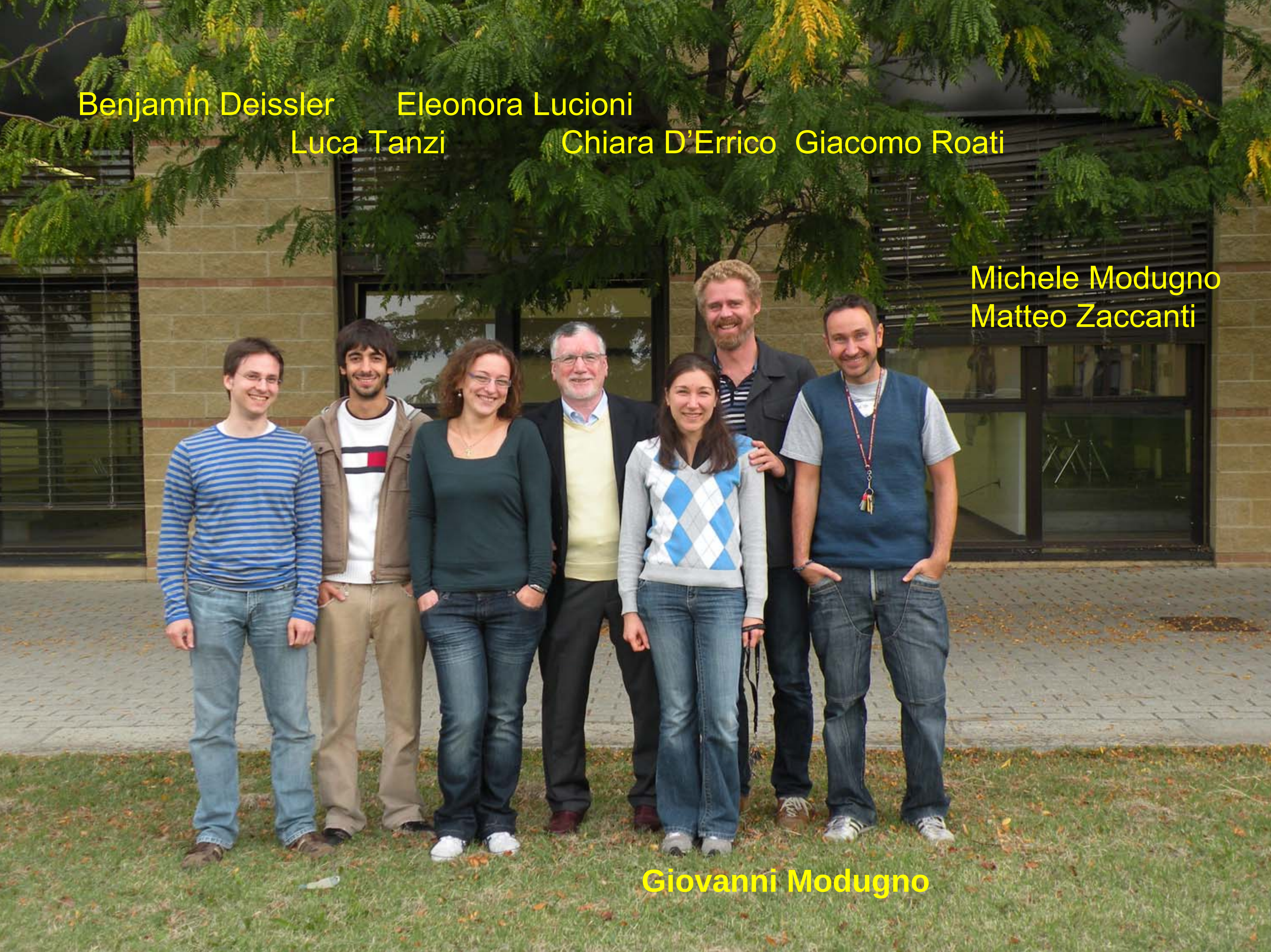
LENS European Laboratory for Nonlinear Spectroscopy,
Dipartimento di Fisica ed Astronomia
Università di Firenze
IstitutoNazionale di Ottica - CNR

Massimo Inguscio

WEAKLY INTERACTING BOSONS
in a disordered lattice

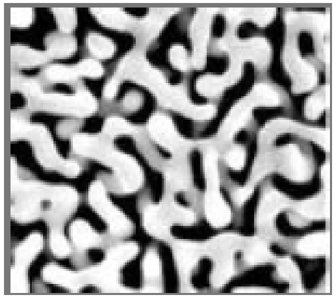
Frontiers of ultra-cold atoms and molecules,
KITP Santa Barbara, October 13, 2010

INO-CNR
Istituto Nazionale di Ottica

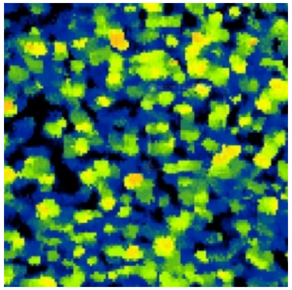

Benjamin Deissler
Eleonora Lucioni
Luca Tanzi
Chiara D'Errico
Giacomo Roati
Michele Modugno
Matteo Zaccanti
Giovanni Modugno

Disorder and Anderson localization

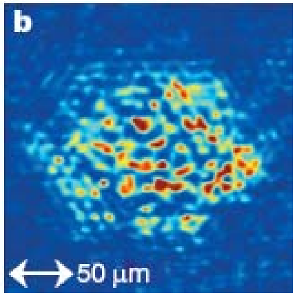
Disorder is ubiquitous in nature. Even if weak, it tends to inhibit transport.



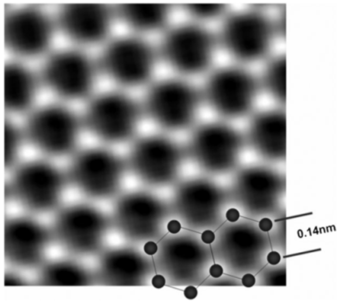
Superfluids in porous media



Granular and thin-film superconductors



Light propagation in random media



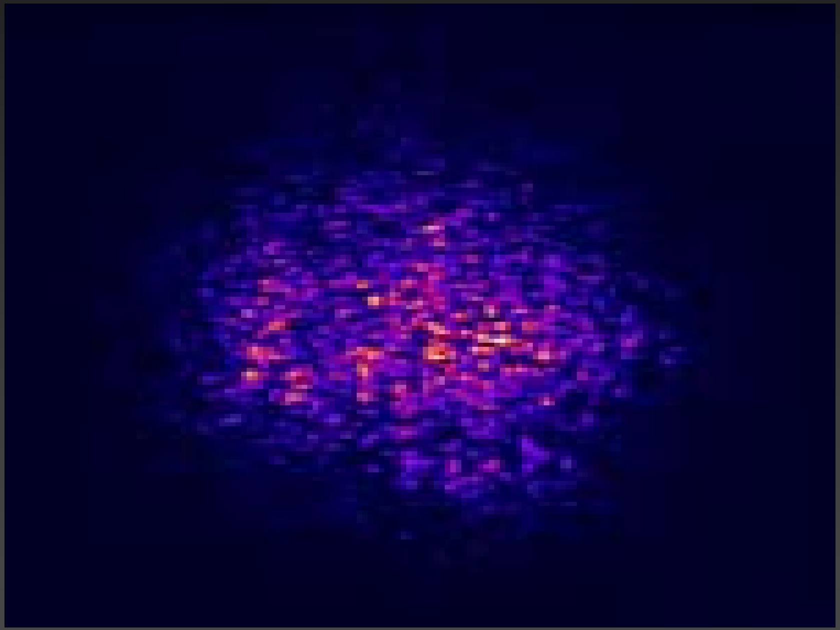
Graphene

Still much has to be understood:

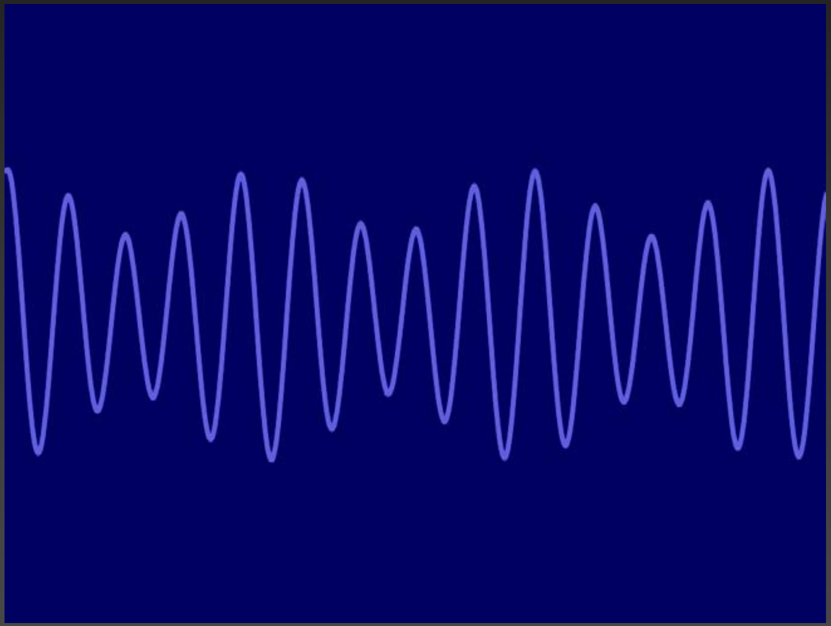
- Single-particle localization and dimensionality
- interplay of disorder and interactions
- strongly correlated systems

How to produce disorder

speckle pattern



bichromatic lattice

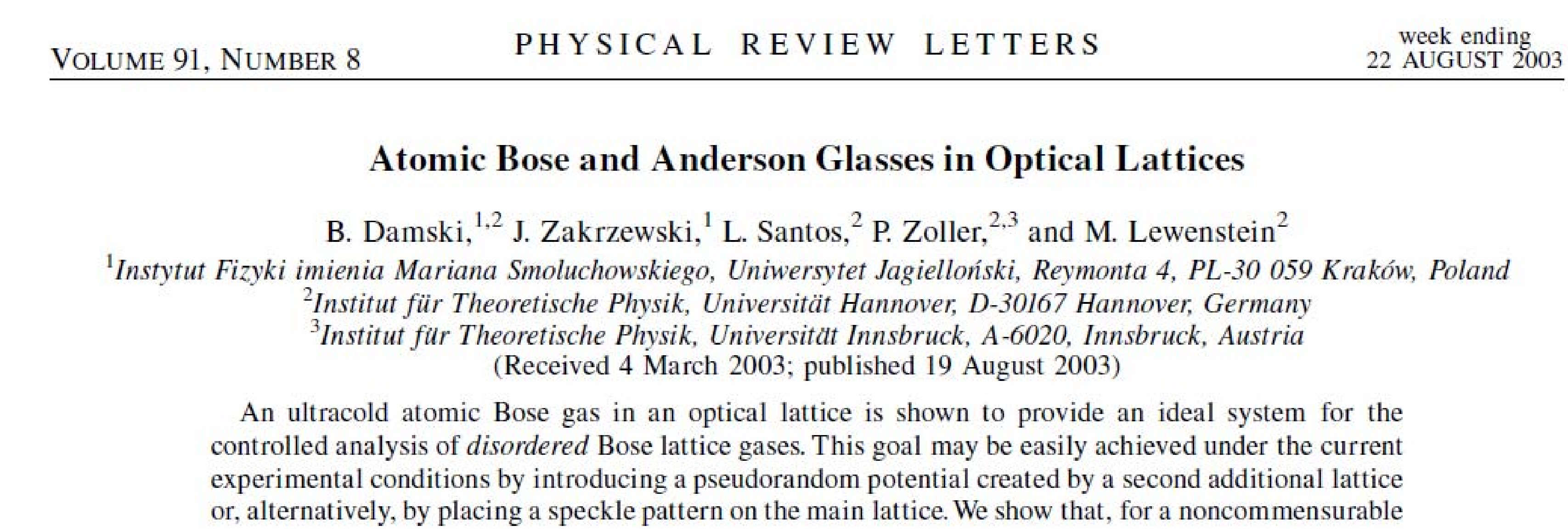


L.Fallani, C.Fort, M.Inguscio

Bose-Einstein condensates in disordered potentials

*Advances Atomic, Molecular and Optical Physics vol **56**, pp 119-160*
edited by E.Arimondo, P.Berman, C.Lin (Academic Press 2008)

Disorder and BEC



VOLUME 91, NUMBER 8 — PHYSICAL REVIEW LETTERS — week ending 22 AUGUST 2003

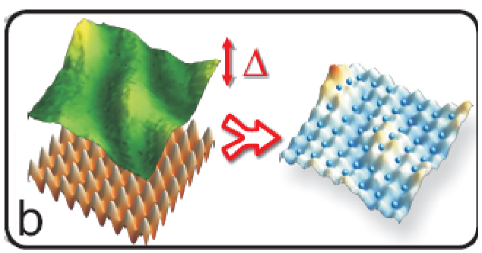
Atomic Bose and Anderson Glasses in Optical Lattices

B. Damski,[1,2] J. Zakrzewski,[1] L. Santos,[2] P. Zoller,[2,3] and M. Lewenstein[2]

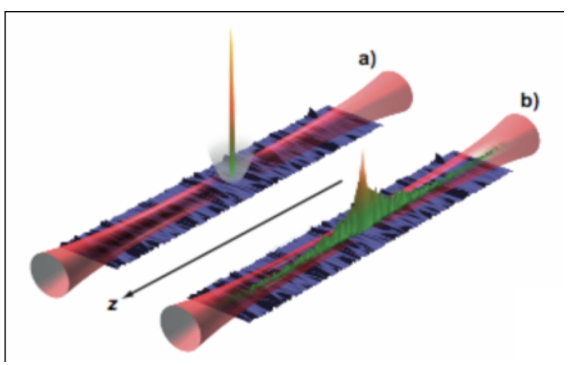
[1]*Instytut Fizyki imienia Mariana Smoluchowskiego, Uniwersytet Jagielloński, Reymonta 4, PL-30 059 Kraków, Poland*
[2]*Institut für Theoretische Physik, Universität Hannover, D-30167 Hannover, Germany*
[3]*Institut für Theoretische Physik, Universität Innsbruck, A-6020, Innsbruck, Austria*

(Received 4 March 2003; published 19 August 2003)

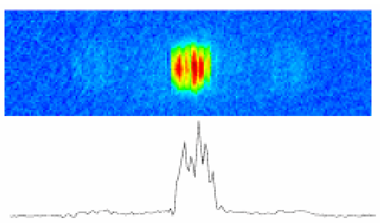
An ultracold atomic Bose gas in an optical lattice is shown to provide an ideal system for the controlled analysis of *disordered* Bose lattice gases. This goal may be easily achieved under the current experimental conditions by introducing a pseudorandom potential created by a second additional lattice or, alternatively, by placing a speckle pattern on the main lattice. We show that, for a noncommensurable



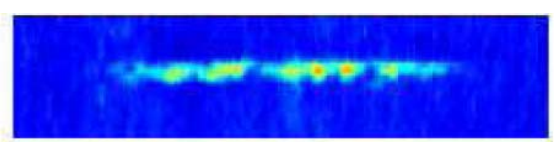
Urbana



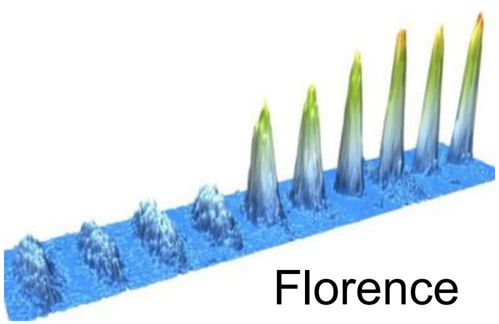
Paris



Hannover



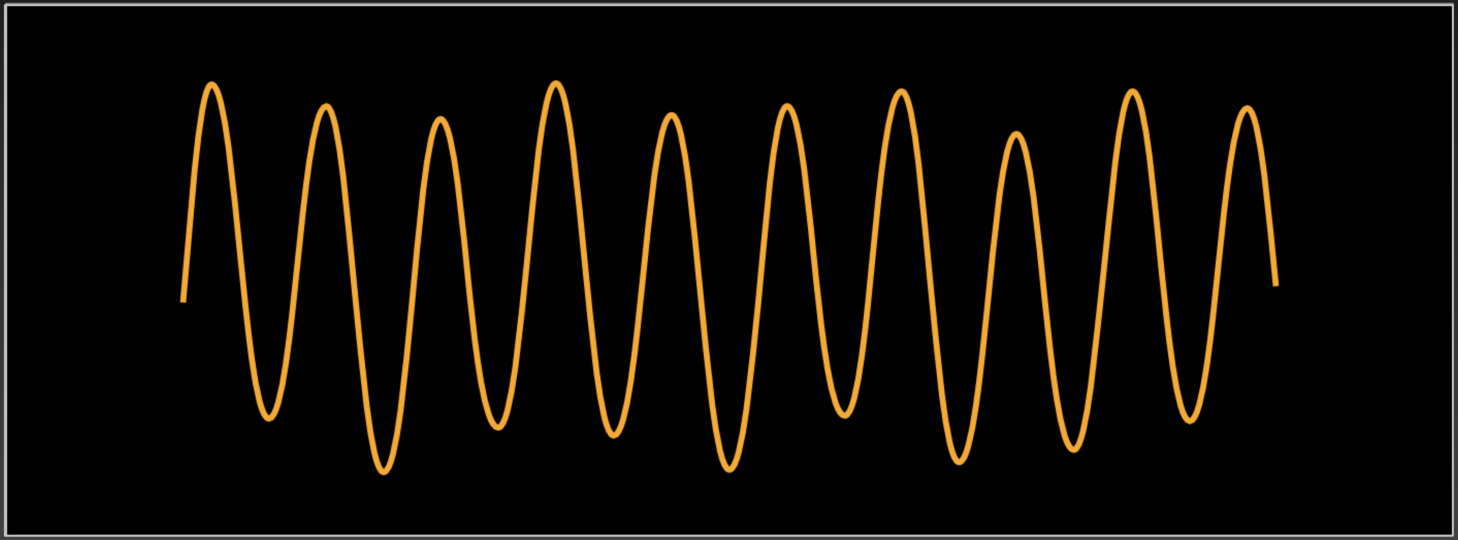
Rice U.



Florence

BICHROMATIC OPTICAL LATTICE – **our non standard optical lattice**

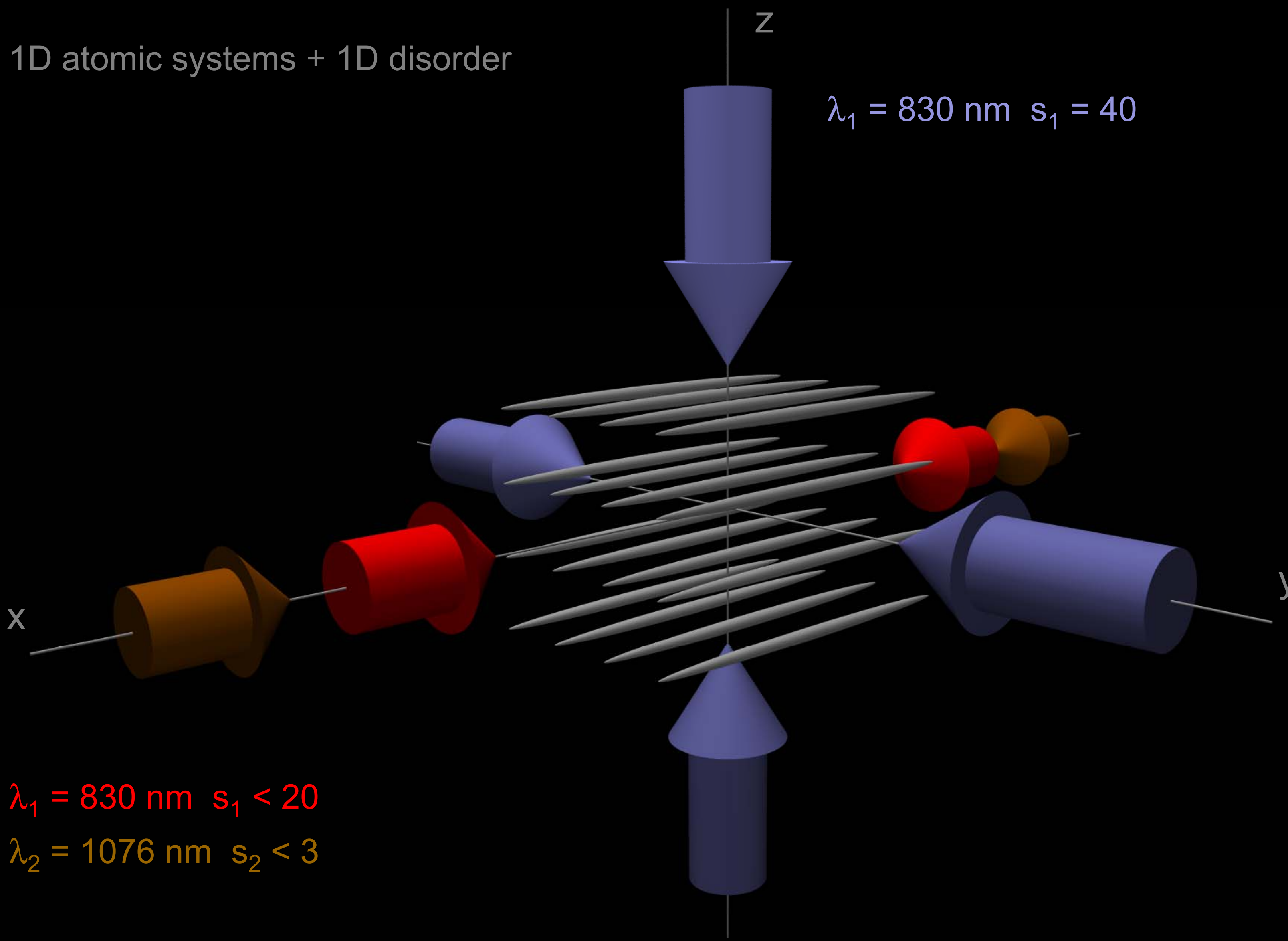
Adding a weak incommensurate optical lattice...



