KITP, Santa Barbara, 11 October 2010

# SPIN FLUCTUATIONS AND MAGNETIC SUSCEPTIBILITY OF A NEARLY FERROMAGNETIC FERMI GAS

**Alessio Recati and Sandro Stringari**

arXiv:1007.4504

**University of Trento**

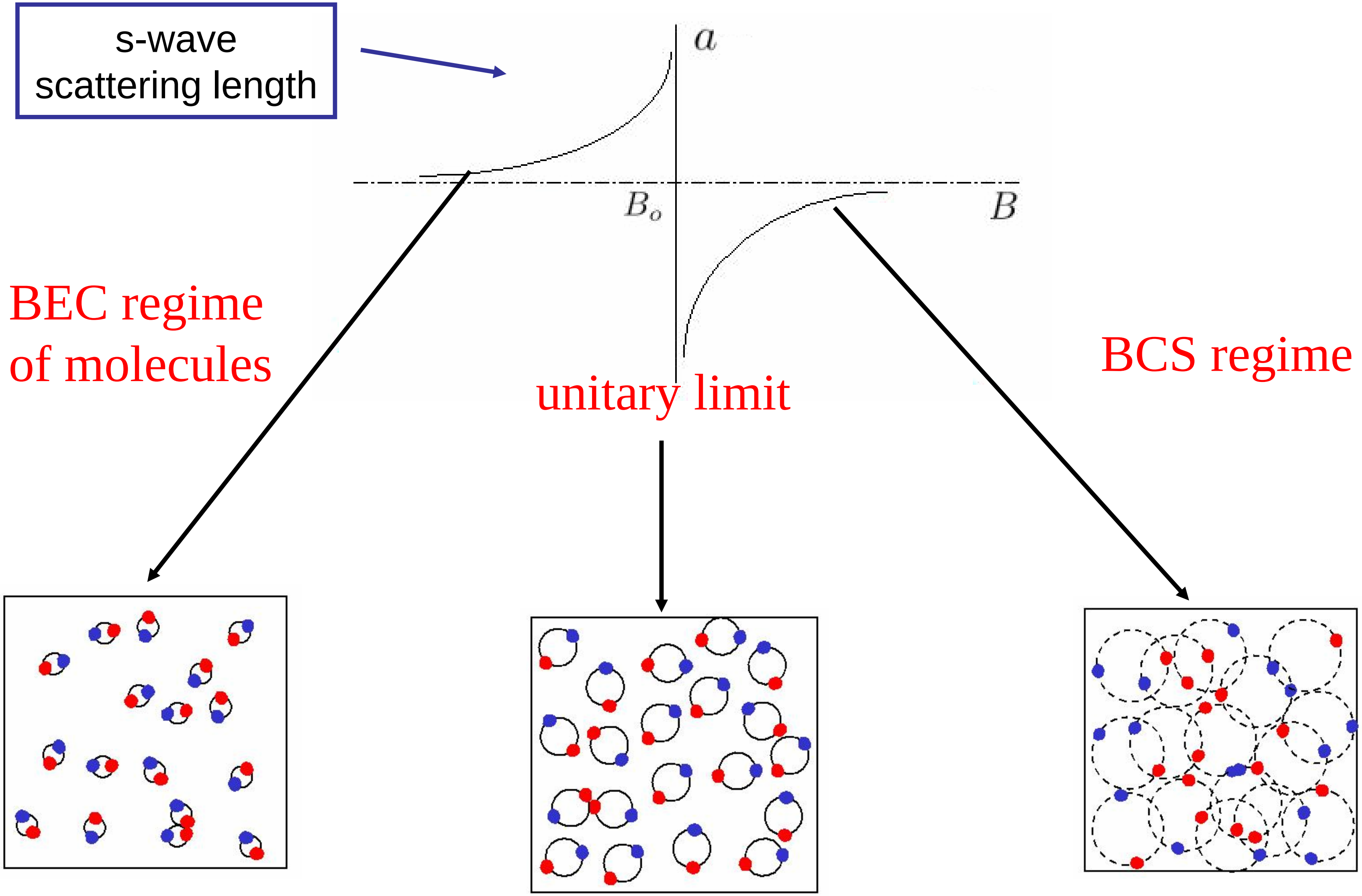
**CNR-INO**

**Experimental realization of BEC-BCS crossover:**

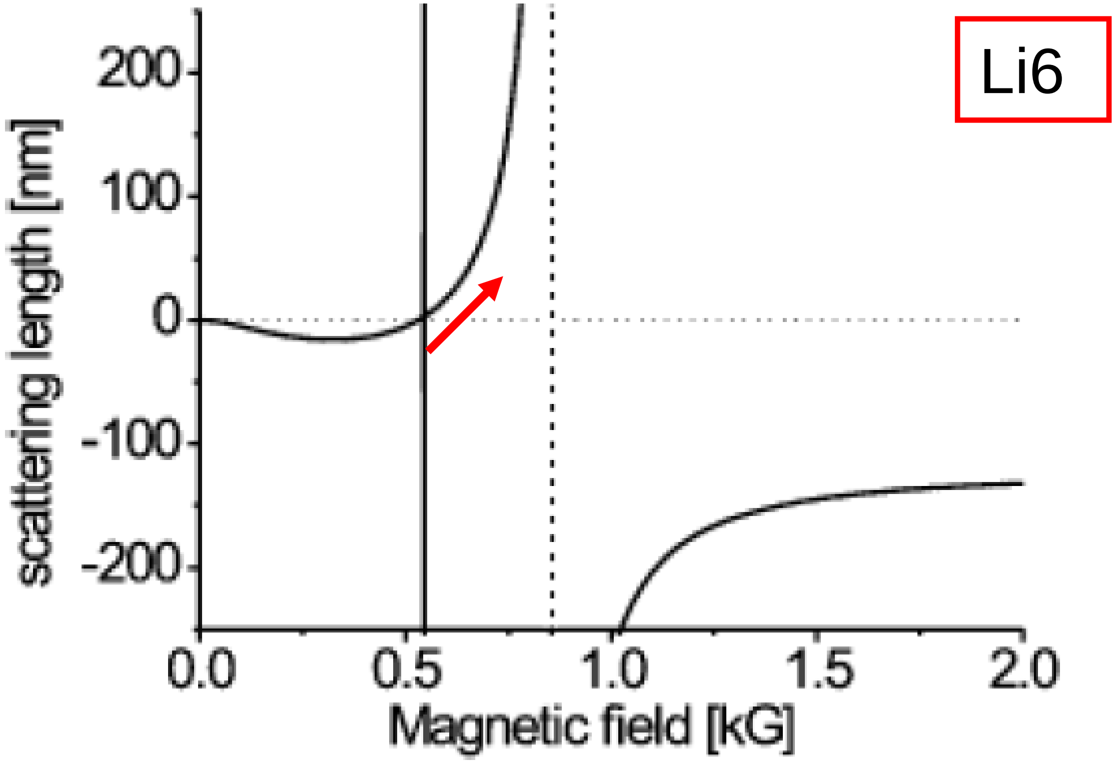
- **one of the most important achievements of ultracold atomic physics**

