Holographic tensor networks in 2d CFT

arXiv:2206.03414 and to appear with Jeevan Chandra

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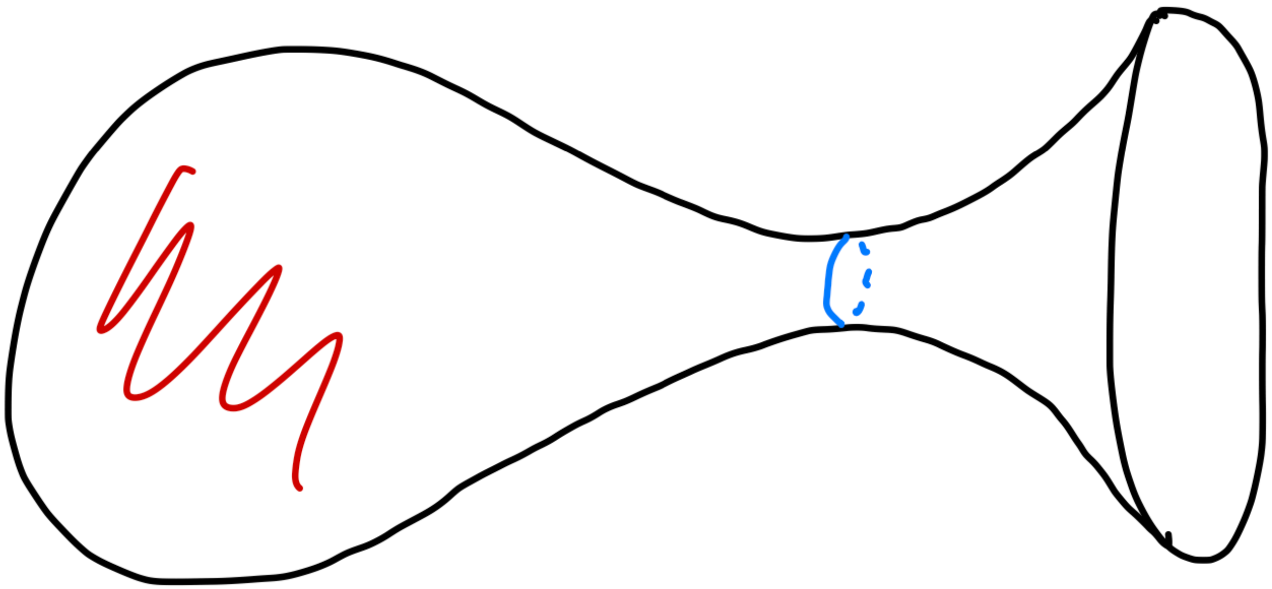
Gravity from Algebra **✦ KITP ✦ January 23, 2023**