Experimental geometry

L.Fallani et al. Phys.Rev.Lett. 98, 130404 (2007)

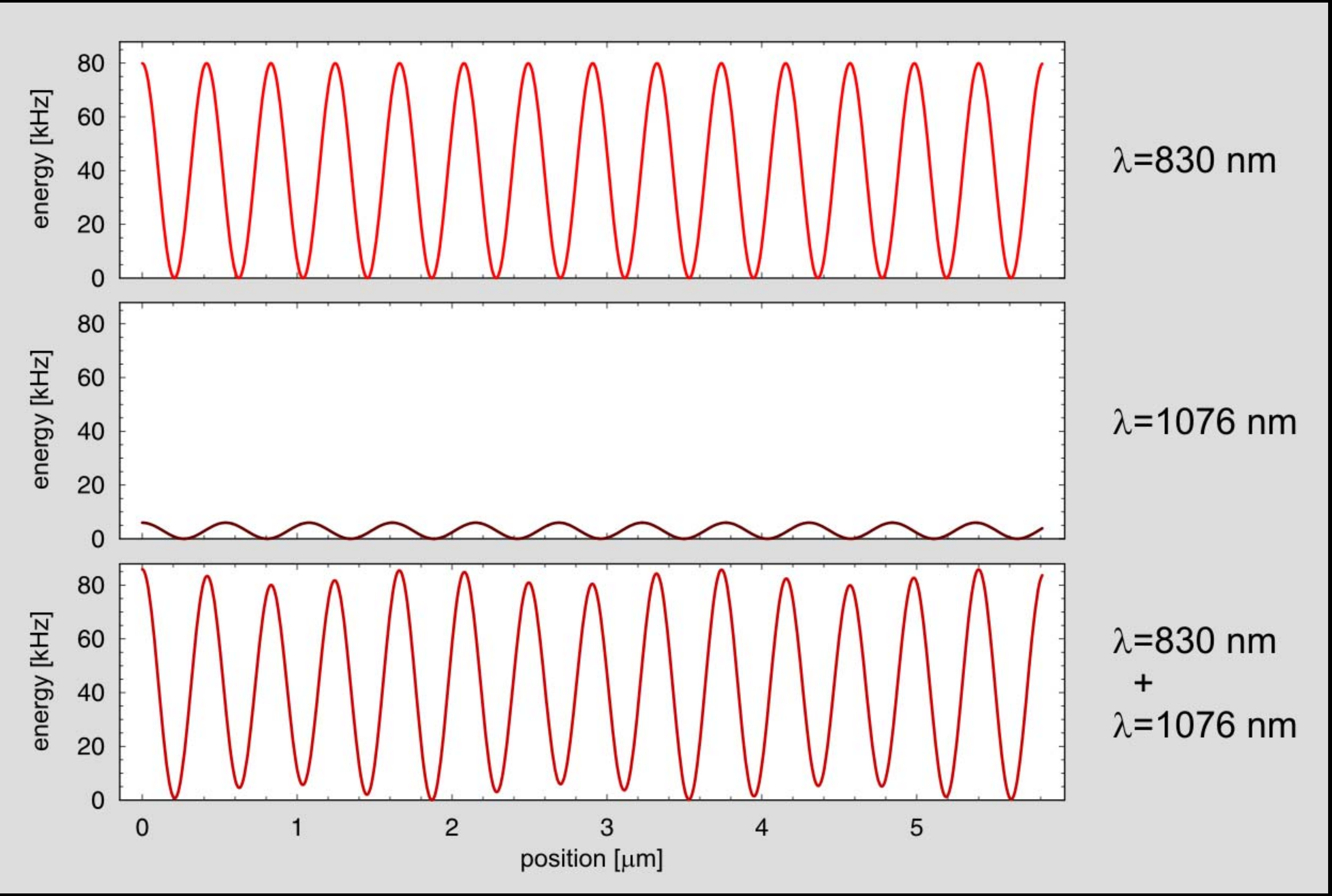
1D atomic systems + 1D disorder



λ_1 = 830 nm s_1 < 20

λ_2 = 1076 nm s_2 < 3

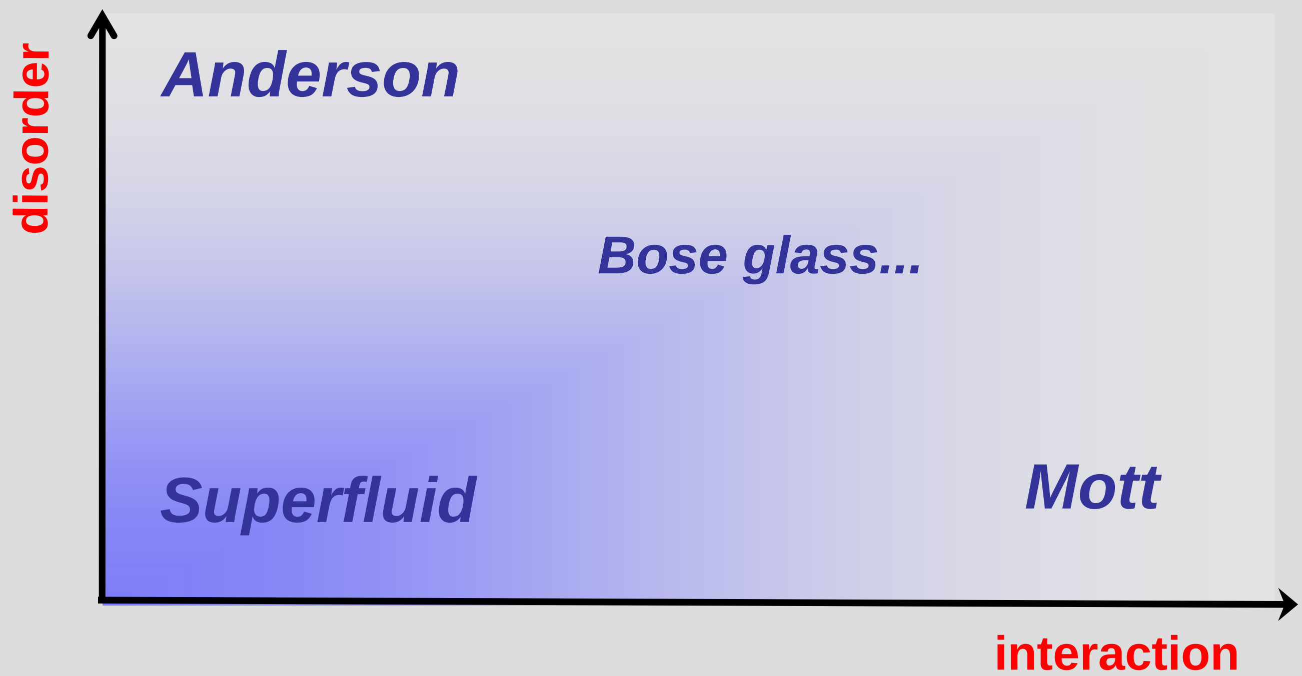
The bichromatic lattice



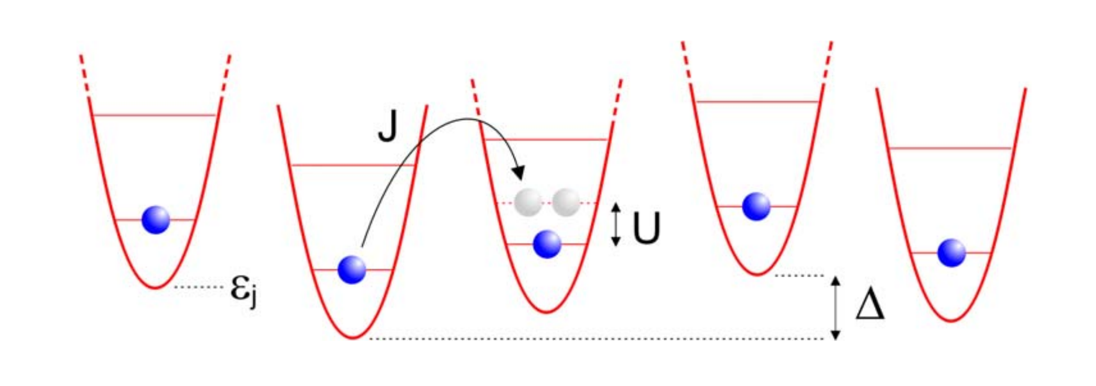
Disordered interacting bosons

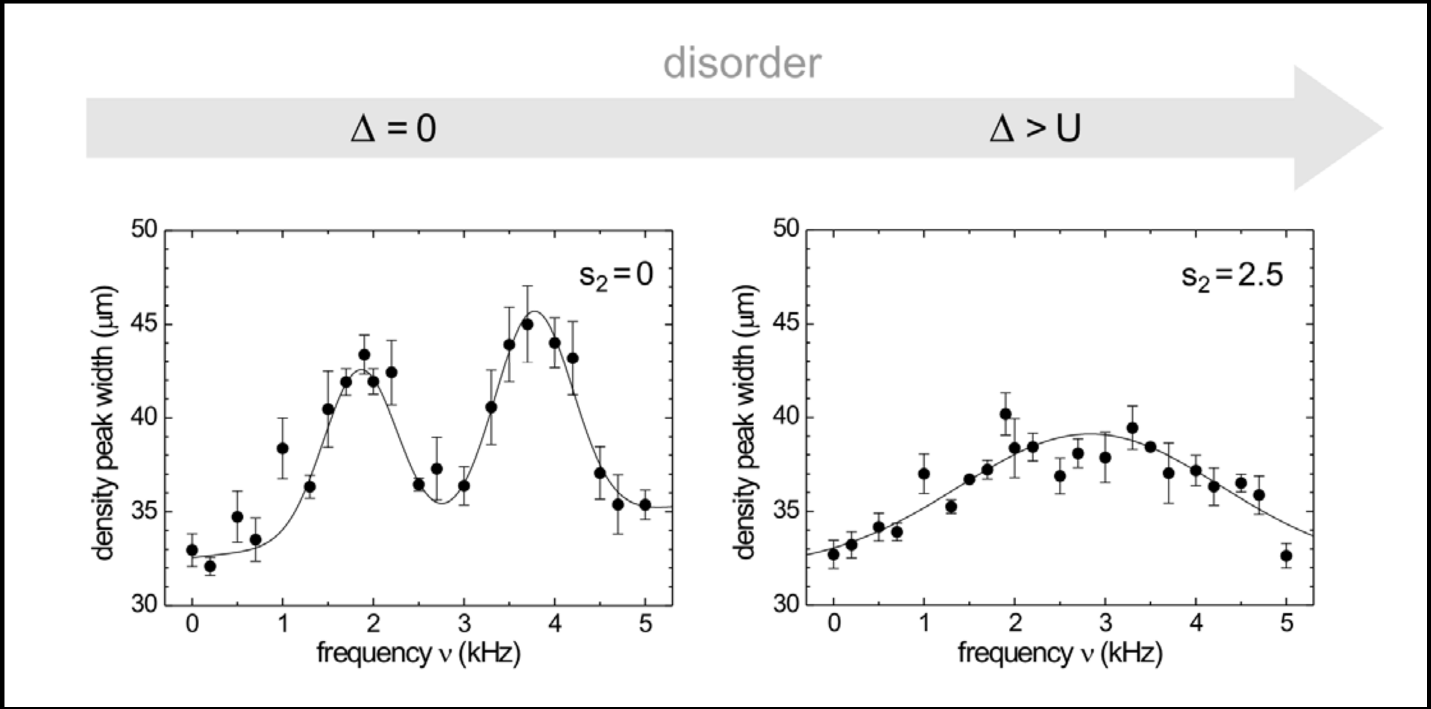
Subtle interplay between disorder and interactions
(granular superconductors, superfluid He in porous media, high-T_c, …)

Phase diagram of disordered interacting bosons



Adding disorder to a Mott Insulator



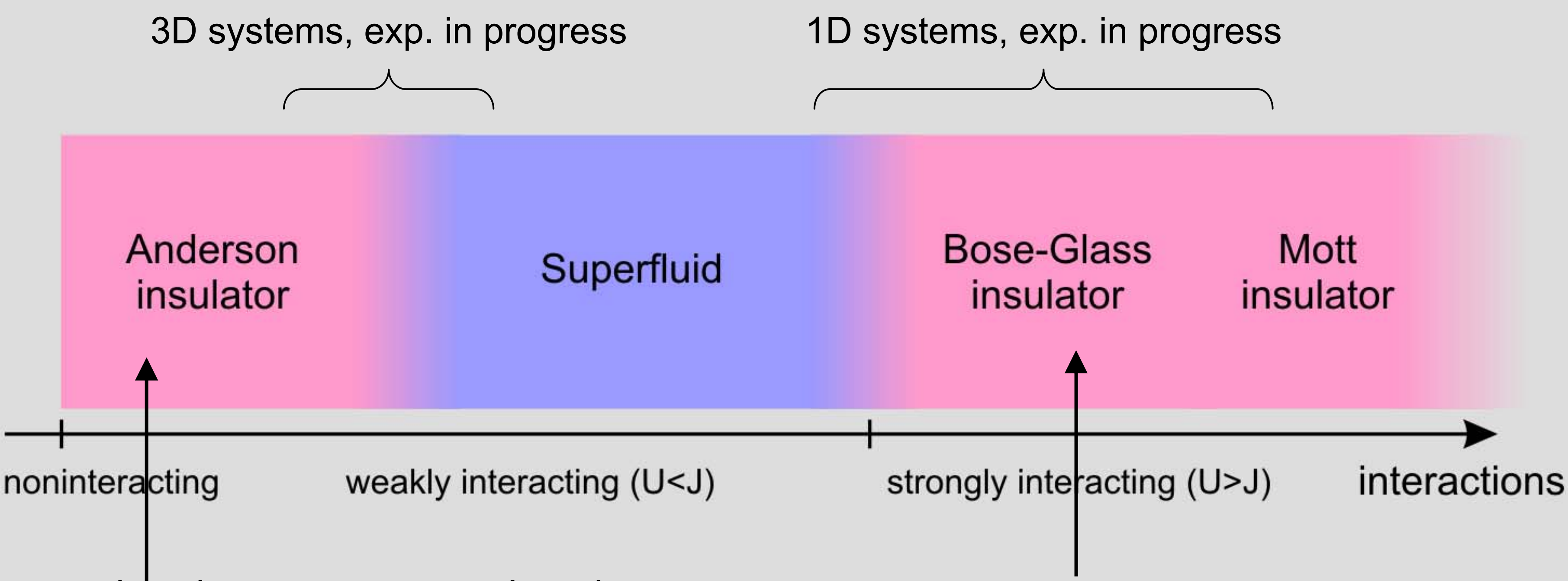


BOSE GLASS

(Fallani 2007)

Disordered interacting bosons

Experiments are in progress to investigate the whole phase diagram of lattice bosons in presence of disorder

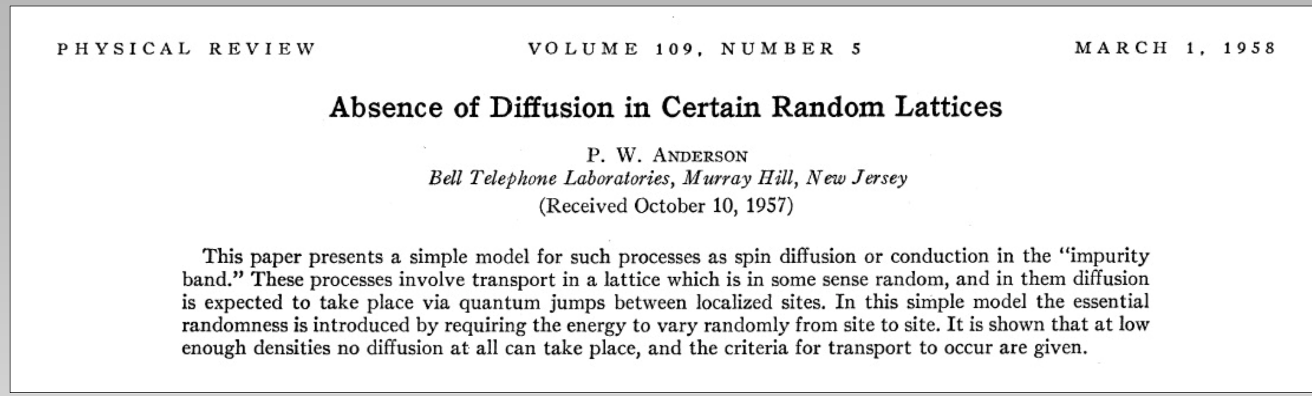


*Roati et al, Nature **453**, 895 (2008)*
*Billy et al, Nature **453**, 891 (2008)*
R.Hulet (Rice)

*Fallani et al, Phys. Rev. Lett. **98**, 130404 (2007)*

*White et al (Urbana) PRL **102**, 055301 (2009)*
*Pasienski et al (Urbana) Nature Phys **6**, 677 (2010)*

Anderson localization



PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

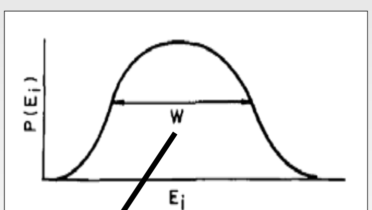
P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

Single particle tight binding model with random on-site energies (electrons in a crystal lattice)

$$i\dot{a}_j = E_j a_j + \sum_{k\neq j} V_{jk} a_k.$$



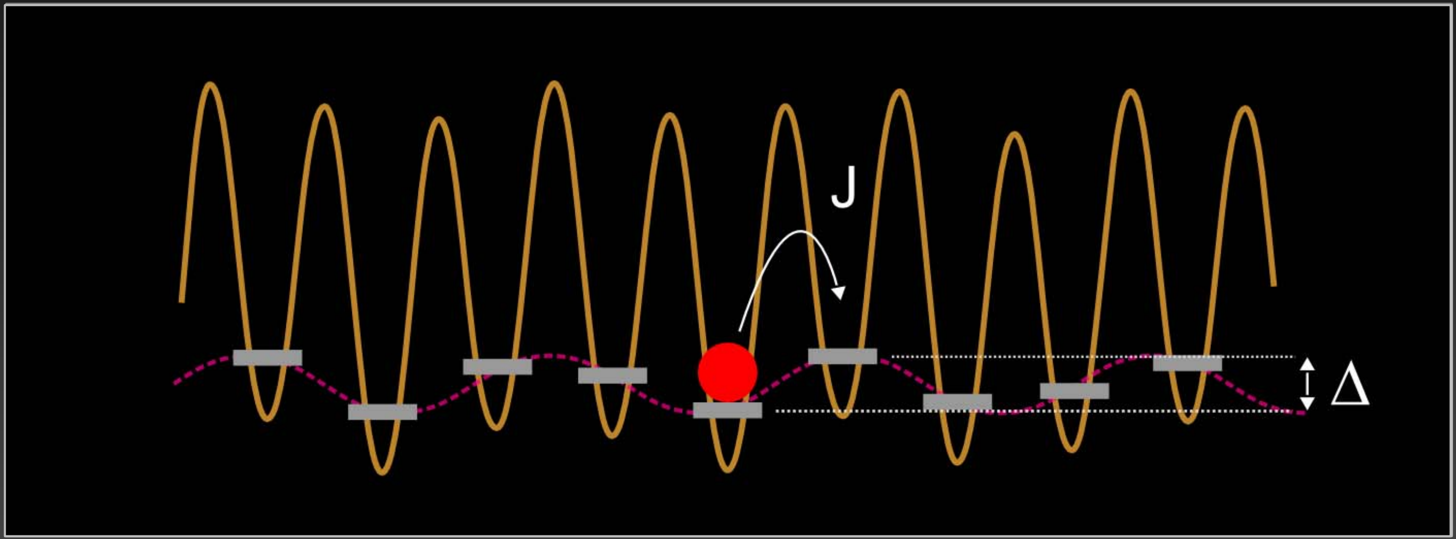
tunneling (J)

amplitude of disorder (Δ)

No diffusion for $V < V_c \sim W$

Aubry-André model with cold atoms!

