Holographic cameras: an eye for the bulk

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on 2211.11791 and earlier work with:

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'Gravity from algebra', KITP, January 2023

Large-N operator algebras

Consider a large-N theory in a classical state Ψ: $\mathcal{O} = \overset{\sim N_c}{\langle \mathcal{O} \rangle_{\Psi}} + \delta\mathcal{O}$.

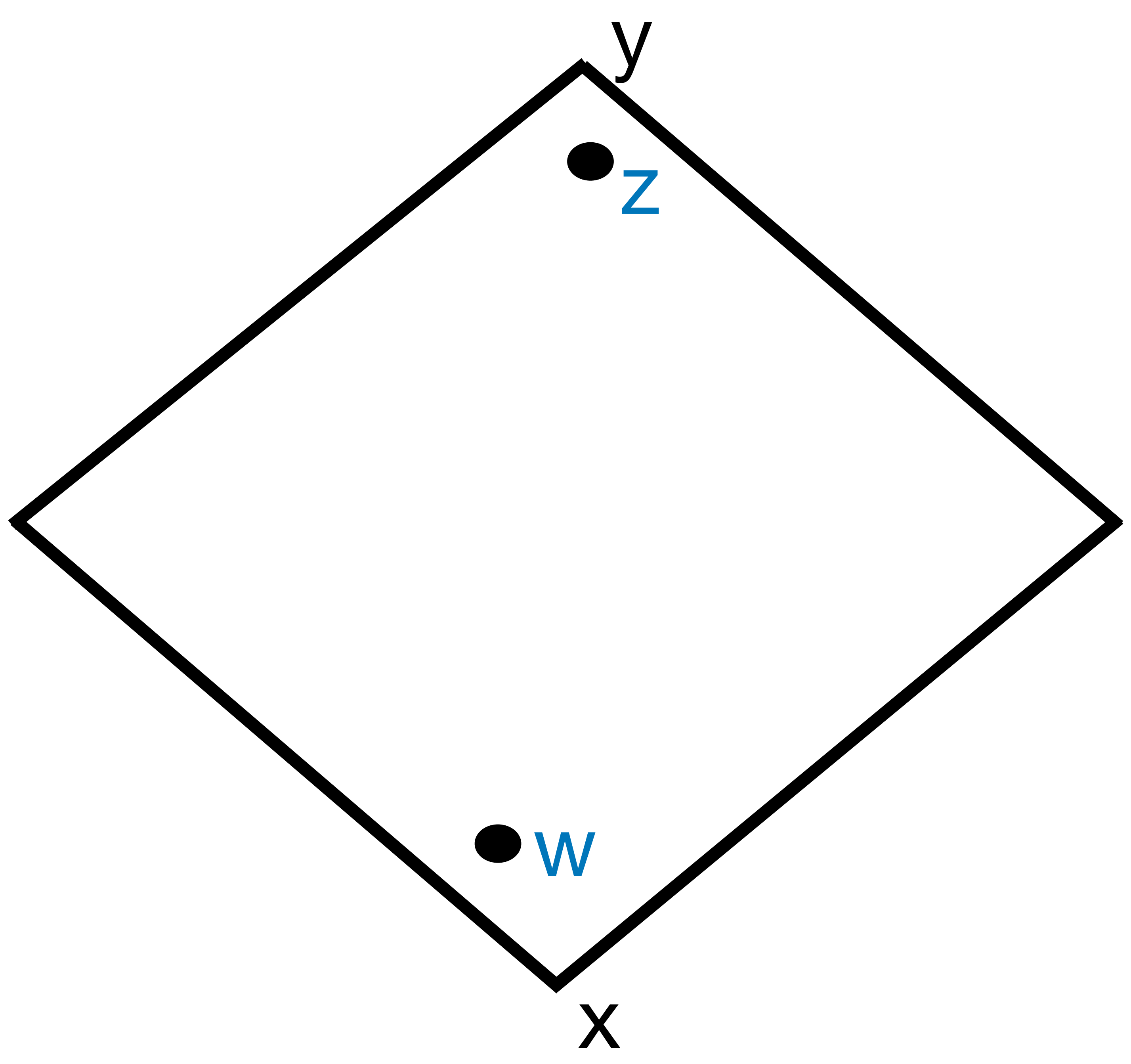
The effective OPE for small fluctuations is state-dependent:

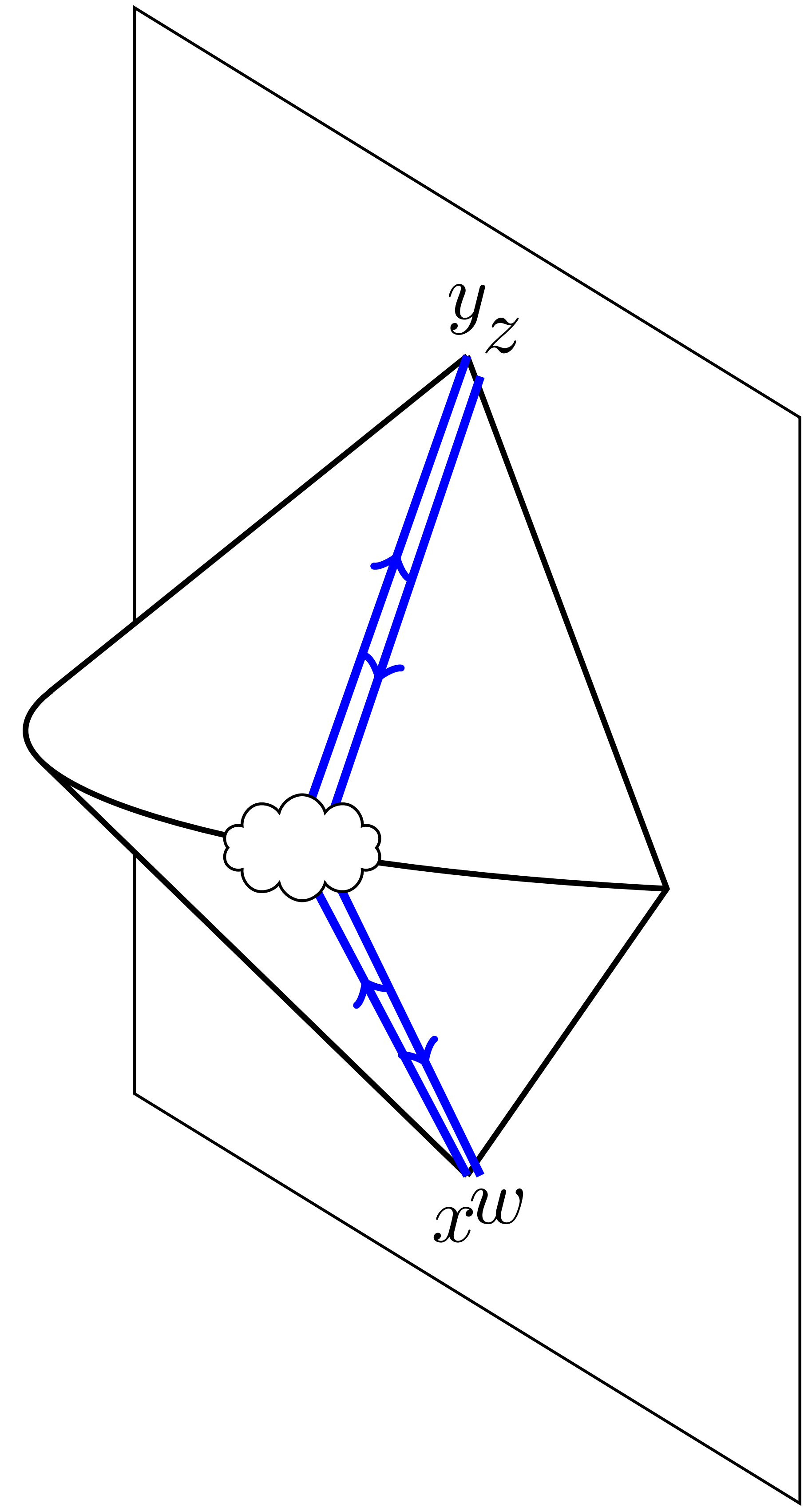
$$[\delta\mathcal{O}(x), \delta\mathcal{O}(y)] = F_{ij}[\Psi]\, \mathbb{1} + \frac{1}{N_c} \sum_k F_{ij}^k[\Psi]\ \delta\mathcal{O}_k + \frac{1}{N_c^2} \sum_{k,k'} F_{ij}^{kk'}[\Psi]\ \delta\mathcal{O}_k \delta\mathcal{O}_{k'} + \dots$$

Identity coefficient $F_{ij}[\Psi]$ and 2-pt functions encode interesting bulk info.

[Leutheusser and Liu ’21
Chandrasekaran, Penington& Witten ’22, …]

today: double-trace coefficient $F^{\mathcal{O}(z)\mathcal{O}(w)}_{\mathcal{O}(x)\mathcal{O}(y)}[\Psi]$ $\simeq$ bulk scattering amplitudes near edge of causal wedge





Reveals bulk geometry very straightforwardly.

* $\langle 0 | a_{\text{in}} b_{\text{out}} a^\dagger_{\text{in}} b^\dagger_{\text{out}} | 0 \rangle$

Outline

1. Motivation

2. Four-point correlators in excited states
   -Eikonal approximation & conformal Regge theory

3. Sample images:
   - planar black holes
   - non-holographic theories

« CFT with large $\overset{C_T?}{N}$ & large $\Delta_{\text{gap}}^{\text{higher-spin}}$ $\longrightarrow S_{\text{bulk}} =$ Einstein+matter+small »

[Heemskerk,Penedones,Polchinski& Sully ’09]

Now relatively understood for <TTTT> in AdS vacuum.

It would be *nice* to get full nonlinear theory (black holes, …)

First step: what’s bulk geometry in boundary language?

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[Heemskerk,Penedones,Polchinski& Sully '09]

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It would be *nice* to get full nonlinear theory (black holes, …)

First step: what's bulk geometry in boundary language?

(of course, integrating bulk EOMs can 'reconstruct' bulk metric from $\langle T^{\mu\nu}(x)\rangle_\Psi$.
but we would prefer to *measure* the metric, not *calculate* it.)

→ situations where we're not sure about bulk EFT?

→ finite-coupling QFTs whose duals we don't know?

Desirably, measure ‘bulk metric’:

- using simple observables

  *that satisfy linear consistency conditions (positive OPE, crossing, etc.)

- from large signal ~1/N, not exponentially small
- with high spatial resolution (ie. to string length << R_{AdS})
- *only when a metric exists* (ie. not for 3d Ising)
- in state&theory-independent way, and fun.

proposal: $\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle_{\Psi}$

EFT constraints in vacuum AdS:

[Camanho, Edelstein, Maldacena& Zhiboedov '14]
...[SCH,Mazac,Rastelli&Simmons-Duffin '21]
...[SCH,Li,Parra-Martinez&Simmons-Duffin '22][2]

$$S_{\text{bulk}} = \frac{1}{16\pi G}\int\left(R + \Lambda + \lambda_{\text{GB}}\text{Riem}^2 + g_3\text{Riem}^3 + g_4\text{Riem}^4 + \ldots\right)$$

If $g_3 \sim 1/M^4$, expect gradient expansion to hold for $\partial \ll M$.

To bound coefficients, scatter gravitons with energies $\Delta_{\text{gap}}/L \sim M$ (<<M_{pl}).

