Extreme AdS(/CFT?)

Thomas Van Riet, Leuven University, Belgium.

Based on

Arxiv: 2207.13692 with **S. Aguilar-Gutierrez** (Leuven)**, K. Parmentier** (Columbia, NY).

Arxiv: 2212.06169 with **G. Shiu, F. Tonioni** (UW Madison), **V. Van Hemelryck** (Leuven)

And older work.

Questions

Vanilla top-down (understood) AdS/CFT pairs seem to feature

$$AdS_d \times X_n$$

With d>2 and X a compact 11-d or 10-d dimensional space with same size radius as AdS.

The extremes:

1. **Can we have d=1? What does that even mean?**

2. **Can we make X small as we want in AdS units? If so, what is the dual CFT?**

"AdS1/CFT0?" A supergravity view

- Prominent top-down AdS/CFT dualities: IIB on AdS_5 x S^5 vs N=4 SYM , 11d supergravity on AdS_4 x S^7 vs ABJM, on $AdS_7 x S^4$ vs (2,0) CFT in 6d + variations on this with less SUSY.

- These are all the “conformal brane examples”: D3, M2, M5. All give Freund-Rubin vacua as near horizons. What about the other D-branes? Singular near horizons → Non conformal holography? The dual field theories are 10d SYM compactified on n-torus. Holography less understood.

- **Extreme** examples of **non-conformal** “holography” are the matrix descriptions of M-theory and IIB strings:

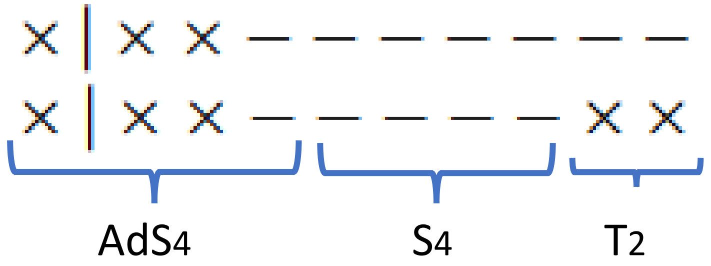
  BFSS matrix quantum mechanics ‘near horizon geometry of D0s’.
  IKKT matrix integrals (0d field theory) as ‘near horizon geometry of D(-1)s’.

(High T regime of BFSS = IKKT)

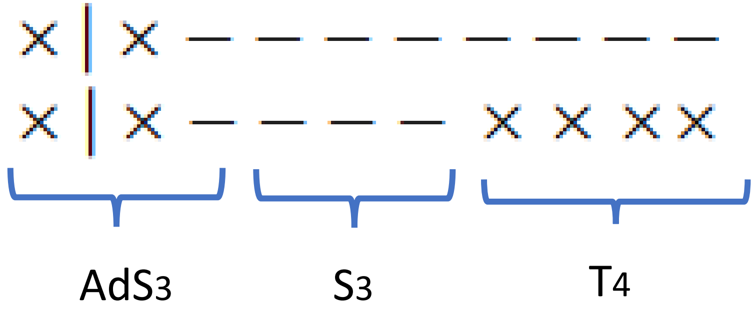
Non-conformal branes: their AdS vacua and dual CFTs?

The near horizon of all Dp branes with p different from 3 is singular. But, we can get AdS from bound states of branes:

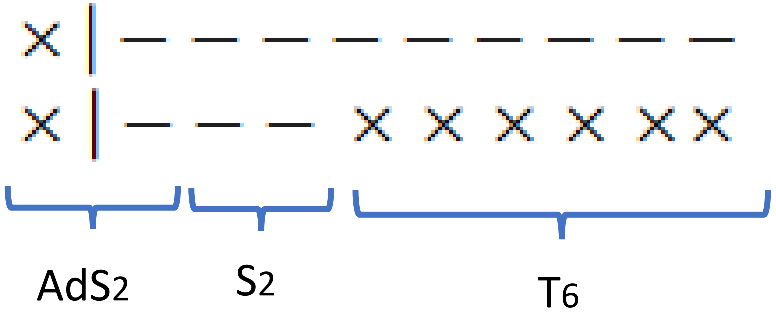
- $AdS_4 \times S^4 \times T^2$ from D2-D4:



- $AdS_3 \times S^3 \times T^4$ from D1-D5:



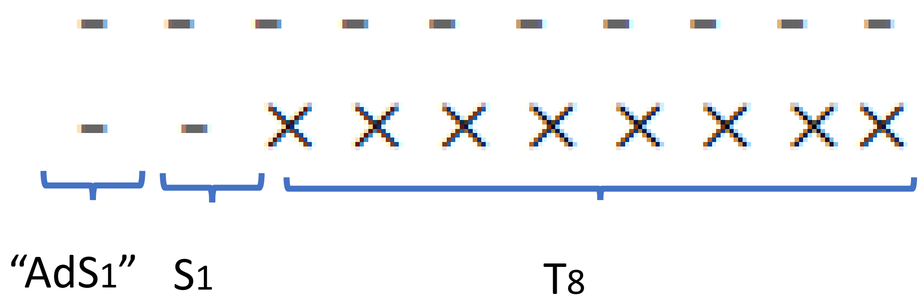
- $AdS_2 \times S^2 \times T^6$ from D0-D6:



→ Each time we see a brane and its magnetic dual. Clearly AdS_5 x S^5 is the top of the example since the magnetic dual to a D3 is a D3. So we find a chain AdS_d x S_d x $T^{(10-2d)}$ from D(d-2)-D(8-d) bound states. For **odd** d these are **SUSY** bound states and have known CFT duals.

→ The natural extension of this chain suggests a last possible entry:

- AdS_1 x S^1 x T^8 from D(-1)-D7:



Supergravity solutions for brane bound states follow Tseytlin's 'harmonic product rule' [Tseytlin 1996, Bergshoeff et al 1996] . Eg D1-D5 solution:

$$ds_{10}^2 = H_1^{-3/4} H_5^{-1/4} \left(-dt^2 + dx_1^2\right) + H_1^{1/4} H_5^{-1/4} \left(dx_2^2 + \ldots + dx_5^2\right) + H_1^{1/4} H_5^{3/4} \left(dr^2 + r^2 d\Omega_3^2\right) ;$$

D1 worldvolume

D5 worldvolume

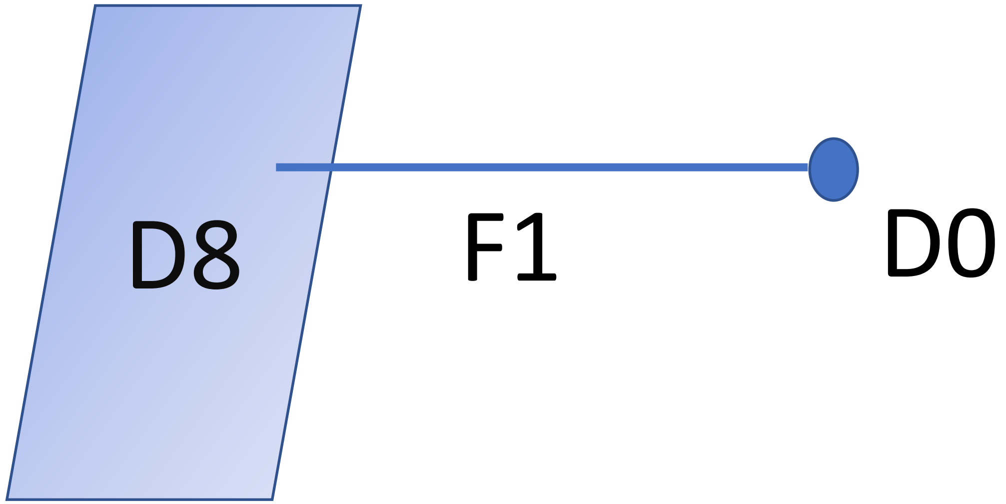
The near horizon gives $AdS_3 \times S^3 \times R^4$ (or T^4)

→ But D0-D8 with this harmonic product rule does **not solve the EOM**. *Similarly, D(-1)/D7 with harmonic product rule does not (T-duality).*

Why?

This is a version of the Hanany-Witten [1996] (or Freed-Witten) effect. A D0-D8 bound state necessarily has a string in between them [Bergshoeff et al 2003, Massar et al 1999]

	0	1	2	3	4	5	6	7	8	9
D0 :	×	–	–	–	–	–	–	–	–	–
D8 :	×	×	×	×	×	×	×	×	×	–
F1 :	×	–	–	–	–	–	–	–	–	×



Unfortunately, formal T duality of the D0-F1-D8 solution does not help in finding D(-1)/D7

→ Instead the 'branes within branes' idea [Douglas] does seem to help here. Example:

D(-1)/ D3 boundstates can be understood as gauge instantons inside D3 worldvolume.

We apply this to our sought for D(-1)/D7 bound state. We start with a class of gauge instantons in 8d YM:

$$\star(F \wedge F) = F \wedge F\,, \qquad k \sim \mathrm{Tr}(F \wedge F \wedge F \wedge F)\,.$$

The 8D action, in Einstein frame, contains:

$$S \supset \int \mathrm{d}x^8 \sqrt{g}\left(g_s N + \mathrm{Tr}(F^2) + \frac{1}{g_s}\mathrm{Tr}(t_8 F^4)\right).$$

The first term is the D7 tension, the second looks like a **D3 brane tension** and the last is the sought for D(-1) tension.

