Nonlinear quantum hydrodynamics
~
shocks, superfluid counterflow, and novel types of solitons

P. Engels
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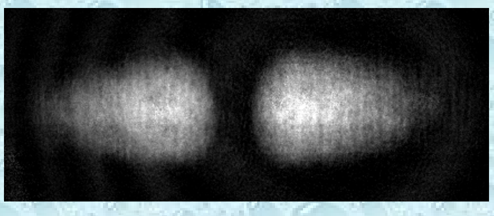
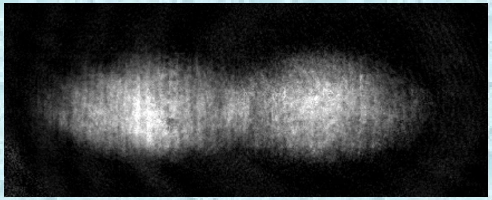
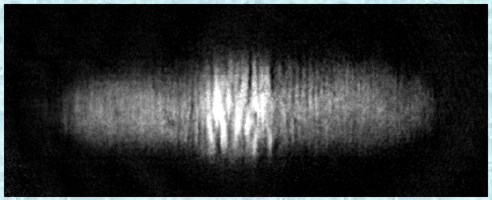
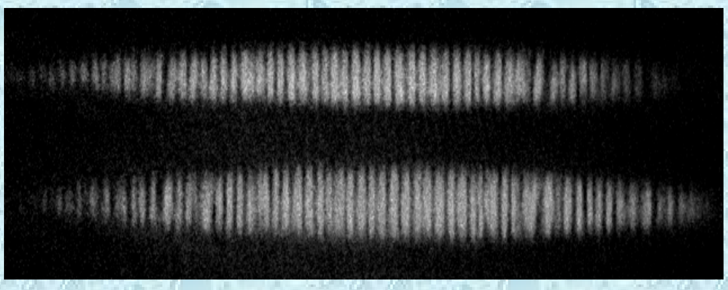
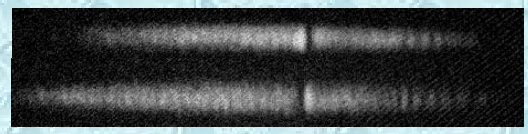
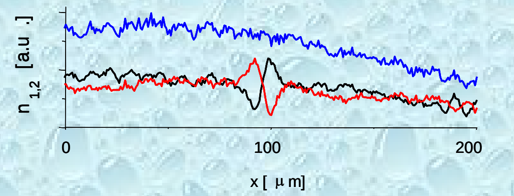
http://www.physics.wsu.edu/Reseach/engels/index.html

$$$ NSF, ARO $$$

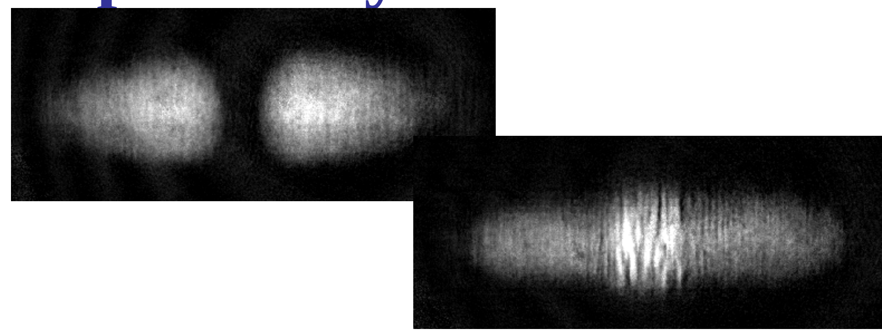
JiaJia Chang, Chris Hamner (WSU)
Mark Hoefer (NCSU)

Further AMO theory at WSU:
D. Blume, C. Zhang

Outline

• **Brief intro: Nonlinear quantum hydrodynamics**

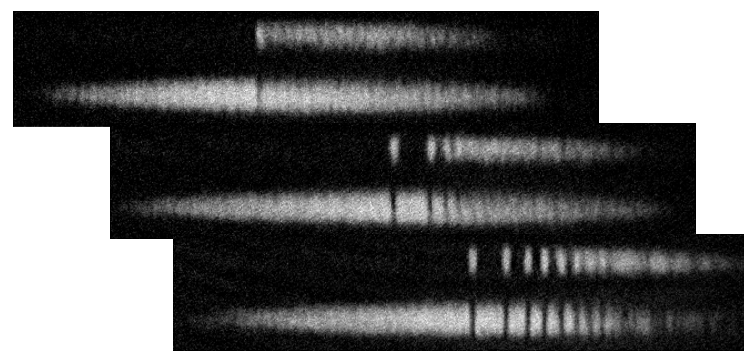
• **Hydrodynamics in single-component systems: dispersive dynamics**



• **binary BECs:**

- **counterflow induced modulational instability**

- **harnessing MI to create dark-bright solitons**



- **novel types of solitons**

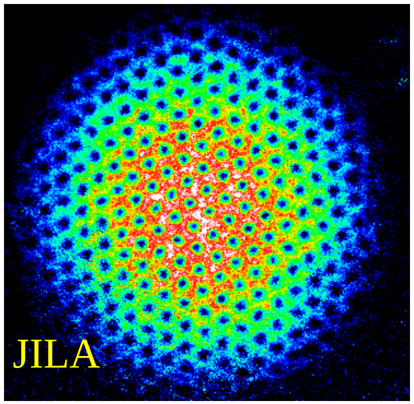
Intro: dilute-gas BEC and hydrodynamics

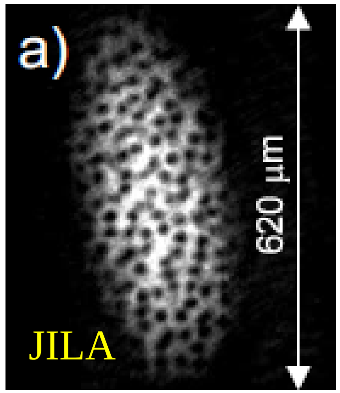
~ and just a little bit of theory from
an experimentalist's perspective ~

The peculiar flow of superfluids

BEC provide a novel and quite unique tool with which the studies of superflow can be pushed into new regimes.

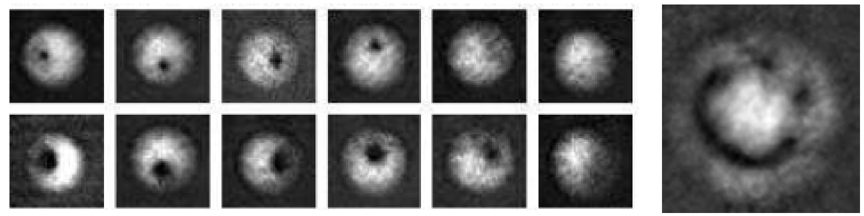
Vortex lattices in BEC



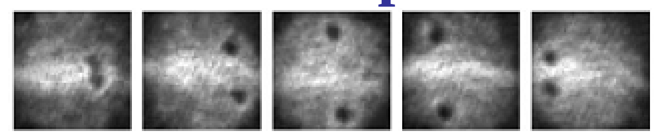


Giant vortex clusters

JILA

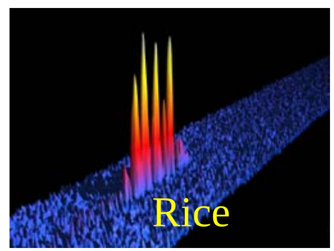


Vortex dipoles

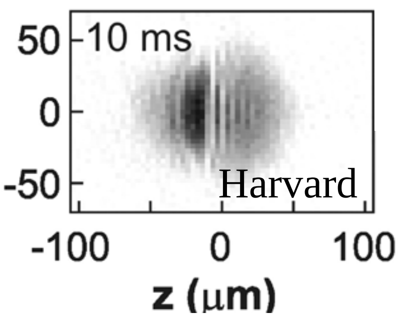


B. P. Anderson, OSC Arizona
(also: D. Hall, Amherst)

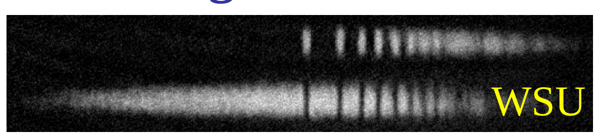
Solitons



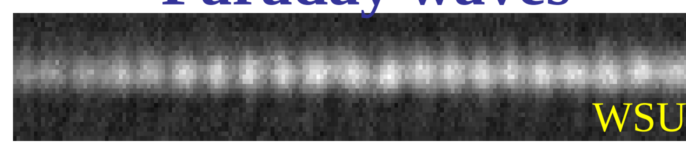
Dispersive shocks



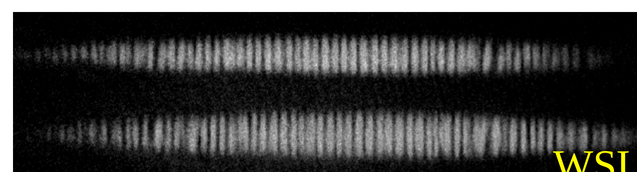
Dark-bright soliton trains



Faraday waves



Modulational instability

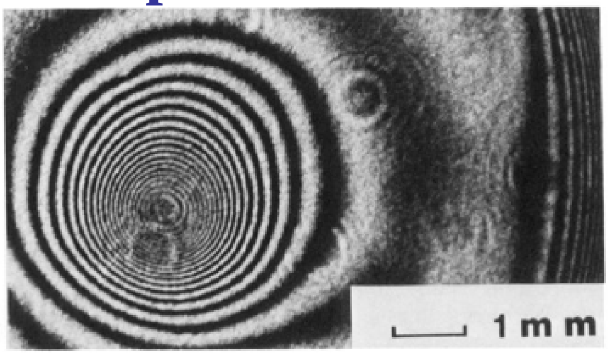


Numerous **spectacular results** **(and apologies for many omissions on this slide!)**

The peculiar flow of superfluids

The underlying nonlinear concepts are fairly general and applicable to a variety of different systems!

Superfluid He II

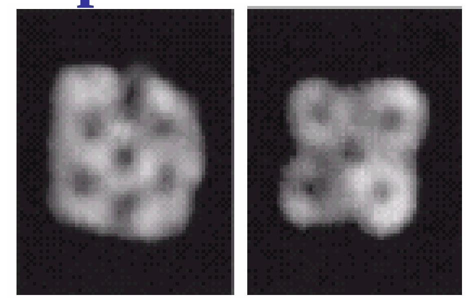


Marston and Fairbank,
Phys. Rev. Lett. 39, 1208–1211 (1977)

Solitons in water

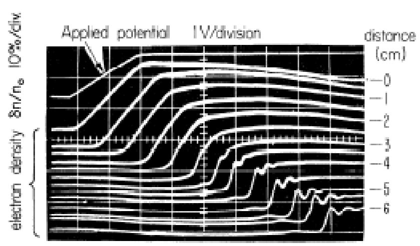
Dugald Duncan/Heriot-Watt University

Optical vortices



Scheuer, Orenstein
Science 285, 230 1999
(output of a VCSEL)

Shocks in plasma

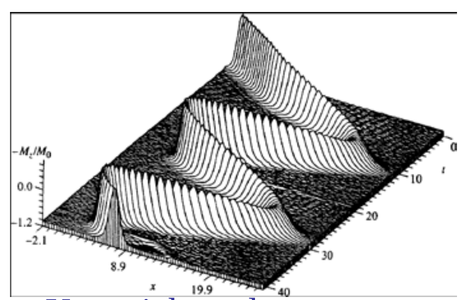


Taylor et al., PRL **24**, 206 (1970)

… and many others!

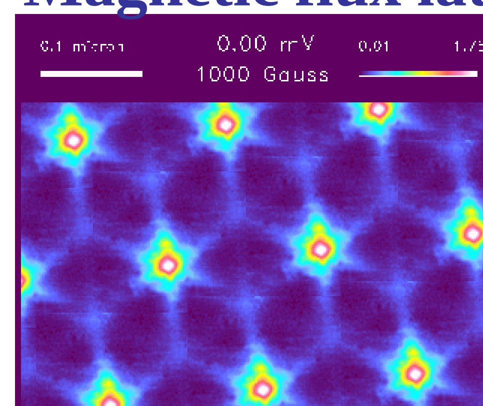
Connections to condensed matter

Solitons in magnets



Kosevich et al.,
Journal of experimental and theoretical physics, Vol. 87, N. 2 (1998)

Magnetic flux lattice



Bell Labs

Gross-Pitaevskii equation:

(an "extension" of Schroedinger equation that includes atomic interactions):

$$\left(-\frac{\hbar^2}{2m}\Delta + V_{extern} + \frac{4\pi\hbar^2 a}{m}\left|\psi(\vec{r})\right|^2\right)\psi(\vec{r}) = \mu\psi(\vec{r})$$

Kinetic energy term, similar to diffractive or dispersive term in optics.

Potential energy

Atomic interaction term, similar to Kerr-type nonlinearity in optics (→ "optical hydrodynamics", nonlinear photonic lattices e.g. in Jason Fleischer's group, Princeton, and others).

Alternatively: To emphasize the hydrodynamic point of view: Rewrite the equation in terms of velocity and density

$$\psi(\vec{x}) = A(\vec{x})\cdot e^{i\phi(\vec{x})}$$

Density: $n(\vec{x}) = A(\vec{x})^2$

Velocity: $\vec{v}(\vec{x}) = \frac{\hbar}{m}\nabla\phi(\vec{x})$

A hydrodynamic perspective… shock waves, dispersive effects etc.…

Quantum vs. classical hydrodynamics

Quantum hydrodynamics:

$$m\left(\frac{\partial v}{\partial t}+\nabla\frac{v^2}{2}\right)=-\nabla\frac{4\pi\hbar^2 a}{m}n-\nabla V_{extern}+\nabla\left(\frac{\hbar^2}{2m\sqrt{n}}\nabla^2\sqrt{n}\right)$$

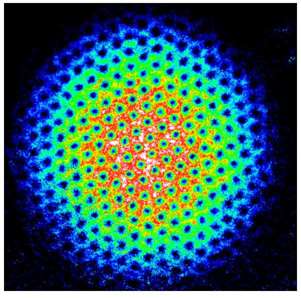
Navier Stokes equation (classical):

$$m\left(\frac{\partial v}{\partial t}+\nabla\frac{v^2}{2}+(\nabla\times v)\times v\right)=-\frac{1}{n}\nabla p-\nabla V_{extern}+\eta\Delta v$$

For an irrotational fluid, this term vanishes! $\nabla\times v=0$

Indeed, since $\vec{v}(\vec{x})=\frac{\hbar}{m}\nabla\phi(\vec{x})$ **the quantum flow is irrotational.**

… at least as long as the phase is not singular!
Otherwise: vortices!



Quantum vs. classical hydrodynamics

Quantum hydrodynamics:

$$m\left(\frac{\partial v}{\partial t}+\nabla\frac{v^2}{2}\right)=-\nabla\frac{4\pi\hbar^2 a}{m}n-\nabla V_{extern}+\nabla\left(\frac{\hbar^2}{2m\sqrt{n}}\nabla^2\sqrt{n}\right)$$

Dispersive (3rd order derivative)

Navier Stokes equation (classical):

$$m\left(\frac{\partial v}{\partial t}+\nabla\frac{v^2}{2}+(\nabla\times v)\times v\right)=-\frac{1}{n}\nabla p-\nabla V_{extern}+\eta\Delta v$$

Dissipative (2nd order derivative)

This is an important difference.
→ Important consequences when gradients are steep.
E.g.: quantum shocks are dispersive shocks, not dissipative shocks, and thus have a rich structure.