Adding a weak incommensurate optical lattice...



$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \sum_j \epsilon_j n_j$$

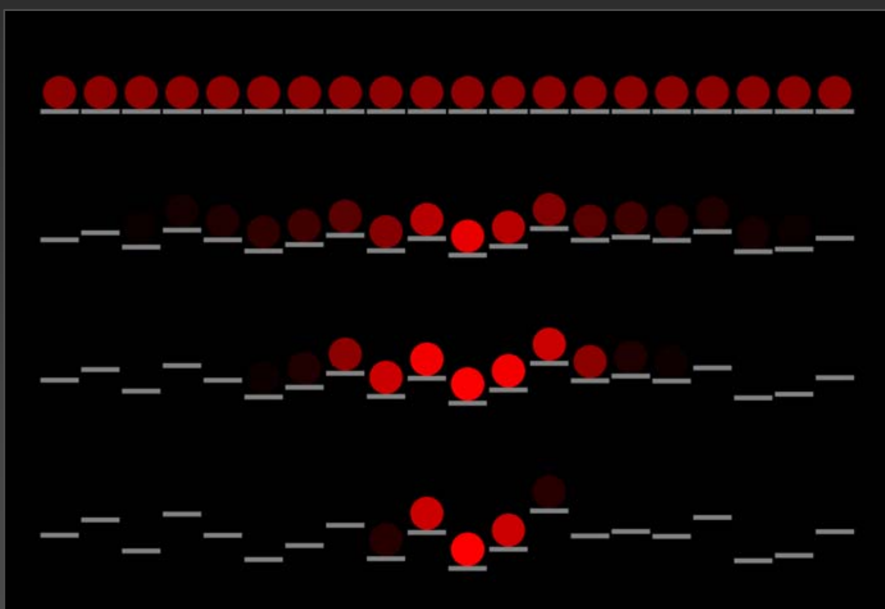
The second lattice controls the site energies $\epsilon_j = \Delta \cos(2\pi\beta j)$

Localization models

Localization depends on the kind of disorder and dimensionality!

1D Anderson model

$$\epsilon_j = \Delta \,\mathrm{Rand}(1)$$

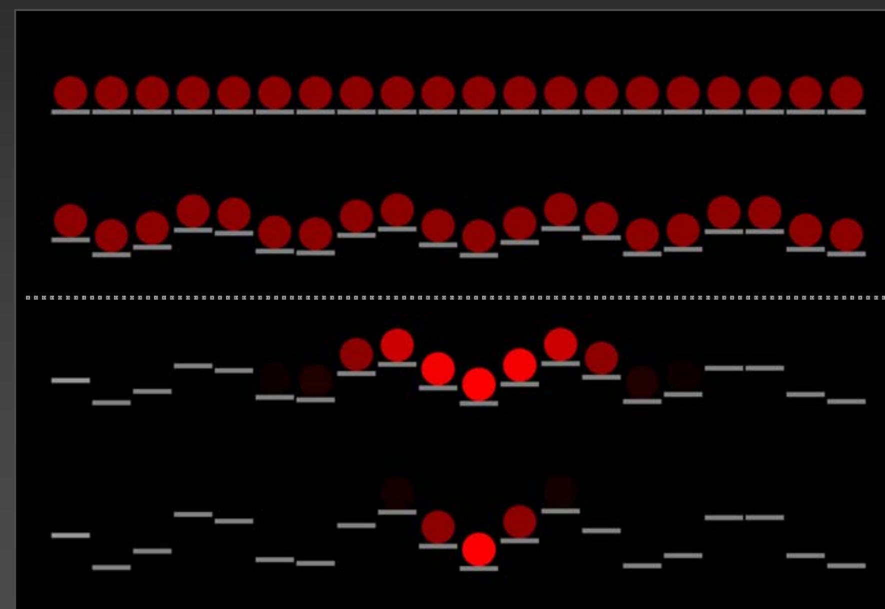


pure random

localization for any Δ

1D Aubry-André model

$$\epsilon_j = \Delta \cos(2\pi\beta j)$$



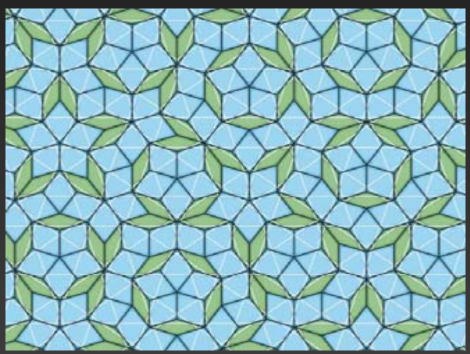
quasiperiodic

localization transition at finite $\Delta = 2J$

The bichromatic lattice

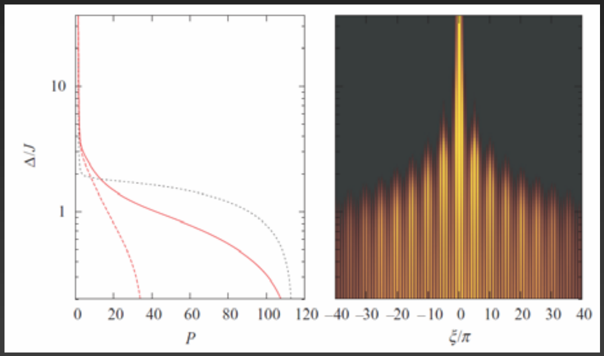
The physics of bi-periodic systems interpolates between periodic systems and disordered systems, as the degree of incommensurability is changed.

- Quasi-crystals



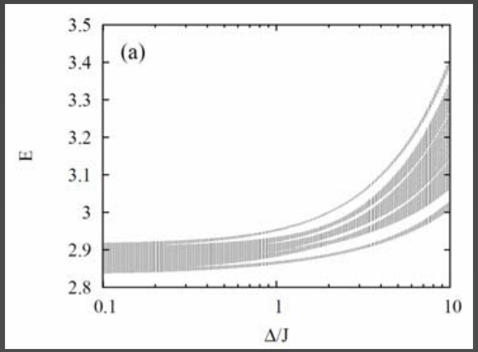
The physics of quasicrystals, ed. P. J. Steinhardt and S. Ostlund (World Scientific, 1987)

- Localization transition in 1D



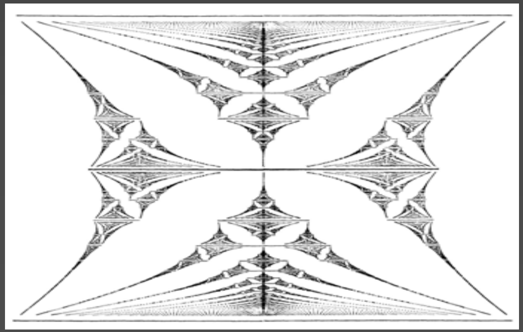
S. Aubry, G. André, *Ann. Isr. Phys. Soc.* **3**, 133 (1980)
M. Modugno, *New J. Phys.* **11**, 033023 (2009).

- Energy bands



M. Modugno, *New J. Phys.* **11**, 033023 (2009).

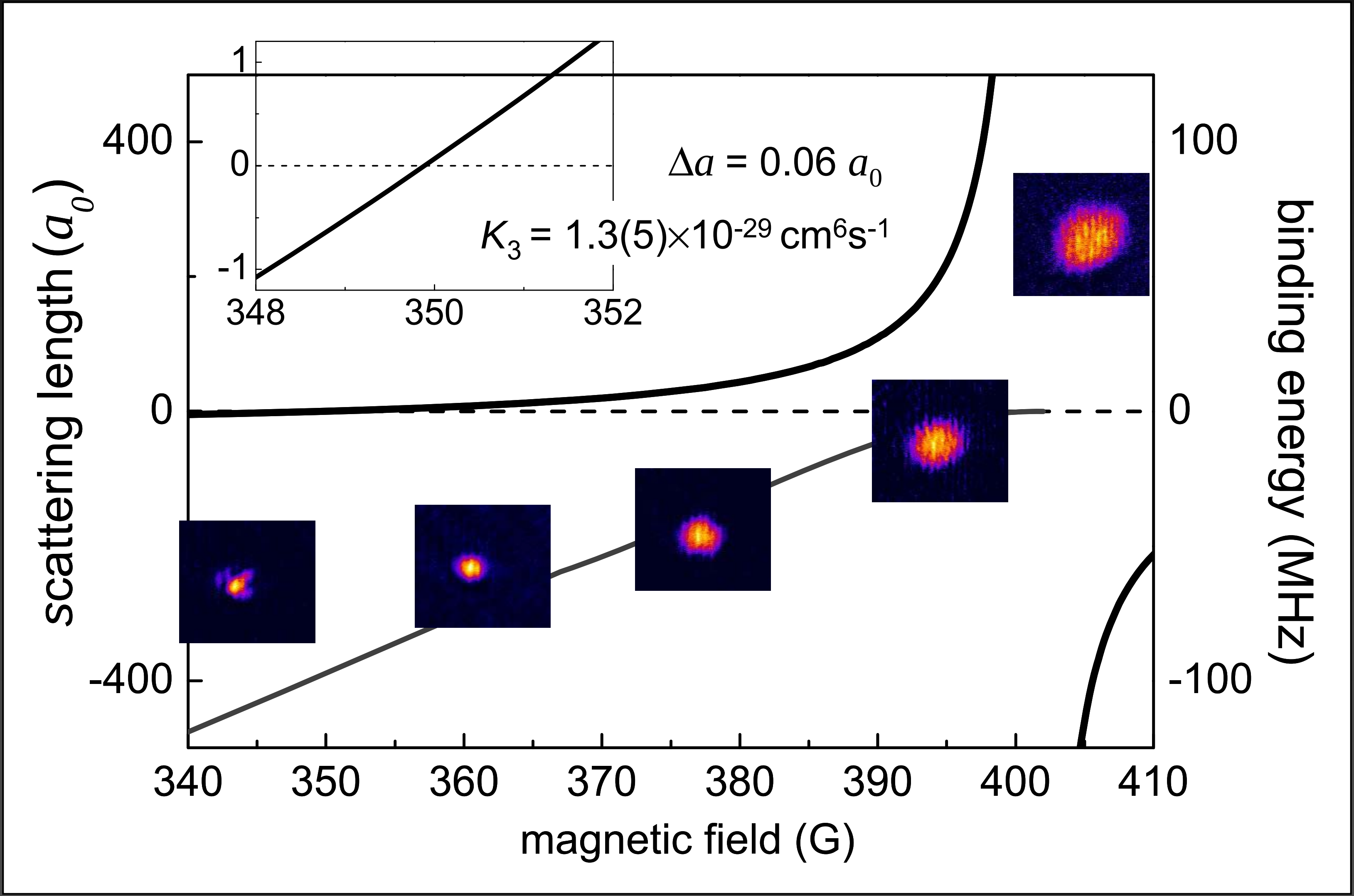
- Fractal critical behavior



D. R. Hofstadter, *Phys. Rev. B* **14**, 2239 (1976).

^{39}K BEC with tunable interactions

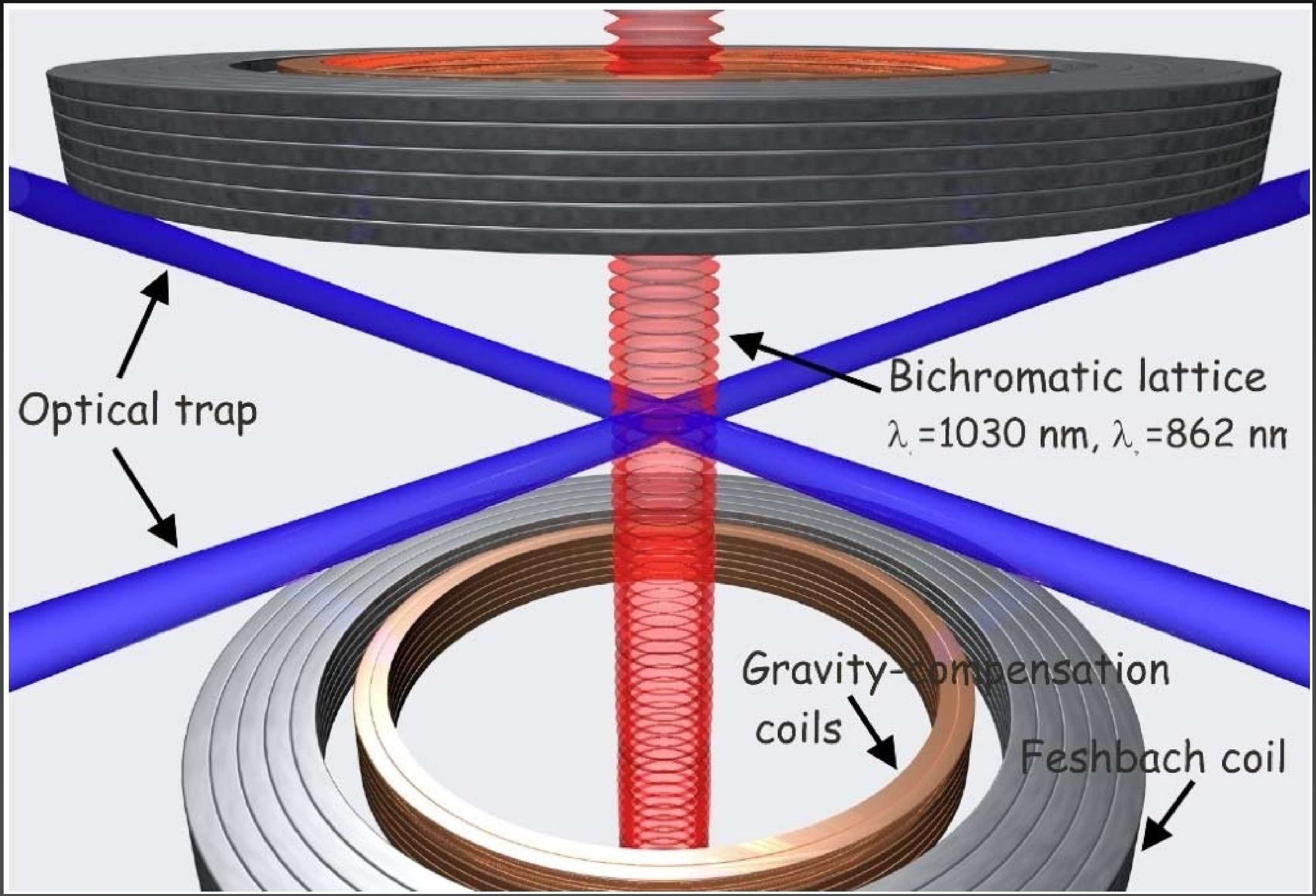
*G. Roati et al., PRL **99**, 010403 (2007)*



Interferometry: *Fattori et al., PRL **100**, 080405 (2008)* Dipolar effects: *Fattori et al., PRL **101**, 190405 (2008)*