- **valuable contribution to our understanding of (high Tc) superconductivity**

# Many-body features of a Fermi gas near a Feshbach resonance

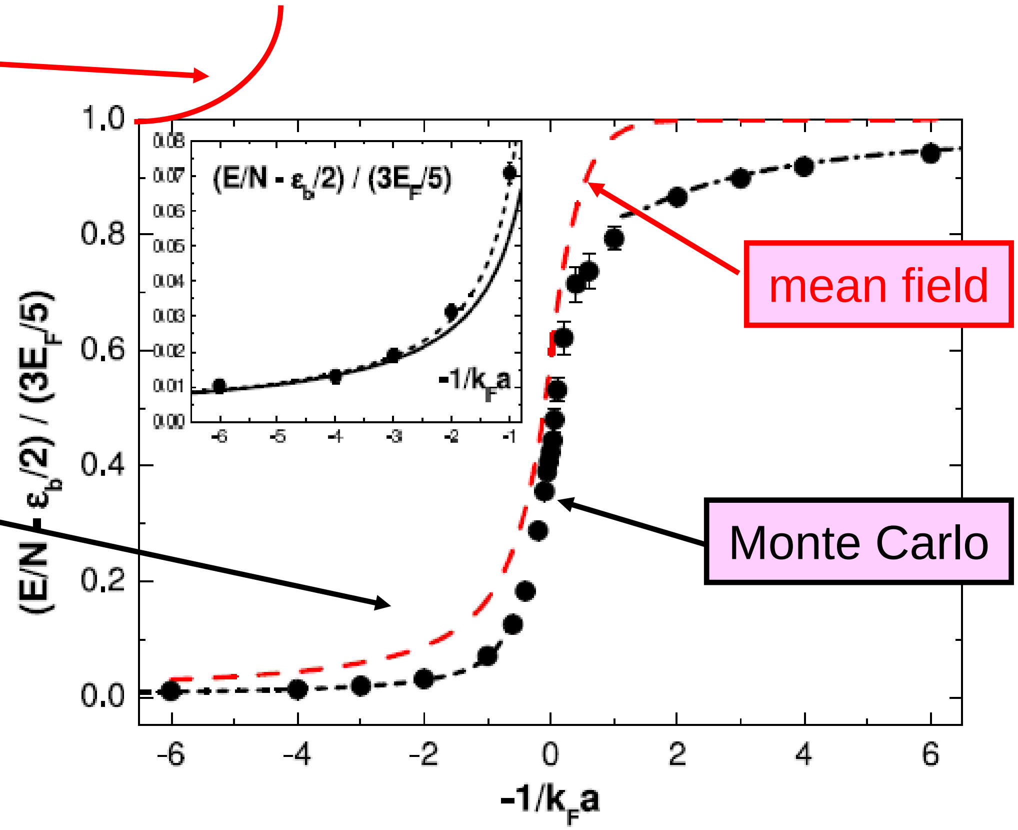


# On the BEC side additional branch occurs




**What happens if value of a is increased adiabatically starting from a=0 ?**

Metastable branch
small a: Fermi gas of
**hard spheres**
(Lee-Yang 1957)



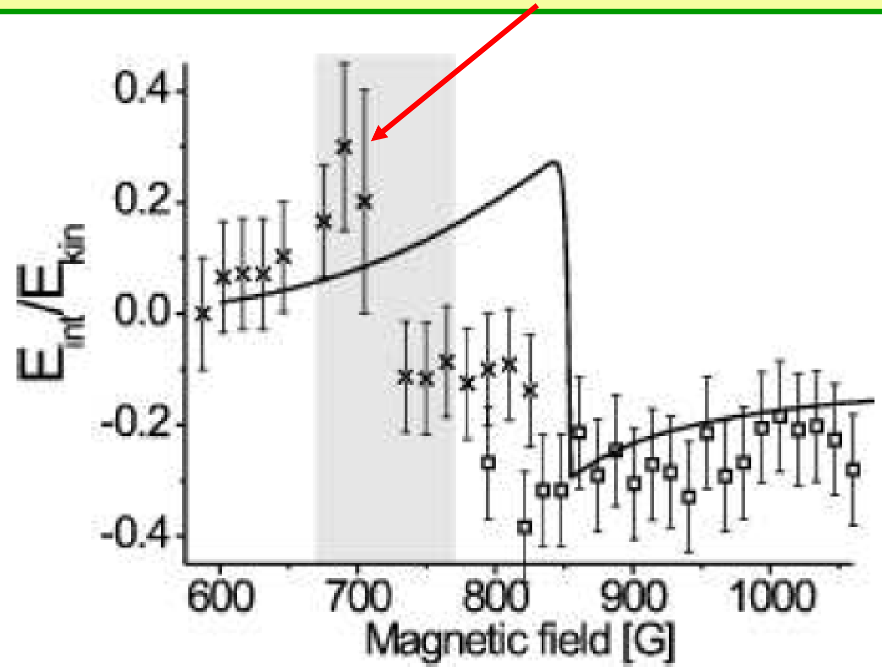

Superfluid molecular
BEC branch
Eq of State now
measured with high
precision
(see Salomon's talk)

Possible scenarios for the fate (**large a**) of the repulsive branch

- **Solidification** (liquid-solid transition in He3)
- **Transition** to superfluid **molecular BEC** phase
- Itinerant **ferromagnetism** (Mit experiment)
- Metastable **combination** of **FM** and **SF**

First experimental evidence for repulsive branch (ENS 2003)

(**interaction energy is positive** !!)




