Quantum phase transition from a Luttinger liquid to a gas of cold polar molecules

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Liquid-gas transition

Typical phase diagram

Helium

Solid: maintains volume and shape
Liquid: maintains volume
Gas: fills all available volume
Outline

Zero-temperature liquid-gas transition:
- polar molecules in a helical optical trap

Phase diagram:
- no multi-atomic gases
- finite pressure

Second order transition
Zero-temperature liquid-gas transition

Cold gases exist at $T=0$ at weak intermolecular interaction = low density

![Interaction energy vs. distance graph](image)

A liquid would emerge in the presence of a Lennard-Jones type interaction

Interaction engineering with cold atoms

Feshbach resonance: modeled by a delta-function
Dipole forces: sign does not depend on distance
Polar molecules in a helical trap

a) Helical lattice

b) Interaction $V(s)$ in units of $p^3/(\pi R)^3$ as a function of distance in units of $\pi R$

Electric field polarizes molecules with dipole moment $p$ along the axis of the helix. Phase transition at a critical electric field.

Gas occupies all available volume. The volume of the liquid depends on the interaction strength.
Helical lattice

Circular polarized beam + linearly polarized side beams

Y. K. Pang et al., Opt. Express 13, 7615 (2005);
Conditions

\[ \frac{\hbar^2}{2MR^2} \propto \frac{p^2}{R^3} \]

for realistic parameters: \( M \) on the order of 100 a.u., \( p \) on the order of 1 Debye, \( R \) on the order of a micron.

\[ \Delta_E \propto \frac{(pE)^2}{E_0 - \hbar \omega + i\Gamma} \]
laser optical intensity on the order of ten kW per square cm. [S. Kotochigova and E. Tiesinga, Phys. Rev. A 73, 041405 (2006)]

\[ T < \frac{\hbar^2}{MR^2} \propto 10 \text{ nK} \]

Effective Hamiltonian

\[ H = -\sum_i \frac{\hbar^2}{2M} \frac{\partial^2}{\partial s_i^2} + \sum_{i>j} V(s_i - s_j) \]

from adiabatic approximation

\[ 2\pi L_z + dp_z \] plays the role of the 1D momentum
Luttinger liquid

We focus on $P=0$

Weak interaction: dilute gas of independent particles

Strong interaction: nearest neighbor interaction dominates and can be approximated by a harmonic potential

$$S_0 = \frac{1}{2} \int d\tau \left[ \sum_k M s_k^2 + \sum_k K (s_k - s_{k+1} - \hbar)^2 \right]$$

$$\langle (s_{n+k}(t) - s_n(t) - k\hbar)^2 \rangle = \frac{\hbar \ln k}{\pi \sqrt{KM}}$$
Phase transition


\[ S = \int d\tau dx [\hbar \Psi^* \partial_\tau \Psi - \frac{\hbar^2}{2m} \Psi^* \nabla^2 \Psi - \mu |\Psi|^2 + g |\Psi|^4] \]


Exact results

\[ V(x) = +\infty, x < a; V(x) = -AU(x), x > a \]

No bound states for small nonzero \( A \). Variational proof. Number all particles from left to right. Set masses of particles with even numbers to infinity and prove that there is still no bound states. This can be reduced to a single-particle problem.
Exact results

\[ H = -\sum_i \frac{\hbar^2}{2M} \frac{\partial^2}{\partial s_i^2} + \sum_{i>j} V(s_i - s_j) \]

\[ H_{12} = -\frac{\hbar^2}{M} \frac{d^2}{d\Delta_1^2} + V(\Delta_1); \Delta_1 = s_2 - s_1 \]

\[ H_{12}\psi_2(\Delta_1) = \varepsilon_2 \psi_2; \text{two-particle ground state, } \varepsilon_2 < 0 \]

\[ H_{123} = -\sum_{k=1}^3 \frac{\hbar^2}{2M} \frac{\partial^2}{\partial s_k^2} + V(\Delta_1) + V(\Delta_2) + V(\Delta_1 + \Delta_2) \]

\[ \psi_3(s_1, s_2, s_3) = \psi_2(\Delta_1)\psi_2(\Delta_2) \]

\[ \langle \psi_3 \mid H_{123} \mid \psi_3 \rangle = \int d\Delta_1 d\Delta_2 \psi_2(\Delta_1)\psi_2(\Delta_2)[V(\Delta_1) + V(\Delta_2) - \frac{\hbar^2}{M} \left( \frac{\partial^2}{\partial \Delta_1^2} + \frac{\partial^2}{\partial \Delta_2^2} \right) + \frac{\hbar^2}{M} \frac{\partial^2}{\partial \Delta_1 \partial \Delta_2} + V(\Delta_1 + \Delta_2)]\psi_2(\Delta_1)\psi_2(\Delta_2) \]

\[ = 2\varepsilon_2 + \text{square of the integral of a full derivative} + \text{negative} \]

Lower energy per particle in a three-particle state than in the two-particle ground state. A generalization of this argument proves that the transition occurs directly from a monoatomic gas to a condensed state.
Variational method

Neglect all interactions except nearest neighbors. We expect universal behavior near the transition. For analytical calculations use the Morse potential

$$V(s) = A \{ \exp(-2\alpha[s-h]) - 2 \exp(-\alpha[s-h]) \}$$

$$\psi_{\text{VAR}} = \prod_{k=1}^{N-1} \psi_k(\Delta_k)$$

$$E = -A[1 - \alpha \hbar / \sqrt{4MA}]^2 \propto (A - A_c)^2, \ A > A_c$$

$$E = 0, \ A < A_c$$

$$\rho \propto (A - A_c)$$

Second order liquid-gas transition? Critical point at zero pressure.
Critical point

\[ H \rightarrow H - P(s_N - s_1) \]

Second order liquid-gas transition? Critical point at zero pressure
Summary

- Cold polar molecules in helical optical lattices exhibit quantum liquid-gas transition at critical electric field
- Direct transition between a monoatomic gas and a liquid
- Second order?
- Is the transition at the dimer formation threshold?