

The solution of the dirty bosons problem



Nikolay Prokof'ev (UMass, Amherst)



Boris Svistunov (UMass, Amherst)

Lode Pollet (Harvard, ETH)



Matthias Troyer (ETH)



Victor Gurarie (U. Colorado, Boulder)



Theorem of inclusions: Phase transitions in disordered systems;
 \forall system, \forall dimension

Superfluid -- Bose/Mott Glass -- Mott insulator transitions:

Absence of direct SF-MI transition

Harris criterion & vortex phase mechanism in $d=1,2$

$T=0$ phase diagram in $d=3$

Weakly interacting gas in a random speckle potential:

Critical temperature. Do we have any analytic theory?

Theorem of Inclusions

Mathematical background required

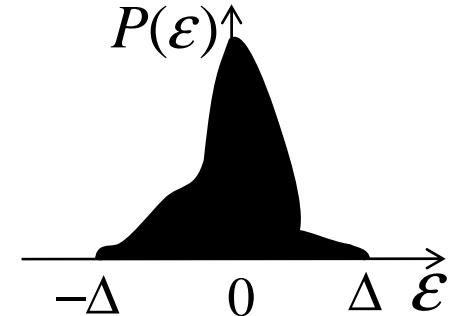
Lemma:

Product of finite number of non-zero numbers is non-zero

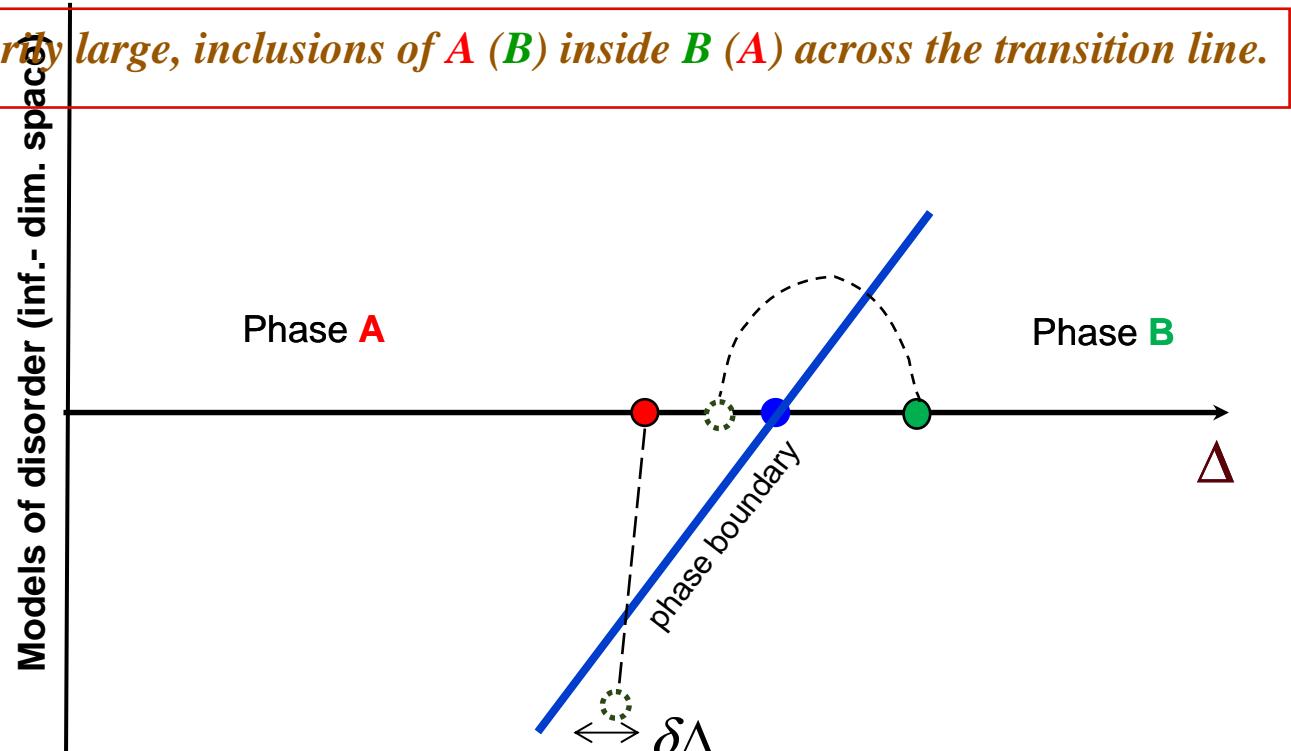
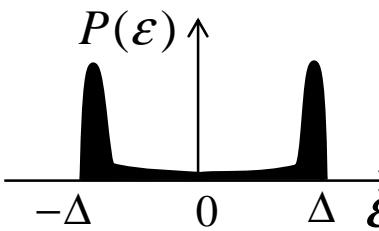
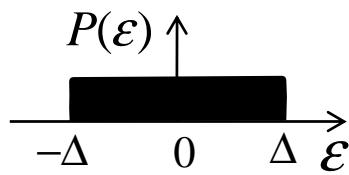
Theorem of Inclusions A

Def: Generic disorder = non-zero prob. density $\forall \varepsilon_i \in (-\Delta, \Delta)$

For an arbitrary transition in a system with generic disorder with transition point Δ_C depending on disorder properties