Schematic sum rules, valid for any $p \leq \Delta_{\text{gap}}$:

$$\frac{G_N}{p^2 + \frac{d^2}{4}} = \sum_{J,\Delta\geq\Delta_{\text{gap}}} c^2_{J,\Delta}P_J(1 - 2p^2/\Delta^2), \qquad G_N g_3^2 = \sum_{J,\Delta>\Delta_{\text{gap}}} \frac{\text{similar}}{\Delta^6}$$

Example results: $g_3^2 \leq \dfrac{25\log\Delta_{\text{gap}}}{\Delta^8_{\text{gap}}}$ in AdS4, $\left|\dfrac{c-a}{c}\right| \leq \dfrac{\sim 23}{\Delta^2_{\text{gap}}}$ in AdS5xX.

holographic cannon

Start with $|\Psi\rangle$ = arbitrary state of boundary QFT

Q: « add an energetic $\mathscr{O}$ excitation and measure energy density. »

holographic cannon

Start with $|\Psi\rangle$ = arbitrary state of boundary QFT

Q: « add an energetic $\mathcal{O}$ excitation and measure energy density. »

A: excite: $|\Psi\rangle \mapsto \int d^d x \, \psi_{p,L}(x) \, \mathcal{O}(x)|\Psi\rangle$

$\psi_{p,L}(x) \sim e^{i\omega t - i\vec{p}\cdot\vec{x} - \frac{|x|^2}{2L^2}}$

plane wave with Gaussian envelope

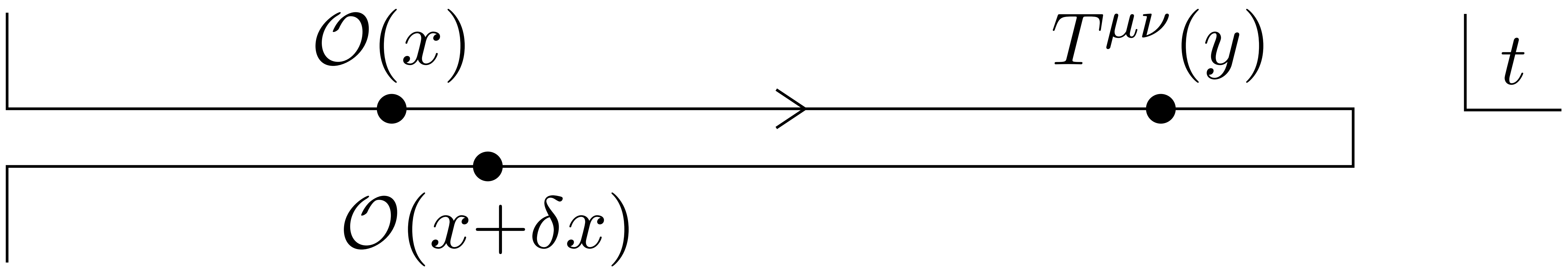
…then take expectation value:

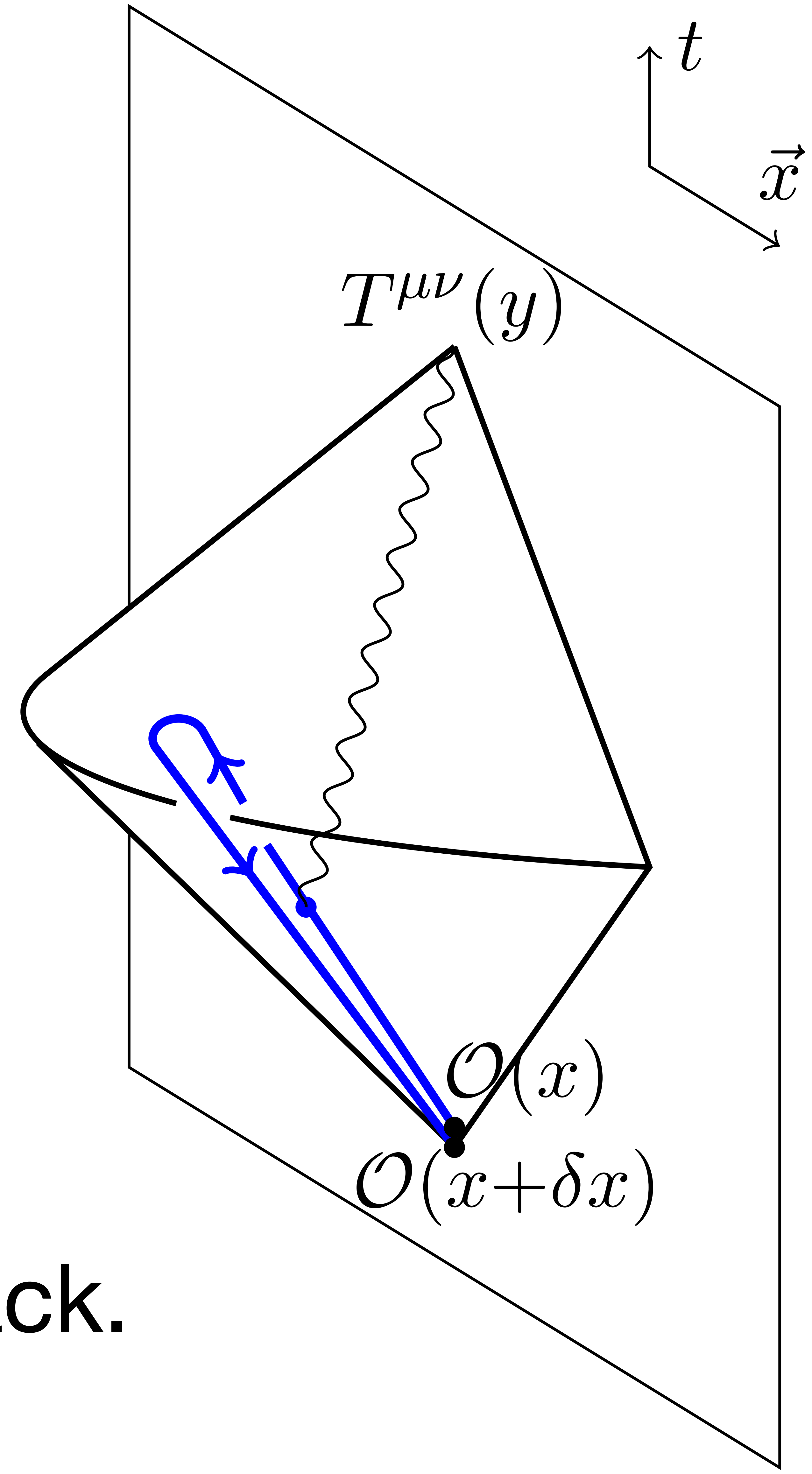
$$\langle T \rangle_{\mathcal{O}|\Psi\rangle} = \int_{x',x} \psi_p^*(x')\psi_p(x) \, \langle \Psi | \mathcal{O}(x') T^{\mu\nu}(y) \mathcal{O}(x) | \Psi \rangle$$

cf: [Hoffman& Maldacena ’08]
[Arnold, Vaman ’11]

notice operator ordering:

$$\langle \Psi | \; \mathcal{O}(x+\delta x) \; T^{\mu\nu}(y) \; \mathcal{O}(x) \; | \Psi \rangle$$





excitation can't go directly from x to $x + \delta x$:

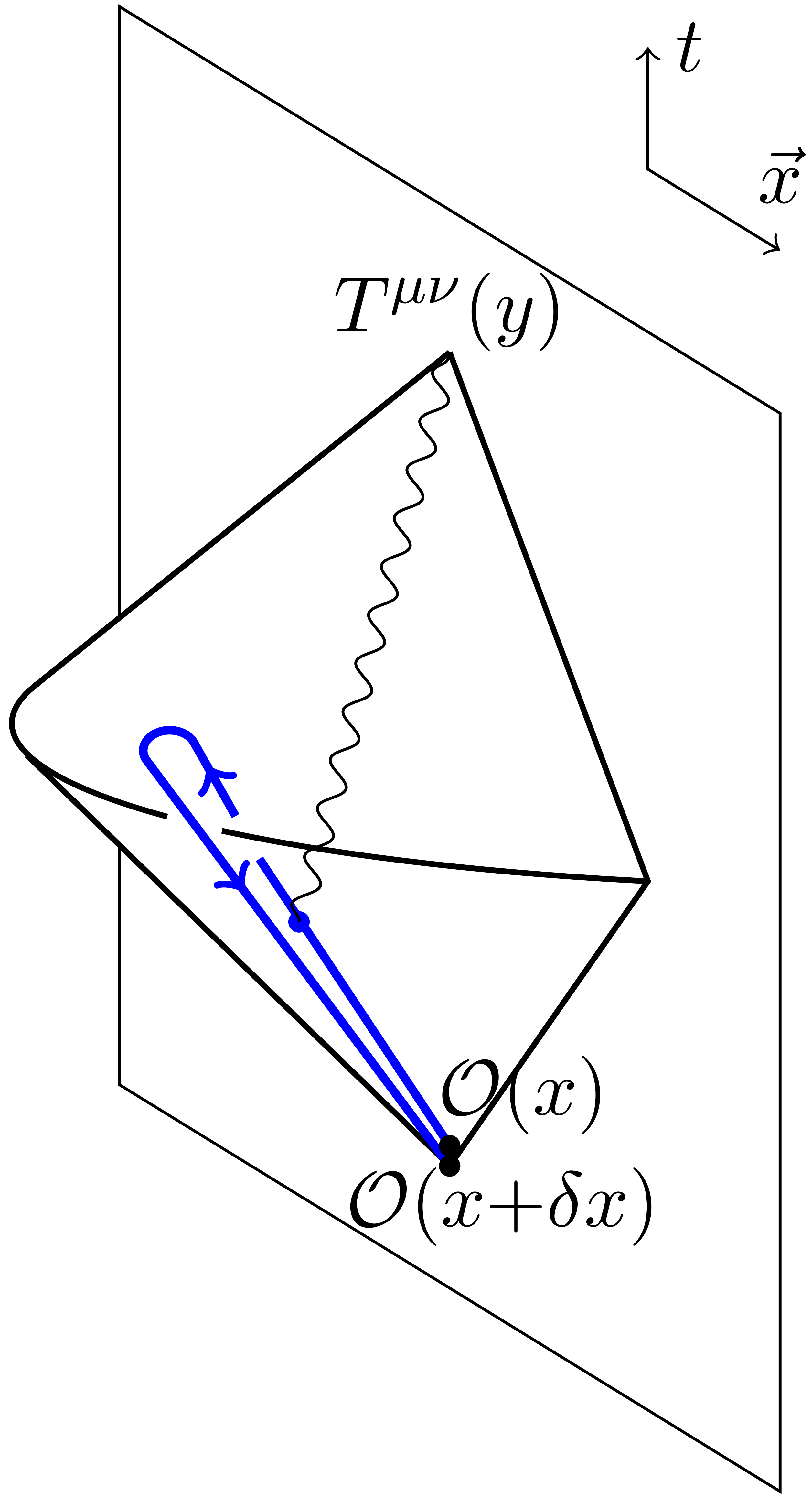
first it must exit past lightcone of y, then come back.

p timelike&large* $\Rightarrow \mathcal{O}$ follows bulk null geodesic

p controls shooting direction precisely:

$\mathcal{O}\mathcal{O} \simeq$ one bulk null geodesic

$\langle T \rangle_{\mathcal{O}|\Psi\rangle}$ displays an expanding shell of energy.



cf: [Hoffman& Maldacena ’08]
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*compared with |x-y| and bulk features