Pure-state black hole



$$\rho = |\Psi\rangle\langle\Psi|$$

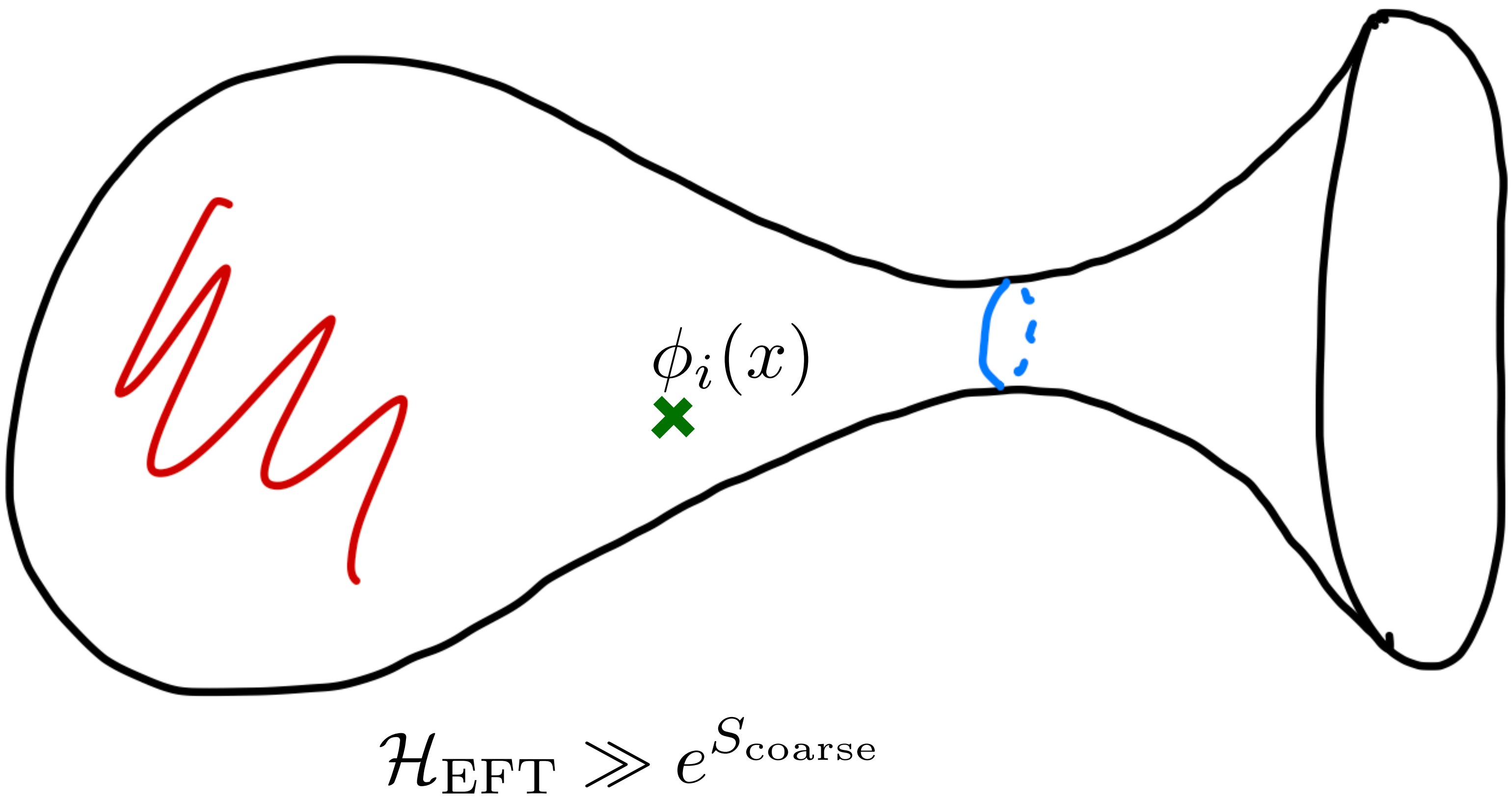
Expected properties:

1. Coarse-grained entropy $$\text{“}S_{\text{coarse}}\text{”} = \frac{1}{4G}\text{area}(\gamma)$$

