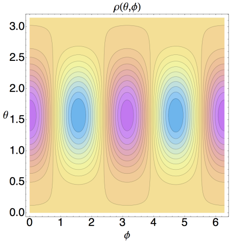
Gravitational turbulent instability of AdS

Jorge E. Santos



Santa Barbara - University of California

In collaboration with
O. J. C. Dias (Saclay) and G. T. Horowitz (Santa Barbara)

Anti-de Sitter spacetime - 1/2

Anti-de Sitter space is a maximally symmetric solution to

$$S = \frac{1}{16\pi G}\int \mathrm{d}^d x\sqrt{-g}\left[R + \frac{(d-1)(d-2)}{L^2}\right],$$

which in global coordinates can be expressed as

$$\mathrm{d}s^2 \equiv \bar{g}_{ab}\mathrm{d}x^a\mathrm{d}x^b = -\left(\frac{r^2}{L^2}+1\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\frac{r^2}{L^2}+1} + r^2\mathrm{d}\Omega^2_{d-2}.$$

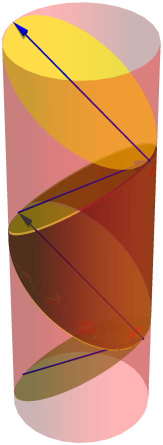
The Poincaré coordinates

$$\mathrm{d}s^2 = R^2(-\mathrm{d}\tau^2 + \mathrm{d}\mathbf{x}\cdot\mathrm{d}\mathbf{x}) + \frac{L^2\mathrm{d}R^2}{R^2}$$

do not cover the entire spacetime.

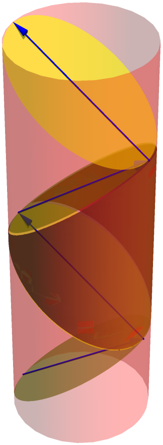
Anti-de Sitter spacetime - 2/2

Conformally, AdS looks like the interior of a cylinder



Anti-de Sitter spacetime - 2/2

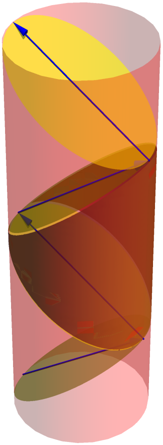
Conformally, AdS looks like the interior of a cylinder



- Poincaré coordinates cover the brown-shaded region.

Anti-de Sitter spacetime - 2/2

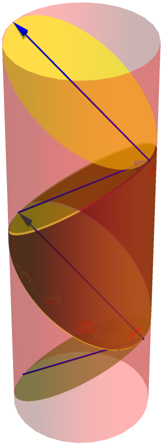
Conformally, AdS looks like the interior of a cylinder



- Poincaré coordinates cover the brown-shaded region.
- The instability described in this talk will occur in global AdS only.

Anti-de Sitter spacetime - 2/2

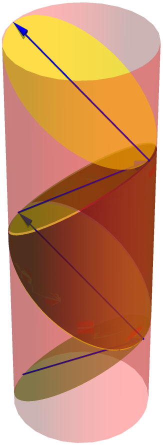
Conformally, AdS looks like the interior of a cylinder



- Poincaré coordinates cover the brown-shaded region.
- The instability described in this talk will occur in global AdS only.
- The dual field theory lives on $\mathbb{R}_t \times S^{d-2}$.

Anti-de Sitter spacetime - 2/2

Conformally, AdS looks like the interior of a cylinder



- Poincaré coordinates cover the brown-shaded region.
- The instability described in this talk will occur in global AdS only.
- The dual field theory lives on $\mathbb{R}_t \times S^{d-2}$.
- With energy preserving boundary conditions, waves bounce off infinity and return in finite time.

The stability problem for spacetimes in general relativity

The setup:

Consider a spacetime $(\mathcal{M}, g)$, together with prescribed boundary conditions $\mathcal{B}$ if timelike boundary exists.

The stability problem for spacetimes in general relativity

The setup:

Consider a spacetime $(\mathcal{M}, g)$, together with prescribed boundary conditions $\mathcal{B}$ if timelike boundary exists.

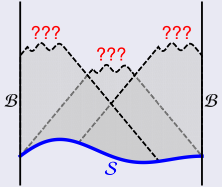
- Take small perturbations (in a suitable sense) on a Cauchy surface $\mathcal{S}$.

The stability problem for spacetimes in general relativity

The setup:

Consider a spacetime $(\mathcal{M}, g)$, together with prescribed boundary conditions $\mathcal{B}$ if timelike boundary exists.

- Take small perturbations (in a suitable sense) on a Cauchy surface $\mathcal{S}$.
- Does the solution spacetime $(\mathcal{M}, g')$ that arises still has the same asymptotic causal structure as $(\mathcal{M}, g)$?

