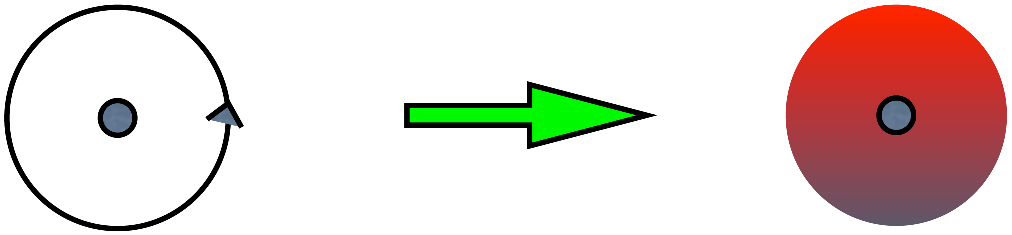
$E \sim kT$

Structure at horizon, no information problem

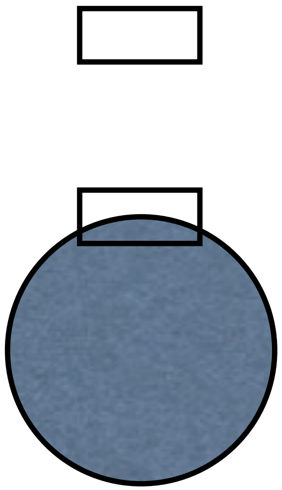


$E \gg kT$

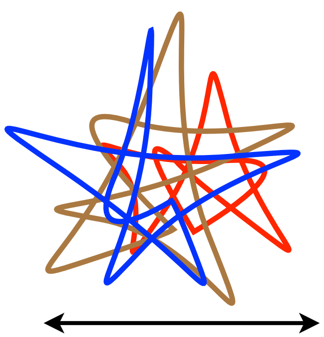
High energy impacts generate collective vibrations that can be approximated for short times by traditional geometry



needed a radical change
in the picture of the horizon



Equivalence principle fails
at the natural size of the
object



$N^\alpha \ l_p$

Bound states in string theory are ‘horizon sized’, large phase space leads to spreading of wavefunctions over a vast phase space:
Cannot localize on a ‘shell’ wavefunction