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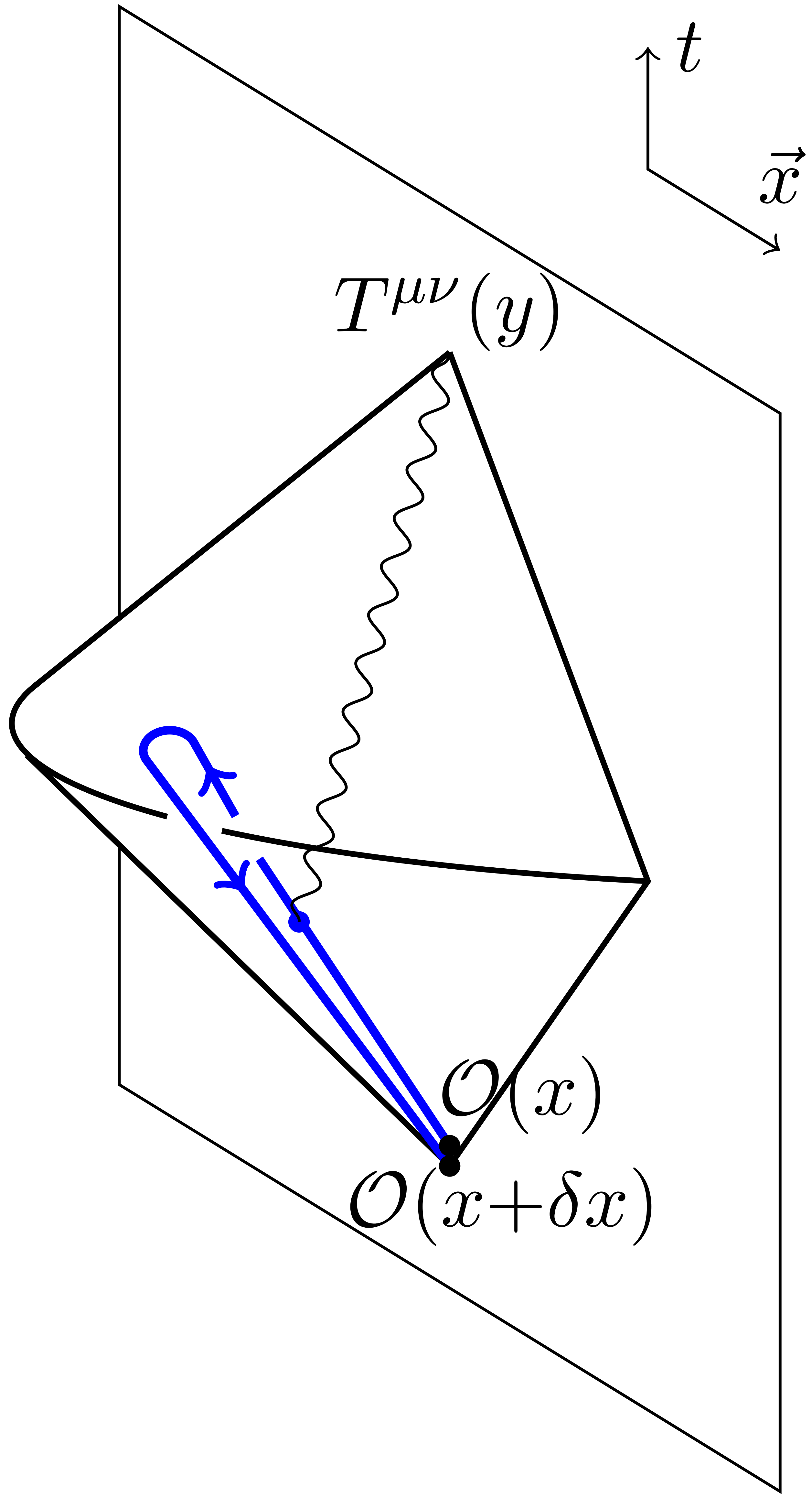
We'll now improve the 'camera' part.



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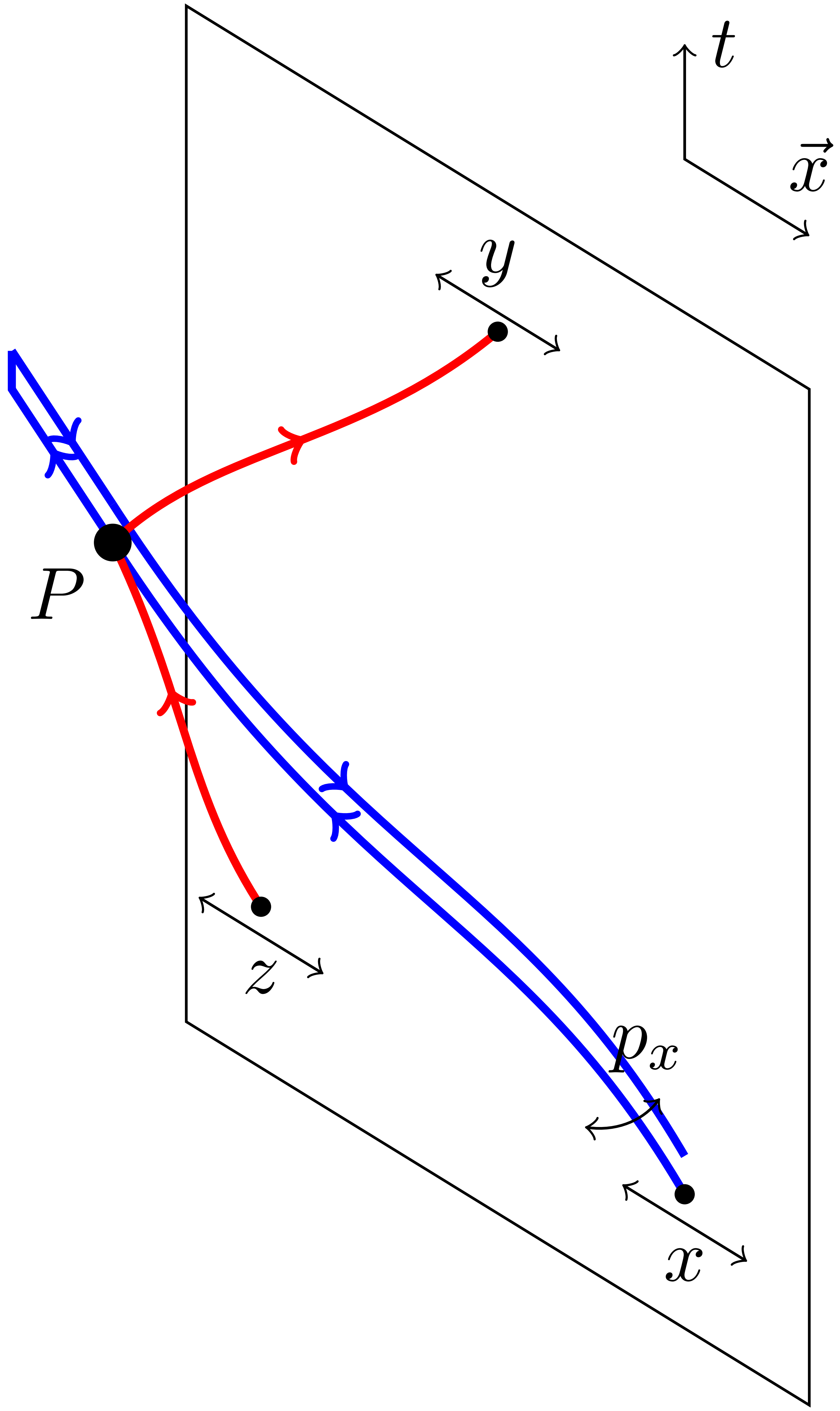
*compared with |x-y| and bulk features

a. Radar camera:

$$\int_{\delta x} \psi_{p,L}(\delta x)\,\langle \Psi \,|\, \mathcal{O}(x+\delta x)\ \mathcal{O}'(y)\mathcal{O}'(z)\ \mathcal{O}(x)\,|\,\Psi\rangle$$

- Send pulse from z, record reflection off P at y



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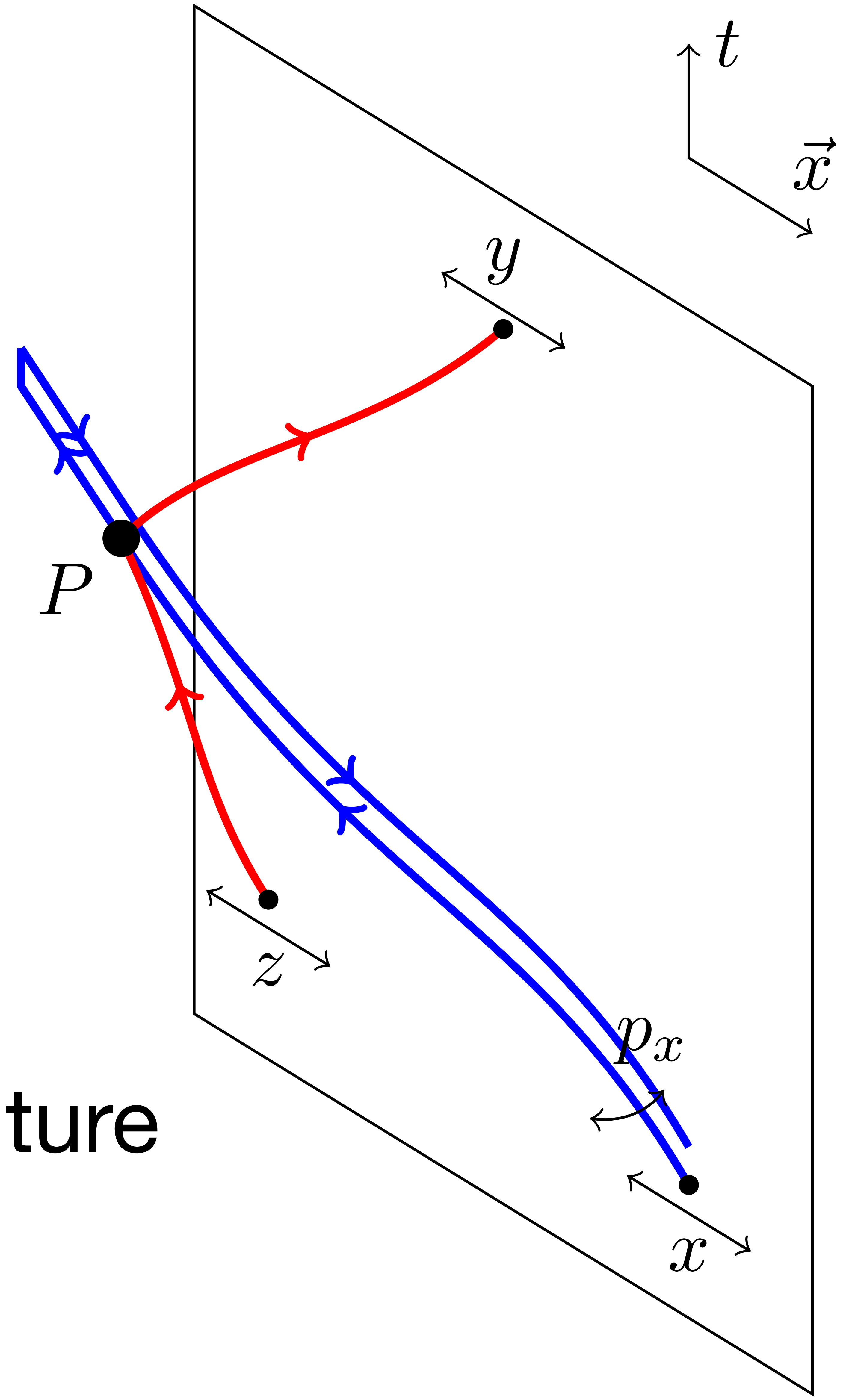


- Send pulse from z, record reflection off P at y

- Ideal regime: three null geodesics
- Signal = singularity as y → lightcone of P
- Similar to ‘bulk point’ *but* we don’t track $\mathcal{O}$’s future

