Quantum critical behavior in driven and strongly interacting Rydberg gases

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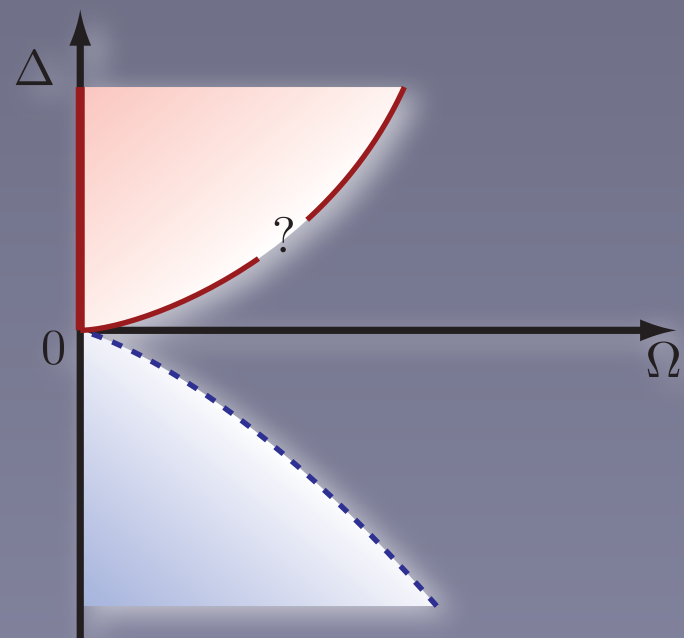
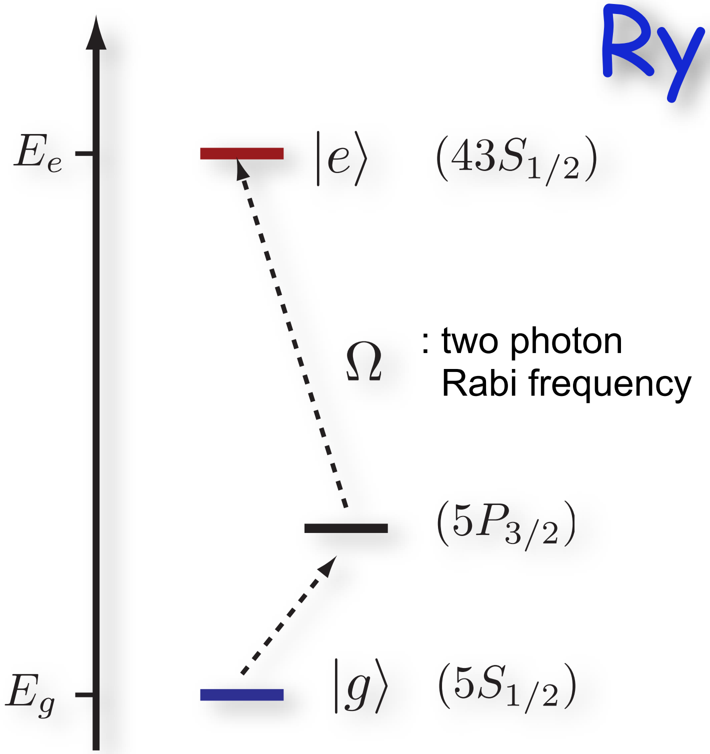


H. Weimer, → Harvard
J. Honer

Exp. collaboration:
T. Pfau, R. Löw

SFB TRR21:
Tailored quantum matter

Rydberg atoms



Rydberg excitation

- excitation of an atom into a state with high principal quantum number n

- well defined quantum number

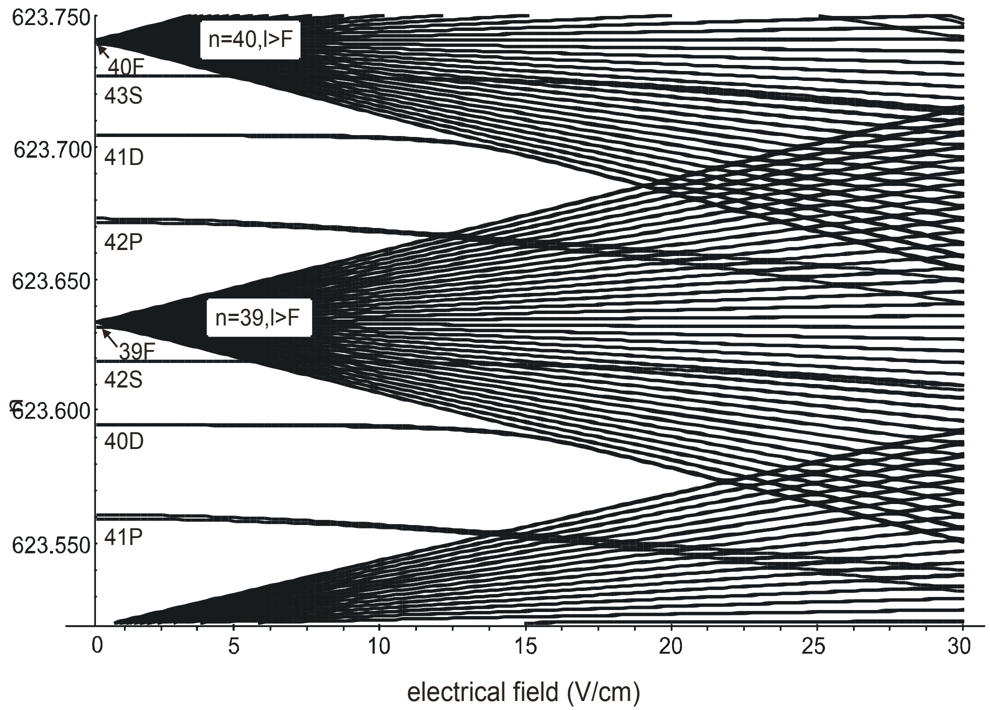
- finite life time

- “frozen” Rydberg gas

- large dipole moments

$$d \sim e\, a_0\, n^2$$

strong Rydberg-Rydberg interactions



Rydberg excitations

Rydberg-Rydberg interaction

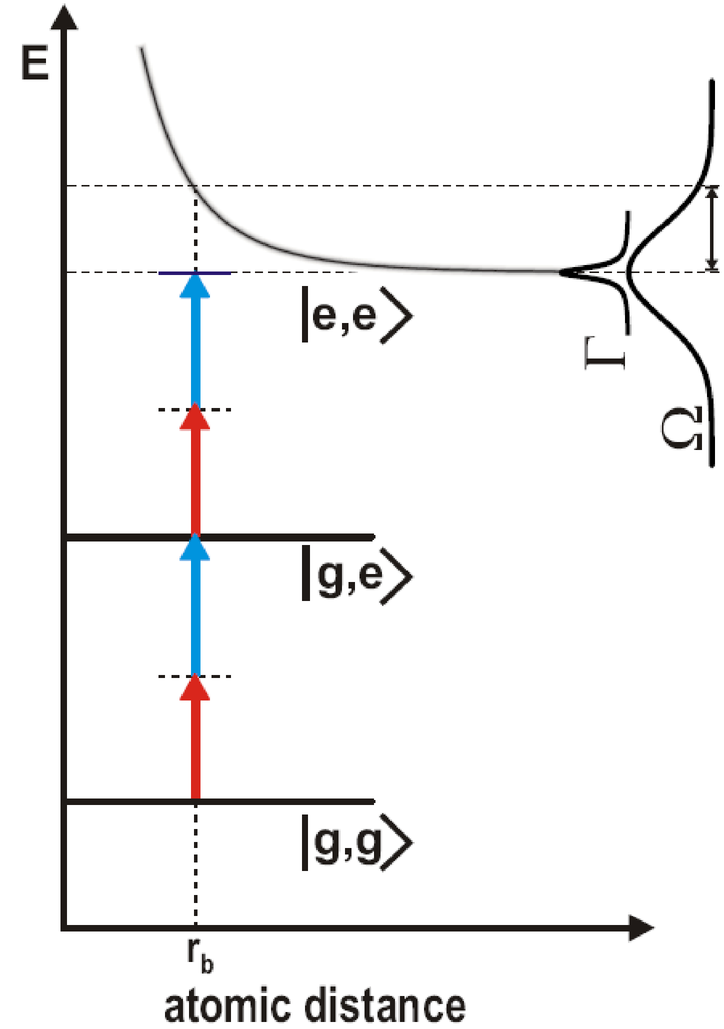
- strong van der Waals interactions for s-wave states
    - depending on n attractive or repulsive
    - $C_6 \sim n^{11}$
- dipole-dipole interactions in presence of an electric field

$$d \sim n^2 e a_0$$

Blockade phenomena

- once a Rydberg atom is excited, further excitatons are shifted out of resonance
- Blockade radius

$$r_b = \sqrt[6]{C_6/\hbar\Omega}$$

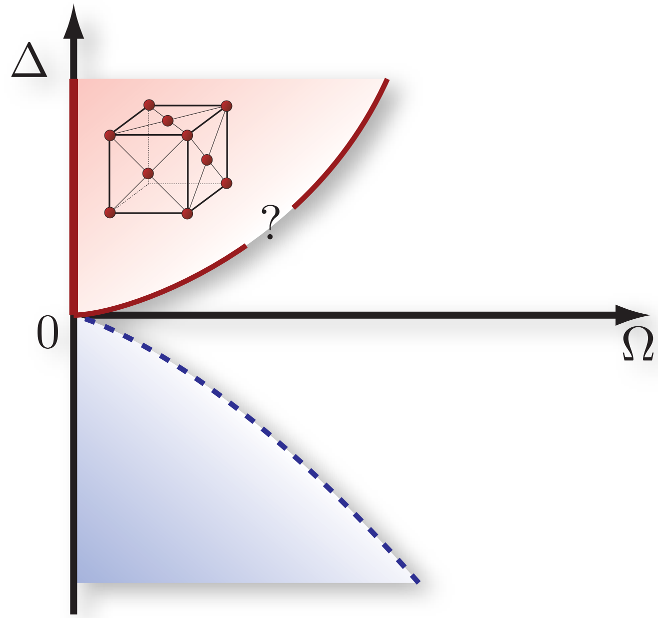


Exp: T. F. Gallagher, Charlottesville; M. Weidemüller, Freiburg; P. Pillet, Orsay; van den Heuvell, Amsterdam; P. Gould, Storrs; T. Pfau Stuttgart, A. Browaeys, P. Grangier, Orsay; M. Saffman.
Th: Robicheaux and Hernández, Ates, Pohl, Pattard, Rost, Stanojevic and Côté,