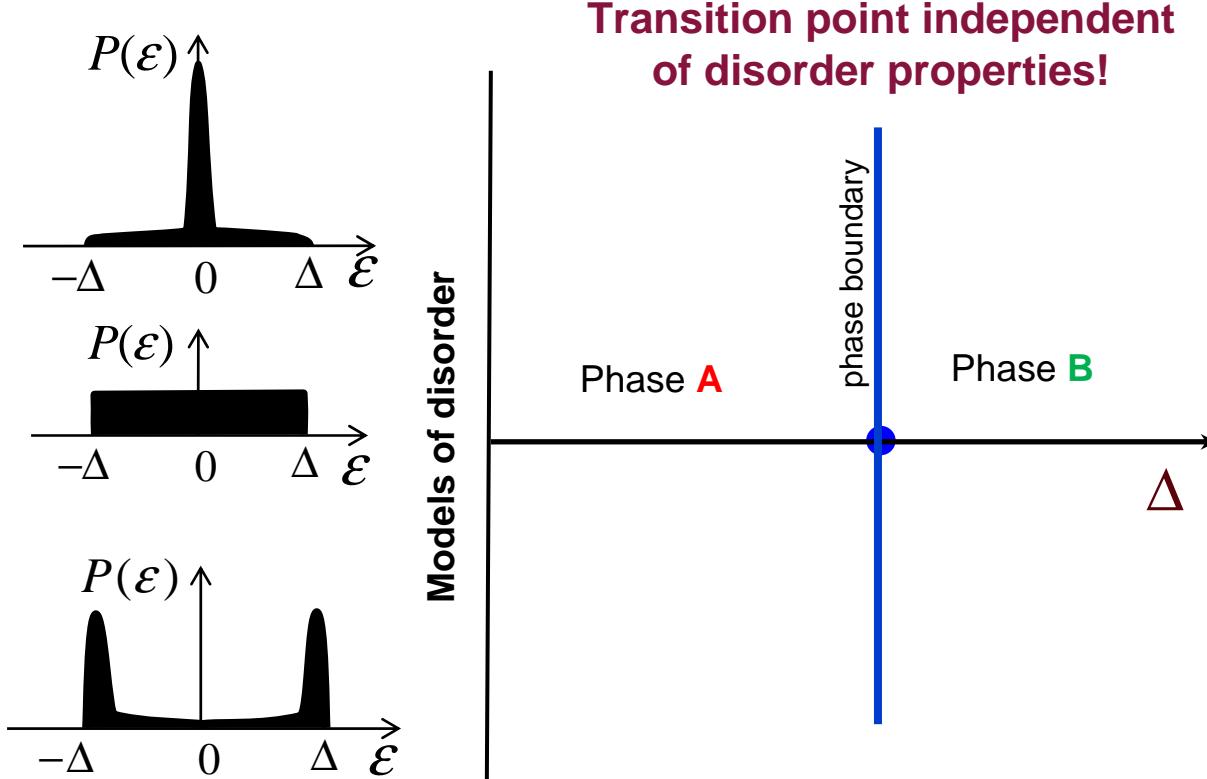


there exist rare, but arbitrarily large, inclusions of A (B) inside B (A) across the transition line.



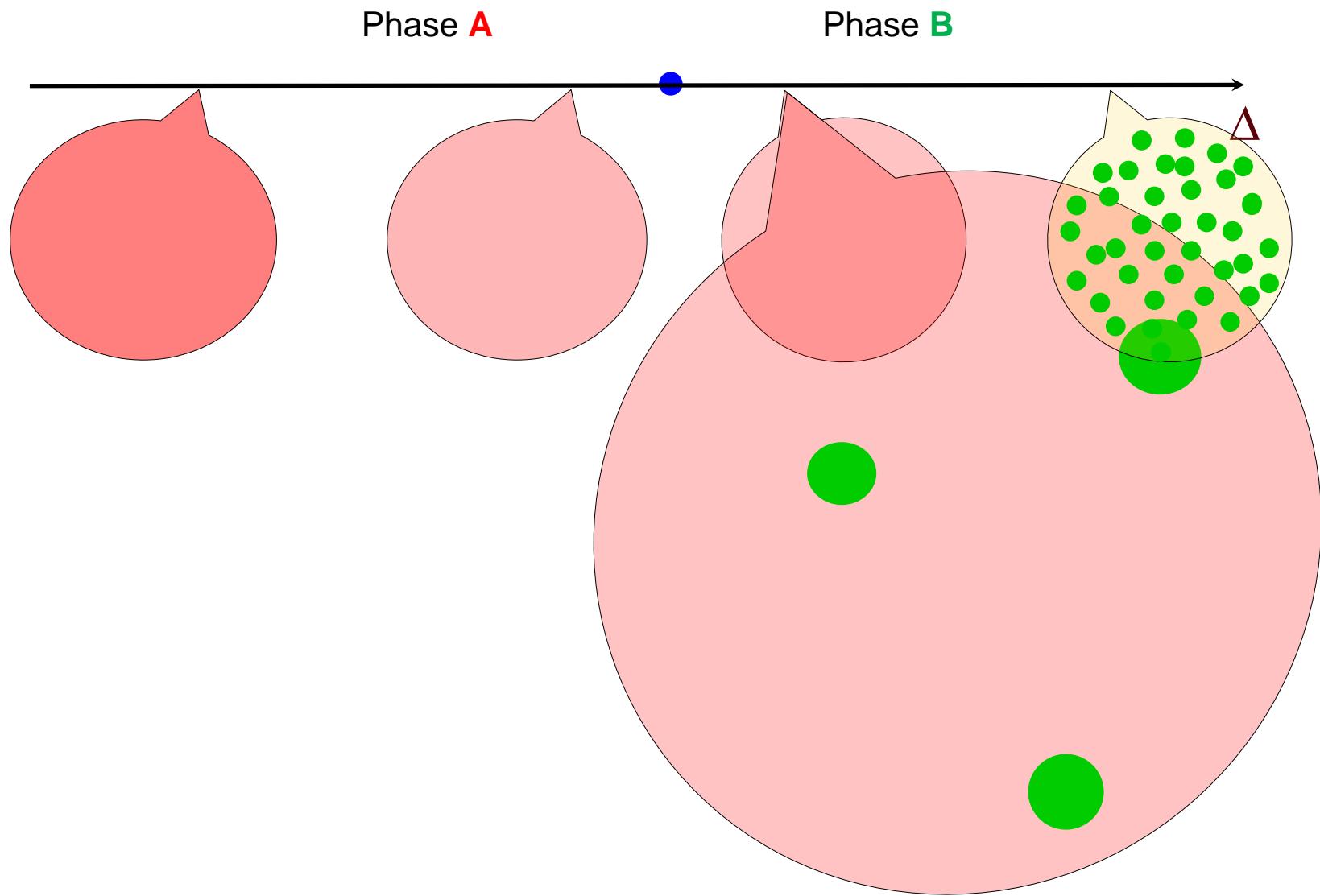
There exist rare, but arbitrarily large, inclusions of A (B) inside B (A) across the transition line.

Theorem of Inclusions B



Transition is driven by statistically rare fluctuations when locally disorder emulates a regular external field with amplitude Δ , e.g. $\epsilon_i = \Delta = E_{MI\,GAP} / 2$

= Griffiths type transition



Consequences:

- For generic lines: if A is **gapless** then B is **gapless** too and vice versa .
- All transitions between **gapfull** and **gapless** phases are of the **Griffiths type**.
- All phases next to the **gapfull** one are **insulating**
 - **SF-to-Mott insulator** transition is forbidden (any D).
 - an intermediate phase (BG) must separate the two
- For generic lines: if A is **superfluid** then B is
 - **compressible** (in the absence of particle-hole symmetry)
 - **gapless (possibly incompressible** in particle-hole symmetric case)

Bose Hubbard model with $\varepsilon_i \in (-\Delta, \Delta)$ at $\langle n_i \rangle = 1$ (or other integer)

$$H = -t \sum_{\langle ij \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i^2 - \sum_i (\mu - \varepsilon_i) n_i$$