T. Bourdel et al. PRL 2003

Recent experiments at MIT (Jo et al. Science, 2009) suggest occurrence of a transition of the repulsive branch to a **ferromagnetic** phase when the scattering length is positive and exceeds a **critical value**

**However**: Spin domains are **not observed** above the transition

Rich (and not exhausted) debate about nature of the FM phase in resonantly interacting Fermi gases (mainly on role of **metastablity** and **adiabaticity**)

Ho, Conduit, Simons, Zhai, Altman, Demler …

**Assuming equilibrium** one can compare experiment with predictions of many-body theories (mean field **Stoner** model, **Monte Carlo** simulations)

## Stoner model

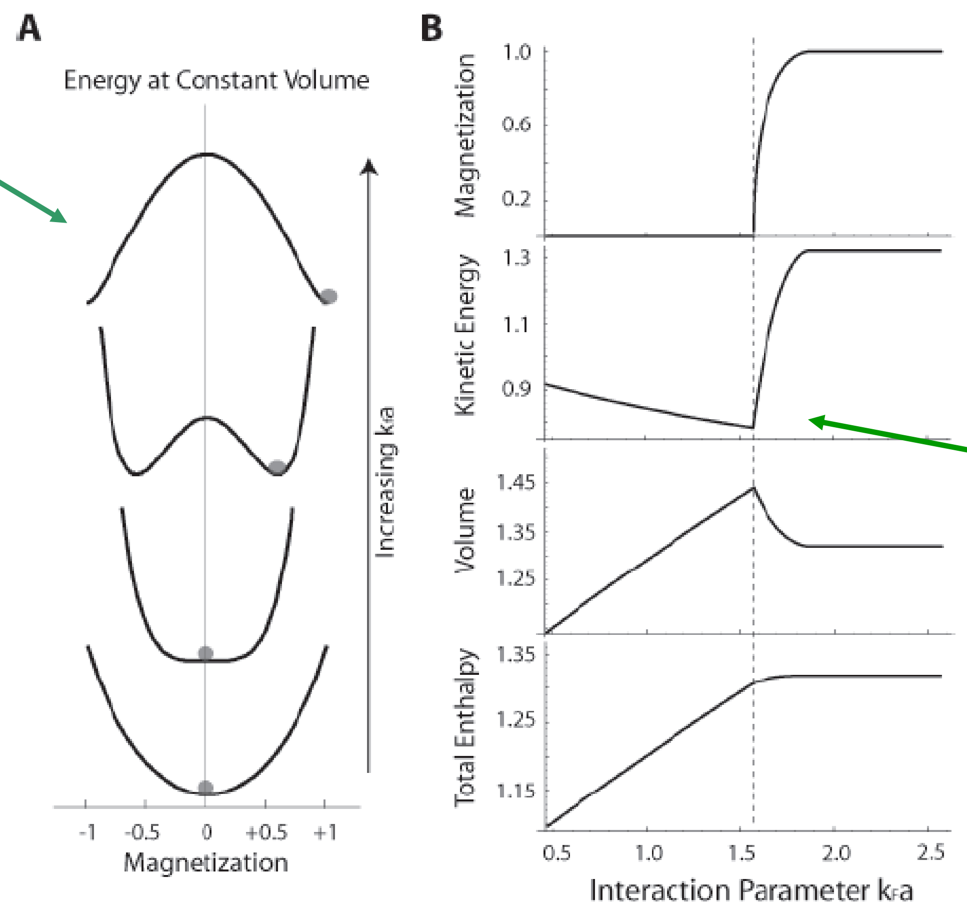
based on mean field density functional

$$E(M) = \int d\vec{r}\left[\frac{3\hbar^2}{10\,m}\left(6\pi^2\right)^{2/3}\left(n_\uparrow^{5/3} + n_\downarrow^{5/3}\right) + \frac{4\pi\hbar^2 a}{m} n_\uparrow n_\downarrow\right]$$

Where $M = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}$ is gas magnetization

# PREDICTIONS OF **STONER MODEL**

E(M) at constant volume




Physical quantities at constant pressure

Minimum in kinetic energy at the FM transition

Magnetic instability (Ferromagnetism) occurs at $k_F a = \pi / 2 \approx 1.6$ (second order transition)

Magnetic instability is associated with **divergent behavior** of magnetic suceptibility