Outline

Many body phenomena

- quantum phase transition
- universal scaling

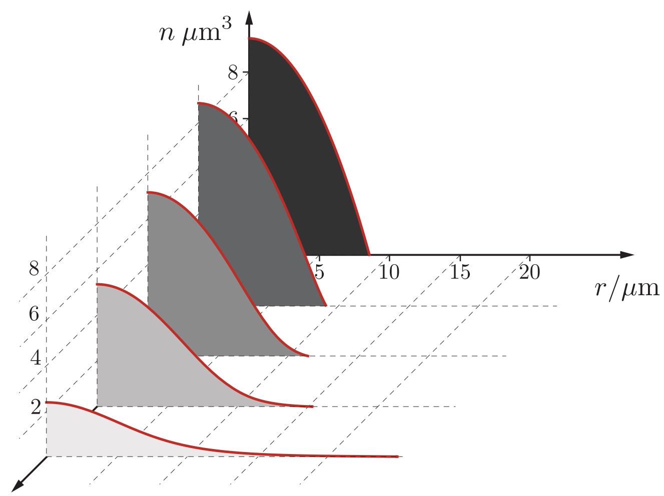


Crystalline phase

- floating solid in one-dimension

Tool for designing interactions

- collective many-body interaction

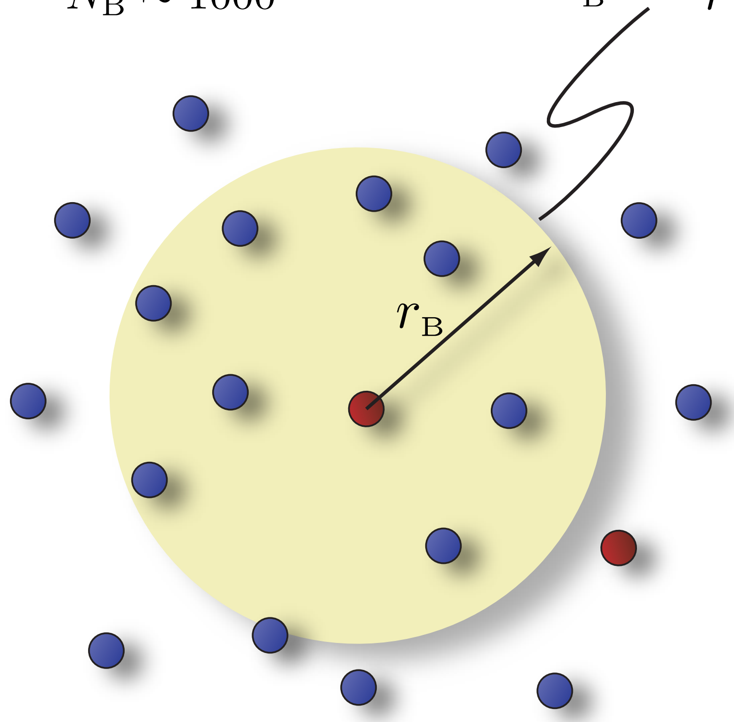


Collective behavior

Evidence for coherent collective Rydberg excitations in the strong blockade regime
(T. Pfau, Phys. Rev. Lett. 99, 163601 2007)

Number of particles within blockade radius
$N_{\mathrm{B}} \sim 1000$

Blockade radius
$r_{\mathrm{B}} \sim 5\mu\mathrm{m}$



- ultracold atomic gas:
  above BEC transition
- resonant Rydberg excitation
- “frozen” Rydberg gas:
  no motion of the atoms
- strong blockade regime:

$$r_{\mathrm{B}} = (C_6/\hbar\Omega)^{1/6} \gg a = n^{-1/3}$$

Blockade radius — interparticle distance

Collective many-body phenomena

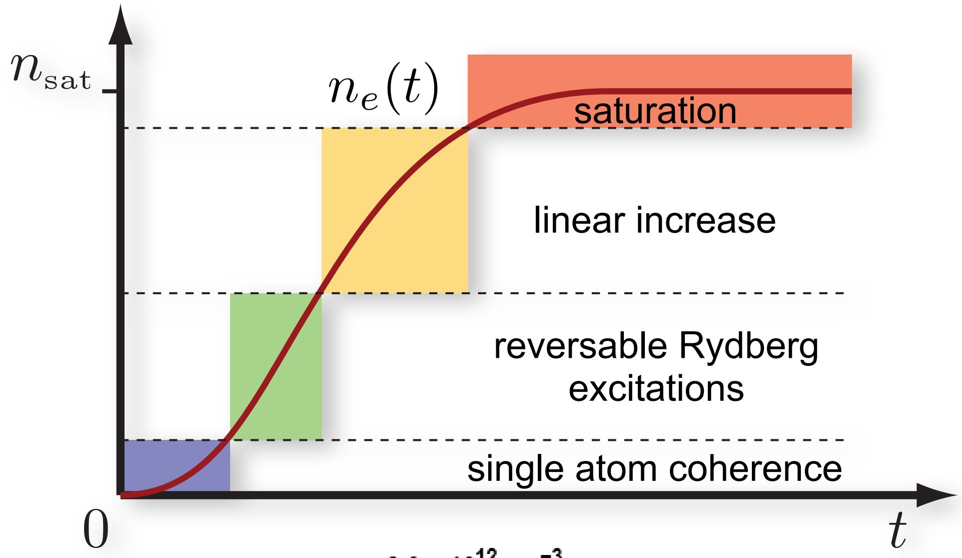
Saturation

Characteristic time evolution

- initial condition:
  all atoms in ground state

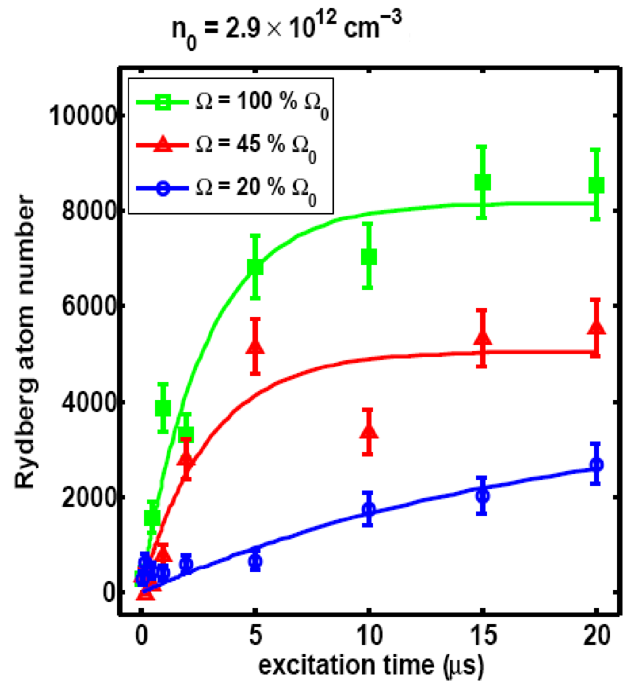
- switching on of laser:

  - single atom coherence on short time scales
  - intermediate regime with Blockade effects
  - saturation in a steady state



Equilibrium state on long time scales

- relation to ground state of the Hamiltonian?
- “thermal” equilibrium state?



(T. Pfau, Phys. Rev. Lett. 99, 163601 2007
R. Löw, *et al* 2009)

Hamiltonian

Effective spin system

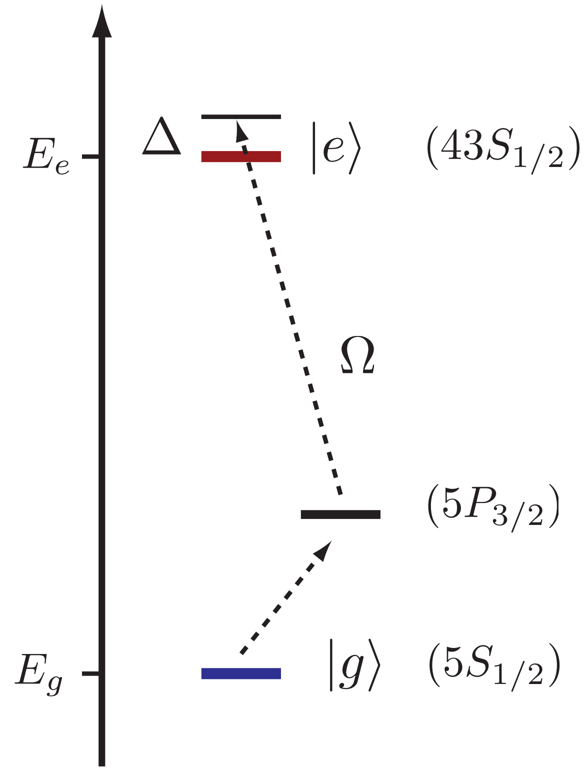
- rotating wave approximation (rotating frame)

- mapping to spin-1/2 system
$$|\uparrow\rangle_i = |e\rangle_i$$
$$|\downarrow\rangle_i = |g\rangle_i$$

$$\sigma_i^z = |e\rangle\langle e|_i - |g\rangle\langle g|_i$$
$$\sigma_i^x = |e\rangle\langle g|_i + |g\rangle\langle e|_i$$

- number of excited Rydberg atoms

$$P_i^e = (\sigma_i^z + 1)/2$$
$$n_e = \sum_i P_i^e$$



Hamiltonian

$$H = \frac{C_6}{2} \sum_{i\neq j} \frac{P_i^e P_j^e}{|\mathbf{r}_i - \mathbf{r}_j|^6} + \frac{\hbar\Omega}{2} \sum_i \sigma_i^x - \frac{\hbar\Delta}{2} \sum_i \sigma_i^z$$

