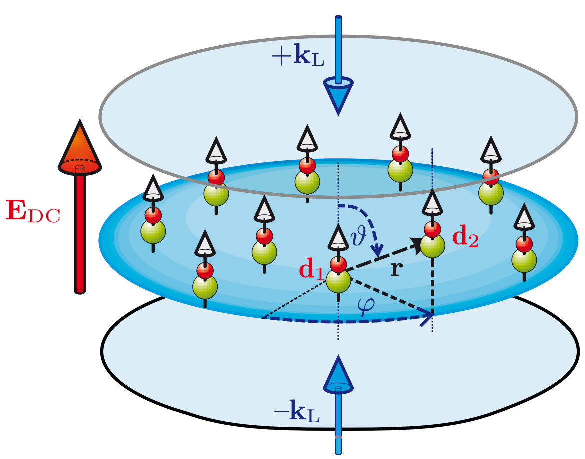
Announced Title:

Cold Atoms and Molecules:
Optical Lattices
→ Self-assembled Lattices

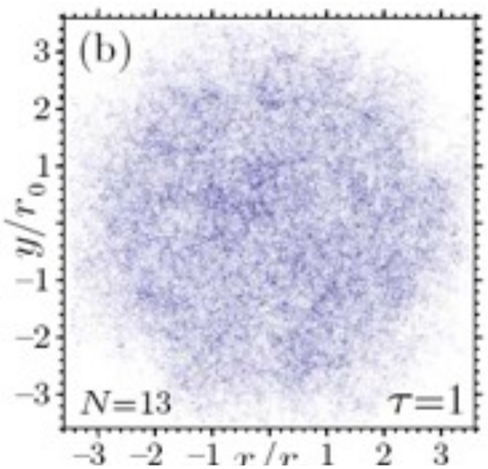
Guido Pupillo (Innsbruck)

Friday Morning Session

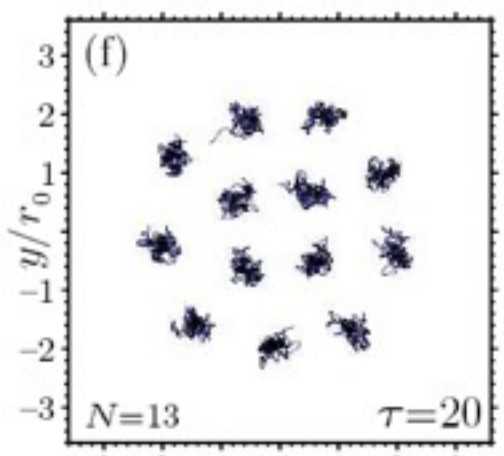
- **Polar Molecules & “Rydberg dressed” atoms**
  - dipolar quantum gases in 2D & “shaped” interaction potentials



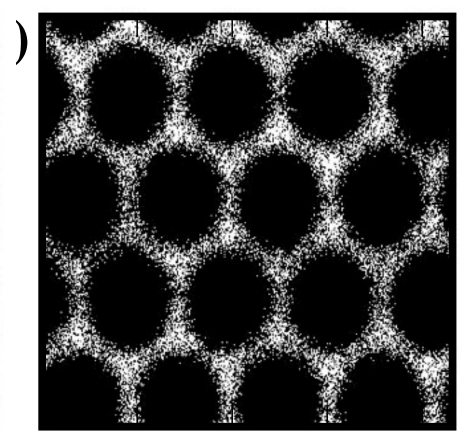
Superfluid



Crystals



Supersolid Droplets



Open Many Body Quantum Systems: Atoms in Optical Lattices

Peter Zoller

UNIVERSITY OF INNSBRUCK

IQOQI
AUSTRIAN ACADEMY OF SCIENCES

Innsbruck:
A. Daley
S. Diehl
A. Micheli
M. Müller (PhD)
A. Kantian (PhD→ Geneva)
I. Lesanovsky (→ Nottingham)
B. Kraus

collaborations:
H.P. Büchler (Stuttgart)

OPTICAL LATTICE EMULATOR OLE

NAMEQUAM
Nanodesigning of atomic and molecular quantum matter

FWF Der Wissenschaftsfonds.

Outline

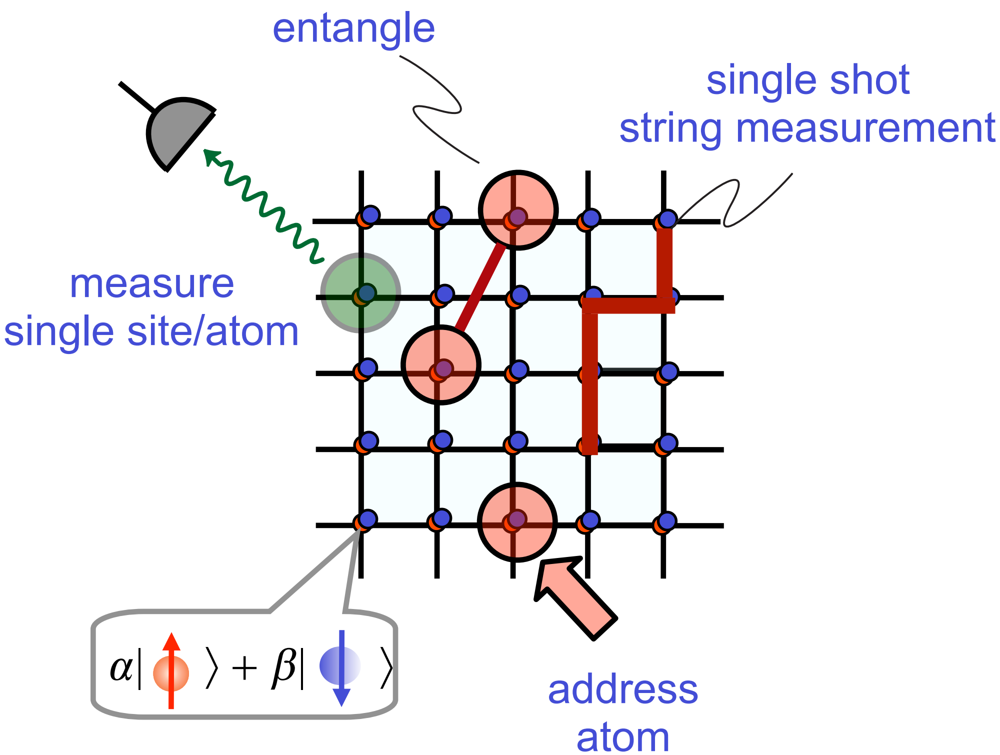
- **Open System Dynamics → Desired Many Body State**
  - d-wave pairing by dissipation

