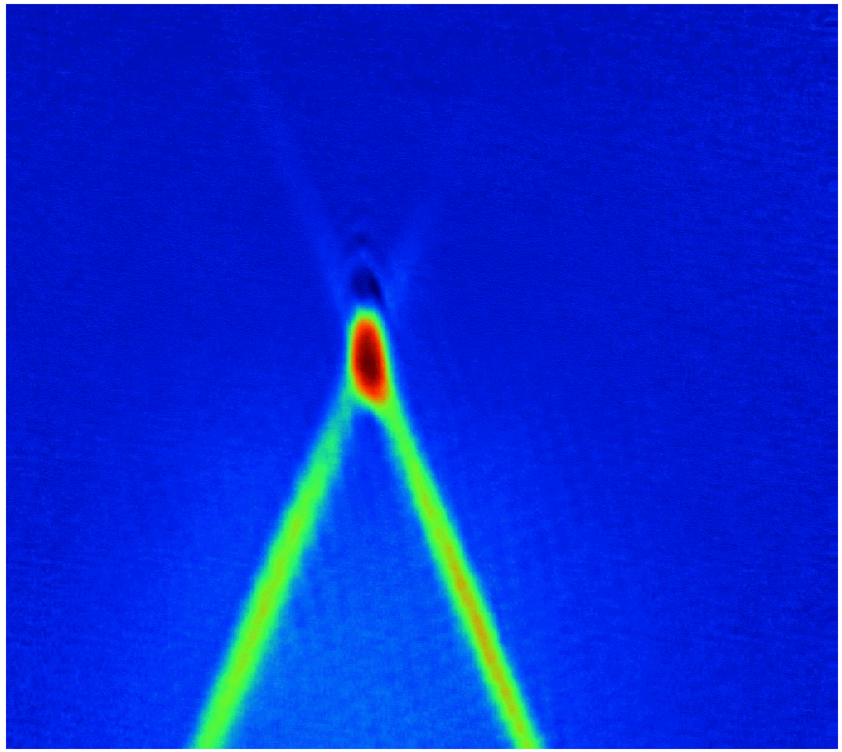
Thermodynamics of a Tunable Fermi Gas



C. Salomon

Laboratoire Kastler Brossel
Physique quantique et applications

KITP, October 11, 2010

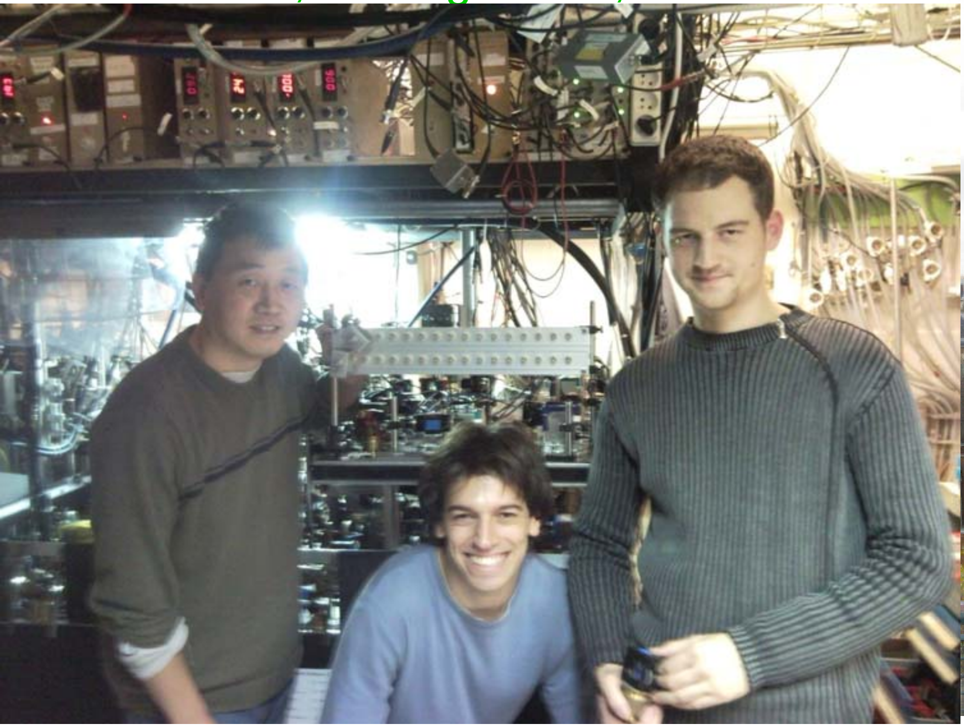
The ENS Fermi Gas Team

^{6}Li-^{7}Li

S. Nascimbène, N. Navon,
K. Jiang, L. Tarruell, F. Chevy, C. S.
K. Günter, K. Magalhães,

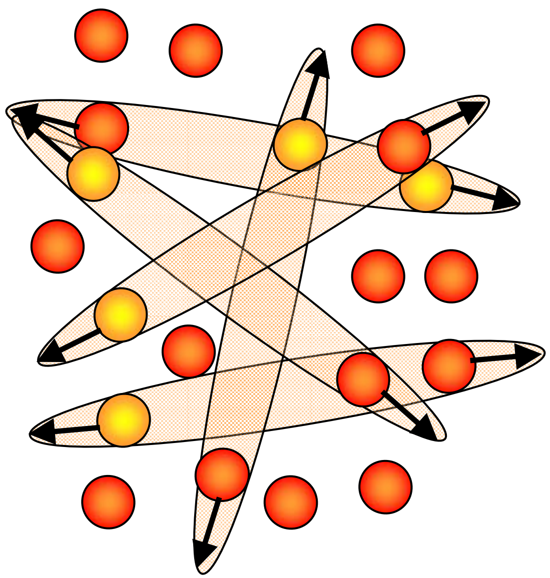
^{6}Li-^{40}K

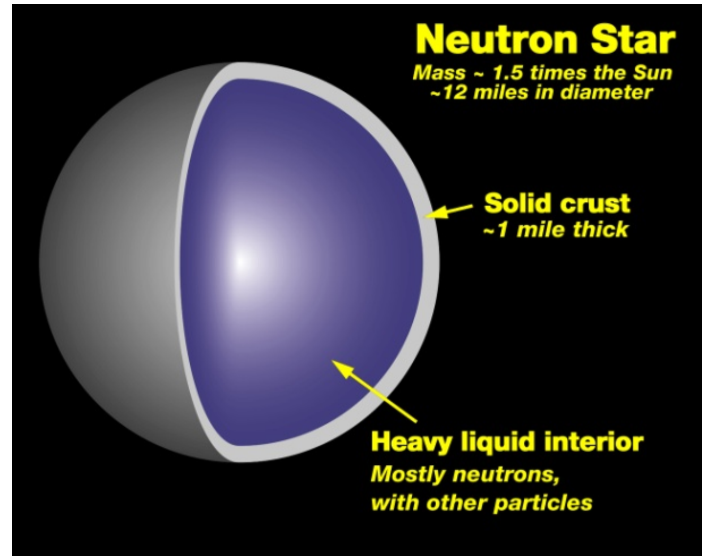
A. Ridinger, T. Salez, S. Chaudhuri,
U. Eismann, D. Wilkowski, F. Chevy,
Y. Castin, M. Antezza, C. Salomon



Theory collaborators: D. Petrov, G. Shlyapnikov , R. Papoular,
J. Dalibard, R. Combescot, C. Mora
S. Stringari, S. Giorgini, I. Carusotto, C. Lobo,
L. Dao, O. Parcollet, C. Kollath, J.S. Bernier, L. De Leo, A. Georges, T. Giamarchi