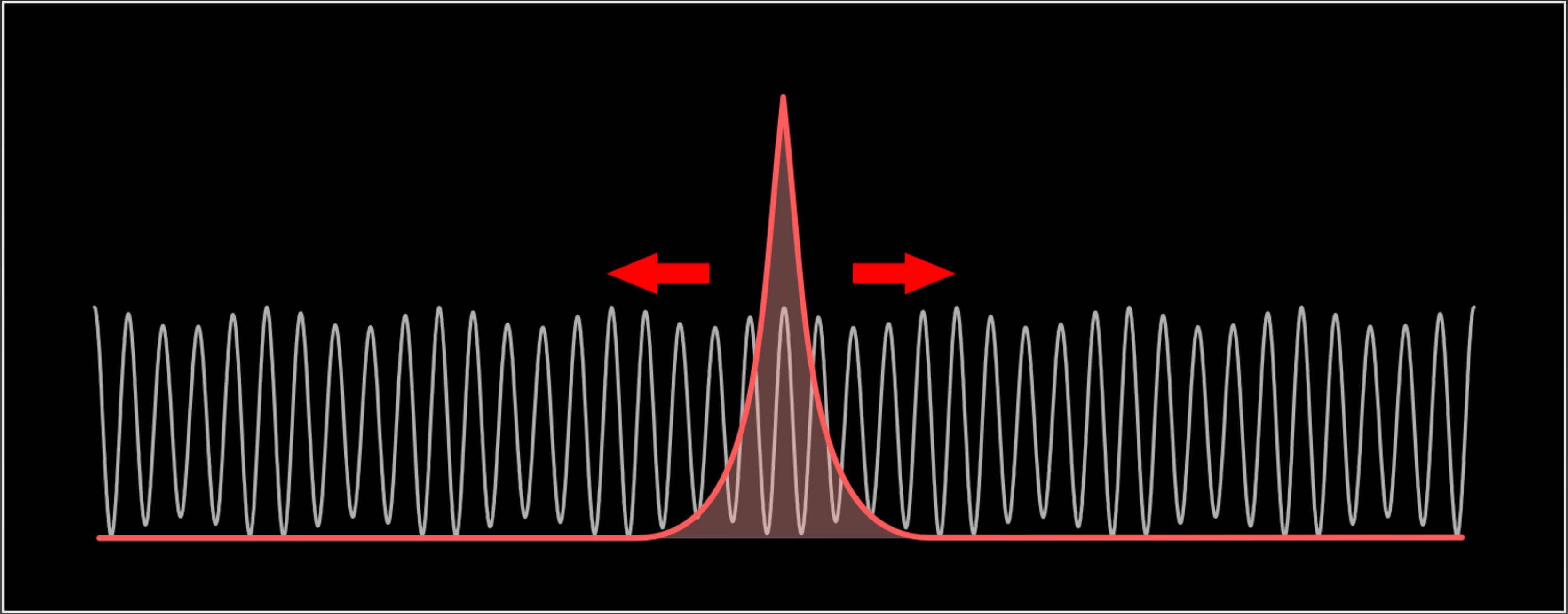
Experimental scheme

*G. Roati et al., Nature **453**, 895 (2008)*



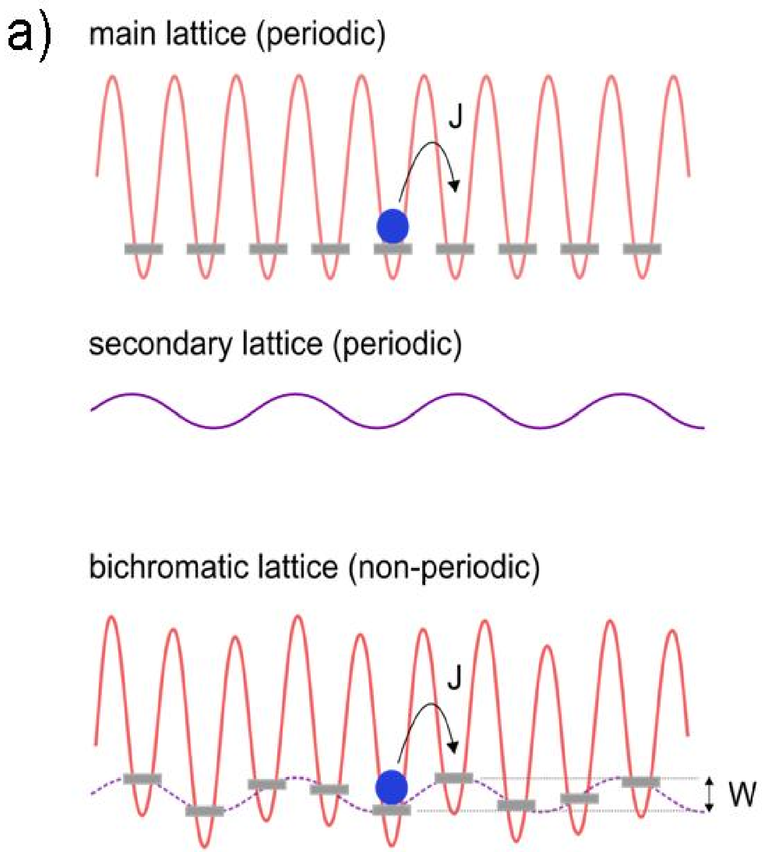
Probing the **transport** properties

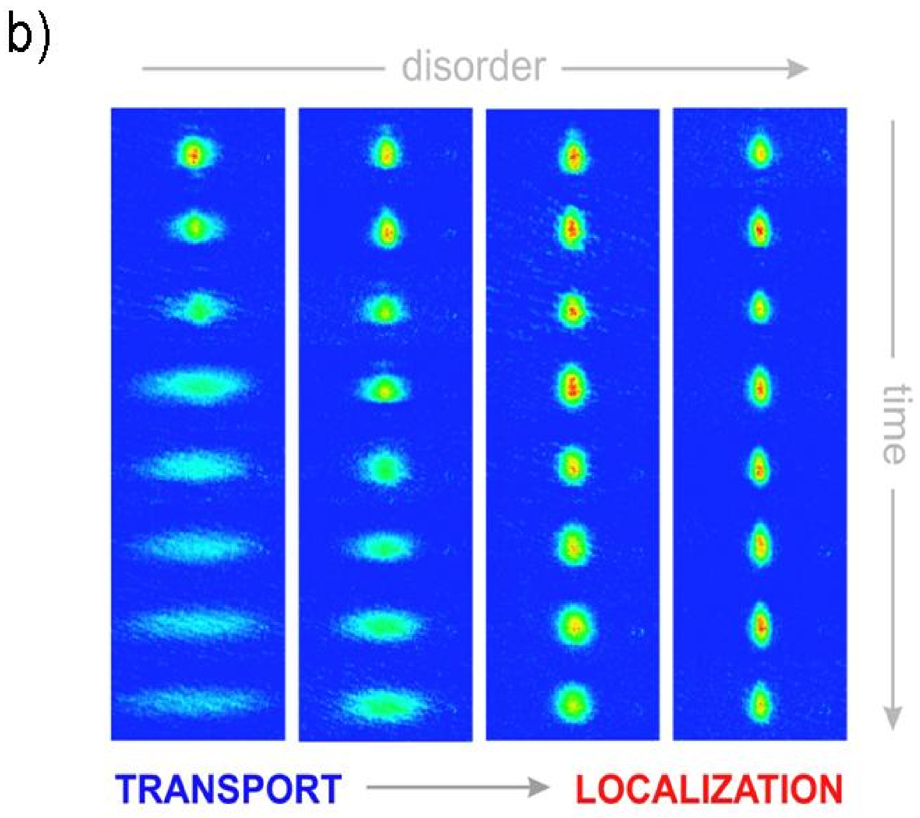
The noninteracting BEC is initially confined in a harmonic trap and then left free to expand in the bichromatic lattice



Absence of diffusion

*G. Roati et al., Nature **453**, 895 (2008)*

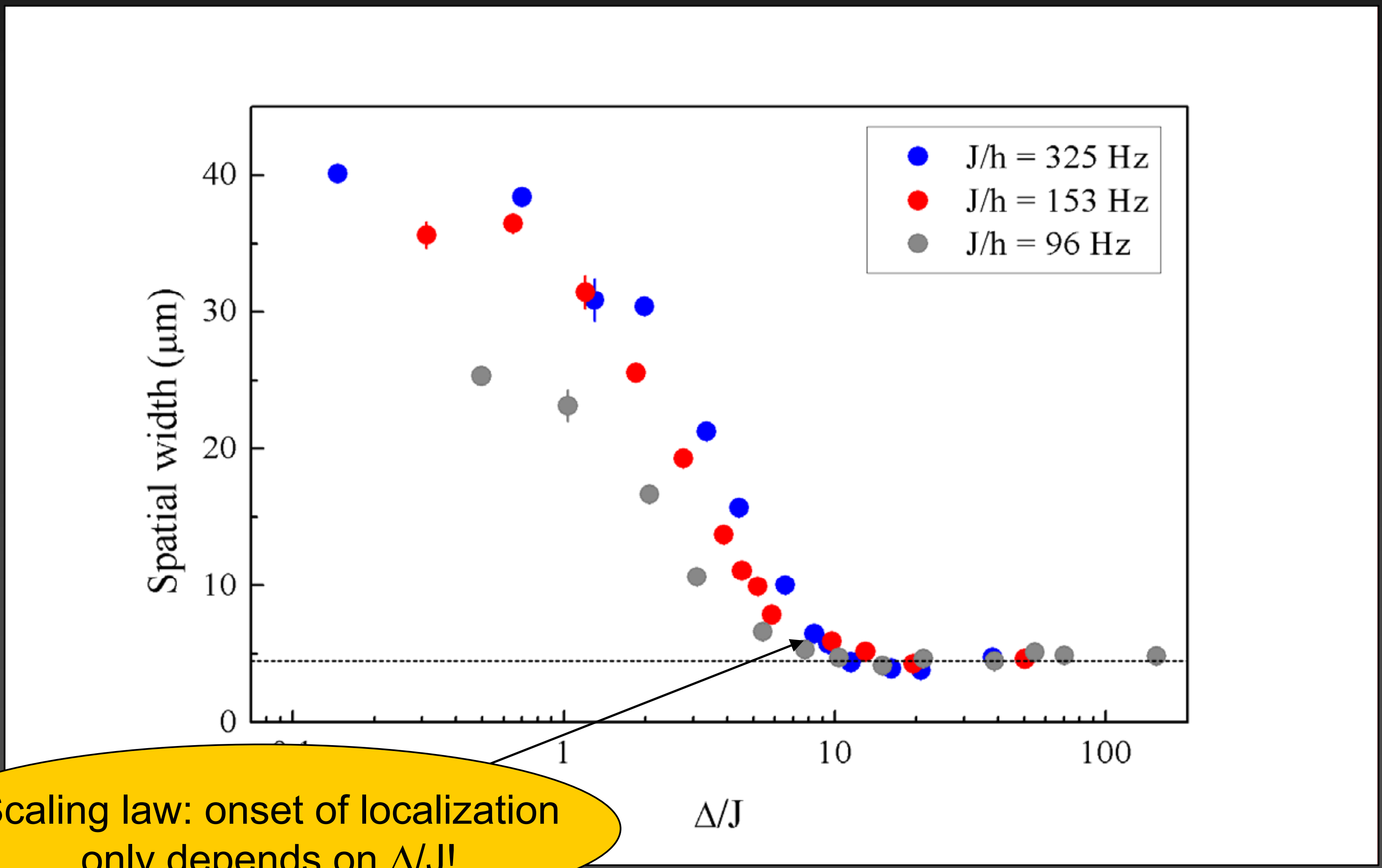




Expansion in the bichromatic lattice

*G. Roati et al., Nature **453**, 895 (2008)*

Size of the condensate after 750 ms expansion in the bichromatic lattice:



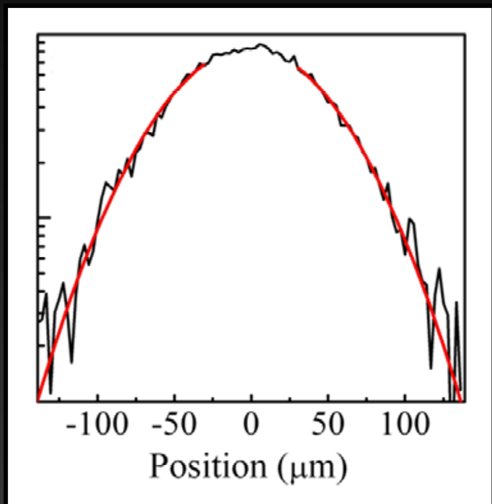
Scaling law: onset of localization only depends on Δ/J!

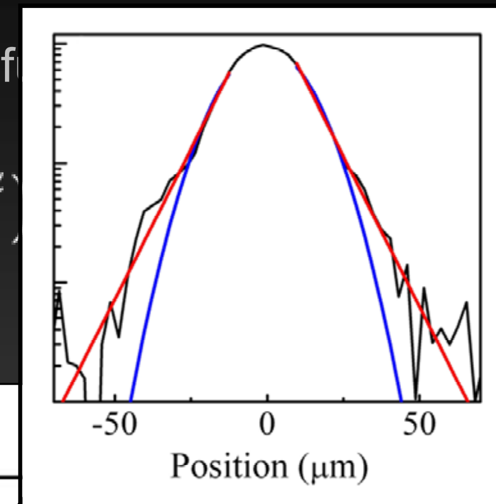
Exponential localization

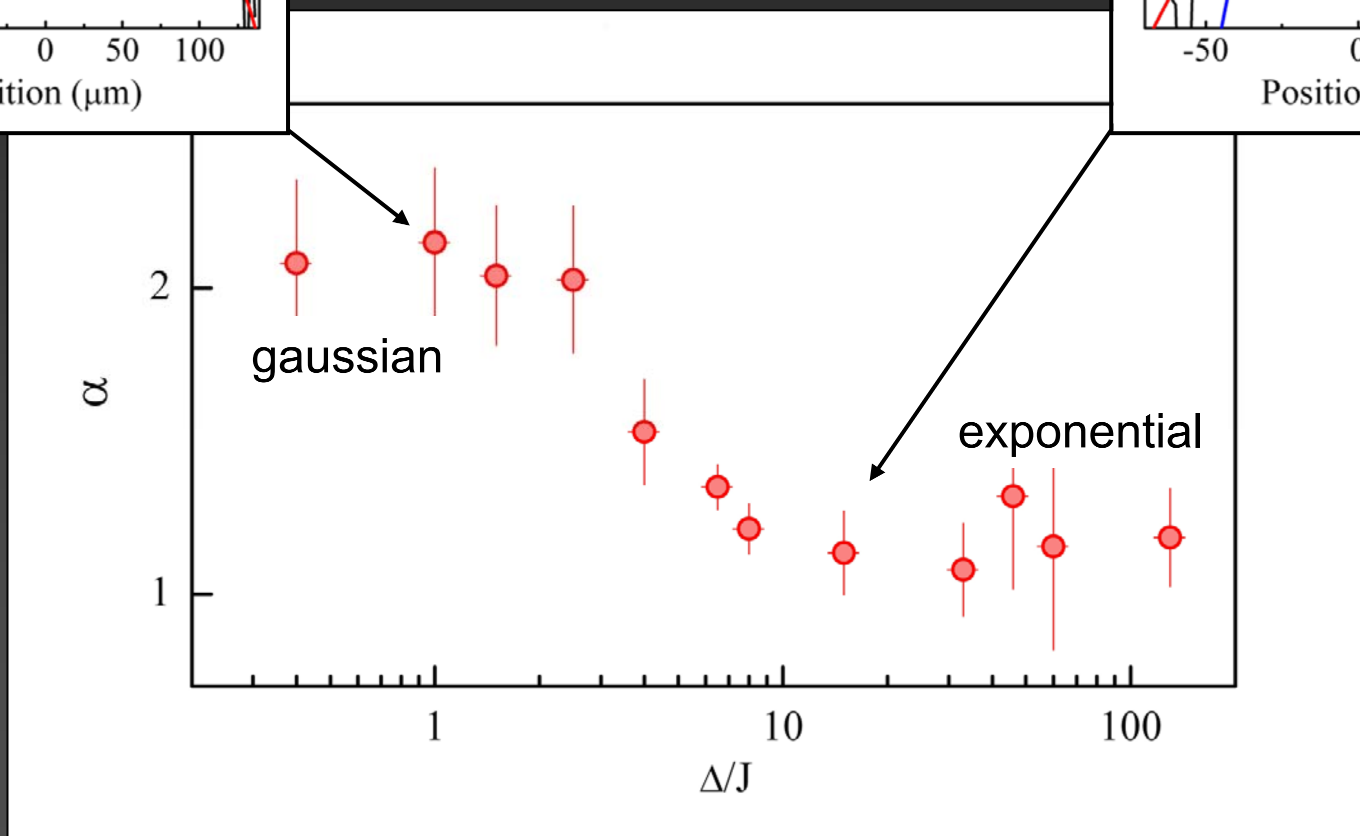
G. Roati et al., Nature ***453****, 895 (2008)*

...bution with a generalized exponential f...

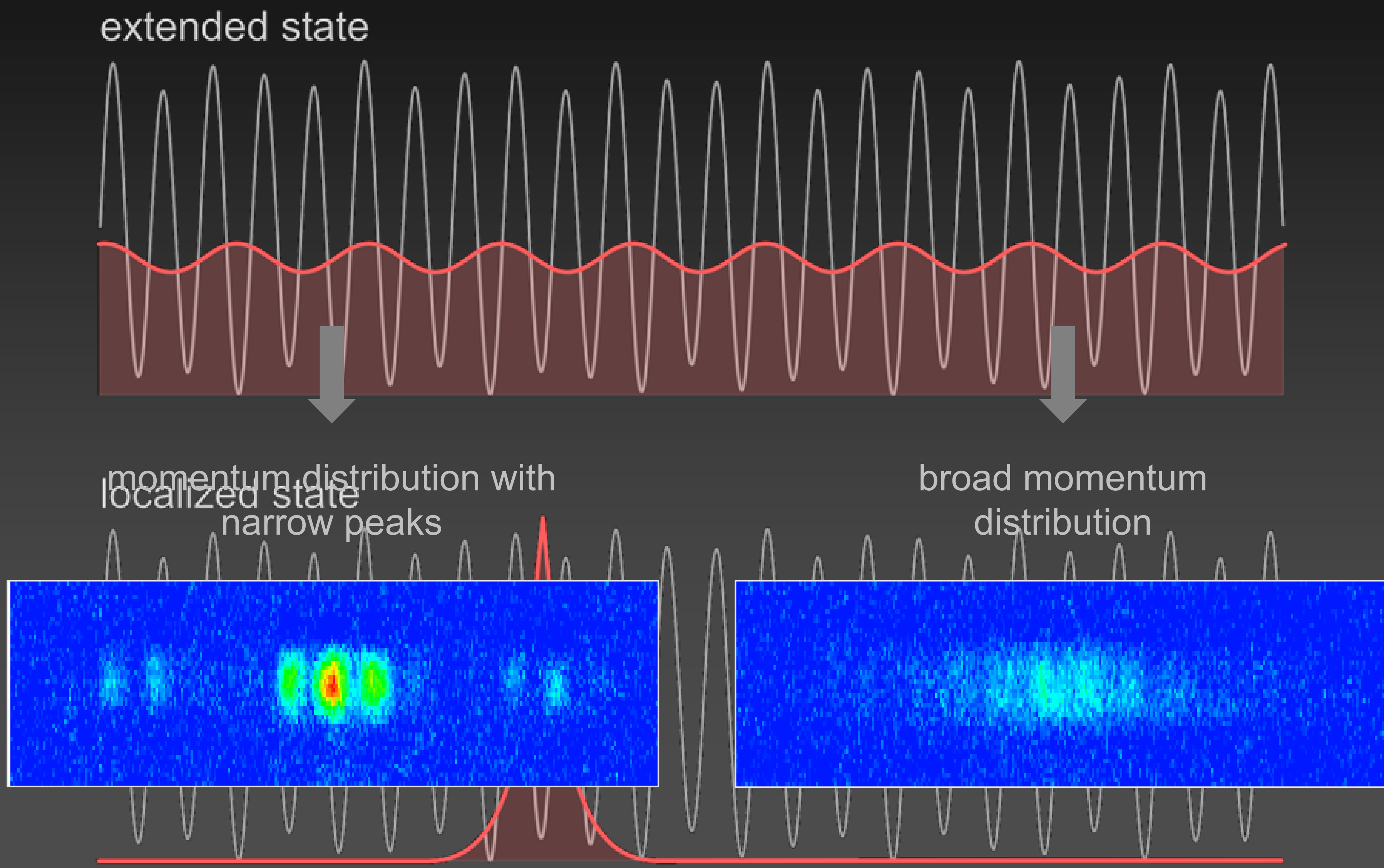
$$n(x) = A \exp\left(-\gamma\,(x - x_0)^{\alpha}\right)$$





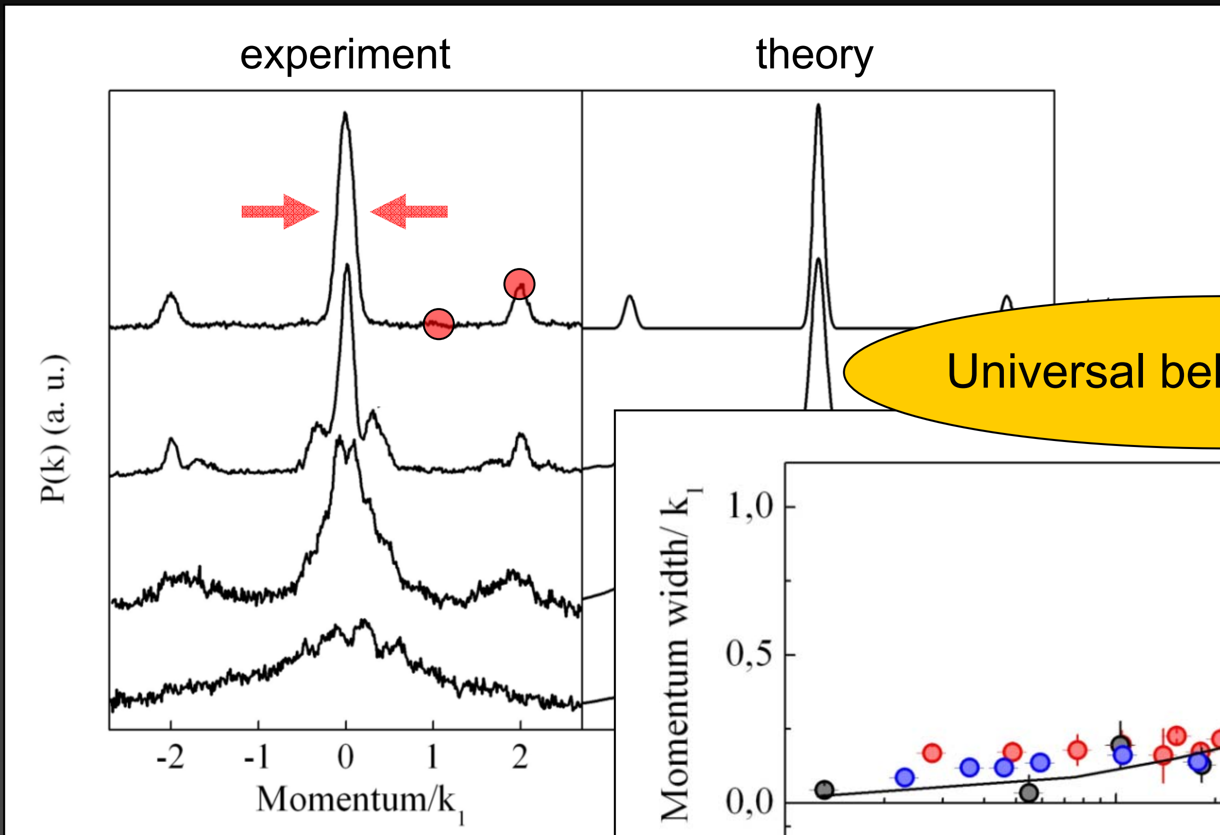


Momentum distribution of the localized states



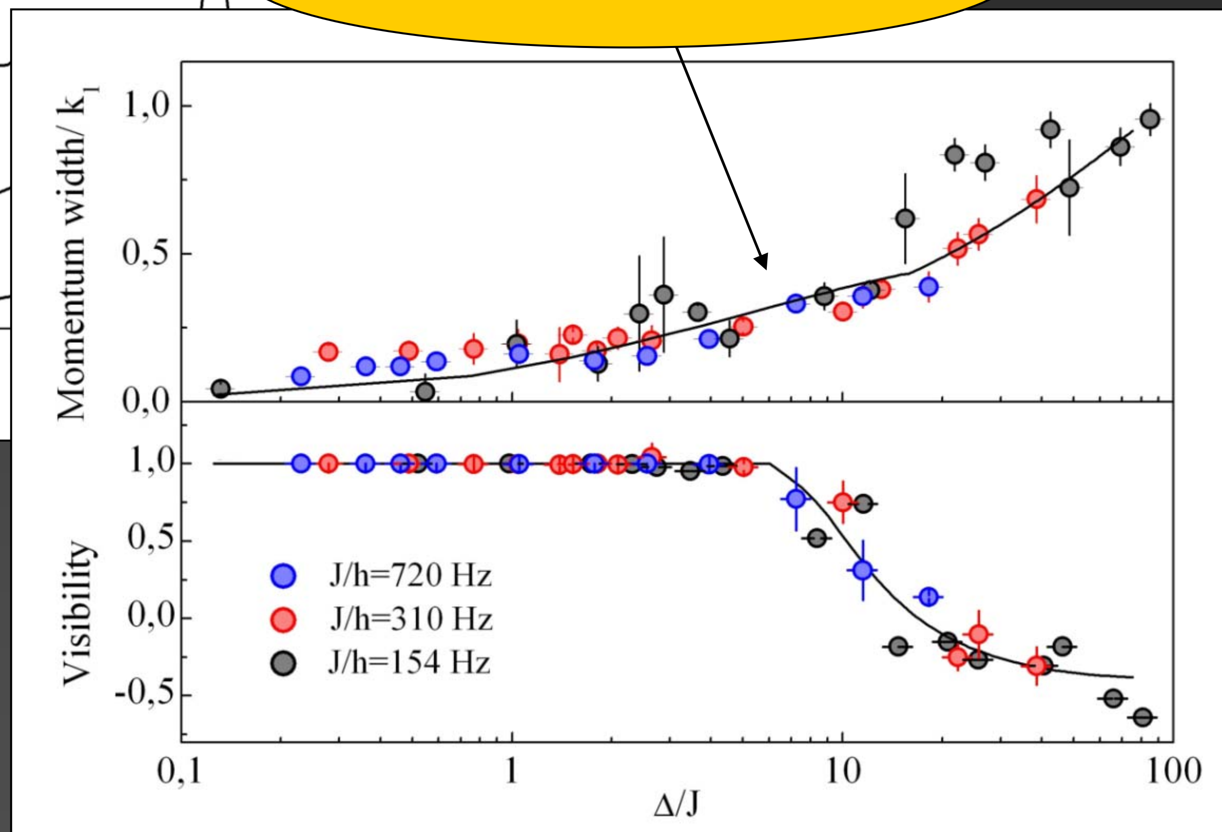
Momentum distribution

*G. Roati et al., Nature **453**, 895 (2008)*



Density distribution after time-of-flight of the initial stationary state

Universal behavior with Δ/J!