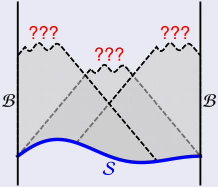


The stability problem for spacetimes in general relativity

The setup:

Consider a spacetime $(\mathcal{M}, g)$, together with prescribed boundary conditions $\mathcal{B}$ if timelike boundary exists.

- Take small perturbations (in a suitable sense) on a Cauchy surface $\mathcal{S}$.
- Does the solution spacetime $(\mathcal{M}, g')$ that arises still has the same asymptotic causal structure as $(\mathcal{M}, g)$?
- If so, can we bound the "difference" between the asymptotic form of g and g' in terms of initial data defined on $\mathcal{S}$?

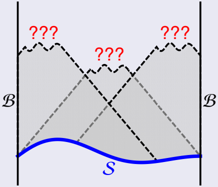


The stability problem for spacetimes in general relativity

The setup:

Consider a spacetime $(\mathcal{M}, g)$, together with prescribed boundary conditions $\mathcal{B}$ if timelike boundary exists.

- Take small perturbations (in a suitable sense) on a Cauchy surface $\mathcal{S}$.
- Does the solution spacetime $(\mathcal{M}, g')$ that arises still has the same asymptotic causal structure as $(\mathcal{M}, g)$?
- If so, can we bound the "difference" between the asymptotic form of g and g' in terms of initial data defined on $\mathcal{S}$?



In particular, if a geodesically complete spacetime is perturbed, does it remain "complete"?

Minkowski, dS and AdS spacetimes

- At the linear level, Anti de-Sitter space-time appears just as stable as the Minkowski or de-Sitter spacetimes.

Minkowski, dS and AdS spacetimes

- At the linear level, Anti de-Sitter space-time appears just as stable as the Minkowski or de-Sitter spacetimes.
- For the Minkowski and de-Sitter spacetimes, it has been shown that small, but finite, perturbations remain small - D. Christodoulou and S. Klainerman '93 and Friedrich '86.

Minkowski, dS and AdS spacetimes

- At the linear level, Anti de-Sitter space-time appears just as stable as the Minkowski or de-Sitter spacetimes.
- For the Minkowski and de-Sitter spacetimes, it has been shown that small, but finite, perturbations remain small - D. Christodoulou and S. Klainerman '93 and Friedrich '86.
- Why has this not been shown for Anti de-Sitter?

Minkowski, dS and AdS spacetimes

- At the linear level, Anti de-Sitter space-time appears just as stable as the Minkowski or de-Sitter spacetimes.
- For the Minkowski and de-Sitter spacetimes, it has been shown that small, but finite, perturbations remain small - D. Christodoulou and S. Klainerman '93 and Friedrich '86.
- Why has this not been shown for Anti de-Sitter?
  - It is just *not* true!

Minkowski, dS and AdS spacetimes

- At the linear level, Anti de-Sitter space-time appears just as stable as the Minkowski or de-Sitter spacetimes.
- For the Minkowski and de-Sitter spacetimes, it has been shown that small, but finite, perturbations remain small - D. Christodoulou and S. Klainerman '93 and Friedrich '86.
- Why has this not been shown for Anti de-Sitter?
  - It is just *not* true!

Claim:

Generic small (but finite) perturbations of AdS become large and eventually form black holes.

Minkowski, dS and AdS spacetimes

- At the linear level, Anti de-Sitter space-time appears just as stable as the Minkowski or de-Sitter spacetimes.
- For the Minkowski and de-Sitter spacetimes, it has been shown that small, but finite, perturbations remain small - D. Christodoulou and S. Klainerman '93 and Friedrich '86.
- Why has this not been shown for Anti de-Sitter?
  - It is just *not* true!

Claim:

Generic small (but finite) perturbations of AdS become large and eventually form black holes.

- The energy cascades from low to high frequency modes in a manner reminiscent of the onset of turbulence.

Folklore

- Doesn't this claim contradict the fact that Anti de-Sitter is supersymmetric?

Folklore

- Doesn't this claim contradict the fact that Anti de-Sitter is supersymmetric?
- Doesn't this contradict the fact that there is a positivity of energy theorem for Anti de-Sitter?

Folklore

- Doesn't this claim contradict the fact that Anti de-Sitter is supersymmetric?
- Doesn't this contradict the fact that there is a positivity of energy theorem for Anti de-Sitter?

NO :

Folklore

- Doesn't this claim contradict the fact that Anti de-Sitter is supersymmetric?
- Doesn't this contradict the fact that there is a positivity of energy theorem for Anti de-Sitter?

