What can black holes tell us about microstates?

Ashoke Sen

Harish-Chandra Research Institute, Allahabad, India

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Introduction

Bekenstein and Hawking gave a universal formula for the black hole entropy:

$$S_{BH} = \frac{A}{4G}$$

– valid in classical, two derivative theory of gravity.

For a class of extremal supersymmetric black holes in string theory we now have a microscopic understanding of this entropy in the limit of large charges.

Strominger, Vafa; ...

$$S_{BH}(Q) = \ln d_{micro}(Q)$$

$d_{micro}(Q)$: number of quantum states carrying a given set of charges $Q \equiv \{Q_1, Q_2, \ldots\}$.

More precisely

$$d_{micro}(Q) = Tr(-1)^{2J}, \quad J = \text{angular momentum}$$

with the trace taken over states carrying charges Q, after factoring out the center of mass d.o.f.

– protected from quantum corrections.

Questions

1. Can quantum gravity give us a prescription for computing the exact black hole entropy taking into account higher derivative and quantum corrections?

2. Can quantum gravity tell us about other statistical properties of the microstates besides degeneracy?

Any affirmative answer can be tested against explicit microscopic results, known in theories with 16 or 32 unbroken supersymmetries.

Plan

1. General formalism for computing extremal black hole entropy.

2. Tests against microscopic results:

   i) Logarithmic corrections to the entropy

   ii) Sign of the index

   iii) Twisted index

3. Non-supersymmetric, non-extremal black holes.

In the extremal limit a black hole acquires an infinite throat described by an AdS_2 factor, separating the horizon from the asymptotic space-time.

Full throat geometry:

$$AdS_2 \times K$$

K includes the angular coordinates of space-time as well as the 6 dimensional space on which we have compactified string theory.

The metric on the throat:

$$ds^2 = a^2 \left[-r^2 dt^2 + \frac{dr^2}{r^2} \right] + ds_K^2$$

The original horizon is towards $r \to 0$.

Original asymptotic space-time is towards $r \to \infty$.

Strategy: Analyze euclidean path integral on this space-time.

Euclidean continuation of the throat metric: $t \to -i\theta$

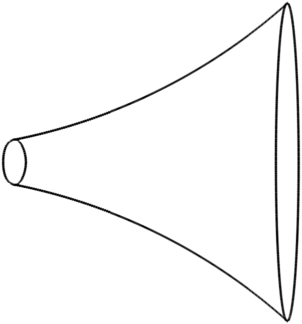
$$ds^2 \;=\; a^2 \left(r^2\, d\theta^2 + \frac{dr^2}{r^2} \right) + ds_K^2$$

Since we can change the period of θ by a rescaling $r \to \lambda r, \theta \to \theta/\lambda$, we choose

$$\theta \equiv \theta + 2\pi$$

The boundary of AdS_2 is at $r = \infty$.

Regularize the infinite volume of AdS_2 by putting a cut-off $r \leq r_0$.



This makes the AdS_2 boundary have a finite length

$$L = 2\pi\, a\, r_0$$

1. Define the partition function:

$$Z_{bulk} = \int D\varphi \, \exp[-\text{Action} - \text{boundary terms}]$$

φ: set of all fields in the theory

Boundary condition: For large r the field configuration should approach the near horizon geometry of the black hole.

In the interior we allow all fluctuations including fluctuations of the topology.

2. From the point of view of the observer at the boundary $r = r_0$:

$$Z_{bulk} = \mathrm{Tr}\left(e^{-L\,H}\right)$$

H: Hamiltonian, L: boundary length

3. As $L \to \infty$

$$Z_{bulk} = Tr(e^{-LH}) \to d_0 \, e^{-L \, E_0}$$

(d_0, E_0): (degeneracy, energy) of ground state.

We identify $(\ln d_0)$ as the quantum corrected extremal black hole entropy S_{macro}

4. Thus we can define S_{macro} as

$$S_{macro} = \lim_{L\to\infty} \left(1 - L\frac{d}{dL}\right) \ln\left(Z_{bulk}\right)$$

5. On AdS_2, deformations of the gauge charges are non-normalizable modes and hence are not allowed fluctuations.

$\Rightarrow$ S_{macro} is the quantum corrected entropy in the microcanonical ensemble.

$\Rightarrow$ forces us to to include a Gibbons-Hawking type boundary term in the path integral

$$\exp[-i \sum_k q_k \oint_{\partial(AdS_2)} d\theta\, A_\theta^{(k)}]$$

$A_\mu^{(k)}$: gauge fields on AdS_2.