Superfluid (SF)

$$zt > U$$

Mott insulator (MI)
gapped insulator

$$U \square zt$$

Bose glass (BG)
compressible insulator

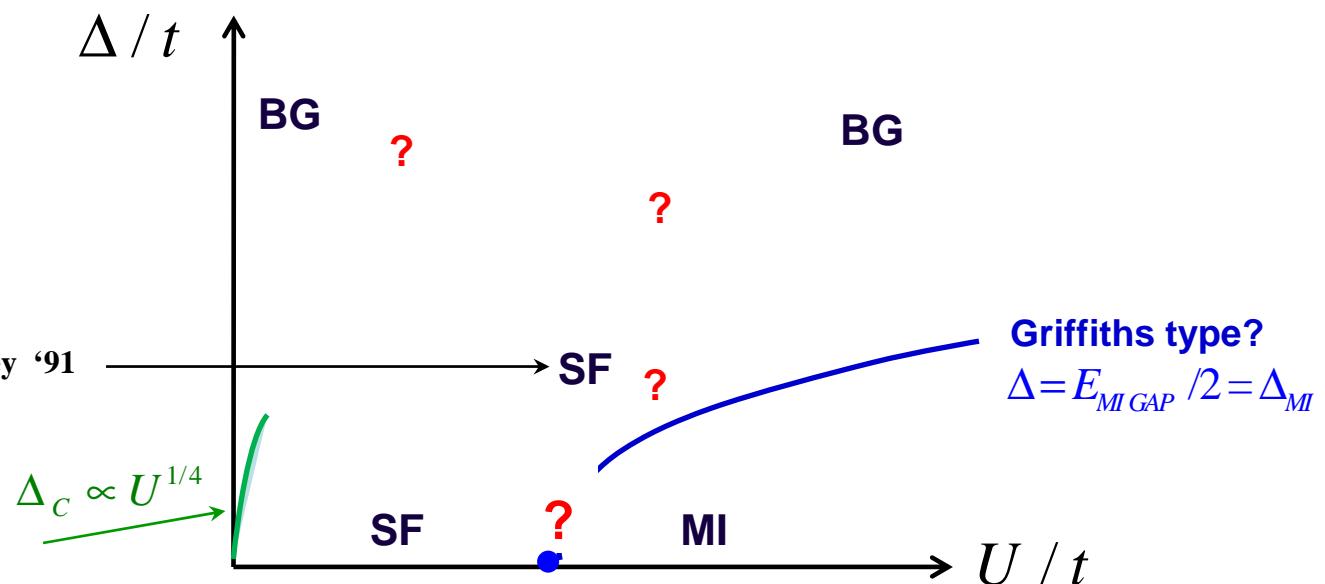
$$\Delta \neq 0$$

T. Giamarchi, H. Schulz, '87

M.P.A. Fisher, P.B. Weichman,
G. Grinstein, D.S. Fisher, '89

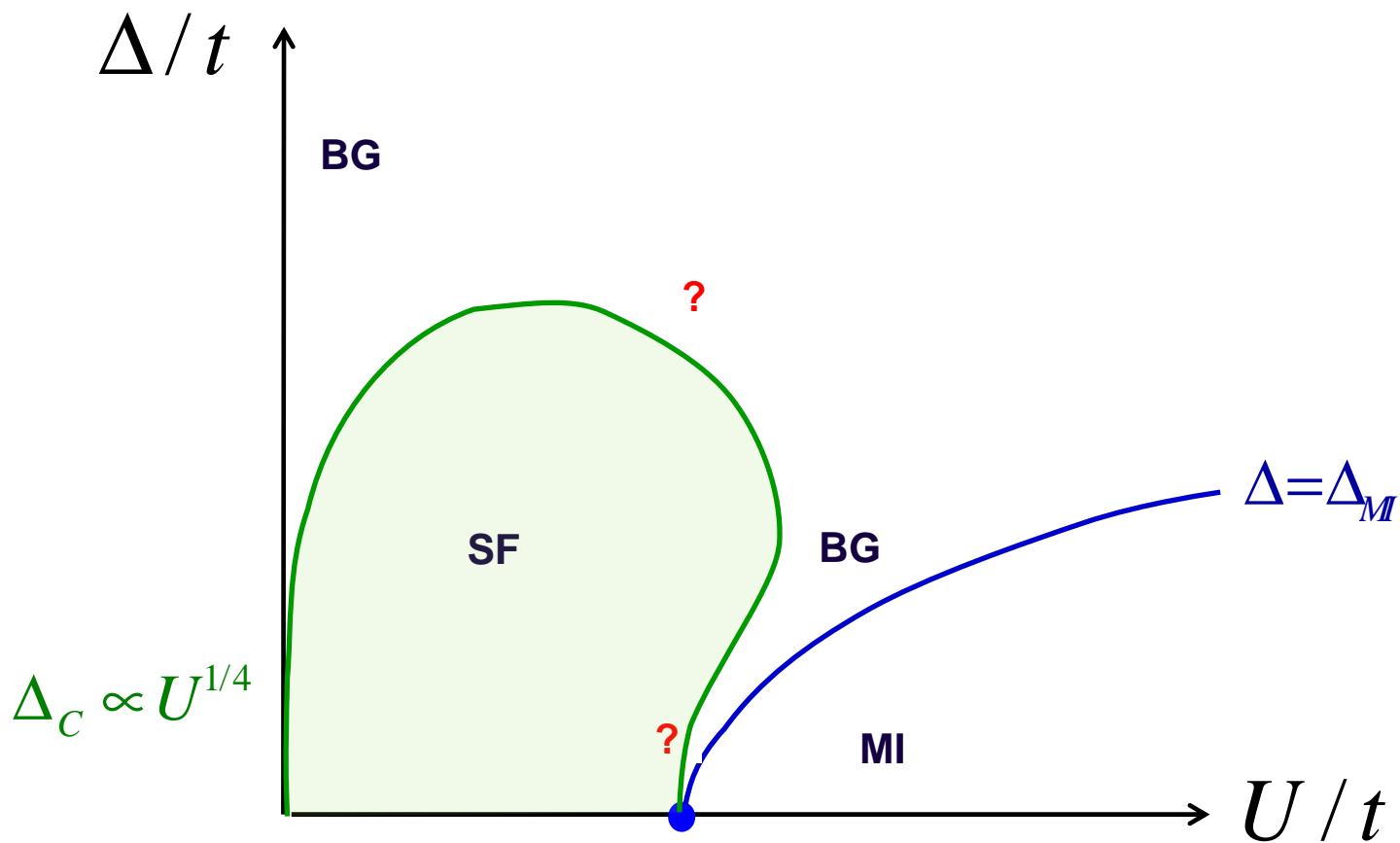
W. Krauth, N. Trivedi, D. Ceperley '91

G. Falco, T. Nattermann, V.
Pokrovsky '08



Bose Hubbard model with $\varepsilon_i \in (-\Delta, \Delta)$ at $\langle n_i \rangle = 1$ (or other integer)

$$H = -t \sum_{\langle ij \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i^2 - \sum_i (\mu - \varepsilon_i) n_i$$