NO :

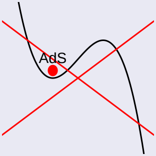
- Positivity energy theorem: if matter satisfies the dominant energy condition, then $E \geq 0$ for all nonsingular, asymptotically AdS initial data, being zero for AdS only.

Folklore

- Doesn't this claim contradict the fact that Anti de-Sitter is supersymmetric?
- Doesn't this contradict the fact that there is a positivity of energy theorem for Anti de-Sitter?

NO :

- Positivity energy theorem: if matter satisfies the dominant energy condition, then $E \geq 0$ for all nonsingular, asymptotically AdS initial data, being zero for AdS only.
  - This ensures that AdS cannot decay.

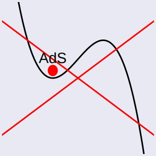


Folklore

- Doesn't this claim contradict the fact that Anti de-Sitter is supersymmetric?
- Doesn't this contradict the fact that there is a positivity of energy theorem for Anti de-Sitter?

NO :

- Positivity energy theorem: if matter satisfies the dominant energy condition, then $E \geq 0$ for all nonsingular, asymptotically AdS initial data, being zero for AdS only.
  - This ensures that AdS cannot decay.
  - It does not ensure that a small amount of energy added to AdS will not generically form a small black hole.

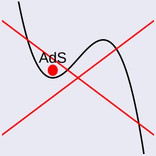


Folklore

- Doesn't this claim contradict the fact that Anti de-Sitter is supersymmetric?
- Doesn't this contradict the fact that there is a positivity of energy theorem for Anti de-Sitter?

NO :

- Positivity energy theorem: if matter satisfies the dominant energy condition, then $E \geq 0$ for all nonsingular, asymptotically AdS initial data, being zero for AdS only.
  - This ensures that AdS cannot decay.
  - It does not ensure that a small amount of energy added to AdS will not generically form a small black hole.
  - That is usually ruled out by arguing that waves disperse. This does not happen in AdS.



Why is AdS unstable?

- AdS acts like a confining finite box. Any generic finite excitation which is added to this box might be expected to explore all configurations consistent with the conserved charges of AdS - including small black holes.

Why is AdS unstable?

- AdS acts like a confining finite box. Any generic finite excitation which is added to this box might be expected to explore all configurations consistent with the conserved charges of AdS - including small black holes.
- Special (fine tuned) solutions need not lead to the formation of black holes.

Why is AdS unstable?

- AdS acts like a confining finite box. Any generic finite excitation which is added to this box might be expected to explore all configurations consistent with the conserved charges of AdS - including small black holes.
- Special (fine tuned) solutions need not lead to the formation of black holes.
  - We will see that for some linearized gravitational *mode* there will be a corresponding nonlinear solution - geon.

Why is AdS unstable?

- AdS acts like a confining finite box. Any generic finite excitation which is added to this box might be expected to explore all configurations consistent with the conserved charges of AdS - including small black holes.
- Special (fine tuned) solutions need not lead to the formation of black holes.
  - We will see that for some linearized gravitational *mode* there will be a corresponding nonlinear solution - geon.
  - These solutions are special since they are exactly periodic in time and invariant under a single continuous symmetry.

Why is AdS unstable?

- AdS acts like a confining finite box. Any generic finite excitation which is added to this box might be expected to explore all configurations consistent with the conserved charges of AdS - including small black holes.
- Special (fine tuned) solutions need not lead to the formation of black holes.
  - We will see that for some linearized gravitational *mode* there will be a corresponding nonlinear solution - geon.
  - These solutions are special since they are exactly periodic in time and invariant under a single continuous symmetry.
  - Geons are analogous to gravitational plane waves.

Why is AdS unstable?

- AdS acts like a confining finite box. Any generic finite excitation which is added to this box might be expected to explore all configurations consistent with the conserved charges of AdS - including small black holes.
- Special (fine tuned) solutions need not lead to the formation of black holes.
  - We will see that for some linearized gravitational *mode* there will be a corresponding nonlinear solution - geon.
  - These solutions are special since they are exactly periodic in time and invariant under a single continuous symmetry.
  - Geons are analogous to gravitational plane waves.
- A perhaps more convincing intuitive picture: colliding exact plane waves produces singularities - Penrose - '71.