2. Isometric transition $$V : \mathcal{H}_{\text{EFT}} \to \mathcal{H}_{\text{CFT}}$$

A map is called *isometric* when $V^{\dagger}V = \mathbb{1}$

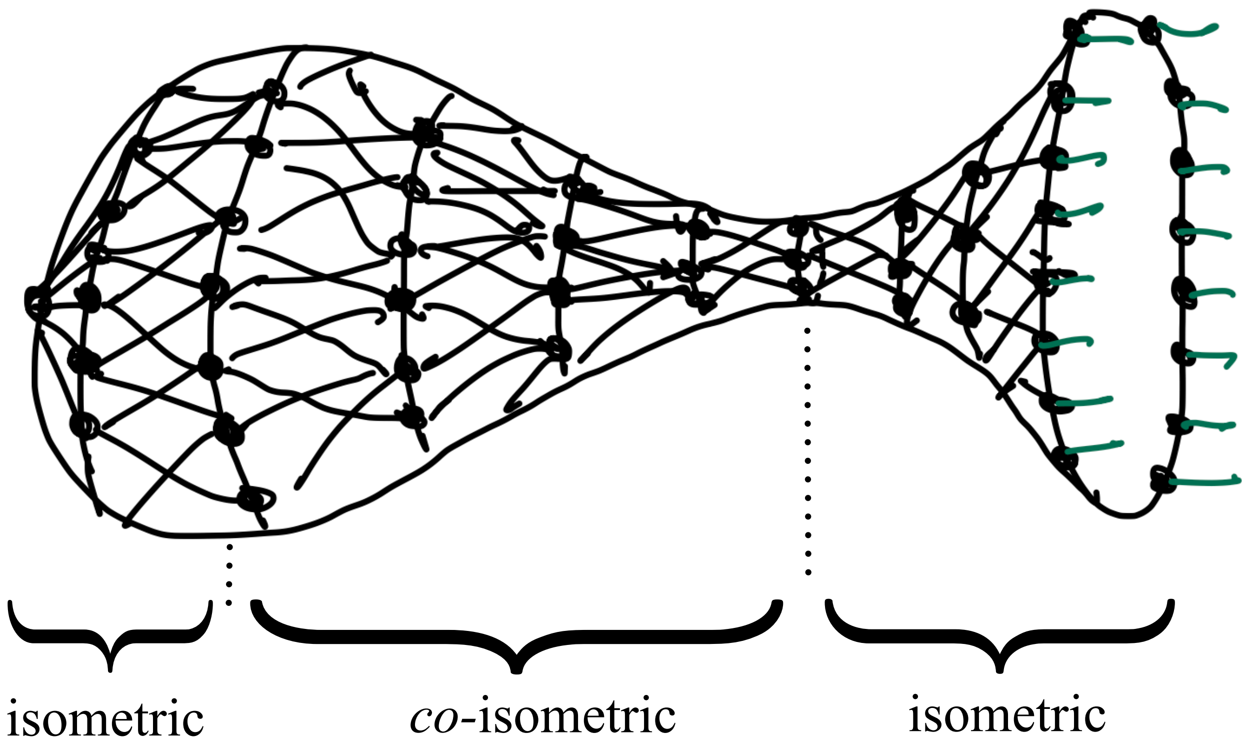
The holographic map is *isometric* outside the horizon, and *non-isometric* inside.