*need $\mathrm{AdS}_{D\geq 3}$ or heavy $\mathcal{O}$

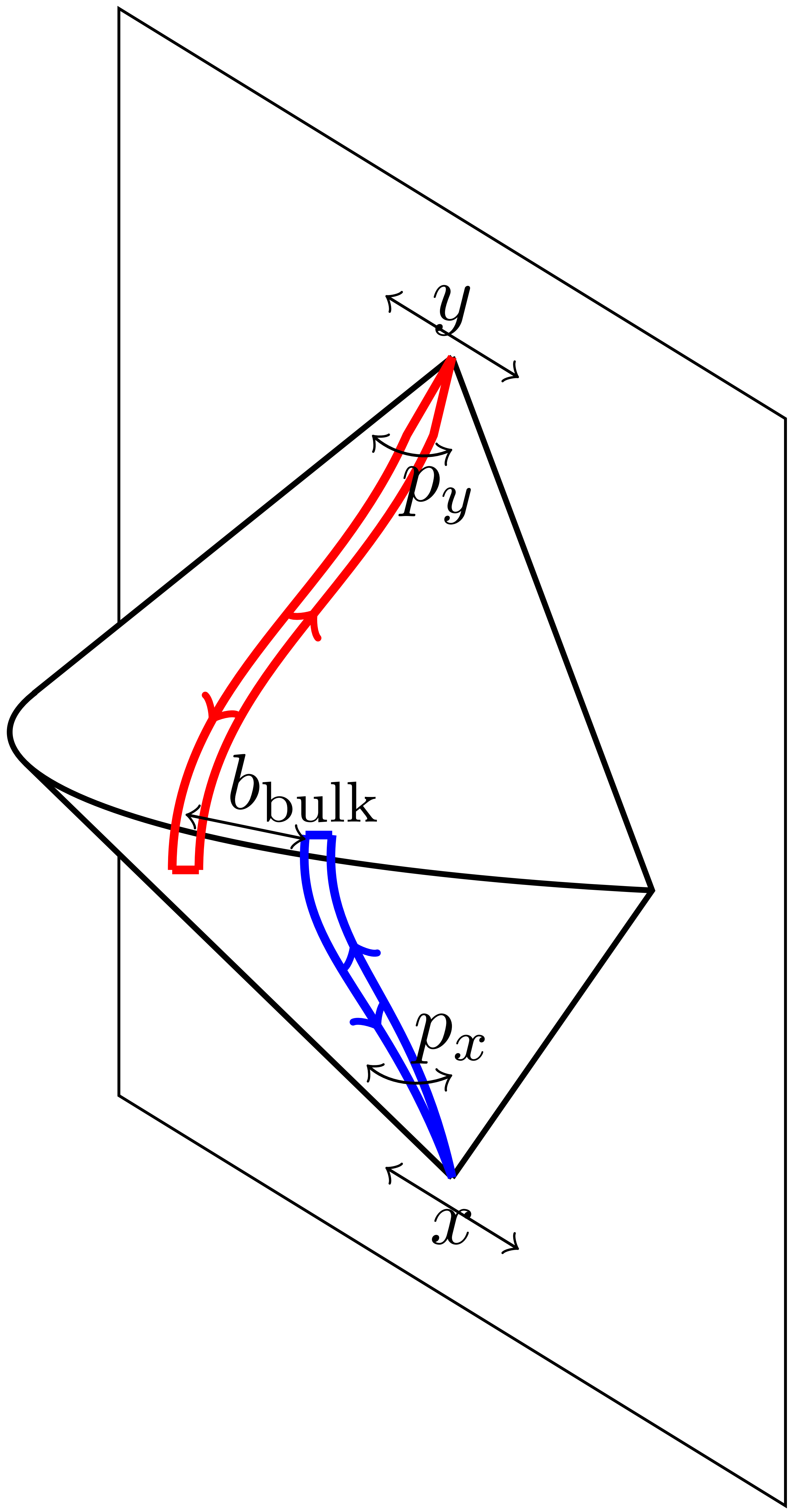
[Maldacena,Simmons-Duffin&Zhiboedov’15]
[Engelhardt& Horowitz ’16]

b. Active camera:

$$\int_{\delta x} \psi_{p_x,L_x}(\delta x) \int_{\delta y} \psi_{p_y,L_y}(\delta y)$$

$$\langle \Psi | \mathcal{O}'(y+\delta y)\ \mathcal{O}(x+\delta x)\ \mathcal{O}'(y)\ \mathcal{O}(x) | \Psi \rangle$$

- OTOC with high energy, early times
- Ideal regime: two null geodesics



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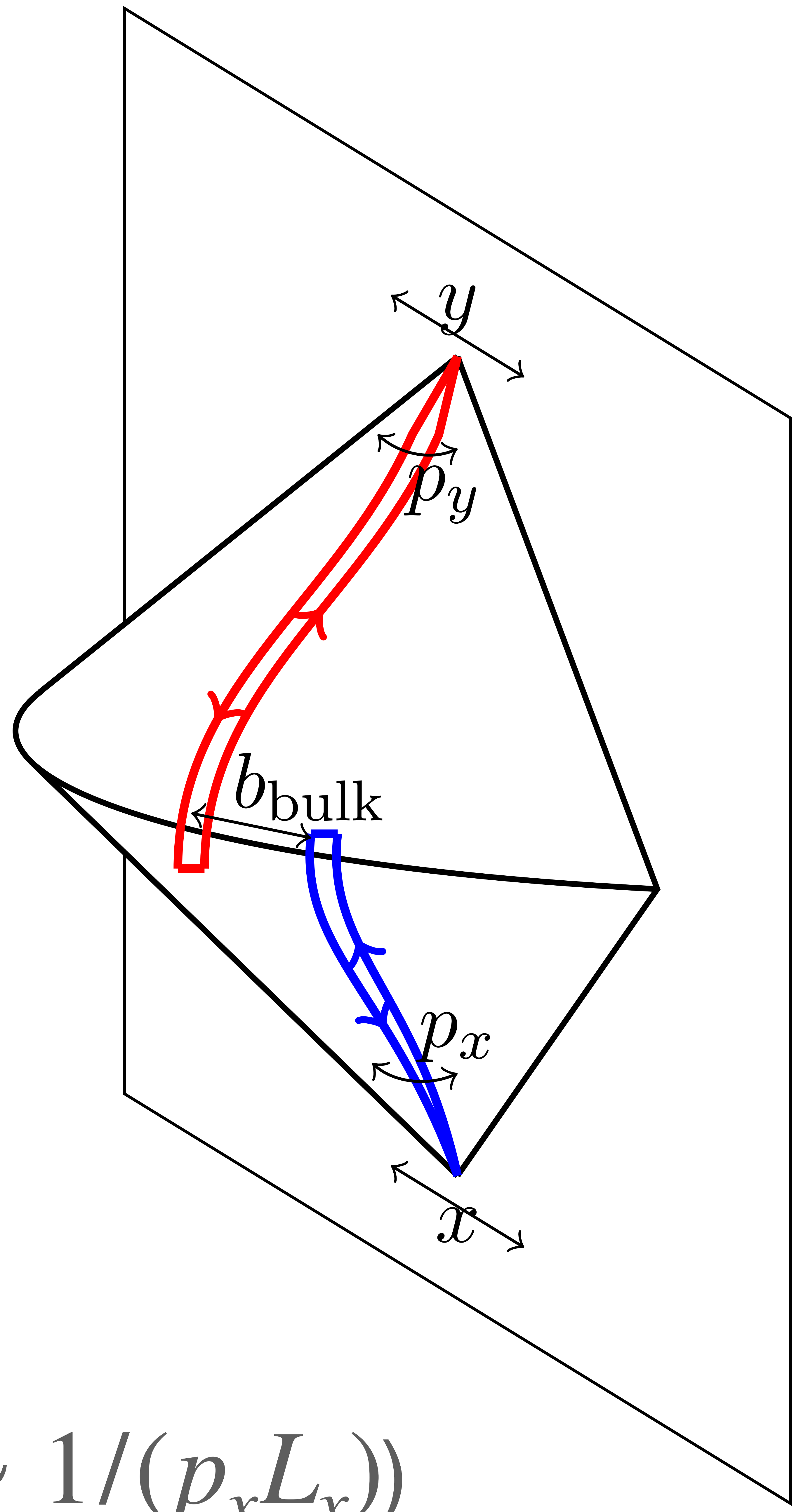
- OTOC with high energy, early times
- Ideal regime: two null geodesics

- Signal = singularity as $b_{\text{bulk}} \to 0$ $(\mathrm{AdS}_{\mathrm{D}\geq 3})$
- Knobs: x^μ, y^μ = spacetime shooting points

  p_x^μ, p_y^μ = shooting directions+energies

  L_x, L_y = Gaussian widths (optics: $\delta\theta \sim 1/(p_x L_x)$)

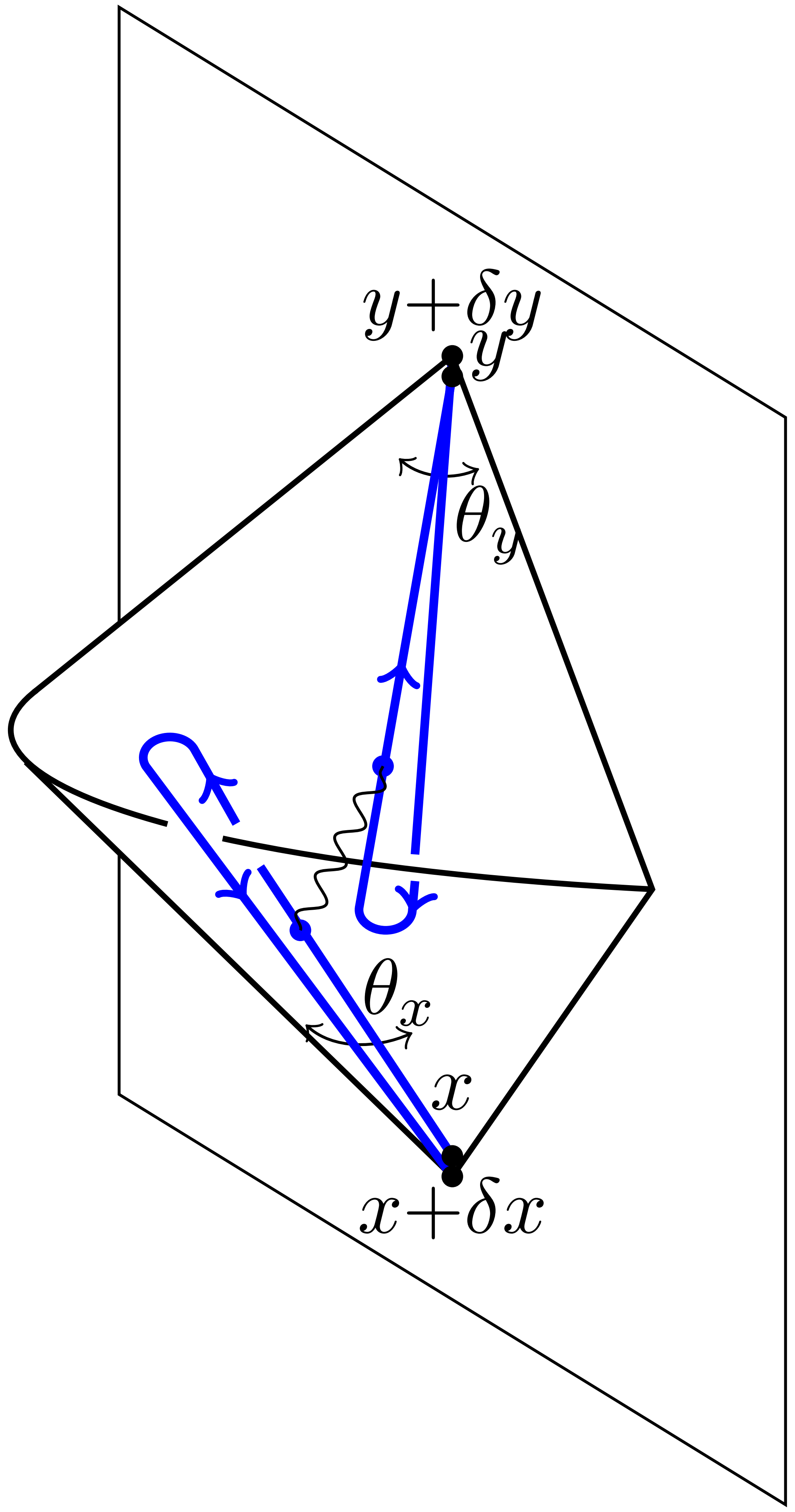


Folded OPE:

$$\mathcal{O}_2(\delta x)\ \mathcal{O}_1(0) \sim (\text{free}) + \int_{H_{d-1}} \frac{\#\ d^{d-1}\theta}{(-\theta\cdot\delta x_-)^{2\Delta+1}} L_\theta[\delta g_{uu}] + \ldots$$

bulk shooting angle

bulk light-transform



leading eikonal approximation:

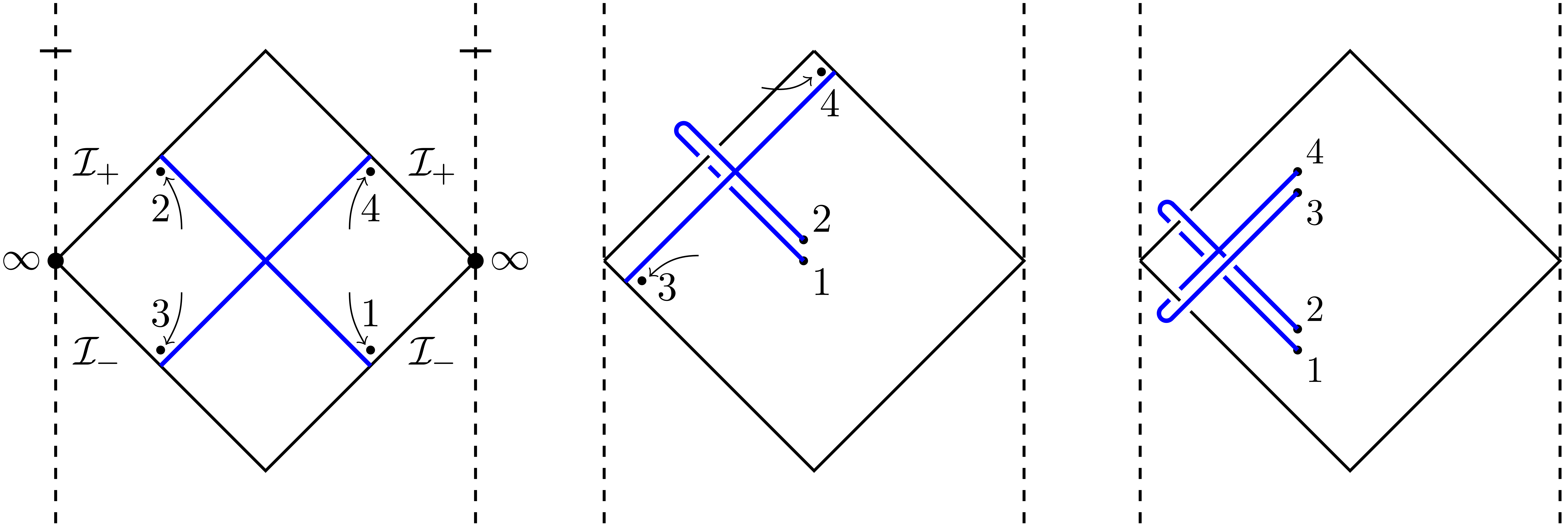
OTOC $\simeq$ graviton exchange between bulk null geodesics

beyond (strings etc): exchange CFT Regge trajectories

*exploit Lorentz sym. of tangent space, not conformal sym.