$\mathbf{r}_i$: particle position

n : averaged particle density

d : dimension of the system

- dimensionless parameter
$$\alpha = \frac{\hbar\Omega}{C_6 n^{6/d}}$$

Phase Diagram

Ground state $\Omega = 0$

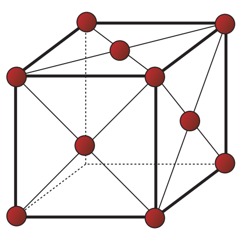
- classical Hamiltonian without quantum fluctuations

$$H = \frac{C_6}{2}\sum_{i\neq j}\frac{P_i^e P_j^e}{|\mathbf{r}_i - \mathbf{r}_j|^6} + \cancel{\frac{\hbar\Omega}{2}\sum_i \sigma_i^x} - \frac{\hbar\Delta}{2}\sum_i \sigma_i^z$$

Crystalline phase

$\Delta > 0, \Omega = 0$

- finite number of excitation: $\langle n_e \rangle > 0$
- crystalline structure: closed sphere packing



Second order quantum phase transition

$$\langle n_e \rangle \sim \Delta^{d/6}$$

Paramagnet, "Vacuum"

$\Delta < 0, \Omega = 0$

- all particles in the ground state: $\langle n_e \rangle = 0$
- initial state of the experiment

Phase Diagram $(\Omega = 0)$

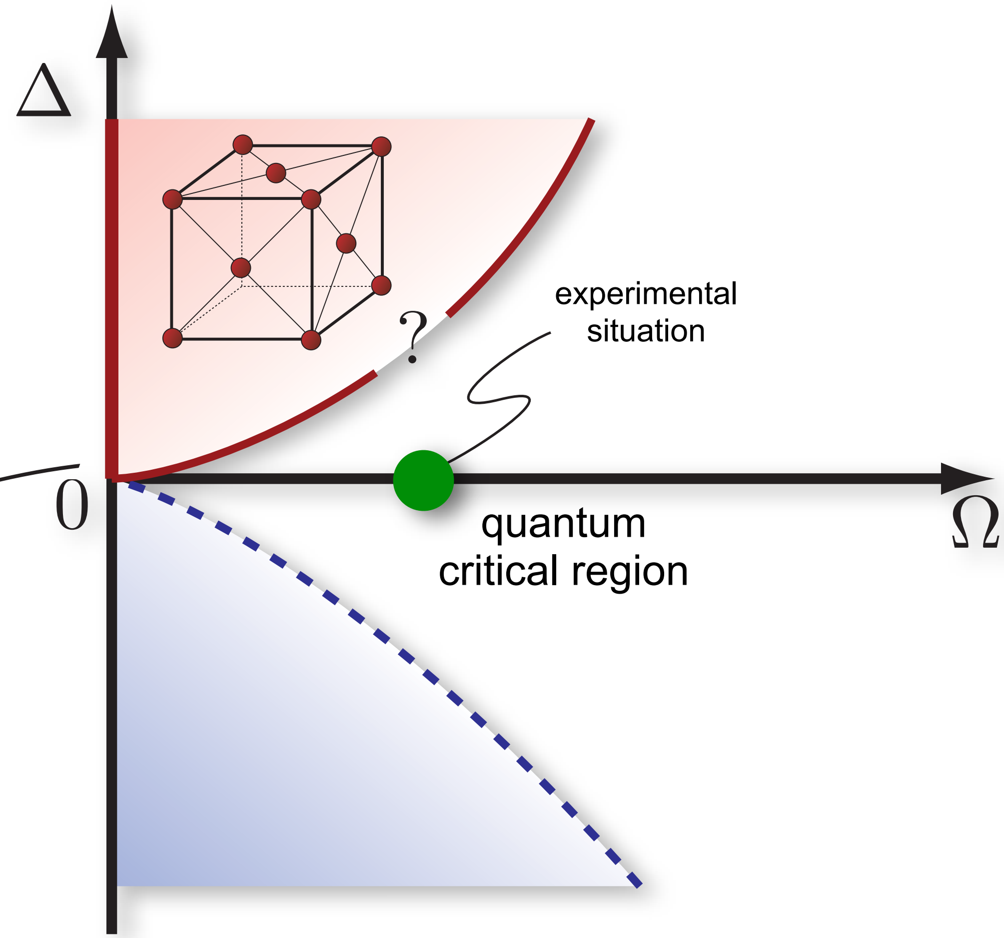
Crystalline phase

$$\Delta > 0, \Omega = 0$$

- finite number of excitation: $\langle n_e \rangle > 0$
- crystalline structure: closed sphere packing
- diagonal long range order

Second order quantum phase transition

$$\langle n_e \rangle \sim \Delta^{d/6}$$



Paramagnet, “Vacuum”

$$\Delta < 0, \Omega = 0$$

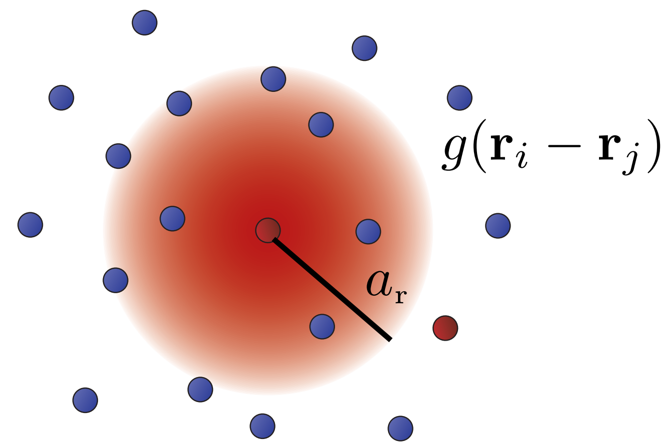
- all particles in the ground state: $\langle n_e \rangle = 0$
- initial state of the experiment

Mean field theory

Approximation

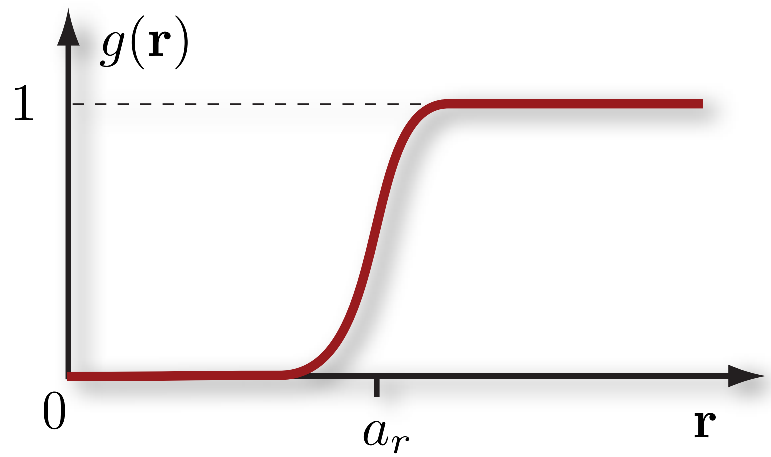
- select a single atom
- surrounded by a bath of atoms
- interaction produces an effective potential

$$h_z = \sum_j g(\mathbf{r}_i, \mathbf{r}_j) \langle P_j \rangle \frac{n^{6/d}}{|\mathbf{r}_i - \mathbf{r}_j|^6}$$



- local Hamiltonian

$$H_i = \frac{\alpha}{2}\sigma_i^z + P_i^e h_z = \frac{\alpha}{2}\sigma_i^x + \frac{h_z}{2}\sigma_i^z + \frac{h_z}{2}$$



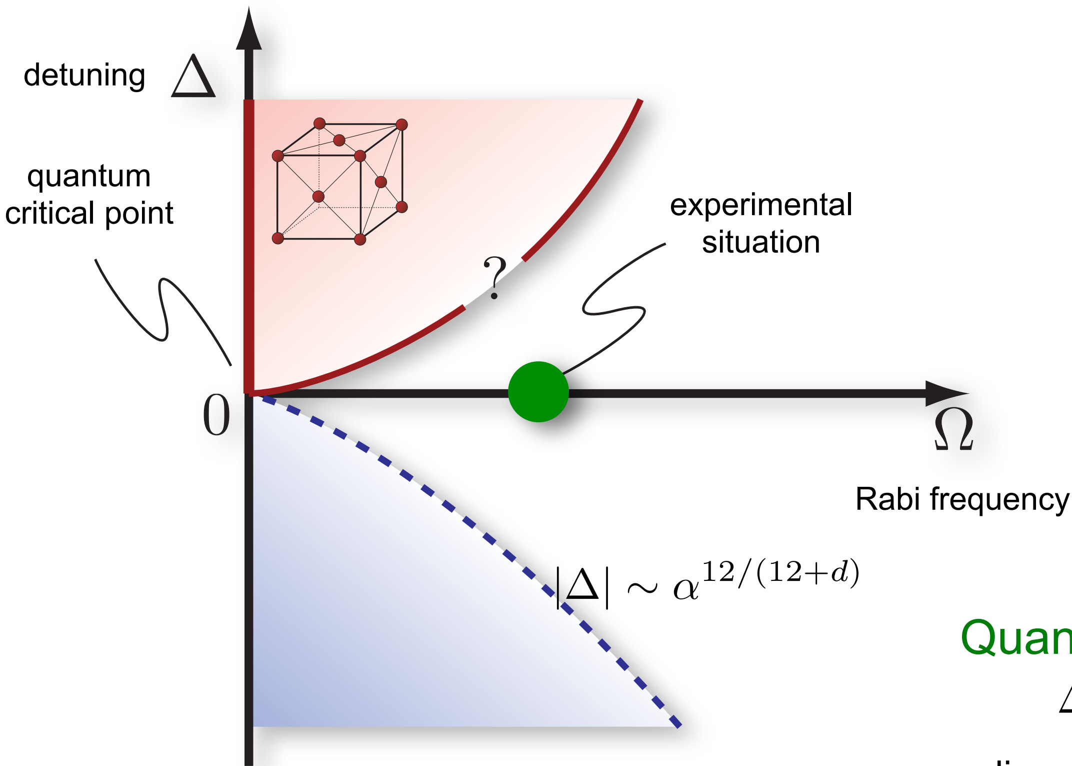
- self-consistency

$$f_e = \langle P_i^e \rangle = \langle P_j^e \rangle$$

Mean-field solution

$$\alpha = f_e^{2d/(12+d)} \left[1 - \frac{\Delta}{f_e^{d/6}} \right]$$

Phase Diagram



Crystalline phase

$$\Delta > 0, \Omega \ll \Delta$$

- Rydberg density: $\langle n_e \rangle \sim n\Delta^{d/6}$

- Open questions:
  - does the crystalline phase survive?
  - phonon spectrum?
  - melting transition?