Perturbative construction - 1/2

- Expand the metric as

$$g = \bar{g} + \sum_i \epsilon^i h^{(i)}.$$

Perturbative construction - 1/2

- Expand the metric as

$$g = \bar{g} + \sum_i \epsilon^i h^{(i)}.$$

- At each order in perturbation theory, the Einstein equations yield:

$$\tilde{\Delta}_L h^{(i)}_{ab} = T^{(i)}_{ab},$$

where $T^{(i)}$ depends on $\{h^{(j \leq i-1)}\}$ and their derivatives and

$$2\tilde{\Delta}_L h^{(i)}_{ab} \equiv -\bar{\nabla}^2 h^{(i)}_{ab} - 2\bar{R}_{a\ b}^{\ c\ d} h^{(i)}_{cd} - \bar{\nabla}_a \bar{\nabla}_b h^{(i)} + 2\bar{\nabla}_{(a}\bar{\nabla}^c h^{(i)}_{b)c}.$$

Perturbative construction - 1/2

- Expand the metric as

$$g = \bar{g} + \sum_i \epsilon^i h^{(i)}.$$

- At each order in perturbation theory, the Einstein equations yield:

$$\tilde{\Delta}_L h_{ab}^{(i)} = T_{ab}^{(i)},$$

where $T^{(i)}$ depends on $\{h^{(j \leq i-1)}\}$ and their derivatives and

$$2\tilde{\Delta}_L h_{ab}^{(i)} \equiv -\bar{\nabla}^2 h_{ab}^{(i)} - 2\bar{R}_a{}^c{}_b{}^d h_{cd}^{(i)} - \bar{\nabla}_a \bar{\nabla}_b h^{(i)} + 2\bar{\nabla}_{(a}\bar{\nabla}^c h_{b)c}^{(i)}.$$

- Any smooth symmetric two-tensor can be expressed as a sum of fundamental building blocks, $\mathcal{T}_{ab}^{\ell m}$, that have definite transformation properties under the $SO(d-1)$ subgroup of AdS.

Perturbative construction - 2/2

- For concreteness, set $d = 4$. Perturbations come in three classes:

Perturbative construction - 2/2

- For concreteness, set $d = 4$. Perturbations come in three classes:
  - Scalar-type perturbations: perturbations are constructed from spherical harmonics on S^2 - $Y_{\ell m}$.
  - Vector-type perturbations: perturbations are constructed from vector harmonics on S^2 - these are $\star_{S^2}\nabla Y_{\ell m}$.
  - Tensor-type perturbations: only exist in $d \geq 5$.

Perturbative construction - 2/2

- For concreteness, set $d = 4$. Perturbations come in three classes:
  - Scalar-type perturbations: perturbations are constructed from spherical harmonics on S^2 - $Y_{\ell m}$.
  - Vector-type perturbations: perturbations are constructed from vector harmonics on S^2 - these are $\star_{S^2} \nabla Y_{\ell m}$.
  - Tensor-type perturbations: only exist in $d \geq 5$.
- We go beyond linear order: need real representation for $Y_{\ell m}$ - $Y^c_{\ell m} = \cos\phi \, \mathrm{L}^m_\ell(\theta)$ and $Y^s_{\ell m} = \sin\phi \, \mathrm{L}^m_\ell(\theta)$.

Perturbative construction - 2/2

- For concreteness, set $d = 4$. Perturbations come in three classes:
  - Scalar-type perturbations: perturbations are constructed from spherical harmonics on S^2 - $Y_{\ell m}$.
  - Vector-type perturbations: perturbations are constructed from vector harmonics on S^2 - these are $\star_{S^2} \nabla Y_{\ell m}$.
  - Tensor-type perturbations: only exist in $d \geq 5$.
- We go beyond linear order: need real representation for $Y_{\ell m}$ - $Y^c_{\ell m} = \cos\phi \, \mathrm{L}^m_\ell(\theta)$ and $Y^s_{\ell m} = \sin\phi \, \mathrm{L}^m_\ell(\theta)$.
- At each order, we can reduce the metric perturbations to 4 gauge invariant functions satisfying (Kodama and Ishibashi '03 for $i = 1$):

$$\Box_2 \Phi^{\alpha,(i)}_{\ell m}(t,r) + V^{(i)}_\ell(r) \Phi^{\alpha,(i)}_{\ell m}(t,r) = \tilde{T}^{\alpha,(i)}_{\ell m}(t,r),$$

where $\alpha \in \{c, s\}$ and $\Box_2$ is the w. op. in the (t,r) orbit space.