[Cornalba, Costa& Penedones '06]
[Costa, Goncalves& Penedones '12]

In CFT vacuum state, OTOC is equivalent to Regge limit and to detectors.



(a) Regge correlator (b) Detectors (c) OTOC

[leaving Poincaré patches: Kravchuk& Simmons-Duffin ’18]

in general, these limits are still structurally similar, but distinct in details.

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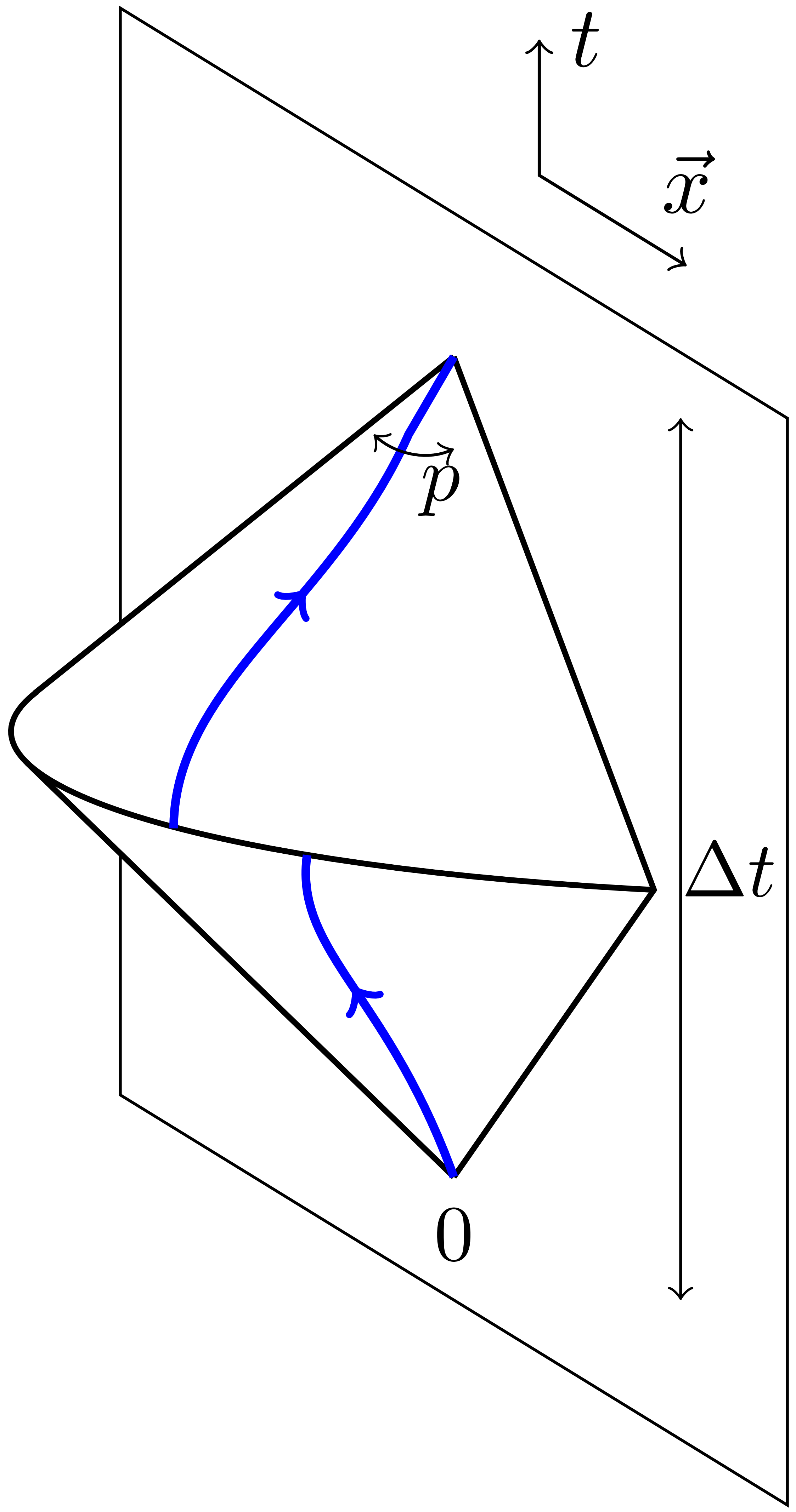
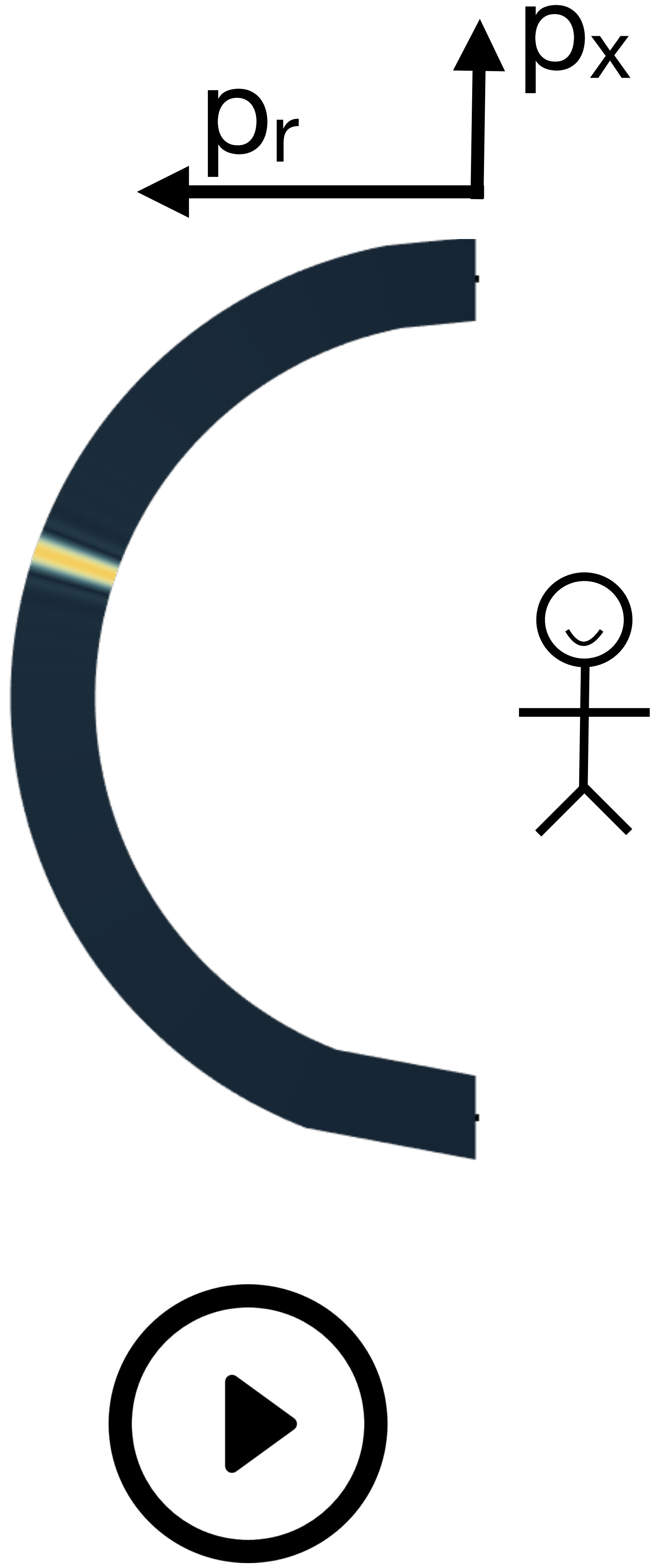
Vacuum AdS$_3$/CFT$_2$

$$p_r = |p| = \sqrt{p_0^2 - p_x^2}$$

|p|L~50





Q: fix shooting angle, vary p.
how does peak move with Δt?

Thermal state in CFT_2

$$\sqrt{z\bar{z}} \to \frac{\pi^2 T_R T_L \;\delta x \delta y}{\sinh(\pi T_R(\Delta t - y)\sinh(\pi T_L(\Delta t + y))}, \quad \sqrt{\frac{z}{\bar{z}}} \to e^{-\varphi_x - \varphi_y}\,\frac{T_R \sinh(\pi T_L(\Delta t + y))}{T_L \sinh(\pi T_R(\Delta t - y))}\,.$$