Dispersive effects in single-component systems

merging and "hole closing" experiments:
lots of dynamics in a relatively simple setting

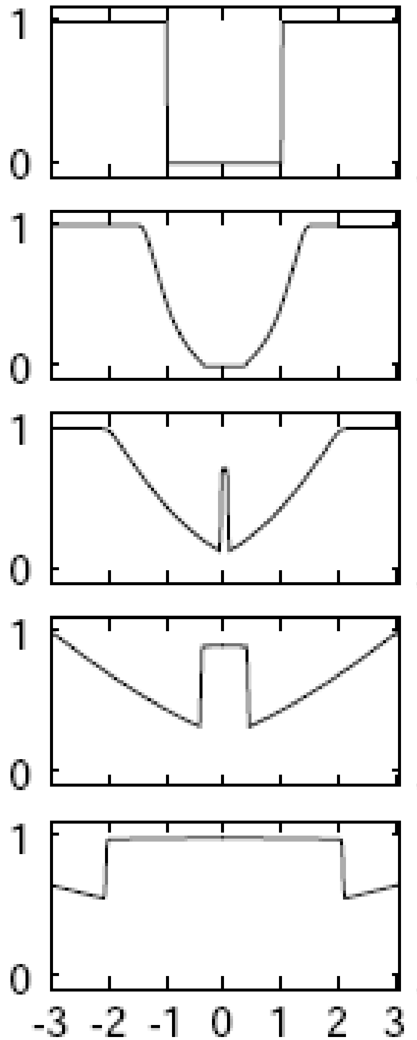
- *Matter-Wave Interference in Bose-Einstein Condensates: a Dispersive Hydrodynamic Approach*, M. A. Hoefer, P. Engels, and J. J. Chang, Physica D, 238, 1311-1320 (2009).

- *Formation of Dispersive Shock Waves by Merging and Splitting Bose-Einstein Condensates*, J. J. Chang, P. Engels, and M. A. Hoefer, Physical Review Letters, 101, 170404 (2008).

classical vs. quantum “hole closing”

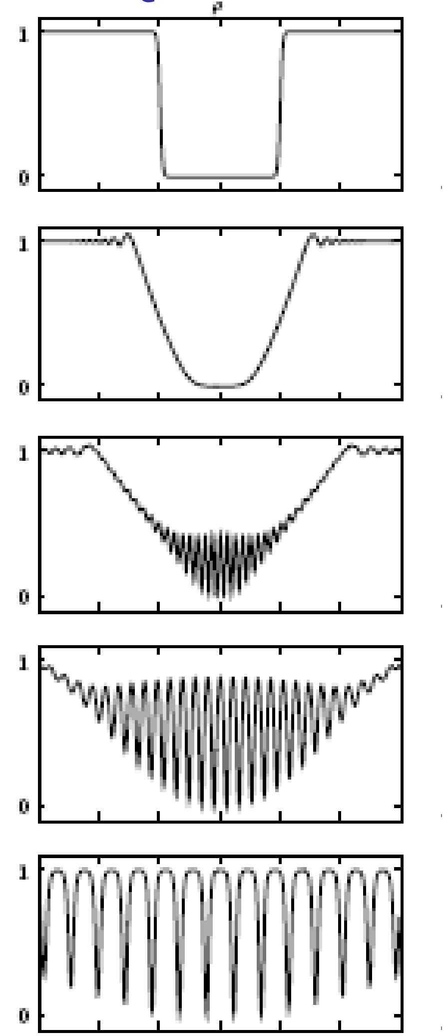
Numerics by M. Hoefer

Classical



Dissipative

Quantum

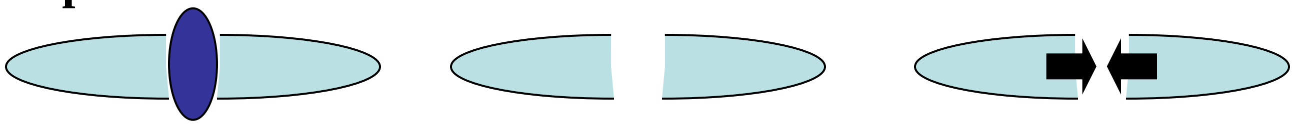


Dispersive

BEC collisions with rather low atom number (20000 atoms)

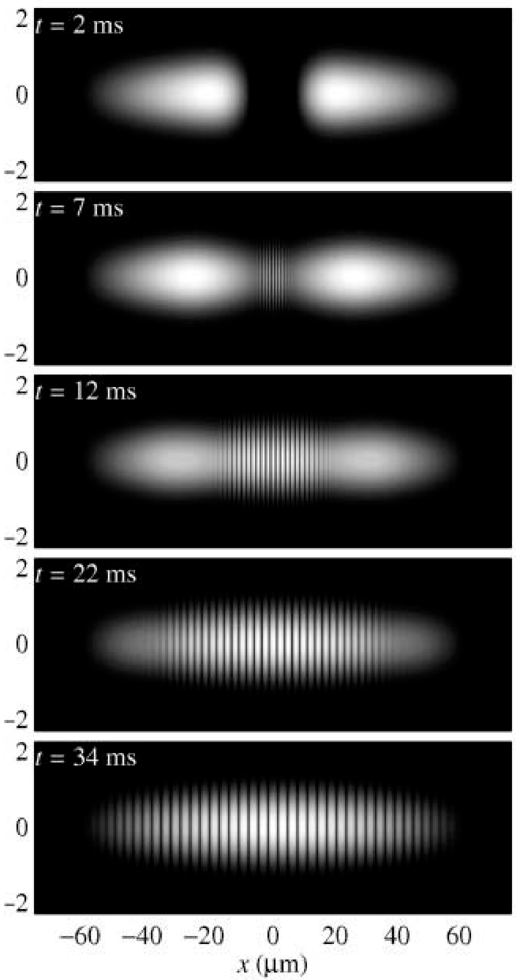
In our experiments, we create an initial gap in a BEC with a repulsive laser.

2 separated BECs | **Turn laser off** | **BECs collide in trap**

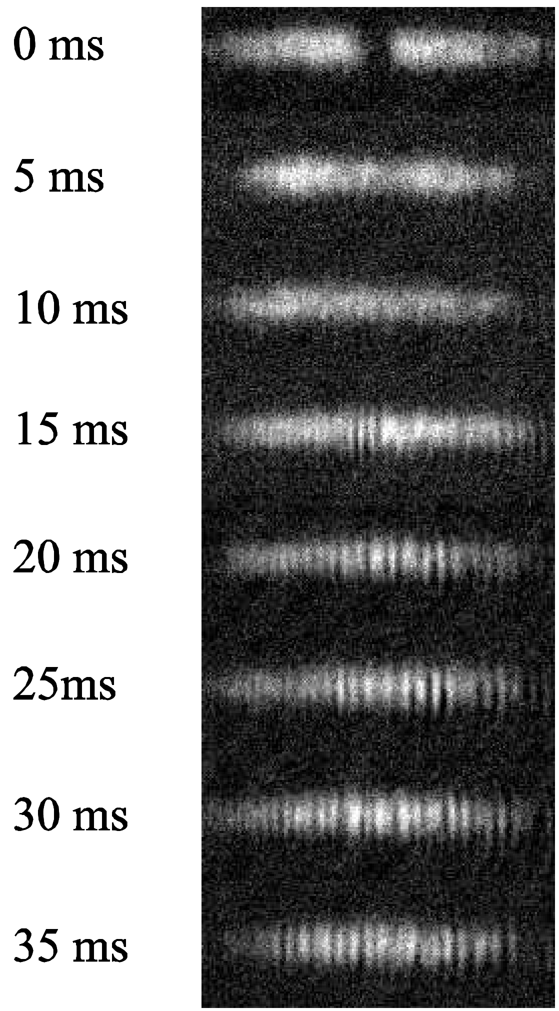


Wait a few ms, then expansion image

Numerics (M. Hoefer)



Experiment



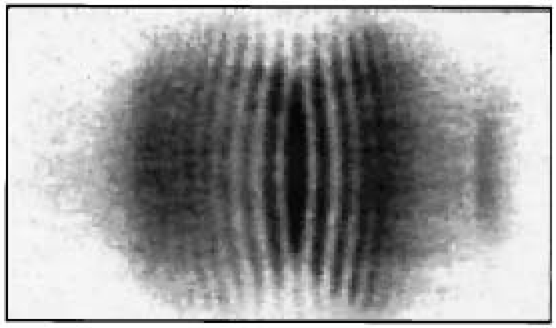
If initial gap is wide enough, we have enough energy to form many soltions. Formation of a *uniform soliton train* as a result of the BEC collision!

M. Hoefer et al., Physica D, 238, 1311-1320 (2009).

See also Weller et al., PRL 101, 130401 (2008); Shomroni et al., Nature Physics 5, 193 (2009);
Theory:
W. P. Reinhardt and C. W. Clark, J. Phys. B 30, L785 (1997), V. A. Brazhnyi and A. M. Kamchatnov, Phys. Rev. A 68, 043614 (2003), B. Damski, Phys. Rev. A 73, 043601 (2006)

BEC collisions

Numerics and experiment show the formation of a uniform soliton train!
How can we understand this?
→ A hydrodynamics perspective of BEC interference.

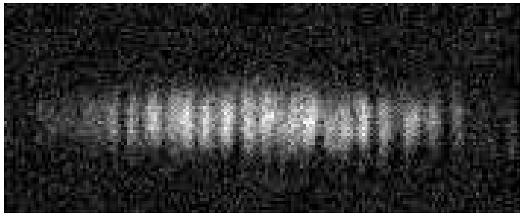


Interference after 40 ms time of flight.
Andrews et al., Science **275**, 637 (1997)

Interference of "noninteracting" BECs leads to a cosine-shaped spatial modulation. (Interference occurred after some time of flight.)

M. R. Andrews et al., Science **275**, 637 (1997); A. Rörl et al., Phys. Rev. Lett. **78**, 4143 (1997); T. Schumm et al., Nature **1**, 57 (2005)

Are these two very different things, or are they related?

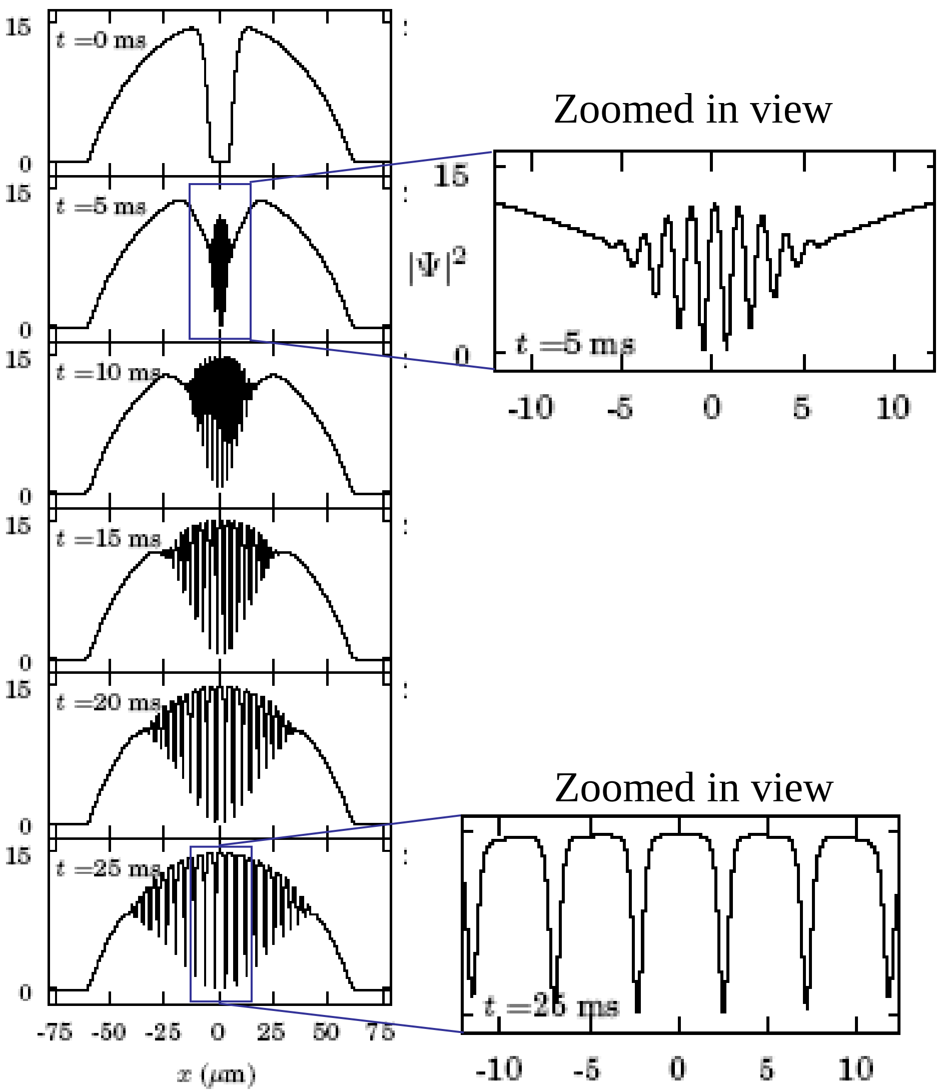


Our case: BEC collision in trap leads to uniform soliton train. Interactions between the atoms are important during the merging!

Dispersive hydrodynamic perspective of matter wave interference in BECs

The two cases are closely related!

Line plots of 3D numerics (M. Hoefer) of our in-trap merging experiments:

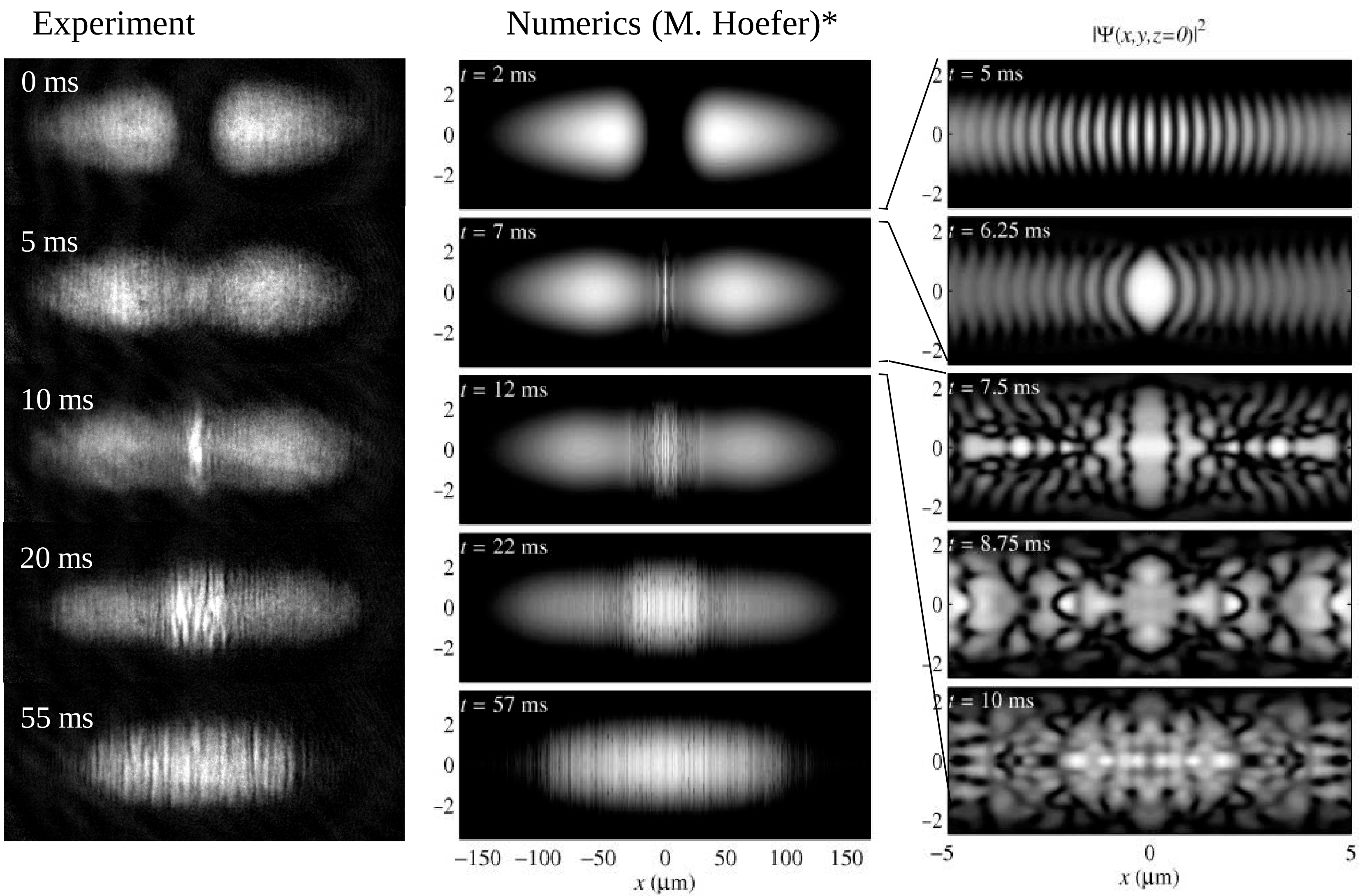


Early during the interaction process, the interference pattern is essentially trigonometric. After a sufficient evolution time, it develops into a soliton train!

