2D Bose Gases:
Current understanding & open questions
- Tunable interactions & Superfluid density

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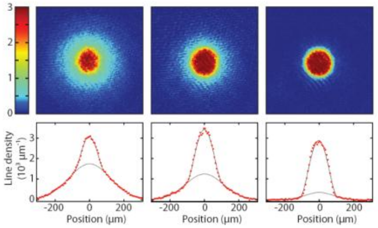
Experiment
Rob Smith – see poster on ^{39}K BEC
Naaman Tammuz
Robbie Campbell
Scott Beattie
Stuart Moulder



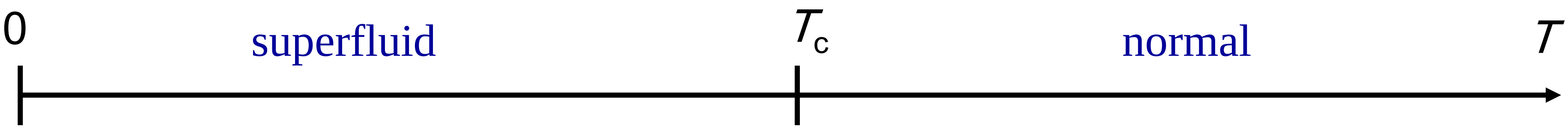
Theory w/
Nigel Cooper
Sebastian John
Jean Dalibard

N.R. Cooper and ZH, PRL **104**, 030401 (2010)
ZH and J. Dalibard, arXiv:0912.1490

KITP, October 14 2010

Berezinskii - Kosterlitz -Thouless (BKT)

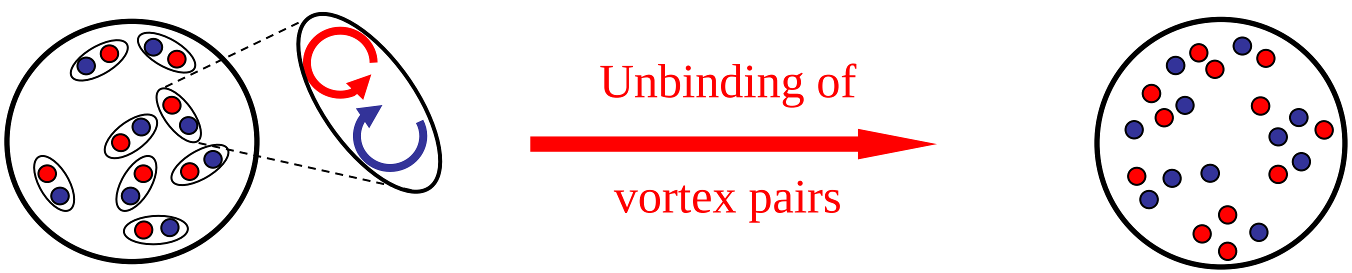
for an infinite uniform 2D system – no BEC



$$n_{\text{superfluid}}\lambda^2 = 4$$

$$\lambda^2 = 2\pi\hbar^2/(mkT)$$

(in addition to phonons…)



Bound vortex-antivortex pairs

Proliferation of free vortices

$$g_1(x,y) = \langle \psi^\dagger(x,y)\,\psi(0)\rangle$$

Algebraic decay

$$g_1(r) \sim r^{-\alpha}$$

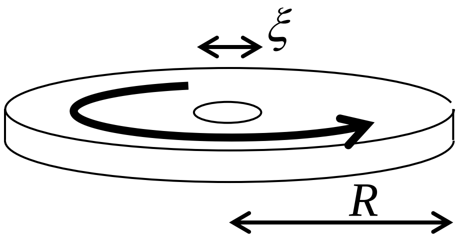
no LRO
either way

Exponential decay

$$g_1(r) \sim \exp(-r/\ell_0)$$

Key quantitative predictions

1. Universal jump in superfluid density $n_S \lambda^2 = 4$ at the transition



$$\frac{E - TS}{k_B T} \sim \frac{1}{2}\left(n_S \lambda^2 - 4\right)\ln\left(R/\xi\right)$$

Theory: Kosterlitz & Nelson 1977
Experiment: Bishop & Reppy 1978

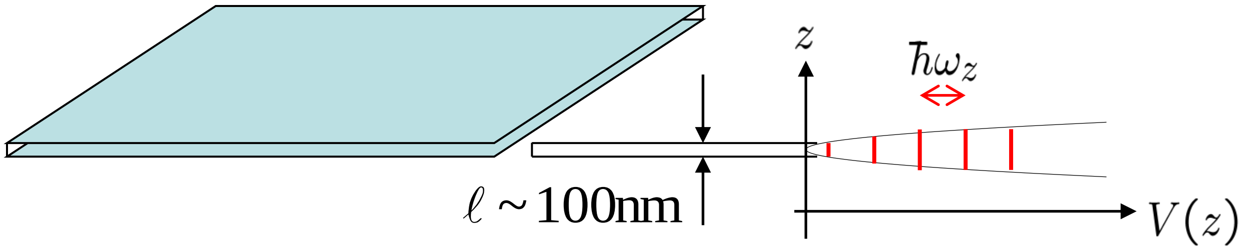
2. Algebraic decay $g_1(r) \propto r^{-\alpha}$ with $\alpha = 1/n_S \lambda^2 \leq 1/4$

3. *Total* critical density depends on interactions $n\lambda^2 = \ln\left(380/\tilde{g}\right)$

$\tilde{g}$ - dimensionless interaction strength

Analytics: Fisher & Hohenberg
Monte-Carlo: Prokof'ev, Svistunov et al.