Geometry as a random tensor network

[Swingle ’12]
[Hayden et al 2016]

$|\Psi\rangle \sim$



?

A random map $V : \mathcal{H}_{\mathrm{small}} \to \mathcal{H}_{\mathrm{big}}$ is approximately isometric.

Goal

Explain these features of pure-state black holes from the dual CFT (mostly in 2d).

Outline

I. CFT states as random tensor networks

II. Isometric transition

Basic point:

Subtle breakdown of the identity block approximation in the black hole interior.

III. Comments and conclusions

Bulk theory: 3d gravity + massive particles

Conjecture: Dual to an average over 2d CFTs with

1. Cardy density of states

2. 3-point coefficients satisfying the eigenstate thermalization hypothesis (ETH) in the form:

$$c_{ijk} = \langle i|O_j|k\rangle$$
$$= T_{ijk}|C_0(h_i, h_j, h_k)|$$

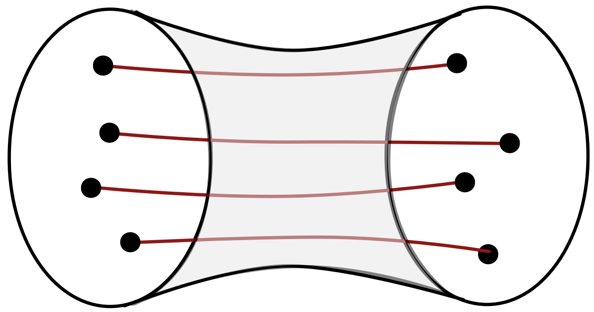
known; from the Virasoro identity block [Collier, Maloney, Maxfield, Tsiares '19]

random with zero mean and unit variance

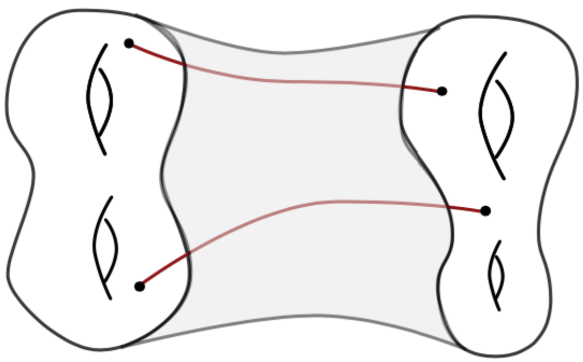
[Chandra, Collier, TH, Maloney '22]

building on [Saad, Shenker, Stanford '19], [Saad '19], [Cotler, Jensen '20], [Belin, de Boer '20]