Mathematically, the elliptic function solution of the nonlinear Schroedinger equation corresponds to linear, trigonometric waves for a small elliptic parameter, and it corresponds to the grey soliton solution for an elliptic parameter approaching 1.

M. Hoefer et al., *Matter wave interference in Bose-Einstein Condensates: A dispersivehydrodynamic perspective*, M. Hoefer et al., Physica D, 238, 1311-1320 (2009).

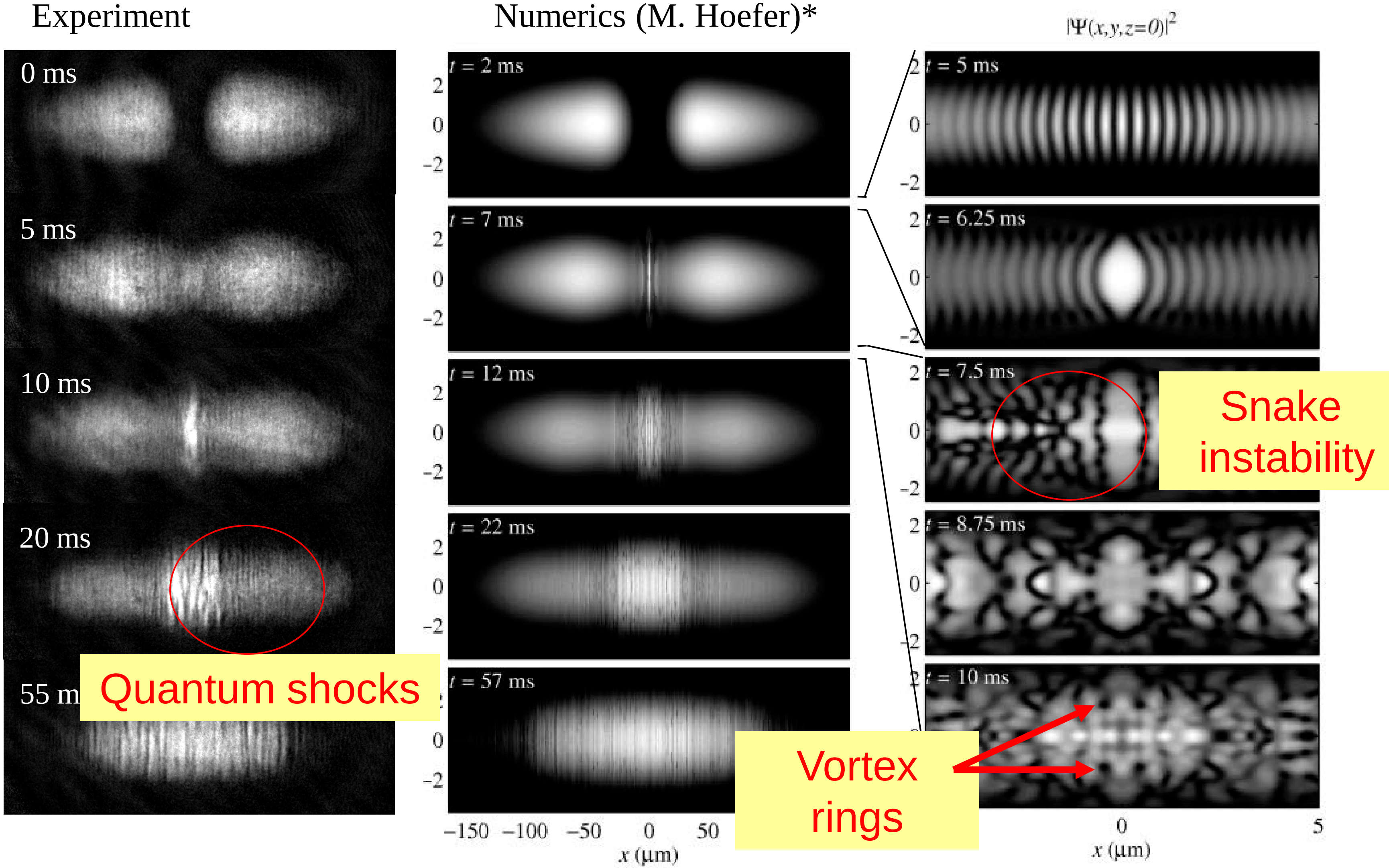
BEC collisions with higher atom number (10^6 atoms)



- **Like in the low-number case, see lots of solitons, but not so uniform**
- **Formation of a pronounced bulge with steep edges, shockfronts**

*** Numerics: no antitrapped expansion was simulated, vertical scale is stretched in figure.**

BEC collisions with higher atom number (10^6 atoms)



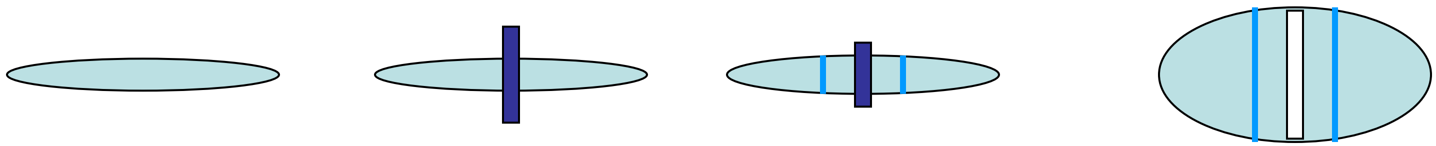
- Like in the low-number case, see lots of solitons, but not so uniform
- Formation of a pronounced bulge with steep edges, shockfronts

* Numerics: no antitrapped expansion was simulated, vertical scale is stretched in figure.

A variation of the theme: turning on a repulsive barrier

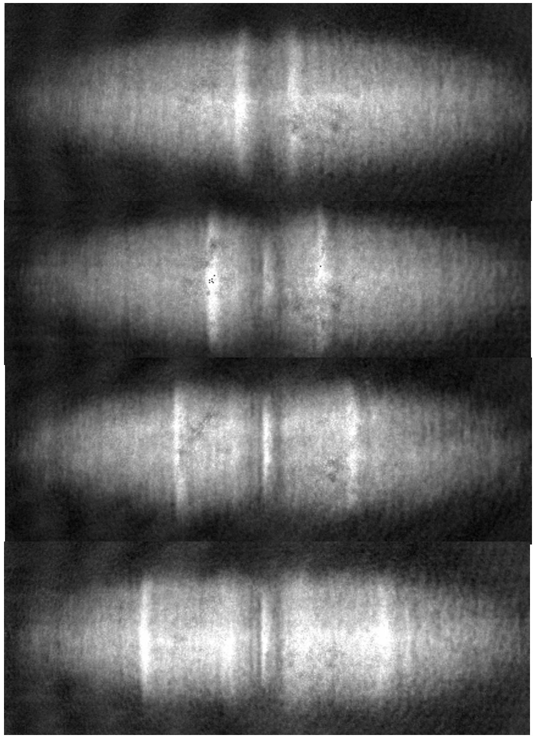
Procedure:

Make a BEC → **Split it (dipole laser)** → **Let evolve in trap** → **Image in expansion**



Weak dipole beam: → sound waves:

In-trap evolution time



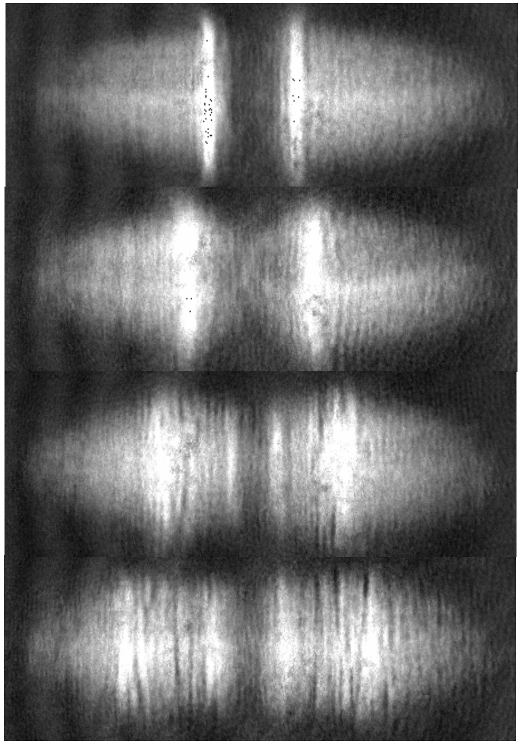
1 ms

4.5 ms

8.5 ms

13.5 ms

Strong dipole beam: → shock/solitons:



1.5 ms

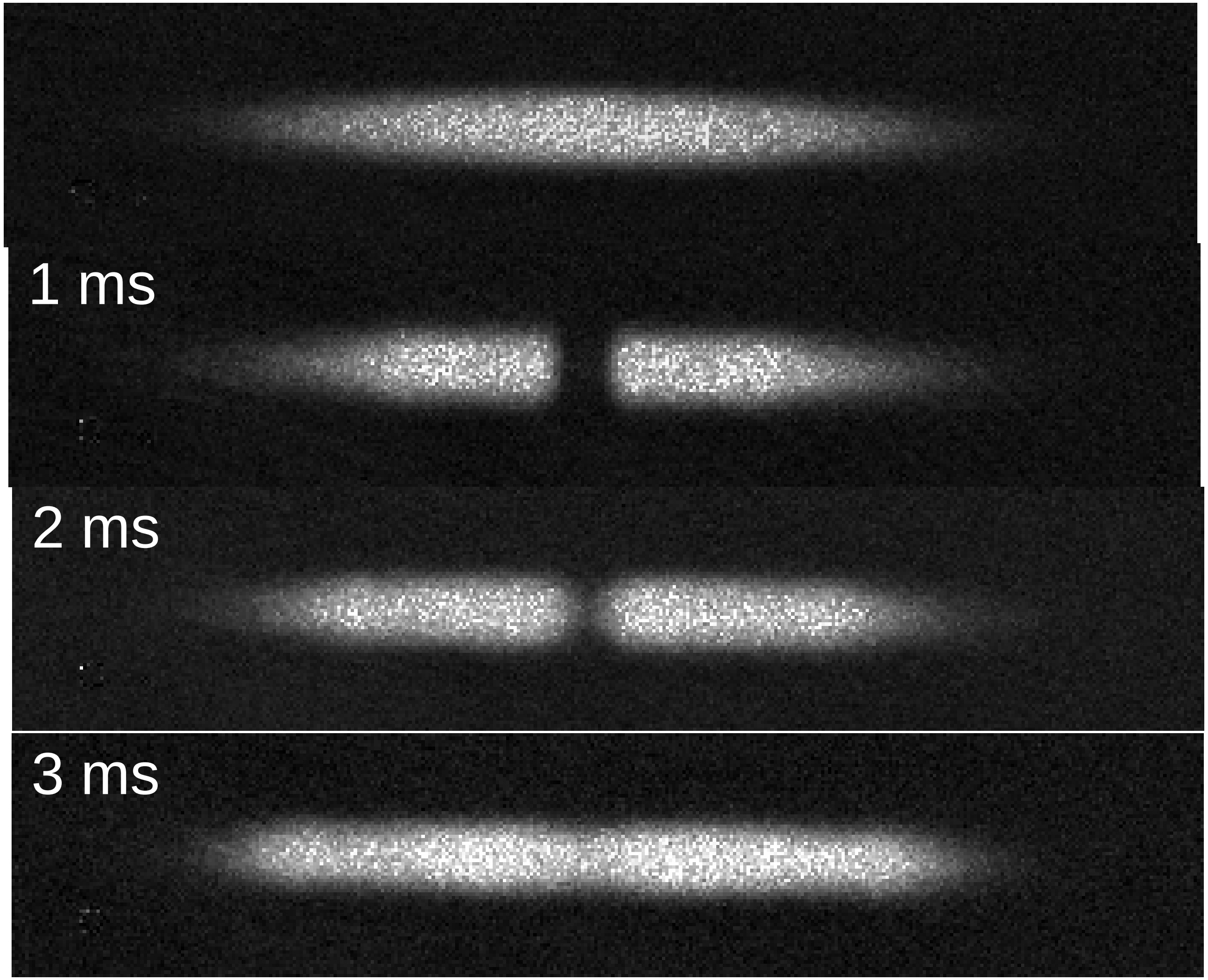
3.5 ms

6.5 ms

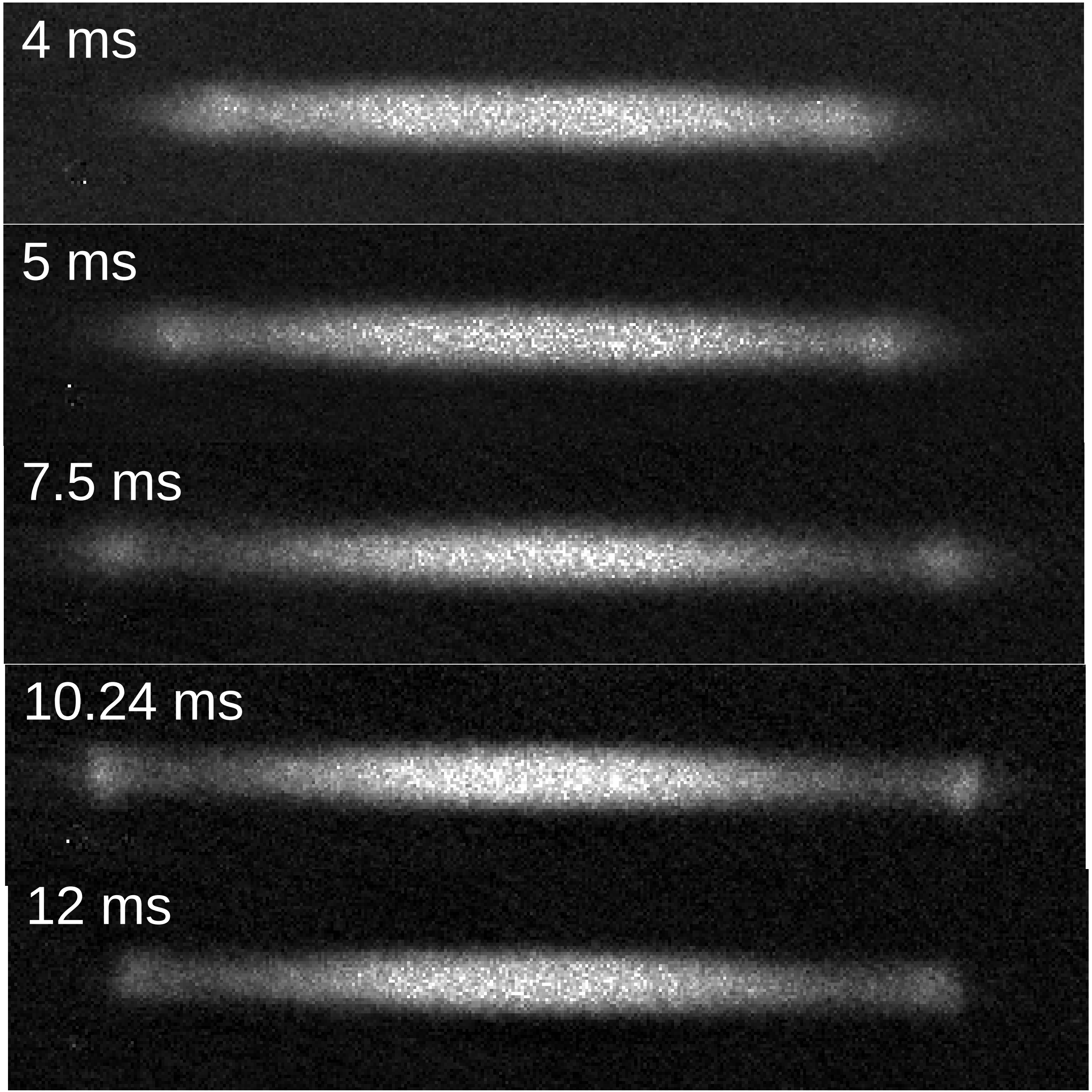
10.5 ms

“Blast wave” experiment with a Fermi cloud

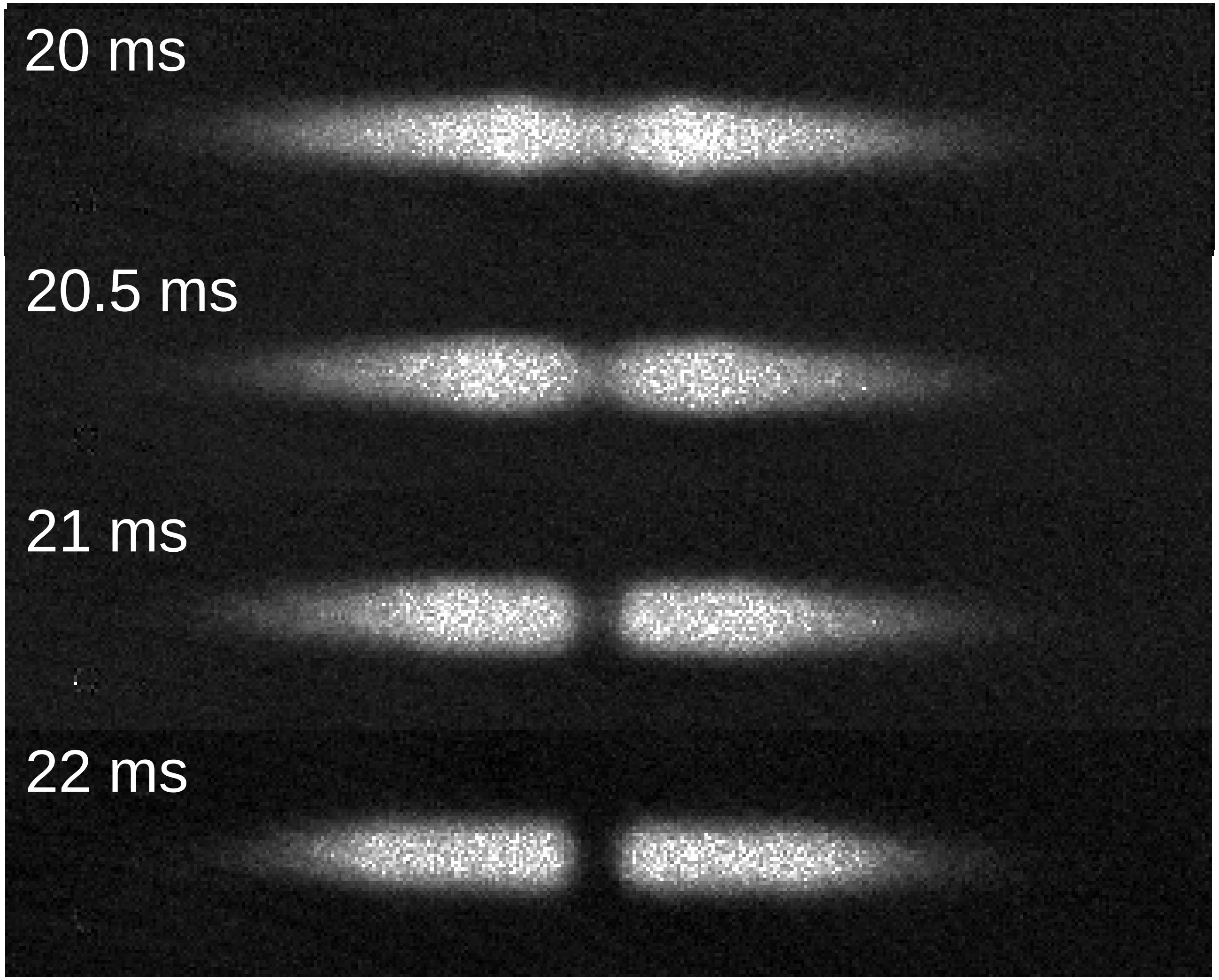
• Create a degenerate Fermi gas of ^{40}K atoms
• Pulse on a repulsive dipole beam focussed onto the center for a short time
• Two wavepackets spread out. What happens when they turn around and collide?



"Blast wave" experiment with a Fermi cloud



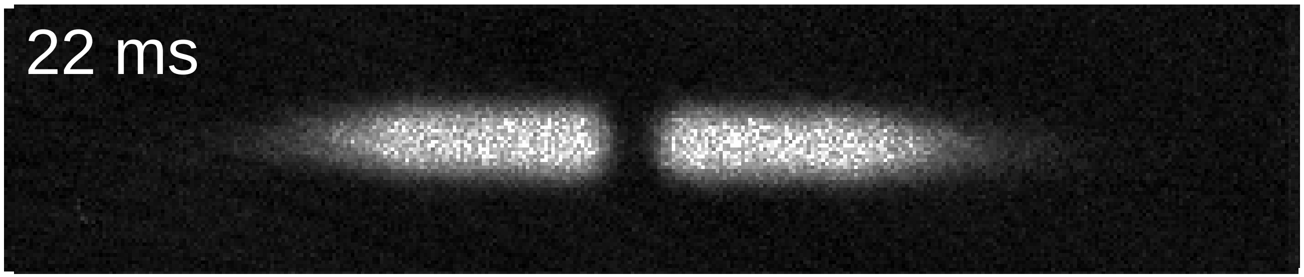
“Blast wave” experiment with a Fermi cloud



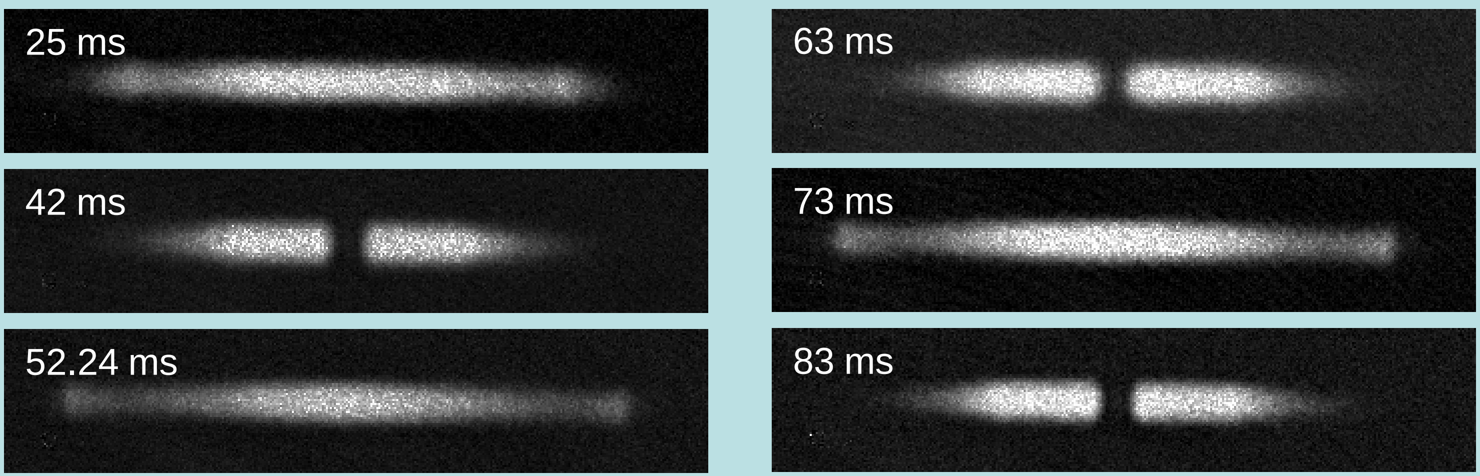
“Blast wave” experiment with a Fermi cloud

- **Note: These experiments are conducted with single-component DFG.**

For sound speed in resonant two-component DFG, see, e.g., J. Joseph et al., PRL 98, 170401 (2007).



Hole opening and closing is observed for many cycles



Quantum shock in degenerate Fermi gases?

Theory: B. Damski, J. Phys. B37 (2004) L85; E. Bettelheim et al., PRL 97, 246402 (2006)

Dynamics of counterflow in binary BECs

C. Hamner, J.J. Chang, P. Engels, M. A. Hoefer, arXiv:1005.2610
M. A. Hoefer, C. Hamner, J.J. Chang, and P. Engels, arXiv:1007.4947

S. Middelkamp et al., Physics Letters A, doi:10.1016/j.physleta.2010.11.025
(application to soliton oscillations)

Dynamics of binary BECs: overview

- **We have now extended these studies to two-component systems (binary BECs)**

- **Relative velocity between the components (i.e., counterflow) is a new degree of freedom not afforded by the single-component system**

- **Depending on the speed of the counterflow we detect:**

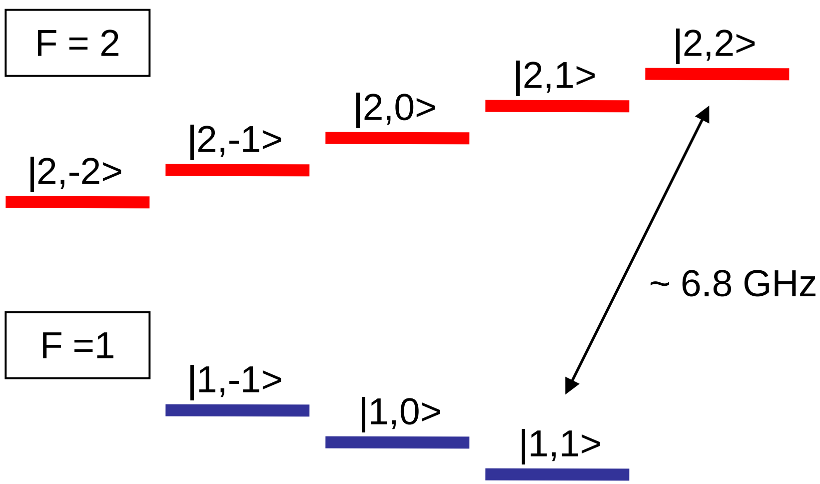
  Modulation instability in *miscible* (!) BEC

  Dark-bright soliton trains

  Novel oscillating dark-dark solitons

Inducing dynamics in binary BECs

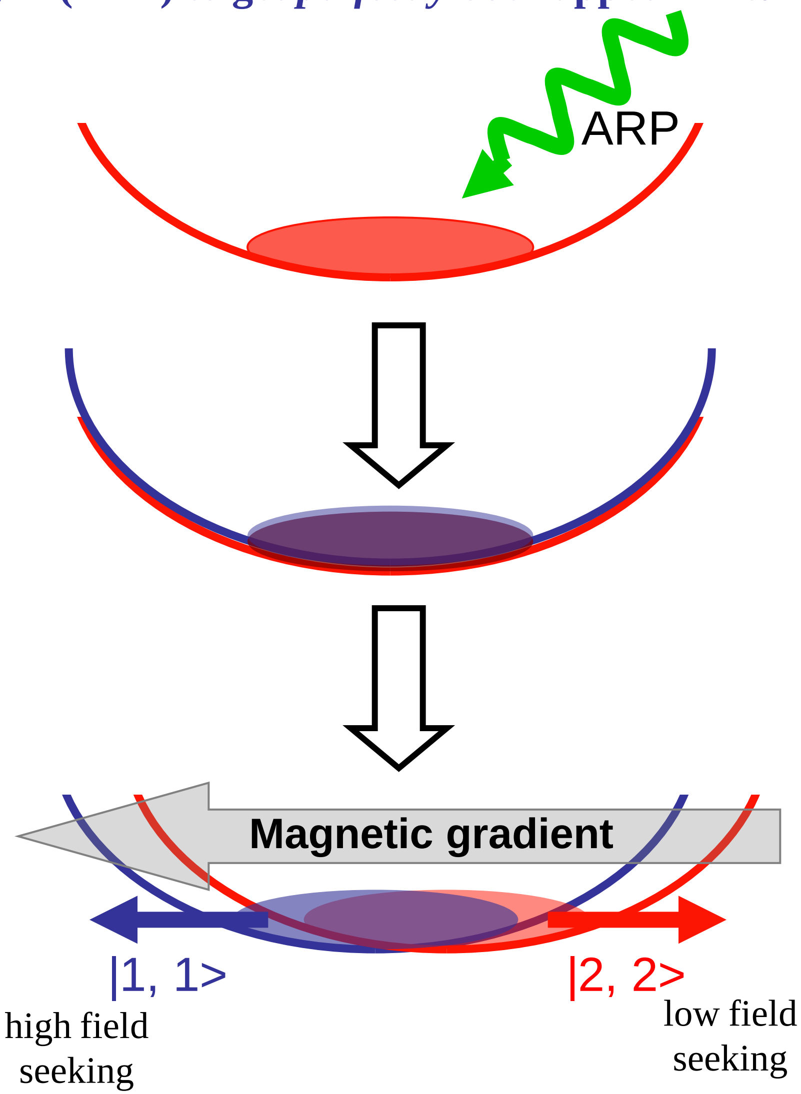
^{87}Rb hyperfine structure: Zeeman splitting



• **External applied magnetic gradient effectively shifts the trap in opposite directions for the two states**
• **We have also used the |1,-1> & |2,-2> states which work in a similar way**

For application to spin gradient demagnetization cooling, see P. Medley et al., arXiv:1006.4674

*** Start with BEC in |2,2> in optical dipole trap**
*** Transfer variable amount of the atoms to |1,1> (ARP) to get *perfectly* overlapped mixture**