Thermal state in CFT_2

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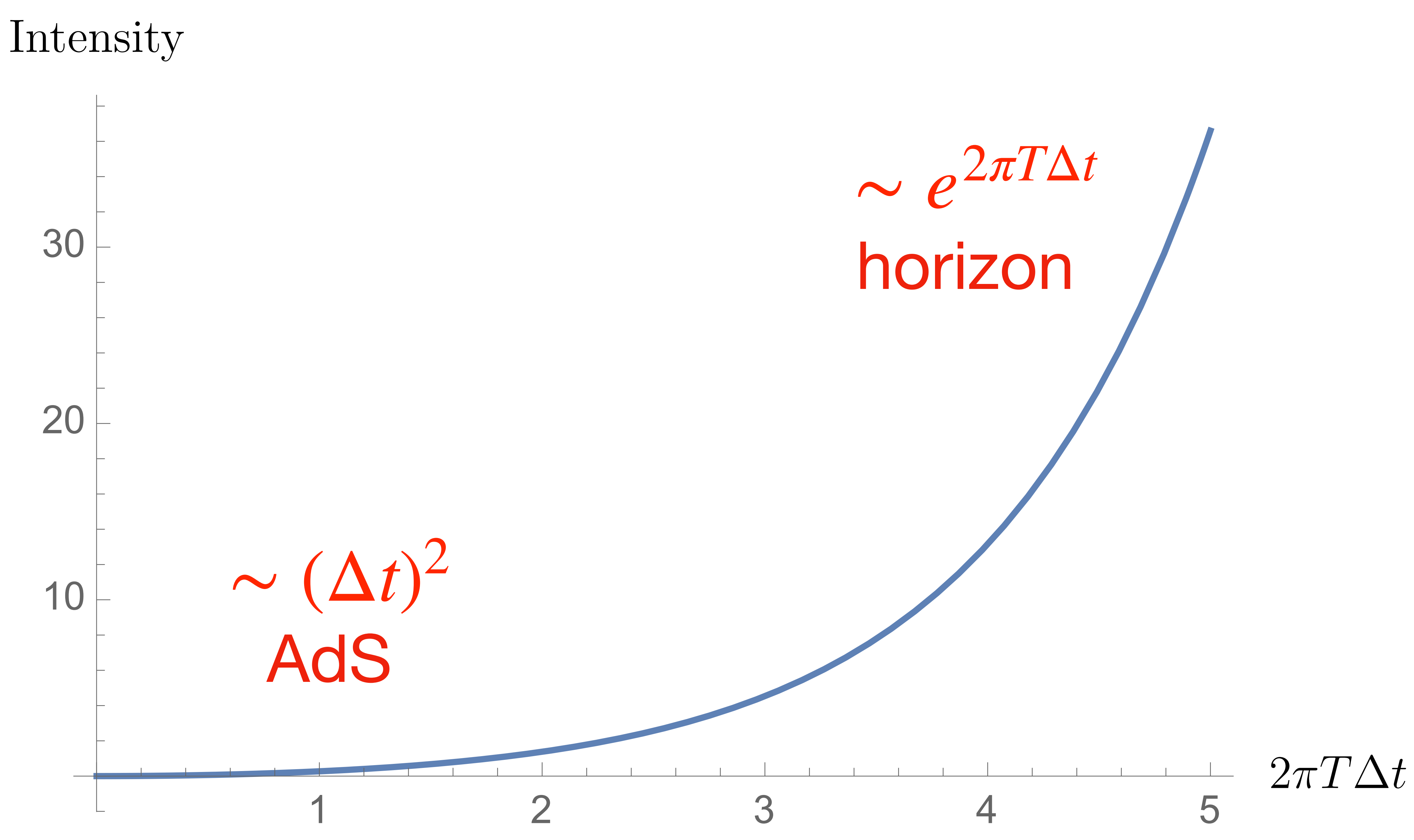
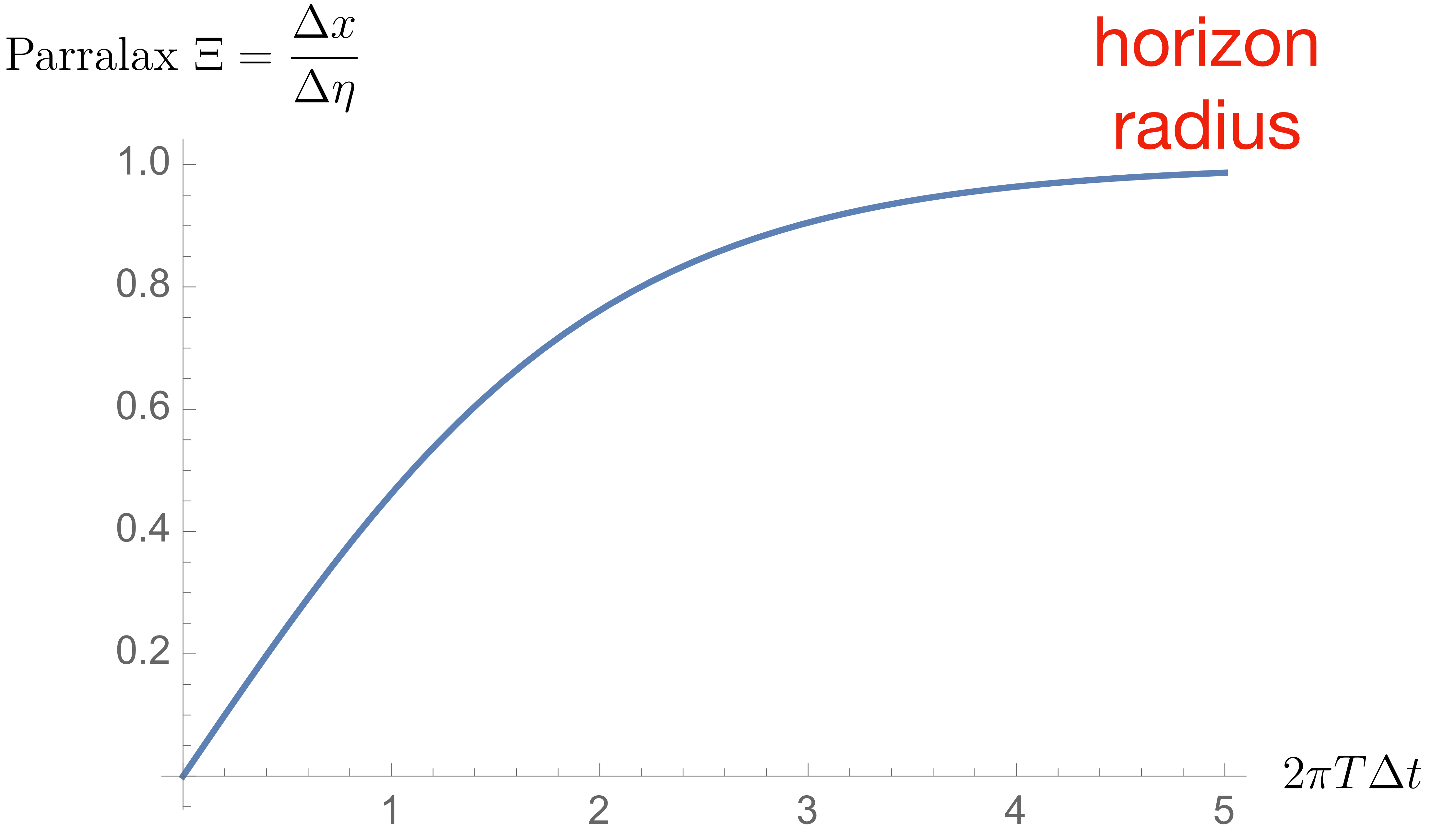
BTZ black hole (AdS_3/CFT_2)





Bulk metric from 4pt function

Metric ansatz: $$ds^2 = \frac{R_{\mathrm{AdS}}^2}{r^2}\left(-A(r)^2 dt^2 + \frac{dr^2}{B(r)} + dx^2\right)$$

OTOC data $\Rightarrow$

geometrical optics

$$r(\Delta t) \propto \frac{I(\Delta t)}{\Xi(\Delta t)^{3-d}} \frac{d\Xi(\Delta t)}{d\Delta t}$$

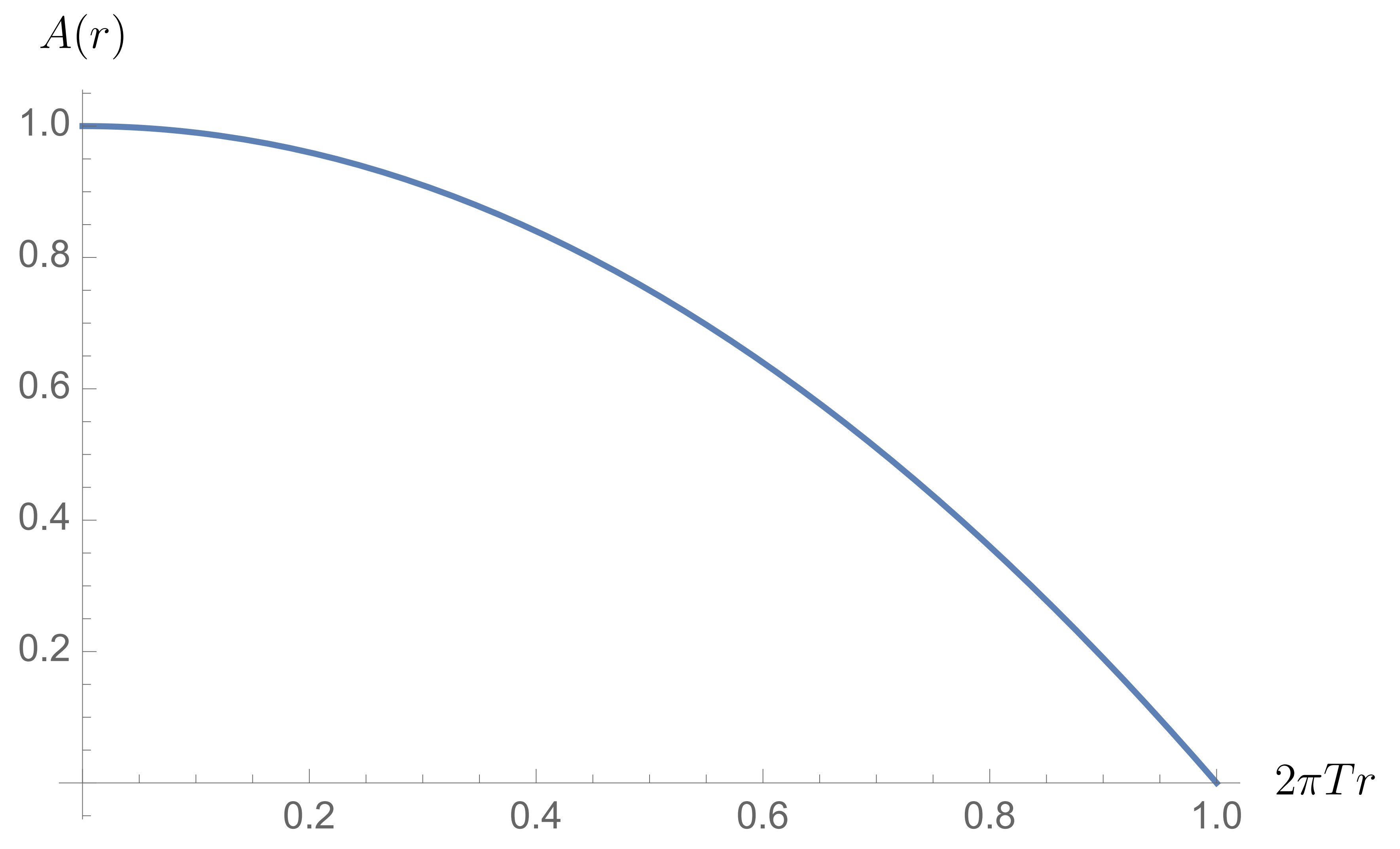
$$A(\Delta t)/r(\Delta t) = \Xi(\Delta t)/I(\Delta t)$$

$$\sqrt{A(r)B(r)} = 2\frac{dr}{d\Delta t}$$

$\Rightarrow$

plot of BTZ black hole metric!

$$A(r) = B(r) = 1 - (2\pi T r)^2$$



For thermal states in CFT$_d$, OTOC data (*intensity& parallax* $I(\Delta t), \Xi(\Delta t)$) very literally measure the bulk metric.

Einstein's equations $\forall r \Leftrightarrow$ OTOC data satisfies $\forall t$:

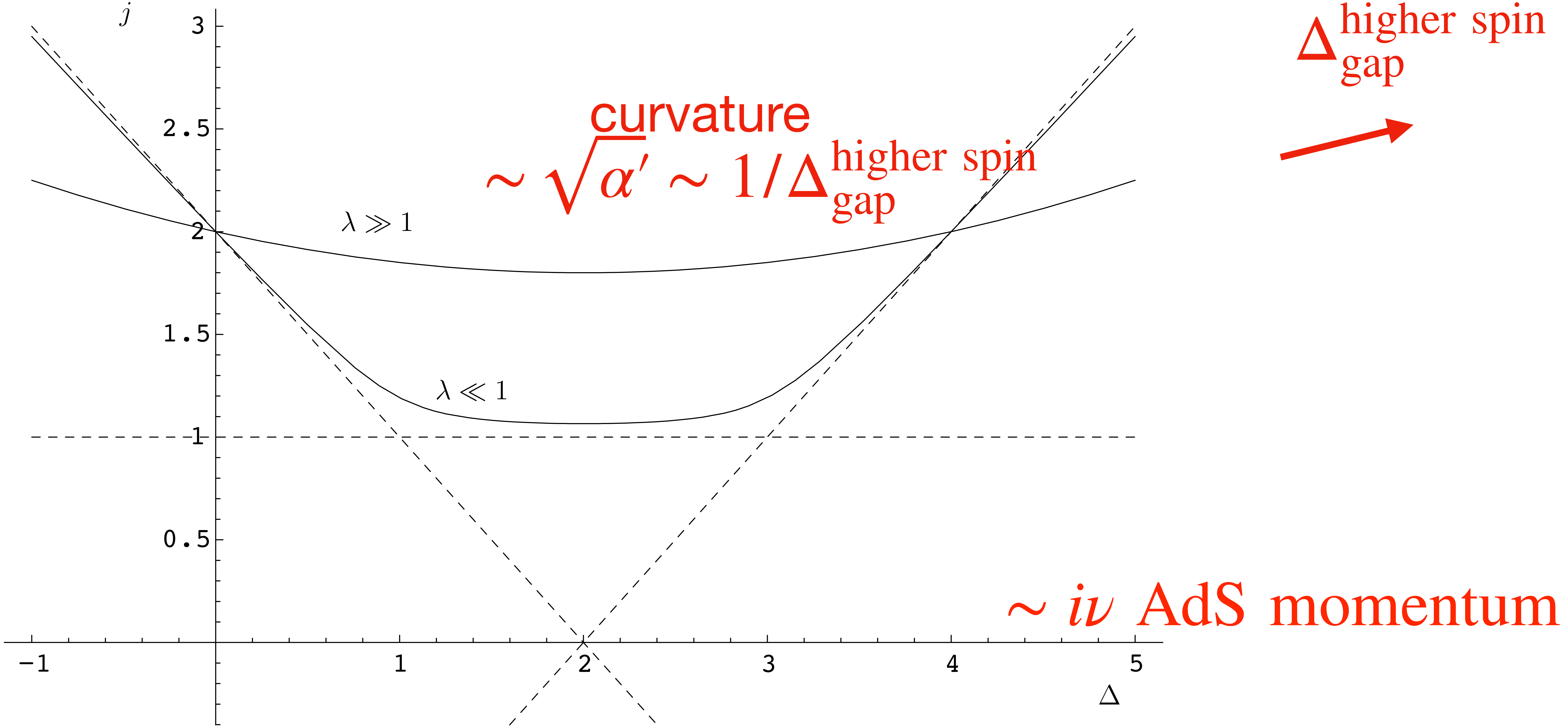
$$\Xi(\Delta t)^{-d}\left(2\frac{d\Xi(\Delta t)}{d\Delta t}-1\right) = \text{Constant}$$