The WZ term contains a term that acts like a D(-1) brane, but also **a D3 brane**!

$$- iC_0 \int \mathrm{Tr}(F \wedge F \wedge F \wedge F \quad + \int C_4 \wedge Tr\,(F \wedge F)$$

This is our “Hanany- Witten” effect. It inspired the following Ansatz:

$$\mathrm{d}s^2 = L_x^2\mathrm{d}x^2 + L_y^2\mathrm{d}y^2 + L_1^2(\mathrm{d}\theta^1)^2 + \ldots + L_8^2(\mathrm{d}\theta^8)^2\,,$$
$$F_1 = \alpha\mathrm{d}x + \mathrm{i}\beta\mathrm{d}y\,,$$
$$F_5 = (1-\mathrm{i}\star)\mathrm{d}\theta^1\wedge\mathrm{d}\theta^2\wedge\mathrm{d}\theta^3\wedge\mathrm{d}\theta^4\wedge(\gamma_1\mathrm{d}x+\delta_1\mathrm{d}y)\,,$$

The F_5 flux Ansatz reflects a SUSY intersection of D3 branes on the T_8

We have a solution if

$$\alpha L_y = \pm\beta L_x\,,$$
$$L_x\delta_1 = \pm\mathrm{i}L_y\gamma_1\,,$$
$$L_{1234}\alpha = \pm\mathrm{i}\gamma_1\,,$$

- Indeed ISD F_1 flux.
- Note that there is a notion of a 'cosmological constant'. Here is the size of the S^1 in the $AdS_1 \times S^1$ factor and it is set by the amount of F_1 flux. In **standard** holography the size of X in AdS x X is the size of AdS. **See second part of talk.**

Supersymmetry in Euclidean IIB?

Tricky [Bergshoeff & Van Proeyen 2000]. Note that $*F_5 = iF_5$. So we get an ambiguity depending on the Hodge duality frame:

$$\text{Option I}: \quad \delta^E_\epsilon \psi_\mu = D_\mu \epsilon + \tfrac{\mathrm{i}}{8} \mathrm{e}^\phi \left(\Gamma^\nu F_\nu + \tfrac{1}{2\cdot 5!} \Gamma^{\nu_1 \dots \nu_5} F_{\nu_1 \dots \nu_5} \right) \Gamma_\mu \epsilon,$$

$$\text{Option II}: \quad \delta^E_\epsilon \psi_\mu = D_\mu \epsilon + \tfrac{\mathrm{i}}{8} \mathrm{e}^\phi \left(\Gamma^\nu F_\nu + \tfrac{\mathrm{i}}{2\cdot 5!} \Gamma^{\nu_1 \dots \nu_5} F_{\nu_1 \dots \nu_5} \right) \Gamma_\mu \epsilon\,.$$

Option II : ¼ BPS with constant Killing spinor. Globally well defined!

→ This needs to be better understood.

What is the actual matrix model? → It was probably constructed in arxiv 2101.01732 :

On the D(–1)/D7-brane systems

M. Billò [a], M. Frau [a], F. Fucito [b], L. Gallot [c], A. Lerda [d] and J.F. Morales [b]

$$S_G = \mathrm{tr}\Big\{\frac{1}{4}D^{IJ}D^{KL}\,\epsilon_{IJKL} + \frac{1}{2}D^2 + \frac{1}{2}[\chi,\xi]^2 + \frac{1}{2}\lambda[\chi,\lambda] + \frac{1}{2}\eta[\chi,\eta] + \frac{1}{4}\lambda^{IJ}[\chi,\lambda^{KL}]\,\epsilon_{IJKL}\Big\}\,,$$

$$S_K = \mathrm{tr}\Big\{-[\xi,B^I][\chi,\overline{B}_I] - [\chi,B^I][\xi,\overline{B}_I] - 2\,M^I[\xi,\overline{M}_I] - 2\,\bar\mu\,\xi\,\mu + \overline{w}\,\xi\,\chi\,w + \overline{w}\,\chi\,\xi\,w + \bar\mu'\,\chi\,\mu' + \bar h'\,h'\Big\}\,,$$

$$S_D = \mathrm{tr}\Big\{-\mathrm{i}\,D\big([B^I,\overline{B}_I] + w\,\overline{w}\big) - D^{IJ}\Big([\overline{B}_I,\overline{B}_J] + \frac{1}{2}\epsilon_{IJKL}\,[B^K,B^L]\Big) + \lambda^{IJ}\big([\overline{B}_I,\overline{M}_J] - [\overline{B}_J,\overline{M}_I] + \epsilon_{IJKL}\,[B^K,M^L]\big) + (\eta+\mathrm{i}\,\lambda)\big([\overline{B}_I,M^I] + w\,\bar\mu\big) + (\eta-\mathrm{i}\,\lambda)\big([B^I,\overline{M}_I] - \mu\,\overline{w}\big)\Big\}\,.$$

It is IKKT plus extra terms from D(-1)/D7 strings. The action for the instanton moduli in 8D SYM.