Quantum critical region

$$\Delta \approx 0, \Omega \gg \Delta$$

- dimensionless parameter $\alpha = \dfrac{\hbar\Omega}{C_6 n^{d/6}}$

- critical phenomena with scaling exponents (mean-field predictions)

$\langle n_e \rangle \sim n\, \alpha^{\nu}$	$\nu = \dfrac{2d}{12+d}$
$\xi \sim \alpha^{-\nu/d}$	: diverging length scale
$\tau \sim \xi^z \sim \alpha^{-z\nu/d}$	: relaxation time $z = 6$

Paramagnet, "Vacuum"

$$\Delta < 0, \Omega \ll \Delta$$

- fluctuations of the excited Rydberg number

- independent Rabi oscillations: large detuning $\langle n_e \rangle \sim \dfrac{\Omega^2}{\Delta^2}$

Quantum phase transition

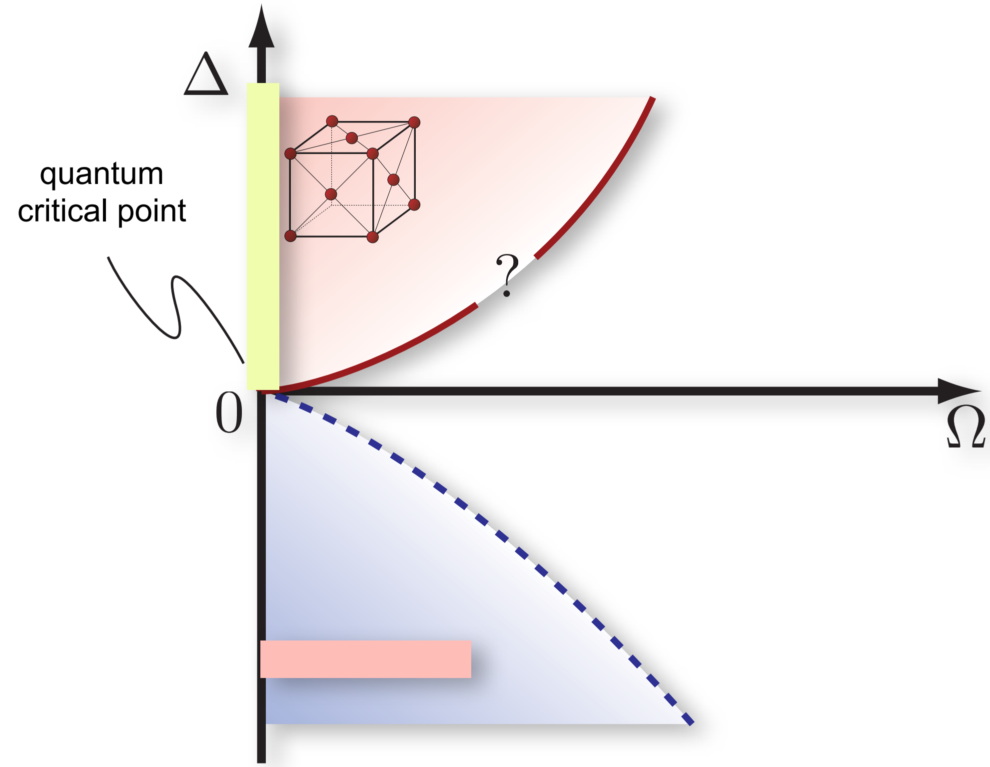
Scaling function

$$\alpha \equiv \frac{\hbar\Omega}{C_6 n^{6/d}} = f_e^{\delta}\chi\left(\frac{\tilde{\Delta}^{\beta}}{f_e}\right)$$

- universal function: χ

- critical exponents: $\delta \quad \beta$

Mean-field exponents correct in all dimensions



Classical Limit: $\Omega = 0$

- crystalline state

$$f_e \sim \tilde{\Delta}^{d/6}$$

$$\beta = \frac{d}{6}$$

Perturbation theory: $\Delta < 0$

$$f_e = \frac{1}{4}\left(\frac{\Omega}{\Delta}\right)^2$$

$$\delta = \frac{12+d}{2d}$$

Local density approximation

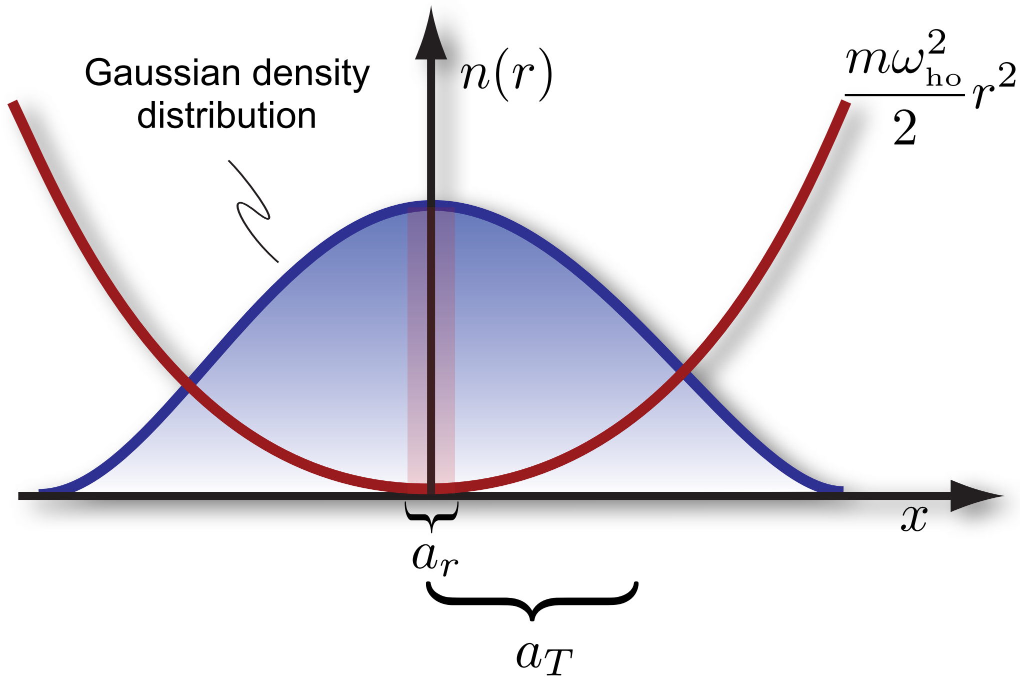
Local density

- harmonic trapping potential
- thermal gas with density distribution

$$n(r) \sim \exp\left(-\frac{m\omega_{\text{ho}}^2}{2T} r^2\right)$$

- smoothly varying trap

$$a_T = \sqrt{T/m\omega_{\text{ho}}^2} \gg a_r = 1/(n f_e)^{1/d}$$



local density approximation

$$N_e = \int d\mathbf{r}\, n(\mathbf{r}) f_r(\alpha) \sim \int d\mathbf{r}\, n(\mathbf{r}) \left[\frac{\hbar\Omega}{C_6 \left(n(\mathbf{r})\right)^{6/d}}\right]^{\nu}$$