Standard Harris criterion:

Random fluctuations in the critical region $\delta\mu \sim \Delta / \sqrt{\xi^d}$

smaller than the distance to the critical point $\delta\mu \ll (\mu - \mu_c) \propto \xi^{-1/\nu}$



$$\nu_{D=d+1} > 2/d$$

At the tip $\delta\mu \sim \Delta \xi^{-d/2} \rightarrow$ critical point fluctuation $\delta U \propto (\delta\mu)^{1/\nu} \propto \Delta^{1/\nu} \xi^{-d/2\nu}$

smaller than the distance to the critical point $\delta U \ll (U - U_c) \propto \xi^{-1/\nu}$



violated in $d=1,2$

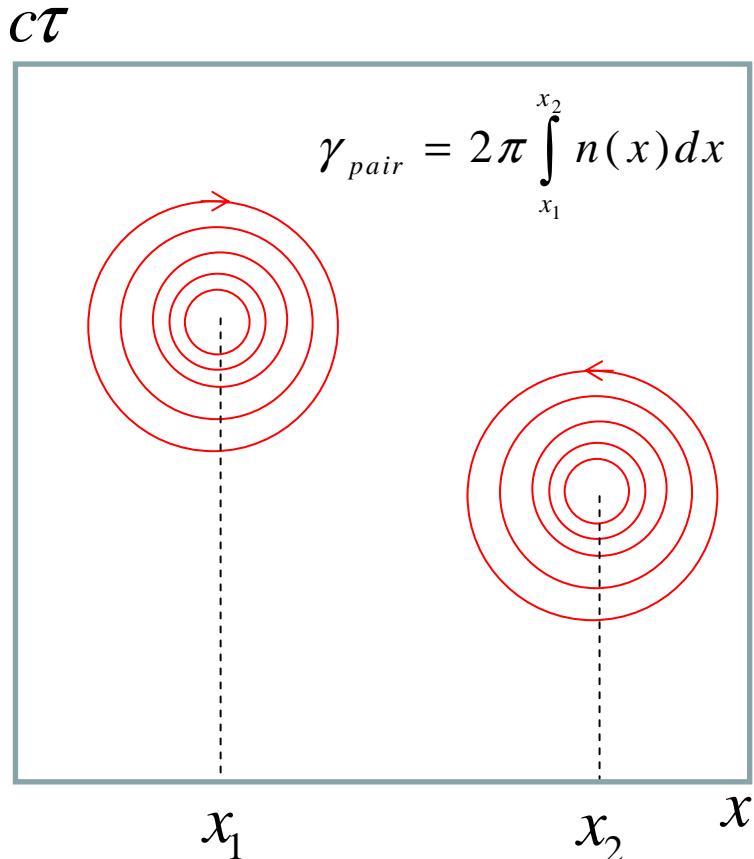
$$d > 2$$

Euclidean SF hydrodynamic action with disorder: $r = (r_{space}, c\tau)$

$$S = \int dr \left\{ in(x) \frac{\partial \varphi}{\partial y} + \frac{K}{2\pi} |\nabla \varphi|^2 \right\}$$

average density at space point x a multiple of 2π
 $\gamma = \text{Im } S = 2\pi \sum_i q_i \int_0^{x_i} n(x) dx$

2D XY-type



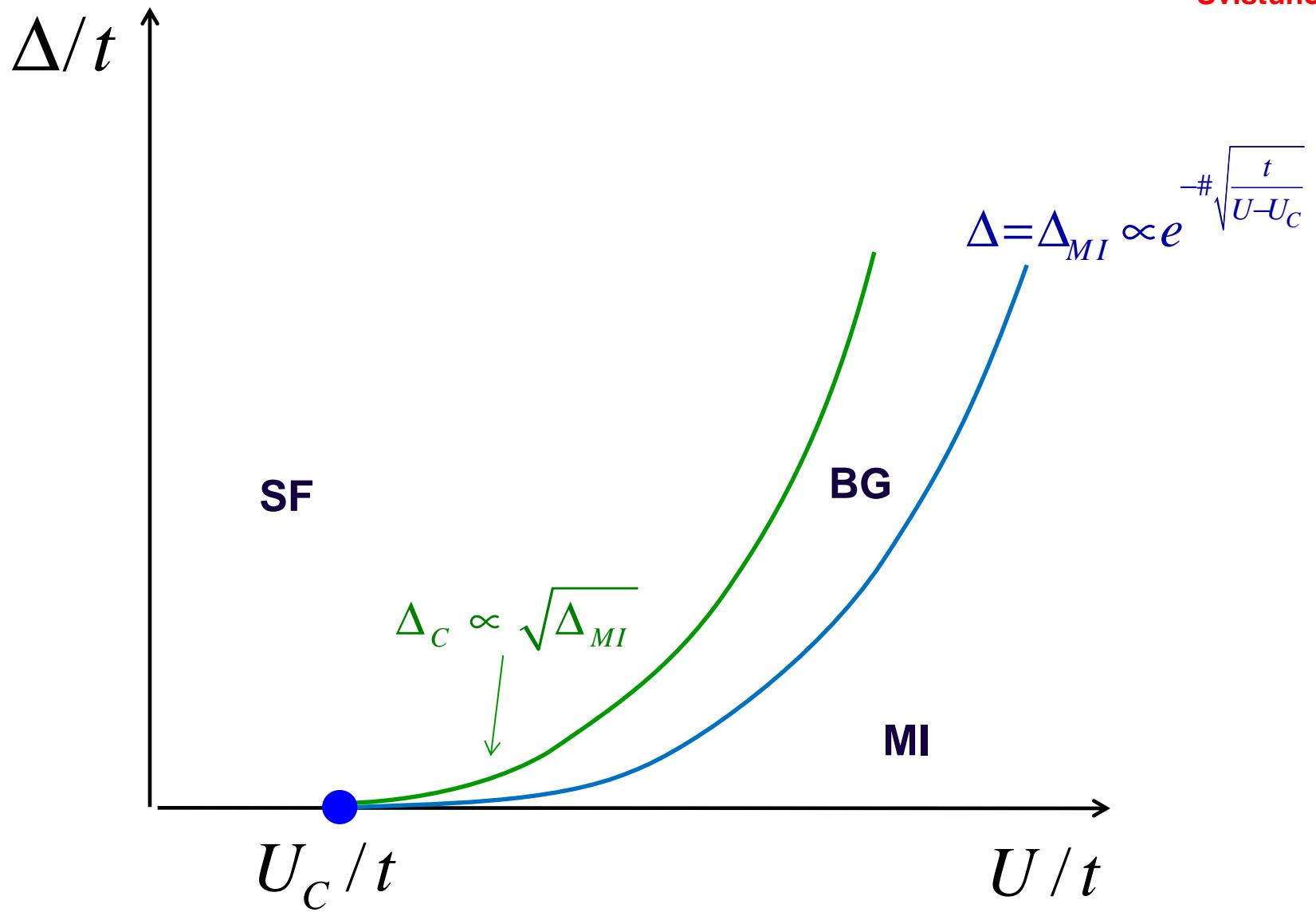
- ideal system with $n(x)=\text{integer} \rightarrow \text{mod}(\gamma, 2\pi) = 0 \rightarrow \text{standard KT (isotropic pairs)}$