Conformal matrix theory?

Consider the following schematic form of a matrix integral

$$Z[\lambda_i] = \int \mathrm{d}M \, \mathrm{e}^{-V(M,\lambda_i)} \, .$$

A possible definition of conformal invariance is invariance with respect to rescaling of the M. The conformal group in D dimensions is SO(D+1,1) so we expect SO(1,1) from the rescalings = rescaling freedom in the "AdS"$_1$ coordinate.

If so, then one expects: $Z[\lambda_i] = \mathrm{const} = 1.$

The independence of Z with respect to the marginal couplings is inspired by independence of the central charge on marginal couplings for CFTs in D>1 .

This in turn leads to vanishing free energy $F = \log Z[\lambda_i] = 0.$

- Holography would then imply vanishing on-shell bulk action. Yes! (computation is short but subtle.)

- Billo et al also mention a version of Hanany-Witten, which is the statement that some of the D7 branes need extra worldvolume flux, beyond the gauge instantons. There is some correspondence with D3 fluxes.

This is clearly only the starting point. We need to develop the holographic dictionary and, in particular, compute Z and verify it is constant. Excellent matrix model problem!

The other extreme: scale separation

$L_{AdS} >> L_{KK}$

Holography? :

- Dual CFTs have only few low lying single trace scalar operators, then a parametric gap!

$$mL_{AdS} = \kappa >> 1 \qquad \Delta = \frac{3}{2} + \frac{1}{2}\sqrt{9 + 4\kappa^2} \gg 1$$

- Even more special: scale separated AdS vacua suited for uplifting have no tachyons, so no relevant deformations: **Dead-end CFTs with huge gap.** This gets close to understanding whether pure AdS gravity has a dual?

Bizar CFTs (See [Polchinski&Silverstein 2009, Alday&Perlmutter 2019]).
But, there are **nogos** and **conjectures** against scale separation **on the gravity side**.

Nogos

Example for 11d compactifications.

Assume no warping for simplicity, then one easily finds;

$$R_4 = -\frac{4}{3}|F_4|^2 - \frac{8}{3}|F_7|^2 \ ,$$
$$R_7 = \frac{5}{3}|F_4|^2 + \frac{7}{3}|F_7|^2 \ .$$

We recognise that $R_4 < 0$ as we expect from Maldacena-Nunez and $R_7 > 0$.

Taking the integrated ratio we find:

$$\boxed{\left|\frac{\int R_7}{\int R_4}\right| = \frac{5\int |F_4|^2 + 7\int |F_7|^2}{4\int |F_4|^2 + 8\int |F_7|^2} \leq \frac{5}{4}}$$

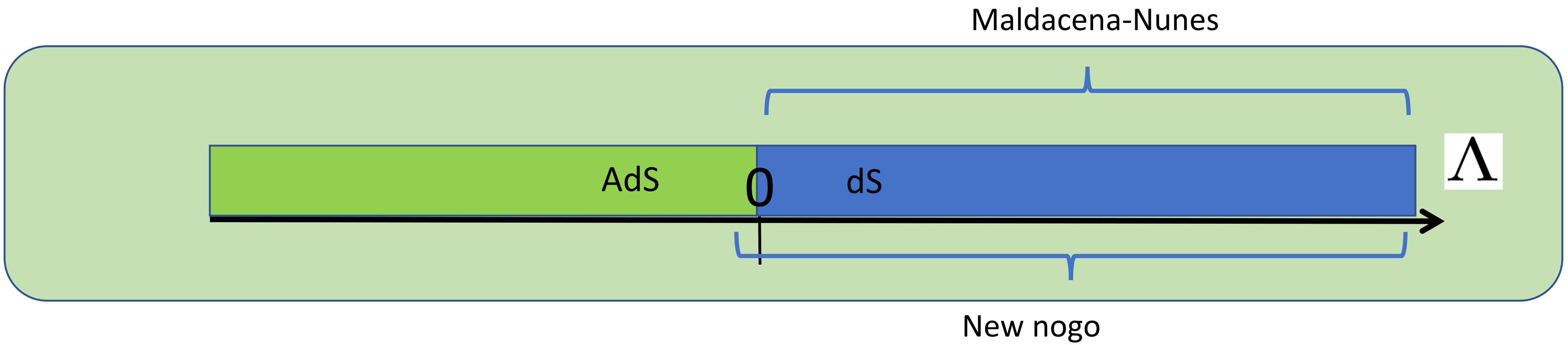
Now define the curvature radius as $L_R^{-2} = \mathrm{vol}_d^{-1} \int \mathrm{d}^d y \sqrt{g_d}\, R_d\,,$