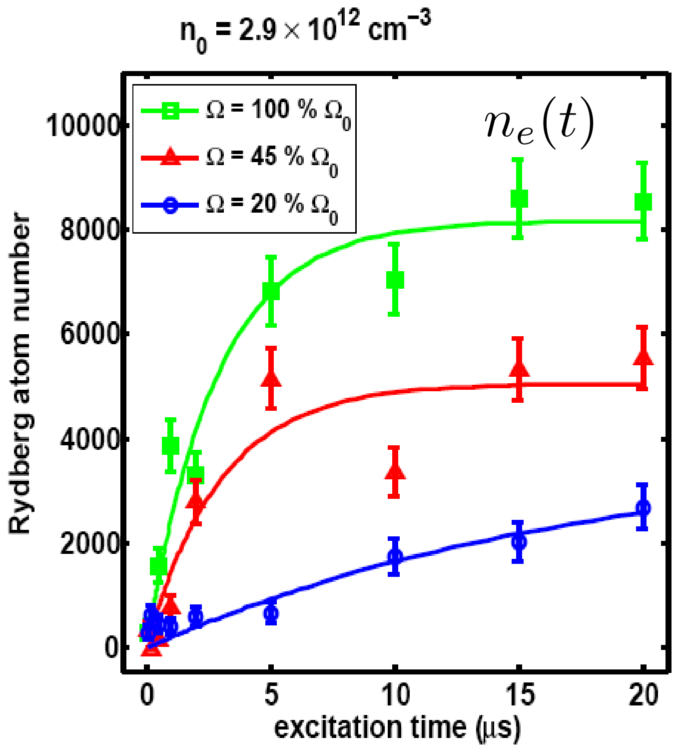
$$\frac{N_e}{N} \sim \alpha^{\nu}$$

density in trap center

: scaling exponent remains invariant

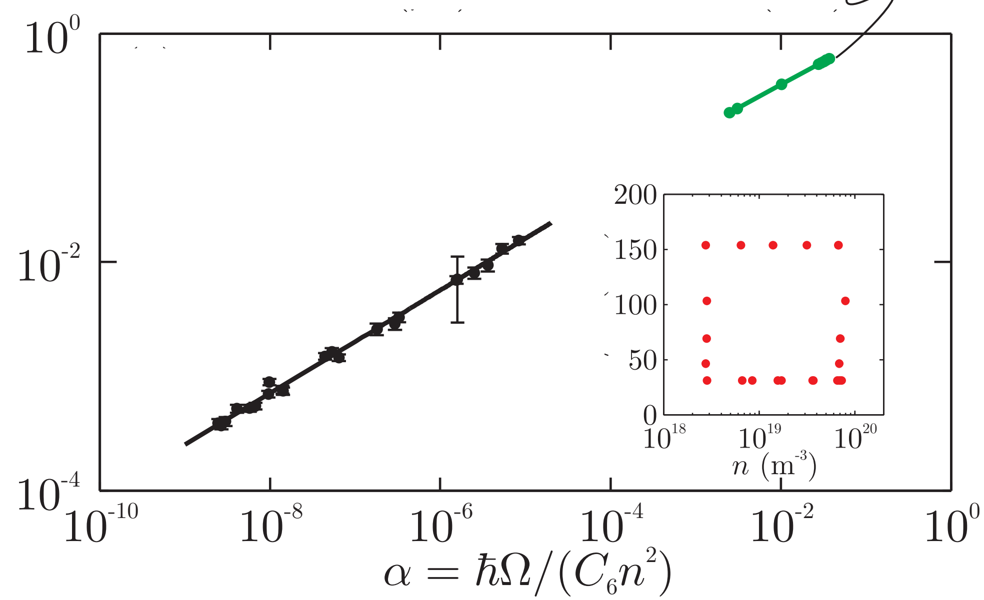
Comparison with experiments

- measurement of the excited Rydberg atoms
  - saturated number of Rydberg atoms
  - time scale for saturation

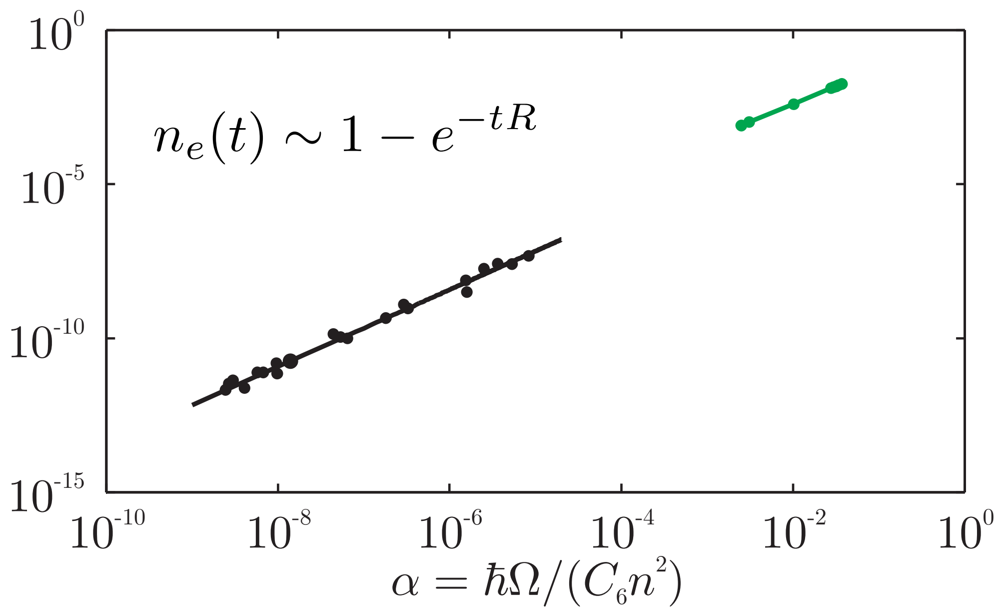


numerical simulations

saturated Rydberg fraction



time scale for saturation



R. Löw *et al*, (2009)

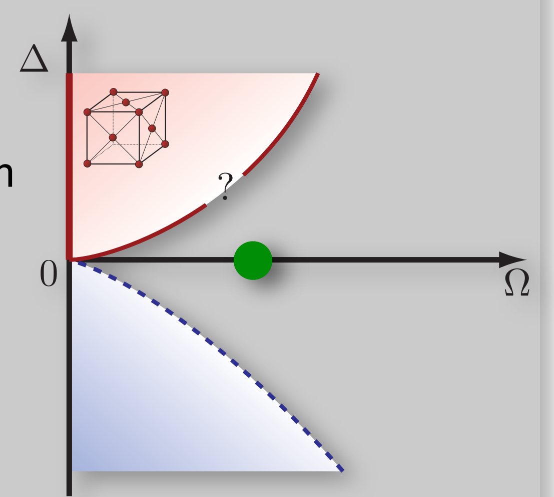
Comparison with experiments

- data collapse	✓
- scaling exponent	✓
- quantitative agreement - cigar shaped trap: cross-over between 3D and 1D	(✓)

	$\gamma \quad (g_r \sim \alpha^\gamma)$	$\mathbf{1/\delta} \quad (f_R \sim \alpha^{1/\delta})$
experiment **[1d]**	1.08 ± 0.01	0.16 ± 0.01
theory γ	$14/13 \approx 1.08$	$2/13 \approx 0.15$
numerical simulation	1.06	0.150 [6]
experiment **[3d]**	1.25 ± 0.03	0.45 ± 0.01
theory γ	$6/5 = 1.2$	$2/5 = 0.4$
numerical simulation	1.15	0.404 [6]

- experimental observation of critical behavior due to a quantum phase transition

- new universality class



- open questions:
  - role of dimension?
  - scaling function?
  - experimental observation of the crystalline correlations?

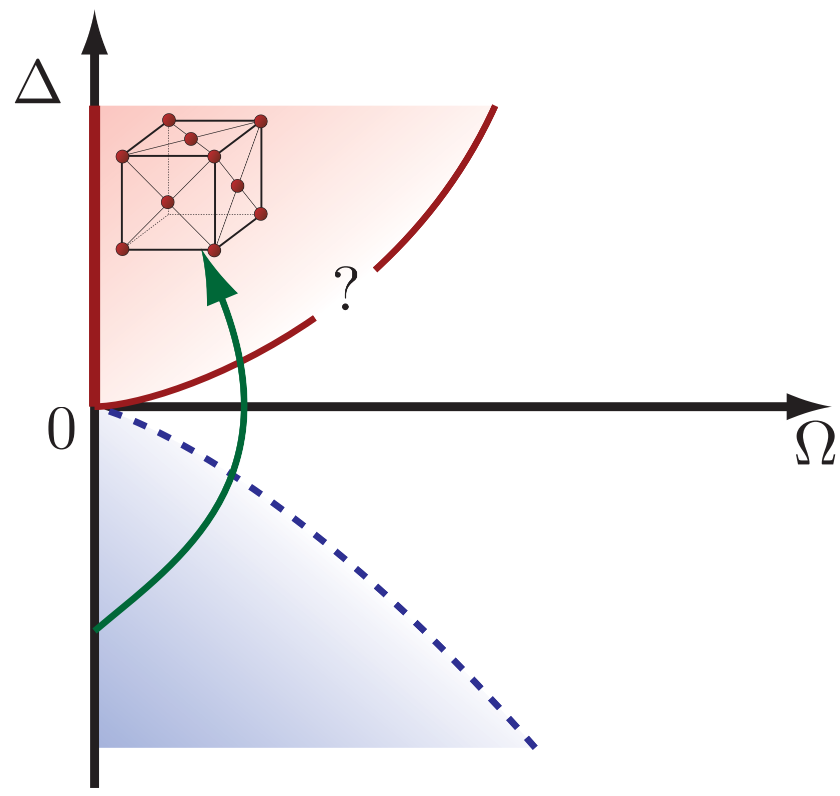
Crystalline phase?

Does the crystalline phase exist?

- adiabatic preparation
  (Pohl et al., PRL 2009
  J. Schachenmayer et al, 2010)

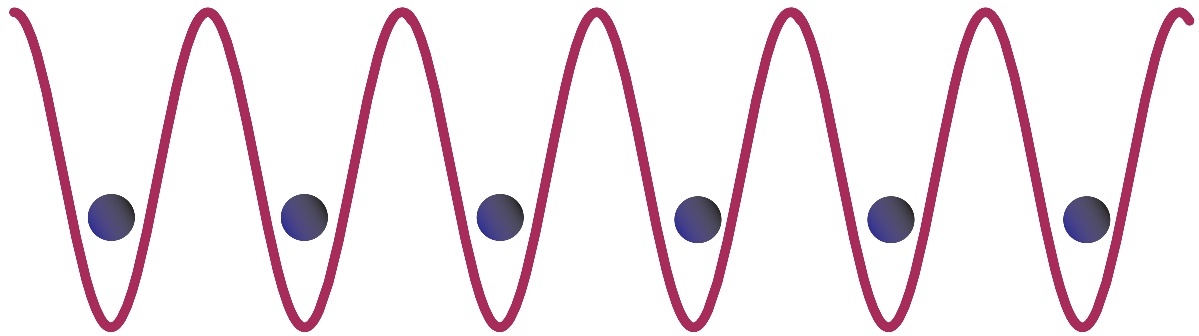
- nature of the phase transition?

- influence of underlying arrangement of atoms?