Perturbative construction - 2/2

- For concreteness, set $d = 4$. Perturbations come in three classes:
  - Scalar-type perturbations: perturbations are constructed from spherical harmonics on S^2 - $Y_{\ell m}$.
  - Vector-type perturbations: perturbations are constructed from vector harmonics on S^2 - these are $\star_{S^2} \nabla Y_{\ell m}$.
  - Tensor-type perturbations: only exist in $d \geq 5$.
- We go beyond linear order: need real representation for $Y_{\ell m}$ - $Y^c_{\ell m} = \cos\phi\, \mathrm{L}^m_\ell(\theta)$ and $Y^s_{\ell m} = \sin\phi\, \mathrm{L}^m_\ell(\theta)$.
- At each order, we can reduce the metric perturbations to 4 gauge invariant functions satisfying (Kodama and Ishibashi '03 for $i = 1$):

$$\Box_2 \Phi^{\alpha,(i)}_{\ell m}(t,r) + V^{(i)}_\ell(r) \Phi^{\alpha,(i)}_{\ell m}(t,r) = \tilde{T}^{\alpha,(i)}_{\ell m}(t,r),$$

where $\alpha \in \{c, s\}$ and $\Box_2$ is the w. op. in the (t, r) orbit space.
- Choice of initial data relates $\Phi^{c,(i)}_{\ell m}$ and $\Phi^{s,(i)}_{\ell m}$: 2 PDEs to solve.

Linear Perturbations

- At the linear level $(i = 1)$ we can further decompose our perturbations as

$$\Phi_{\ell m}^{\alpha,(i)}(t,r) = \Phi_{\ell m}^{\alpha,(i),c}(r)\cos(\omega_\ell t) + \Phi_{\ell m}^{\alpha,(i),s}(r)\sin(\omega_\ell t).$$

Linear Perturbations

- At the linear level $(i = 1)$ we can further decompose our perturbations as

$$\Phi_{\ell m}^{\alpha,(i)}(t,r) = \Phi_{\ell m}^{\alpha,(i),c}(r)\cos(\omega_\ell t) + \Phi_{\ell m}^{\alpha,(i),s}(r)\sin(\omega_\ell t).$$

- Because AdS acts like a confining box, only certain frequencies are allowed to propagate

$$\omega_\ell^2 L^2 = (1 + \ell + 2p)^2,$$

where p is the radial overtone. These are the so-called normal modes of AdS. The fact that $\omega^2 L^2 > 0$ means that AdS is linearly stable.

Linear Perturbations

- At the linear level $(i = 1)$ we can further decompose our perturbations as

$$\Phi_{\ell m}^{\alpha,(i)}(t,r) = \Phi_{\ell m}^{\alpha,(i),c}(r)\cos(\omega_\ell t) + \Phi_{\ell m}^{\alpha,(i),s}(r)\sin(\omega_\ell t).$$

- Because AdS acts like a confining box, only certain frequencies are allowed to propagate

$$\omega_\ell^2 L^2 = (1 + \ell + 2p)^2,$$

where p is the radial overtone. These are the so-called normal modes of AdS. The fact that $\omega^2 L^2 > 0$ means that AdS is linearly stable.

- For simplicity, we will take $p = 0$, in which case one finds

$$\Phi^{\alpha,(1),\kappa}(r) = A^{\alpha,(1),\kappa}\frac{r^{\ell+1}}{(r^2+L^2)^{\frac{\ell+1}{2}}},$$

where $A^{\alpha,(1),\kappa}$ is a normalization constant.

General Structure

1. Start with a given perturbation $\Phi_{\ell m}^{\alpha,(i),\kappa}(r)$, and determine the corresponding $h_{\ell m}^{(i)}(t,r,\theta,\phi)$ through a linear differential map.

General Structure

1. Start with a given perturbation $\Phi^{\alpha,(i),\kappa}_{\ell m}(r)$, and determine the corresponding $h^{(i)}_{\ell m}(t, r, \theta, \phi)$ through a linear differential map.
2. Compute $T^{(i+1)}_{ab}$ and decompose it as a sum of the building blocks $\mathcal{T}^{\ell m}_{ab}$.

General Structure

1. Start with a given perturbation $\Phi^{\alpha,(i),\kappa}_{\ell m}(r)$, and determine the corresponding $h^{(i)}_{\ell m}(t,r,\theta,\phi)$ through a linear differential map.
2. Compute $T^{(i+1)}_{ab}$ and decompose it as a sum of the building blocks $\mathcal{T}^{\ell m}_{ab}$.
3. Compute source term $\tilde{T}^{\alpha,(i+1)}_{\ell m}(t,r)$, and determine $\Phi^{\alpha,(i+1)}_{\ell m}(t,r)$.