Harmonically trapped (quasi-)2D atomic gases



$$\hbar\omega_z > k_B T, \mu$$

$$\ell \approx \sqrt{\hbar / m\omega_z} < \lambda, \xi$$

Interaction strength: $\tilde{g} = \sqrt{8\pi}\frac{a}{\ell}$

- Liquid helium films: $\tilde{g} \sim 1$
- Atomic gases (so far): $\tilde{g} \sim 10^{-2} - 10^{-1}$

NB: Conventional BEC possible in the ideal harmonically trapped 2D Bose gas…

2D s.h.o. density of states
→ saturation of excited states for: $N > N_c = 1.6\left(\frac{kT}{\hbar\omega}\right)^2$

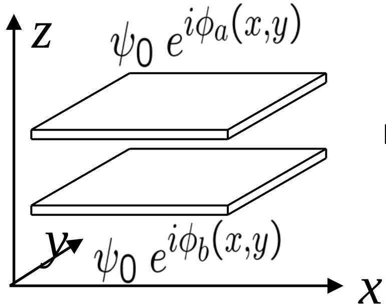
Bagnato – Kleppner 1991

…but requires $n(0)\lambda^2 = \infty$, suppressed by interactions

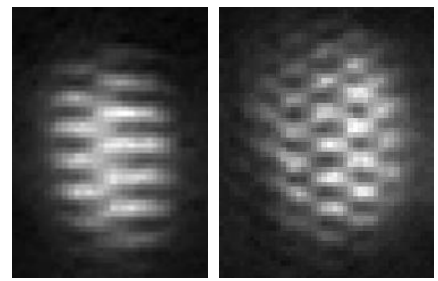
BKT predictions & experiments so far…

Review: ZH and J. Dalibard, arXiv:0912.1490

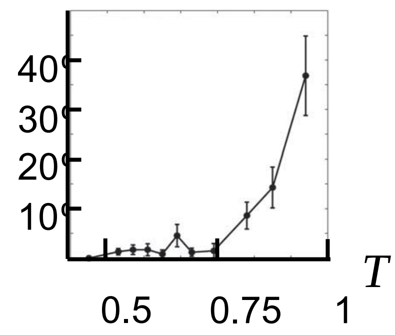
1. Vortices



Time of flight



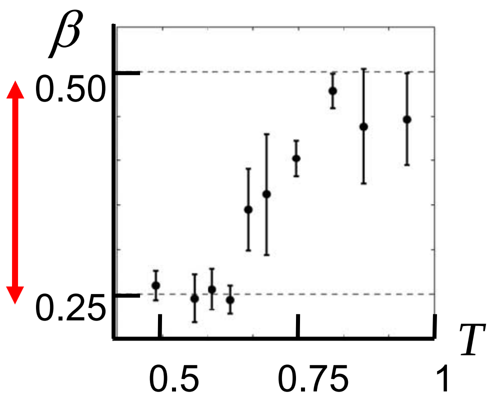
ZH et al, Nature 2006
(also NIST, Harvard)



2. Universal jump

from: $$g_1(r) \sim r^{-1/n_s\lambda^2}$$

Theory: Polkovnikov et al., PNAS 2006
Exp: ZH et al, Nature 2006



But no conventional, transport measurement!

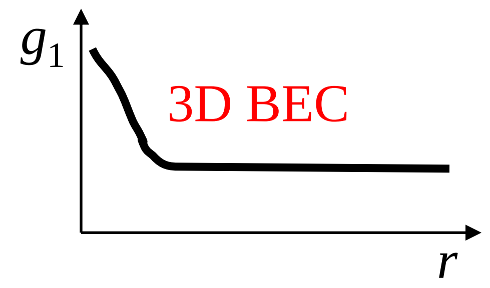
+ "universal" = independent of $\tilde{g}$ - also need tunable interactions!

3. More quantitative stuff (ENS, NIST, JILA, Chicago) :
critical point, LDA, scale invariance, density fluctuations… see Cheng Chin's talk!

Finite size effects & condensation

Condensed fraction:
(Penrose-Onsager)

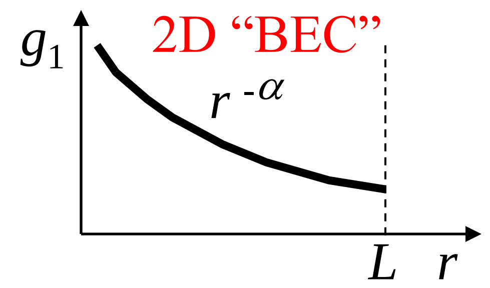
$$n_0 \sim \lim_{r \to \infty} g_1(r)$$



BKT superfluid:
$(\alpha \leq 1/4)$

$$n_0 \sim \left(\xi/L\right)^{\alpha} \sim \left(\tilde{g}N\right)^{-1/8} \geq 0.1$$

($N^{-1/8}$ thanks to Willi Zwerger)



"BEC" ≠ BEC,
but not much

Non-zero n_0: (1) due to finite size, not due to harmonic trap
(2) signature of the BKT phase transition

Thermodynamic limit $\ln\left(L/\xi\right) >> 1$ experimentally impossible

"…the system would have to be bigger than Texas for Mermin-Wagner to apply …"

"*Magnetization as a signature of BKT*," Bramwell – Holdsworth, 1994

So what's the difference between BEC and BKT [condensation]?

1. (B-E) Condensation in an interacting system → superfluidity

2. (BKT) Superfluidity in a finite system → non-zero condensed fraction

3. All systems are both finite and interacting

→ in practice superfluidity and condensation always go hand in hand

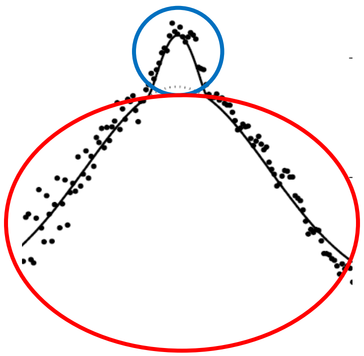
Conceptual difference more in the causes than in the symptoms!

BEC - purely statistical phase transition (the only one!)
doesn't require interactions, driven by saturation of excited states

BKT - fundamentally interaction driven,
condensation without saturation

The not so normal state

$\tilde{g} \sim 10^{-1}$

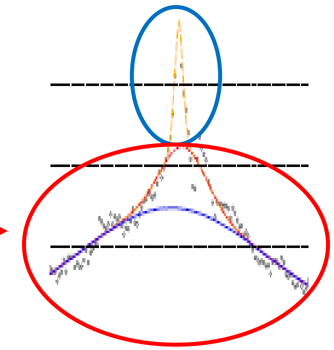


"Gaussian"