Width of the central peak

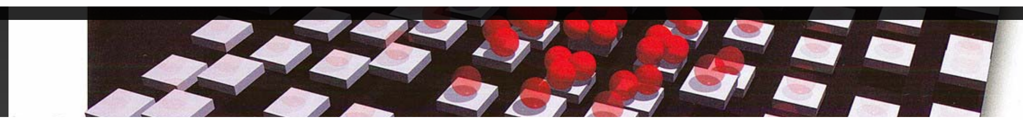
Visibility $$\frac{P(2k_1) - P(k_1)}{P(2k_1) + P(k_1)}$$

feature article

Anderson localization of ultracold atoms

Alain Aspect and Massimo Inguscio

To study localized matter waves, two experimental groups hold a Bose–Einstein condensate in the grip of a disordered but tunable optical potential formed by interfering laser beams.



feature article

Fifty years of Anderson localization

Ad Lagendijk, Bart van Tiggelen, and Diederik S. Wiersma

What began as a prediction about electron diffusion has spawned a rich variety of theories and experiments on the nature of the metal–insulator transition and the behavior of waves—from electromagnetic to seismic—in complex materials.

P. W. Anderson, Nobel lecture (1977)

… about the role of interactions

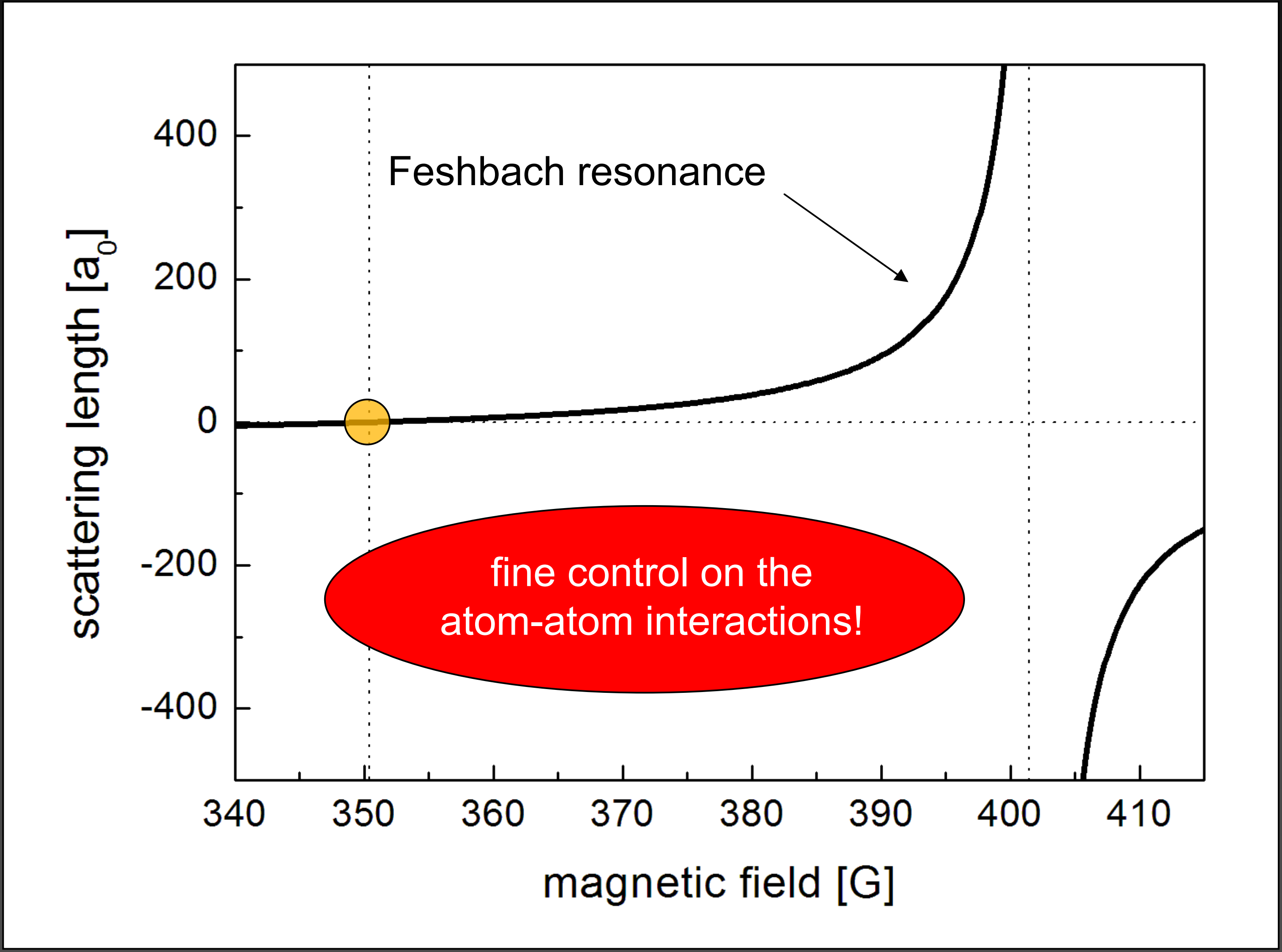
A second reason why I felt discouraged in the early days was that I couldn't fathom how to reinsert *interactions*, and I was afraid they, too, would delocalize.

The realization that, of course, the Mott insulator localizes without randomness, because of interactions, was my liberation on this: one can see easily that Mott and Anderson effects supplement, not destroy, each other…

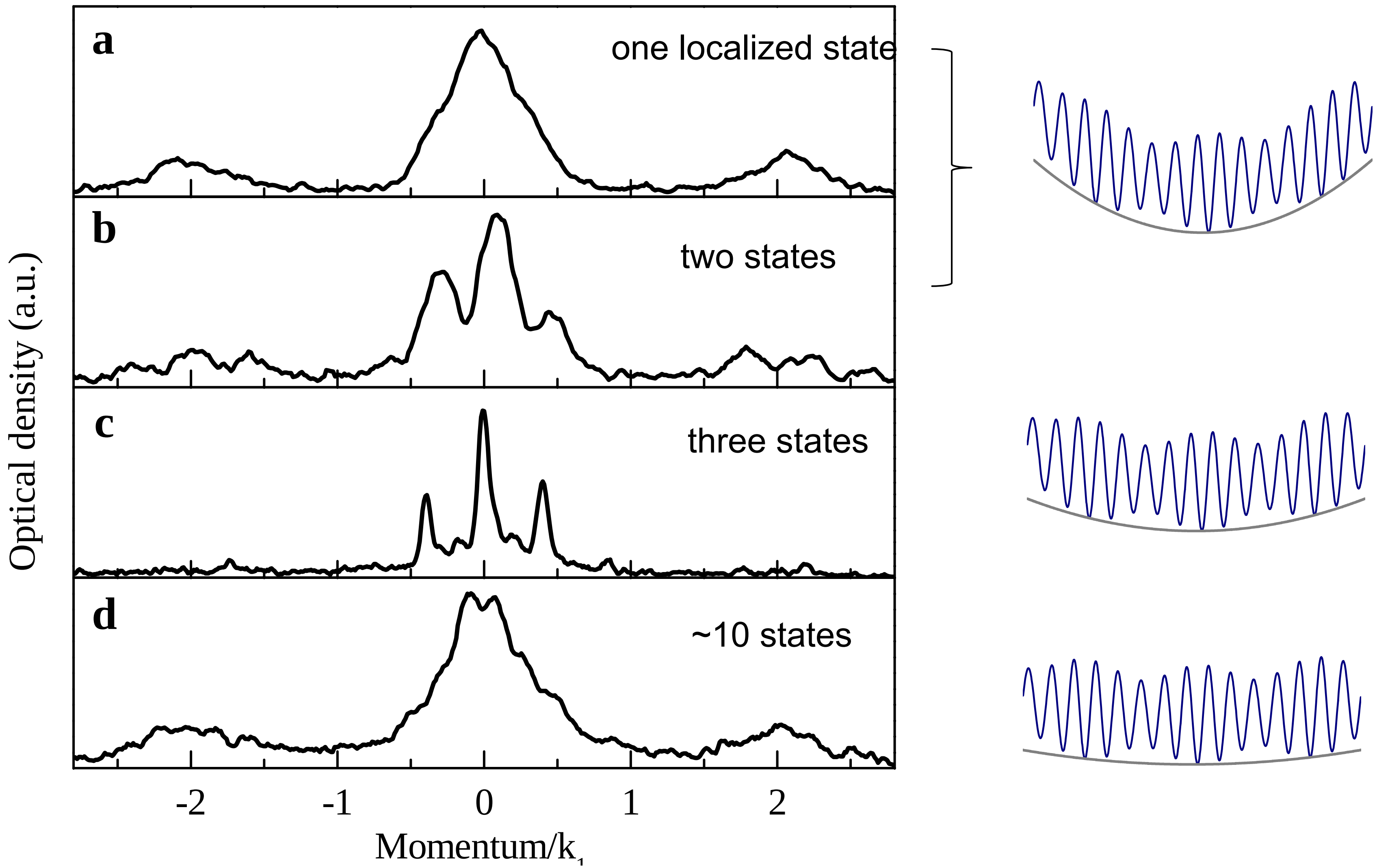
The present excitement of the field for me is that a *theory of localization with interactions* is beginning to appear…
It is remarkable that in almost all cases **interactions play a vital role**, yet many results are not changed too seriously by them.

^{39}K BEC with tunable interactions

*G. Roati et al., PRL **99**, 010403 (2007)*

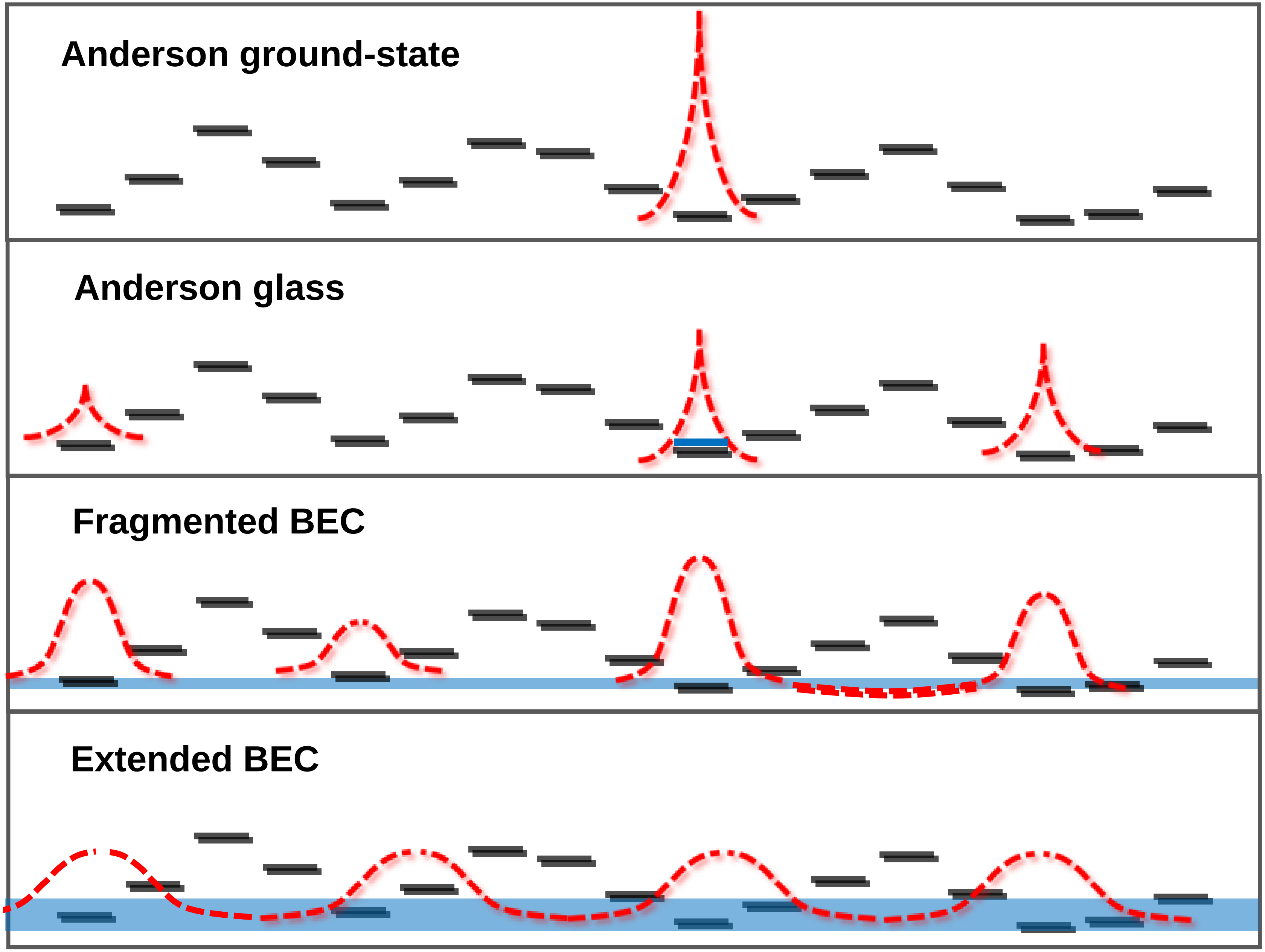


No interactions: an excited Anderson insulator



Reaching the Anderson-localized ground state is very difficult, since $J_{eff} \rightarrow 0$

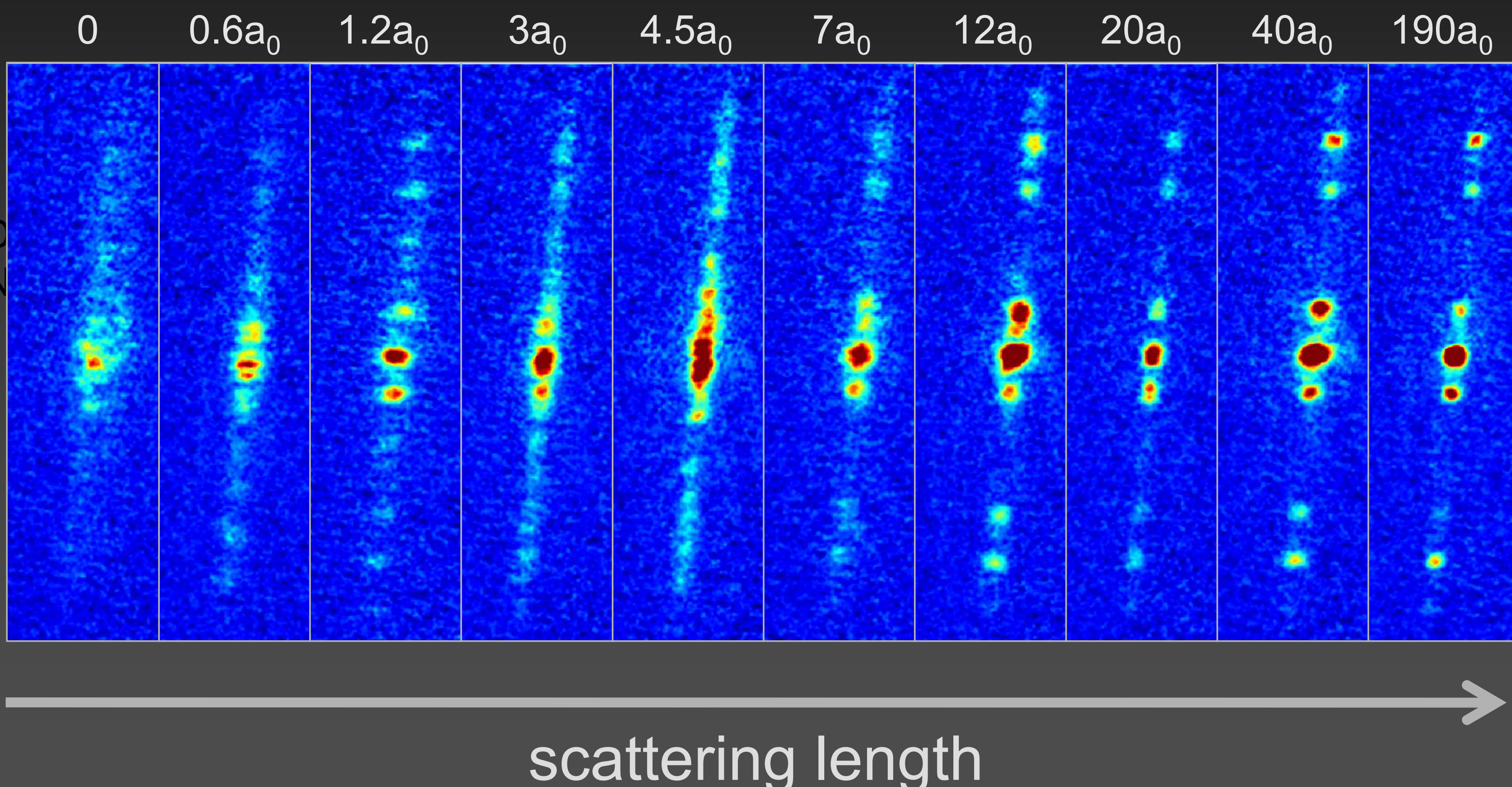
What is the nature of the ground-state with interactions?