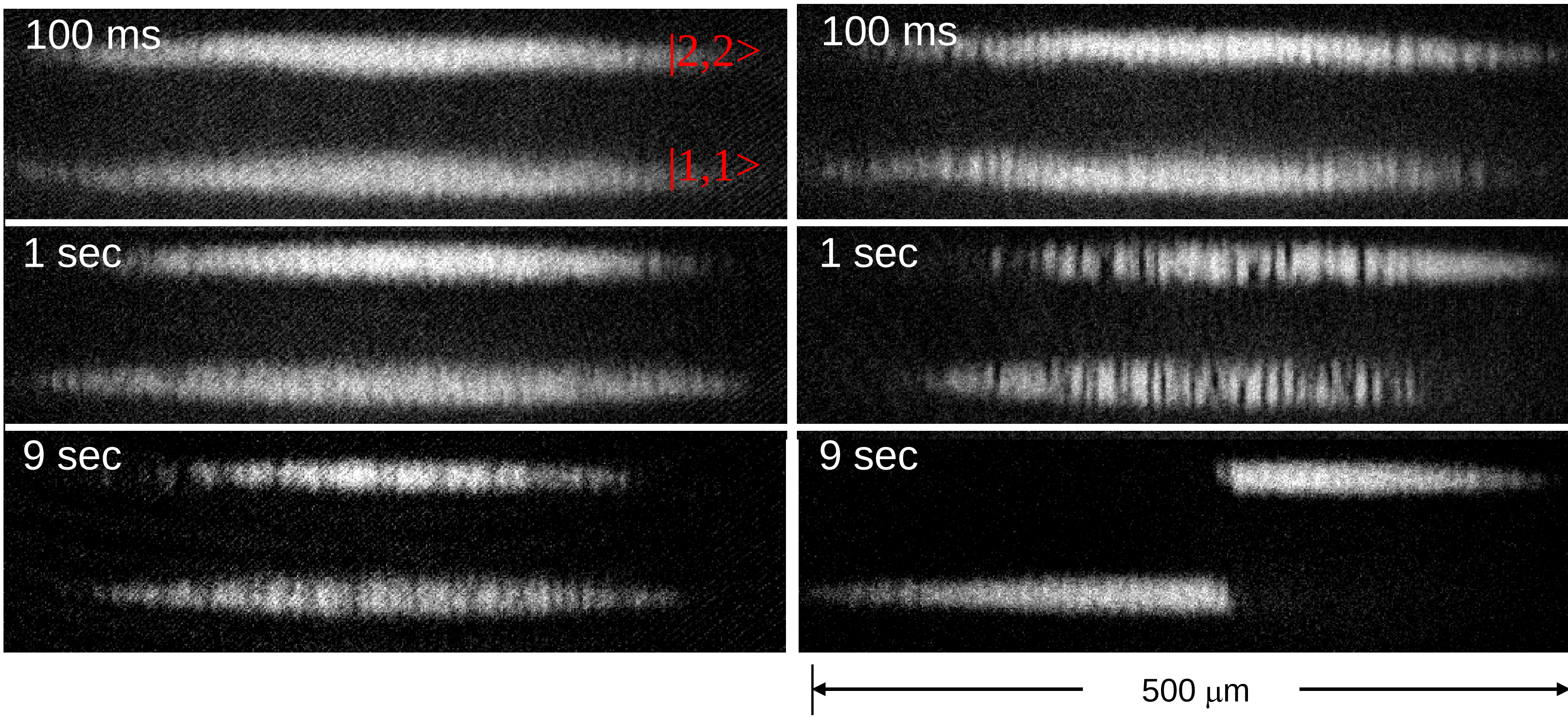


Inducing dynamics in binary BECs

Without axial gradient

With axial gradient, leading to 60 micron relative trap shift

(Note: components are vertically overlapped when in trap.)

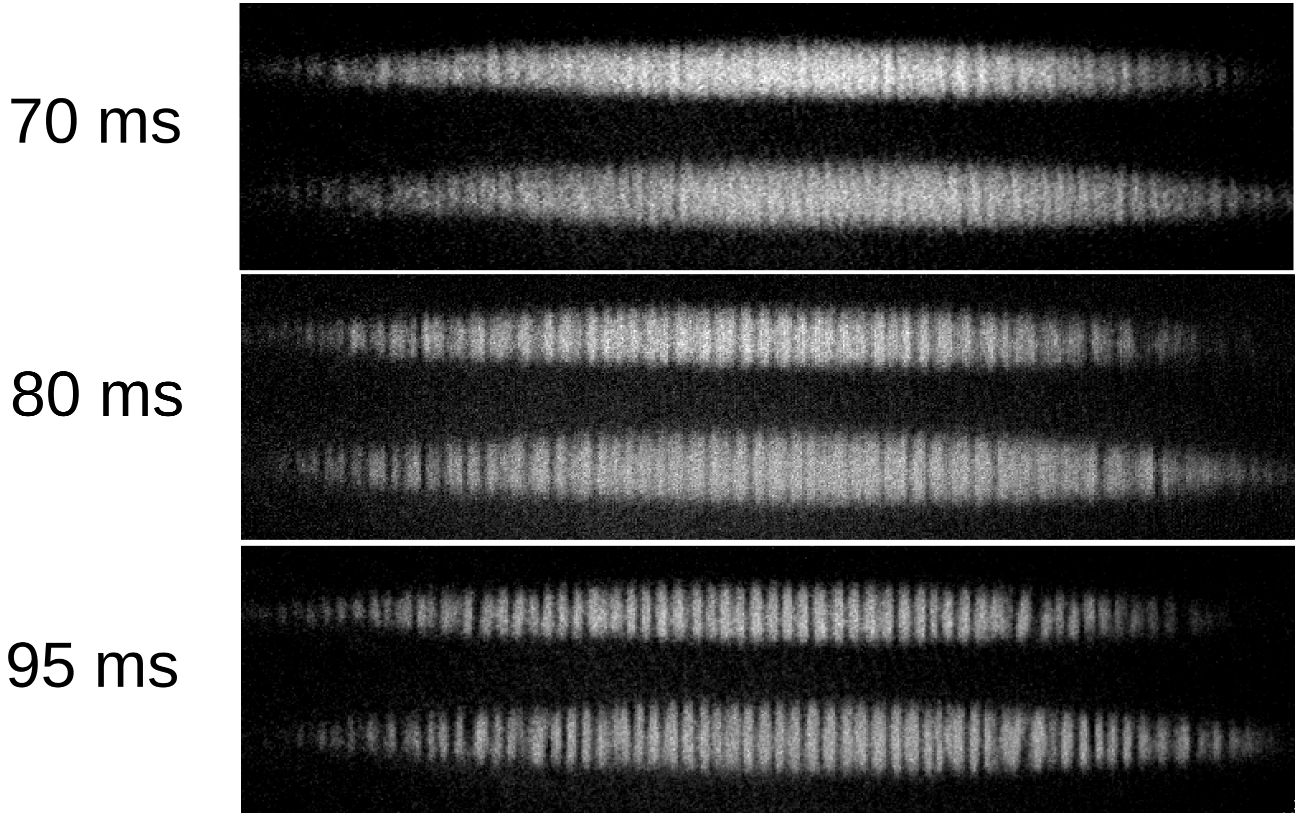


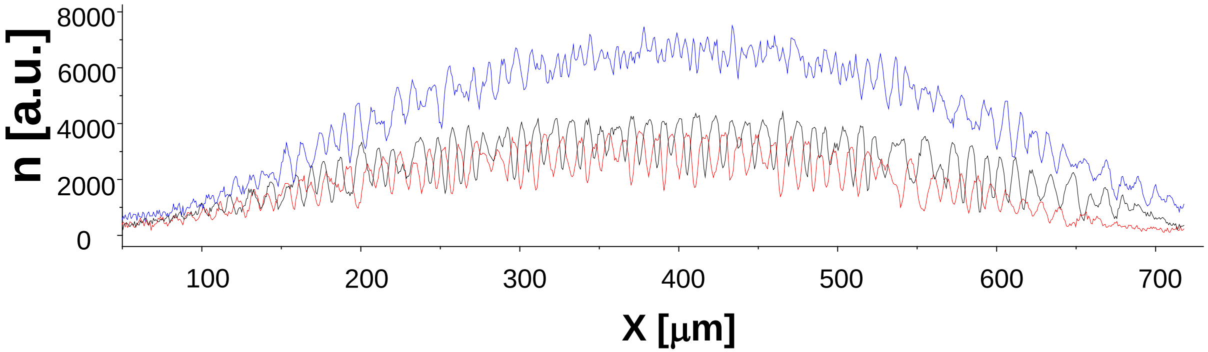
Without a gradient, the mixture is (weakly) miscible.

Even a small relative trap shift can lead to nearly complete demixing.

Counterflow induced modulational instability

Relative trap shift 176 microns (10.7 mG/cm)





Theory: Critical velocity for onset of MI: Method 1

- **Coupled GP equations in 3D (vector NLS equation)**

$$i\hbar\frac{\partial\Psi_2}{\partial t}=(-\frac{\hbar^2}{2m_2}\Delta+\frac{4\pi\hbar^2a_{22}}{m_2}|\Psi_2|^2+\frac{2\pi\hbar^2a_{12}}{m_{12}}|\Psi_1|^2-\mu_2)\Psi_2$$

$$i\hbar\frac{\partial\Psi_1}{\partial t}=(-\frac{\hbar^2}{2m_1}\Delta+\frac{4\pi\hbar^2a_{11}}{m_1}|\Psi_1|^2+\frac{2\pi\hbar^2a_{12}}{m_{12}}|\Psi_2|^2-\mu_1)\Psi_1$$

Hoefer et al., arXiv:1007.4947

C. K. Law et al., PRA 63, 063612 (2001)

Takeuchi et al., PRL 105, 205301 (2010)

J. Ruostekoski and Z. Dutton, PRA 76, 063607 (2007) [lattice system]

- **Consider small perturbations to the plane wave solutions**

$$\Psi_j(\vec{r},t)=\sqrt{\rho_j}e^{i(\nu_j\cdot\vec{r}-\mu_j t)},\qquad \mu_j=\tfrac{1}{2}\nu_j^{\,2}+\rho_j+\sigma_j\rho_{3-j},\qquad \sigma_j=\frac{a_{12}}{a_{jj}}\qquad a_{11}=100.40a_0\qquad a_{12}\approx a_{22}=98.98a_0$$

- **Bogoliubov- deGennes type analysis around the stationary state**

$$\delta\Psi_j=e^{(i/\hbar(m_j\nu_j\cdot\vec{r}-\mu_j t))}\{u_je^{i(\kappa_j\cdot\vec{r}-\omega t))}-w_je^{-i(\kappa_j\cdot\vec{r}-\omega t))}\}$$

- **Examine resulting dispersion relation for imaginary ω occurring in k → 0 region**

For our parameters, one can show $0.1189\le\frac{\mathrm{v}_{\mathrm{cr}}}{\sqrt{\rho_1}}\le 0.1685,$

where the exact value depends on the mixing ratio of the two components.

Theory: Critical velocity for onset of MI: Method 2

M. Hoefer

- **Hydrodynamic equations in 1D: introduce density** $\Psi_j = \sqrt{\rho_j}\, e^{i\phi_j}$

  and velocity $u_j = \dfrac{\partial \phi_j}{\partial x}$

$$\frac{\partial u_1}{\partial t} + \frac{\partial}{\partial x}\left(\tfrac{1}{2}u_1^{\,2} + \rho_1 + \sigma_1\rho_2\right) = \tfrac{1}{4}\frac{\partial}{\partial x}\left(\frac{\frac{\partial^2\rho_1}{\partial x^2}}{\rho_1} - \frac{\left|\frac{\partial\rho_1}{\partial x}\right|^2}{2\rho_1^{\,2}}\right) \qquad \frac{\partial \rho_2}{\partial t} + \frac{\partial}{\partial x}(\rho_2 u_2) = 0$$

$$\frac{\partial u_2}{\partial t} + \frac{\partial}{\partial x}\left(\tfrac{1}{2}u_2^{\,2} + \rho_2 + \sigma_2\rho_1\right) = \tfrac{1}{4}\frac{\partial}{\partial x}\left(\frac{\frac{\partial^2\rho_2}{\partial x^2}}{\rho_2} - \frac{\left|\frac{\partial\rho_2}{\partial x}\right|^2}{2\rho_2^{\,2}}\right) \qquad \frac{\partial \rho_1}{\partial t} + \frac{\partial}{\partial x}(\rho_1 u_1) = 0$$

- **Small wavenumber limit: neglect higher order derivatives above equations (rhs)**

- **Solve for sound speeds** $\rho = \dfrac{\rho_{3-j}}{\rho_j}, \qquad \sigma = \sqrt{\sigma_1\sigma_2} \qquad \sigma_j = \dfrac{a_{12}}{a_{jj}}$

- **Look for relative velocities where a sound speed becomes complex**

$$V_{cr} = \sqrt{w}\,/2 \text{ where w is the smallest, positive real root of :}$$

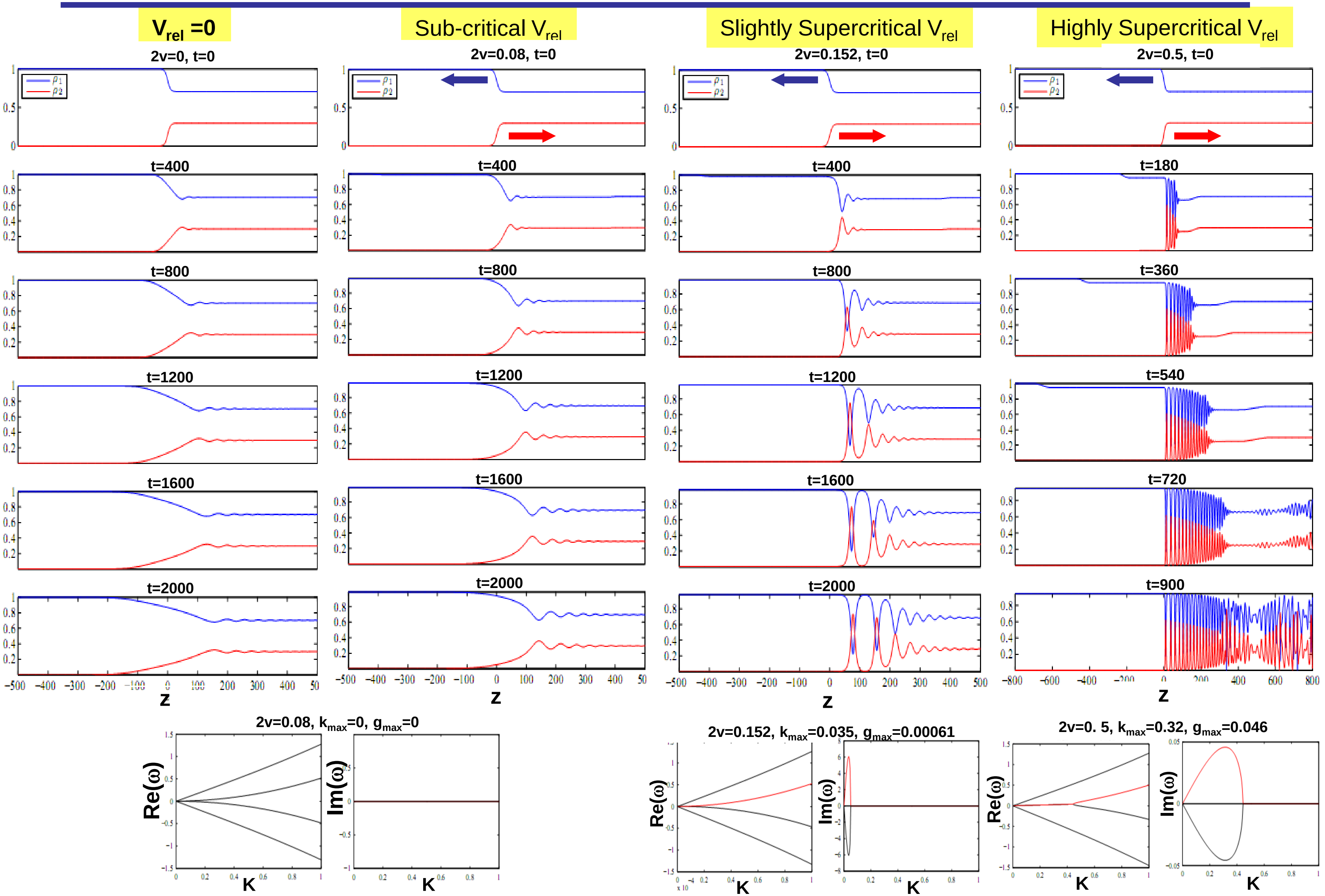
$$(1-\sigma^2)[(\rho-1)^2 + 4\rho\sigma^2]^2 - (1+\rho)[4(1-\rho)^2 - (3+\rho)\sigma^2 + 20\rho\sigma^4]w +$$
$$[2(3+2\rho+3\rho^2) - (3+26\rho+3\rho^2)\sigma^2 + \rho\sigma^4]w^2 - (1+\rho)(4-\sigma^2)w^3 + w^4 = 0$$