- For the external dimensions this defines the Hubble length, aka AdS radius L_{AdS}

- **If we assume that L_{KK} cannot be taken to zero at fixed L_R** → nogo for scale separation. Precise & complete treatment, see [De Luca, Tomasiello, 2104.12773]

- Without o plane sources, internal manifold has positive curvature [Douglas&Kallosh 2010]

We arrive at an extension of the MN nogo to AdS vacua with scale separation [Gautason, Schillo, Williams, VR 2015]



→ Easiest way out: **include negative tension objects** (orientifolds): ***DGKT vacua*** [DeWolfe, Girvayets, Kachru Taylor, 2005] and the like. IIA with O6 planes.

$$\mathrm{vol}_6 \sim n^{3/2}$$
$$g_s \sim n^{-3/4}$$

$$\boxed{\frac{L_{\mathrm{KK}}^2}{L_{\mathrm{AdS}}^2} \sim n^{-1} \to 0.}$$

n is unbounded F4 flux quantum

But, backreaction of **intersecting** O6 planes not well understood and so contrived. *Recent progress* at "first order" in perturbation [Junghans 2020, Marchesano et al 2020]. Although it ignores intersection ☹

→ Less easy way out: find Einstein space for which one can shrink LKK at fixed curvature.

[Collins, Jafferis, Vafa, Xu, Yau, 2201.03660]: Large set of holographic CFTs checked. There is universal upper bound for dimension of first non-trivial spin 2 operator. The internal space for the CFT dual has minimal diameter in AdS units.

***They conjecture* it holds for all CFTs**

But the two ways out can be related! [arXiv 2107.00019, with N. Cribiori, D. Junghans, V. Van Hemelryck and T. Wrase]

→ There exist AdS vacua in IIA with O6 planes on generalized CY that can be scale separated at strong coupling, such that O6 backreaction is small. →lifts to weakly curved pure Freund-Rubin vacua in 11d. *We find a geometry contradicting Collins et al*?

→ Lift is not fully explicit since we only have first-order description of backreacted O6 planes. Work in progress. *But, a priori, seems controlled.*

(Other way out is **Casimir energy**, see eg [De Luca, De Ponti, Mondino, Tomasiello, 2022])

A curious feature

Early investigation on CFT dual to IIA vacua [Aharony et al 2008], but new investigation [Conlon, Ning Revello, 2021] shows **all such operator dimensions in DGKT are integer,** [Apers, Conlon, Ning, Revello, 2022] ([Apers, Montero, VR, Wrase 2022]) based on formalism of [Marchesano, Quirant 2019].

Modulus	Operator dimension Δ
1. $h_-^{1,1}$ saxionic Kähler moduli from J	6
1. $h_-^{1,1}$ axionic Kähler moduli from B_2	5
2. The dilaton direction	10
2. The C_3-axion appearing in W	11
3. $h^{2,1}$ saxionic complex structure moduli from $Re(\Omega)$	1 or 2
3. $h^{2,1}$ massless C_3-axions	3

Table 2: Summary of integer operator dimension of a putative CFT_3 dual for generic supersymmetric DGKT type AdS_4 vacua.

Why? See [Apers 2022] for comments: polynomial shift symmetries in large N limit on AdS side

Swampland Conjectures against scale separation

Strong AdS scale separation conjecture *of* [Lust, Palti, Vafa 2019] claiming ratio of lengthscales is order 1 for **SUSY** AdS vacua. However beautiful refinement by [Buratti et al 2020]: (k is from discrete Z_k 3-form symmetry)

$$L_{KK} = \mathcal{O}(1)\frac{L_{AdS}}{\sqrt{k}}\ .$$

- **Counter example to strong AdS distance conjecture:** *KKLT & LVS in parametric regimes. But especially* ***DGKT vacua*** [DeWolfe, Girvayets, Kachru Taylor, 2005].

- **Counter example** to refined strong AdS distance conjecture by [Buratti et al 2020] ***AdS_3*** *vacua from massive IIA on G2 space with O6 planes* [Farakos, Tringas, VR, 2020] as pointed out in [Apers, Montero, VR, Wrase, 2022]

Argument strong AdS conjecture [Lust, Palti, Vafa] from ***distance conjecture*** *[Ooguri, Vafa]*:

At large geodesic distance Δ in field space from the original vacuum, the mass scale m of a tower of modes becomes lighter as