- **Rydberg Quantum Simulator for Open System Dynamics**
  - Example: Kitaev toric code & stabilizer pumping
  - [an ion trap experiment: “single plaquette”]

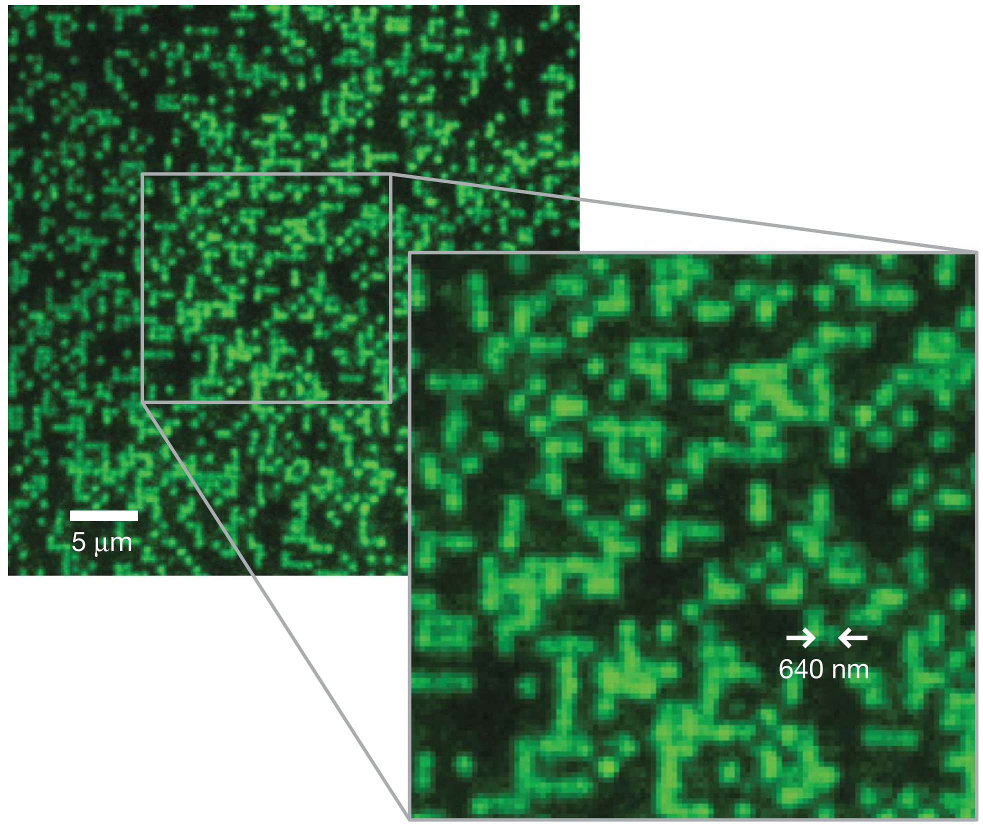
- **Decoherence / heating: spontaneous emission in opt. lattices**

Single Site Imaging (& Addressing)

- **Addressable lattices**



- **Imaging Hubbard models:**
  Harvard, Munich, Chicago, Penn State, ...



Site-resolved imaging of single atoms on a 640-nm-period optical lattice, loaded with a high density BEC

WS Bakr, JI Gillen, A Peng, S Fölling, M Greiner - Nature, 2009

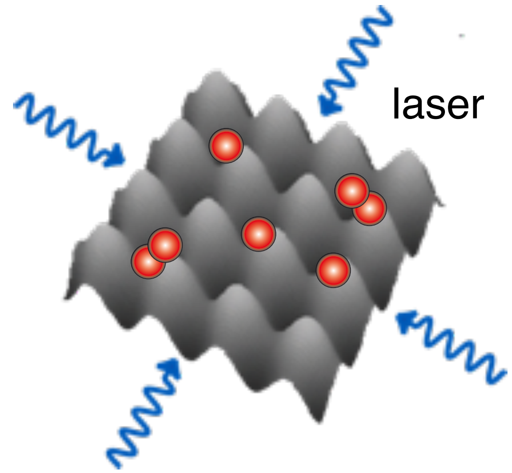
Preparing “interesting” (pure) many body states … & AMO

- **condensed matter physics**

$$\rho \sim e^{-H/k_B T} \xrightarrow{T\to 0} |E_g\rangle \langle E_g|$$

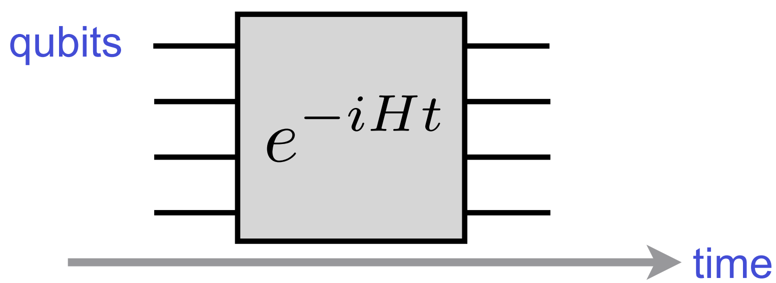
cooling to ground state
strongly correlated states
equilibrium cond mat

- **cold atoms in optical lattices**



quantum simulation
engineering Hamiltonians

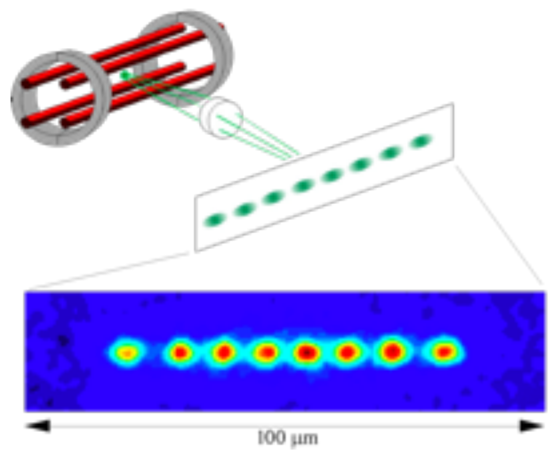
- **quantum computing & resources**



$$|\psi\rangle \to U_t |\psi\rangle$$

coherent Hamiltonian evolution
- quantum gates

- **trapped ions**



general purpose
quantum computing

Preparing "interesting" (pure) many body states … & AMO

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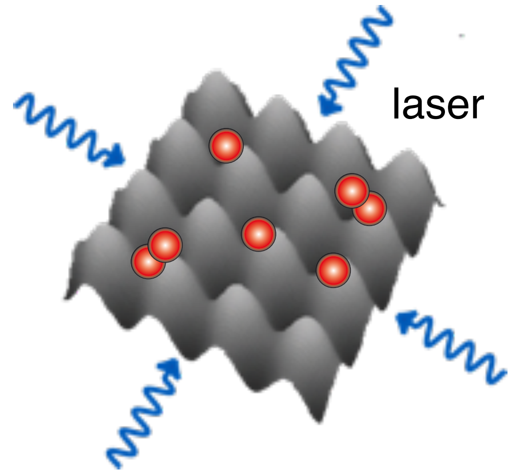