- disordered system $\rightarrow \gamma_{pair R} = 2\pi \int_0^R \delta n(x) dx = 2\pi \kappa(R) \int_0^R \delta \mu(x) dx \propto \left(\frac{\Delta}{U} \right) R^{1/2}$

KT with vertical pairs

One-dimensional diagram tip

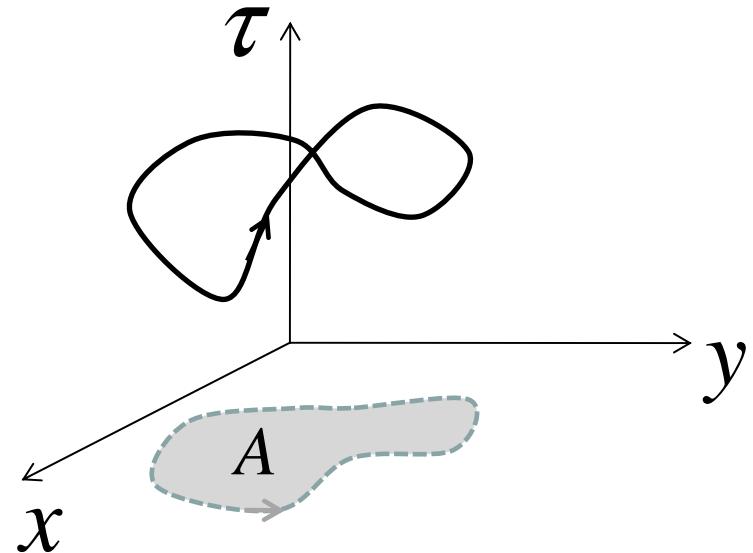
Svistunov'96



Two-dimensional diagram tip

$$\gamma = \text{Im } S = 2\pi \sum_i q_i \oint_{A_i} n(\rho) d^2\rho$$

projected vortex loop area

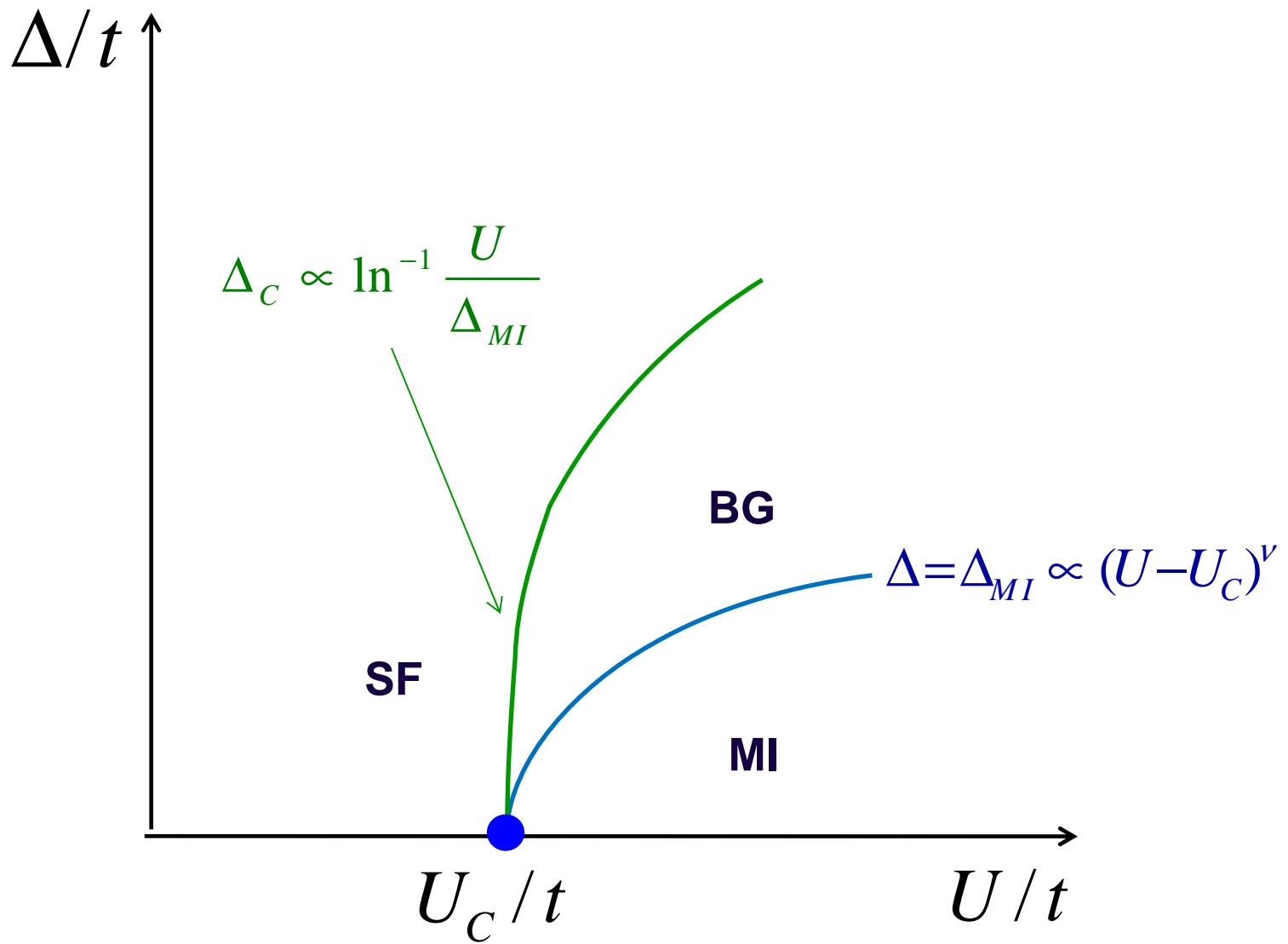


$$\gamma_{loop R} = 2\pi \iint_{A \sqsubset R^2} n(\rho) d^2\rho \approx 2\pi \kappa(R) \iint_{A \sqsubset R^2} \varepsilon(\rho) d^2\rho \propto \kappa(R) \times \Delta R$$