General Structure

1. Start with a given perturbation $\Phi^{\alpha,(i),\kappa}_{\ell m}(r)$, and determine the corresponding $h^{(i)}_{\ell m}(t,r,\theta,\phi)$ through a linear differential map.
2. Compute $T^{(i+1)}_{ab}$ and decompose it as a sum of the building blocks $\mathcal{T}^{\ell m}_{ab}$.
3. Compute source term $\tilde{T}^{\alpha,(i+1)}_{\ell m}(t,r)$, and determine $\Phi^{\alpha,(i+1)}_{\ell m}(t,r)$.
4. If $\tilde{T}^{\alpha,(i+1)}_{\ell m}(t,r)$ has an harmonic time dependence $\cos(\omega\, t)$, then $\Phi^{\alpha,(i+1)}_{\ell m}(t,r)$ will exhibit the same dependence, EXCEPT when ω agrees with one of the normal frequencies of AdS:

$$\Phi^{\alpha,(i+1)}_{\ell m}(t,r) = \Phi^{\alpha,(i+1),c}_{\ell m}(r)\cos(\omega\, t) + \Phi^{\alpha,(i+1),s}_{\ell m}(r)\, t\, \sin(\omega\, t).$$

This mode is said to be resonant.

General Structure

1. Start with a given perturbation $\Phi^{\alpha,(i),\kappa}_{\ell m}(r)$, and determine the corresponding $h^{(i)}_{\ell m}(t,r,\theta,\phi)$ through a linear differential map.
2. Compute $T^{(i+1)}_{ab}$ and decompose it as a sum of the building blocks $\mathcal{T}^{\ell m}_{ab}$.
3. Compute source term $\tilde{T}^{\alpha,(i+1)}_{\ell m}(t,r)$, and determine $\Phi^{\alpha,(i+1)}_{\ell m}(t,r)$.
4. If $\tilde{T}^{\alpha,(i+1)}_{\ell m}(t,r)$ has an harmonic time dependence $\cos(\omega\, t)$, then $\Phi^{\alpha,(i+1)}_{\ell m}(t,r)$ will exhibit the same dependence, EXCEPT when ω agrees with one of the normal frequencies of AdS:

   $$\Phi^{\alpha,(i+1)}_{\ell m}(t,r) = \Phi^{\alpha,(i+1),c}_{\ell m}(r)\cos(\omega\, t) + \Phi^{\alpha,(i+1),s}_{\ell m}(r)\, t\, \sin(\omega\, t).$$

   This mode is said to be resonant.
5. If for a given perturbation one can construct $\Phi^{\alpha,(i)}_{\ell m}$ to any order, without ever introducing a term growing linearly in time, the solution is said to be stable and is unstable otherwise.

Geons

- Start with a single mode $\ell = m = 2$ initial data.

Geons

- Start with a single mode $\ell = m = 2$ initial data.
- At second order there are no resonant modes and the solution can be rendered regular everywhere.

Geons

- Start with a single mode $\ell = m = 2$ initial data.
- At second order there are no resonant modes and the solution can be rendered regular everywhere.
- At third order there is a resonant mode, but one can set the amplitude of the growing mode to zero by changing the frequency slightly

$$\omega L = 3 - \frac{14703}{17920}\epsilon^2.$$

Geons

- Start with a single mode $\ell = m = 2$ initial data.
- At second order there are no resonant modes and the solution can be rendered regular everywhere.
- At third order there is a resonant mode, but one can set the amplitude of the growing mode to zero by changing the frequency slightly

$$\omega L = 3 - \frac{14703}{17920}\epsilon^2.$$

- The structure of the equations indicate that there is only one resonant term at each odd order, and that the amplitude of the growing mode can be set to zero by correcting the frequency.

Geons

- Start with a single mode $\ell = m = 2$ initial data.
- At second order there are no resonant modes and the solution can be rendered regular everywhere.
- At third order there is a resonant mode, but one can set the amplitude of the growing mode to zero by changing the frequency slightly

$$\omega L = 3 - \frac{14703}{17920}\epsilon^2 .$$

- The structure of the equations indicate that there is only one resonant term at each odd order, and that the amplitude of the growing mode can be set to zero by correcting the frequency.
- One can compute the asymptotic charges to fourth order, and they readily obey to the first order of thermodynamics:

$$E_g = \frac{3J_g}{2L}\left(1 - \frac{4901\, J_g}{7560\pi L^2}\right), \quad \omega_2 = \frac{3}{L}\left(1 - \frac{4901\, J_g}{3780\pi L^2}\right),$$

where we defined ϵ by $J_g = \frac{27}{128}\pi L^2 \epsilon^2$.

Colliding Geons - 1/2

- Start with a linear combination of $\ell = m = 2$ and $\ell = m = 4$.

Colliding Geons - 1/2

- Start with a linear combination of $\ell = m = 2$ and $\ell = m = 4$.
- Alike the single mode initial data, at second order there are no resonant modes and the solution can be rendered regular everywhere.