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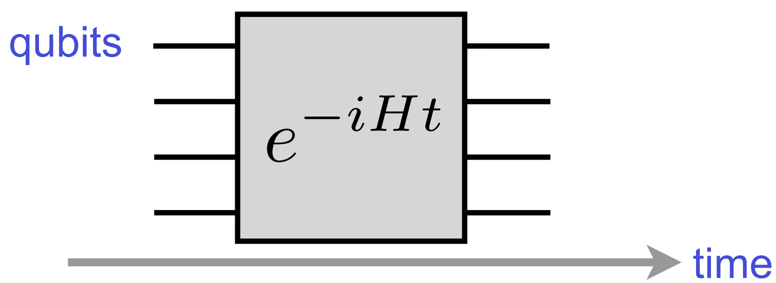

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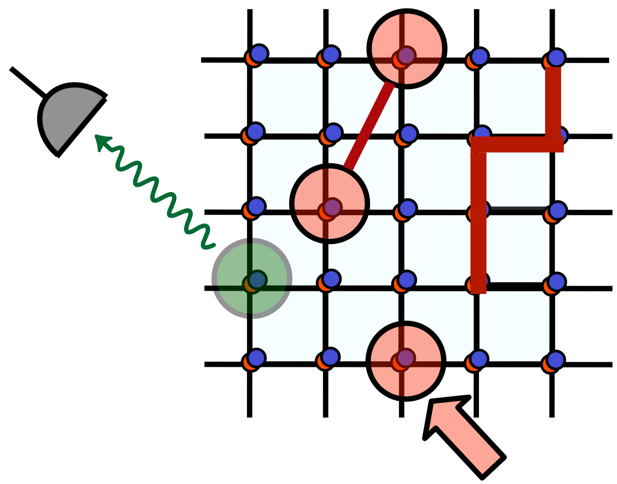
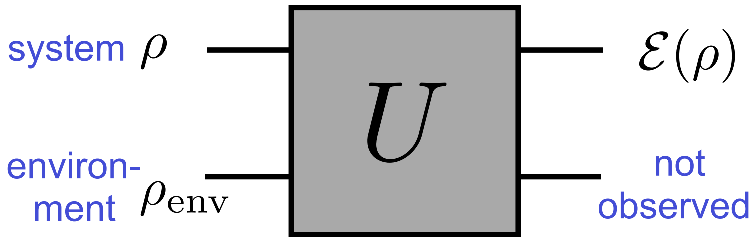


addressable optical lattices

Here ...

- **open quantum system**
  - quantum operations

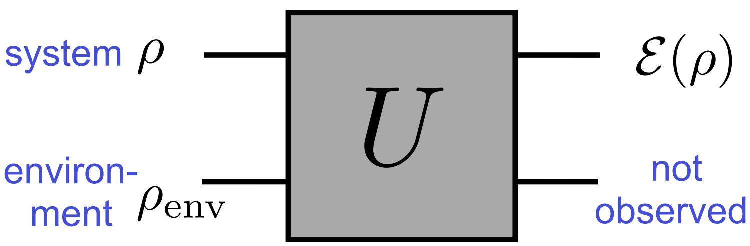


$$\rho \to \mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

entanglement with environment

decoherence ☹

- **open quantum system**
  - quantum operations



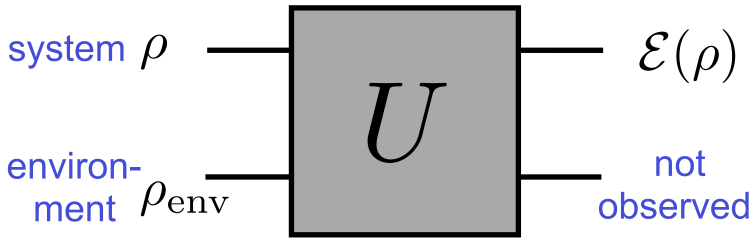
$$\rho \rightarrow \mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

!?

$$= |\psi\rangle \langle\psi|$$

Q.: prepare given pure many body state by open system dynamics?

- **open quantum system**
  - quantum operations



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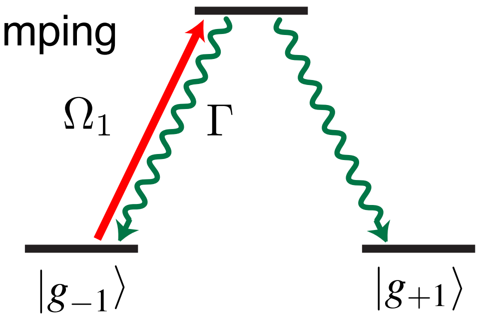
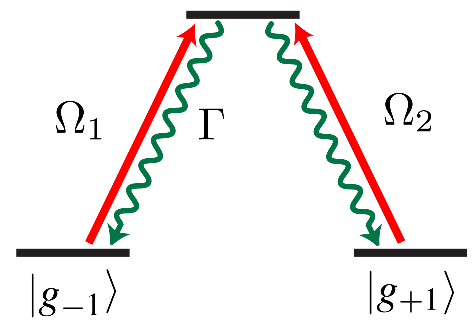
Q.: prepare given pure many body state by open system dynamics?

- **quantum optical systems as *driven open* quantum systems**

optical pumping



Λ-system: EIT, STIRAP, …, VSCPT

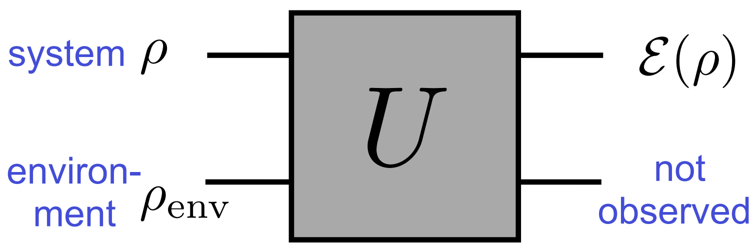


$$\rho(t) \xrightarrow{t\to\infty} |D\rangle\langle D|$$

steady state as pure "dark state"