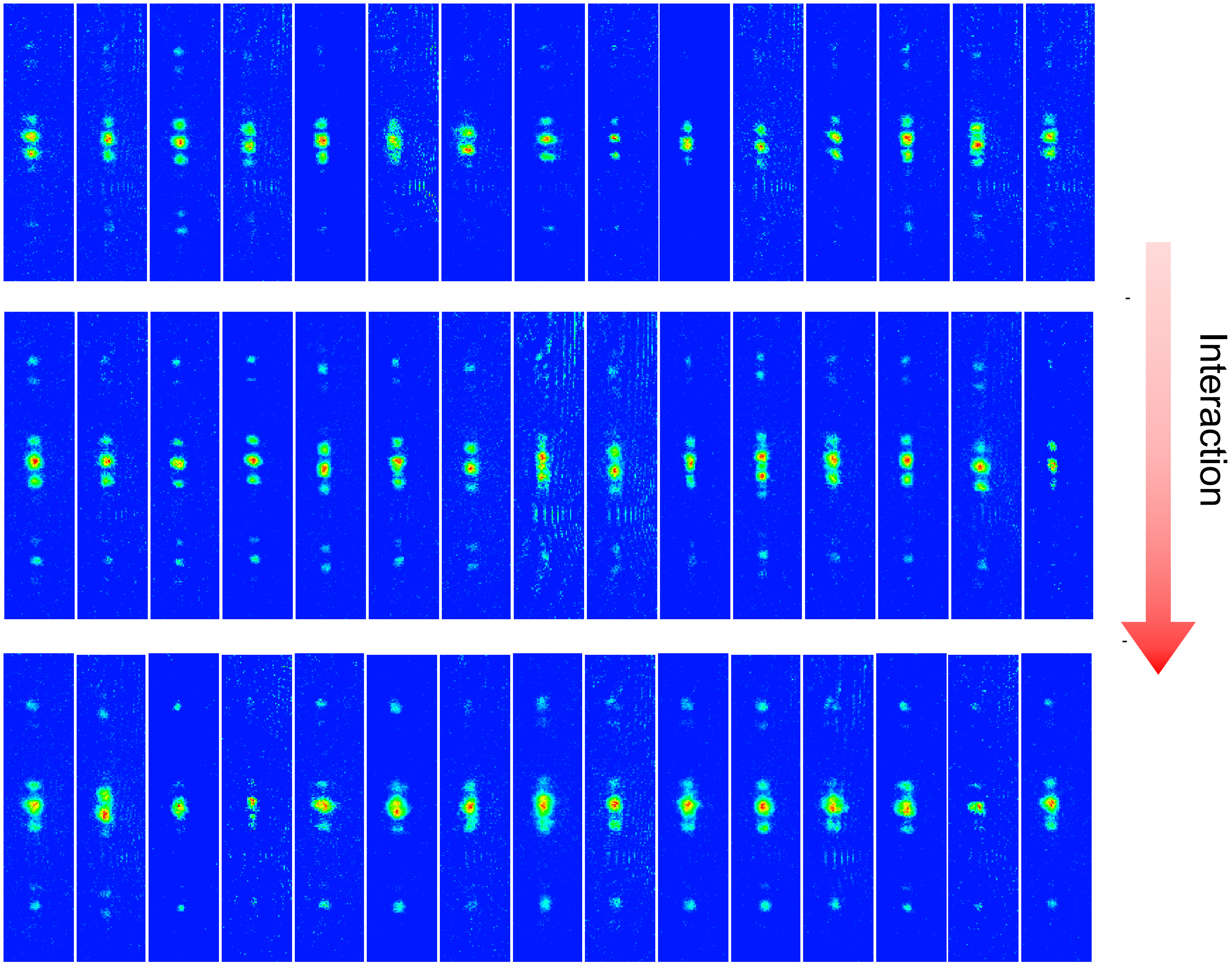
Momentum distribution with interactions

Diessler et al
Nature Phys. 6, 354 (2010)

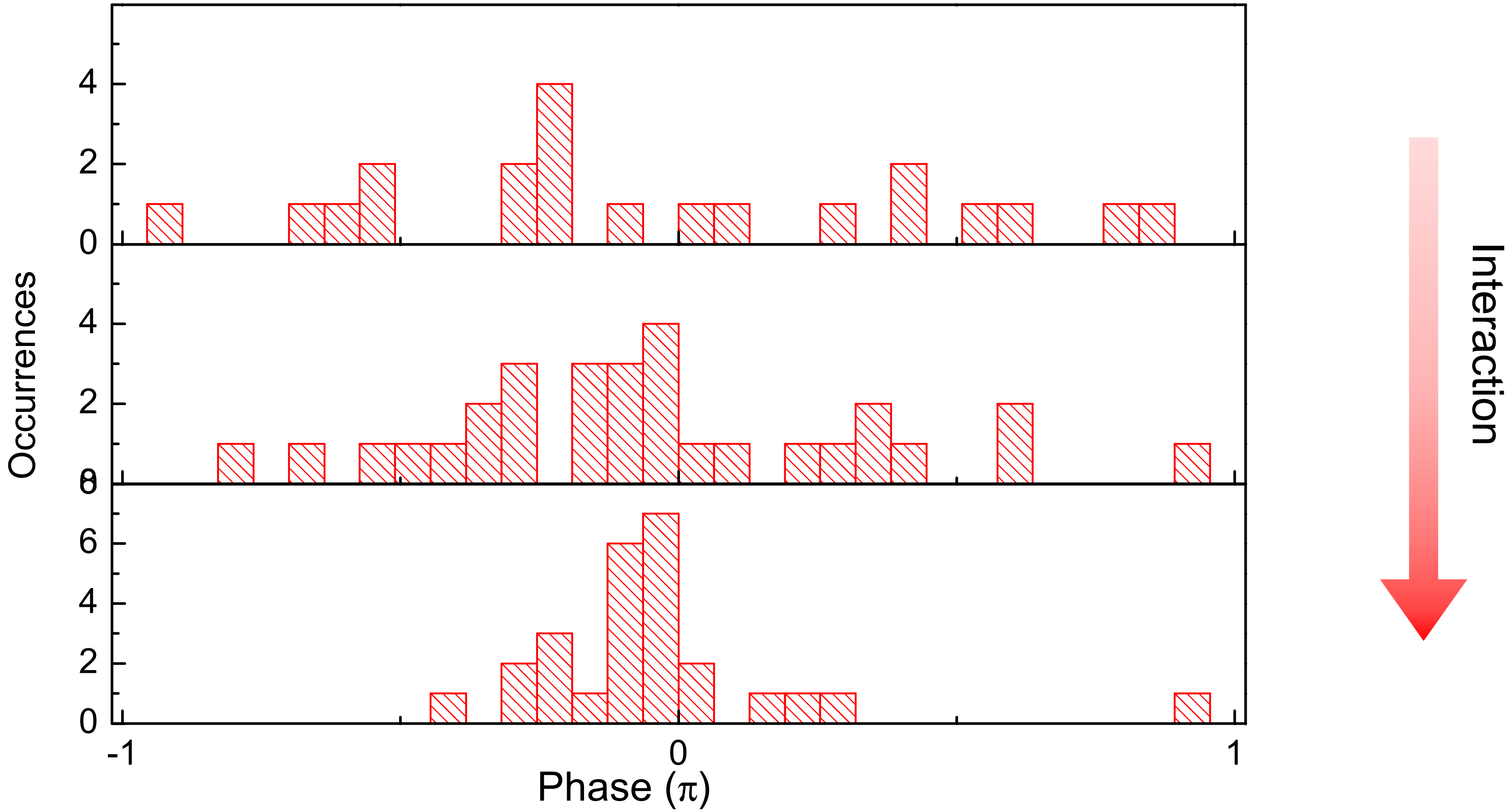
Momentum distributions for $\Delta/J=15$



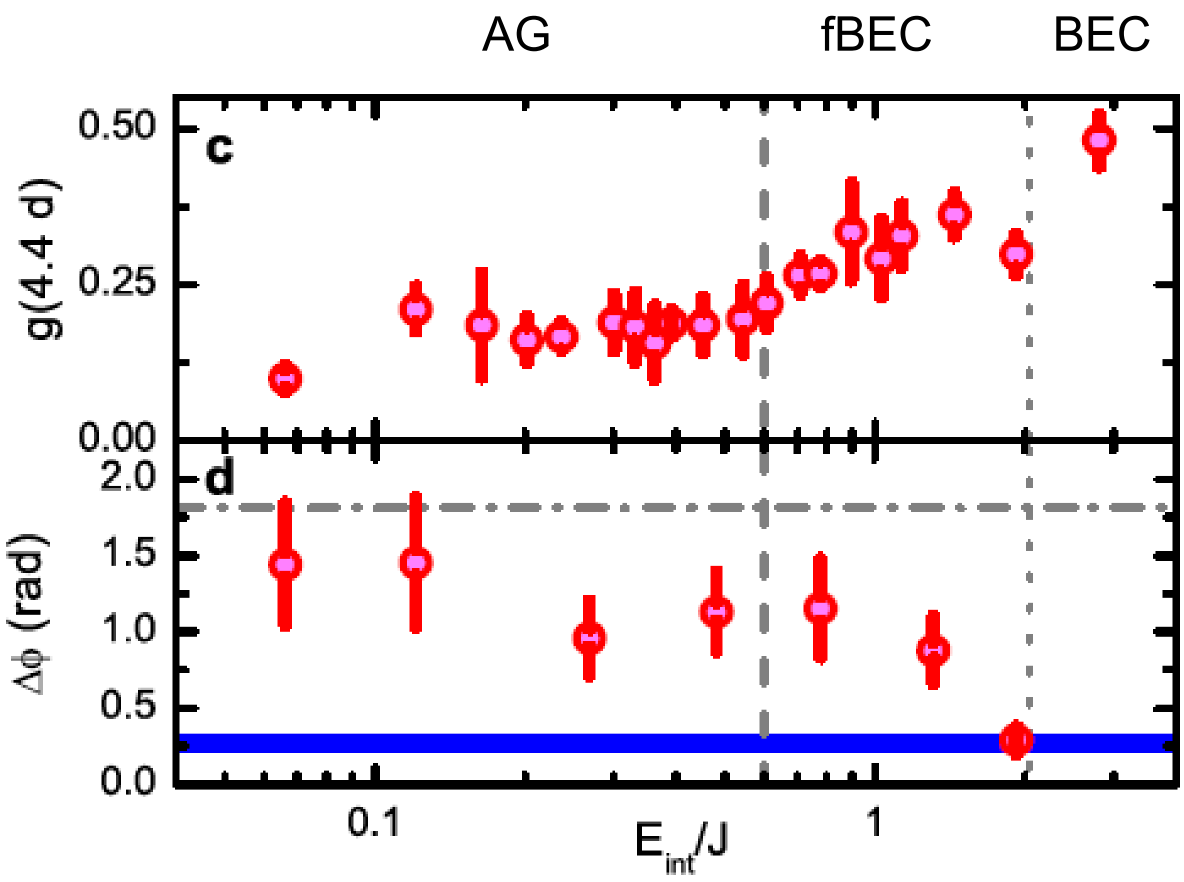
Phase fluctuations



Phase fluctuations



Phase fluctuations

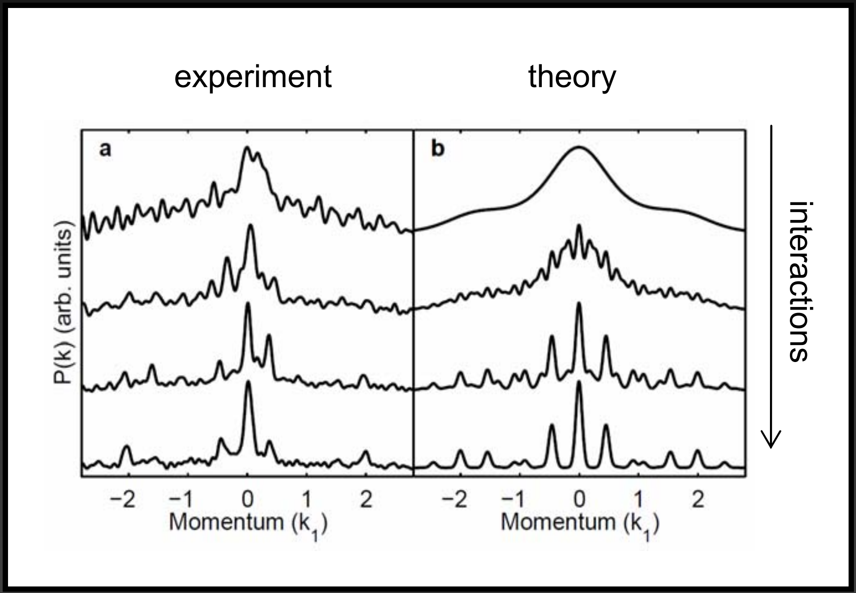


related work at Rice
Chen et al PRA 77, 033632 (2008)

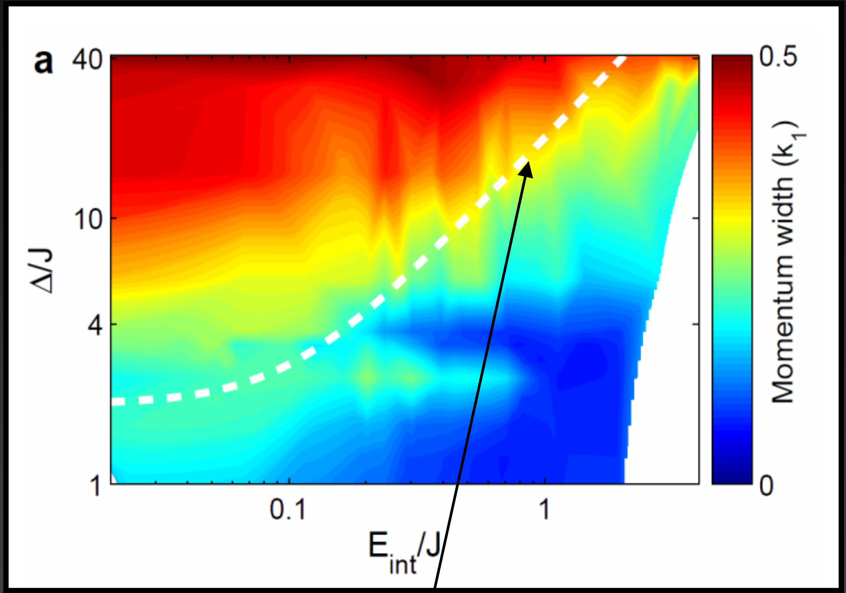
Diessler et al Nature Phys. 6, 354 (2010)

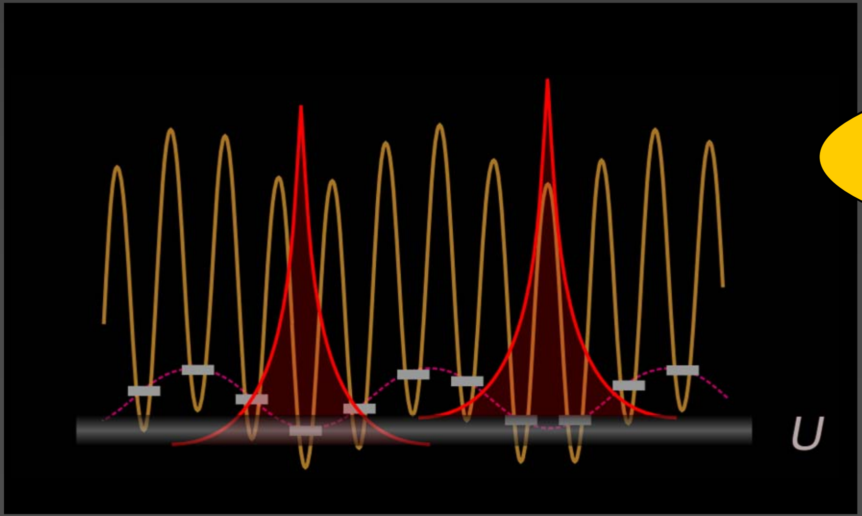
Interaction-induced delocalization

Momentum distribution:

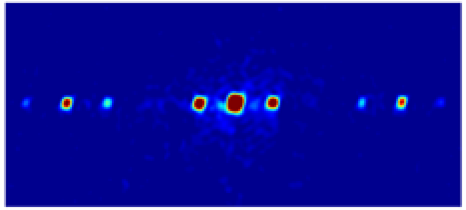


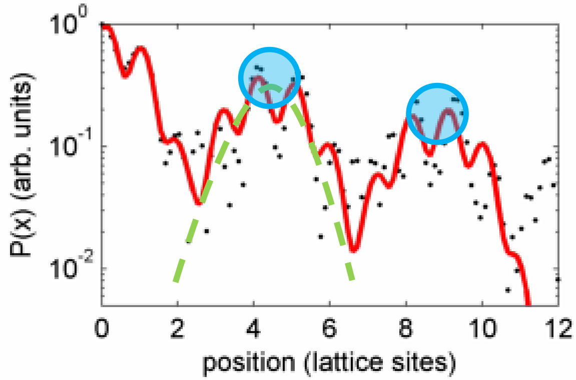
Width of the central peak:





Repulsive interactions shift the localization transition towards larger Δ!





1. Fourier transform $\mathfrak{F}^{-1}\left|\hat{\Psi}(\mathrm{k})\right|$:
average local shape of the wavefunction
Fit to sum of three generalized exponential functions

$$A\exp\left(-\left|\frac{z-z_c}{L}\right|^\alpha\right)$$

exponent α, local length L_s

2. Correlations:
Wiener-Khinchin theorem

$$\mathfrak{F}\{g(x)\} = |F(k)|^2 = \rho(k)$$

gives us spatially averaged correlation function

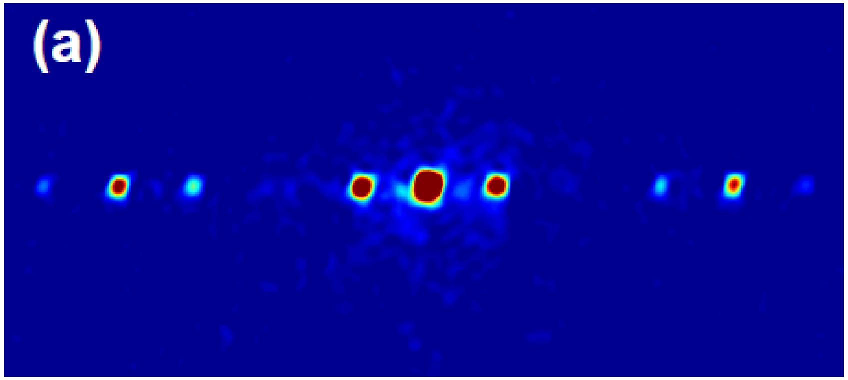
$$\rho(k) \propto \mathfrak{F}^{-1}\int G(x', x+x')\,dx'$$

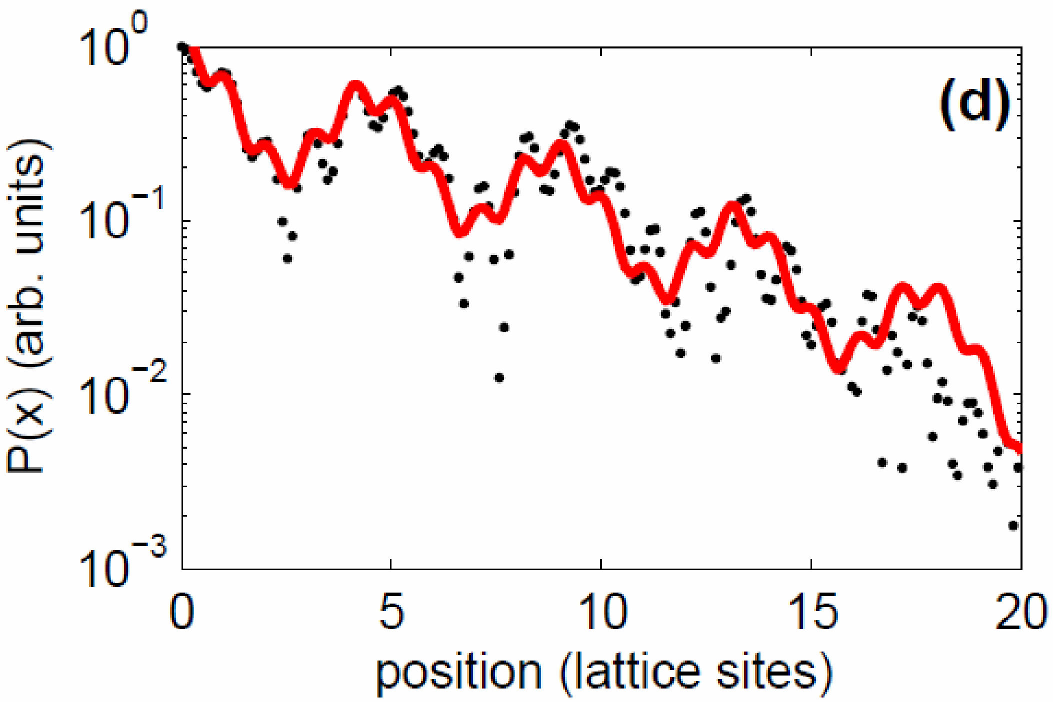
correlation g(4.2d), g(8.4d)