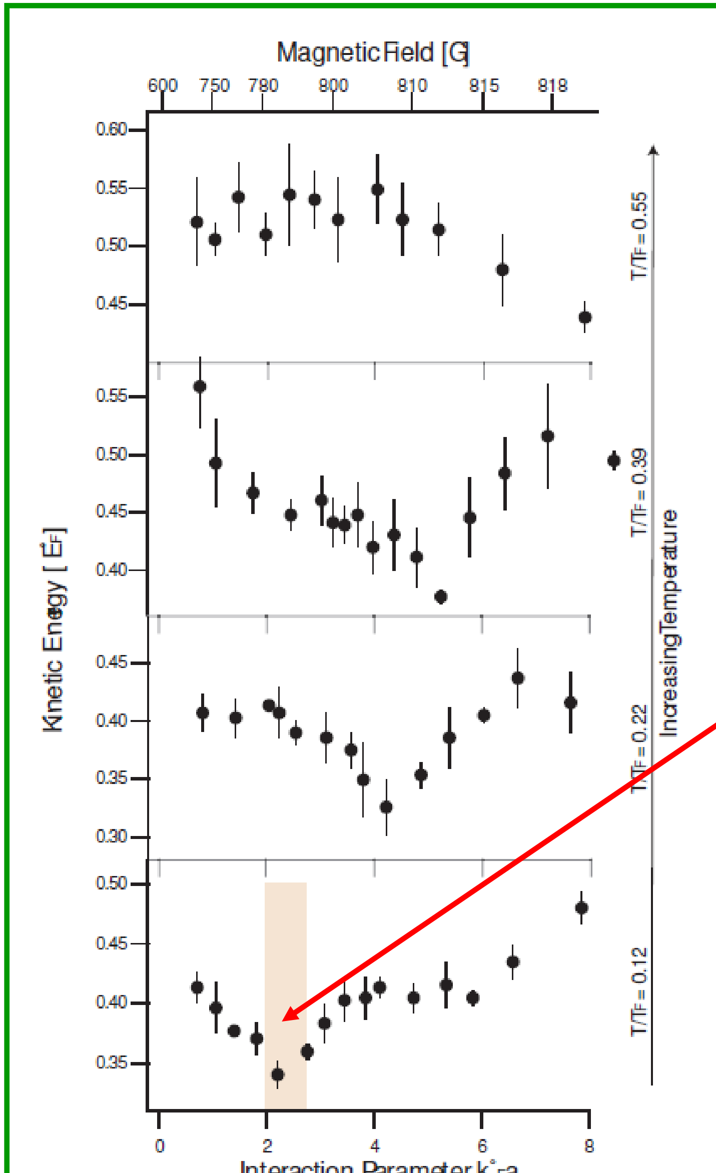
$$\chi = \chi_0 \frac{m^*}{m(1+F_0^a)}$$

Stoner model predicts $$F_0^a = -\frac{2}{\pi} k_F a$$

$$\chi_0 = 3n / 2\varepsilon_F$$ is magnetic susceptibility of ideal Fermi gas

Question:
is **Stoner model valid** for large values of $k_F a$ ?

# MIT experimental results (kinetic energy)




At the lowest temperature kinetic energy exhibits pronounced minimum at

$$k_F^0 a \approx 2$$

$$k_F^0 = (24N)^{1/6} / a_{ho}$$

(Fermi momentum of ideal Trapped Fermi gas)

Recent **MC calculations** of equation of state (Pilati et al., arXiv 1004.1169, Chang et al. arXiv 1004.2680) reveal

important **deviations**

with respect to **Stoner** model ( $k_F a \approx 0.85$ vs $k_F a = 1.6$ )

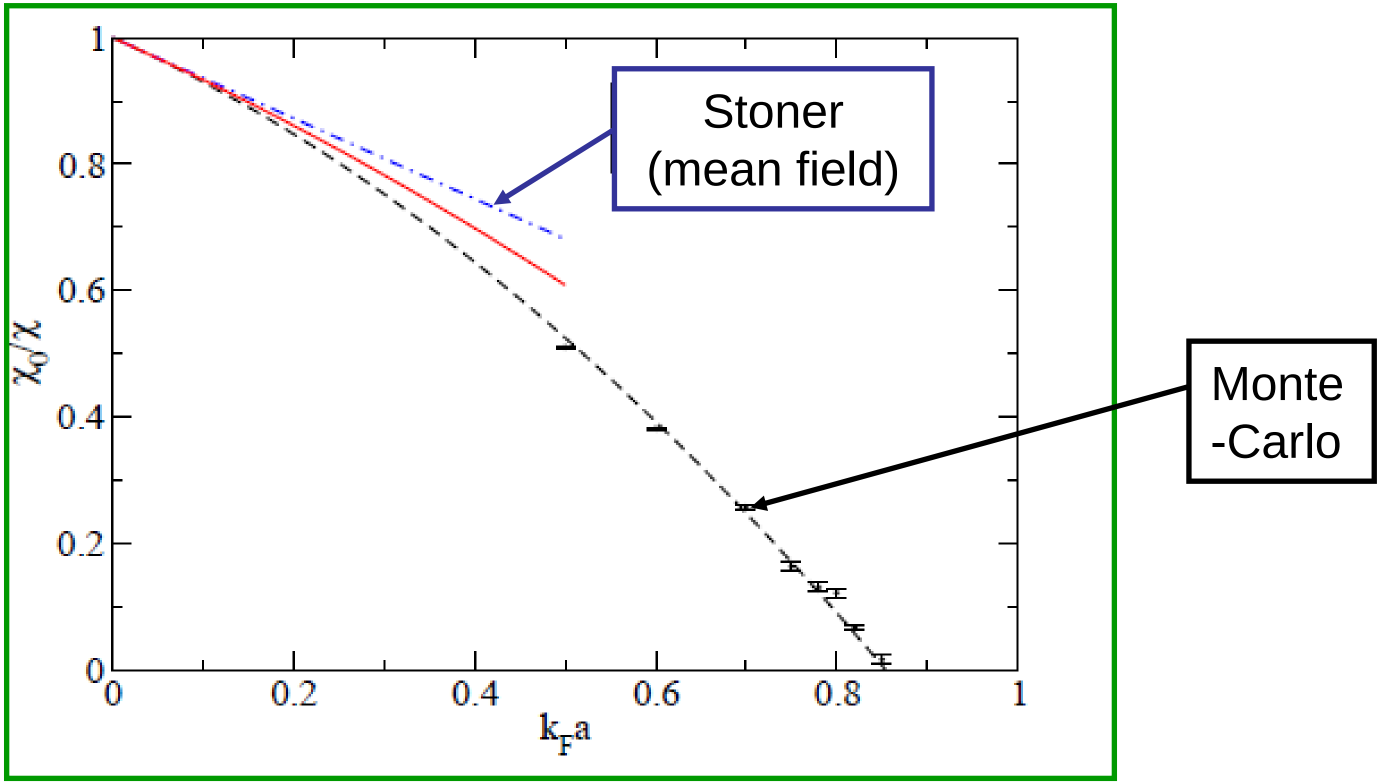
and even more

compared to **experiment** ( $k_F a \approx 2$ )

# Magnetic susceptibility diverges at the FM transition

$$\chi^{-1} = V\partial^2 A / \partial(N_\uparrow - N_\downarrow)^2$$

Inverse susceptibility at T=0




Galitskii expansion

$$\frac{\chi_0}{\chi} = [1 - \frac{2}{\pi} k_F a - \frac{16(2+\ln 2)}{15\pi^2}(k_F a)^2]$$

## Questions

Can we probe **directly** the **magnetic** properties of the gas ?