here: … on a single particle level

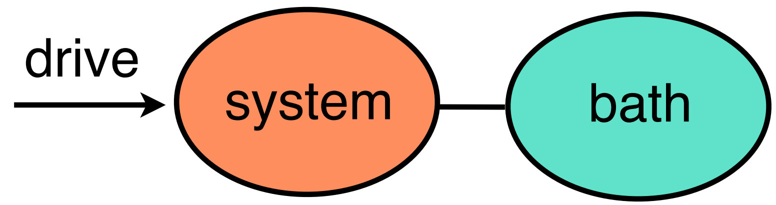
- **open quantum system**
  - quantum operations



$$\rho \rightarrow \mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

$$\overset{!?}{=} |\psi\rangle \langle\psi|$$

- **quantum optical systems as *driven open* quantum systems**



$$\frac{d\rho}{dt} = -i\,[H,\rho] + \mathcal{L}\rho$$ master equation

competing dynamics

$$\rho(t) \xrightarrow{t\to\infty} \rho_{ss}$$ mixed state

$$\overset{!?}{=} \quad |D\rangle \langle D|$$ pure state ("dark state")

steady state

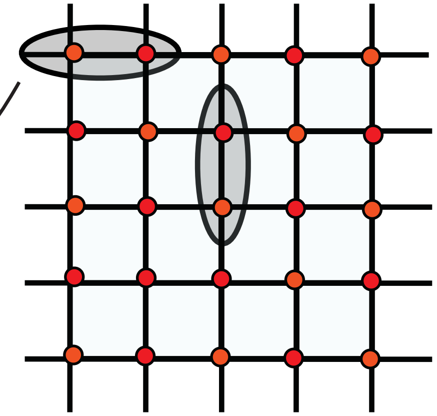
"non-equilibrium cooling"

Q.: prepare given pure many body state by open system dynamics?

Q.: engineer quantum reservoirs & couplings in many body system?

… Many Body "Dark States"

qubits or particles on a lattice



quasi-local Lindblad operators

- master equation

$$\frac{d\rho}{dt} = -i\,[H,\rho] + \mathcal{L}\rho$$

Liouville

$$\mathcal{L}\rho \equiv \sum_{\alpha=1}^{N_c} \kappa_\alpha \left(2c_\alpha \rho c_\alpha^\dagger - c_\alpha^\dagger c_\alpha \rho - \rho c_\alpha^\dagger c_\alpha\right)$$

Lindblad operators

- Engineering $\{H, \mathcal{L}\}$ so that a given $|D\rangle$ is a dark state:

constructing "parent Liouvillians"

$$\text{(i)} \quad \forall\alpha \quad c_\alpha |D\rangle = 0$$
$$\text{(ii)} \quad H|D\rangle = E|D\rangle$$

uniqueness

engineer quantum reservoirs

$$\rho(t) \xrightarrow{t\to\infty} |D\rangle\langle D|$$

B. Kraus et al., PRA 2008

Q.: Identifying and engineering Lindblad operators, new physics … ?

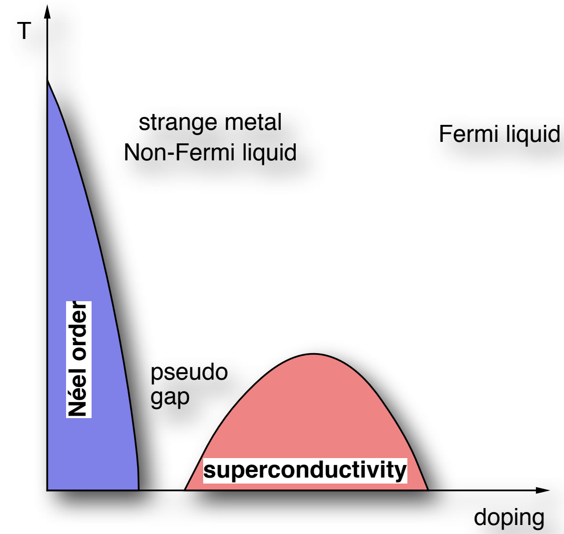
- **Example 1:** d-wave pairing by dissipation

S. Diehl, W. Yi, A. J. Daley, PZ, PRL 2010 (in print)

Motivation: Repulsive 2D fermionic Hubbard model

Hubbard Hamiltonian

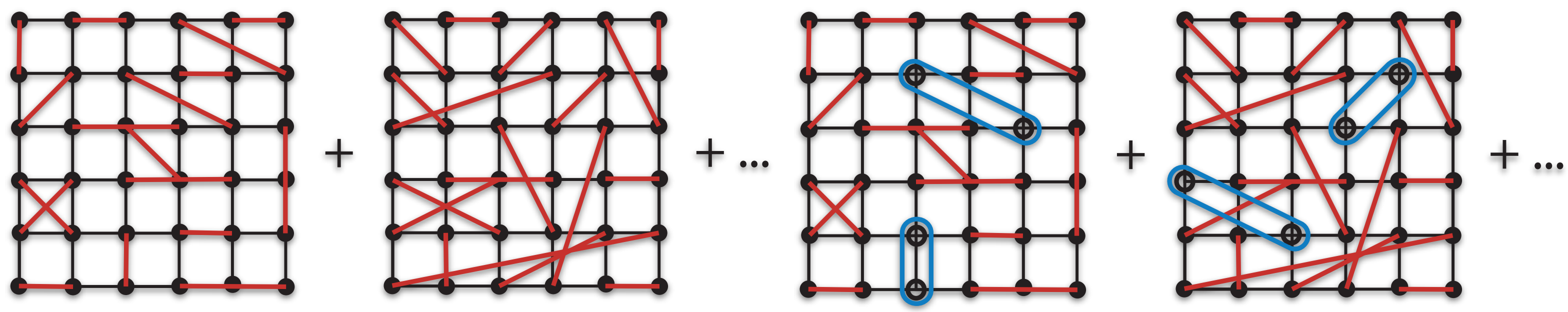
$$H = -t \sum_{i,j,\sigma} (c^{+}_{i\sigma} c_{j\sigma} + c^{+}_{j\sigma} c_{i\sigma}) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$



doped resonant valence bond (RVB) state (?)

half-filling (parent compounds): superposition of singlet coverings

hole-doping: hole pairs condense (BCS)



mean field description: Gutzwiller projected BCS-state with d-wave Cooper pairs

here: pairing due to interactions. Can we induce pairing by coupling to an environment? Can we “cool” to paired states?