Krüger et al, PRL 2007

momentum distribution
in 3D TOF for $N > N_C$

"normal" component
different

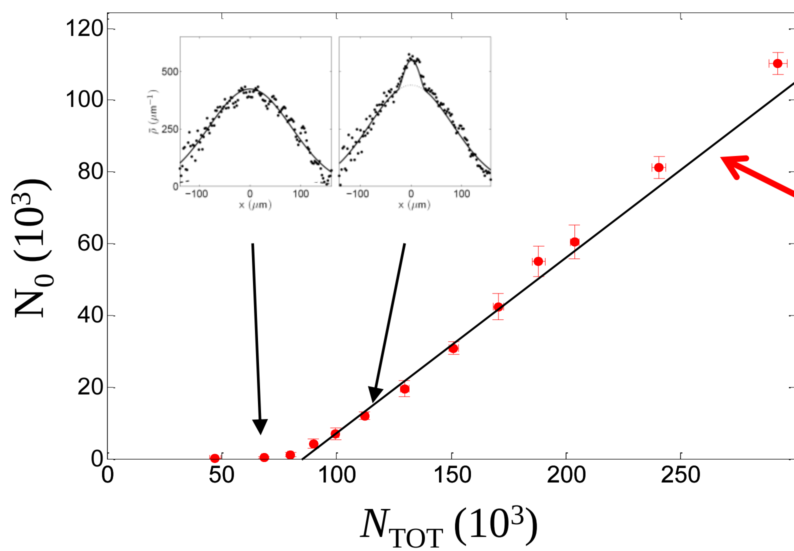


$\tilde{g} \sim 10^{-2}$

"bimodal"

Cladé et al, PRL 2009

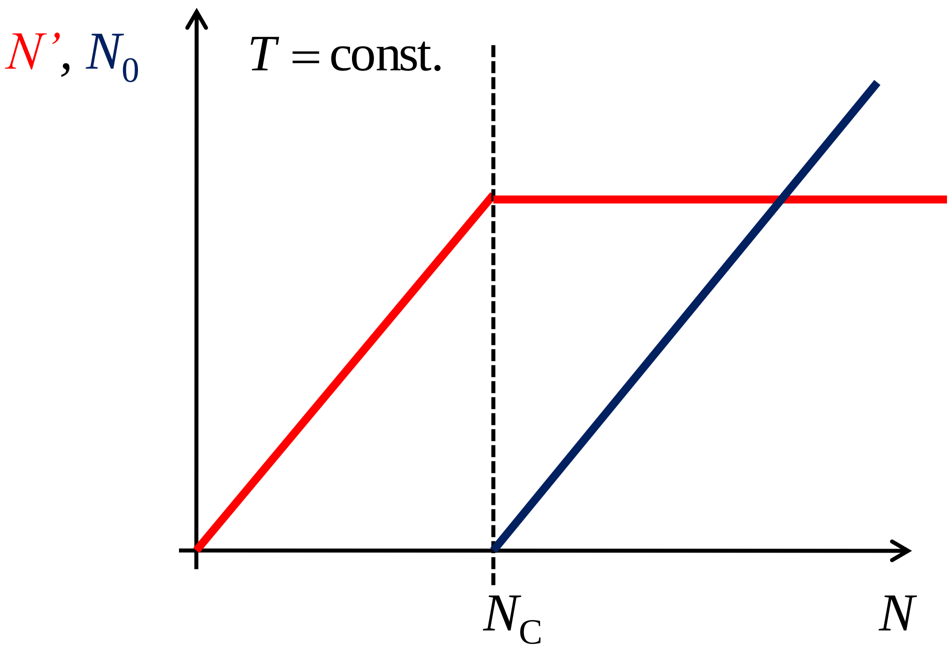
The not so ordinary condensate



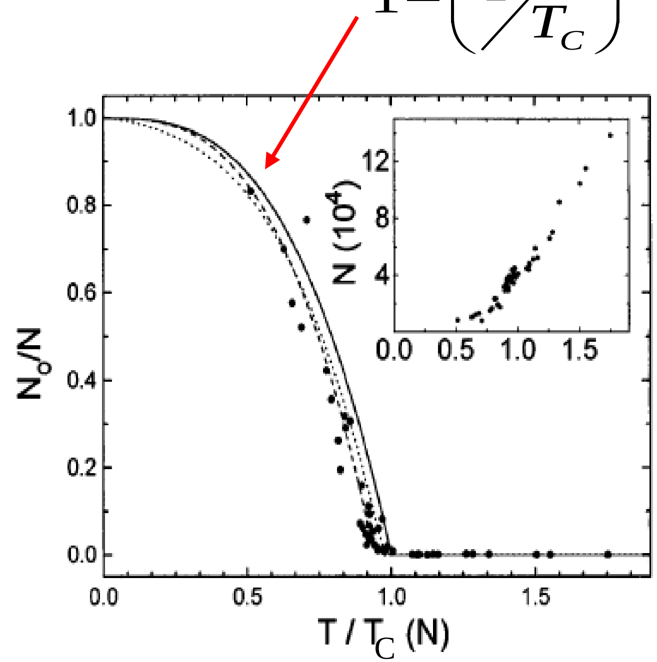
slope < 1
not a saturated gas

Back to 3D…

Textbook Einstein:

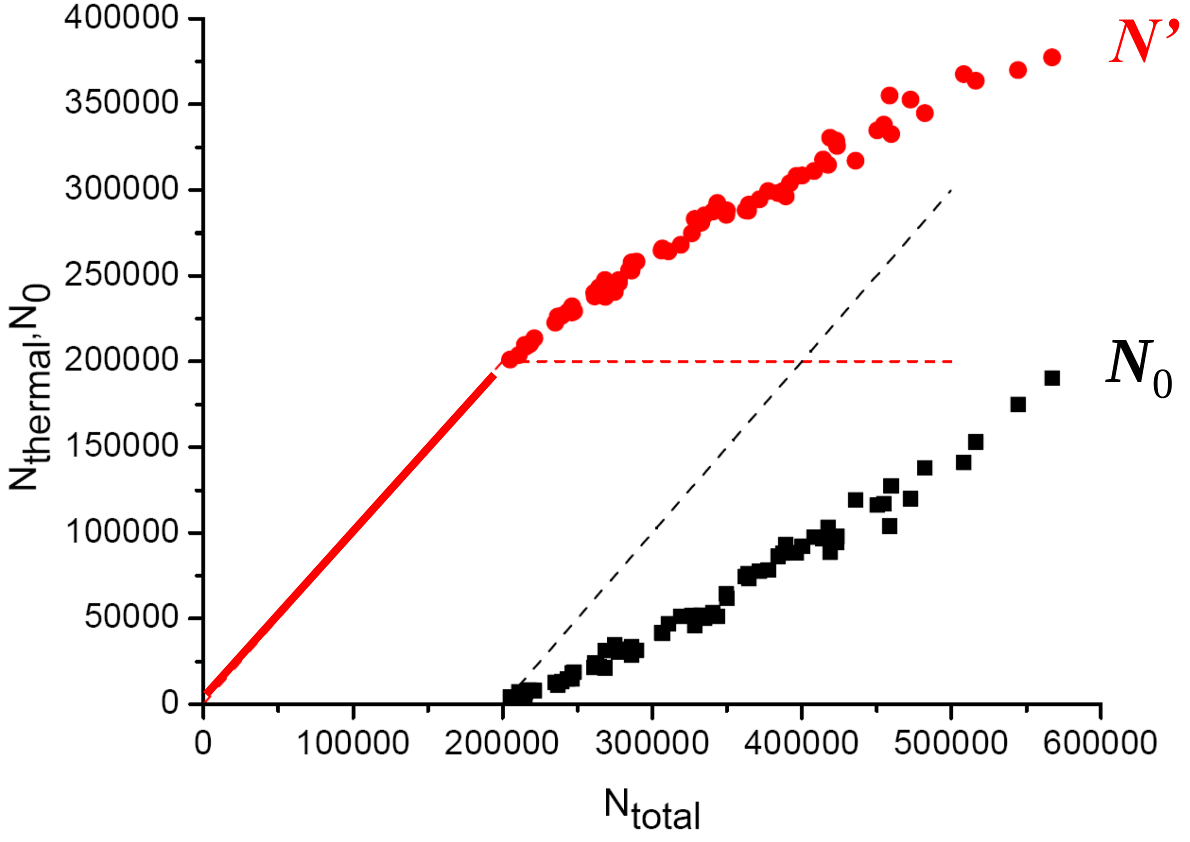


$$1-\left(\frac{T}{T_C}\right)^3$$



Ensher *et al*., PRL 1996

Lack of saturation in a (*trapped*) 3D gas



$T = \text{const.}$

$a = 100 a_0$

Two Big Questions…

Was Einstein actually right?

Yes.

Have we ever really shown it experimentally?

T_C - damn close [Aspect et al., 2004]

Critical exponents [Esslinger et al., 2007]

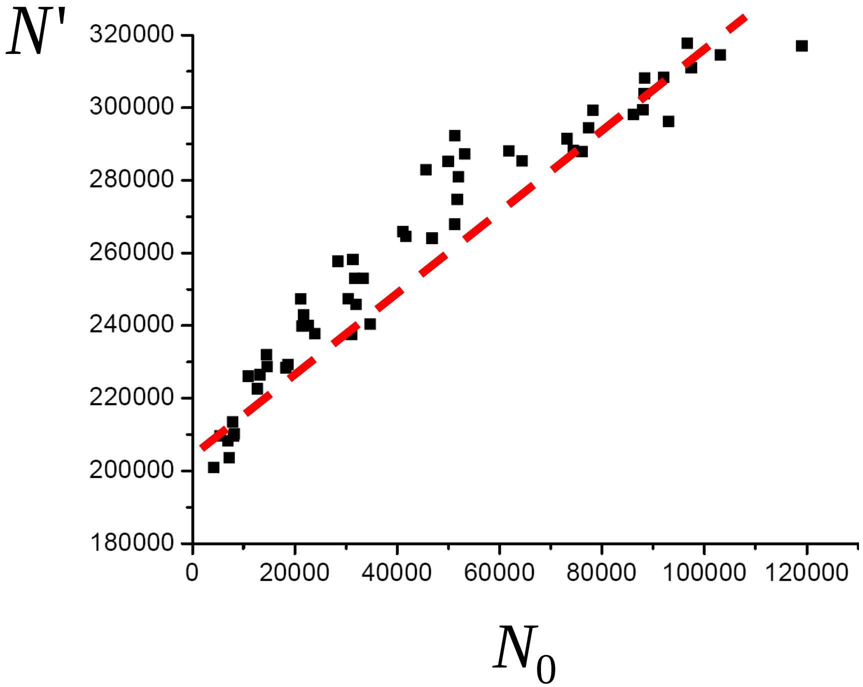
…

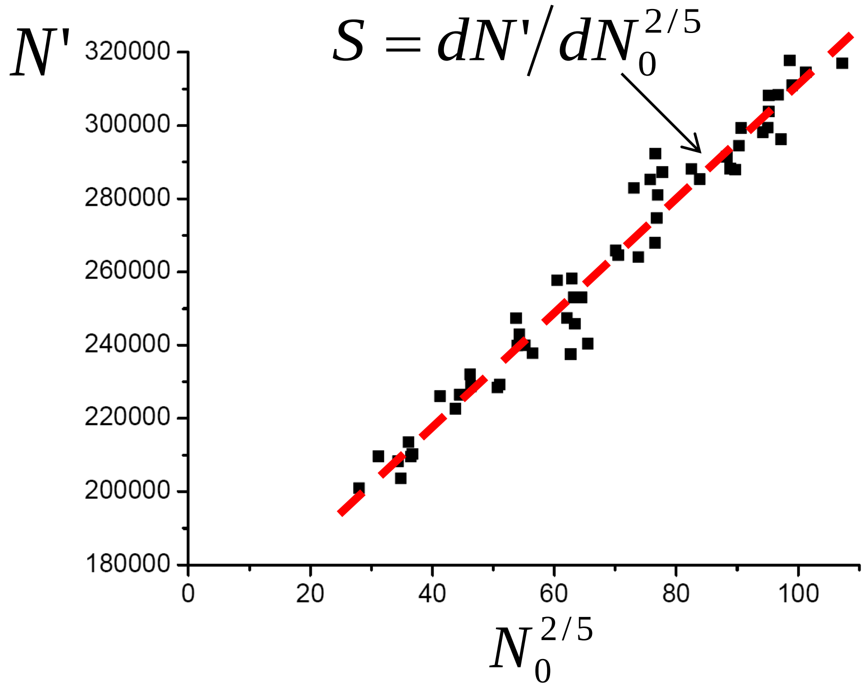
The basic underlying mechanism of

purely statistical saturation of the excited states?

Doesn't really seem so.