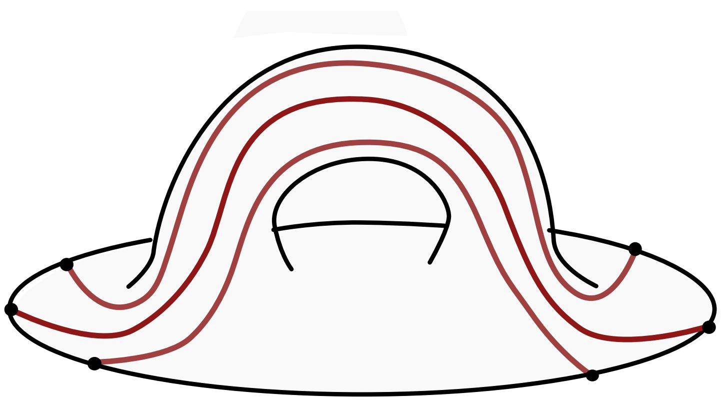
Evidence



$$= \overline{G_4 G_4}$$



$$= \overline{Z Z'}$$



$$\supset \overline{G_6}$$

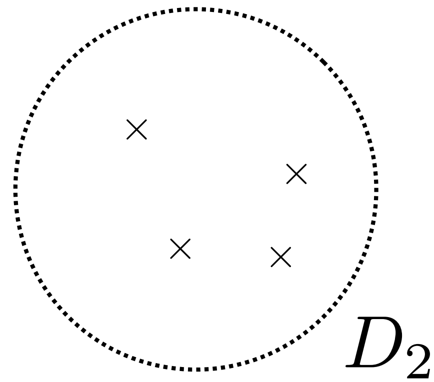
etc.

Consider the CFT state

$$\Psi = A_i(x_1)A_j(x_2)\ldots|0\rangle$$

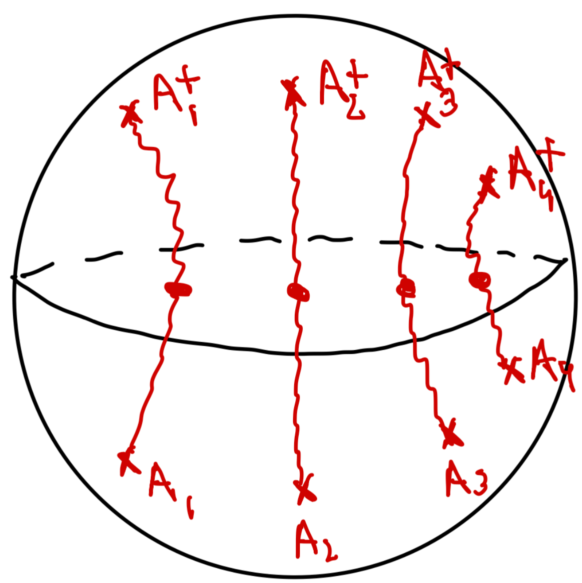
$$h < \frac{c}{24}$$

$$V : \{i, j, \ldots\} \to \mathcal{H}_{\mathrm{CFT}}$$



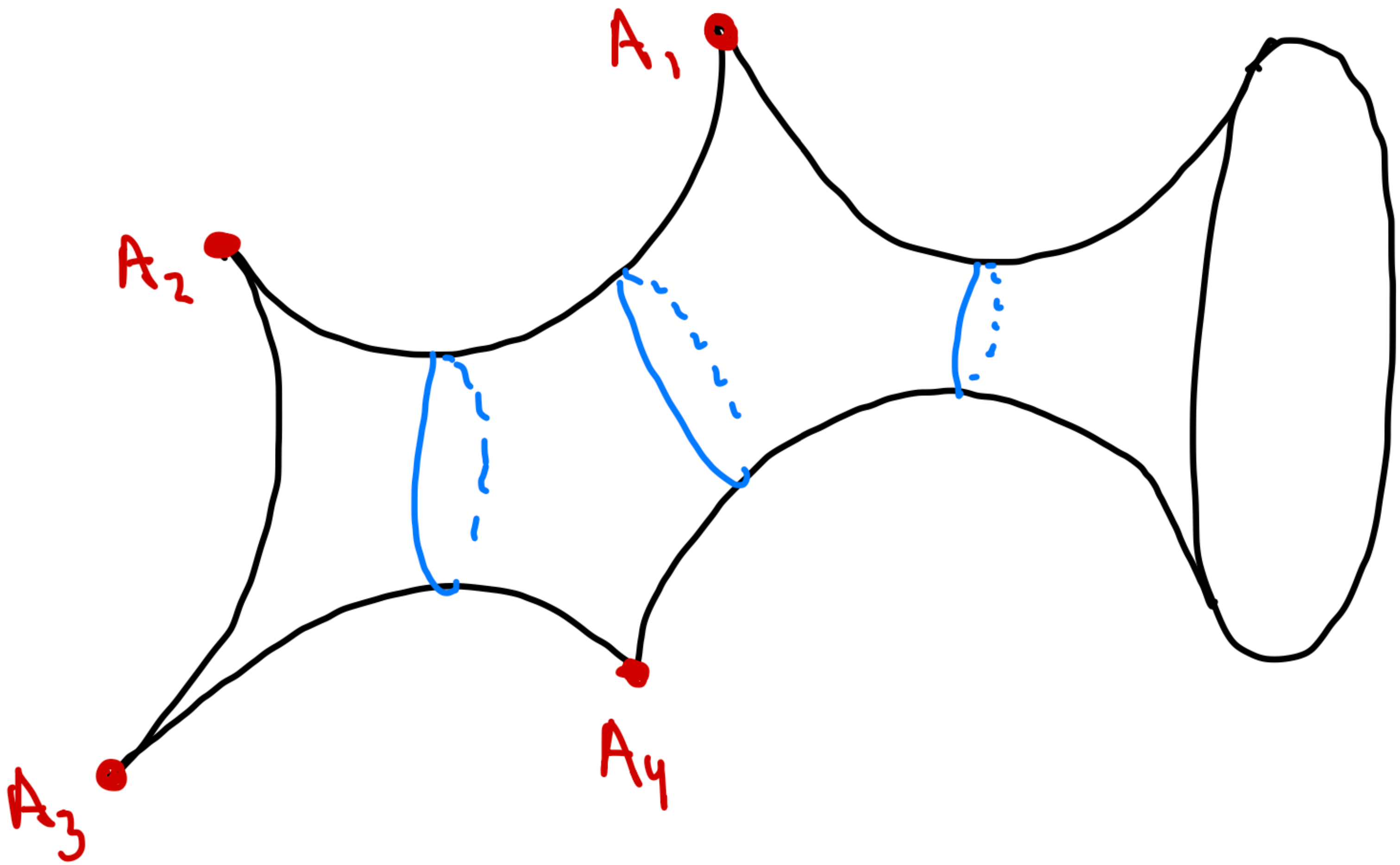
Bulk dual:

$$\langle\Psi|\Psi\rangle =$$



$$ds^2 = d\rho^2 + \cosh^2\rho\, d\Sigma^2_{\mathrm{hyp}}$$

Spatial geometry



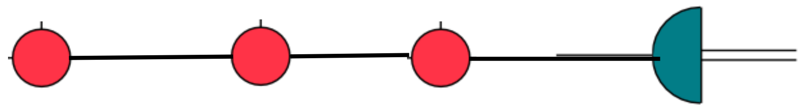
We will rewrite this CFT state as a random tensor network.

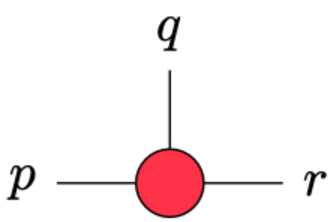
$$|\Psi\rangle = A_4 A_3 A_2 A_1 |0\rangle$$

$$= \sum_{\substack{p,q,r \\ \text{primaries}}} c_{12p} c_{3pq} c_{4qr} \left|\mathcal{B}\right|^2 |r\rangle$$

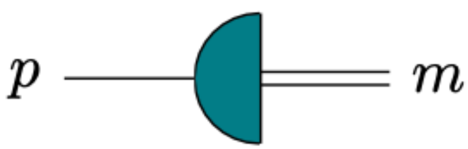
Virasoro OPE block

Now use: Large-c + Saddlepoint truncation + ETH

$|\Psi\rangle_* \approx$ 



$= T_{pqr}$

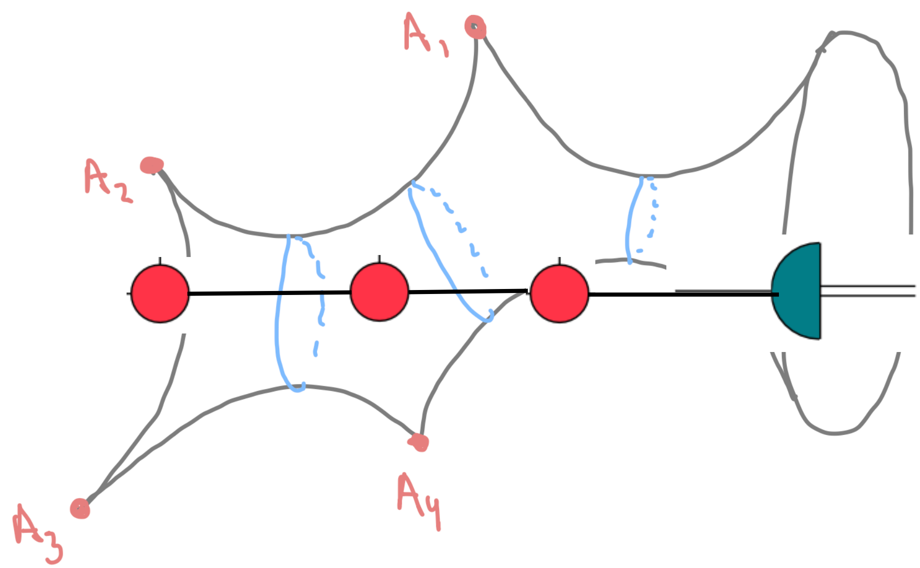


$= \text{ OPE block} \times (C_0 \text{ factors})$

Single lines = $\mathcal{H}_{\text{primaries}}$

Double lines = $\mathcal{H}_{\text{CFT}}$

Claim: This tensor network discretizes the radial direction in the bulk.



I.e., it has the following properties:

1. $${}_*\langle\Psi|\Psi\rangle_* = e^{-S_{\text{bulk}}}$$

2. $$\text{bond dimensions} = \exp\left(\text{Area}/4\right)$$

3. Probes undergo isometric transition at the horizon

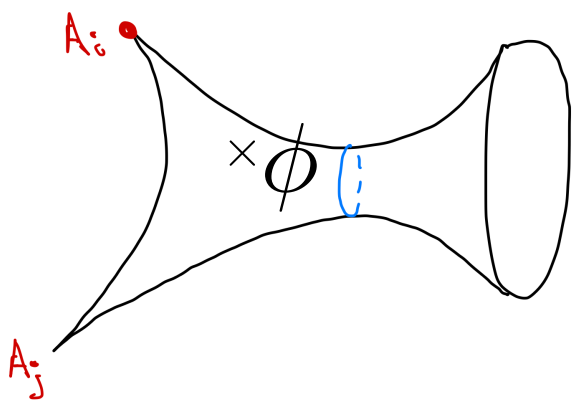
The isometric transition

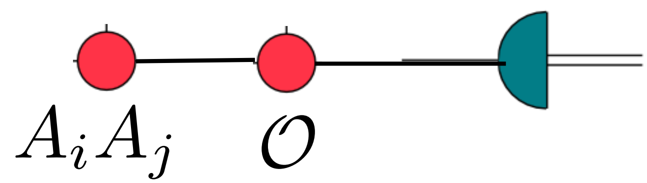
The tensor network realizes/extends a proposal made in

[Verlinde, Verlinde '12]
[Goel, Lam, Turiaci, H. Verlinde '18]
[H. Verlinde et. al, talk at Banff '21]

Take one operator to be a light probe:

$$|\Psi\rangle = \mathcal{O}(x)A_i(x_1)A_j(x_2)|0\rangle$$





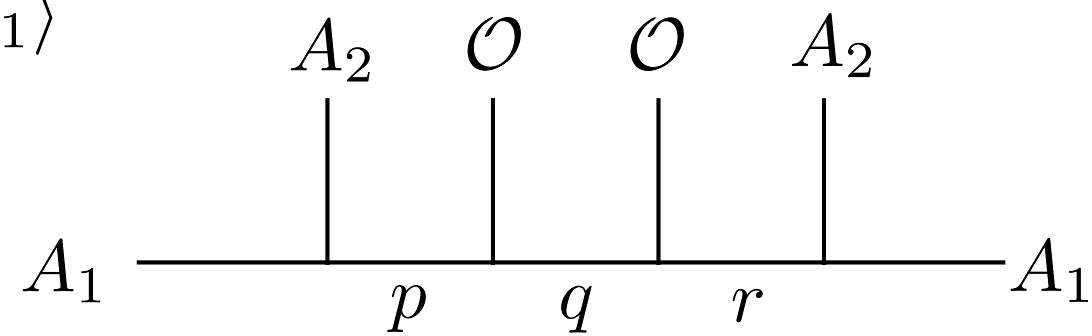
In a random tensor network, the isometric/non-isometric behavior of the code is determined entirely by the dimensions of the bonds.