S. Diehl
A. Daley
Wei Yi

Example 1: d-wave pairing by dissipation

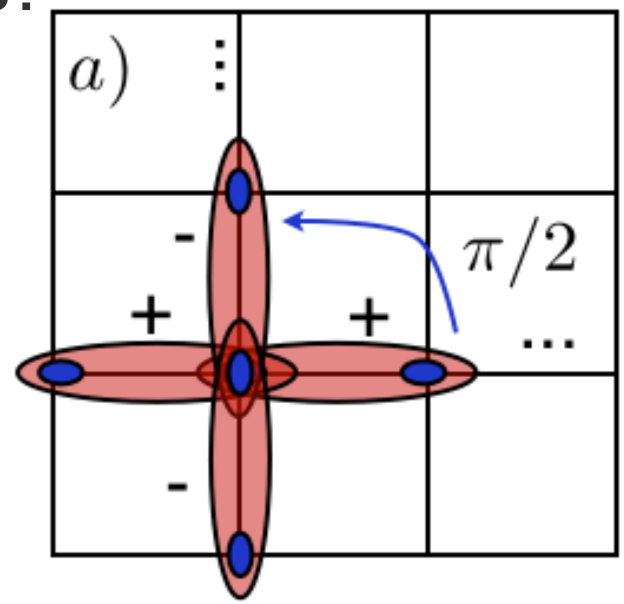
- **What is the *parent Liouvillian* for d-wave BCS?**

- **BCS-type state:** number conserving

$$|\mathrm{BCS}_N\rangle \sim (d^\dagger)^{N/2}|\mathrm{vac}\rangle$$

$$d^\dagger = \sum_{\mathbf{q}} \varphi_{\mathbf{q}} c^\dagger_{\mathbf{q},\uparrow} c^\dagger_{-\mathbf{q},\downarrow}$$

$$= \sum_{i,j} \varphi_{ij} c^\dagger_{i,\uparrow} c^\dagger_{j,\downarrow}$$ offsite pairing



d-wave pair

- **"dissipative d-wave pairs"**

$$\varphi_{q_x,q_y} = -\varphi_{-q_y,q_x} = \varphi_{-q_x,-q_y}$$

$$\varphi_{\mathbf{q}} = \cos q_x - \cos q_y$$

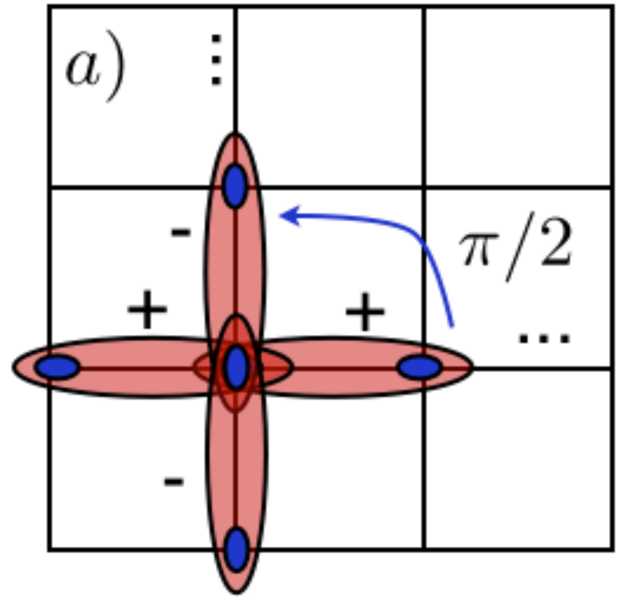
$$\varphi_{ij} = \frac{1}{2}\sum_{\lambda=x,y} \rho_\lambda(\delta_{i,j+\mathbf{e}_\lambda} + \delta_{i,j-\mathbf{e}_\lambda}) \qquad (\rho_x = 1, \rho_y = -1)$$

given the state, we want to find the Lindblad operators: "parent Liouvillian"

- **master equation**

$$\dot{\rho} = -\mathrm{i}H_{\mathrm{eff}}\rho + \mathrm{i}\rho H_{\mathrm{eff}}^{\dagger} + \kappa \sum_{\ell} j_{\ell}\rho j_{\ell}^{\dagger}$$

$$H_{\mathrm{eff}} = H - \tfrac{\mathrm{i}}{2}\kappa \sum_{\ell} j_{\ell}^{\dagger} j_{\ell}$$



- **Lindblad operators**

$$J_i^{\alpha} = \sum_{\lambda=x,y} \rho_{\lambda}(c_{i+\mathbf{e}_{\lambda}}^{\dagger} + c_{i-\mathbf{e}_{\lambda}}^{\dagger})\sigma^{\alpha} c_i \quad (\alpha = \pm, z;\ \alpha = x, y, z)$$

$c_i = (c_{i,\uparrow}, c_{i,\downarrow})^T$

single particle, quasi local

- **effective non-Hermitian Hamiltonian**

$$H_{\mathrm{eff}} = (U - \frac{\mathrm{i}}{2}\kappa) \sum_{i,a=\pm,z} J_i^{a\dagger} J_i^{a}$$

$J_i^a|\mathrm{BCS}_N\rangle = 0$ unique

Rem.: for U>0, d-wave ground state, i.e. we have also constructed a parent Hamiltonian for d-wave pairing

- “near” final BCS state: Bogoliubov-type analysis: (U=0)

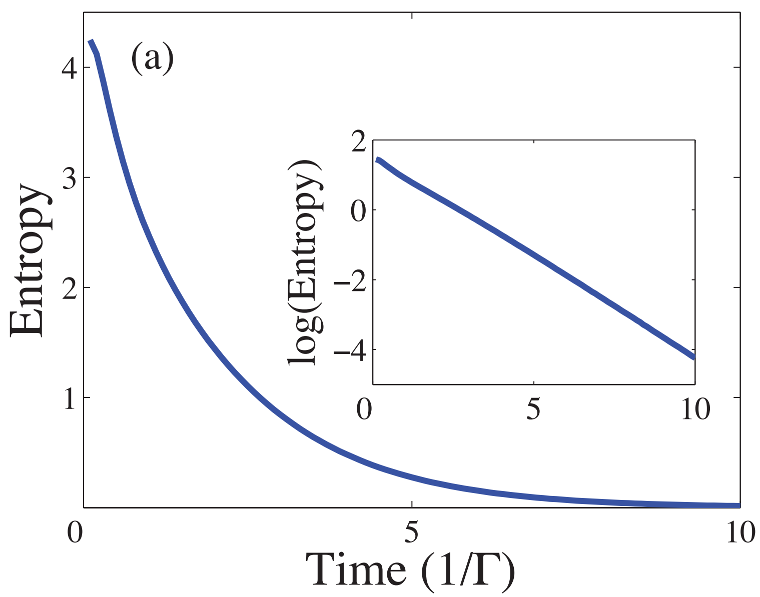
$$H_{\text{eff}} = -\frac{\mathrm{i}}{2}\kappa \sum_{\mathbf{q},\sigma} \left[\tilde{n}(c^\dagger_{\mathbf{q},\sigma} c_{\mathbf{q},\sigma} + |\varphi_{\mathbf{q}}|^2 c_{\mathbf{q},\sigma} c^\dagger_{\mathbf{q},\sigma}) + \tilde{\Delta}_{\mathbf{q}} s_\sigma c_{-(\mathbf{q},\sigma)} c_{\mathbf{q},\sigma} + \text{h.c.} \right]$$

$$= -\frac{\mathrm{i}}{2} \sum_{\mathbf{q},\sigma} \kappa_{\mathbf{q}} \gamma^\dagger_{\mathbf{q},\sigma} \gamma_{\mathbf{q},\sigma},$$ with a “dissipative gap” $$\kappa_{\mathbf{q}} = \kappa\, \tilde{n}\, (1 + \varphi^2_{\mathbf{q}}) \geq \kappa\, \tilde{n}.$$