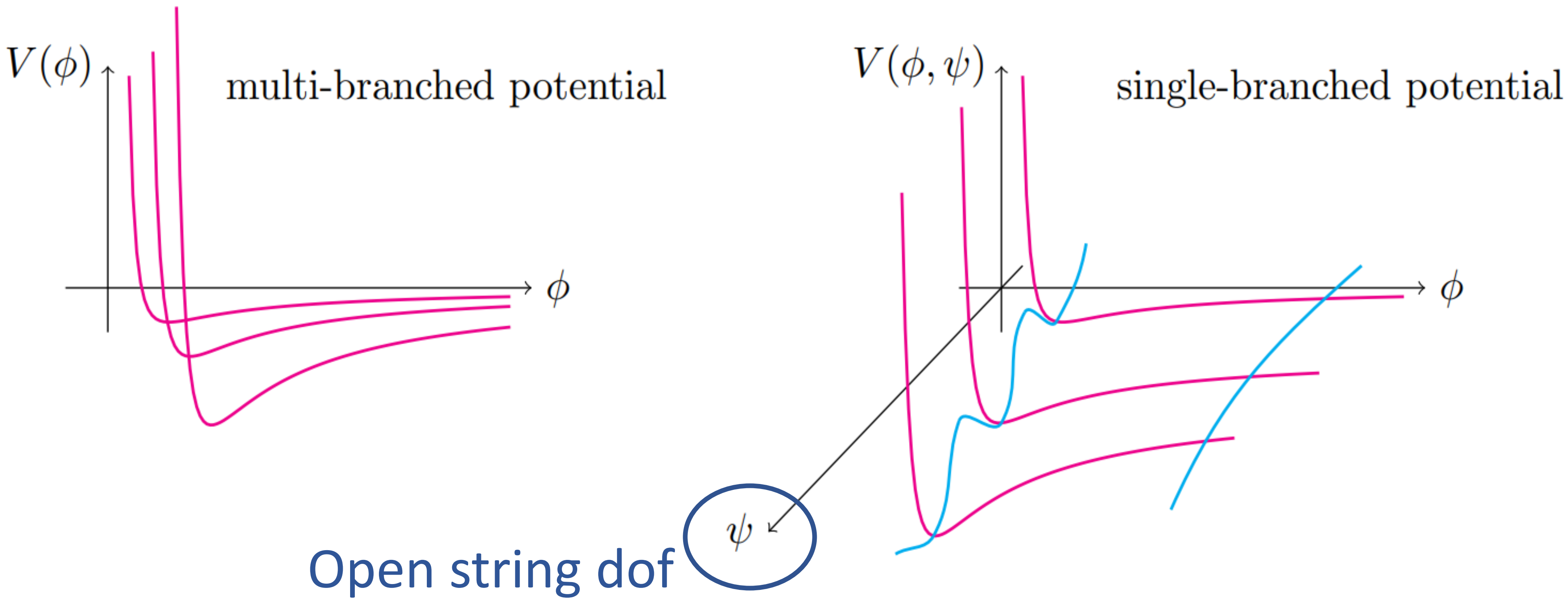
$$m \sim m_0 \, \mathrm{e}^{-\beta\Delta}$$

with β an order-one number.

[Lust, Palti, Vafa], suggested that conjecture also holds for distances travelled in **metric field** and then one finds the ADC. But there is little to no evidence that the distance conjecture applies beyond scalar fields.

We [Shiu, Tonioni, Van Hemelryck, VR 2022] found a **scalar** field that allows us to interpolate between the DGKT fluxes. This field is a brane position. This field makes DGKT vacua obey the distance conjecture!

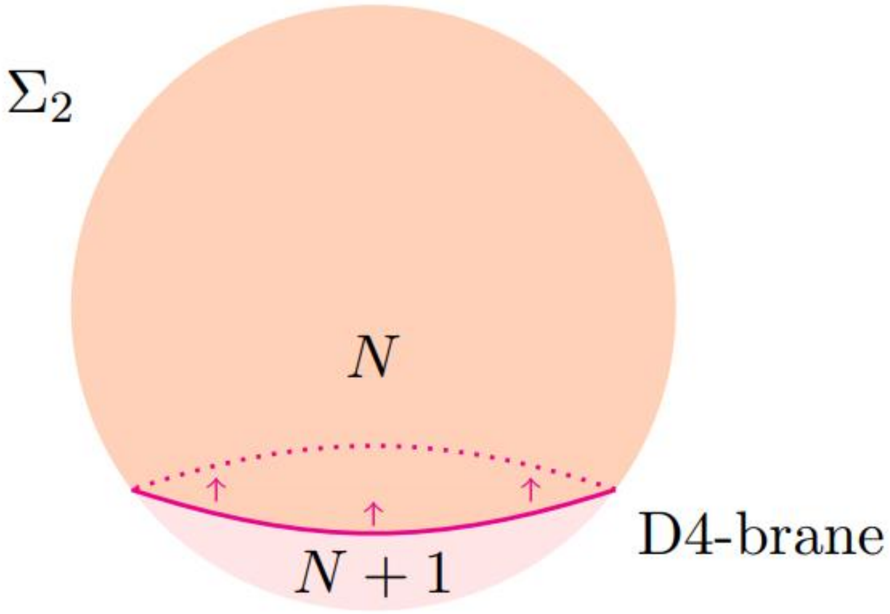
Hopping between flux quanta without domain walls.