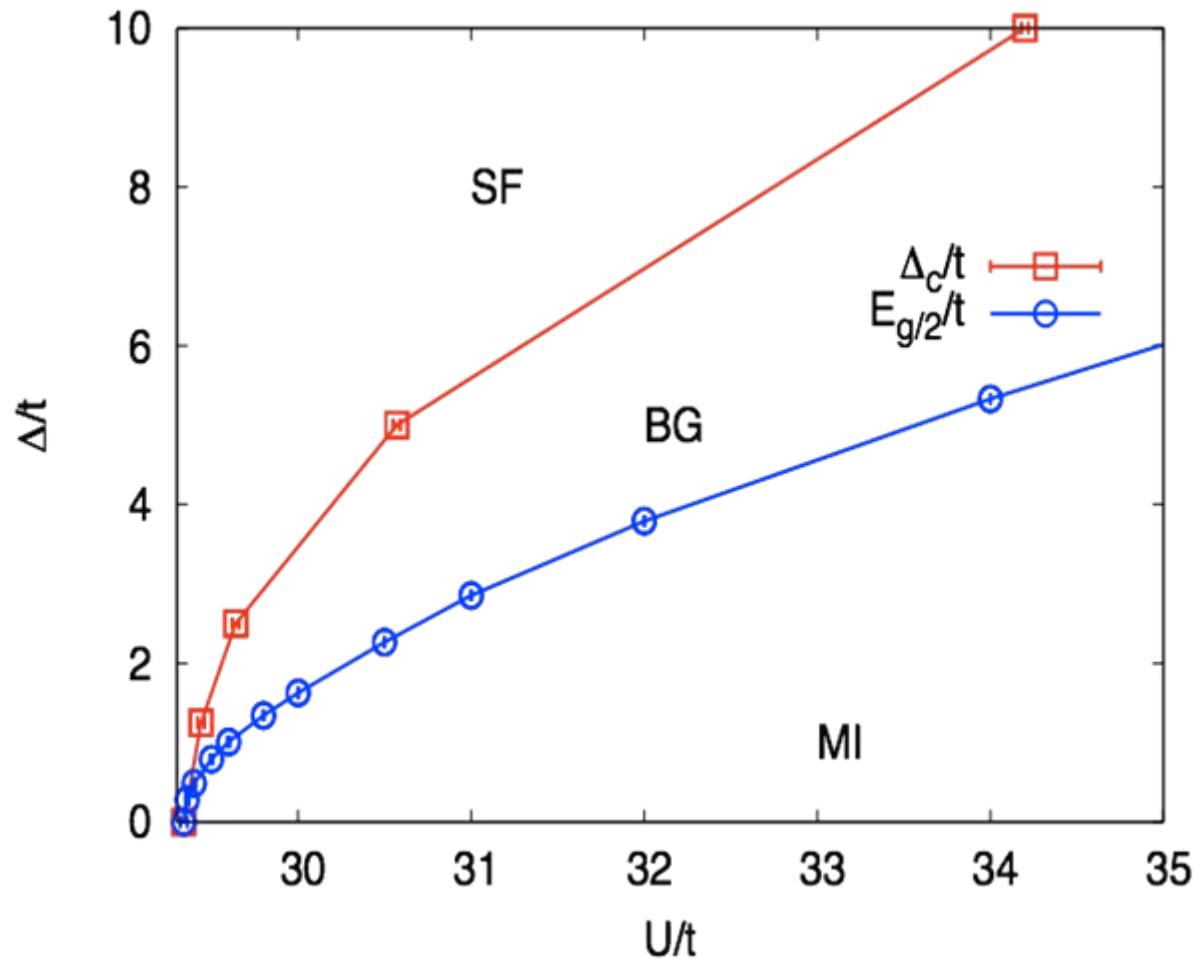
$U(1)$ -criticality in $d+1=3$ has $\kappa(R) \propto 1/R$ \rightarrow different scales contribute equally

$$\gamma_R \square \frac{\Delta}{U} \ln\left(\frac{R}{a}\right) \rightarrow \frac{\Delta}{U} \ln\left(\frac{\xi}{a}\right) \rightarrow \frac{\Delta}{U} \ln\left(\frac{1}{U - U_c}\right)$$