Fermi Gases with Tunable Interactions





Cold atoms, Spin ½

Dilute gas : 10^{14} at/cm^{3}, T=100nK

BEC-BCS crossover

Superfluidity, collective modes,

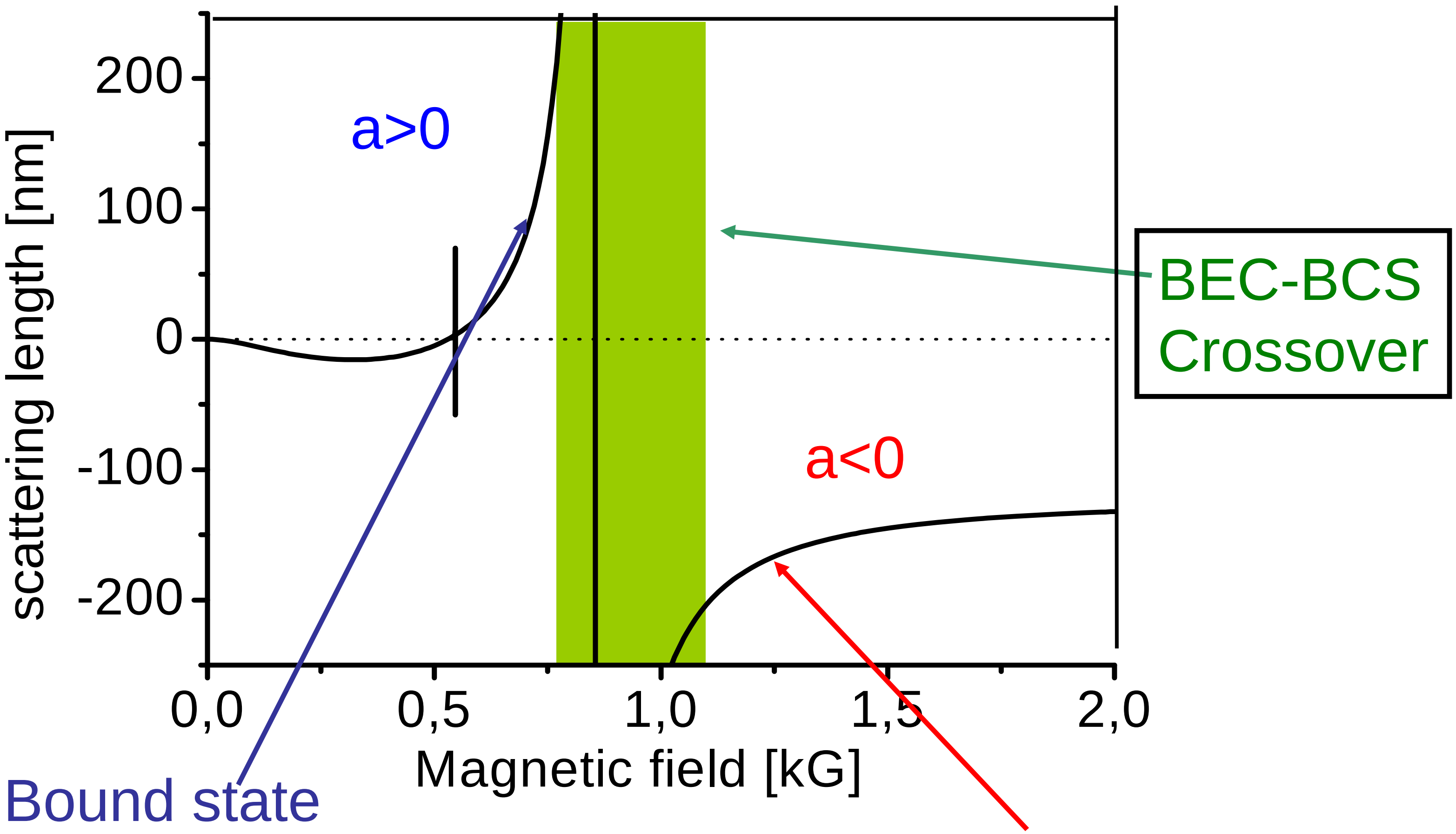
Spin imbalance, exotic phases

Neutron star, Spin ½

a = -18.6 fm, n~2 10^{36}cm^{-3}

- $T_c = 10^{10}$ K , $T = T_F/100$
- $k_F a$ ~ -4,-10,…
- $k_F r_e << 1$

Tuning interactions in Fermi gases
Lithium 6



$$E_B = -\frac{\hbar^2}{ma^2}$$

condensate of molecules

BCS phase

Thermodynamics

$$PV = Nk_B T$$

Is a useful but incomplete equation of state !

Complete information is given by thermodynamic potentials:

Grand potential $$\Omega = -PV = E - TS - \mu N$$

Pressure
Volume
Internal energy
Temperature
Entropy
Chemical potential
Atom number

Equ. of state useful for engines, chemistry, phase transitions,….

We have measured the grand potential of a tunable Fermi gas

S. Nascimbène et al., Nature, **463**, 1057, (Feb. 2010), arxiv 0911.0747
N. Navon et al., Science **328**, 729 (2010)

Thermodynamics of a Fermi gas

In the grand canonical ensemble, the EoS of the *homogeneous* Fermi gas is:

$$\Omega(\mu, a, T) = E - TS - \mu N$$
$$\Omega(\mu, x, T) = -P(\mu, a, T)V$$

Pressure contains all the thermodynamic information

Variables :		
	scattering length	**a**
	temperature	**T**
	chemical potential	**μ**

We build the dimensionless parameters :

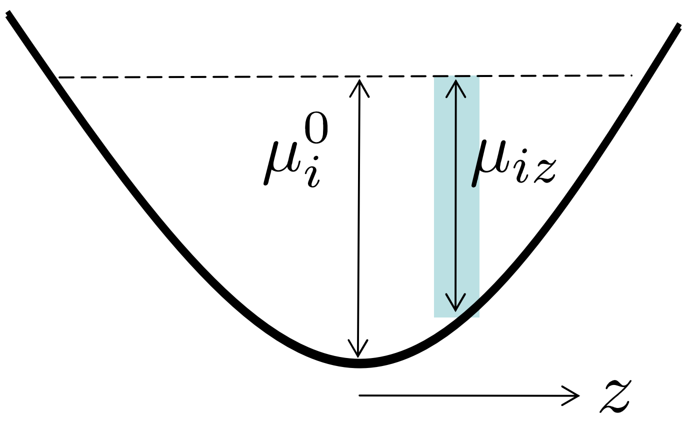
		Canonical analogs
Interaction parameter	$\delta = \frac{\hbar}{\sqrt{2m\mu}a}$	$(k_F a)^{-1}$
Fugacity (inverse)	$\zeta = \exp\left(-\frac{\mu}{k_B T}\right)$	T/T_F

Measuring the EoS of the Homogeneous Gas

Local density approximation:
gas locally homogeneous at

$$\mu_{iz} = \mu_i^0 - \frac{1}{2} m\omega_z^2 z^2$$

i=1, spin up
i=2, spin down



$$V(z) = \frac{1}{2} m\omega_z^2 z^2$$

Measuring the Pressure of the homogeneous Gas

Extraction of the pressure from *in situ* images

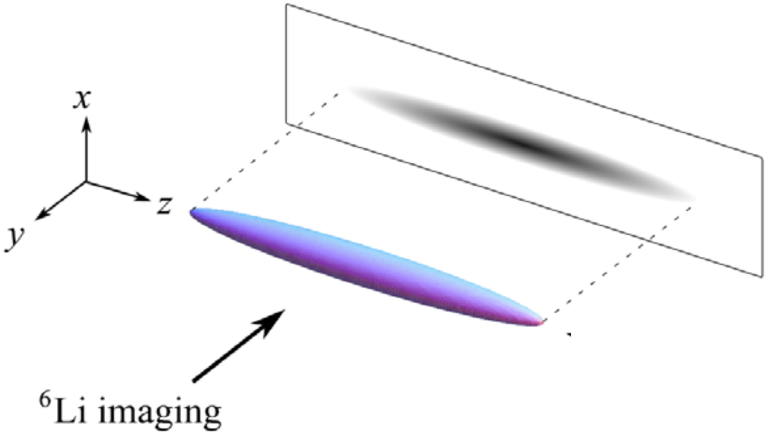
$$P(\mu_{1z}, \mu_{2z}, T) = \frac{m\omega_r^2}{2\pi}(\overline{n}_1(z) + \overline{n}_2(z))$$

Ho, T.L. & Zhou, Q., Nature Physics, 09

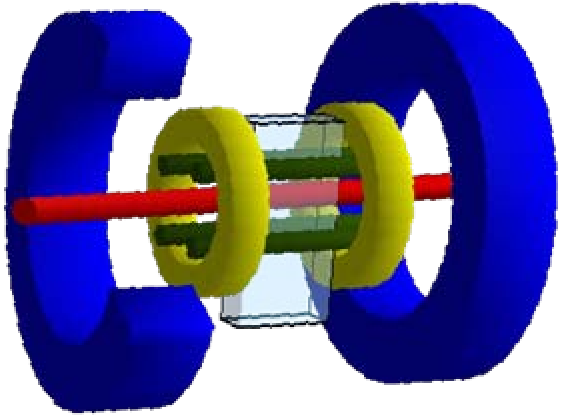
- $\overline{n}_i(z) = \int \mathrm{d}x\mathrm{d}y\, n_i(x,y,z)$

doubly-integrated density profiles

equation of state measured for all values of (μ_{1z}, μ_{2z}, T)



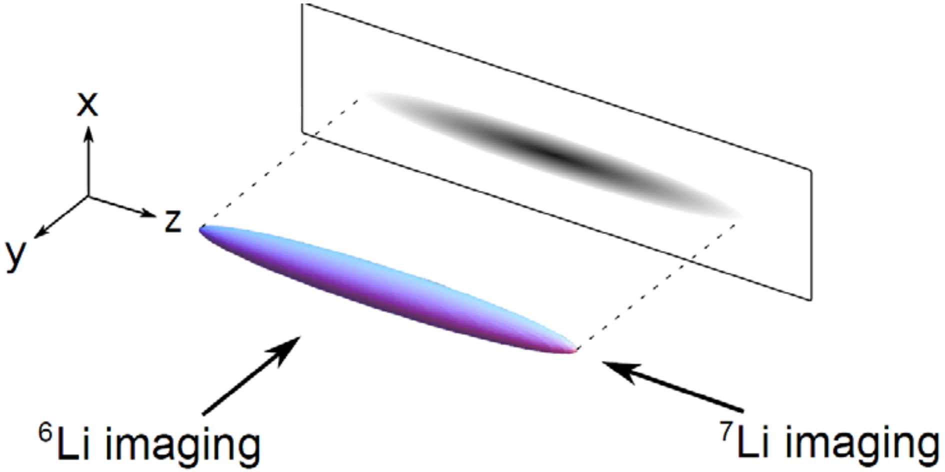
Experimental sequence



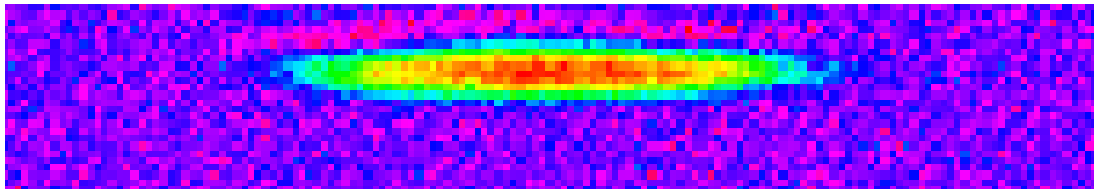
- Loading of ^{6}Li in the optical trap

- Tune magnetic field to Feshbach resonance
- Evaporation of ^{6}Li ^{7}Li mixture