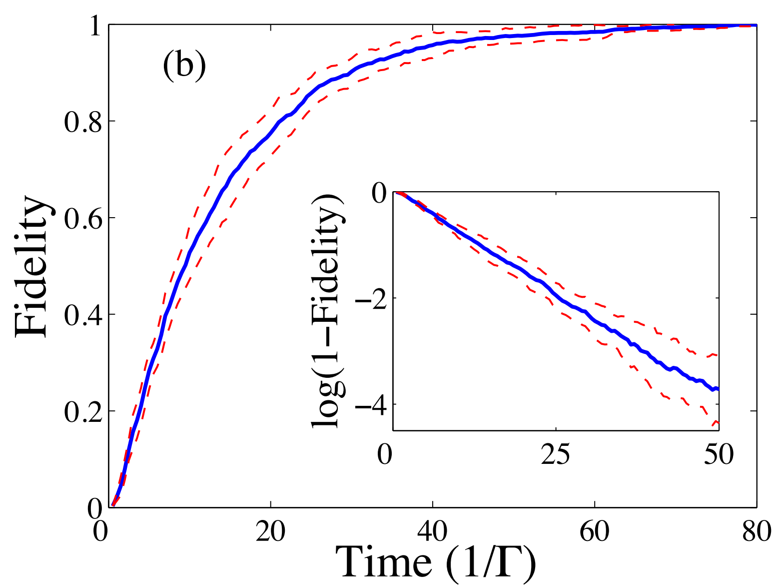
$$\dot{\rho} = -iH_{\text{eff}}\rho + i\rho H^\dagger_{\text{eff}} + \sum_{\mathbf{q},\sigma} \kappa_{\mathbf{q}} \gamma_{\mathbf{q},\sigma} \rho \gamma^\dagger_{\mathbf{q},\sigma}$$ with $$\gamma_{\mathbf{q},\sigma} |\text{BCS}_\theta\rangle = 0$$

- **numerical illustration**

entropy



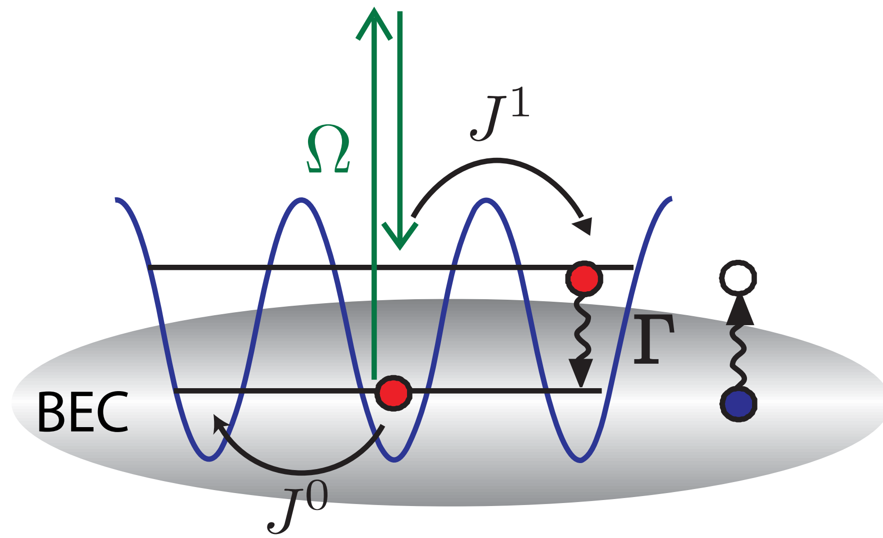
fidelity of d-wave BCS



Monday, October 11, 2010

Remarks:

- **Cold atom implementation:**
  - design atom - reservoir couplings



- **Other interesting states ...**

BEC

S. Diehl et al., Nature Physics 2008
B. Kraus et al., PRA 2008

Laughlin (?)

Exotic Spin models (see below)

H. Weimer et al., Nature Physics 2010

Q.: Identifying and engineering Lindblad operators, new physics … ?

- **Example 2:** driven BEC & dynamical quantum phase transition

$$\frac{d\rho}{dt} = -i\left[H,\rho\right] + \mathcal{L}\rho$$

interactions ⟷ drives to a BEC

dynamical quantum phase transition

nonequilibrium phase diagram

physical implementation with atoms in optical lattice

S. Diehl et al., Nature Physics 2008
B. Kraus et al., PRA 2008
S. Diehl et al. PRL 2010

Rydberg Quantum Simulator for Open System Dynamics

- Exotic spin models with atoms in 2D/3D optical lattices

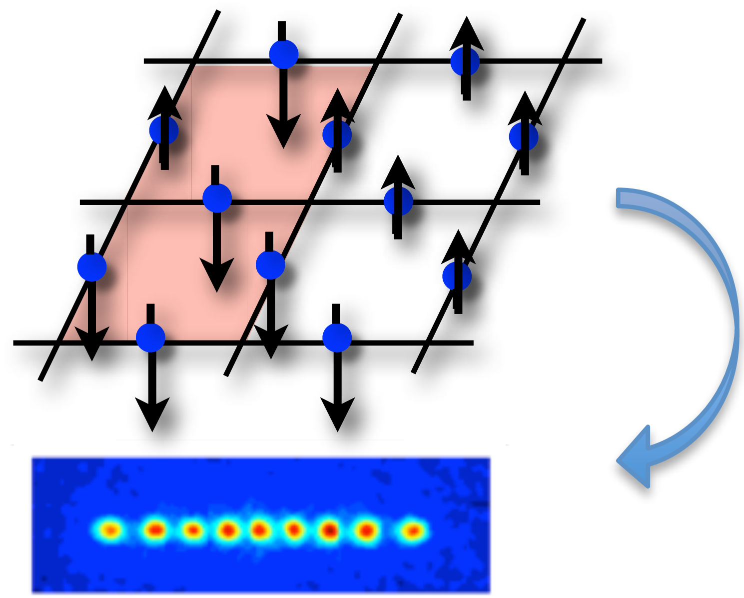
M. Müller, I. Lesanovsky, H. Weimer, H. Büchler, PZ, PRL 2009