One-dimension in optical lattice

Ground state atoms in an optical lattice



- one atom per lattice site

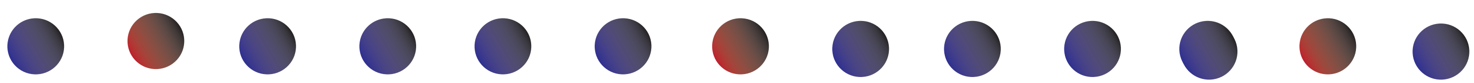
- one-dimensional chain

- Hamiltonian

$$H = -\frac{\hbar\Delta}{2}\sum_i \sigma_z^{(i)} + \frac{\hbar\Omega}{2}\sum_i \sigma_x^{(i)} + \frac{C_6}{a^6}\sum_{i<j}\frac{P_{ee}^{(i)}P_{ee}^{(j)}}{(i-j)^6}$$

lattice spacing

- commensurate solids $\Delta > 0$

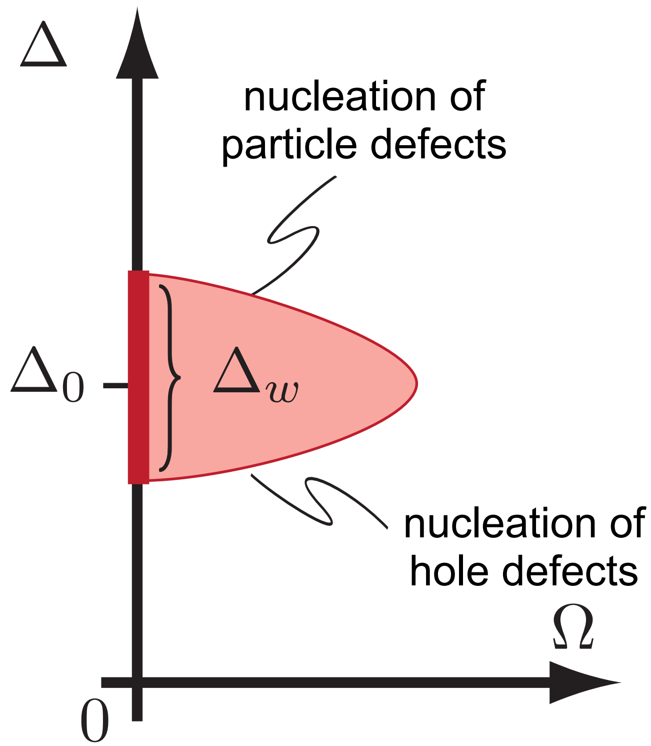


Devils staircase

Ground state ($\Omega = 0$)
(Bak et a.l PRL, 1984)

- complete devils staircase

- Rydberg density: $f = \dfrac{p}{q}$



ground state $q = 5 \quad p = 1$

fractional hole defect

fractional particle defect

- detuning for center of lobe

$$\Delta_0 = 7\zeta(6)\frac{C_6}{a^6}\left(\frac{p}{q}\right)^6$$

- width of the lobe

$$\Delta_w = 42\zeta(7)\frac{C_6}{a^6}\frac{1}{q^7}$$

- dominant lobes for $p = 1$

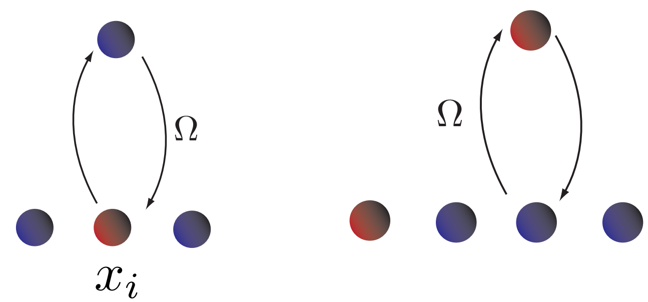
commensurate solid is stable for finite Ω

Commensurate lobes

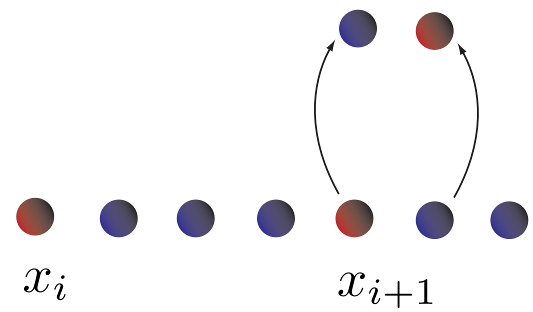
Stability of lobes

- second order perturbation theory in Ω/Δ
- energy shift for ground state and defects
- effective hopping for defects

energy shifts:



hopping energy:



Effective model for defects

- position of Rydberg atom x_i
- defect number at i

$$S_i^z = x_{i+1} - x_i - q$$

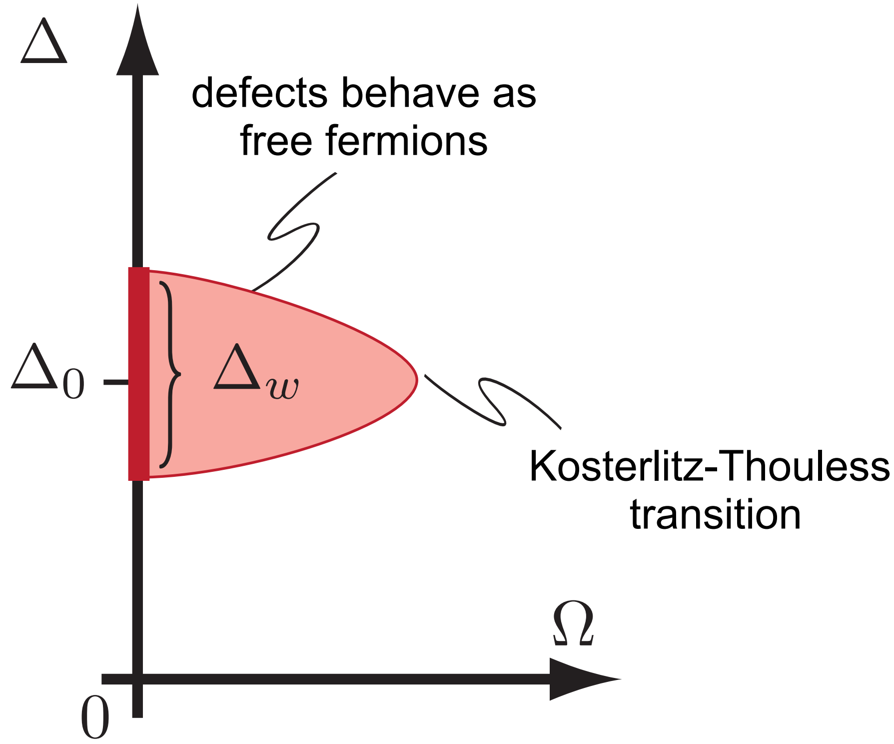
- spin-1 system in a superlattice with spacing q

$$H_{\mathrm{eff}} = \sum_i \left[U (S_i^z)^2 - J S_i^+ S_{i+1}^- + \mathrm{h.c.} - \mu S_i^z \right]$$

interaction	hopping	chemical potential
$U \approx f \Delta_w / 2$	$J \approx \frac{7}{5} \frac{\hbar \Omega^2}{\Delta}$	$\mu = \hbar(\Delta - \Delta_0)$

Phase transition

- effective model remains correct close to the lobe with low defect density



Commensurate-Incommensurate transition

- nucleation of particles-defects
- defects behave as hard-core bosons/ free fermions
- defects described by Luttinger liquid with $K = 1$

Tip of the lobe

- Kosterlitz-Thouless transition
- defects described by Luttinger liquid with $K = 2$
- simultaneous nucleation of particle/hole defects

Novel phase with algebraic correlations

- spin-spin correlations

$$\langle S_i^z S_j^z \rangle \sim 1/|i-j|^{2K}$$

- what are the correlations in the original model? $\langle P_{ee}^{(i)} P_{ee}^{(j)} \rangle$

Structure factor for Rydberg atoms

Correlation function

- mapping of the effective model to the physical quantity

$$\langle P_{ee}^{(0)} P_{ee}^{(j)} \rangle = \frac{1}{q+n} \left\langle \sum_k \delta_{j, N_k + kq} \right\rangle$$

$$= \sum_k \frac{P_k(j-kq)}{q+n}$$

- determined numerically via Monte Carlo with correlated random numbers

- long wave length approach within the Luttinger liquid theory

$$P_k(m) = \frac{1}{\sqrt{2\pi\kappa^2}} e^{-\frac{(m-nk)}{2\kappa^2}}$$

$$\kappa^2 = \langle (N_k - nk)^2 \rangle = \frac{K}{\pi^2} \log(k/b)$$

averaged defect number: $n = \langle S_i^z \rangle$

defect number operator between site 0 and k:

$$N_k = \sum_{i=0}^{k-1} S_i^z = x_k - x_0$$

distribution function: $P_k(m)$

Solid correlations for the Rydberg atoms:

$$\frac{\langle P_{ee}^{(0)} P_{ee}^{(j)} \rangle - \langle P_{ee}^{(0)} \rangle^2}{\langle P_{ee}^{(0)} \rangle^2} = \cos\left(\frac{2\pi j}{n+q}\right) \left[\frac{b(n+q)}{j}\right]^{\frac{2K}{(n+q)^2}}$$

floating solid

Phase diagram

Quantum phase transition

- floating solid with
  algebraic correlations

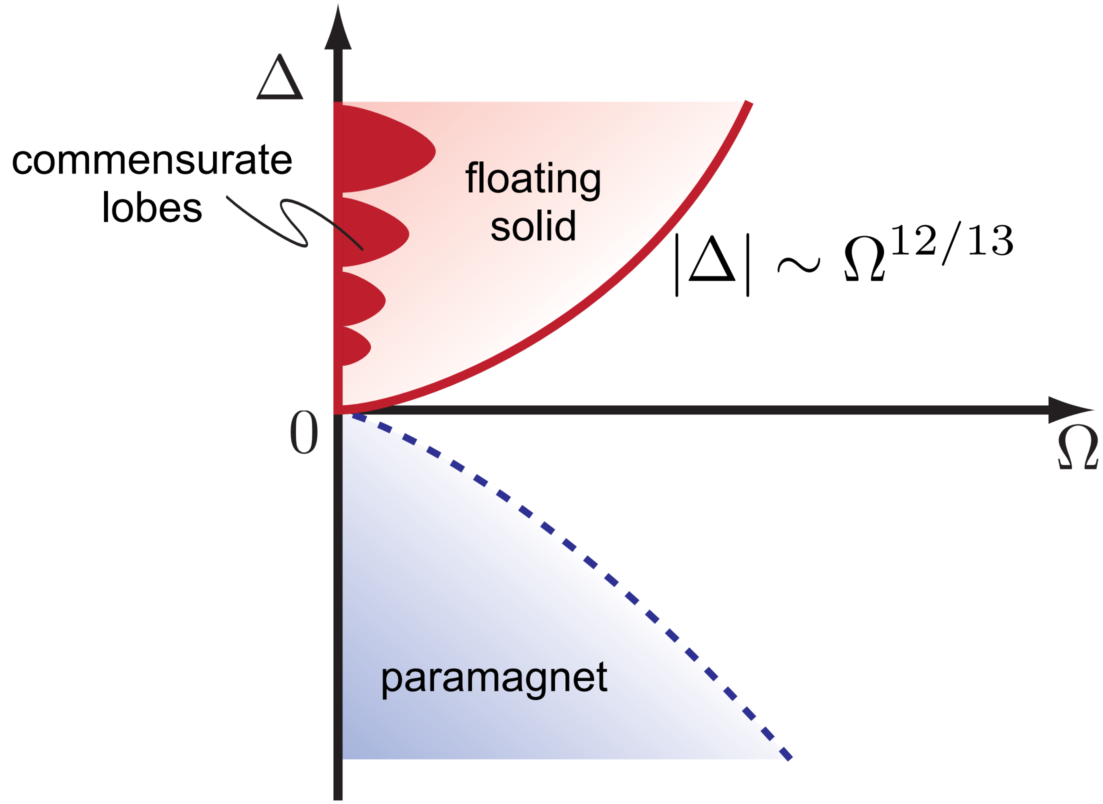
- paramagnet with excitation gap

- break down of the effective model
  in terms of defects:

  - include higher defects

  - multiple defect hopping

  - fluctuations of defects
    per site larger than the
    spacing



mean-field theory

$$\langle n_i^2 \rangle \sim \left(\frac{J_c}{U}\right)^2 \sim q^2$$

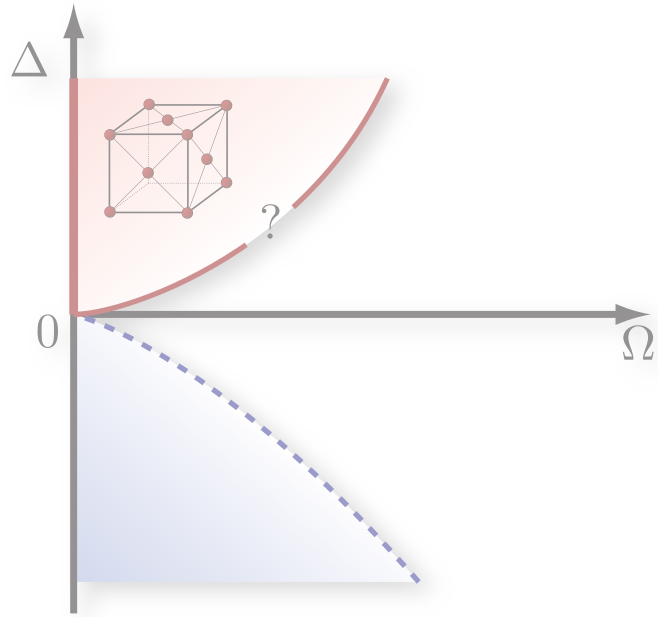
Phase transition to paramagnet:

$$|\Delta| \sim \Omega^{12/13}$$

Outline

Many body phenomena

- quantum phase transition
- universal scaling

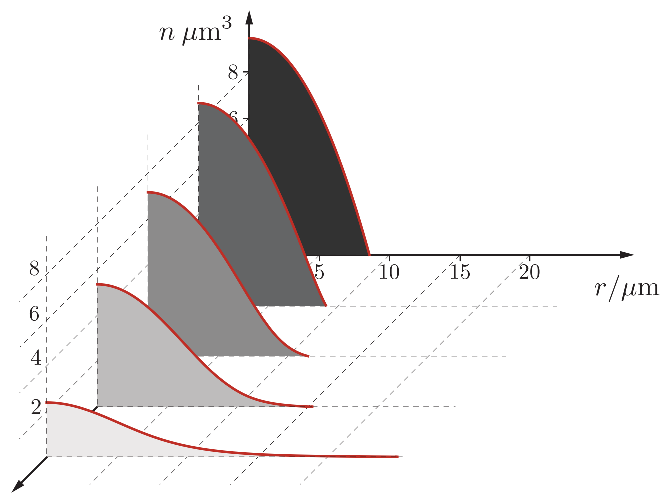


Crystalline phase

- floating solid in one-dimension

Tool for designing interactions

- collective many-body interaction



Rydberg dressing

- weakly dressing with a Rydberg level

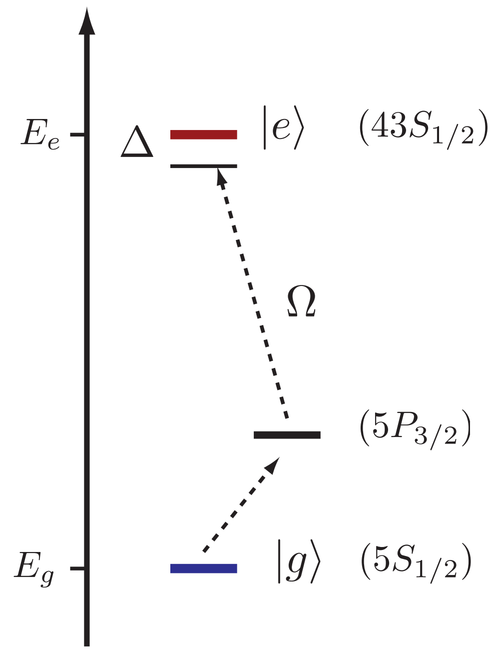
- design ground state interaction for cold atomic gases

$$|d\rangle = \alpha|g\rangle + \beta|e\rangle$$

$$\beta \approx \frac{\Omega}{2\Delta}$$

- spontaneous emission: $\Gamma_{\text{eff}} = \frac{\Omega^2}{4\Delta^2}\Gamma_e$

- allow for motion of the atoms

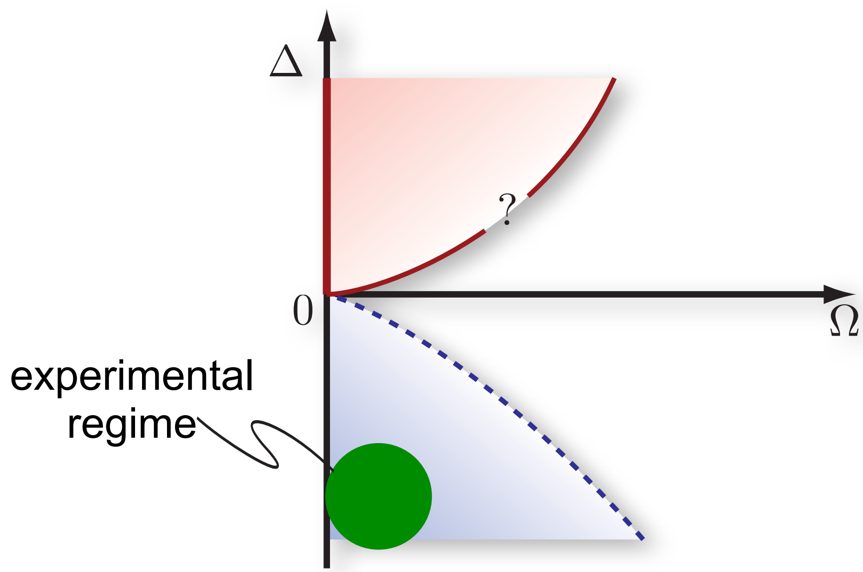


Effective interaction

- Born-Oppenheimer potential

$$V_{\text{eff}}(\mathbf{r}) = \frac{\hbar\Omega^4}{|\Delta|^3}\frac{1}{1+(r/\xi_0)^6}$$

- Blockade radius $\quad \xi_0 = (C_6/2\hbar|\Delta|)^{1/6}$



Supersolid instability?

Roton instability

(T. Pohl, PRL 2009, V. Liu, 2010)

- effective interaction $V_{\mathrm{eff}}(\mathbf{q})$
  negative for $q \sim 1/\xi_0$

- Roton instability within
  Bogoliubov theory

Quantum Monte Carlo

(G. Pupillio, PRL 2010)

- solid with many-particles
  on each lattice site

- superfluid coherence between
  the sites established by tunneling

Influence on a Bose-Einstein condensate

- experimental parameters

$$\xi_0 \sim 2\mu\mathrm{m}$$

- large atomic density

→ collective many-body interaction

Many-body interactions

Two-body interaction

- s-wave scattering length

$$g_{\mathrm{eff}} = \frac{4\pi\hbar^2 a_{\mathrm{eff}}}{m} = \frac{\pi^2}{12}\frac{\hbar\Omega^4}{|\Delta|^3}\xi_0^3$$

- validity of 1 Born approximation

$$\Omega^4/|\Delta|^3 \ll \hbar/m\xi_0^2$$

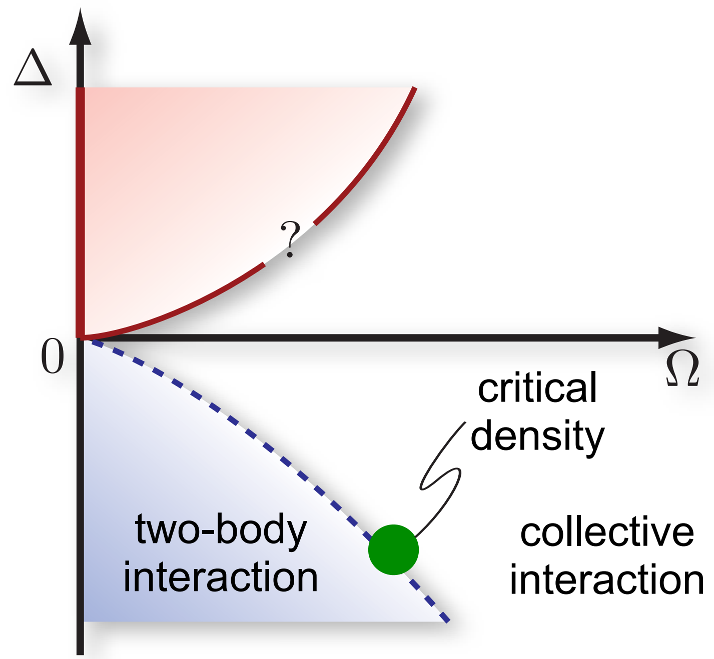
Three-body interactions

- solving Born-Oppenheimer with three-particles
- three-body interactions suppressed by Ω/Δ