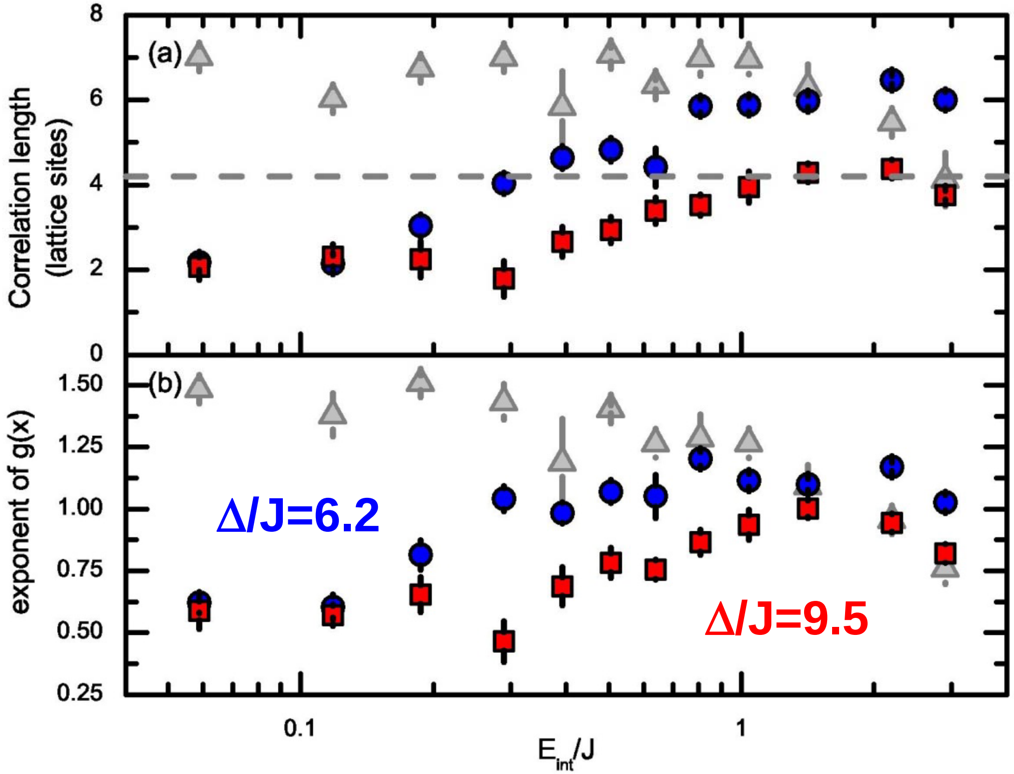
3. Phase of momentum distribution:
measure fluctuations $\Delta\phi$

4. Correlation function:
Fit g(x) with 5 generalized exponentials, amplitudes determined by shape of correlation function
exponent β, correlation length L_g

The correlation function







correlation length increases with E_{int}
(limits from imaging resolution)
→average size of fragments in fragmented BEC increases

exponent increases, as expected (expect β=1 to β=2, discrepancy from thermal component in images)

Change of exponent: crossover from AG to BEC

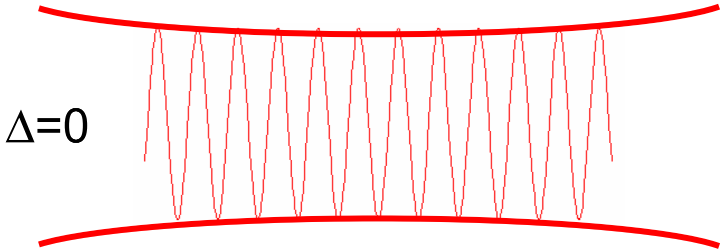
theory:

Fontanesi, Wouters & Savona: PRL **103**, 030403 (2009)
Cetoli & Lundh: PRA **81**, 063635 (2010)

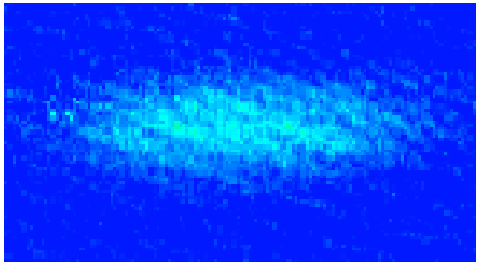
experiment:

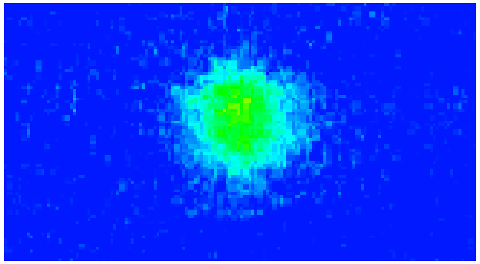
Deissler *et al*., arXiv:1010.0853

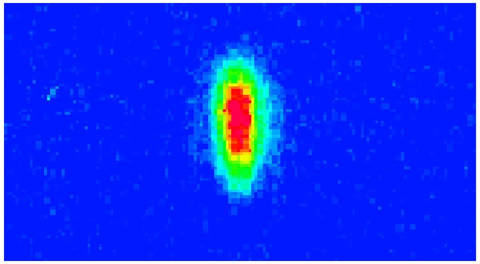
Conductivity



No interactions
Ballistic expansion:
$\langle r^2 \rangle(t) \propto t^2$







Anomalous diffusion

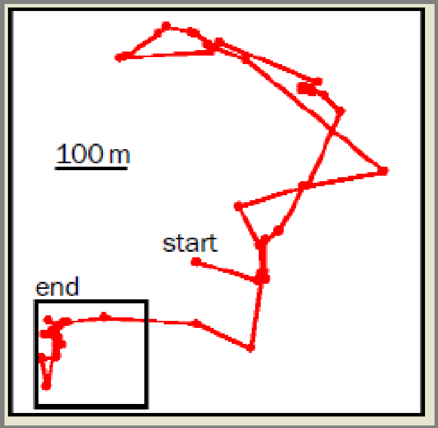
Diffusive expansion $\alpha = 1/2$: (Einstein 1905)

random walk → existence of a mean free path

existence of a mean time between collisions

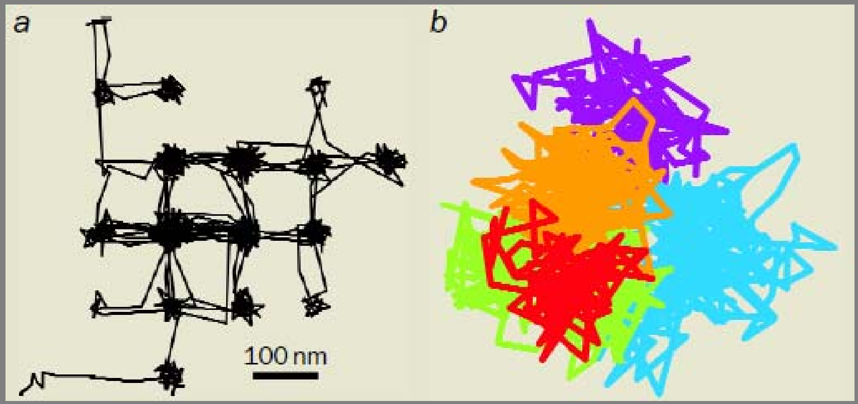
Superdiffusive expansion $\alpha > 1/2$:

Levy flighs → the probability that a particle remains in motion without changing direction for a long time is not negligible (the mean free path is no more well defined)



Subdiffusive expansion $\alpha < 1/2$:

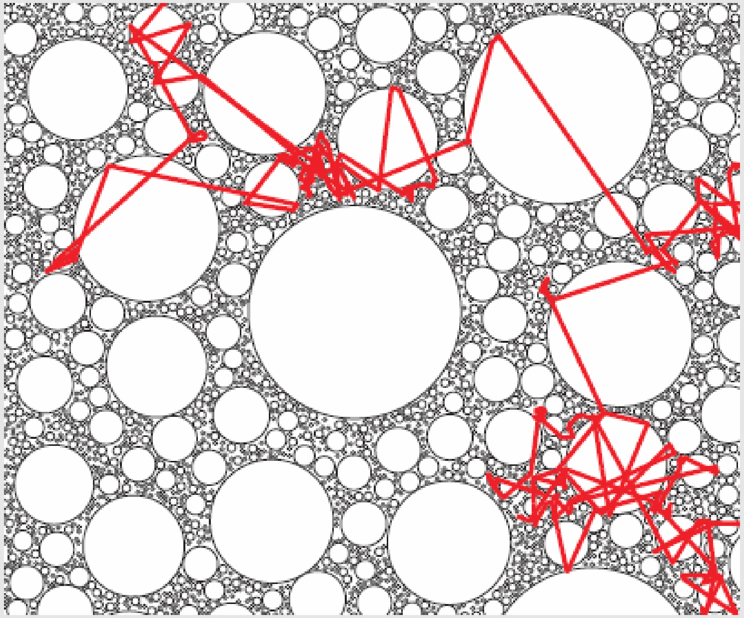
particles spend relatively long time trapped in a zone before jumping in another zone



Many area of interest:
biology, optics, financial…

Physics World August 2005 – Anomalous diffusion spread its wings

Lévy glass



- New optical material in which light performs a Lévy flight
- Anomalous transport properties: optical superdiffuser

Barthelemy, Bertolotti, Wiersma, Nature **453**, 498 (2008)

Many theoretical prediction on an interacting system expanding in a disordered potential for both random disorder and bichromatic lattice

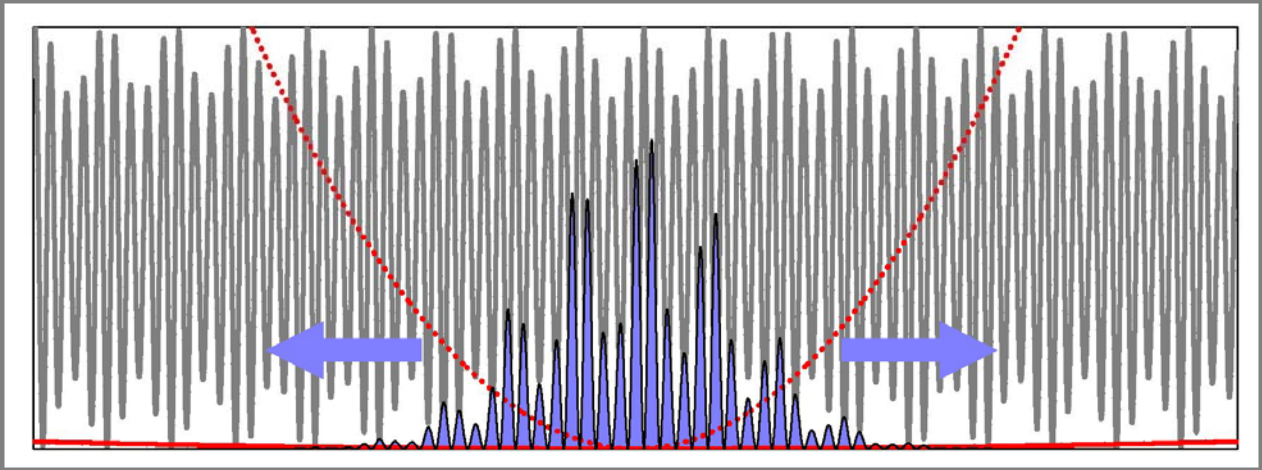
Study of the evolution in time of the width of the wavepacket

$$w(t) = \sqrt{m_2(t)} = \left\{ \sum_j (j - \langle j \rangle)^2 |\psi_j(t)|^2 \right\}^{1/2} \qquad w(t) \approx t^{\alpha}$$

Characterize expansion by exponent α :

$\alpha = 1$	ballistic expansion
$\alpha = 0.5$	diffusion
$\alpha < 0.5$	sub-diffusion
$\alpha = 0$	localization

Dynamics – Expansion in a lattice



1064.4 nm
859.6 nm

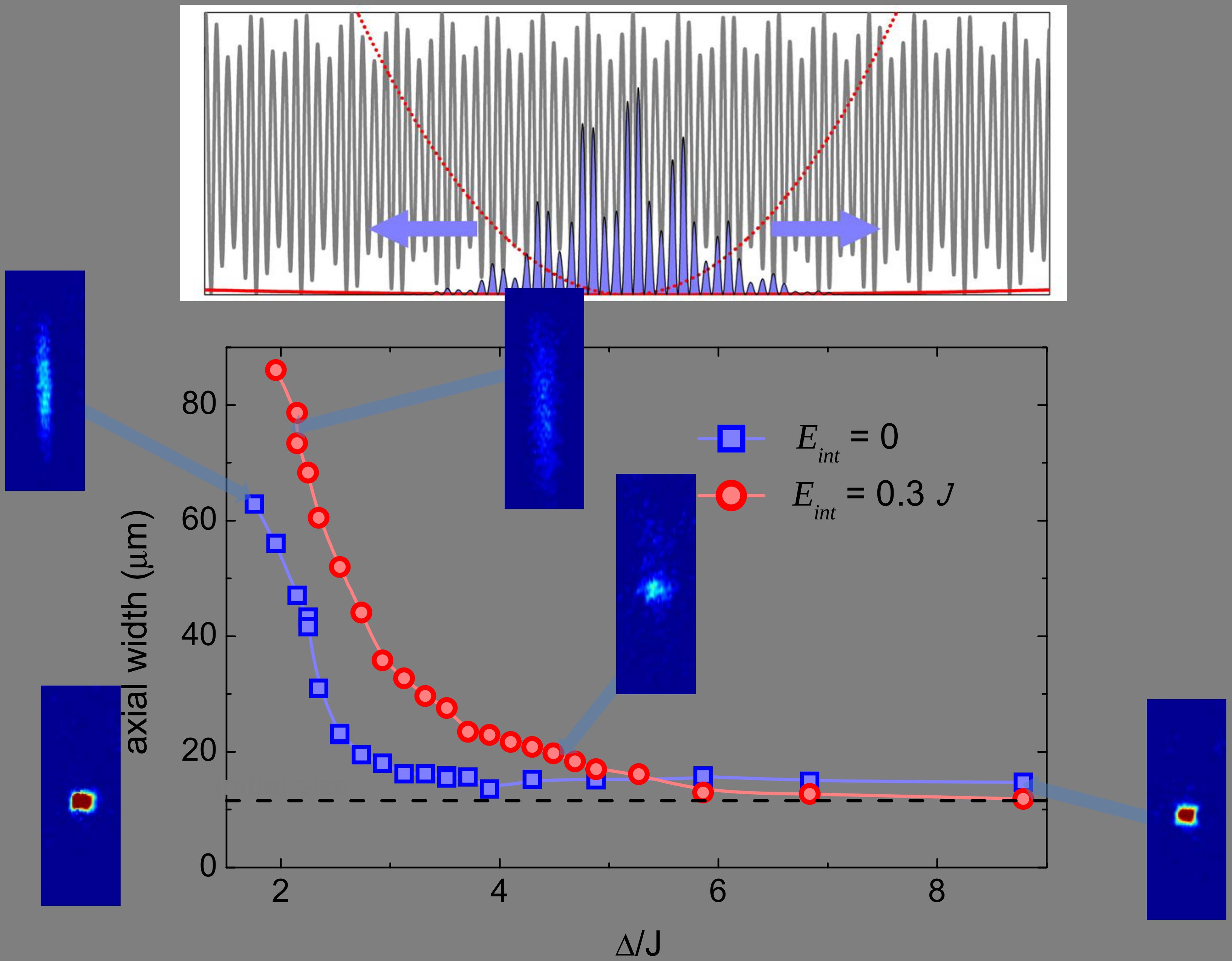
Prepare interacting system in optical trap + lattice,
then release from trap and change interactions simultaneously

Initial state always the same
Radial confinement ≈ 50 Hz

many theoretical predictions:
Shepelyansky: PRL **70**, 1787 (1993)
Shapiro: PRL **99**, 060602 (2007)
Pikovsky & Shepelyansky: PRL **100**, 094101 (2008)
Flach *et al.*: PRL **102**, 024101 (2009)
Larcher *et al.*: PRA **80**, 053606 (2009)
…

Transport properties

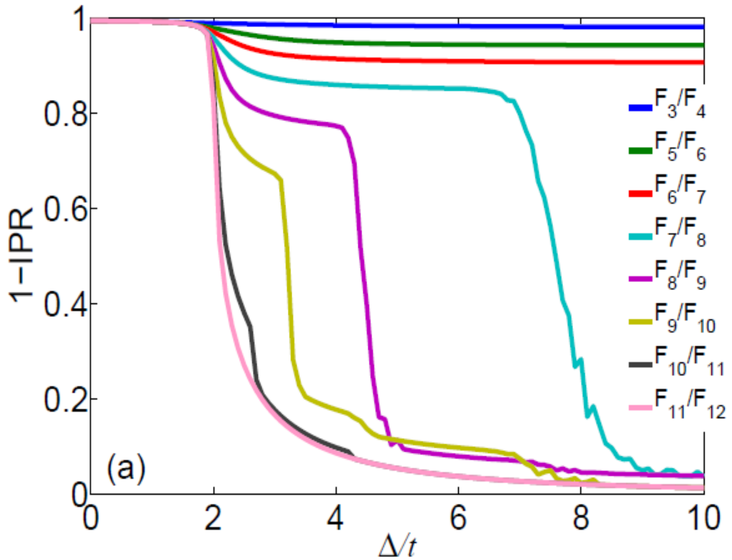
size after 10 seconds of expansion



Localization transition for β approximating an irrational number through the

$$F_n = 1, 1, 2, 3, 5, 8, \ldots$$

$$\lim_{n\to\infty} F_{n+1}/F_n = (1+\sqrt{5})/2$$



(courtesy of A. Minguzzi)

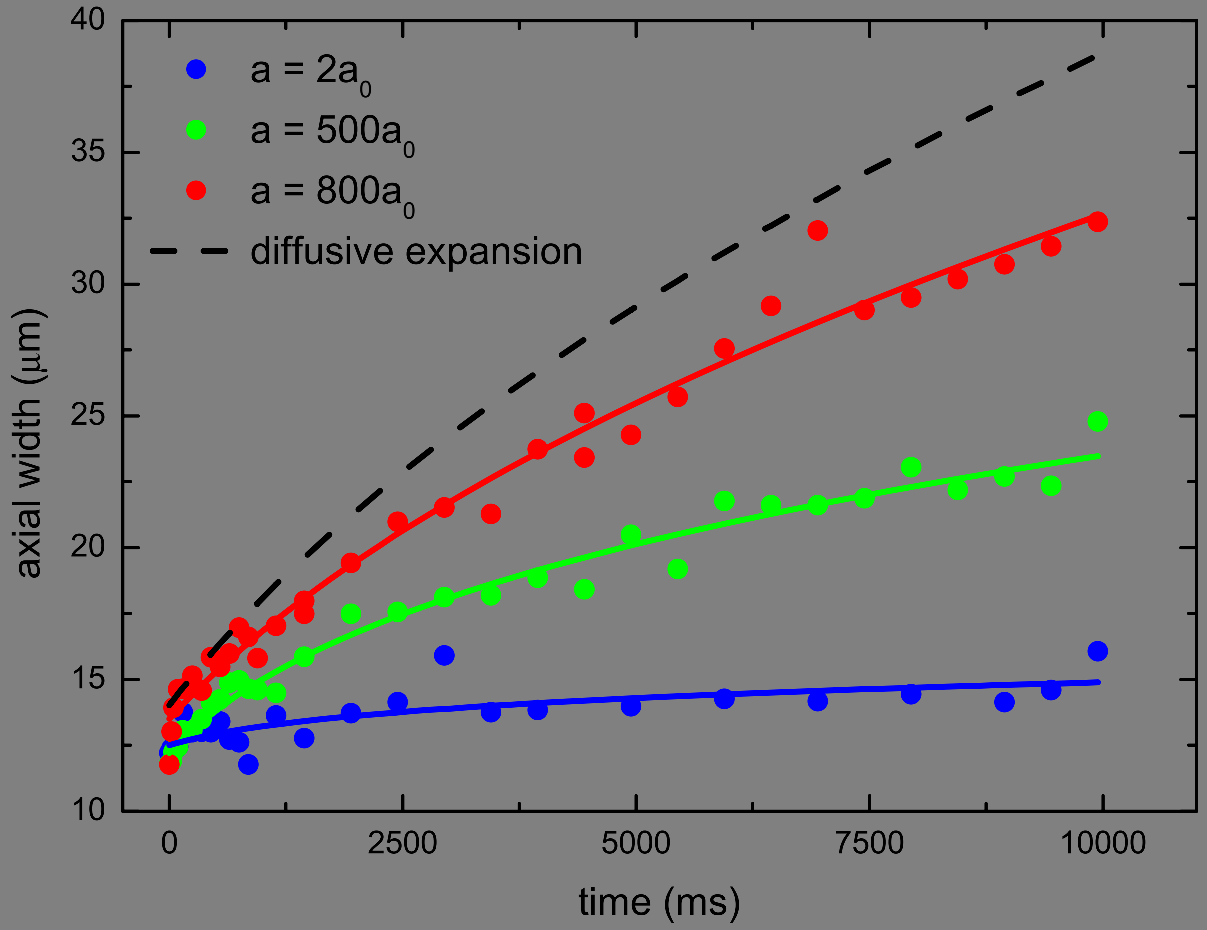
Expansion in a lattice

incoherent hopping between localized states

Characterize expansion by exponent α:

α = 1: ballistic expansion
α = 0.5: diffusion
α < 0.5: sub-diffusion

Size as a function of time

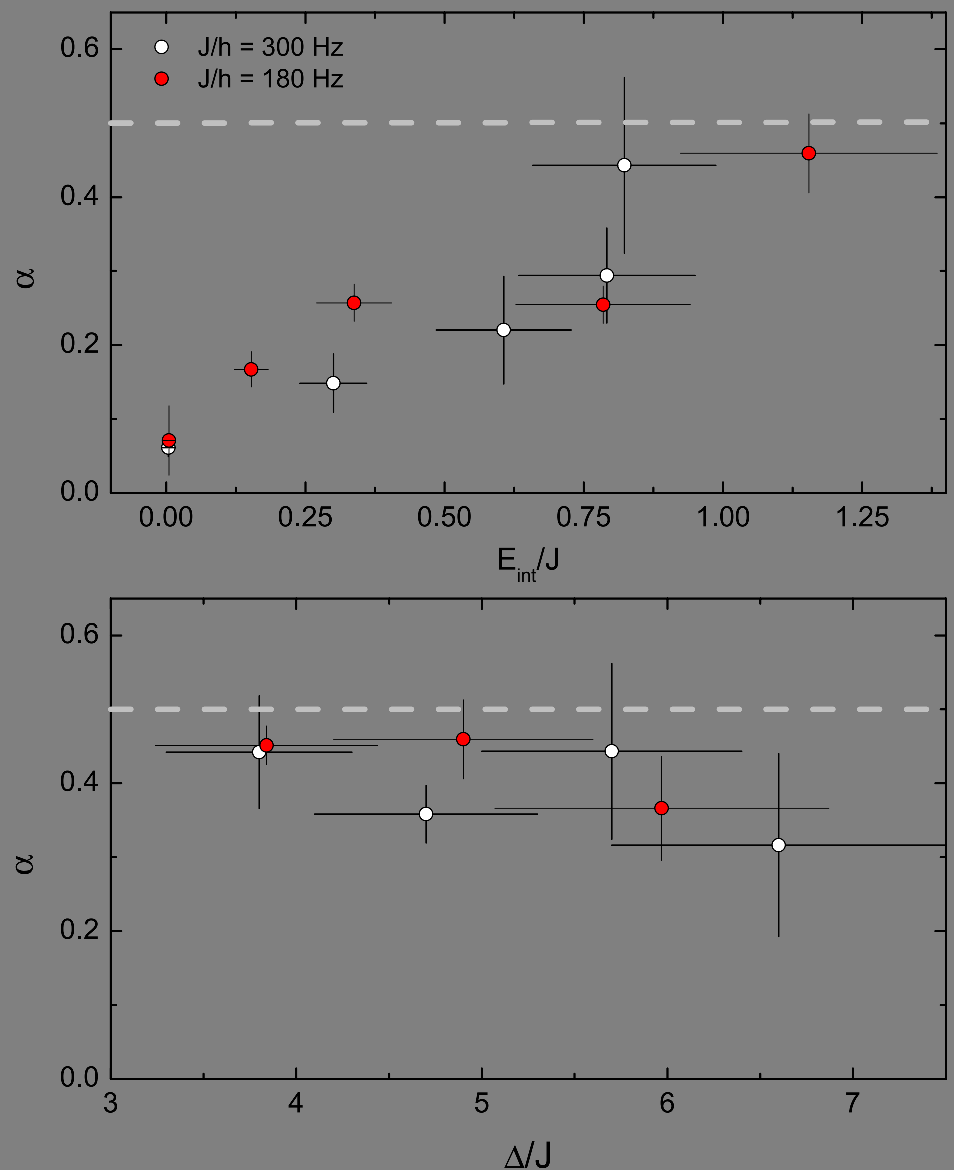


fit curves to

$$\sigma = \sigma_0 \left(1 + \frac{t}{t_0}\right)^{\alpha}$$

Expansion in a lattice

Dependence of α on interactions and on disorder



Expansion mechanisms:
resonances between states (interaction energy enables coupling of states within localization volume)

Observe:

- subdiffusive expansion ($\alpha < 0.5$)
- α increases with E_{int}/J, decreases with Δ/J
- α saturates at ≈ 0.4

larger than predictions for random potential, comparable to Larcher *et al*. for quasiperiodic → effect of correlated potential?

Eleonora Lucioni *et al*.,
in preparation

Transport properties

PRL 102, 024101 (2009) PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2009

Universal Spreading of Wave Packets in Disordered Nonlinear Systems

S. Flach, D. O. Krimer, and Ch. Skokos

Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Strasse 38, D-01187 Dresden, Germany

(Received 30 May 2008; published 14 January 2009)

In the absence of nonlinearity all eigenmodes of a chain with disorder are spatially localized (Anderson localization). The width of the eigenvalue spectrum and the average eigenvalue spacing inside the localization volume set two frequency scales. An initially localized wave packet spreads in the presence of nonlinearity. Nonlinearity introduces frequency shifts, which define three different evolution outcomes: (i) localization as a transient, with subsequent subdiffusion; (ii) the absence of the transient and immediate subdiffusion; (iii) self-trapping of a part of the packet and subdiffusion of the remainder. The subdiffusive spreading is due to a finite number of packet modes being resonant. This number does not change on aver-

PHYSICAL REVIEW A **80**, 053606 (2009)

Effects of interaction on the diffusion of atomic matter waves in one-dimensional quasiperiodic potentials

M. Larcher,[1] F. Dalfovo,[1] and M. Modugno[2]

[1]*CNR INFM-BEC and Dipartimento di Fisica, Università di Trento, 38050 Povo, Italy*

[2]*LENS and Dipartimento di Fisica, Università di Firenze, Via N. Carrara 1, 50019 Sesto Fiorentino, Italy*

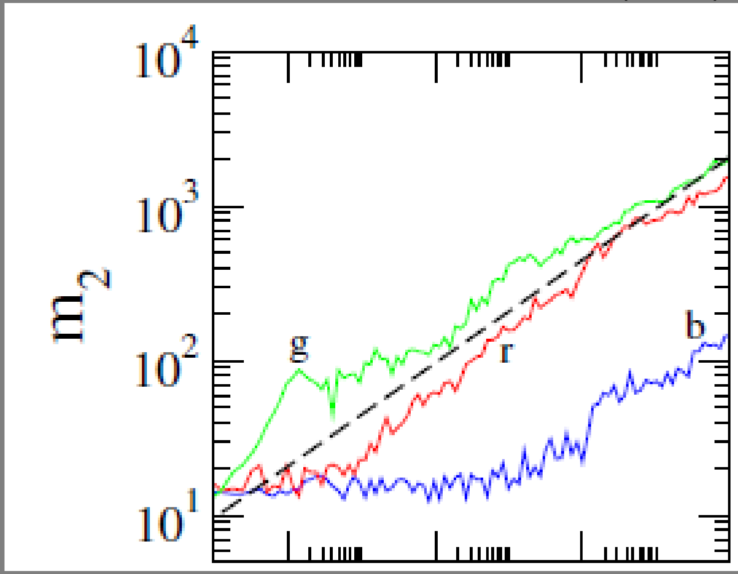
(Received 9 September 2009; published 9 November 2009)

We study the behavior of an ultracold atomic gas of bosons in a bichromatic lattice, where the weaker lattice is used as a source of disorder. We numerically solve a discretized mean-field equation, which generalizes the one-dimensional Aubry-Andrè model for particles in a quasiperiodic potential by including the interaction between atoms. We compare the results for commensurate and incommensurate lattices. We investigate the role of the initial shape of the wave packet as well as the interplay between two competing effects of the interaction,

Expansion: theory and simulation

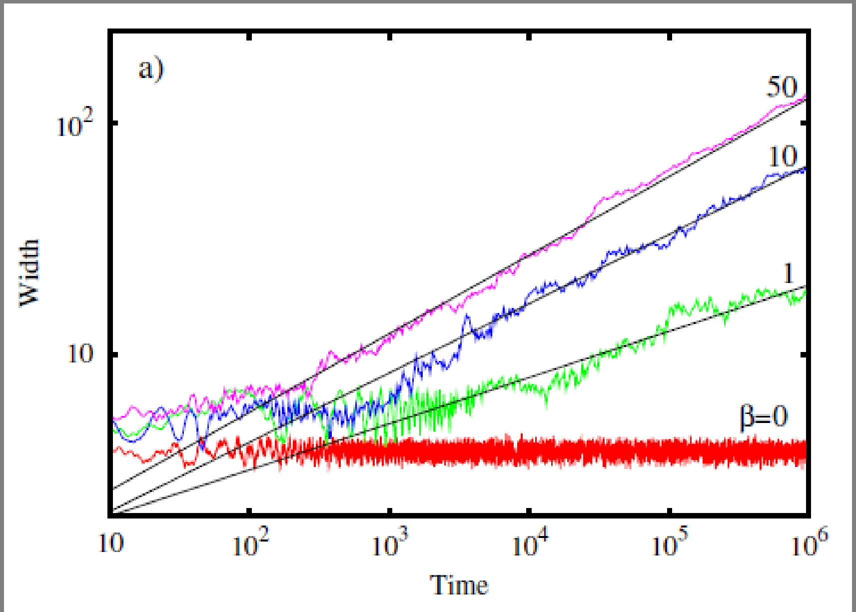
Random disorder

Flach *et al.*: PRL **102**, 024101 (2009)



Bichromatic lattice

Larcher *et al.*: PRA **80**, 053606 (2009)



Subdiffusion: α = 1/6
independent on the disorder
and on the interaction strenght

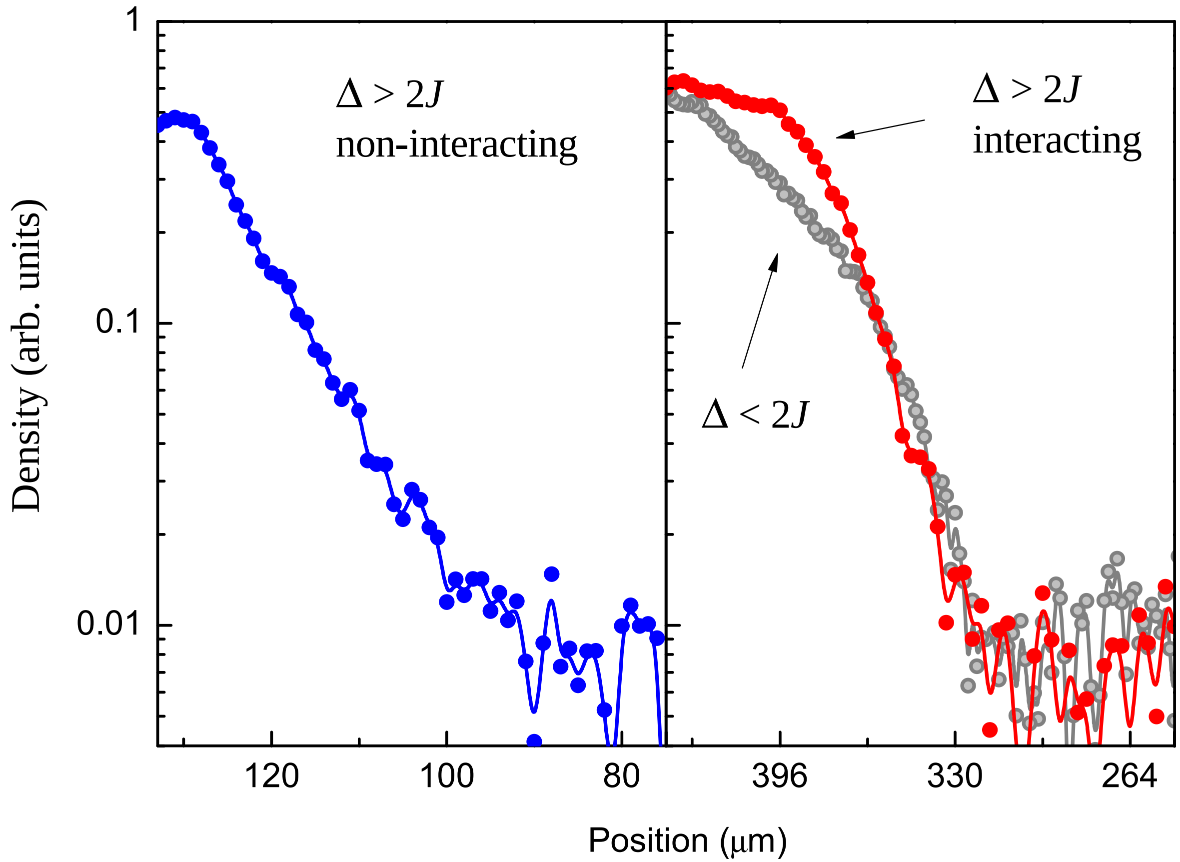
Subdiffusion: α <1/2
dependent on both disorder strength
and interaction strength

Many other theoretical works:

- Shepelyansky PRL **70**, 1787 (1993)
- Shapiro PRL **99**, 060602 (2007)
- Pikovsky & Shepelyansky PRL **100**, 094101 (2008)
- …

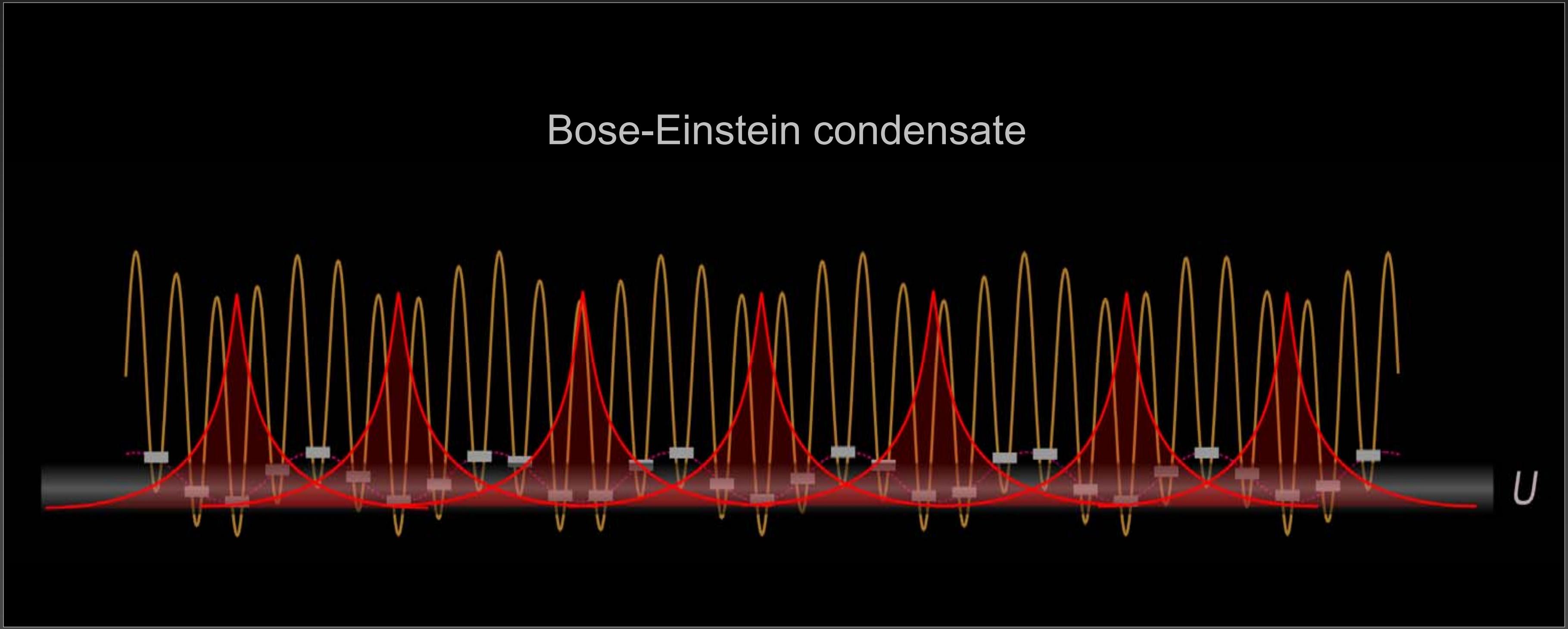
Transport properties

The interaction is larger at the center: flat-top profile

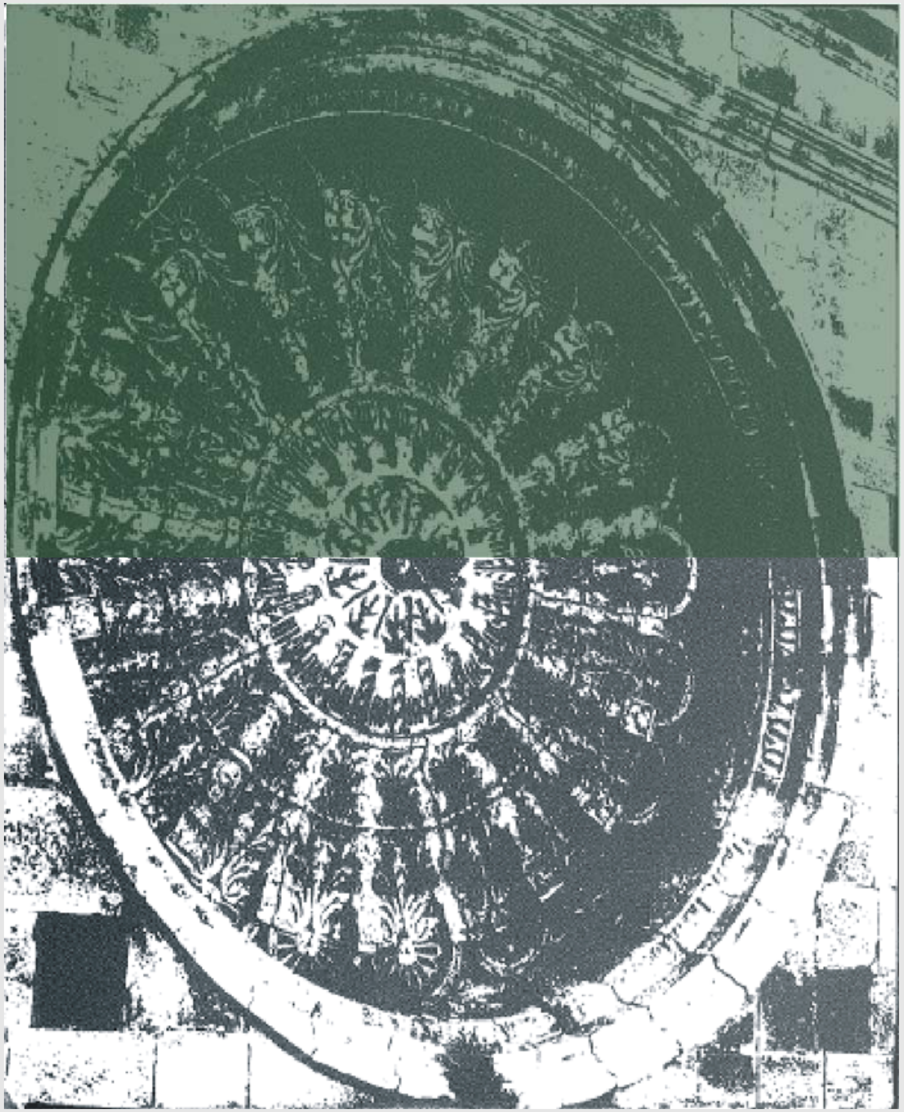


Interaction-induced delocalization

Delocalization transition for increasing interaction strength



BEAUTIFULNESS OF DISODER (controlled by hands) …



LECCE, Puglia
Santa Croce

for Martin's new landascapes

2D and 3D, new schemes for disorder time changing disorder, more fancy lattices, Fermions, Fermionized bosons, Mixtures, coupling with photonic xtals, polar molecules, new diagnostics…K, Li, Yb…