H. Weimer, M. Müller, I. Lesanovsky, PZ, H. Büchler, Nature Physics 2010

- Experimental demonstration of basic concepts: trapped ions

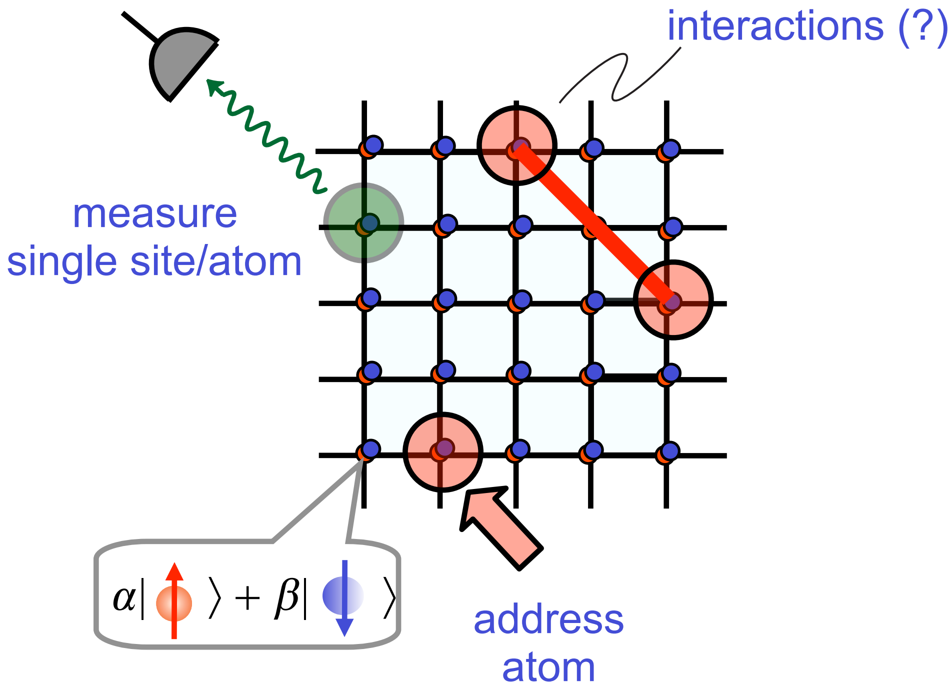
J. Barreiro, M. Müller, ..., PZ, R. Blatt, draft

2D optical lattice



1D string of ions
(in present form non-scalable)

Single Site Imaging (& Addressing)



large spacing optical lattices:

✓ single site / atom addressing

✓ small tunneling: ~~Hubbard models~~

spin models

✓ long distance interactions (?)

Rydberg interactions

“Rydberg Quantum Simulator” for Spin Systems

- **Large spacing optical lattices:**

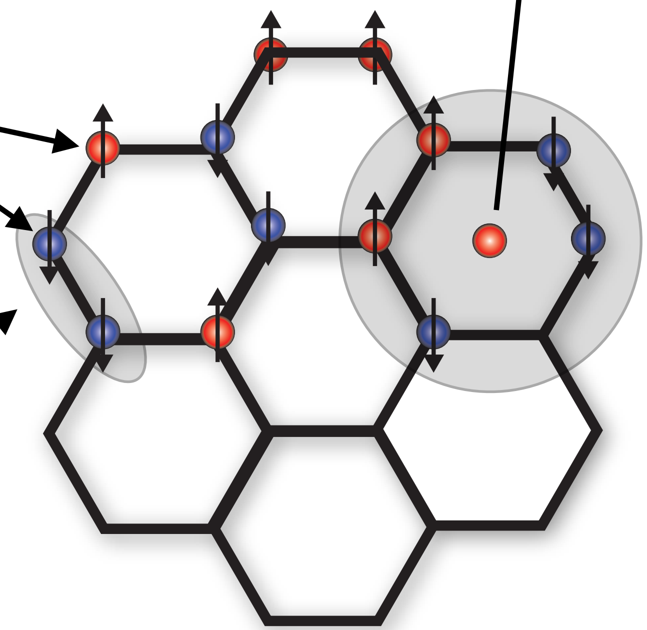
  ✓addressable spins

- **Rydberg - Rydberg interaction**

  ✓strong / long distance interactions

- **Goal: Exotic Spin Models**

  ✓coherent n-spin interactions
  ✓dissipative “cooling” dynamics

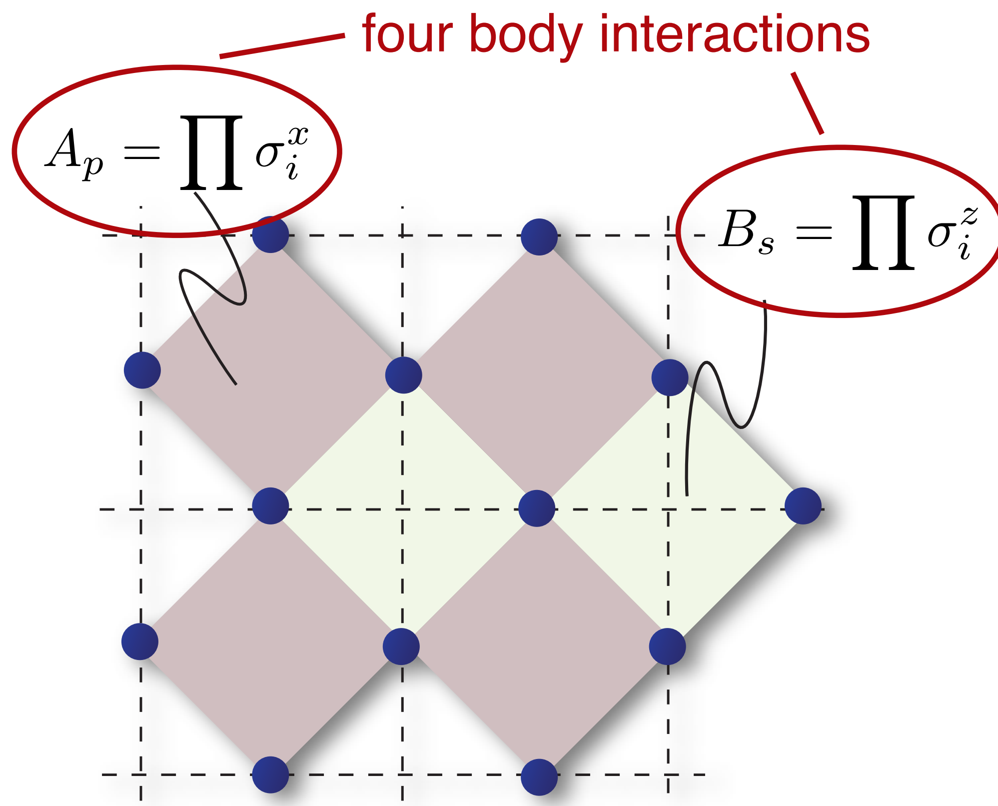


M. Müller et al., PRL 2009
H. Weimer et al., Nature Physics 2010

provides implementation for ...