Collective blockade phenomena

- density of excited Rydberg atoms: $\dfrac{\Omega^2}{4\Delta^2}n$
- allowed distance between Rydberg atoms: ξ_0
- critical density

$$n_c = 4\frac{\Delta^4}{\Omega^4\xi_0^3}$$



Generalized Gross-Pitaevskii equation

Gross-Pitaevskii equation

- Bose-Einstein condensate:
  homogenous density

- interaction described by
  energy functional for internal structure

$$i\hbar\partial_t\psi = \left\{ -\frac{\hbar^2}{2m}\Delta + V(\mathbf{r}) + \partial_n E_{\text{eff}}[n] \right\}\psi$$

- Hamiltonian for internal structure

$$H = \frac{C_6}{2}\sum_{i\neq j}\frac{n_i^e n_j^e}{|\mathbf{r}_i - \mathbf{r}_j|^6} + \hbar\Omega\sum_i \sigma_i^x + \hbar\Delta\sum_i \sigma_i^z$$

- ground state $|0\rangle$

$$E_{\text{eff}}[n] = \langle 0|H|0\rangle$$

: mean field theory

Low densities: $n \ll n_c = 4\frac{\Delta^4}{\Omega^4\xi_0^3}$

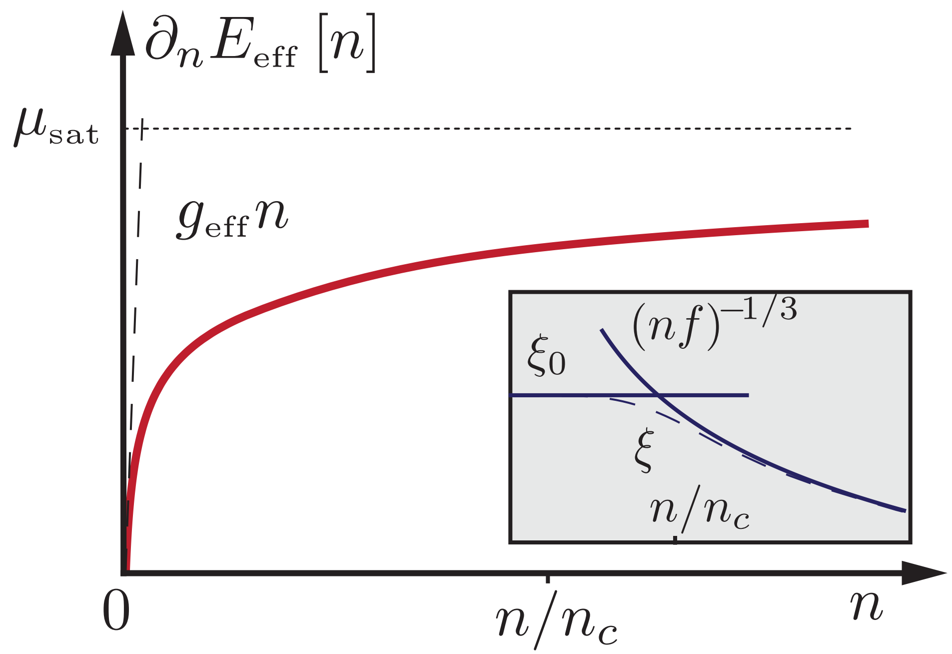
- two-body interaction

$$E_{\text{eff}}[n] = \frac{g_{\text{eff}} n^2}{2}$$

High densities: $n \gg n_c = 4\frac{\Delta^4}{\Omega^4\xi_0^3}$

- saturation on chemical potential:
  all atoms are within the Blockade radius

$$E_{\text{eff}}[n] = \mu_{\text{sat}} n$$

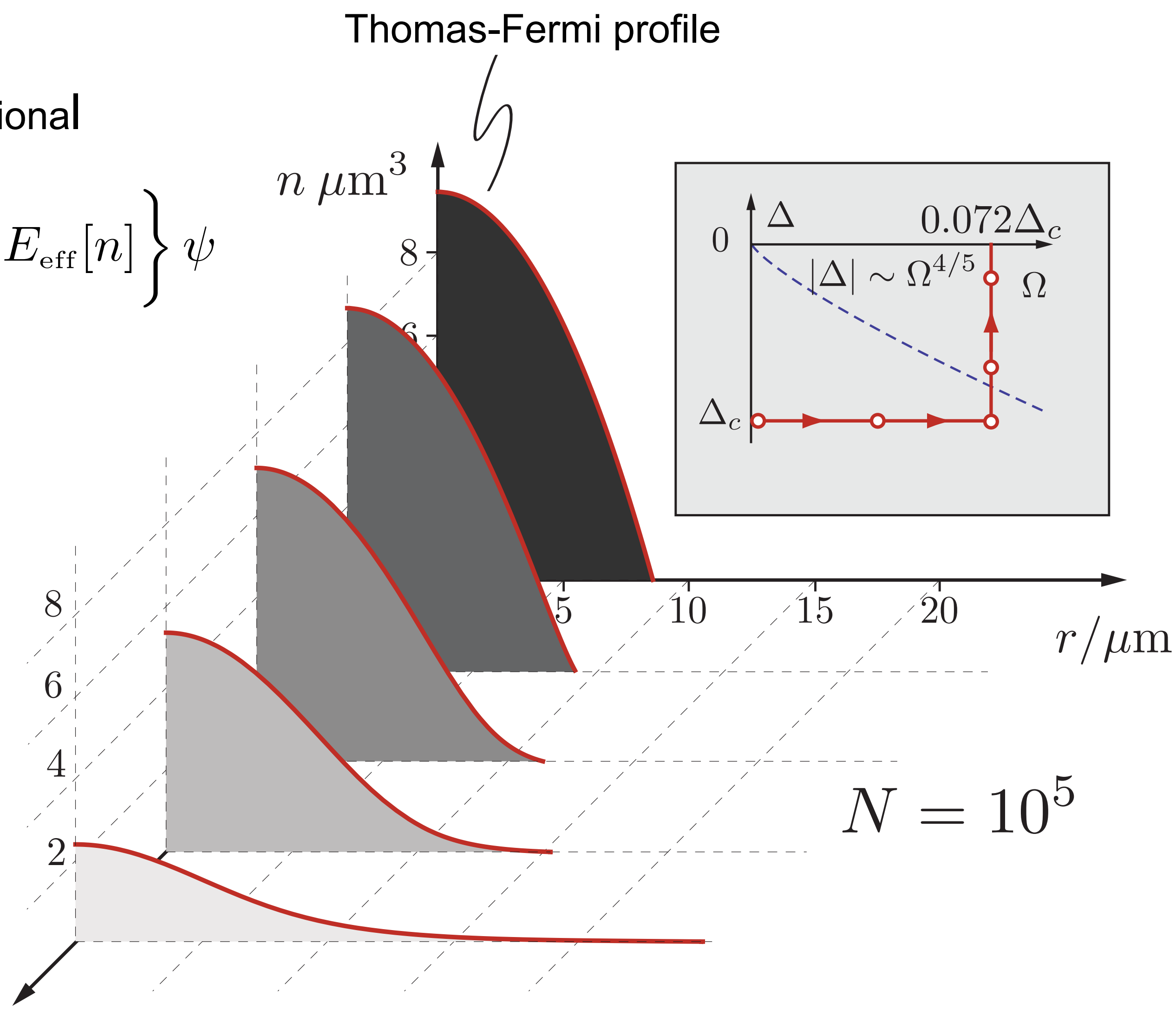


Generalized Gross-Pitaevskii equation

Gross-Pitaevskii equation

- interaction described by energy functional

$$i\hbar\partial_t\psi = \left\{-\frac{\hbar^2}{2m}\Delta + V(\mathbf{r}) + \partial_n E_{\text{eff}}[n]\right\}\psi$$



$N = 10^5$

Parameters

- Rydberg level: $\text{Rb}\ |35s\rangle$
- life-time: $\gamma \sim 6\text{Hz}$
- blue laser: $\Omega_r = 22\text{MHz}$
- two-photon Rabi frequency

  $\Omega = 7.8\text{kHz}$

  $\Delta = 107\text{kHz}$

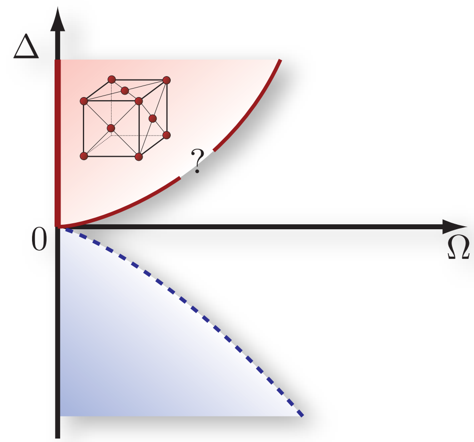
- induced s-wave scattering length: $a_{\text{eff}} = 49.5\text{nm}$

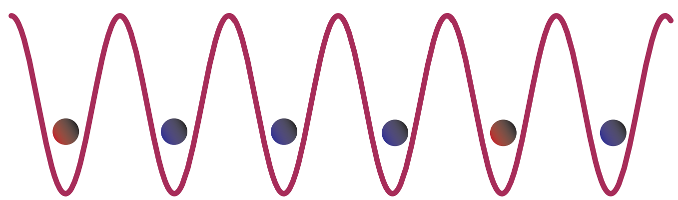
Conclusion

Van der Waals blockade

- complex quantum many-body system

- critical phenomena with universal scaling exponents





Crystalline phase

- floating solid in one-dimension

- does the solid survive higher dimensions?

Tool for designing interactions

- Rydberg dressed interactions for cold atomic gases

- is a supersolid experimentally realizable?