- Image of ^{6}Li *in-situ*
- Image of ^{7}Li *in time of flight*



Spin balanced Unitary Fermi Gas

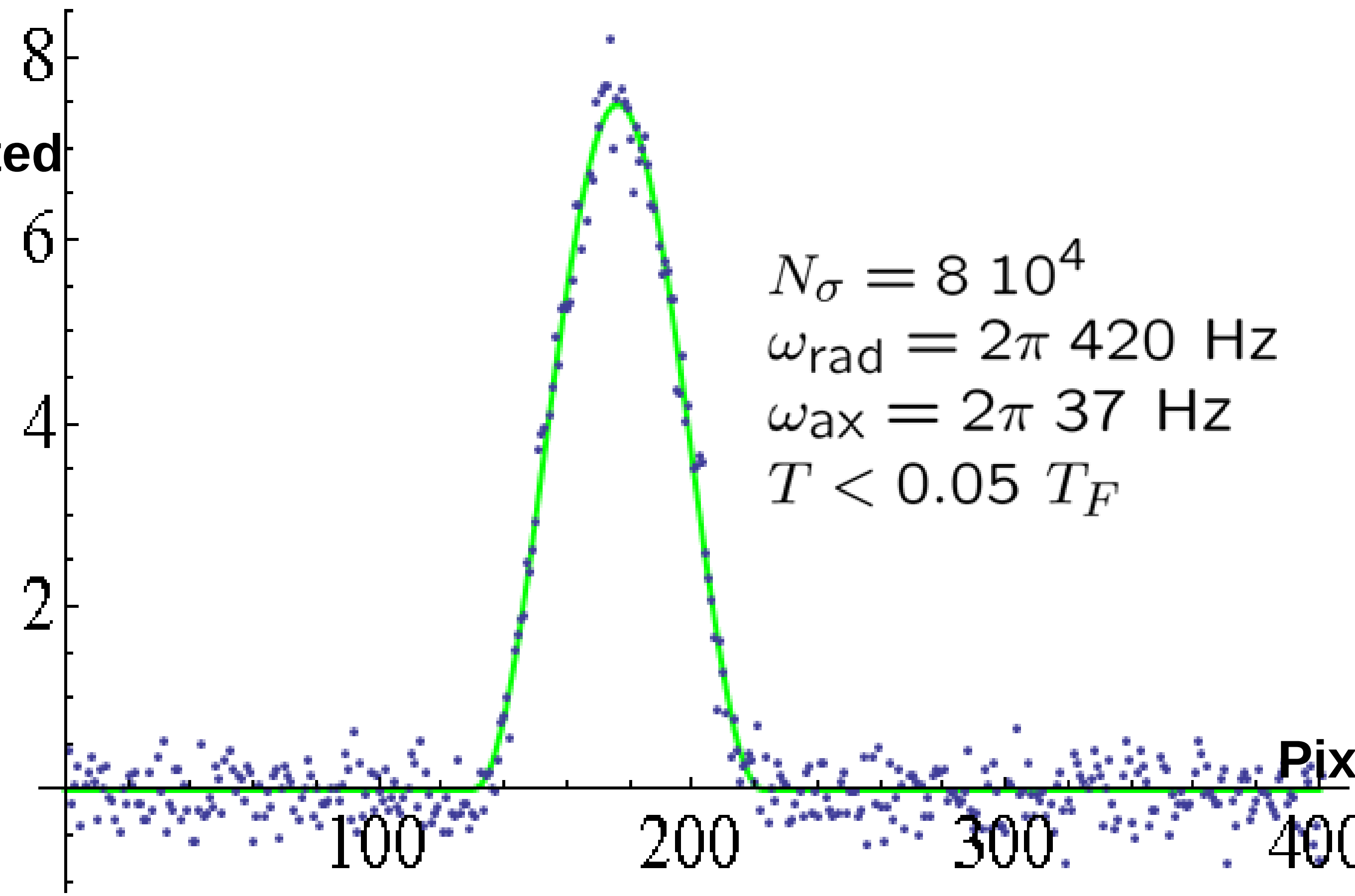


$a = \infty$

Pressure

Doubly integrated Density

(a.u)



The Equation of State at unitarity

$$1/k_F\, a = 0$$

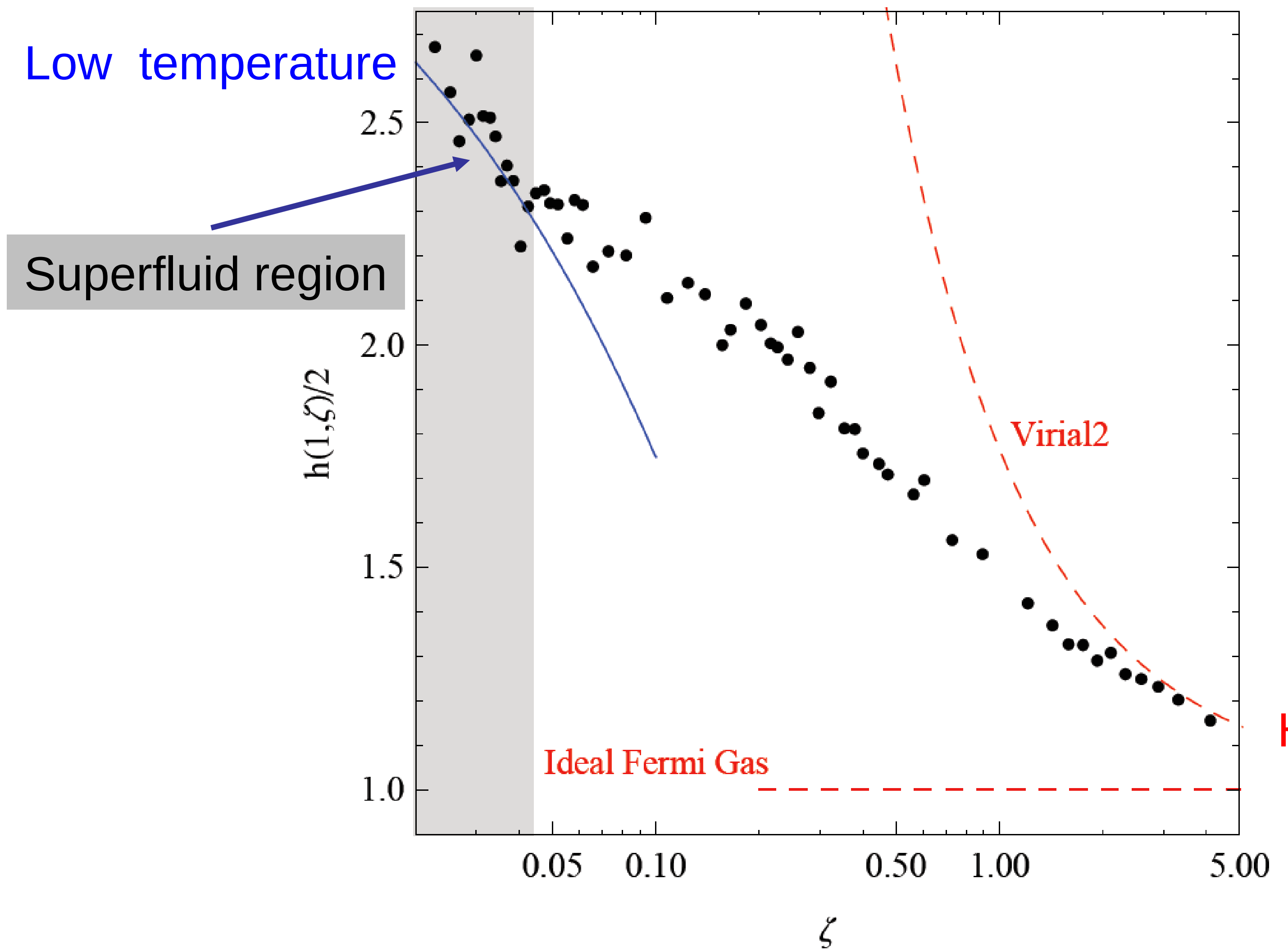
Thermodynamics is universal

J. Ho, E. Mueller, ‘04

S. Nascimbene et al., Nature, **463**, 1057, (Feb. 2010)

Equation of state of balanced gas

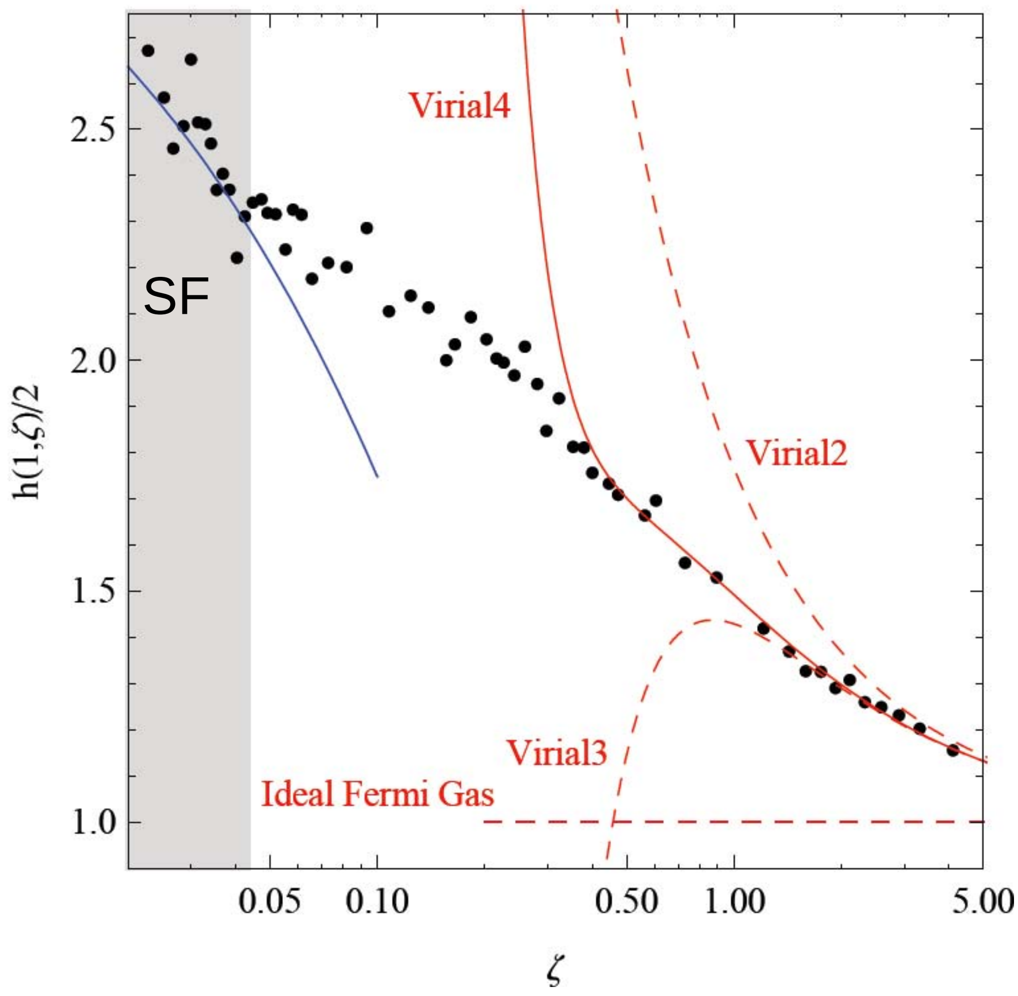
$$P(\mu, T) = P_1(\mu, T)h(1, \zeta)$$



$$\zeta = \exp\left(-\frac{\mu_1}{k_B T}\right)$$

High T : virial expansion

$$\frac{h(1,\zeta)}{2} = \frac{\sum_{n=1}^{\infty}\left(\left(-1\right)^{n+1}n^{-5/2} + b_n\right)\zeta^{-n}}{\sum_{n=1}^{\infty}(-1)^{n+1}n^{-5/2}\zeta^{-n}}$$