[Hayden et al., '16]

Therefore the CFT makes the following prediction:

$$\langle A_1^\dagger A_2^\dagger \mathcal{O}^\dagger \mathcal{O} A_2 A_1 \rangle$$

$$= \sum_{p,q,r}$$



The holographic map is isometric iff, at the saddlepoint,

$$S(E_p) \leq S(E_q)$$

Crucially: *Primary* energies, not total

If the map is non-isometric then the Virasoro identity approximation

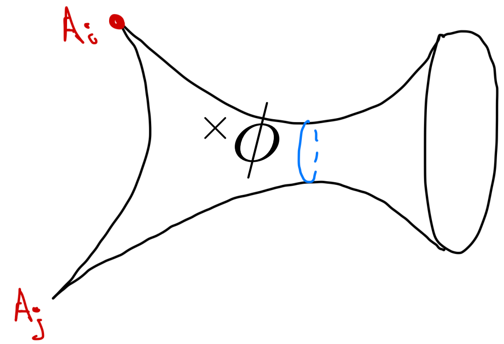
$$\mathcal{O}^\dagger \mathcal{O} \approx \mathbb{1}_{\mathrm{Vir}}$$

must break down.

But this breakdown can be detected only in complicated superpositions.

The nontrivial test

We must show that the transition occurs exactly when the particle crosses the apparent horizon.



Sketch of the calculation

Bulk geometry

==> $T(z)$

==> Fuchsian equation; monodromies $M_i \in PSL(2,\mathbb{R})$

==> Saddlepoint weights in the conformal block

It works.

Conclude

If the dual particle is inside the horizon, then the code is non-isometric.

Aside

In the analytic bootstrap, we often consider limits where

$$\mathcal{O}^\dagger \mathcal{O} \approx \mathbb{1}$$

Are there other situations, besides black hole interiors, where there is an entropic reason this cannot hold inside superpositions?

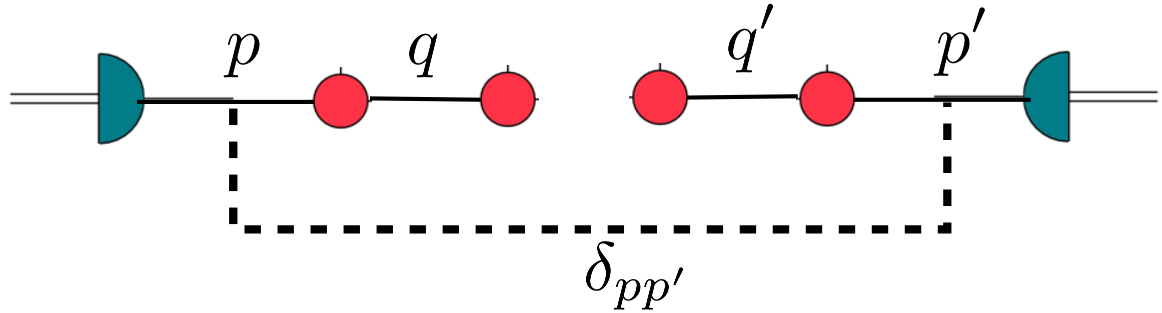
Do non-holographic CFTs have an isometric transition?