- **This explains the uniform counterflow. What about the behavior at a density jump?**

Theory of soliton generation from density jump in single-component, attractive BEC: see A. M. Kamchatnov et al., Phys. Lett. A 319, 406 (2003)

Harnessing MI: 1D Numerics density plot

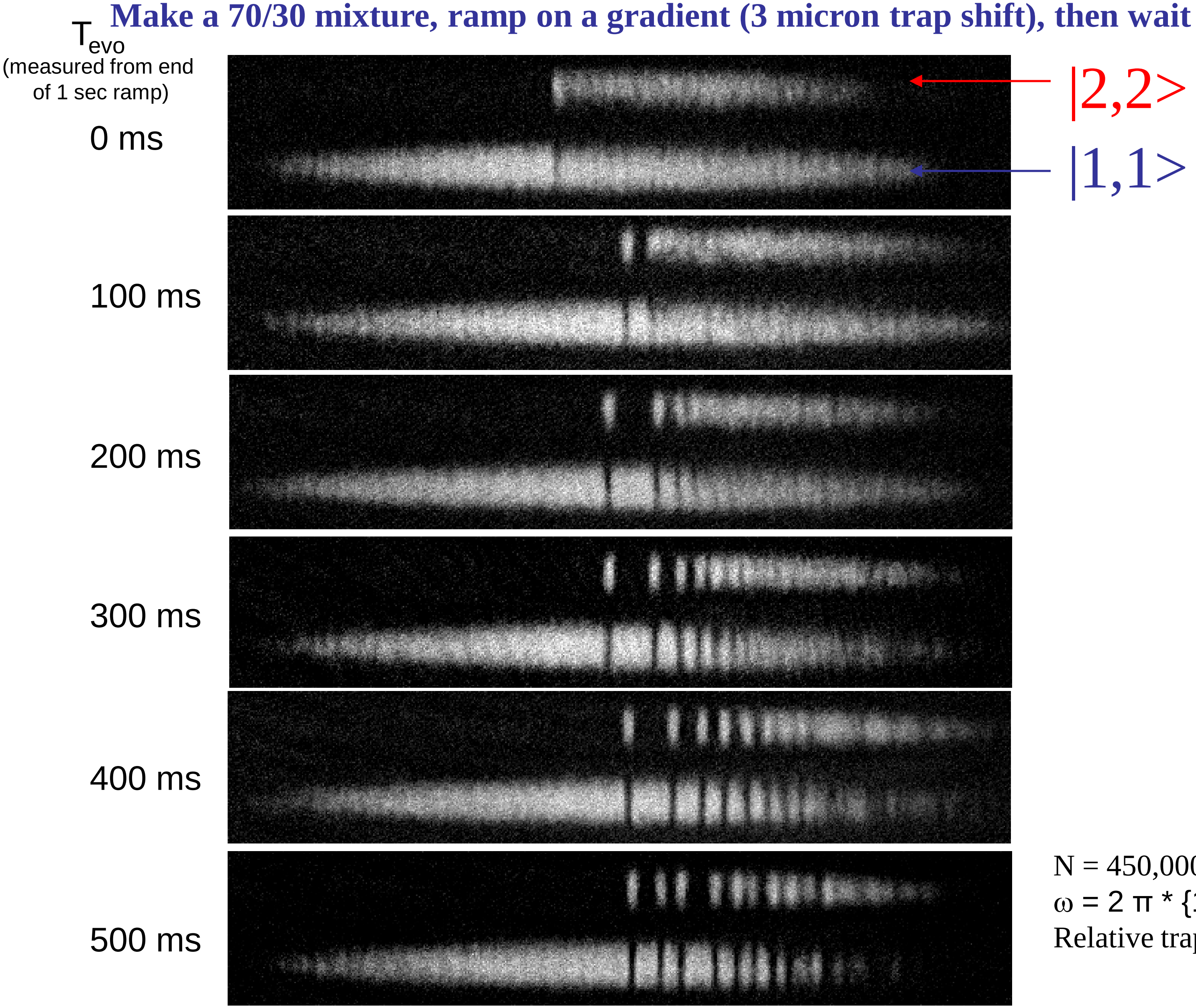
M. Hoefer



Harnessing MI: Experiment

arXiv:1005.2610

Make a 70/30 mixture, ramp on a gradient (3 micron trap shift), then wait t_{evo} and image

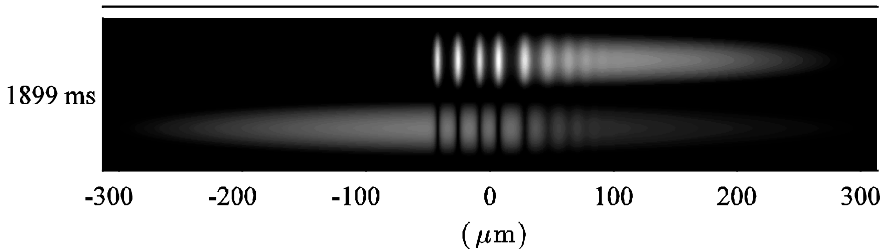


Note: components are vertically overlapped when in trap.

N = 450,000
ω = 2 π * {1.2, 174, 120} Hz
Relative trap shift 3 microns

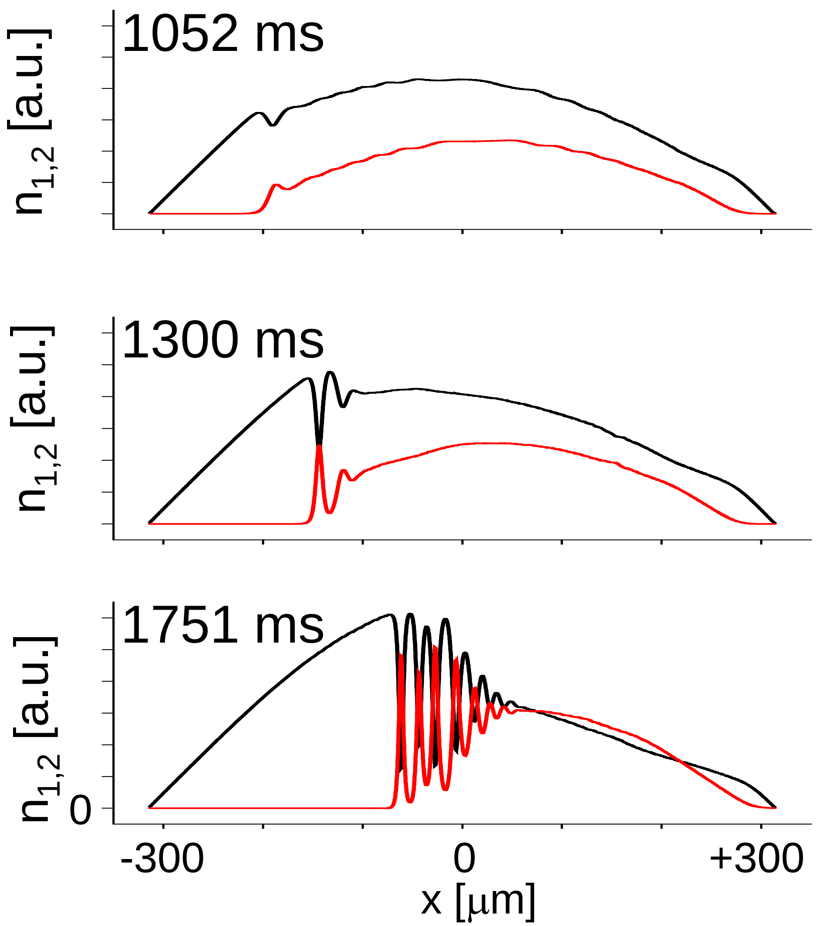
Harnessing MI: 3D Numerics

M. Hoefer

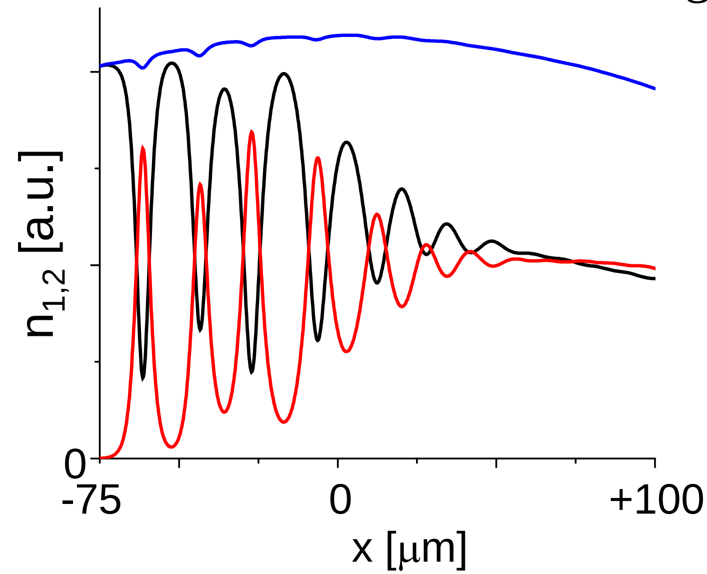


<movie simulation DB train generation>

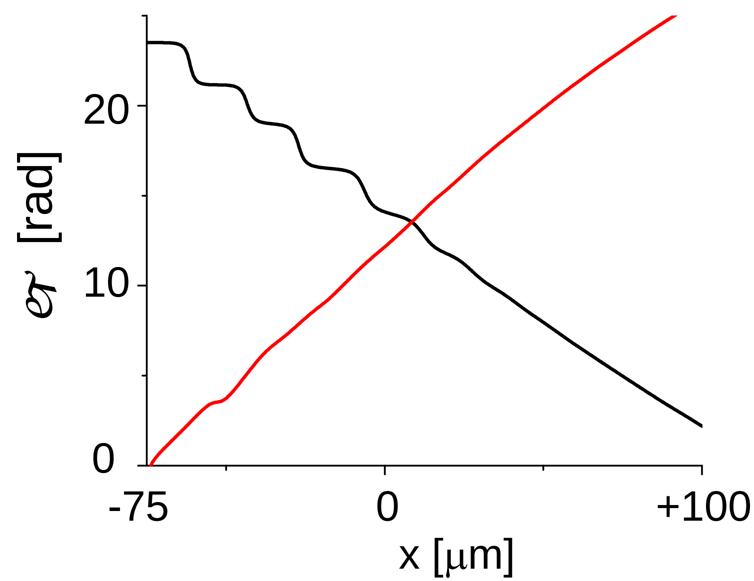
Some integrated cross sections:



Zoomed-in view of soliton region:



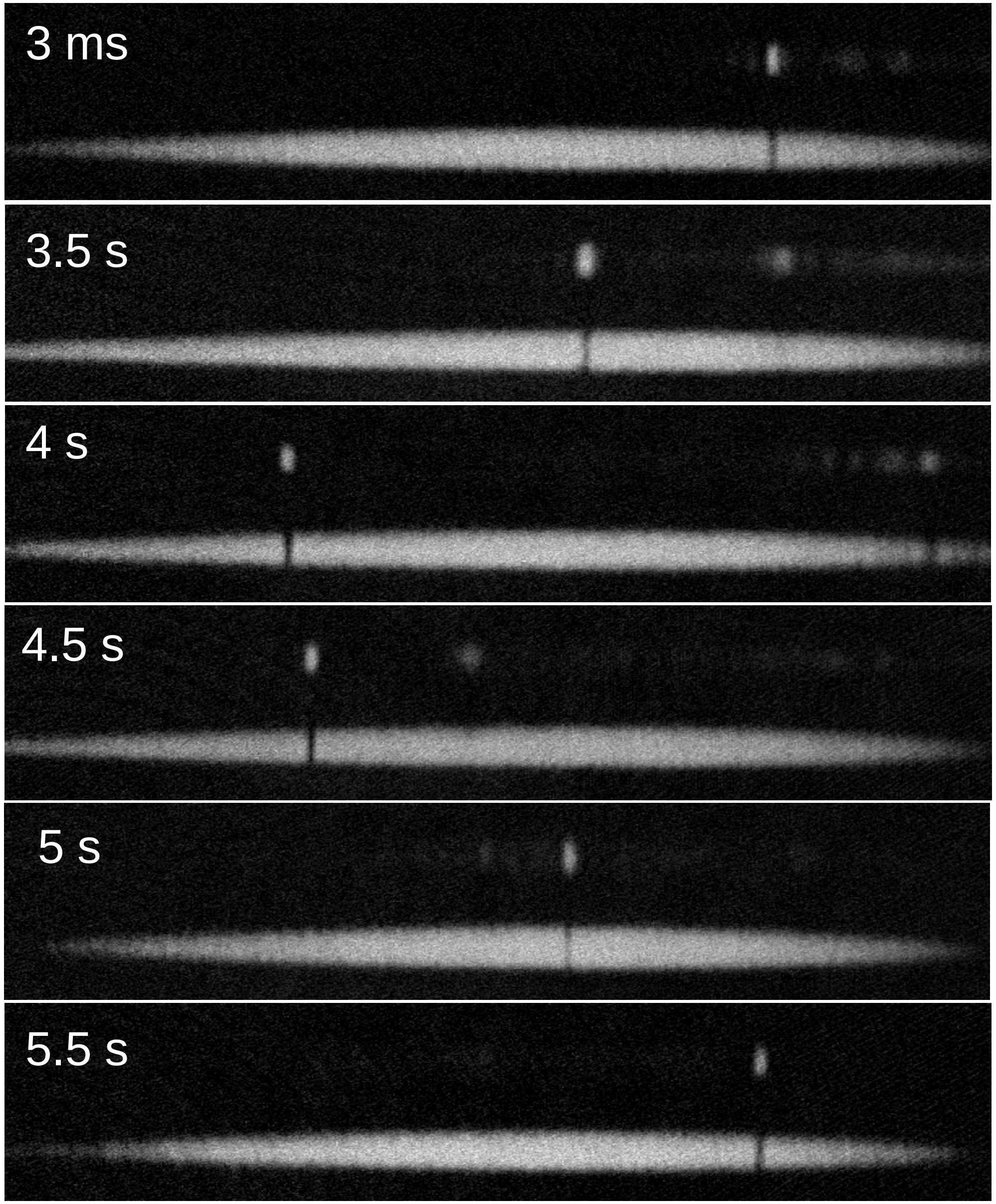
Phase behavior in soliton region:



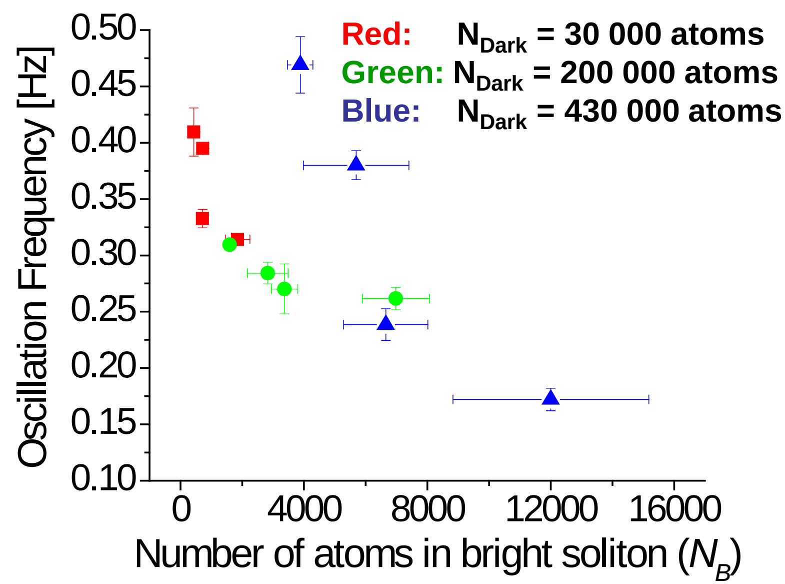
Application: Dark-bright soliton oscillation in a trap

(Note: here we used |1,-1> and |2,-2> states)

S. Middelkamp et al., Physics Letters A, doi:10.1016/j.physleta.2010.11.025



Dark-bright soltions are very slow (compare: ω_{ax} = 1.3 Hz)! Our dark-bright solitons have a very long lifetime!