$$b_3 = -0.35(2)$$

$$b_3^{\text{th}} = -0.355$$

X. Liu et al., PRL 102, 160401 (2009)

$$b_3^{\text{th}} = 1.05$$

G. Rupak, PRL 98, 90403 (2007)

$$b_4 = 0.096(15)$$

No theoretical prediction
→ 4-body problem

Comparison with Many-Body Theories (1)

Diagram. MC

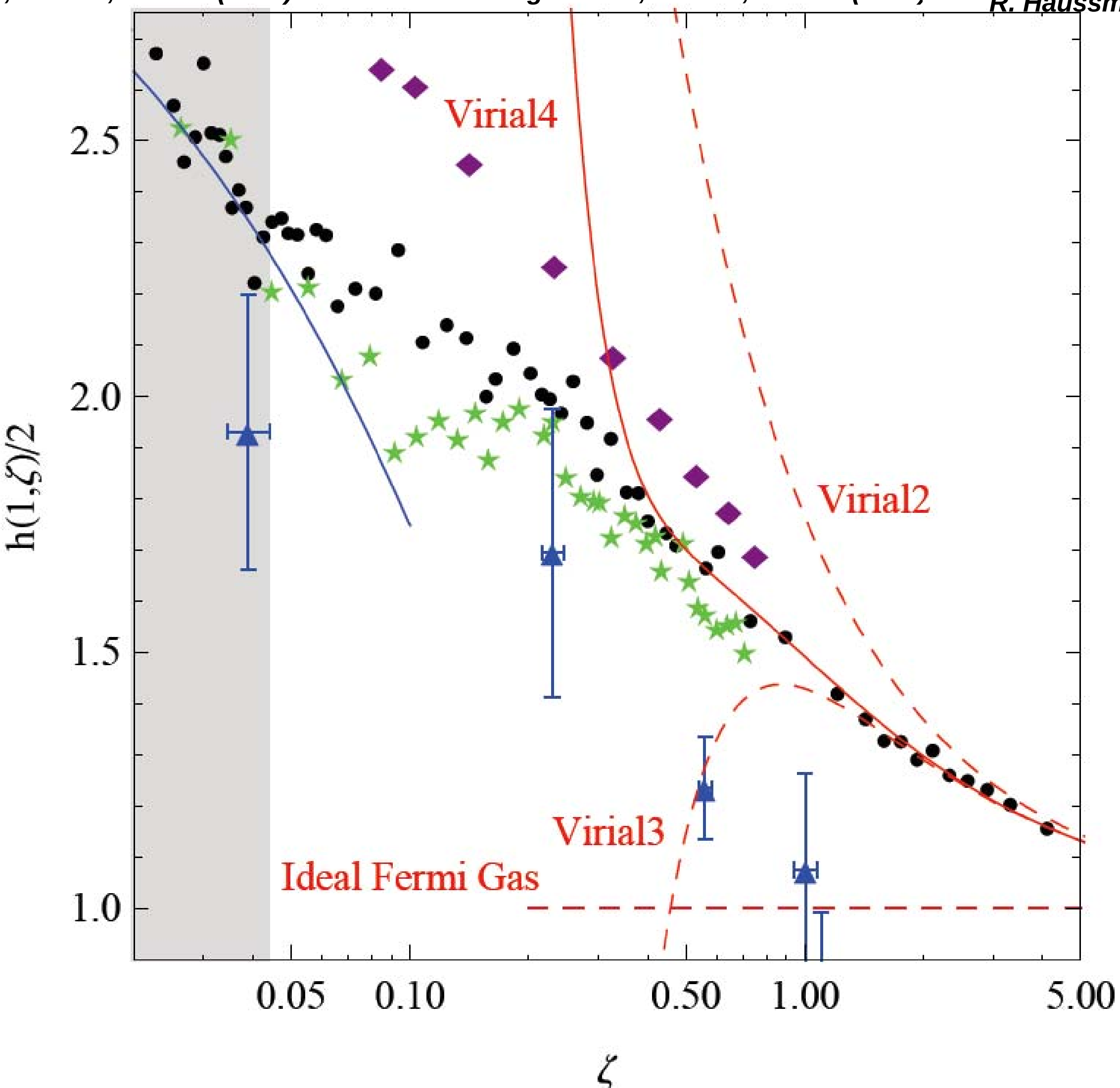
E. Burovski et. al., PRL 96, 160402 (2006)

QMC

A. Bulgac et al., PRL 99, 120401 (2006)

Diagram.+analytic

R. Haussmann. et al., PRA 75, 023610 (2007)



Comparison with Many-Body Theories (2)

Diagram. MC

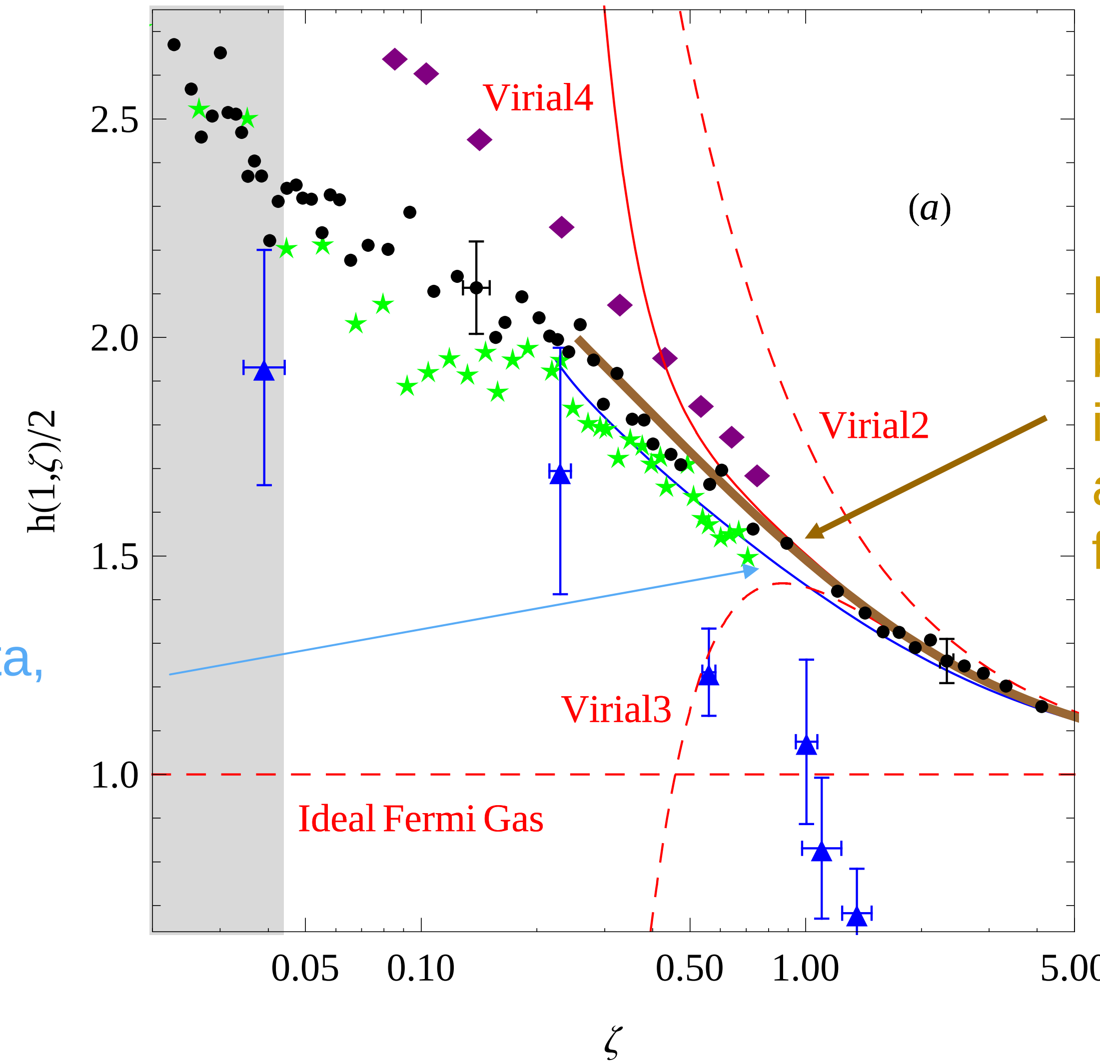
E. Burovski et. al., PRL 96, 160402 (2006)