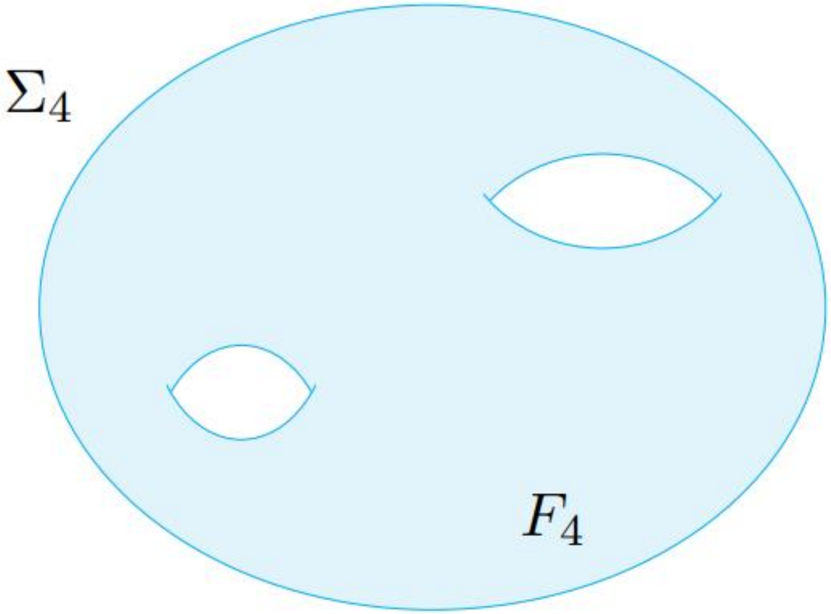


This field is still within the EFT since the wobbles in the psi direction are parametrically smaller than the vacuum energy:

$$\frac{V(\psi)}{V_{\text{flux}}} \sim N^{-1/2},$$

…for DGKT this field is a D4 position.





I gave two arguments in favor of **scale separation**:

1. The M-theory example: Freund Rubin like.
2. Saturation of distance conjecture for IIA DGKT vacua

Yet, scale separation seems hard to achieve explicitly in 11/10d language.

All suggestions have **minimal SUSY**. With more SUSY there are two firmly established Swampland conjectures that **rule it out.**

1. Magnetic WGC [Cribiori, Dall' Agata 2022]: *For SUSY 4d AdS vacua preserving Q>4, no scale separation if magnetic WGC holds.*

→Probably extends to all (any d) SUSY AdS vacua with more than 4 Q's. If so, no scale separation for SUSY vacua in D>4
→ AdS/CFT proof using charge bounds?

2. **Extreme scale separation**, meaning X is vanishing small at fixed M_{planck}, can be ruled out for AdS vacua with gauged R-symmetry! The R-symmetry becomes a global, unbroken symmetry in that limit. [Martinec, Montero, Vafa 2022]

Also note [Alday, Perlmutter, 2019] : **Holographic Hierarchy Conjecture**

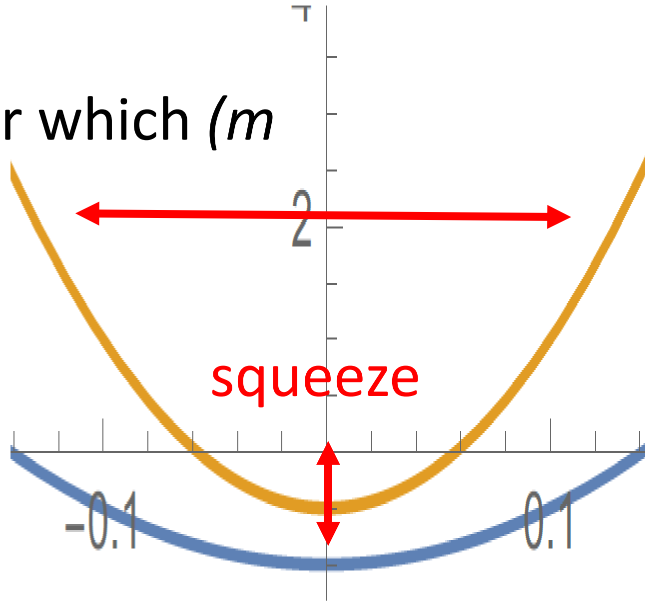
CFT has large higher spin gap + no global symmetry → scale separated AdS vacuum.

Conclusions:

Trying to achieve extremes. Much work is needed, especially on the CFT side of things. Simple mathematical statements about geometries and matrix models are suggested. Proofs would be nice.