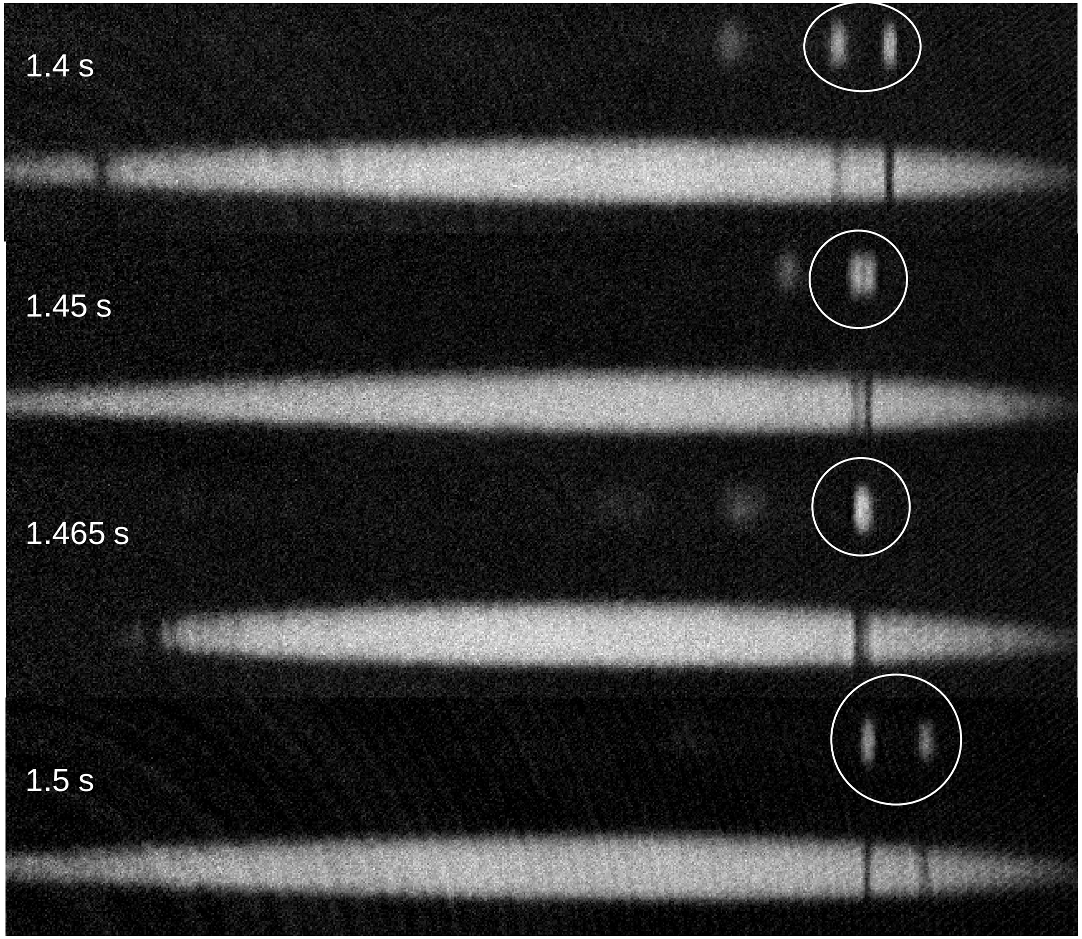


For related data from the Sengstock group, see Becker et al., Nature Physics 4, 496-501 (2008). Theory: See, e.g., Busch and Anglin, Phys. Rev. Lett. 87, 010401 (2001)

Application: Dark-bright soliton oscillation in a trap

S. Middelkamp et al., Physics Letters A, doi:10.1016/j.physleta.2010.11.025

Now use slightly more atoms in |2,-2>:



Solitons maintain their character as separate, individual entities even through a collision

For theory see, e.g.,

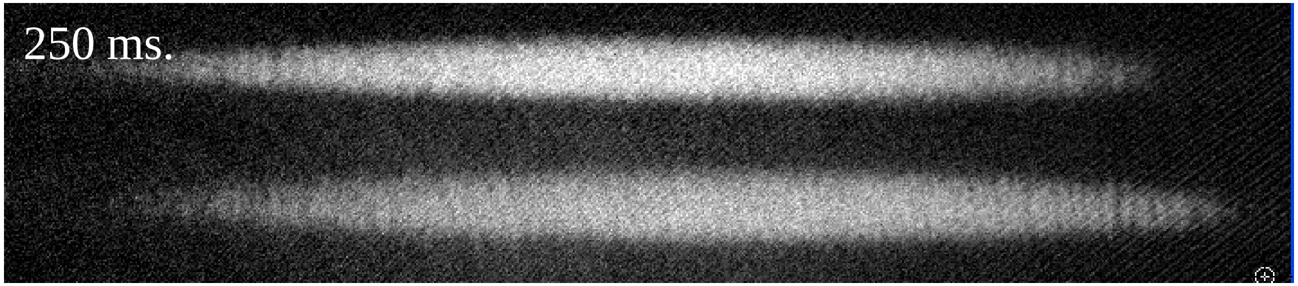
Busch and Anglin, Phys. Rev. Lett. 87, 010401 (2001)

Sheppard and Kivshar, Phys. Rev. E 55, 4773 (1997)

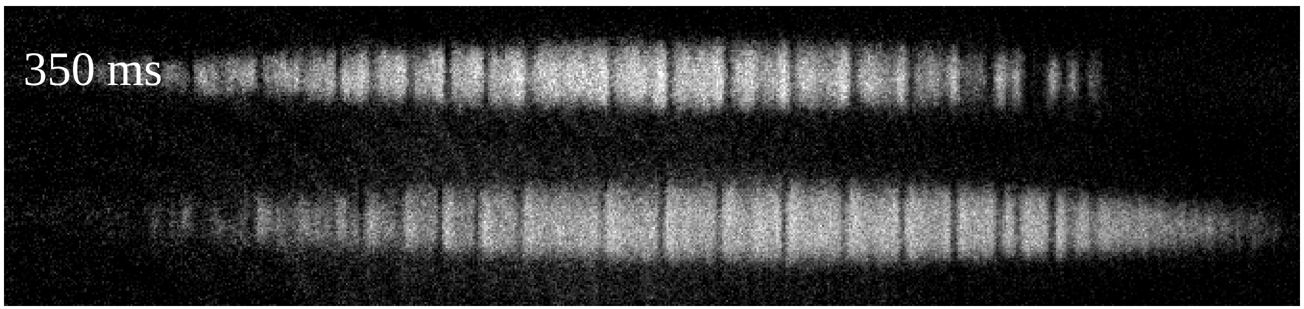
Novel soliton structures

Sparse MI pattern (using intermediate gradients)

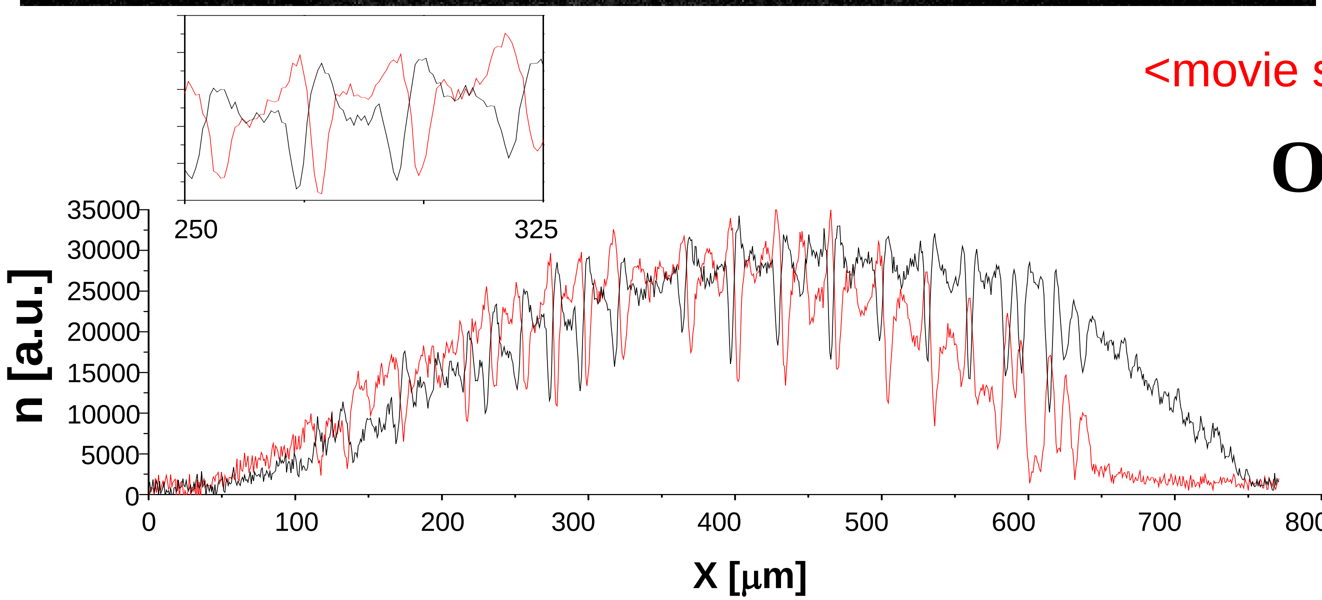
Relative trap shift 23 microns



Smooth counterflow



Spares MI pattern



<movie simulation sparse MI>

Oscillating dark-dark solitons

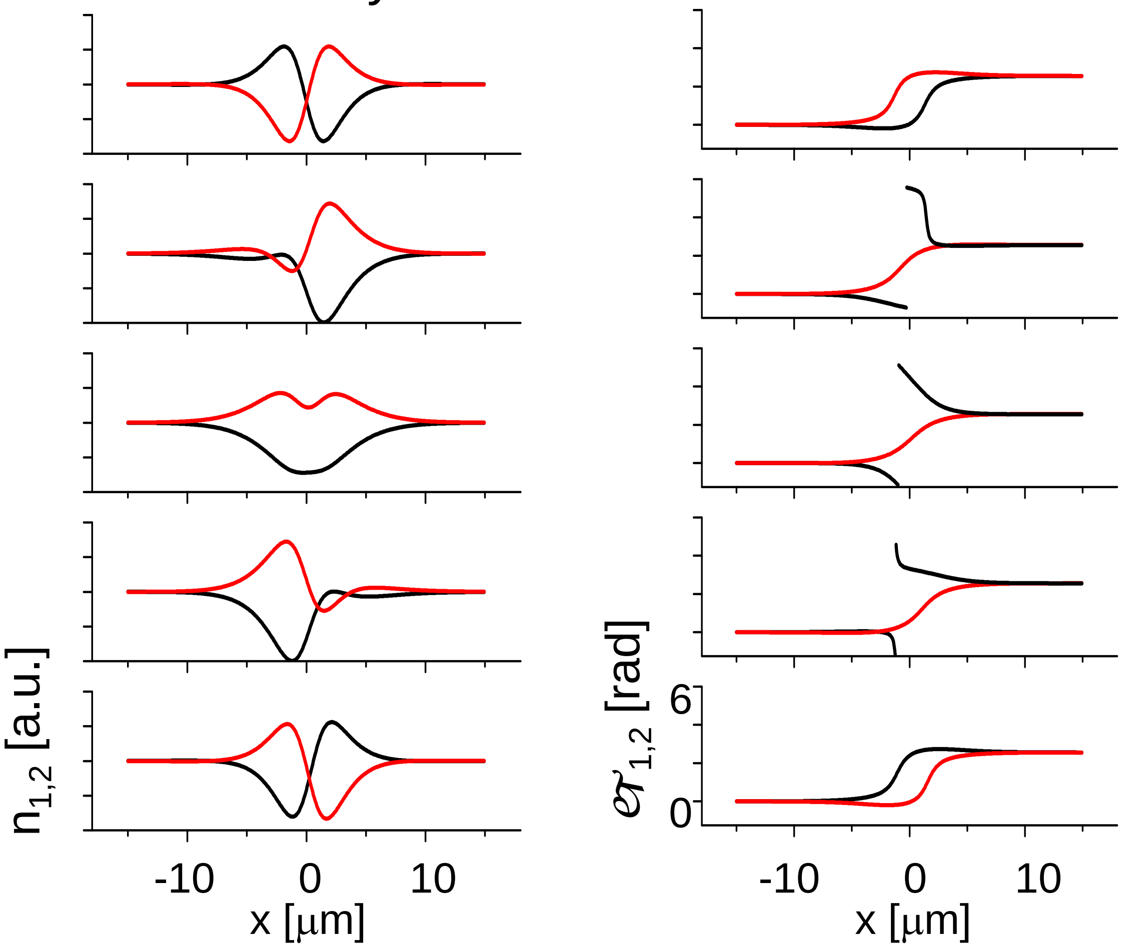
For theory, see also
Q.-H. Park and H. J. Shin, PRE 61, 3093 (2000)
Z. Dutton and C. W. Clark, PRA 71, 063618 (2005)
H. Susanto et al., PRA 75, 055601 (2007)

Oscillating dark-dark solitons: dynamics and phase

Simplified model for a homogenous system were all scattering lengths are the same.