$$I(\Delta t)\Xi(\Delta t)^{d-4}\ \frac{d\Xi(\Delta t)}{dt} = \text{Constant}$$

what do these mean, from the CFT perspective?

To probe small scales, we need large bulk momentum.

Resolution is limited by $\Delta_{\text{gap}}^{\text{higher-spin}}$ $(\sim\sqrt{\alpha'})$.



[Brower,Polchinski,Strasser& Tan ’06]

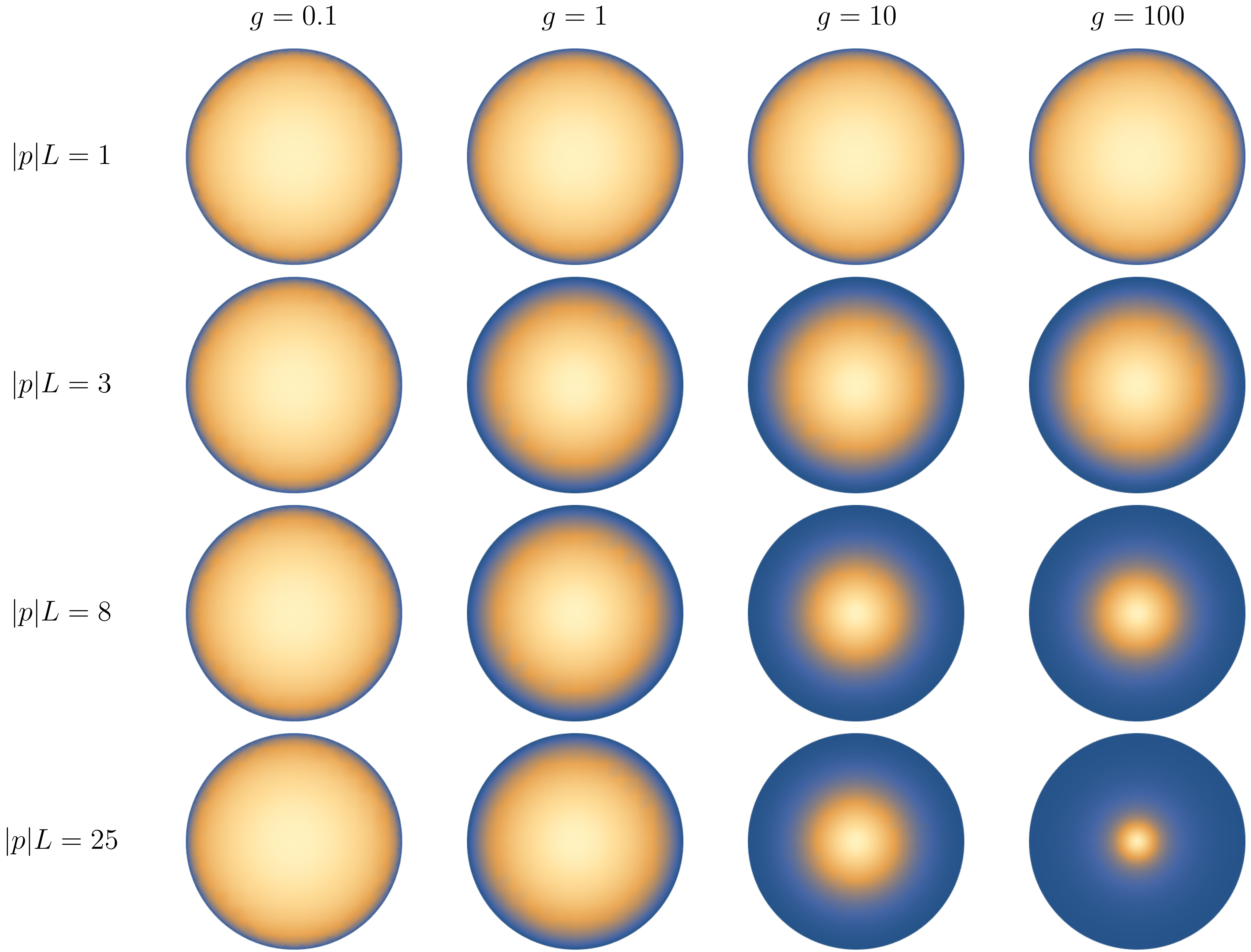
planar
N=4 SYM

weak coupling

Stronger coupling

$g = \frac{\sqrt{\lambda}}{4\pi}$

Finer optics



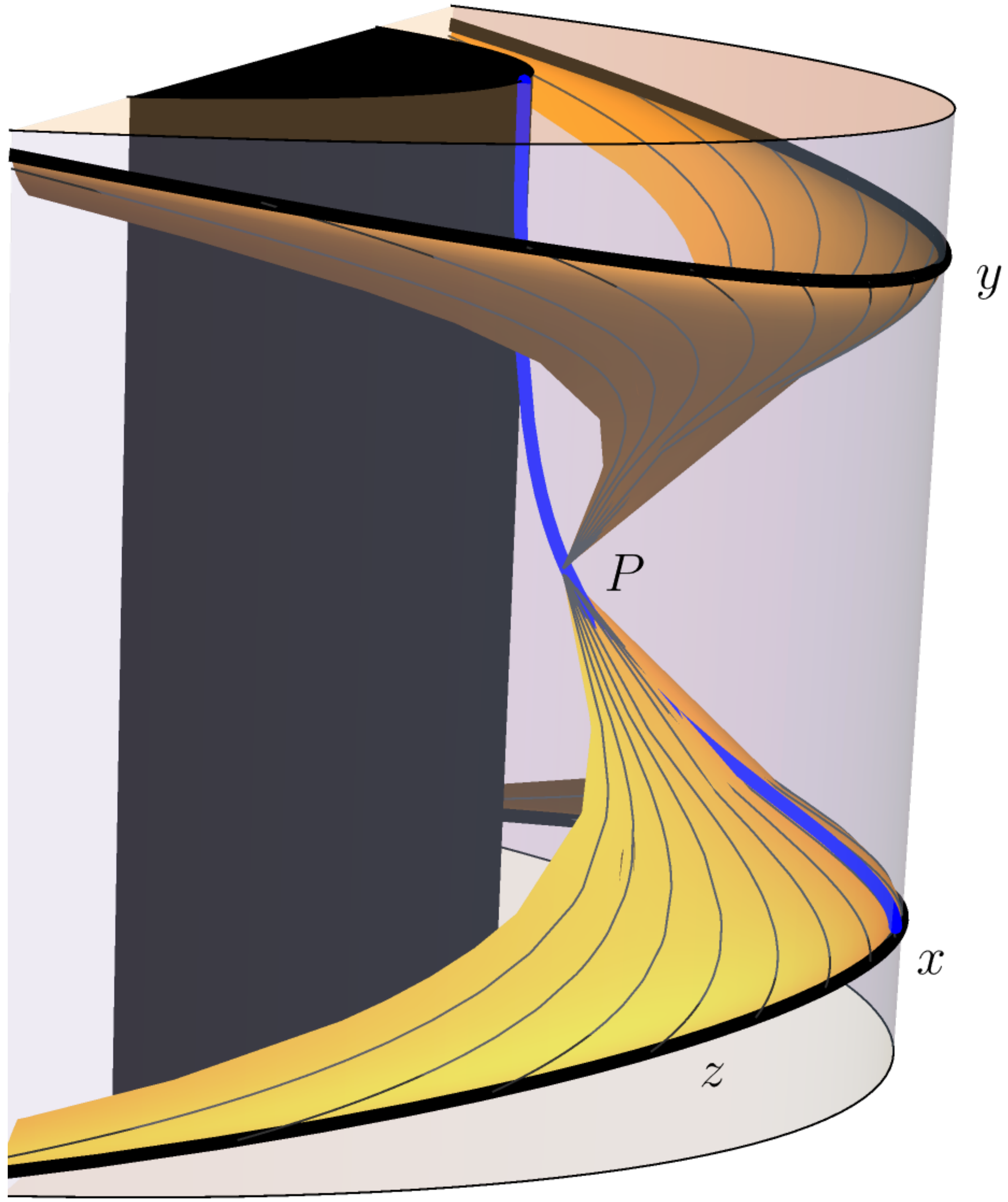
Summary

- Four-point correlators $\leftrightarrow$ *local* bulk metric.
- Works in arbitrary states; images only sharp in holographic states & theories.

Many open questions…

- Other bulk fields?
- More interesting/extreme geometries: when/how does it break?
- Bootstrap:
  - derive finite-T metric from consistency of $\langle TTTT \rangle_\beta$?
  - what to do at large*ish* N, λ ?
- Flat space version: image asymptotically flat geometries?
- Add 'modular' lenses to image behind causal horizons?