APPENDIX

<u>AdS moduli conjecture</u> [Gautason, Van Hemelryck, VR 2018] : AdS vacua for which *(m is mass of lightest scalar)*



$$m_\phi L_{AdS} \gg 1$$

Are in the Swampland. (Difference is that it is a single scalar, not a tower).

→ Much weaker then conjectures against scale sep. KKLT, LVS, DGKT satisfy this. Only counterexample is bttom-up model by Kallosh-Linde.

→ Inspired from difficulties in finding dS and holographic CFTs.

$$2\pi F = \begin{pmatrix} 0 & f_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -f_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -f_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -f_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -f_4 & 0 \end{pmatrix} \mathbb{1}_{N\times N}$$

SUSY requires

$$\prod_I \frac{1 - \mathrm{i} f_I}{1 + \mathrm{i} f_I} = 1 \,,$$

Out of these 4 constrained numbers f_I , the symmetric combinations $f_I f_J$ will appear in Tr(F ∧ F) sourcing the D3 fluxes. We indeed have similarly six possible F5 flux terms.

Remarkable: In IIA Romans supergravity on CY with fluxes and O6 planes we can achieve moduli stabilization& scale separation with arbitrary good control! [DeWolfe et al 2005]. No alpha' corrections or quantum corrections.

$$\mathrm{vol}_6 \sim n^{3/2} \qquad g_s \sim n^{-3/4} \qquad \boxed{\frac{L^2_{\mathrm{KK}}}{L^2_{\mathrm{AdS}}} \sim n^{-1} \to 0.}$$

n is F4 flux quantum and is **not** bounded by tadpoles.

Backreaction of **intersecting** O6 planes not well understood. No worries in large volume, weak coupling limit [Baines, VR 2020]? *However, now O6 planes intersect:*

$\otimes$	$\otimes$	$\otimes$	—	—	—
$\otimes$	—	—	$\otimes$	$\otimes$	—
—	$\otimes$	—	—	$\otimes$	$\otimes$
—	—	$\otimes$	$\otimes$	—	$\otimes$

Despite certain beliefs intersecting brane solutions in SUGRA are **not known**, only upon partial smearing.

fluxes

$$H_3 = 2m \,\mathrm{Re}\,\Omega,$$
$$g_s F_0 = 5m,$$
$$g_s F_2 = \frac{\tilde{m}}{3} J + \mathrm{i}\mathcal{W}_2,$$
$$g_s F_4 = \frac{3}{2} m J \wedge J,$$
$$g_s F_6 = 3\tilde{m}\, \mathrm{dvol}_6.$$

geometry

$$\mathrm{d}J = 2\tilde{m}\,\mathrm{Re}\,\Omega\,,$$
$$\mathrm{d}\Omega = -\frac{4}{3}\mathrm{i}\,\tilde{m} J \wedge J + \mathcal{W}_2 \wedge J\,,$$
$$\frac{1}{L_H^2} = m^2 + \tilde{m}^2\,.$$

Sources

$$g_s j_3 = \mathrm{i}\,\mathrm{d}\mathcal{W}_2 + \left(\frac{2}{3}\tilde{m}^2 - 10m^2\right)\mathrm{Re}\,\Omega.$$

This source term represents the O6: $dF_2 = F_0 H_3 + j_3$

Non-conformal branes and AdS vacua.

Back to the original chain: easy to see why the chain AdS_d x S_d x $T^{(10\text{-}d)}$ exists from the viewpoint of dimensional reduction? Consider again (in general D dimensions))

$$S[A] = \int \left(\star R - \tfrac{1}{2} \star \mathrm{d}\phi \wedge \mathrm{d}\phi - \tfrac{1}{2} \mathrm{e}^{a\phi} \star F_n \wedge F_n\right).$$

Reduction Ansatz
$$\mathrm{d}s^2 = \mathrm{e}^{2\alpha\varphi}\mathrm{d}s_d^2 + \mathrm{e}^{2\beta\varphi}\mathrm{d}s_n^2\,,$$

When n=p we can have both magnetic flux (inside compact dimensions) and electric flux (inside non-compact dimensions) for the field strength F_n. If internal space has curvature R_n we then generate 3 terms in the lower dimensional scalar potential

$$V(\varphi,\phi) = \frac{1}{2}Q_E^2 \mathrm{e}^{-a\phi+(\alpha d-\beta n)\varphi} + \frac{1}{2}Q_M^2 \mathrm{e}^{a\phi+(\alpha d-\beta n)\varphi} - R_n \mathrm{e}^{(\alpha d+n\beta-2\beta)\varphi}\,.$$

Whenever both QE and QM are non-zero we generate an AdS vacuum since the dilaton will not be runaway anymore.