QMC

A. Bulgac et al., PRL 99, 120401 (2006)

Diagram.+analytic

R. Haussmann. et al., PRA 75, 023610 (2007)

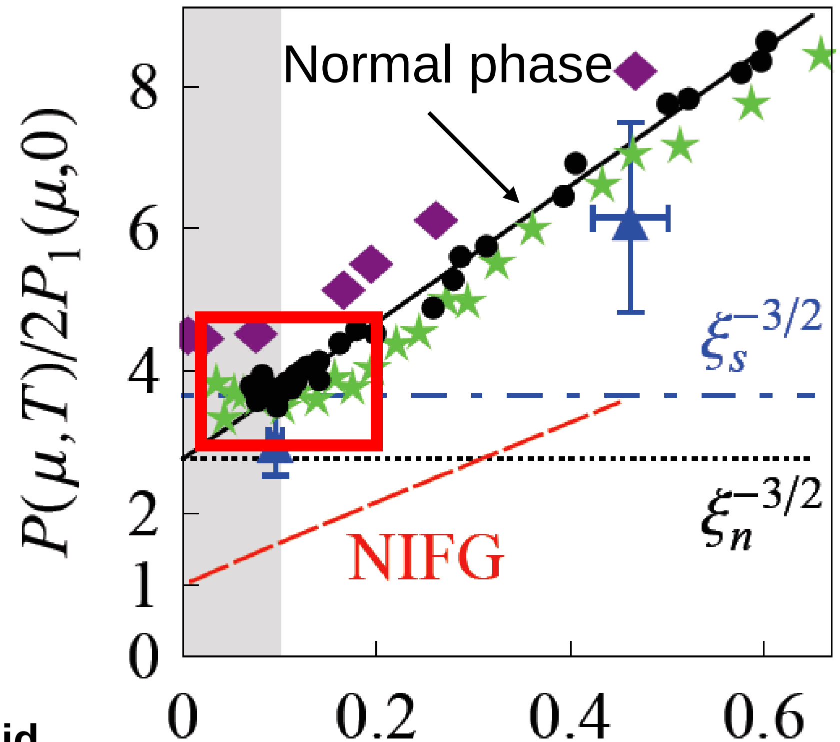


New Q. MC by Amherst in good agreement for $\varsigma \geq 0.2$

R. Combescot, Alzetta, Leyronas, PRA, 09

Low Temperature

- ● Exp. data
- ▲ *B. Svistunov, Prokofiev, 2006*
- ★ *A. Bulgac et al., PRL 99, 120401 (2006)*
- ◆ *R. Haussmann. et al., PRA 75, 023610 (2007)*



Superfluid at T = 0

$$P_s(\mu,0) = \xi_s^{-3/2} 2P_1(\mu,0)$$

$$\xi_s = 0.42$$

Normal phase : Landau theory of the Fermi liquid

$$P(\mu,T) = 2P_1(\mu,0)\left(\xi_n^{-3/2} + \frac{5\pi^2}{8}\xi_n^{-1/2}\frac{m^*}{m}\left(\frac{k_B T}{\mu}\right)^2\right)$$

we find : $\xi_n = 0.51(2)$ $\qquad \xi_n^{\text{th}} = 0.56$

$$m^*/m = 1.13(3)$$

C. Lobo et al., PRL 97, 200403 (2006)

Normal-Superfluid phase transition

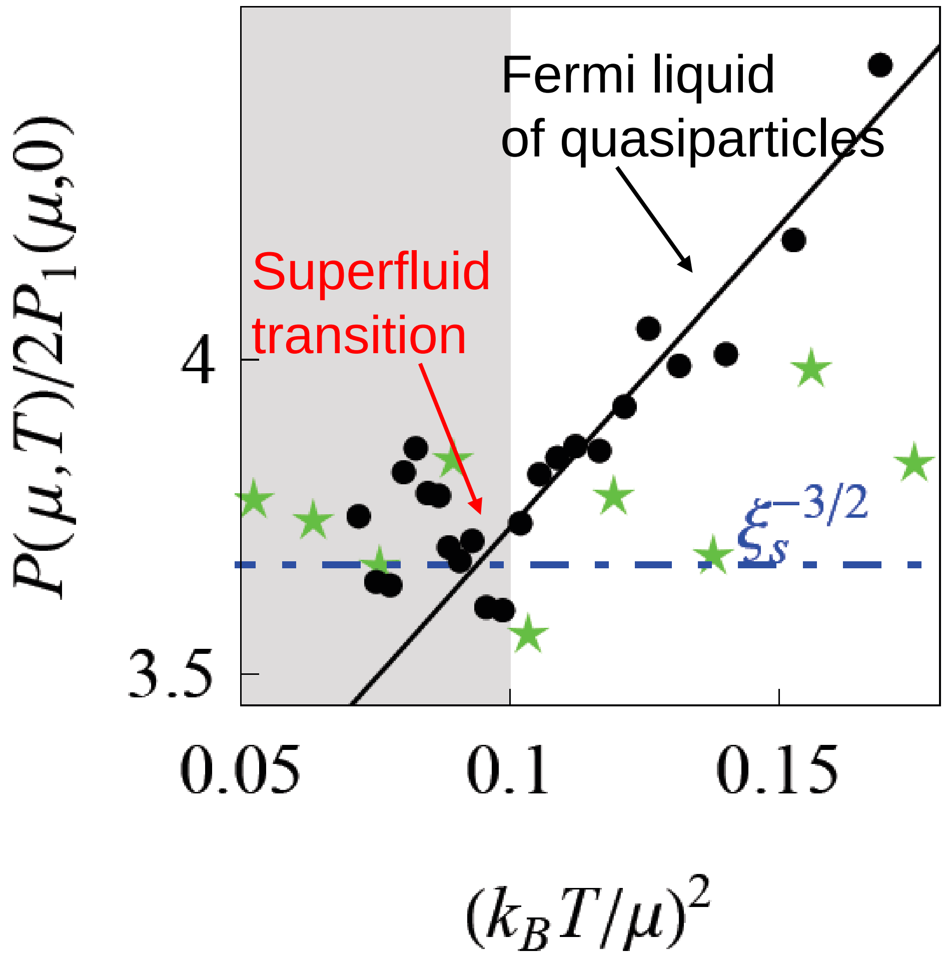
We find the critical parameters

$$(k_B T/\mu)_c = 0.32(3)$$

$0.32(2)$	▲	*E. Burovski et. al., PRL 96, 160402 (2006)*
0.24		*K.B. Gubbels and H.T.C Stoof, PRL 100, 140407 (2008)*
0. 32	★	*A. Bulgac et al., PRA, 78, (2008)*
0.41	◆	*R. Haussmann. et al., PRA 75, 023610 (2007)*

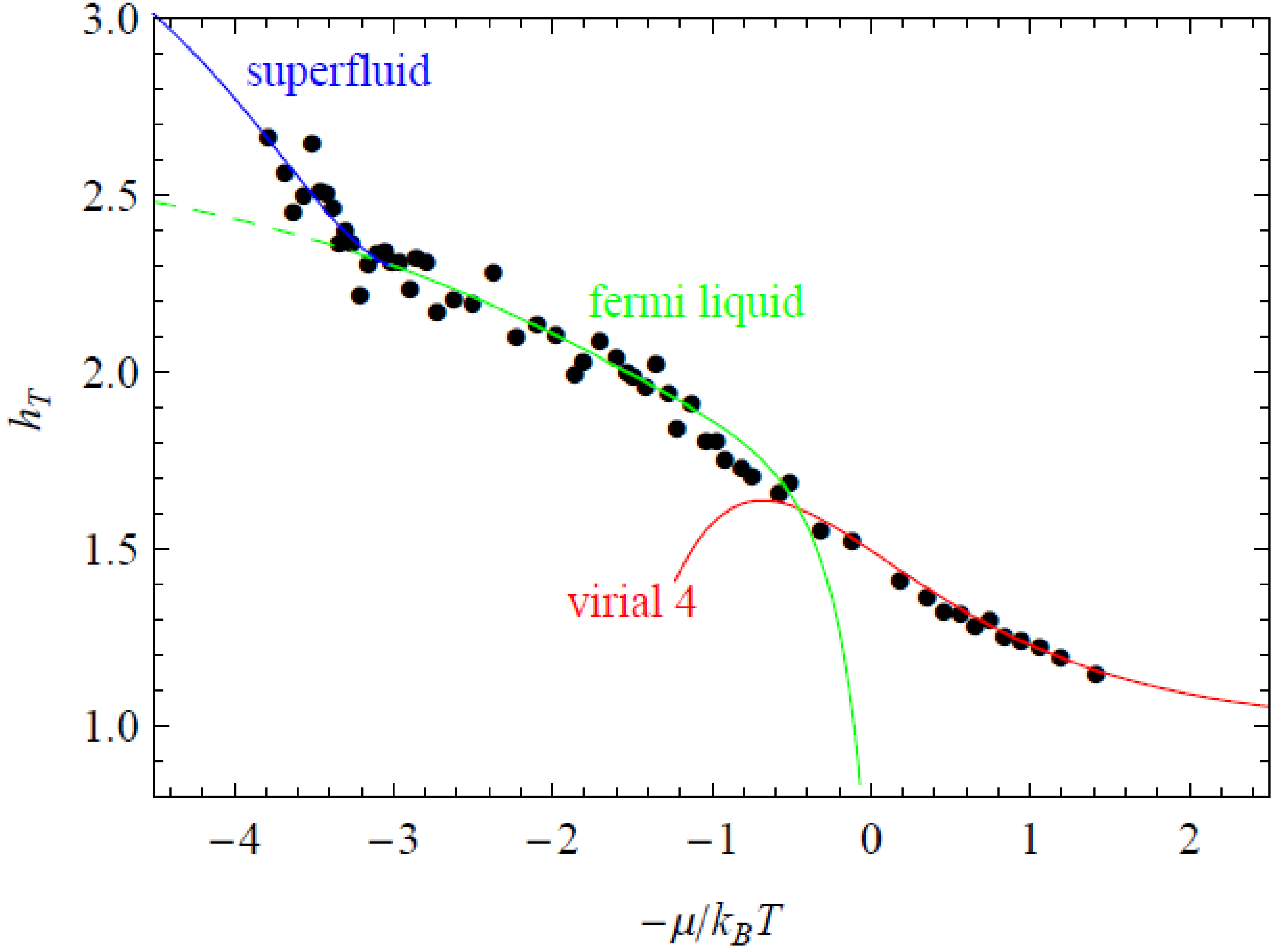
$(\mu/E_F)_c = 0.49\ (2)$

also $T_c = 0.157(15) T_F$



Good agreement with theory, with Riedl et al.,
and with M. Horikoshi, *et al. Science* **327**, 442 (2010);