Colliding Geons - 1/2

- Start with a linear combination of $\ell = m = 2$ and $\ell = m = 4$.
- Alike the single mode initial data, at second order there are no resonant modes and the solution can be rendered regular everywhere.
- At third order, there are four resonant modes:

Colliding Geons - 1/2

- Start with a linear combination of $\ell = m = 2$ and $\ell = m = 4$.
- Alike the single mode initial data, at second order there are no resonant modes and the solution can be rendered regular everywhere.
- At third order, there are four resonant modes:
    - The amplitude of the growing modes in two of the resonant modes can be removed by adjusting the frequency of the initial data ($\omega_2 \, L = 3$ and $\omega_4 \, L = 5$) just like we did for the single mode initial data.

Colliding Geons - 1/2

- Start with a linear combination of $\ell = m = 2$ and $\ell = m = 4$.
- Alike the single mode initial data, at second order there are no resonant modes and the solution can be rendered regular everywhere.
- At third order, there are four resonant modes:
  - The amplitude of the growing modes in two of the resonant modes can be removed by adjusting the frequency of the initial data ($\omega_2 \, L = 3$ and $\omega_4 \, L = 5$) just like we did for the single mode initial data.
  - The amplitude of the growing mode of smallest frequency is automatically zero.

Colliding Geons - 1/2

- Start with a linear combination of $\ell = m = 2$ and $\ell = m = 4$.
- Alike the single mode initial data, at second order there are no resonant modes and the solution can be rendered regular everywhere.
- At third order, there are four resonant modes:
  - The amplitude of the growing modes in two of the resonant modes can be removed by adjusting the frequency of the initial data ($\omega_2 \, L = 3$ and $\omega_4 \, L = 5$) just like we did for the single mode initial data.
  - The amplitude of the growing mode of smallest frequency is automatically zero.
  - The amplitude of the growing mode with the largest frequency cannot be set to zero ($\omega L = 7$, $\ell = m = 6$)!

Colliding Geons - 1/2

- Start with a linear combination of $\ell = m = 2$ and $\ell = m = 4$.
- Alike the single mode initial data, at second order there are no resonant modes and the solution can be rendered regular everywhere.
- At third order, there are four resonant modes:
  - The amplitude of the growing modes in two of the resonant modes can be removed by adjusting the frequency of the initial data ($\omega_2\, L = 3$ and $\omega_4\, L = 5$) just like we did for the single mode initial data.
  - The amplitude of the growing mode of smallest frequency is automatically zero.
  - The amplitude of the growing mode with the largest frequency cannot be set to zero ($\omega L = 7$, $\ell = m = 6$)!

AdS is nonlinearly unstable!

Colliding Geons - 2/2

- The frequency of the growing mode is higher than any of the frequencies we started with!

Colliding Geons - 2/2

- The frequency of the growing mode is higher than any of the frequencies we started with!
- The "energy" is transferred to modes of higher frequency.

Colliding Geons - 2/2

- The frequency of the growing mode is higher than any of the frequencies we started with!
- The "energy" is transferred to modes of higher frequency.
- Expect this to continue. When the $\omega L = 7$, $\ell = m = 6$ mode grows, it will source even higher frequency modes with growing amplitude.

Colliding Geons - 2/2

- The frequency of the growing mode is higher than any of the frequencies we started with!
- The “energy” is transferred to modes of higher frequency.
- Expect this to continue. When the $\omega L = 7$, $\ell = m = 6$ mode grows, it will source even higher frequency modes with growing amplitude.

Conjecture:

The endpoint of this instability is a rotating black hole.

Colliding Geons - 2/2

- The frequency of the growing mode is higher than any of the frequencies we started with!
- The "energy" is transferred to modes of higher frequency.
- Expect this to continue. When the $\omega L = 7$, $\ell = m = 6$ mode grows, it will source even higher frequency modes with growing amplitude.

Conjecture:

The endpoint of this instability is a rotating black hole.

- Spherical scalar field collapse in AdS - Bizon and Rostworowski, '11.

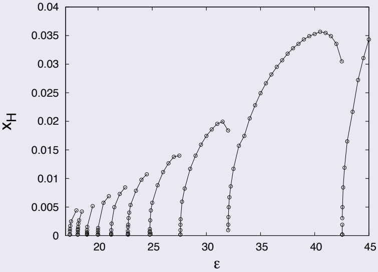
Colliding Geons - 2/2

- The frequency of the growing mode is higher than any of the frequencies we started with!
- The "energy" is transferred to modes of higher frequency.
- Expect this to continue. When the $\omega L = 7,\ \ell = m = 6$ mode grows, it will source even higher frequency modes with growing amplitude.

Conjecture:

The endpoint of this instability is a rotating black hole.