- **Spin fluctuations** **(CORRELATIONS)**

- Frequency of **spin oscillations** **(DYNAMICS)**
  (first exp at Jila 2002)

- Measurement of the **equation of state** of imbalanced
  Fermi gas **(THERMODYNAMICS)**

**Spin fluctuations**:
potentially powerful tool to explore the nature of quantum phases:

Examples:

- **Fermi liquid** (Fermi gas with small a>0)
- **Itinerant ferromagnetism** (large a>0)
- **Gap** in Fermi superfluids (unitary Fermi gas)
- **AF and FM** in optical traps

…..

At finite T spin fluctuations provide direct access to the **magnetic susceptibility**

# Spin fluctuations and magnetic susceptibility

At finite temperature statistical mechanics provides general relationship between spin fluctuations and **magnetic susceptibility**:

$$\frac{\Delta(N_\uparrow - N_\downarrow)^2}{N} = k_B T \frac{\chi(T)}{n}$$

**Spin thermal fluctuations**

Analog of result for density fluctuations in terms of **isothermal compressibility**

$$\frac{\Delta N^2}{N} = k_B T n k_T$$

**Density thermal fluctuations**

N : number of atoms in small subvolume of whole cloud

At **low T** thermal fluctuations are suppresed and **quantum** fluctuations become more and more **important**

At zero temperature only **quantum fluctuations** survive. They exhibit **weaker N dependence** with respect to thermal fluctuations.
(Astrakarchick, Combescot, Pitaevskii (PRA 2007))

For systems exhibiting static structure factor with linear q behavior:

$$\frac{\Delta(N_\uparrow - N_\downarrow)^2}{N} = 2\alpha\left(\frac{12}{\pi^4 N}\right)^{1/3} \ln(CN^{1/3})$$

- $\alpha$ defined by small q- behavior $S^a(q) \to \alpha q / q_F$ of static structure factor (FT of two-body correlation function)

- C: cut-off fixed by short range behavior of S(q) (ideal Fermi gas: $\alpha = 3/2^{7/3}\,; C = 8.3$

- Holds for **normal Fermi liquids** (example: repulsive gas before FM transition)

- For Fermi superfluids existence of gap implies $S^a(q) \propto q^2$ and hence even weaker N dependence

# Thermal vs. Quantum Fluctuations

$$\frac{\Delta(N_\uparrow - N_\downarrow)^2}{N} = k_B T \frac{\chi(T)}{n}$$ ← thermal

$$\frac{\Delta(N_\uparrow - N_\downarrow)^2}{N} = 2\alpha\left(\frac{12}{\pi^4 N}\right)^{1/3} \ln(CN^{1/3})$$ ← quantum

Comparison shows that, for the ideal Fermi gas at $T = 0.1T_F$ , (where we can use $\chi_0(T) \approx 3n/2k_B T_F$ ) quantum fluctuations become important for $N < 10^3$