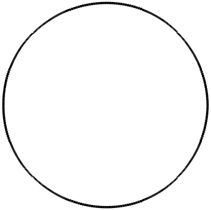
q_k: associated electric charge

Classical limit

The dominant saddle point that contributes to Z_{bulk} is the euclidean black hole:

$$ds^2 = a^2\left[(r^2-1)d\theta^2 + \frac{dr^2}{r^2-1}\right] + ds_K^2$$

$$= a^2(d\eta^2 + \sinh^2 d\theta^2) + ds_K^2 \qquad r = \cosh\eta$$



$r = r_0$

One can show that in the classical limit

$$S_{macro} \Rightarrow S_{wald}$$

1. Logarithmic corrections to entropy

Typically the leading entropy S_{BH} is a homogeneous function of the various charges Q_i.

e.g. in D=4

$$S_{BH}(\Lambda Q) = \Lambda^2 S_{BH}(Q)$$

Logarithmic corrections: correction to the entropy $\propto \ln \Lambda$ in the limit of large Λ.

These arise from one loop correction to the leading saddle point result for Z_{bulk} from loops of massless fields.

Banerjee, Gupta, A.S.; Banerjee, Gupta, Mandal, A.S.; A.S; Ferrara, Marrani; Bhattacharyya, Panda, A.S.

Mann, Solodukhin; Fursaev; ...

Consider a spherically symmetric extremal black hole in D=4 with horizon size a

$$\mathbf{ds^2 = a^2\left(\frac{dr^2}{r^2-1} + (r^2-1)d\theta^2 + d\psi^2 + \sin^2\psi d\phi^2\right) + ds^2_{compact}}$$

There are also fluxes through AdS_2 and S^2 and vev of moduli fields.

When the charges scale uniformly by a large number Λ then

$$a \sim \Lambda$$

keeping other moduli, e.g. $ds^2_{compact}$, fixed.

$\{\kappa_n\}$: eigenvalues of the kinetic operator of four dimensional massless fields in the near horizon background.

Non-zero mode contribution to Z_{bulk}:

$$\prod_n{}' \kappa_n^{-1/2} = \exp\left[-\frac{1}{2}\sum_n{}' \ln \kappa_n\right]$$

$'$: remove zero modes from the product / sum

We can use heat kernel expansion to determine the terms in the exponent proportional to $\ln a$.

– arise from modes with eigenvalues $<< m_{pl}^2$ and hence are insensitive to the ultraviolet cut-off.

Zero mode contribution:

The zero modes are normalizable deformations generated by gauge transformations with non-normalizable transformation parameters.

i) Change integration variables from fields to gauge transformation parameters.

ii) Find a-dependence of the jacobian as well as the a-dependence of the range of integration of the gauge transformation parameters.

$\Rightarrow$ determines total a-dependence of Z_{bulk} from integration over the zero modes.

Final results:

S. Banerjee, Gupta, Mandal, A.S.; Ferrara, Marrani; A.S.

The theory	scaling of charges	logarithmic contribution	microscopic
$\mathcal{N}=4$ with n_V matter	$Q_i \sim \Lambda$, $A_H \sim \Lambda^2$	0	$\surd$
$\mathcal{N}=8$	$Q_i \sim \Lambda$, $A_H \sim \Lambda^2$	$-8\ln\Lambda$	$\surd$
$\mathcal{N}=2$ with n_V vector and n_H hyper	$Q_i \sim \Lambda$, $A_H \sim \Lambda^2$	$\frac{1}{6}(23+n_H-n_V)\ln\Lambda$	?*
$\mathcal{N}=6$	$Q_i \sim \Lambda$, $A_H \sim \Lambda^2$	$-4\ln\Lambda$	?
$\mathcal{N}=5$	$Q_i \sim \Lambda$, $A_H \sim \Lambda^2$	$-2\ln\Lambda$	?
$\mathcal{N}=3$ with n_V matter	$Q_i \sim \Lambda$, $A_H \sim \Lambda^2$	$2\ln\Lambda$	?
BMPV in type IIB on $T^5/\mathbb{Z}_N$ or $K3\times S^1/\mathbb{Z}_N$ with n_V vectors	$Q_1, Q_5, n \sim \Lambda$; $J \sim \Lambda^{3/2}$, $A_H \sim \Lambda^{3/2}$	$-\frac{1}{4}(n_V-3)\ln\Lambda$	$\surd$
BMPV in type IIB on $T^5/\mathbb{Z}_N$ or $K3\times S^1/\mathbb{Z}_N$ with n_V vectors	$Q_1, Q_5, n \sim \Lambda$; $J=0$, $A_H \sim \Lambda^{3/2}$	$-\frac{1}{4}(n_V+3)\ln\Lambda$	$\surd$

*: various proposals exist but no definite result

Ooguri, Strominger, Vafa; Cardoso, de Wit, Mahapatra; Denef, Moore; David; Cardoso, de Wit, Mahapatra

2. Sign of the index

4 unbroken supersymmetry $\Rightarrow$ extremal black holes in D=4 are rotationally invariant $\Rightarrow$ have $J = 0$.