- Spherical scalar field collapse in AdS - Bizon and Rostworowski, '11.
- No matter how small you make the initial amplitude, the curvature at the origin grows and you eventually form a small black hole.



Field theory implications - 1/2:

The fact that one evolves to a state of maximum entropy can be viewed as thermalization - not in the canonical ensemble, but in the microcanonical.

Field theory implications - 1/2:

The fact that one evolves to a state of maximum entropy can be viewed as thermalization - not in the canonical ensemble, but in the microcanonical.

All theories with a gravity dual will show this cascade of energy like the onset of turbulence.

Field theory implications - 1/2:

The fact that one evolves to a state of maximum entropy can be viewed as thermalization - not in the canonical ensemble, but in the microcanonical.

All theories with a gravity dual will show this cascade of energy like the onset of turbulence.

Puzzle:

In 2+1 dimensions, classical turbulence has an inverse energy cascade due to an extra conserved quantity - the enstrophy. Our results indicate that in a strongly coupled quantum theory, there is a standard energy cascade.

Field theory implications - 1/2:

The fact that one evolves to a state of maximum entropy can be viewed as thermalization - not in the canonical ensemble, but in the microcanonical.

All theories with a gravity dual will show this cascade of energy like the onset of turbulence.

Puzzle:

In 2+1 dimensions, classical turbulence has an inverse energy cascade due to an extra conserved quantity - the enstrophy. Our results indicate that in a strongly coupled quantum theory, there is a standard energy cascade.

Solution: This intuition comes from solving the Navier Stokes equations in 2+1 dimensions: quantum turbulence is different.

Field theory implications - 2/2:

- Perhaps more intriguing, from the CFT perspective, is the existence of Geons.

Field theory implications - 2/2:

- Perhaps more intriguing, from the CFT perspective, is the existence of Geons.
- At the linear level, these are spin 2 excitation.

Field theory implications - 2/2:

- Perhaps more intriguing, from the CFT perspective, is the existence of Geons.
- At the linear level, these are spin 2 excitation.
- A nonlinear geon is like a bose condensate of these excitations.

Field theory implications - 2/2:

- Perhaps more intriguing, from the CFT perspective, is the existence of Geons.
- At the linear level, these are spin 2 excitation.
- A nonlinear geon is like a bose condensate of these excitations.

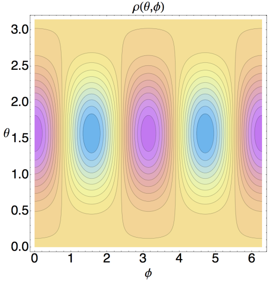
These high energy states do NOT thermalize!!

Field theory implications - 2/2:

- Perhaps more intriguing, from the CFT perspective, is the existence of Geons.
- At the linear level, these are spin 2 excitation.
- A nonlinear geon is like a bose condensate of these excitations.

These high energy states do NOT thermalize!!

- The boundary stress-tensor contains regions of negative and positive energy density around the equator:



Field theory implications - 2/2:

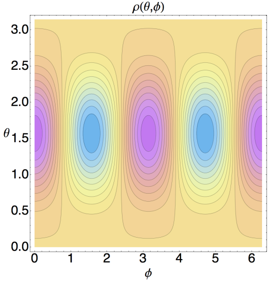
- Perhaps more intriguing, from the CFT perspective, is the existence of Geons.
- At the linear level, these are spin 2 excitation.
- A nonlinear geon is like a bose condensate of these excitations.

These high energy states do NOT thermalize!!

- The boundary stress-tensor contains regions of negative and positive energy density around the equator:
  - It is invariant under

$$K = \frac{\partial}{\partial t} + \frac{\omega}{m}\frac{\partial}{\partial \phi},$$

which is timelike near the poles but spacelike near the equator.



Conclusions & Open questions

Conclusions:

- Anti-de Sitter spacetime is nonlinearly unstable: generic small perturbations become large and (probably) form black holes.
- For some linearized gravity mode, there is an exact, nonsingular geon.
- Dual field theory shows generic turbulent cascade to maximum entropy state but there are special states (geons) that do not thermalize

Conclusions & Open questions

Conclusions:

- Anti-de Sitter spacetime is nonlinearly unstable: generic small perturbations become large and (probably) form black holes.
- For some linearized gravity mode, there is an exact, nonsingular geon.
- Dual field theory shows generic turbulent cascade to maximum entropy state but there are special states (geons) that do not thermalize

Open questions:

- Prove a singularity theorem for anti-de Sitter.
- Understand the space of CFT states that do not thermalize.
- Find the endpoint (if any) of time evolution of the anti-de Sitter turbulent instability!