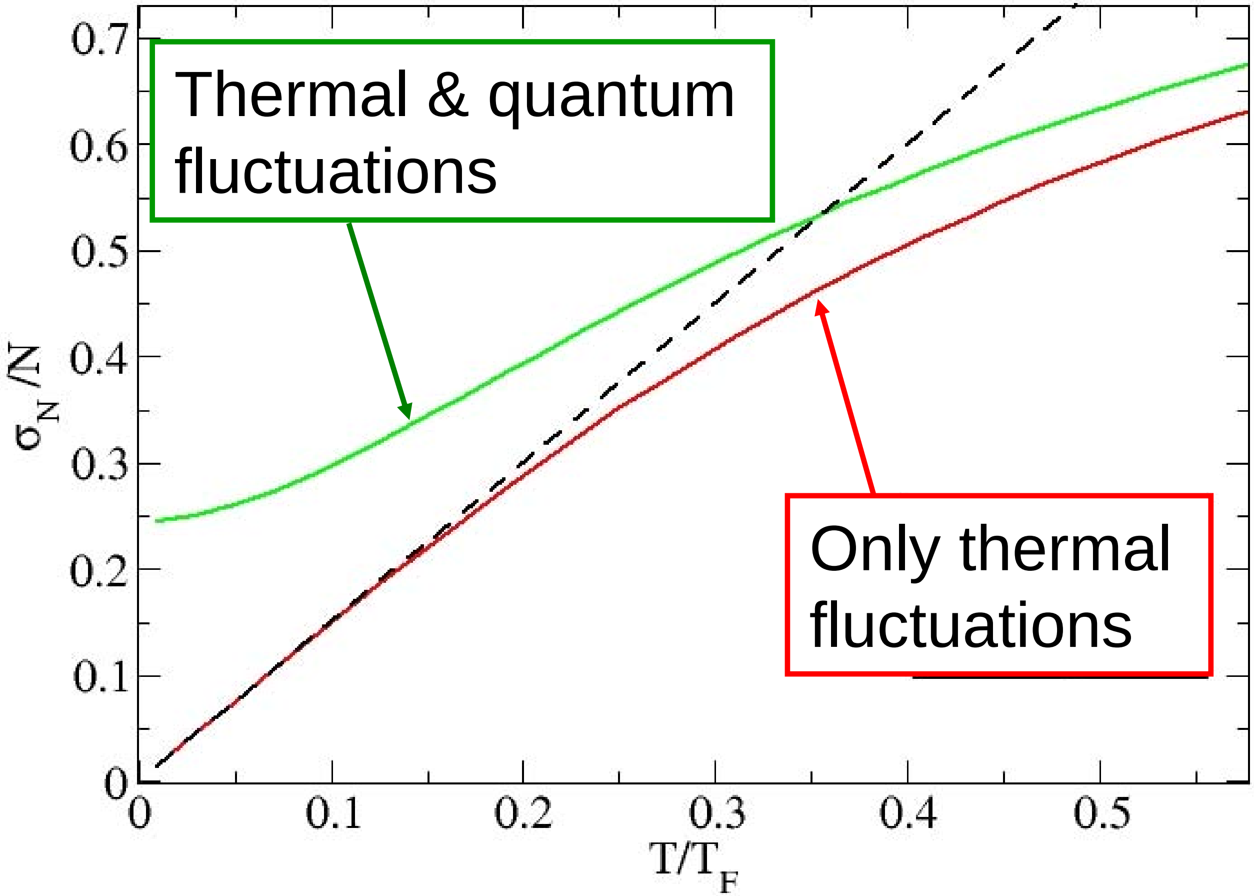
## Some questions

Is transition between thermal and quantum fluctuations measurable in actual experiments ?

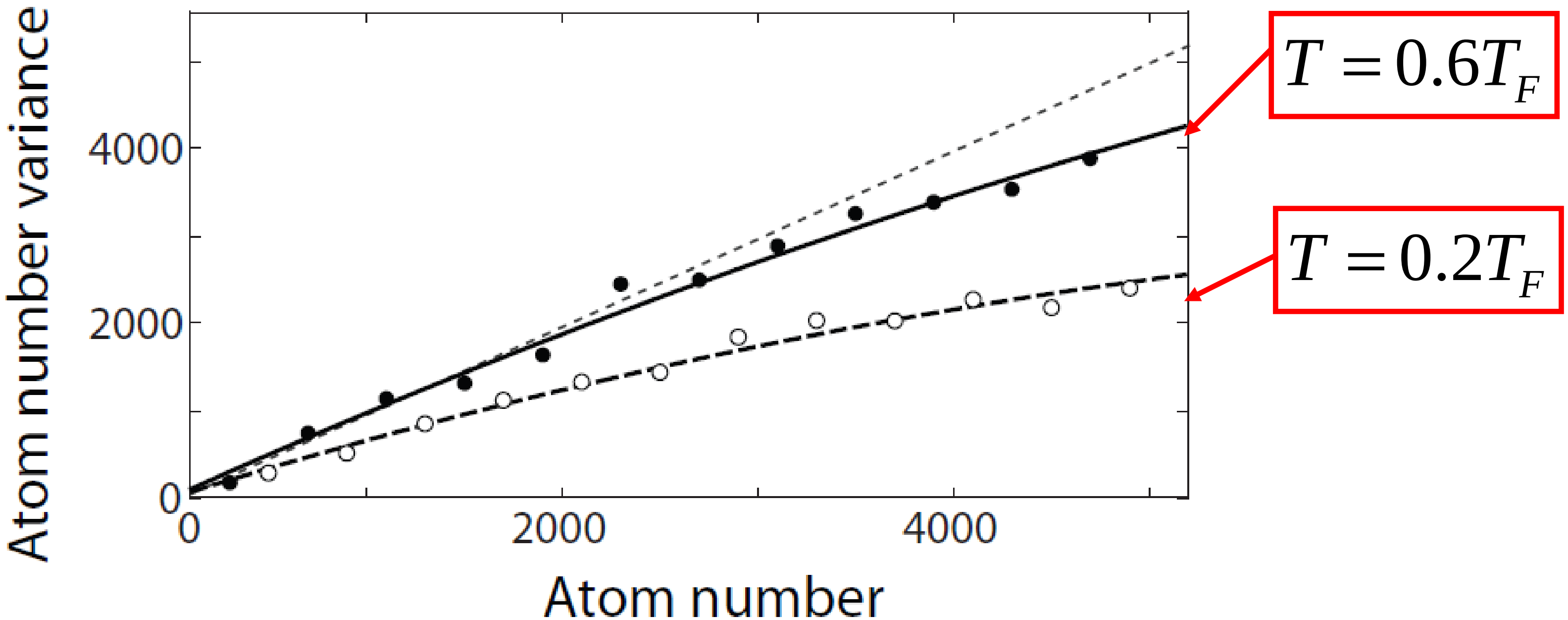
What is the behavior of quantum fluctuations near FM transition?

The simplest example: the ideal Fermi gas

N=100



**Density fluctuations** measured at MIT in **ideal Fermi gas** (Sanner et al. arXiv 1005.1309)




At **low T** one observes stronger deviations from linear N dependence

Fluctuations for **smaller N** measured at Zurich (Mueller et al. arXiv 1005.0302)

## Behavior of spin quantum fluctuations near FM transition?

First answer provided by Landau theory of Fermi liquids.

Ignore Landau parameters with $l \geq 1$ .