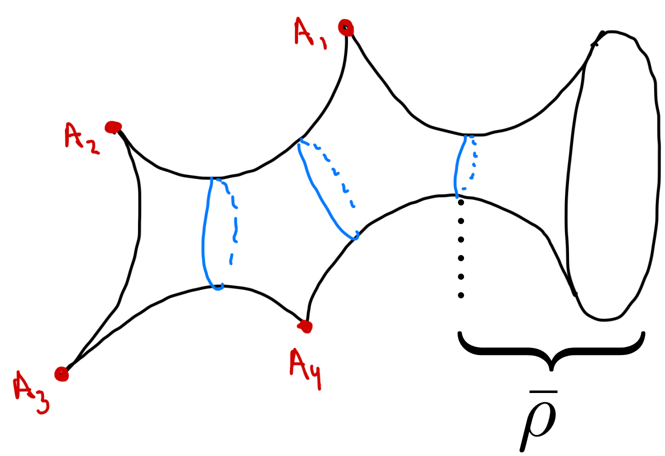
Comments and Conclusion

Wormholes

Coarse-grained states

$\bar{\rho} =$

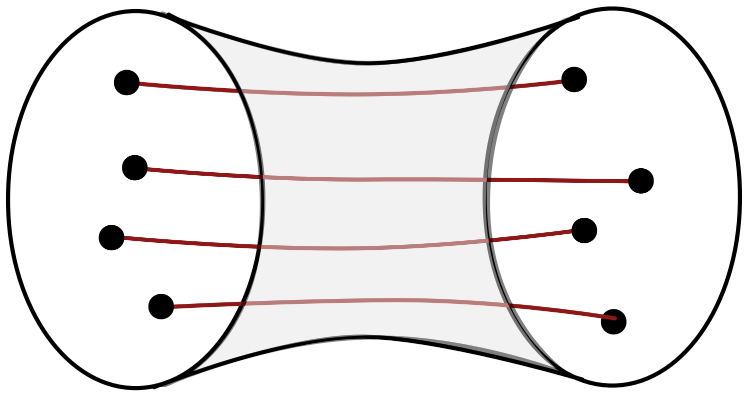




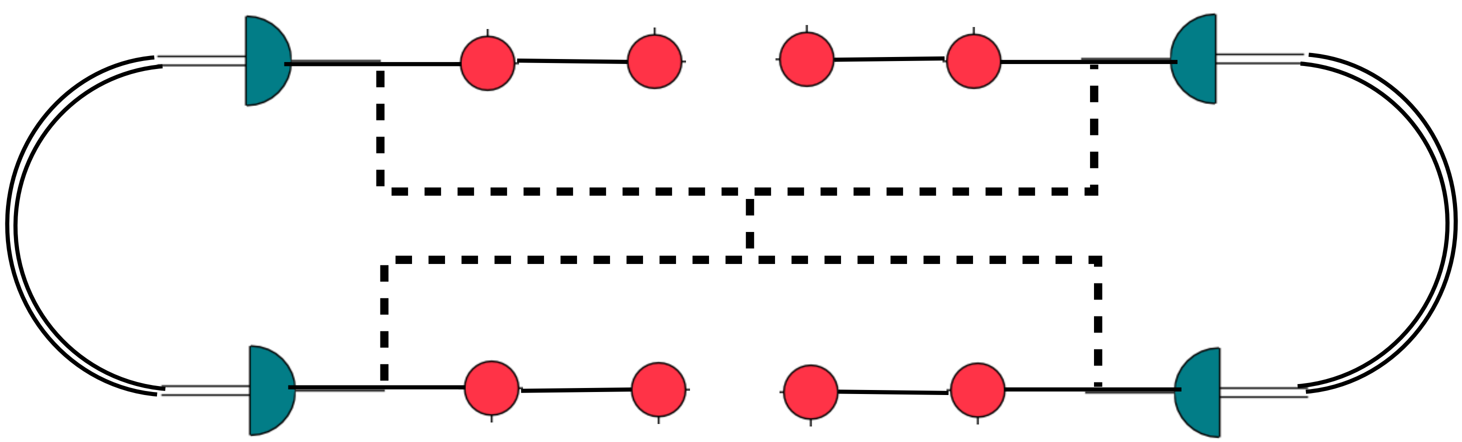
Wormholes

Come from gluing together multiple networks in a similar way.

$$\overline{G_4 G_4} =$$



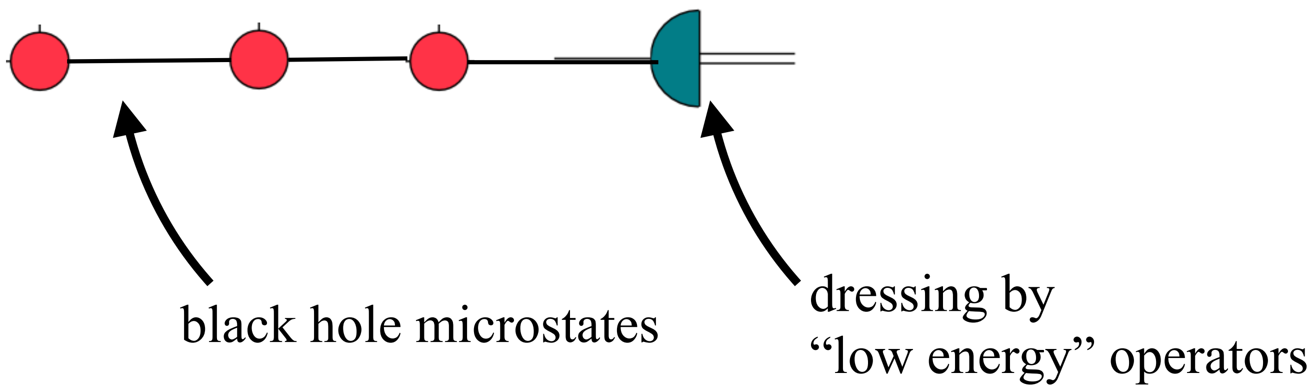
$$= \operatorname{tr} \bar{\rho}^2$$



(*caveat: fixed-area)

Comments on higher dimensions

What should the tensor network look like in higher dimensions?



Similarly, to describe higher-dimensional gravity as an ensemble average, it seems necessary to first "factor out" the low energy theory.

The low energy theory is not random; only the UV is averaged.

the end.