Non-saturation "slopes"

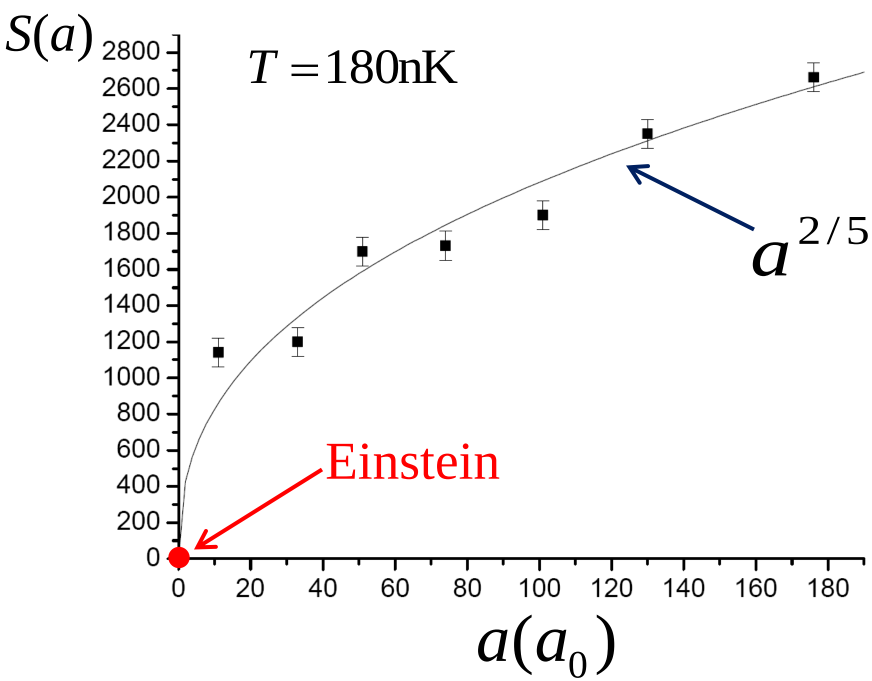


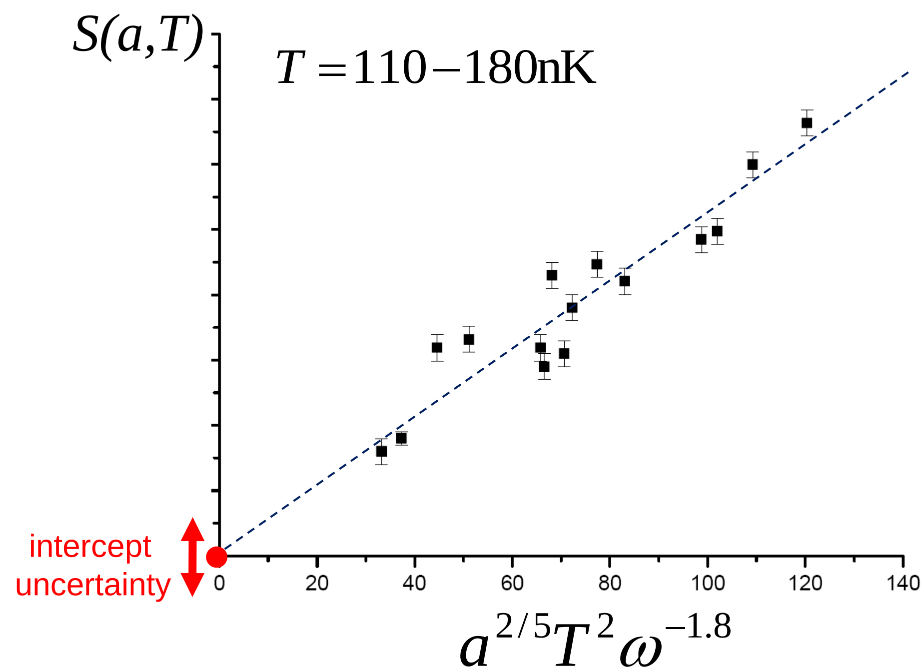


$$N^{'} = a * T^3 + b * \mu T^2$$

in line with Stringari & co. circa 1996

Tuning interactions





…Einstein (very probably) cleared!

(Tunable 2D hopefully next time we meet…)

How to measure the superfluid fraction of an atomic gas?

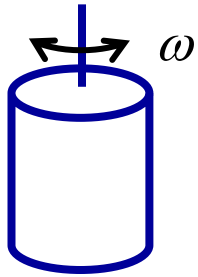
ZH & Nigel Cooper + Sebastian John

N.R. Cooper and ZH, PRL **104**, 030401 (2010)
S. John et al., coming out soon

Inspired by experiments of Ian Spielman et al. + theories of many

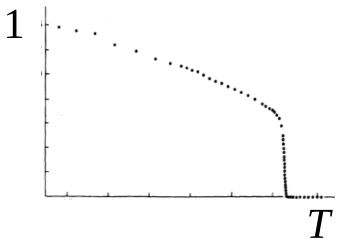
NB: Another recent idea - Ho and Zhou, Nature Phys. **6**, 66 (2010)

Superfluid fraction



$$\frac{\rho_s}{\rho} \equiv 1 - \lim_{\omega \to 0} \left(\frac{\langle L \rangle}{I_{\rm cl}\,\omega} \right)$$

Liquid He: 3D Andronikashvili 1946
2D Bishop & Reppy 1978



Atomic gases: vortices, critical velocity, persistent currents…
not quantitative measurements of SF fraction

good at rotating, but not so good at measuring mass flow

Why do we care?

weakly interacting 3D $n_s \approx n_0$

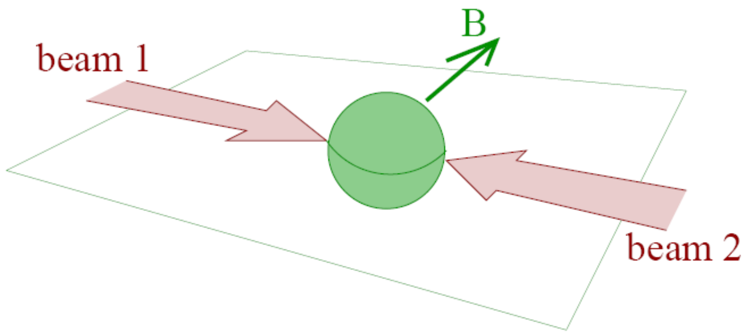
2D and/or strong interactions $n_s \neq n_0$