Thus

A.S.; Dabholkar, Gomes, Murthy, A.S.

$$Tr(-1)^{2J} = Tr(1) = \text{degeneracy} = \exp[S_{macro}]$$

This leads to the conclusion that

$$Tr(-1)^{2J} = \exp[S_{macro}] \quad \text{and} \quad Tr(-1)^{2J} > 0$$

for a supersymmetric black hole in D=4.

$$\mathrm{Tr}(-1)^{2J} = \exp[S_{macro}] \quad \text{and} \quad \mathrm{Tr}(-1)^{2J} > 0$$

In the microscopic counting, performed at weak coupling, the spectrum may be different, but the protected index $\mathrm{Tr}(-1)^{2J}$ should continue to be positive, and equal to $\exp[S_{macro}]$.

Positivity of $\mathrm{Tr}(-1)^{2J}$ has been tested in many supersymmetric string theories where the microscopic index is known exactly.

A.S.

This holds even for finite charges, after subtracting the contribution to the index from multi-centered black holes.

So far there are no counterexamples.

Some microscopic results for the index of single centered black holes in heterotic string theory on T^6

$(Q^2, P^2)\backslash Q.P$	2	3	4	5	6	7
(2,2)	648	0	0	0	0	0
(2,4)	50064	0	0	0	0	0
(2,6)	1127472	25353	0	0	0	0
(4,4)	3859456	561576	12800	0	0	0
(4,6)	110910300	18458000	1127472	0	0	0
(6,6)	4173501828	920577636	110910300	8533821	153900	0
(2,10)	185738352	16844421	16491600	0	0	0

No negative index

Blue entries represent states with $Q^2P^2 - (Q.P)^2 < 0$ for which there are no classical single centered black hole solutions.

3. Twisted index

A.S.

Suppose the theory has a discrete $\mathbb{Z}_N$ symmetry generated by g, which is not part of a continuous gauge symmetry, and which commutes with the unbroken supersymmetries of the black hole.

What is the value of the protected index

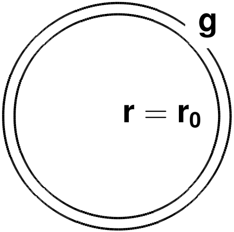
$$\mathrm{Tr}\{(-1)^{2J} g\}$$

for a supersymmetric black hole?

By our earlier argument

$$\mathrm{Tr}\{(-1)^{2J} g\} = \mathrm{Tr}\, g$$

For computing Tr(g) in the euclidean path integral approach, we need to evaluate the partition function with a g-twisted boundary condition along the euclidean time circle.



The euclidean black hole which gave dominant contribution to Z_{bulk} is no longer allowed since the time circle, along which we have g-twist, is contractible.

However an appropriate $\mathbb{Z}_N$ orbifold of the original saddle point geometry does contribute to the path integral.

The corresponding action is 1/N times the action of an Euclidean black hole.

This leads to the prediction:

$$\mathrm{Tr}\{(-1)^{2J} g\} \sim \exp[S_{BH}/N]$$

This has been verified in a wide class of supersymmetric string theories where exact microscopic results for many twisted indices are known.

Other tests

4. Black holes have also been used to compute $\mathcal{O}(1)$ correction to the entropy due to one loop contribution from massive string loops.

Cardoso, de Wit, Kappeli, Mohaupt; David, Jatkar, A.S.

5. Progress has been made towards evaluating Z_{bulk} using localization techniques.

N. Banerjee, S. Banerjee, Gupta, Mandal, A.S.; Dabhokar, Gomes, Murthy

Given the success of this program we can extend it to the study of logarithmic corrections to non-supersymmetric black hole entropy.

1. Non-supersymmetric extremal black hole:

The same algorithm works since the near horizon geometry contains an AdS_2 factor.

2. Non-extremal black holes:

In this case the euclidean path integral gives the partition function in the grand canonical ensemble.

Gibbons, Hawking

We need to remove the thermal gas contribution to the partition function and then take Laplace transform to get the microcanonical entropy.

Extremal black holes

Extremal Kerr in pure gravity has a correction

$$\frac{16}{45} \ln A_H$$

Extremal Kerr-Newmann in Einstein-Maxwell theory has a more complicated formula for the logarithmic correction.

Bhattacharyya, Panda, A.S.

Can Kerr/CFT correspondence explain these microscopically?

Guica, Hartman, Song, Strominger

Note: Cardy limit formula is not useful here and even for the extremal supersymmetric black holes.

Non-extremal black holes

Example: In D=4 the Schwarzschild black hole entropy has logarithmic correction

$$\frac{77}{90} \ln A_H$$

– counts number of rotationally invariant states in unit mass range.

(related earlier work by Fursaev, Solodukhin)

In contrast loop quantum gravity analysis of the black hole entropy gives a logarithmic correction of $-\ln A_H$

Majumdar, Kaul; Meissner; Ghosh, Mitra; Engle, Noui, Perez, Pranzetti; ...

???

Conclusion

Quantum gravity is capable of explaining detailed statistical properties of the microstates.

Besides giving a systematic procedure for computing corrections to the leading Bekenstein-Hawking result, it also predicts certain properties of the microstates which are quite surprising from the point of view of the microscopic description.

– sign of the index

– asymptotic growth of the twisted index