From behavior of dynamic response function one extracts $S^a(q) \to \alpha q / q_F$ with

$$\alpha = \frac{3}{2^{4/3}} \int_0^1 \frac{\lambda d\lambda}{\left(1 - F_0^a(1 - \frac{\lambda}{2}\ln\frac{1+\lambda}{1-\lambda})\right)^2 + (\lambda F_0^a / 2)^2}$$

Near the FM transition quantum fluctuations diverge logaritmically

$$\frac{\Delta(N_\uparrow - N_\downarrow)^2}{N} \propto \ln \frac{1}{1+F_0^a}$$

while thermal fluctuations (at low T) diverge as

$$\frac{\Delta(N_\uparrow - N_\downarrow)^2}{N} \propto \frac{1}{1+F_0^a}$$

follows from

$$\Delta(N_\uparrow - N_\downarrow)^2 / N \propto \chi \ ; \ and \quad \chi = \chi_0 \frac{m^*}{m(1+F_0^a)}$$

($F_0^a \to -1$ at the FM transition)

Questions

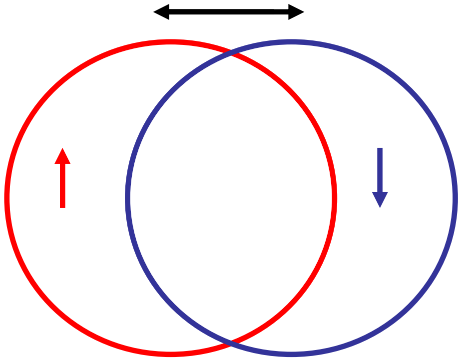
Can we probe **directly** the **magnetic** properties of the gas ?

- **Spin fluctuations** **(CORRELATIONS)**

- Frequency of **spin oscillations** **(DYNAMICS)**
  (first exp at Jila 2002)

- Measurement of the **equation of state** of imbalanced
  Fermi gas **(THERMODYNAMICS)**

# Spin dipole frequency of a trapped repulsive Fermi gas



(Analog of giant dipole resonance of nuclear physics)

In **uniform** Fermi liquids spin oscillation is **Landau damped** if $F_0^a < 0$

In **harmonic trap** dipole oscillation is **not** Landau damped because of absence of s.p excitations below oscillator frequency

# Sum rule approach to the spin dipole frequency of a trapped repulsive Fermi gas

$$\hbar^2 \omega_{spin}^2 = \frac{m_1(D)}{m_{-1}(D)}$$

Rigorous upper bound to the lowest frequency

excited by the spin dipole operator $D = \sum_{i\uparrow} z_i - \sum_{i\downarrow} z_i$

$$m_k(D) = \sum_n (E_n - E_0)^k \, |\langle 0 | D | n \rangle|^2$$

are k-moments of dipole dynamic structure factor

Energy-weighted sum rule is given by Thomas Reich-Kuhn sum rule

$$m_1(D) = \frac{1}{2} < [D,[H,D]] >= N\hbar^2 / 2m$$

Inverse energy-weighted sum rule can be derived in the local density approximation (LDA) starting from energy functional

$$E(n_\uparrow, n_\downarrow) = \int d\vec{r}\, \varepsilon(n_\uparrow, n_\downarrow) - \lambda \int d\vec{r} z (n_\uparrow - n_\downarrow)$$

to evaluate spin dipole polarizability. By expanding $\varepsilon(n_\uparrow, n_\downarrow)$ up to terms quadratic in $(n_\uparrow - n_\downarrow)^2$

one finds

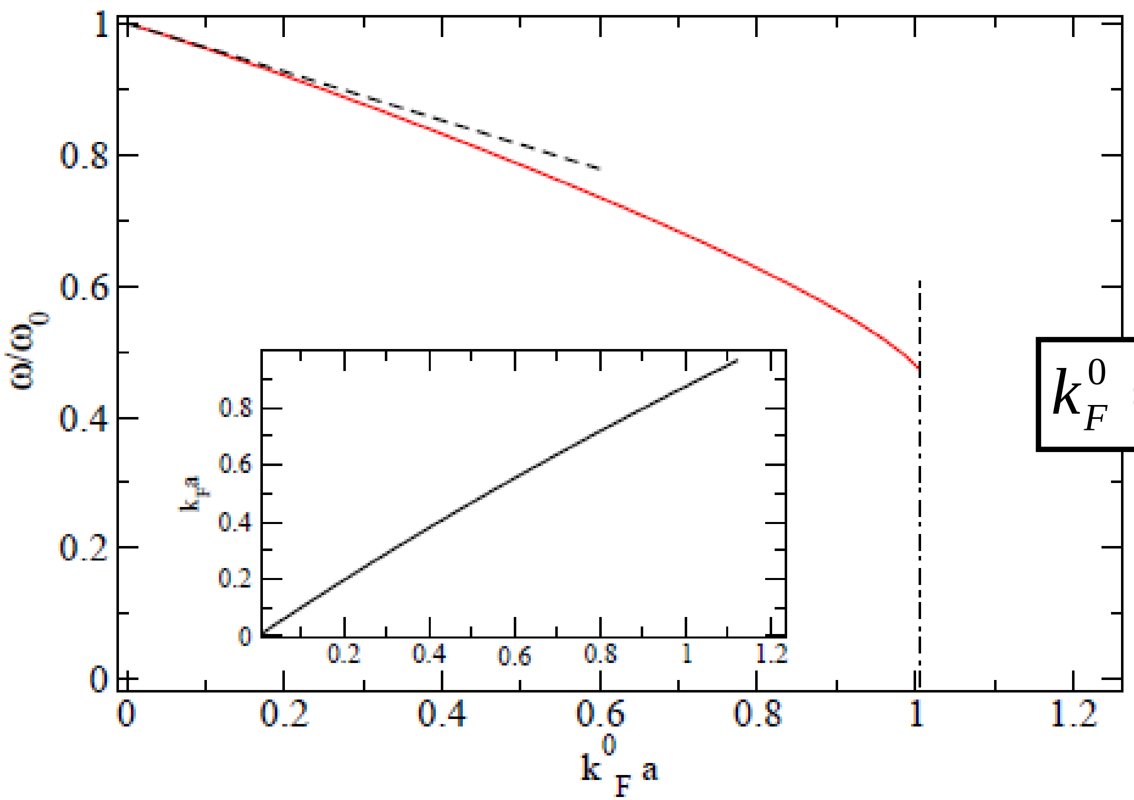
$$m_{-1}(D) = \frac{1}{2\lambda} \int d\vec{r} z \delta(n_\uparrow - n_\downarrow) = \frac{1}{2} \int d\vec{r} z^2 \chi$$

with $\chi^{-1} = \partial^2 \varepsilon(n_\uparrow, n_\downarrow) / \partial(n_\uparrow - n_\downarrow)^2$ inverse susceptibility of uniform matter

# Results for spin dipole frequency

$$\omega_{spin}^2 = \frac{N}{m\int d\vec{r} z^2 n^2 \chi_M(n)}$$

Value of $\chi$ from MC calculations of Pilati et al.




$$k_F^0 = (24N)^{1/6} / a_{ho}$$

# Summary

- **Spin fluctuations** of a repulsive Fermi gas exhibit key features near **FM transition.**

- **Thermal** fluctuations provide info about **magnetic suceptibility**, **quantum** fluctuations (small T, small N) fixed by low q behavior of spin **structure factor**.