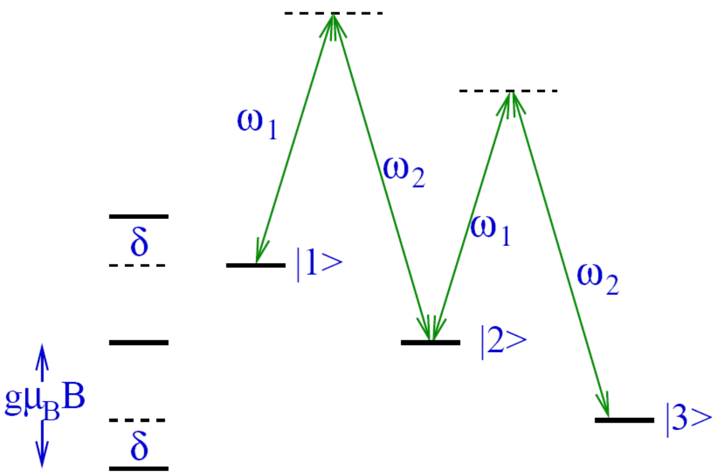
Superfluid fraction of an atomic gas

Basic idea: Generate transport using optically induced gauge potentials

→ measure the Andronikashvili/Leggett superfluid fraction spectroscopically

3 internal (hyperfine) atomic states coupled by two (Raman) lasers:





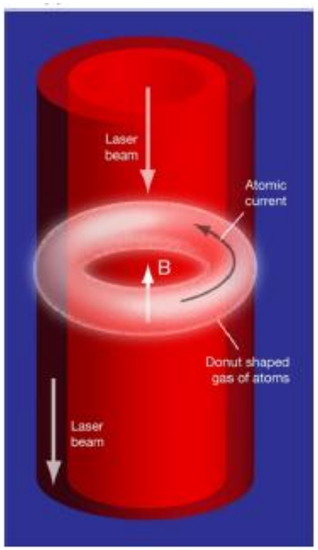
$$\Delta k = k_1 - k_2 \simeq 2k_r$$

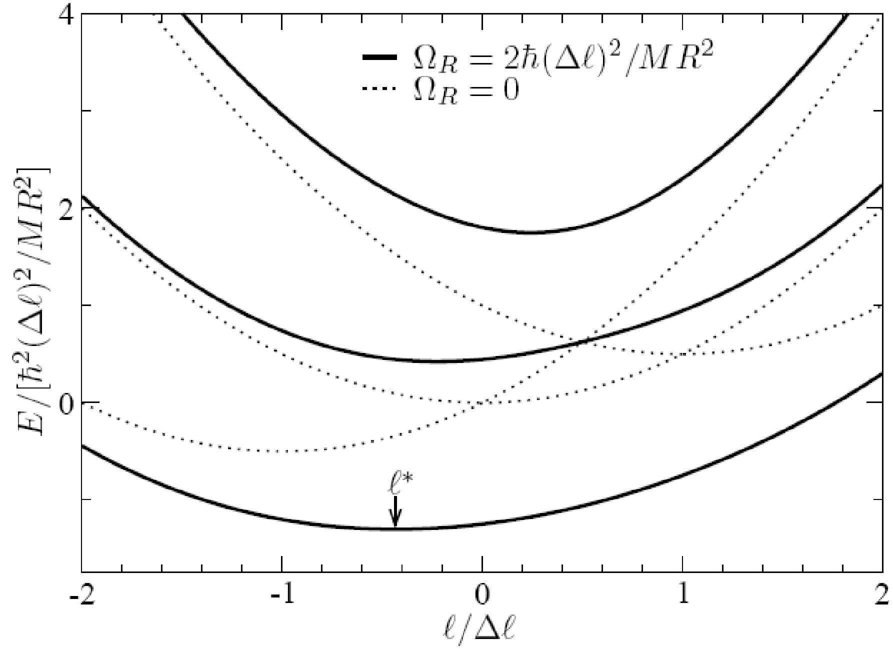
$$\delta = g\mu_B B/\hbar - (\omega_2 - \omega_1)$$

→ Coupling between internal and external states (in this picture linear momentum)

Superfluid fraction – Ring geometry

$R >> \Delta R$





co-propagating laser beams,
with orbital angular momentum

$$\Delta\ell = \ell_2 - \ell_1$$

$$E \simeq E_0 + \frac{\hbar^2}{M^* R^2}\left(\frac{\ell^2}{2} - \ell\,\ell^*\right)$$

$$\ell^* \simeq -\sqrt{2}\frac{\delta}{\Omega_R}\Delta\ell + \mathcal{O}(1/\Omega_R^2)$$

lab behaves like
a rotating frame with:

$$\omega_{\text{eff}} \equiv \frac{\hbar\ell^*}{M^* R^2}$$

Superfluid fraction – Spectroscopy

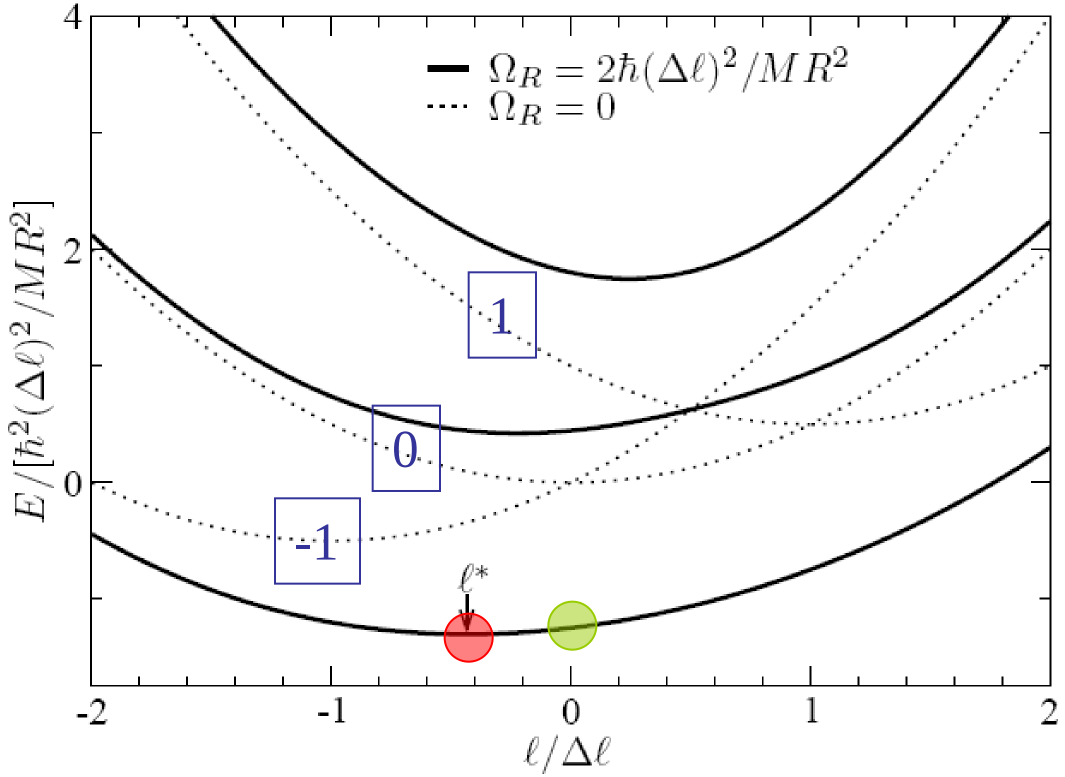
$$\frac{\rho_s}{\rho} \equiv 1 - \lim_{\omega_{\text{eff}} \to 0} \left(\frac{\langle L \rangle}{I_{\text{cl}} \omega_{\text{eff}}} \right) \qquad [I_{\text{cl}}\omega_{\text{eff}} = NM^* R^2 \omega_{\text{eff}} = N\hbar\ell^*]$$

Normal fluid: $\langle L \rangle / (\hbar N) = \ell^*$

(at rest in the lab frame)

Superfluid: $\langle L \rangle = 0$

(rotating in the lab frame)



Spectroscopic signature: $|\psi_{-1}|^2 - |\psi_1|^2 \equiv \Delta p_0 + \Delta p' \ell + \mathcal{O}(\ell^2)$

Spectroscopic signal

spin imbalance: $$\Delta p \equiv \frac{N_{-1} - N_1}{N} = \frac{\sum_\ell \langle n_\ell \rangle \left[|\psi_{-1}|^2 - |\psi_1|^2 \right]}{\sum_\ell \langle n_\ell \rangle}$$

$$\frac{\rho_s}{\rho} = 1 - \lim_{\ell^* \to 0} \left(\frac{\Delta p - \Delta p_0}{\ell^* \Delta p'} \right) + \mathcal{O}(\mu / \hbar \Omega_R)$$

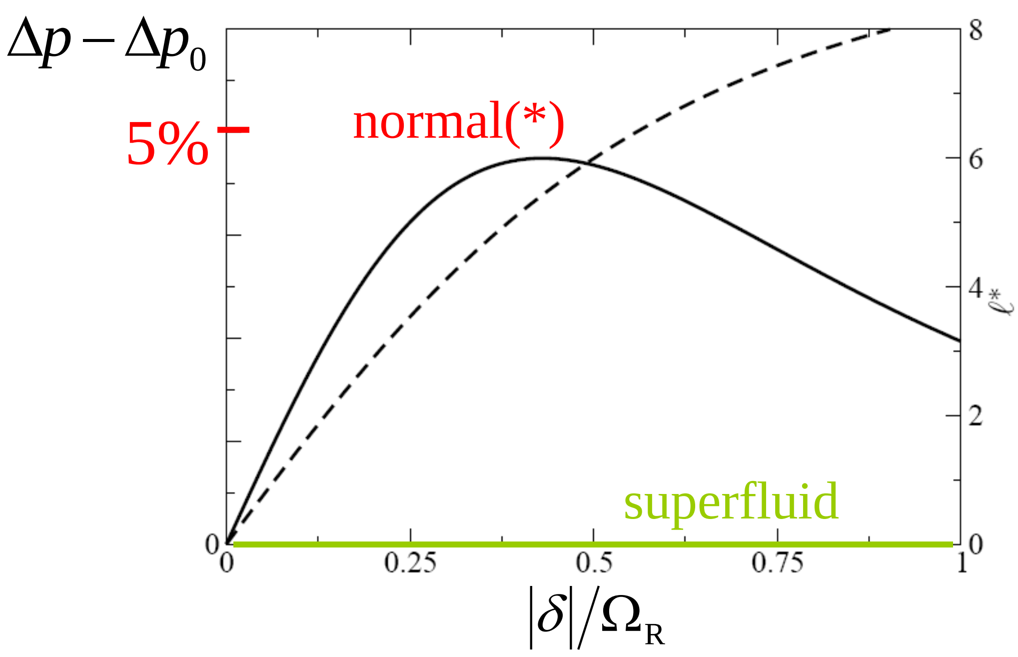
e.g. for ^{23}Na:

$R = 10\,\mu\text{m}$

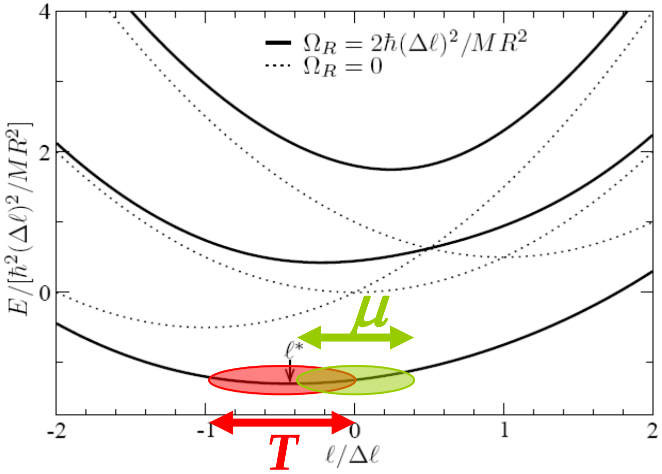
$\Omega_R \simeq 2\pi \times 4.4\,\text{kHz}$