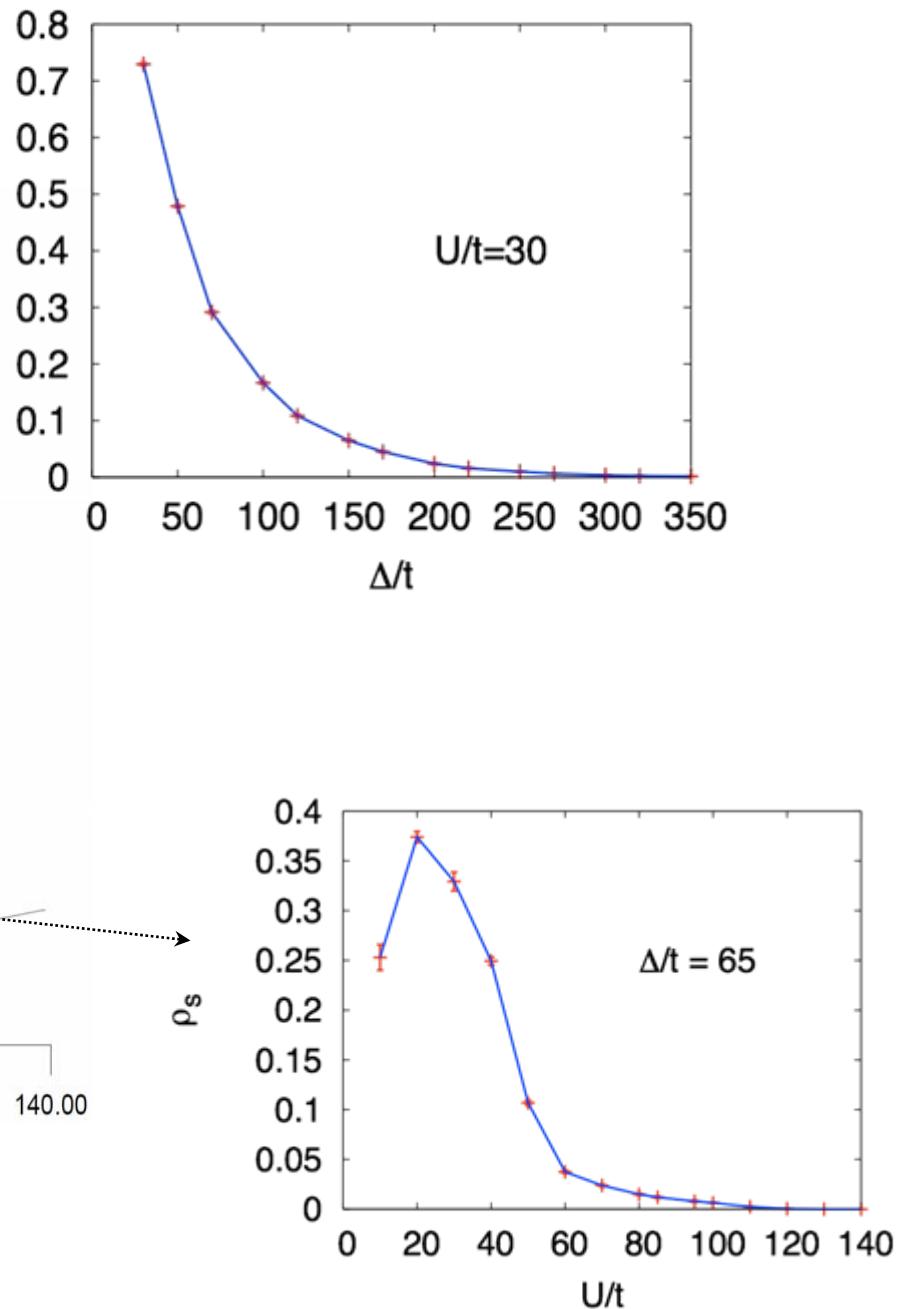
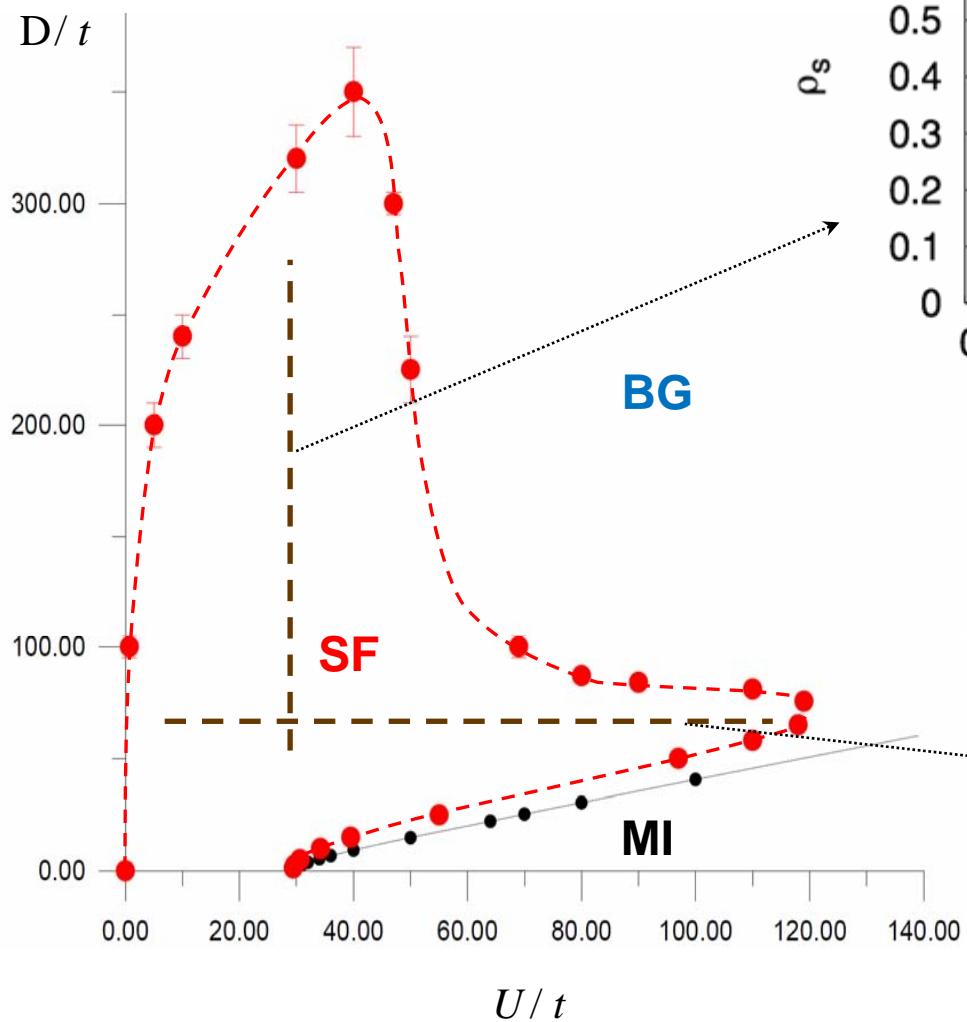
Two-dimensional diagram tip



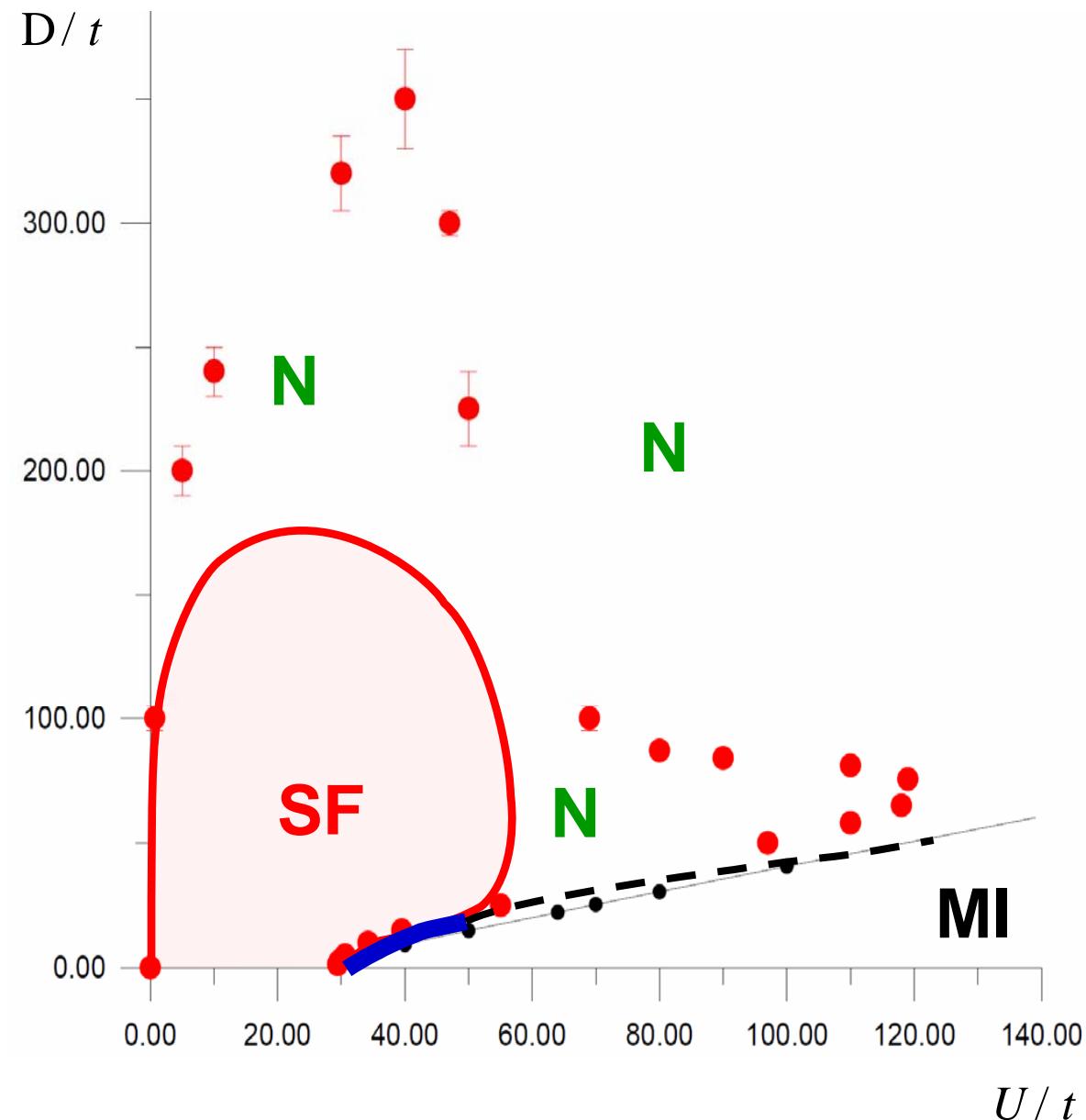
3D phase diagram tip (Worm Algorithm Monte Carlo)