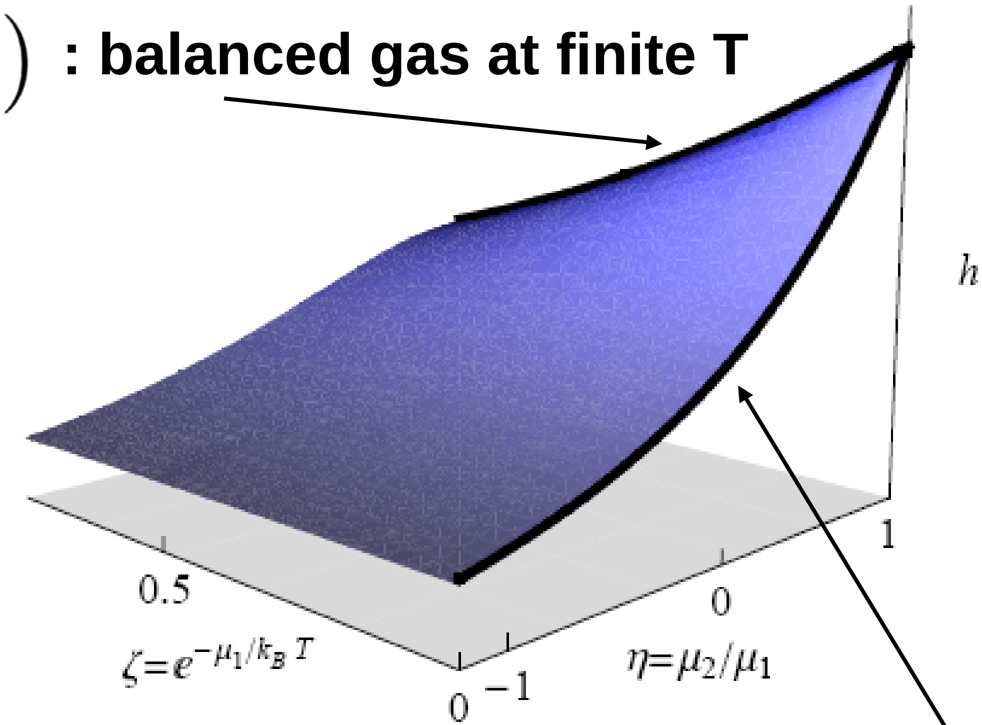
Summary (1): balanced gas at finite T



Exploring the spin imbalanced gas at zero temperature

$$P(\mu_1, \mu_2, T) = P_1(\mu_1, T) h(\eta, \zeta)$$

$(1, \zeta)$ **: balanced gas at finite T**



$$\eta = \mu_2/\mu_1$$

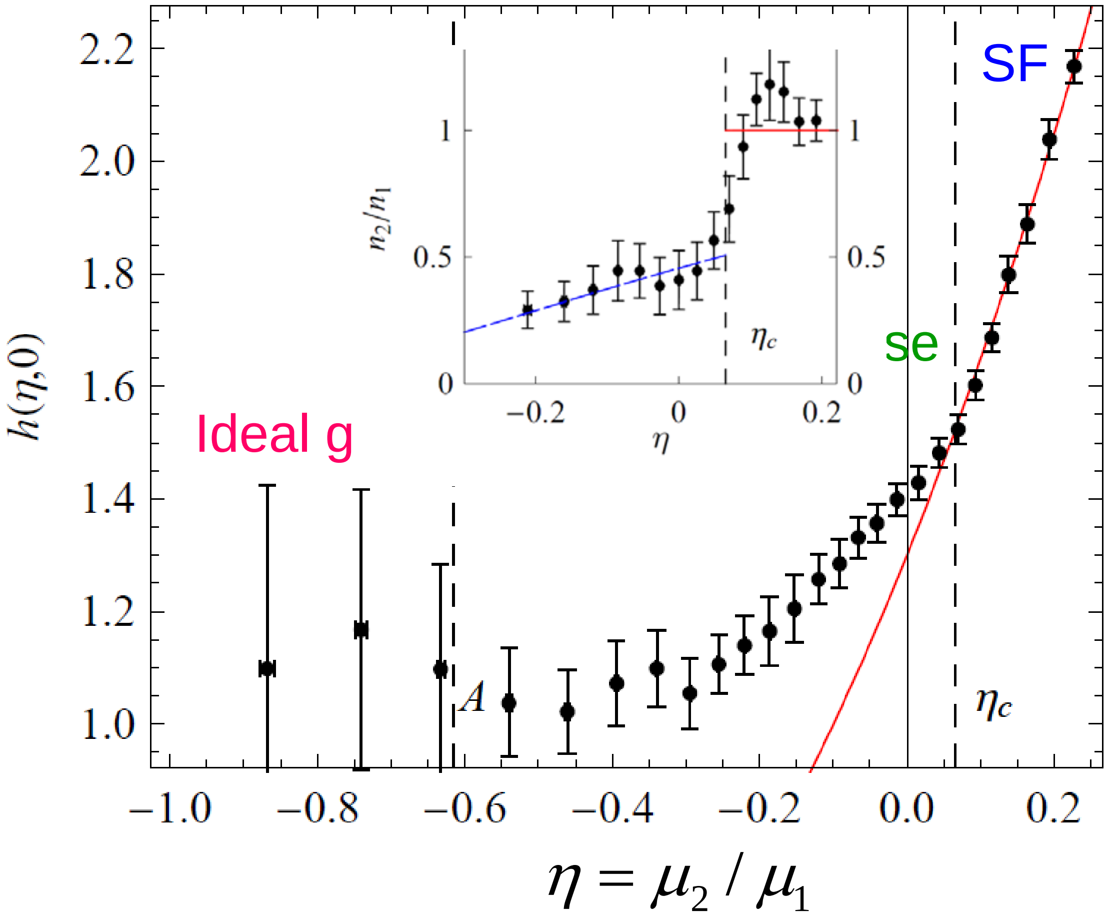
$$\zeta = \exp\left(-\frac{\mu_1}{k_B T}\right)$$

Inverse of the fugacity

$(\eta, 0)$ **: imbalanced gas at T=0**

MIT ’06,: 3 phases, RICE ’06: 2 phases, ENS ’09: 3 phases

Equation of state h(η, 0) i.e.T=0



$$h_s(\eta,0) = \frac{1}{(2\xi_s)^{3/2}}(1+\eta)^{5/2}$$

Deviation from h_s at

$$\boxed{\eta_c = 0.065(20)}$$

T=0 SF-Normal Phase Transition

$\eta_c = 0.02$ **Fixed-Node MC**

$\eta_c = 0.03(2)$

MIT: Y. Shin, PRA 08,

EoS and phase diagram

$$h(\eta,0) = \begin{cases} \frac{1}{(2\xi_s)^{3/2}}(1+\eta)^{5/2} & \text{if } \eta > \eta_c \\ h_n(\eta,0) & \text{if } A < \eta < \eta_c \\ 1 & \text{if } \eta < A \end{cases}$$

The Equation of State
in the BEC-BCS crossover

$$1/k_F\, a \neq 0$$

The ground state: T=0

N. Navon, S. Nascimbène, F. Chevy, and C. Salomon,
Science **328**, 729 (2010)

Ground state of a tunable Fermi gas

- Single-component Fermi gas:

$$P_0(\mu_1) = \frac{1}{15\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\mu_1^{5/2}$$