- Frequency of **spin dipole oscillation** sensitive to behavior of magnetic susceptibility. Exhibits **significant quenching** with respect to ideal gas value also far from the FM transition (test of **Normal Fermi liquid theory**)

# Open questions and perspectives

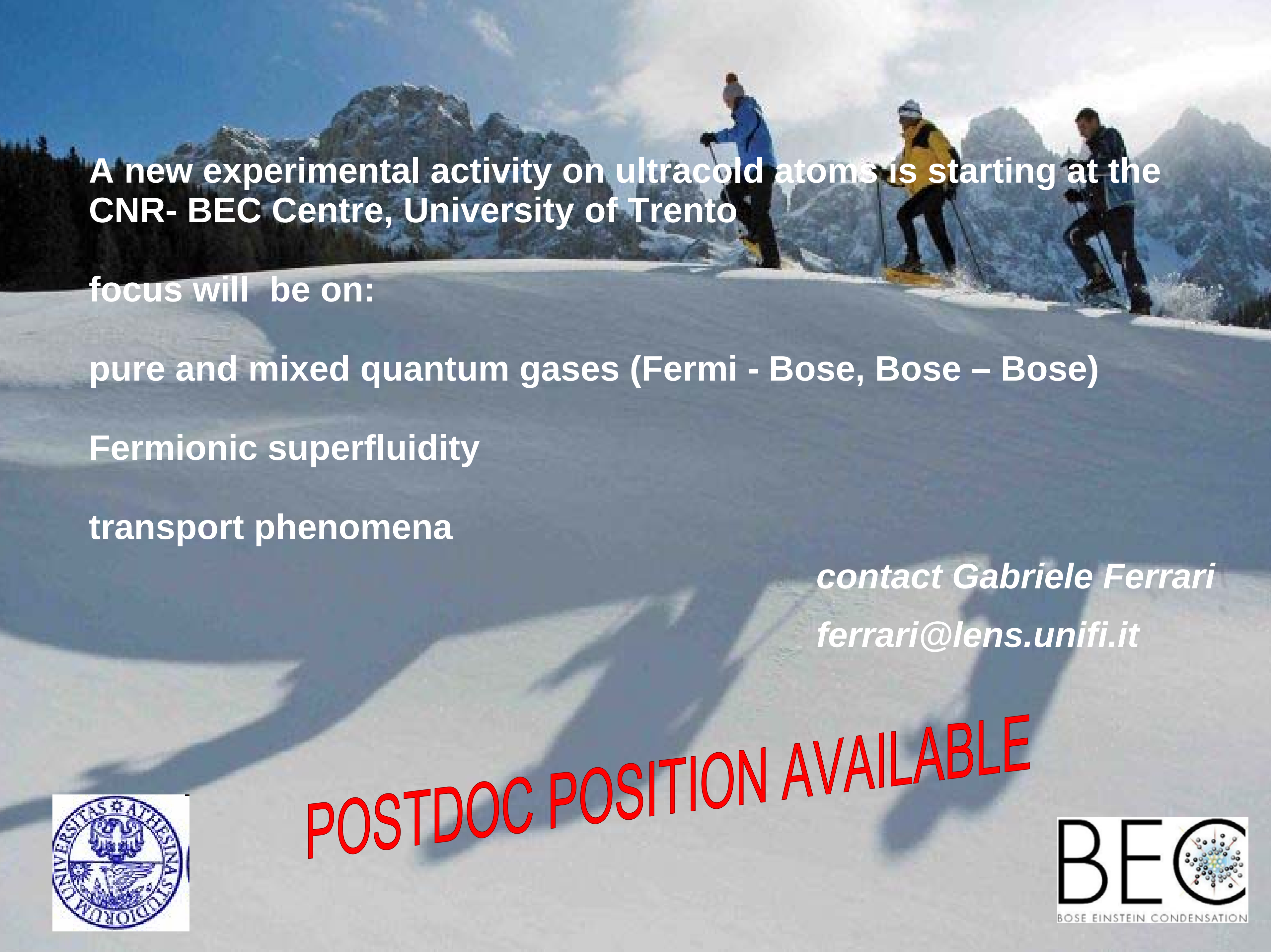
- Effect of **metastability** and molecular formation on spin fluctuations.

- Identification of **quantum** fluctuations as **surface effect**. Shape dependence of subvolumes explored by laser.

- **Collisional damping** of spin oscillations (T-dependence) (connection with spin diffusion in Zwierlein talk)

- Spin fluctuations in other quantum phases (**Fermi superfluid along the BEC-BCS crossover**)

See recent MIT paper arXiv:1010.1874v1, 9 Oct 2010


A new experimental activity on ultracold atoms is starting at the CNR- BEC Centre, University of Trento
focus will be on:
pure and mixed quantum gases (Fermi - Bose, Bose – Bose)
Fermionic superfluidity
transport phenomena
contact Gabriele Ferrari
ferrari@lens.unifi.it
POSTDOC POSITION AVAILABLE
UNIVERSITAS ATHESINA STUDIORUM
BEC
BOSE EINSTEIN CONDENSATION

Sasso Piatto m 2960
(Dolomites)
October 1st 2010