$\Delta\ell = 10$

(*) or relaxed superfluid



Temperature and interactions



Normal/superfluid distributions broadened by temperature/interactions

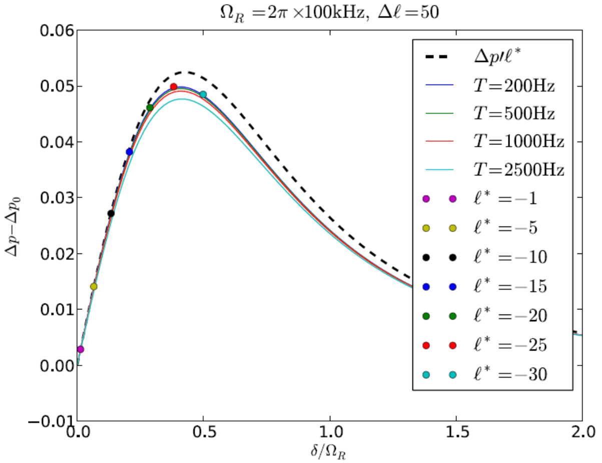
$$\Delta p_\ell = (\Delta p_\ell)_{\text{linear}} + \boxed{c'\ell^2} + \ldots$$

$$E = E_{\text{parabolic}} + \boxed{c\ell^3} + \ldots$$

All perfect for $\Omega_R \to \infty$, but signal $1/\Omega_R \to 0$

details: S. John et al, coming soon.

bottom line: all seems OK.



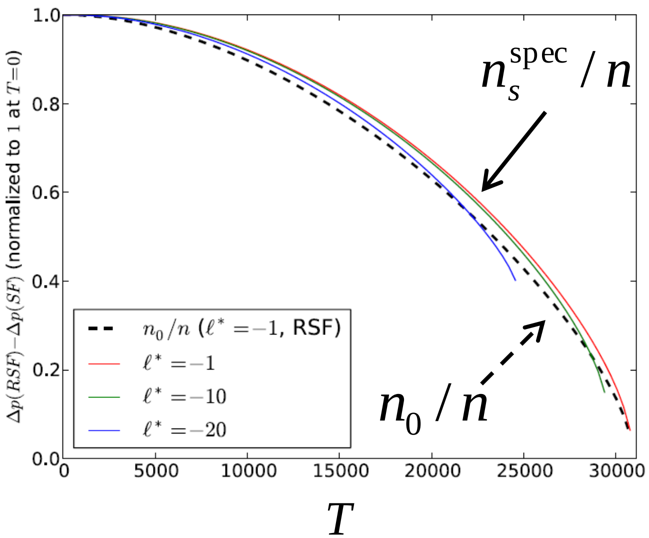
What do you actually measure?

In a single shot – just the spin imbalance Δp, nothing to compare to

One idea - cooling and rotating don't commute for a superfluid!

Does it make sense?

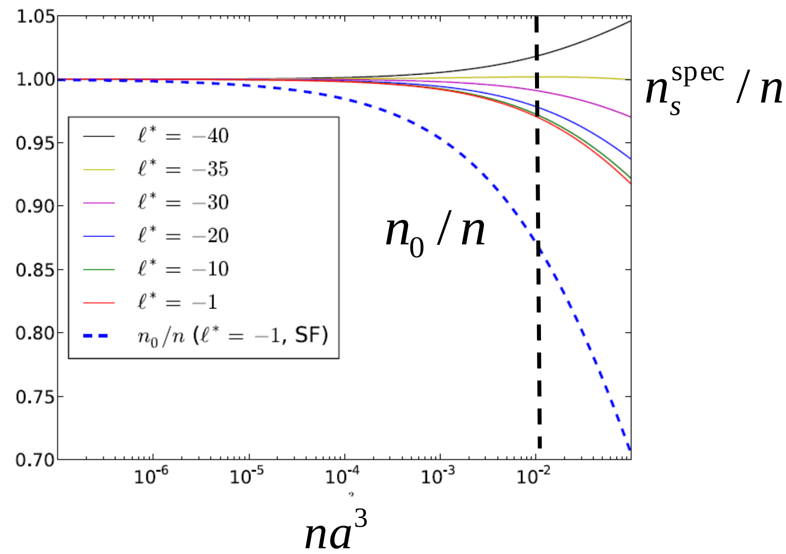
weak interactions, vary T



$$n_s^{\text{spec}} \approx n_0 \quad ✓$$

Is it really useful?

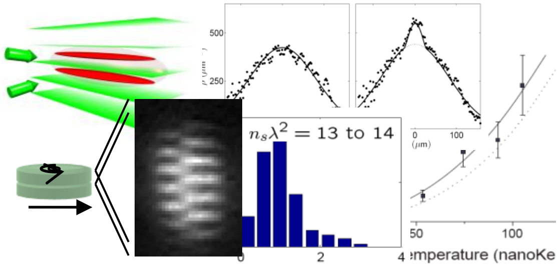
T=0, tune interactions



$$@\, na^3 = 10^{-2}: \quad 1 - n_0/n = 12\% \qquad n_s/n = 100\%$$

$$|1 - n_s^{\text{spec}}/n| \leq 3\%$$

Summary

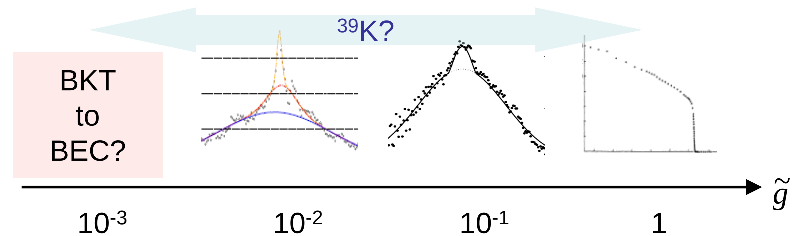


2D so far (ENS, JILA, NIST, Chicago…):

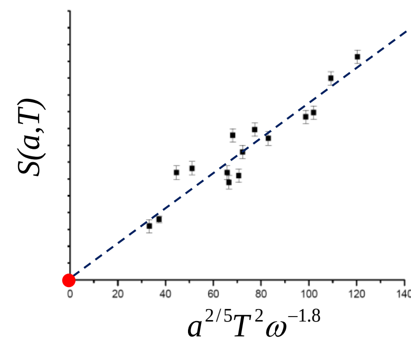
Vortices, critical point, (quasi-)coherence…

Review: ZH and J. Dalibard, arXiv:0912.1490

Our new experiment
in Cambridge (Rob's poster):



3D intermezzo: Einstein right!



Need a measurement of superfluid density

N.R. Cooper and ZH, PRL **104**, 030401 (2010)
S. John et al, coming out soon