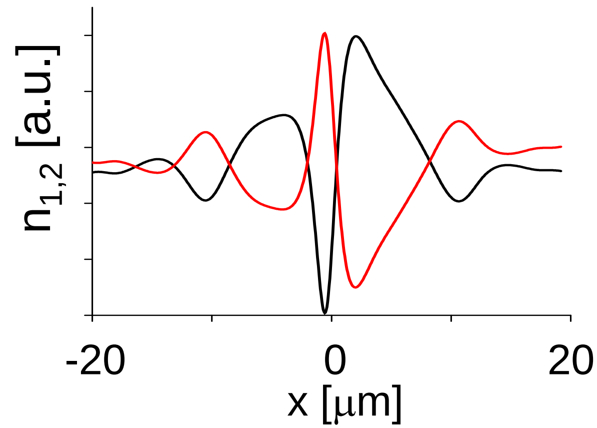
Density

Phase

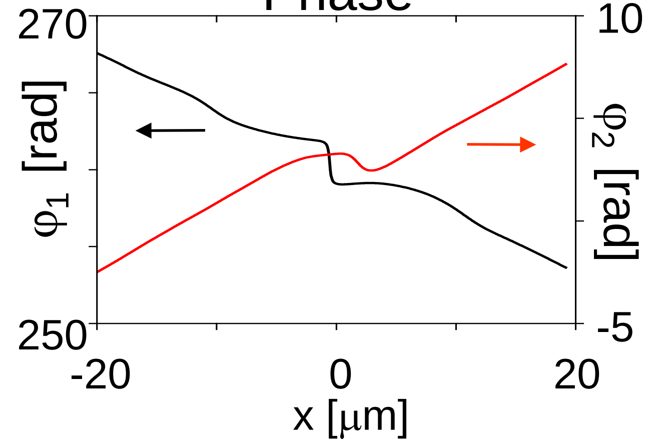


Numerical similation with experimental scattering lengths and trap geometry

Density

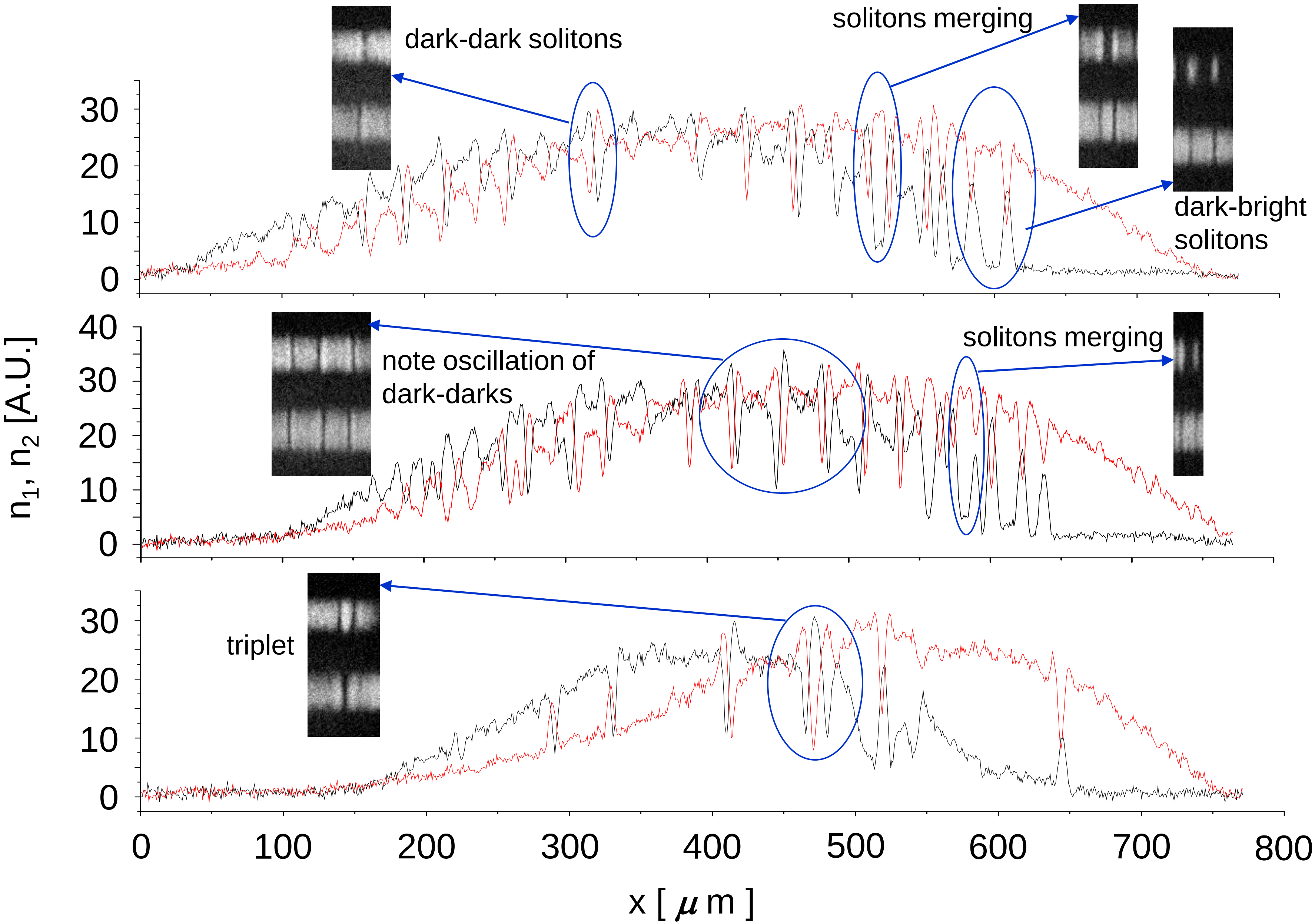


Phase



Simulations by M. Hoefer

The soliton zoo (experimental images)

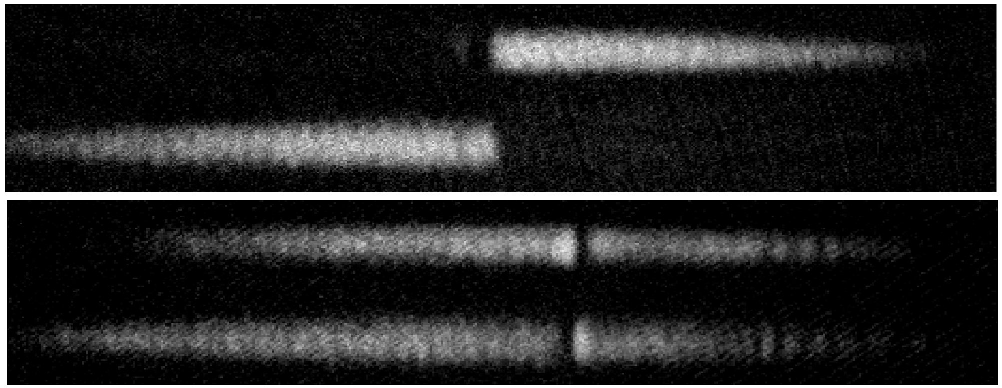


Oscillating dark-dark solitons during remixing

0 ms: applied gradient is tuned off

0 ms

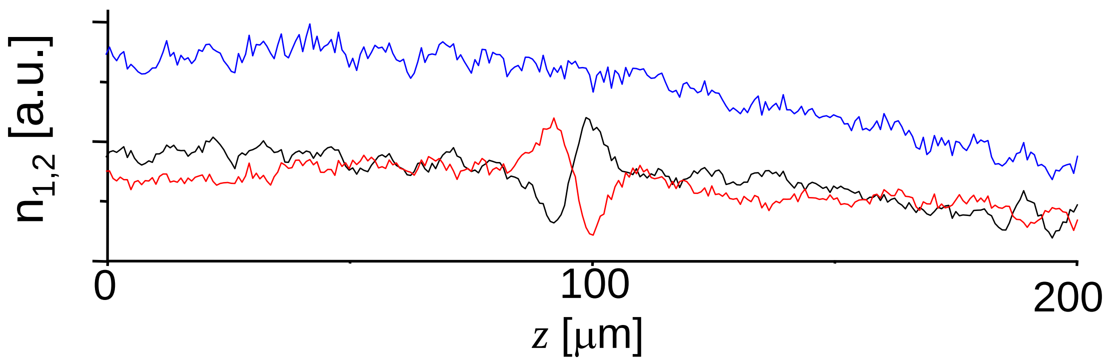
1000 ms



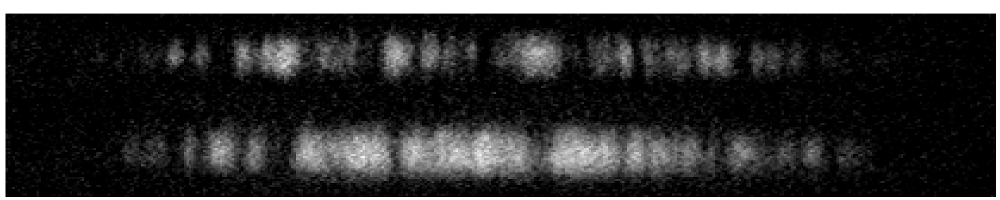
|2,2>

|1,1>

Zoomed-in integrated cross section at 1000 ms:



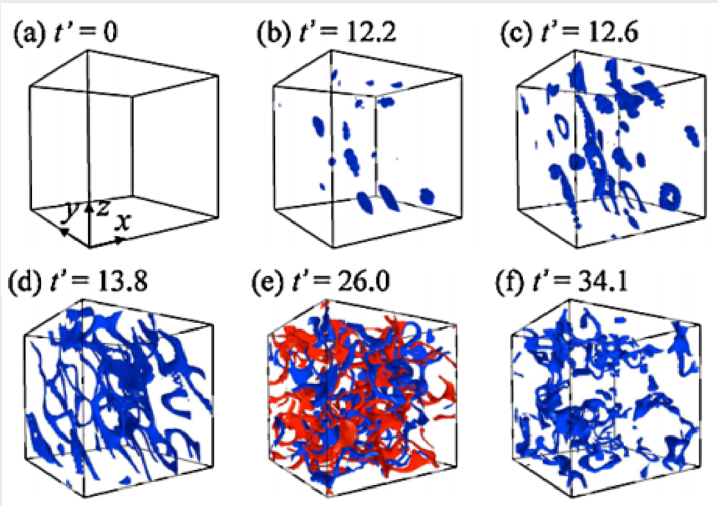
9000 ms



Outlook: Binary quantum turbulence arising from countersuperflow instability

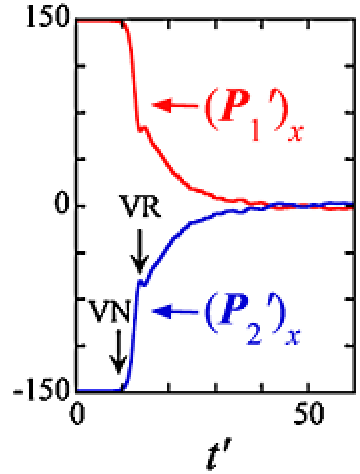
Takeuchi, Ishino, Tsubota, PRL **105**, 205301 (2010)

Vortex tangle formation and decay:



Isosurfaces of density $n_1 = 0.05\, n_0$

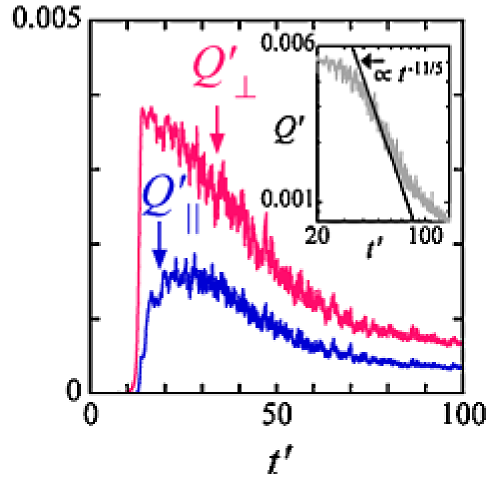
Momentum exchange:



VN: vortex nucleations

VR: vortex reconnections

Enstrophy:



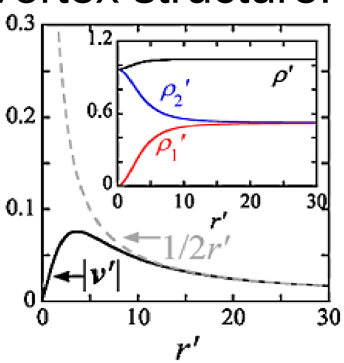
$$\vec{Q} = \frac{1}{2V} \int \omega^2 d\vec{r}$$

Initial enstrophy decay

$$\propto t^{-11/5}$$

Similar to classical turbulence

Vortex structure:



Vortex in one component is filled by other component →velocity field is continuous

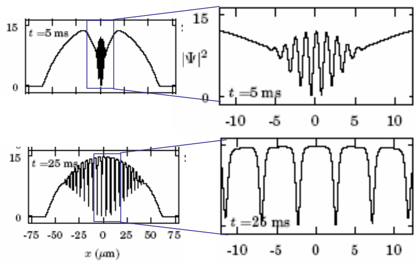
See also:

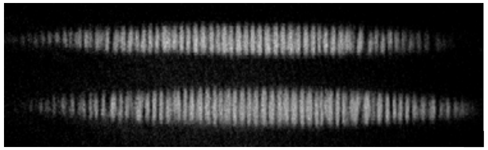
Kasamatsu, Tsubota, and Ueda, Int. J. Mod. Phys. **B19**, 1835 (2005)

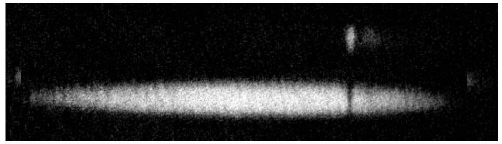
For turbulence in single component BEC, see also E. A. L. Henn et al. PRL 103, 045301 (2009)

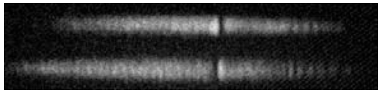
Conclusions

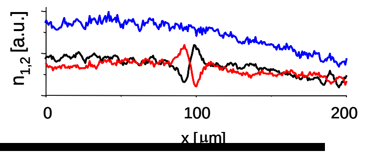
- single component BEC: from interference to soliton trains
- binary BEC: counterflow induced MI
  - harnessing MI to create dark-bright solitons
  - novel types of solitons





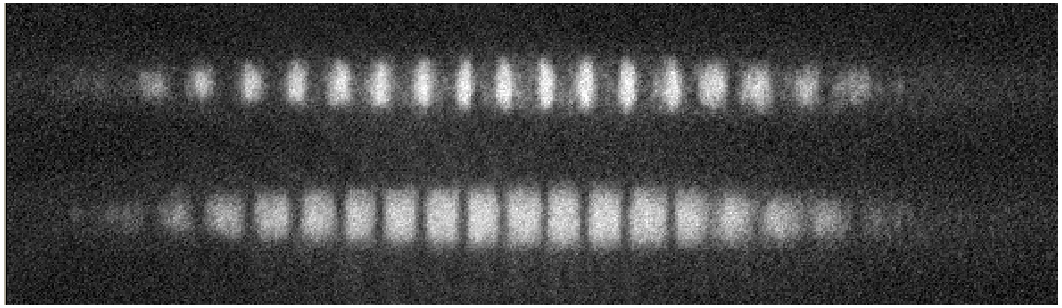




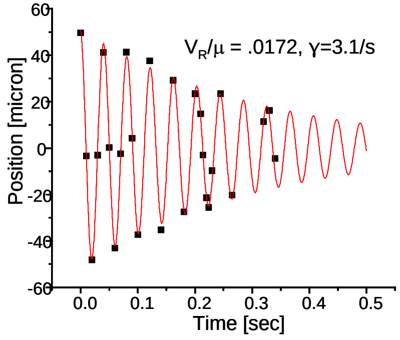


Further projects:

- phase winding a BEC into a soliton train



- disorder in Fermi systems and incommensurate superlattices



→ Open for discussions during the week!