Geometry of radar camera



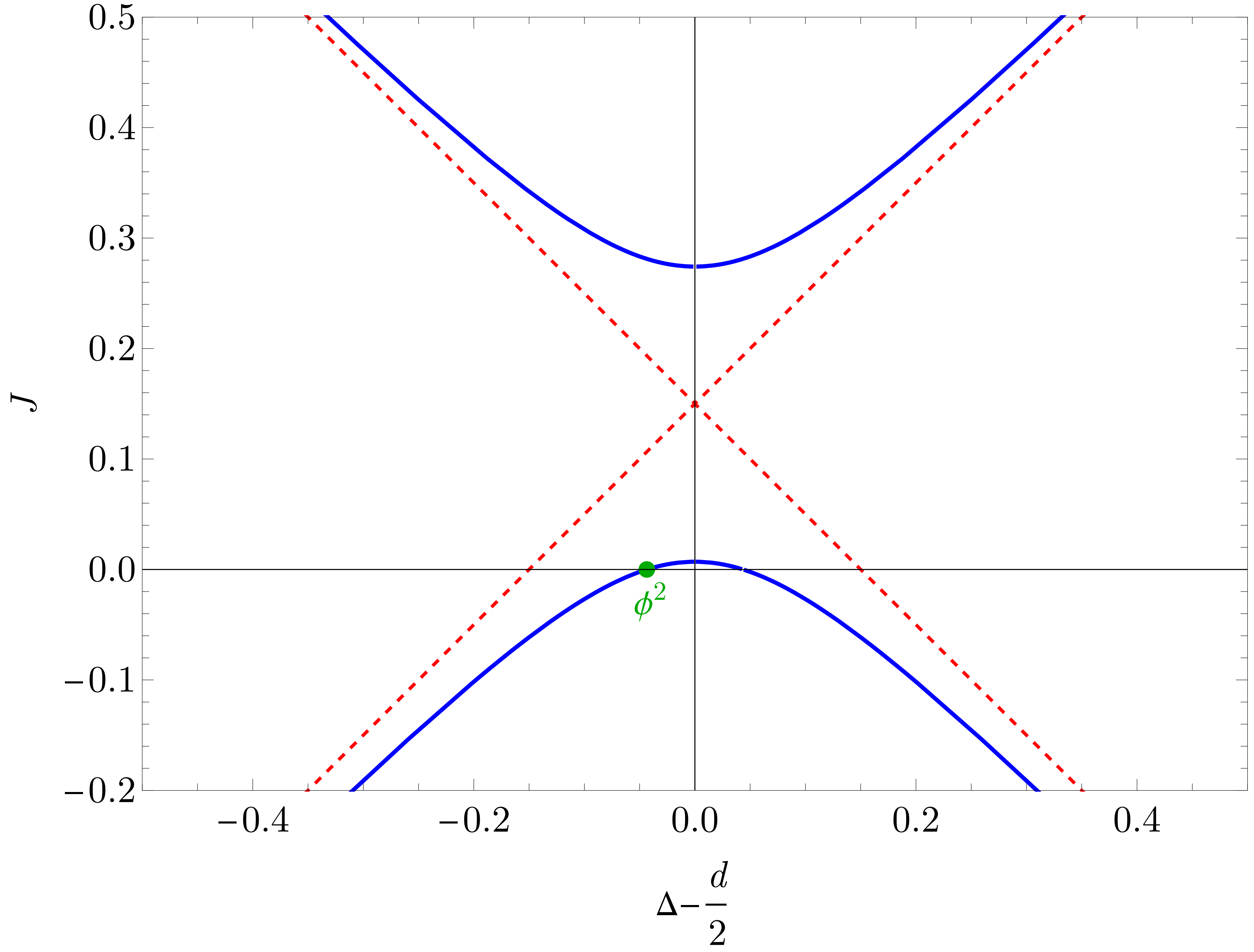
a bulk point P can be probed using:
-Any point y on its future lightcone
-Any points x,z on its past lightcone

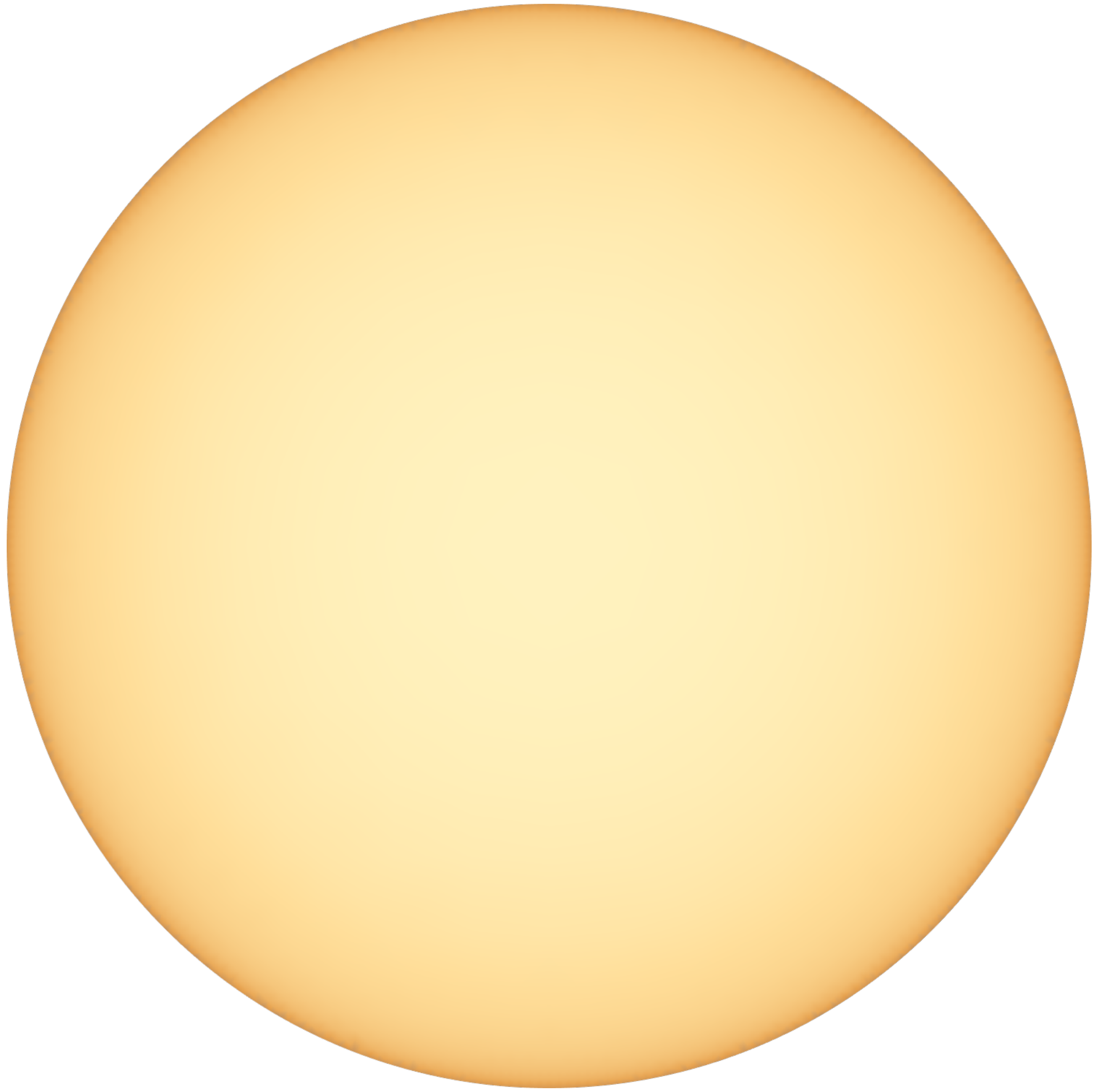
use parallax from x,y *or* z to get depth.

Particle sent from x isn't recovered:
easy to access near-horizon region.

lightcone cuts: [Engelhardt& Horowitz '16]

critical Wilson-Fisher theory / 3D Ising





leading Regge trajectory too steep to access to high momenta.
No hint of local bulk physics.

Today we'll treat bulk fields as interacting primarily via gravity.

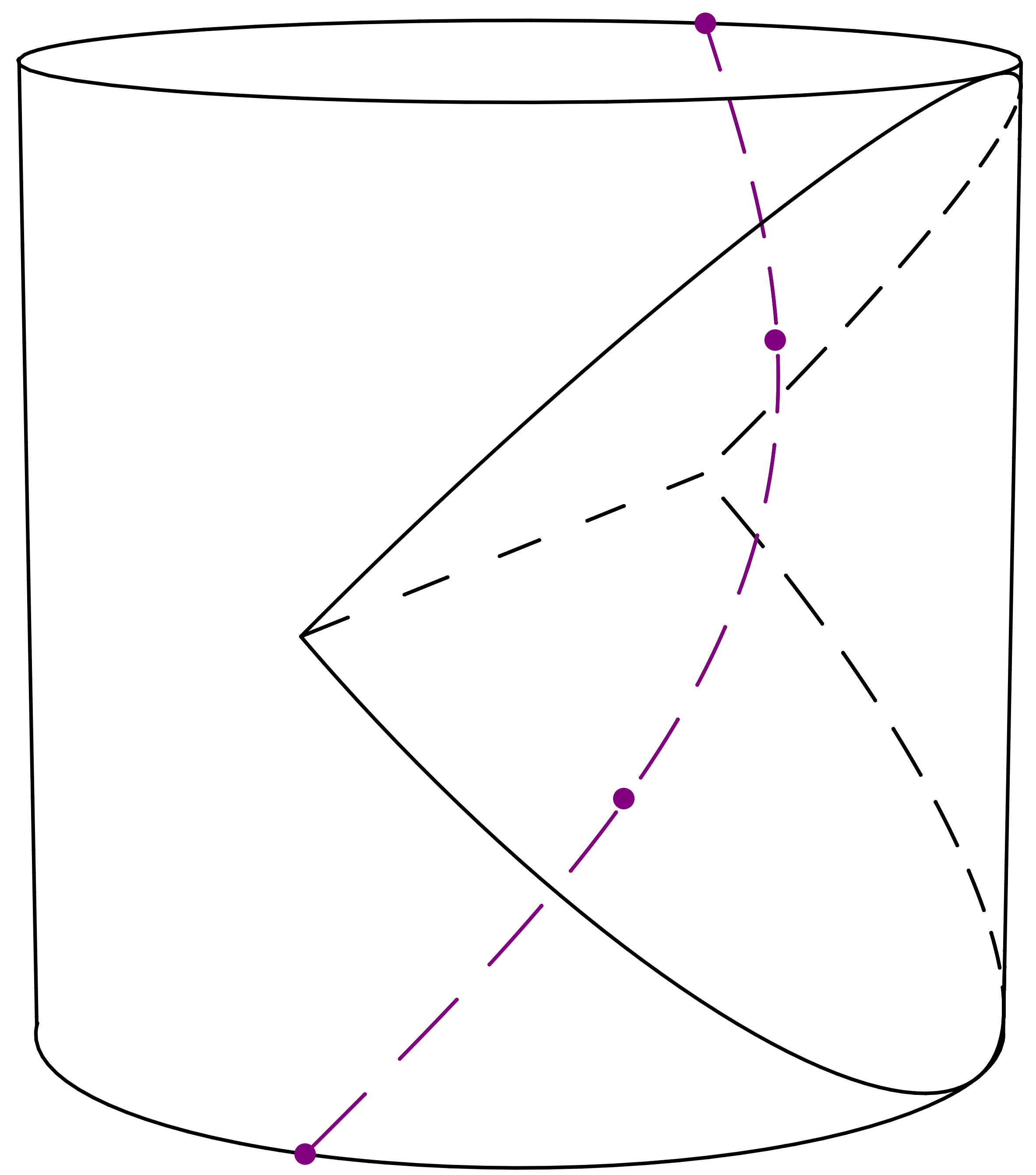
This is certainly true for gravitons, but not necessarily for other fields.

[Apolo, Belin, Castro& Keller '22]

‘Linear inverses' need exponential accuracy to detect local features:

$$K_{\mathrm{HKLL}}(x, r\,|\,y) \propto \int d\nu \; K_\nu(x, r\,|\,y)$$

AdS momentum $\qquad \sim e^{+\pi\nu}$



physical reason: one-particle states can have exponential small wavefunction near boundary.

[HKLL ’06: AdS-Rindler kernel]

[Bousso, Freivogel, Leichenauer, Rosenhaus, Zukowski ’12]

Conformal Regge theory

J=2 for graviton

$$\frac{\delta x \cdot I \cdot \delta y}{|\delta x||\delta y|} \sim b_{\text{bulk}}$$

$$\mathcal{G}(z,\bar{z})^{\circlearrowleft} - 1 \approx 2\pi i \int_0^\infty \frac{d\nu}{2\pi} \rho(\nu)\alpha(\nu)\ (z\bar{z})^{\frac{1-J(\nu)}{2}} \mathcal{P}_{\frac{2-d}{2}+i\nu}\left(\frac{z+\bar{z}}{2\sqrt{z\bar{z}}}\right)$$

Crucial fact: $\alpha \lesssim e^{-\pi|\nu|}$. (Known in examples.)
Integral commutes with Fourier transform.

$$G(0,p,L;e,p',L')\big|_{\text{CRT}} \approx 1 - i\int_0^\infty \frac{d\nu}{2\pi}\rho(\nu)\tilde{\alpha}(\nu)\ (|p||p'|)^{J(\nu)-1}\,\mathcal{P}_{\frac{d-2}{2}+i\nu}(\hat{p}\cdot\mathcal{I}_e\cdot\hat{p}')e^{-\frac{\nu^2}{2}\sigma_{\text{eff}}^2}$$

$$\tilde{\alpha} = \frac{4\pi G_N^{(d+1)}}{\nu^2 + \frac{d^2}{4}}$$

~flat space amplitude.
easy to add stringy effects.

$$\sigma_{\text{eff}} \sim \frac{1}{|p|L}$$

optical limit on angular resolution