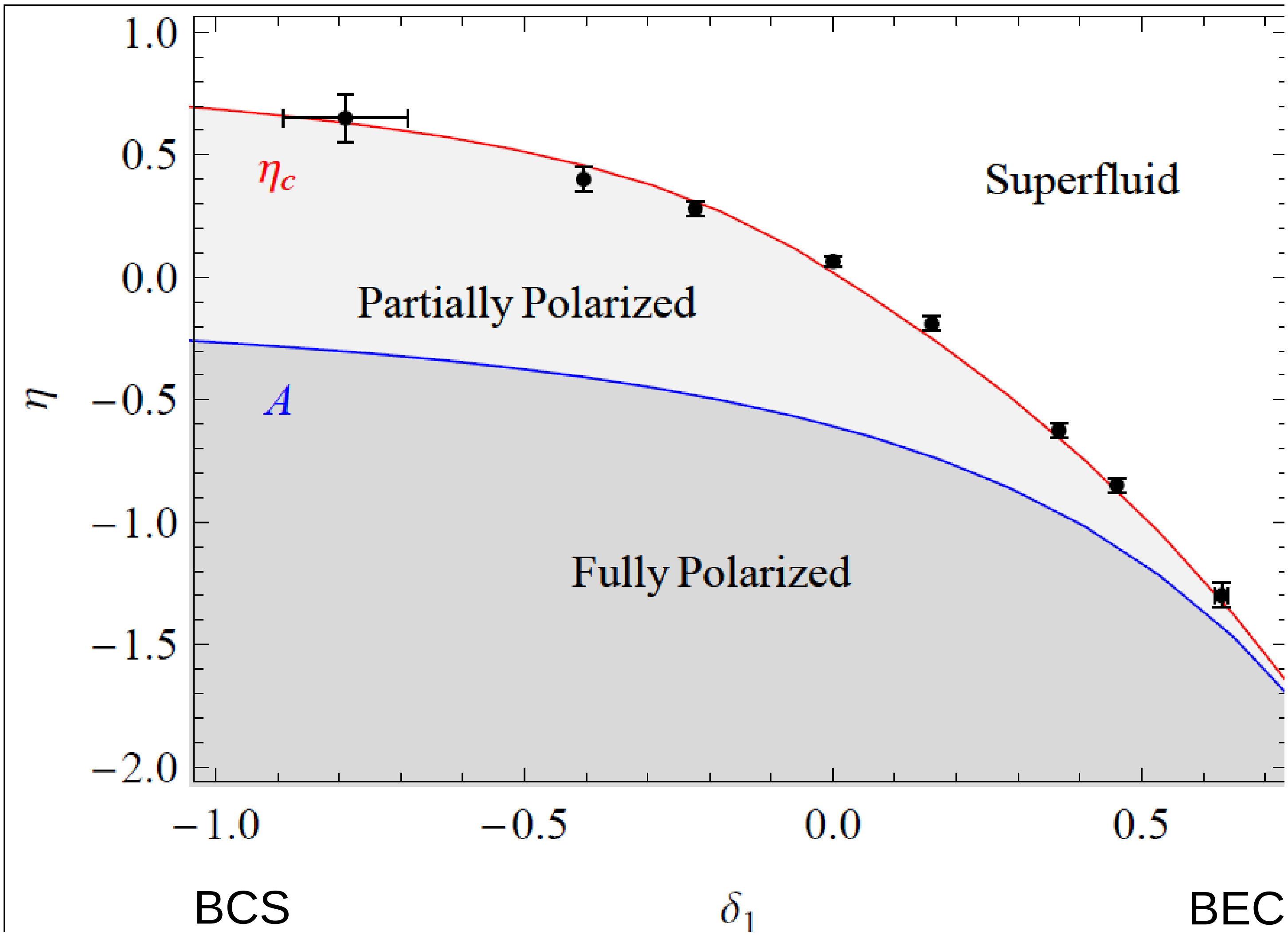
- Two-component Fermi gas

$$P(\mu_1, \mu_2, a) = P_0(\mu_1) h\left(\delta_1 = \frac{\hbar}{\sqrt{2m\mu_1}a}, \eta = \frac{\mu_2}{\mu_1}\right)$$

δ_1: grand-canonical analog of $1/k_{F1}a$

η: chemical potential imbalance

Phase diagram



Superfluid Equation of State

Full pairing:

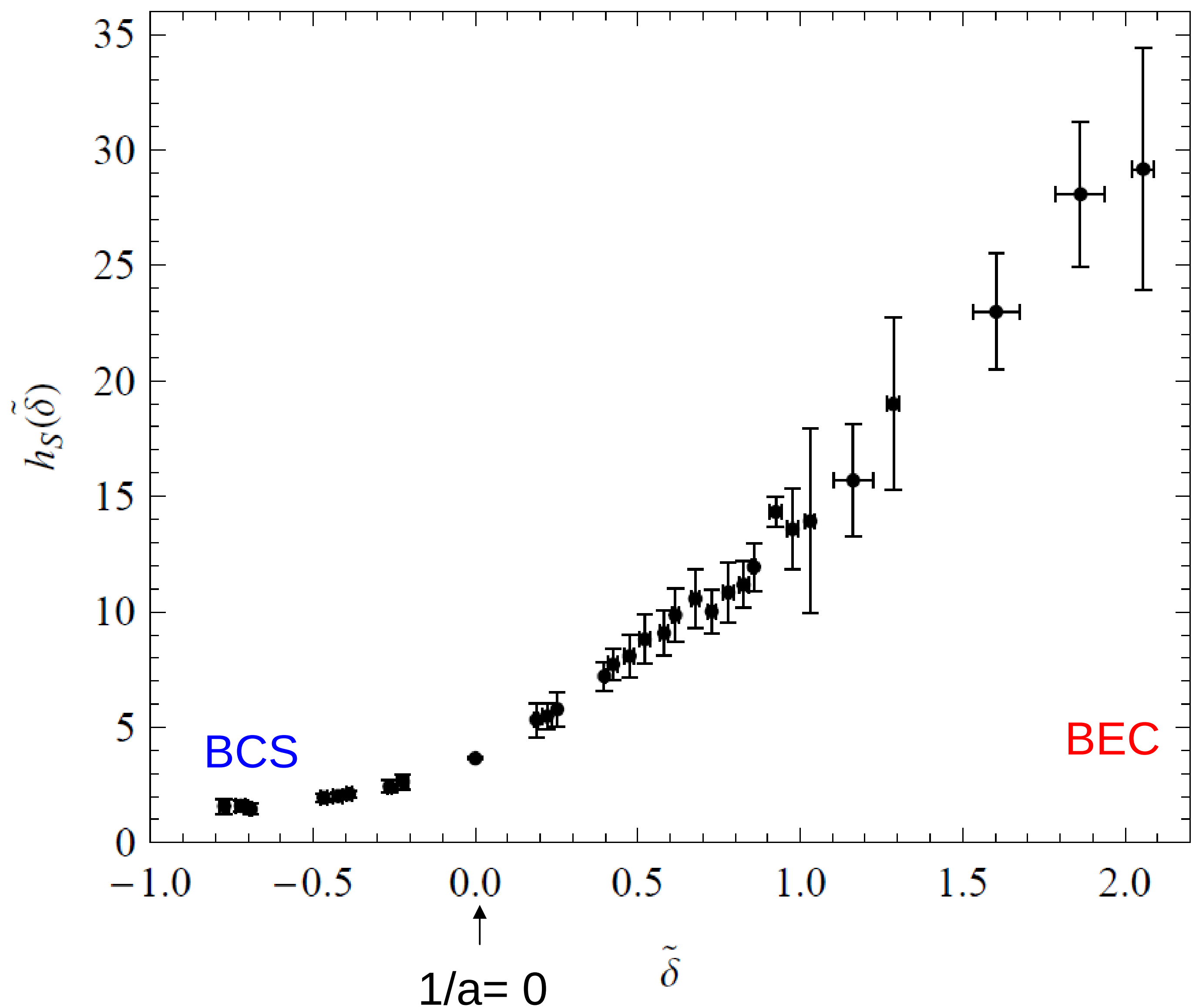
$$n_1 = \frac{\partial P}{\partial \mu_1} = n_2 = \frac{\partial P}{\partial \mu_2} \quad \Rightarrow \quad P(\mu_1, \mu_2, a) = P\left(\frac{\mu_1+\mu_2}{2}, \frac{\mu_1+\mu_2}{2}, a\right)$$

Symmetric parametrization:

$$P(\mu_1, \mu_2, a) = P_0\left(\frac{\mu_1+\mu_2}{2}\right) h_S(\widetilde{\delta})$$

$$\widetilde{\delta} = \frac{\hbar}{\sqrt{2m\left(\frac{\mu_1+\mu_2}{2} - E_b/2\right)}a}, \quad E_b = \begin{cases} -\frac{\hbar^2}{ma^2} & (a>0) \\ 0 & (a<0) \end{cases}$$

Superfluid Equation of State in the Crossover



Asymptotic behaviors

BCS limit:

$$E = \frac{3}{5} N E_F \left(1 + \frac{10}{9\pi} k_F a + \frac{4(11 - 2\log 2)}{21\pi^2} (k_F a)^2 \dots \right)$$

mean-field

Lee-Yang correction

BEC limit

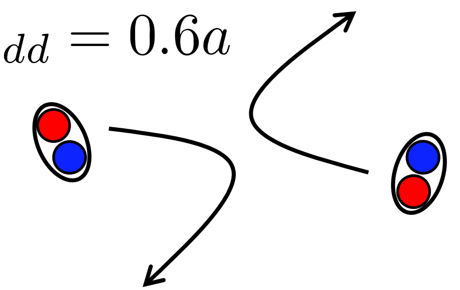
$$E = \frac{N}{2} E_b + N \frac{\pi \hbar^2 a_{dd}}{2m} n \left(1 + \frac{128}{15\sqrt{\pi}} \sqrt{n a_{dd}^3} + \dots \right)$$

molecular binding energy

mean-field

Lee-Huang-Yang correction

$$a_{dd} = 0.6a$$



Unitary limit

$$E = \frac{3}{5} N E_F \left(\xi_s - \zeta \frac{1}{k_F a} + \dots \right)$$

$$\mu = \xi_s E_F$$

$$C = \frac{2\zeta}{5\pi} k_F^4$$

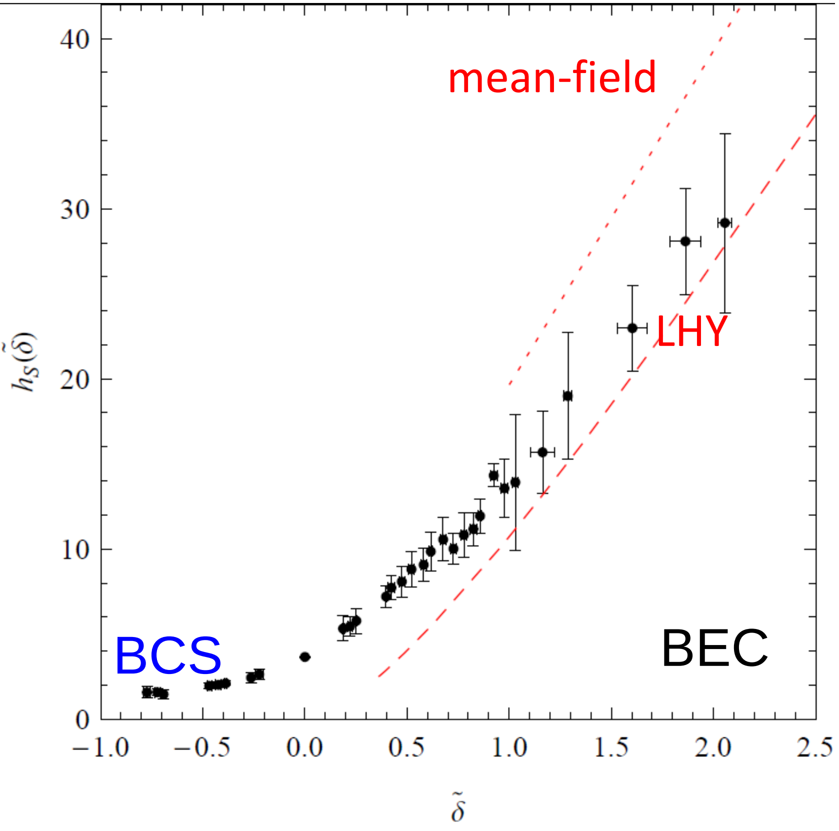
We get: ξ_s= 0.41(1)

contact coefficient

ζ= 0.93(5)