Robust SF region, but ... fragile superfluidity !

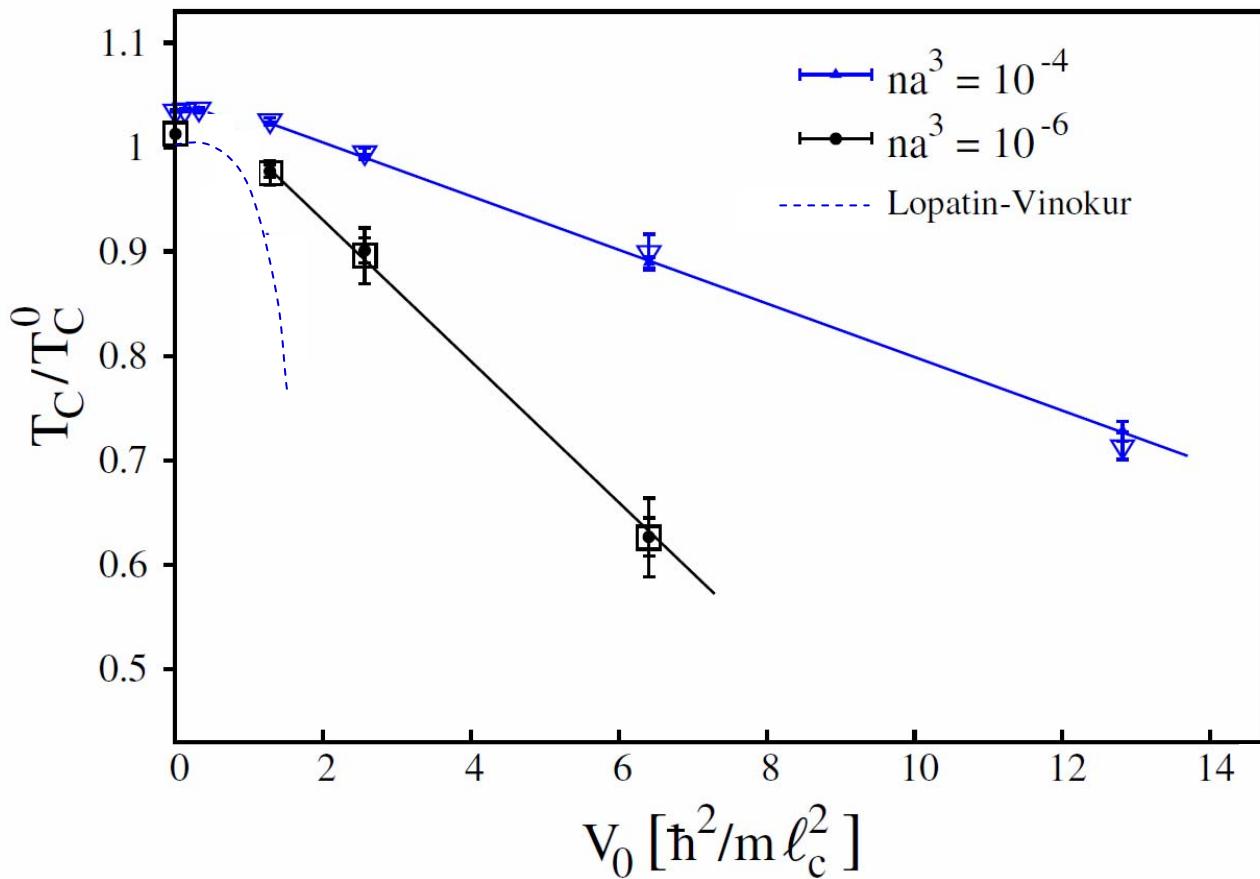


Reality of finite traps
 with $N \sim 10^7$ particles
 at temperature $T \sim t$



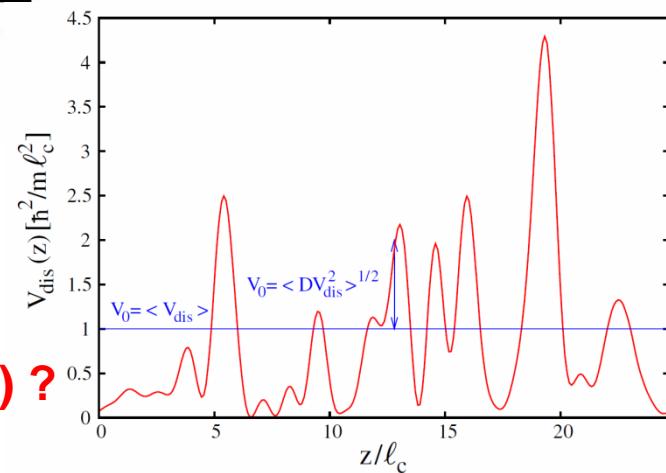
Critical temperature dependence on speckle disorder

S. Pilati, S. Giorgini, NP '09



$$n' (4\pi l_c^3 / 3) @ 1$$

i.e. $a_s = l_c$



Theoretical understanding of $T_c(D)$ (Bose or Fermi) ?

Conclusions:

1. Theorem of inclusions & its consequences apply to \forall system in \forall dim.
2. SF-MI transition is forbidden but will be observed for weak disorder at the SF-BG boundary
3. Proper understanding of the SF-BG boundary in $d=3$ and critical temperature dependence on disorder in the WIBG is still lacking (though known).