Measurement of the Lee-Huang-Yang correction

$$E = \frac{N}{2}E_b + N\frac{\pi\hbar^2 a_{dd}}{2m}n\left(1 + \frac{128}{15\sqrt{\pi}}\sqrt{na_{dd}^3} + \dots\right)$$



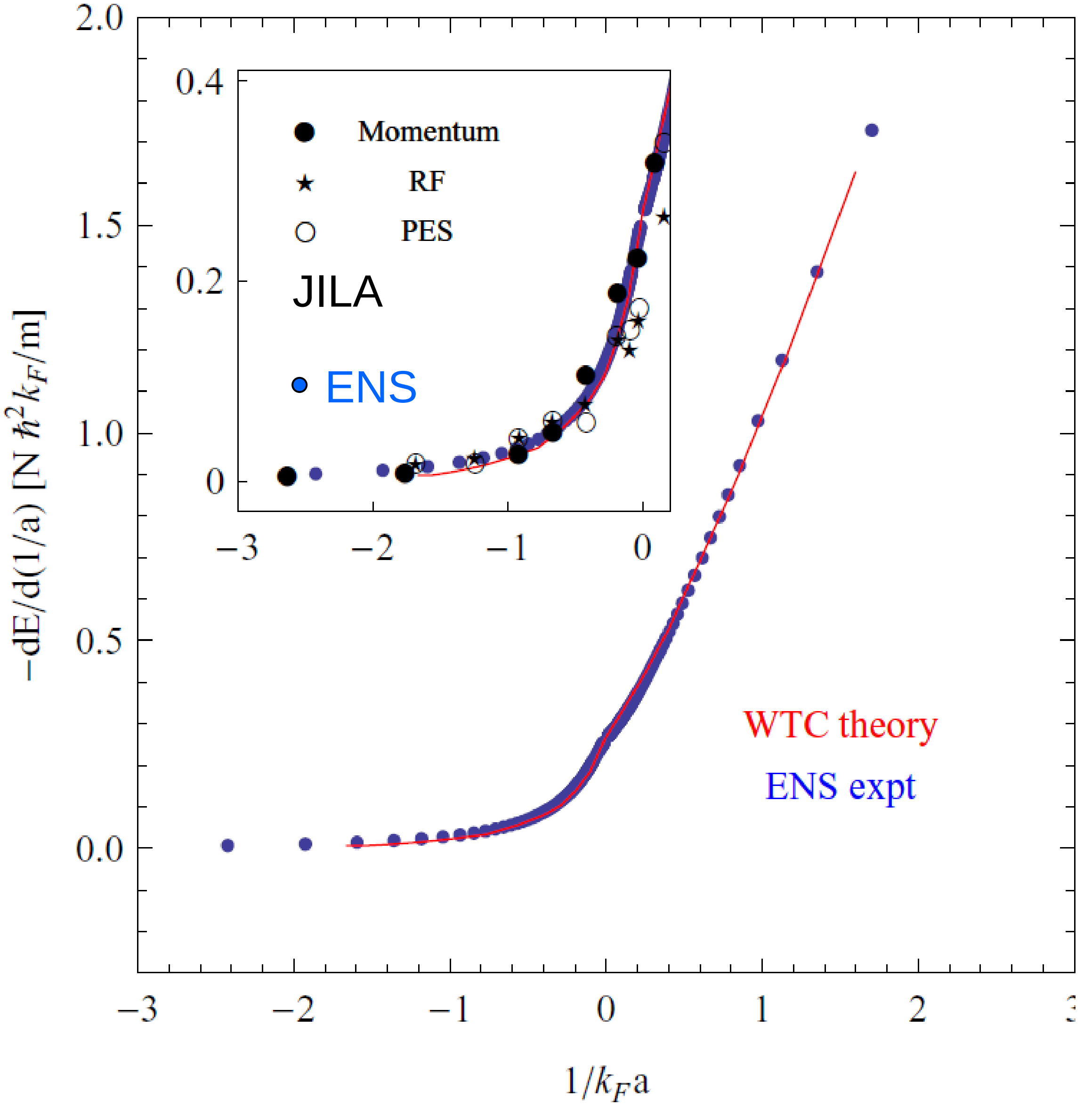
Fit of the LHY coefficient: 4.4(5)

theory: $\frac{128}{15\sqrt{\pi}} \simeq 4.81$

No effect of the composite nature of the dimers

X. Leyronas *et al*, PRL **99**, 170402 (2007)

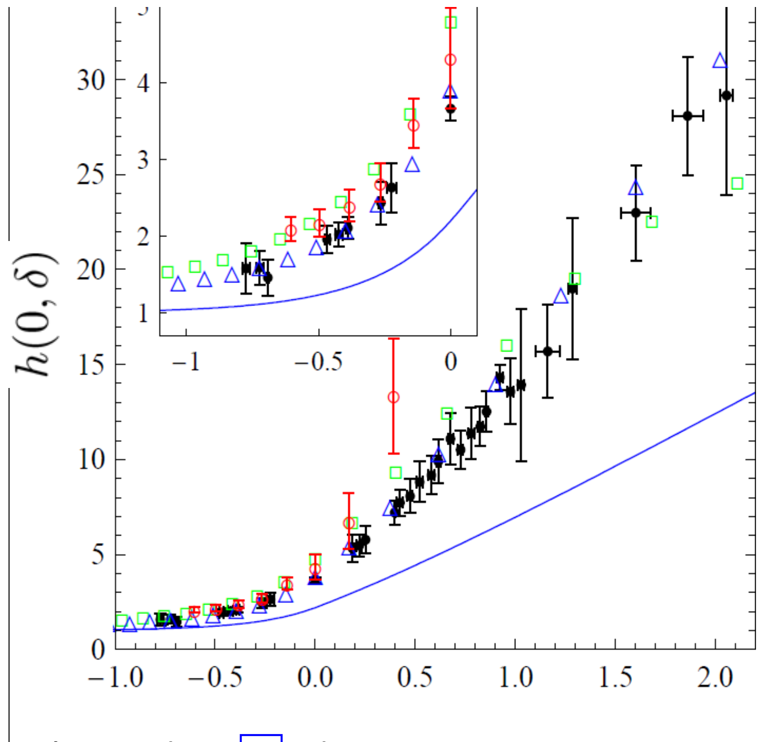
Contact coefficient



Direct Comparison to Many-Body Theories

Grand-Canonical – Canonical Ensemble

$$P(\mu, a, T=0) = P_1(\mu, 0)h(0,\delta)$$

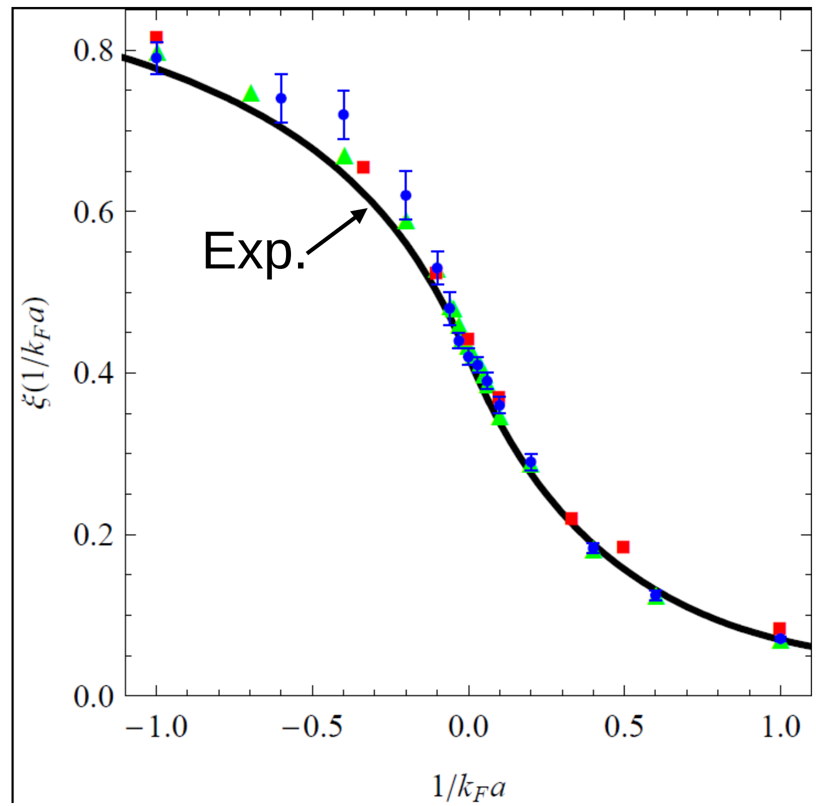


△ Nozières-Schmitt-Rink approximation
Hu *et al*, EPL **74**, 574 (2006)

□ Diagrammatic theory
Haussmann *et al*, PRA **75**, 23610 (2007)

○ Quantum Monte Carlo
Bulgac *et al*, PRA **78**, 23625 (2008)

$$E = \frac{N}{2}E_b + \frac{3}{5}NE_F\xi\left(\frac{1}{k_F a}\right)$$



Fixed-Node Monte-Carlo theories

■ Chang *et al*, PRA **70**, 43602 (2004)

● Astrakharchik *et al*, PRL **93**, 200404 (2004)

▲ Pilati *et al*, PRL **100**, 030401 (2008)

Conclusion - Perspectives

- **EOS of a uniform Fermi gas at unitarity in two sectors**

  1) balanced gas at finite T

  2) T = 0 imbalanced gas

- **Precision Test of Many-body Theories**

- **EoS in the BEC-BCS crossover at T=0**

  - **First quantitative measurement of Lee-Huang-Yang quantum corrections and Lee-Yang on BCS side**

  - **Simple description of the normal phase as two ideal gases on BEC and unitary; breakdown on BCS side**

- **Next: Mapping the EOS in the complete (η, ζ) space**

→ imbalanced gas at finite T , mass imbalance

- **Lattice experiments**