A. Kitaev, Fault-tolerant quantum computation by anyons (2003,2006)
E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, Topological Quantum Memory (2002)

Kitaev's Toric Code



Stabilizer states: "toric code"

- set of stabilizer operators: local, commuting

$$A_p|\psi\rangle = |\psi\rangle$$

$$B_s|\psi\rangle = |\psi\rangle$$

- ground state of the Hamiltonian

$$H = -\sum_p A_p - \sum_s B_s$$

- topological phase with anyonic excitations:
  - "magnetic" excitation: $A_p|m\rangle = -|m\rangle$
  - "charge" excitation: $B_s|c\rangle = -|c\rangle$
  - abelian statistics

(alternative methods for Kitaev model (two-body interaction)
polar molecules, Micheli et al 2006; optical lattices, Demler et al 2004)

Kitaev's Toric Code

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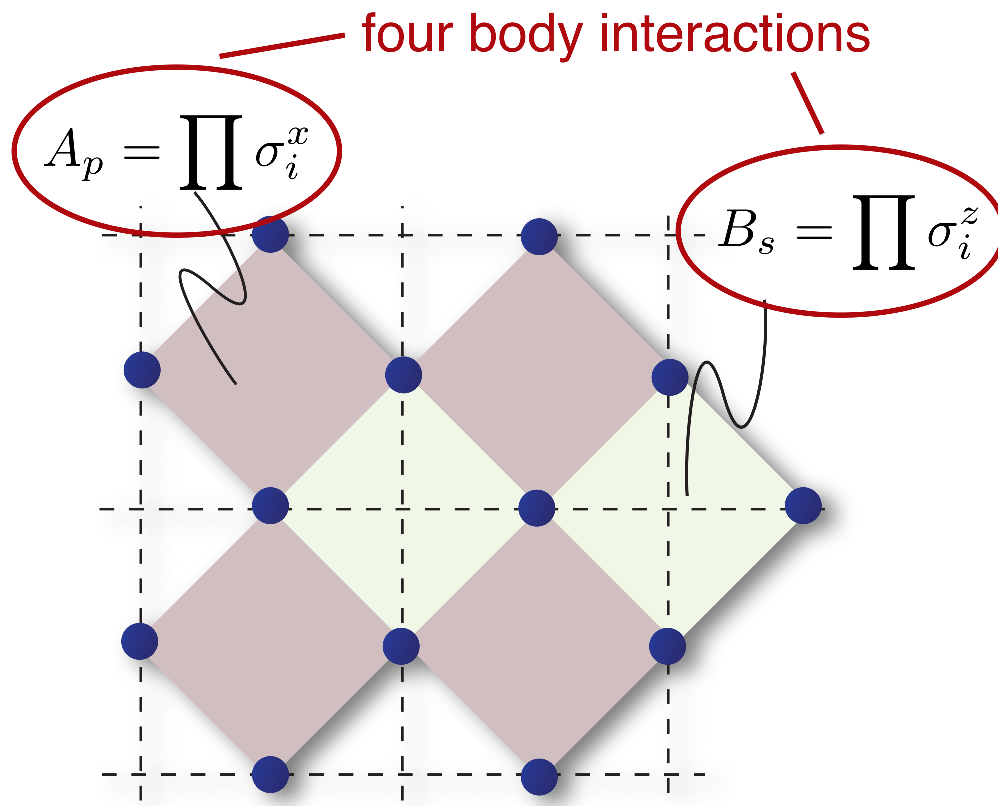

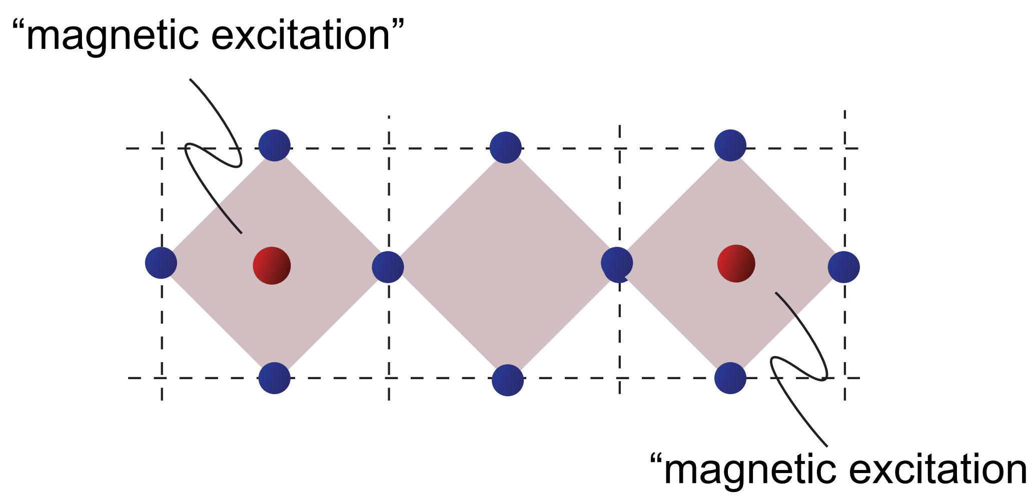

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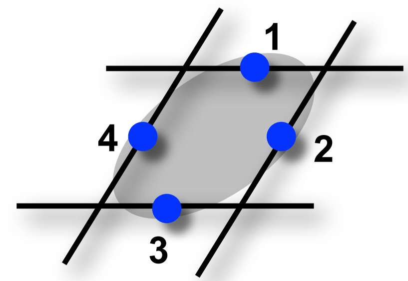
A Toy Model



Goal: On a single plaquette ...

- Lindblad master equation

$$\frac{d}{dt}\rho = -i\,[H,\rho] + \gamma\left(c\rho c^\dagger - \frac{1}{2}c^\dagger c\rho - \rho\frac{1}{2}c^\dagger c\right)$$

- Coherent evolution: Hamiltonian

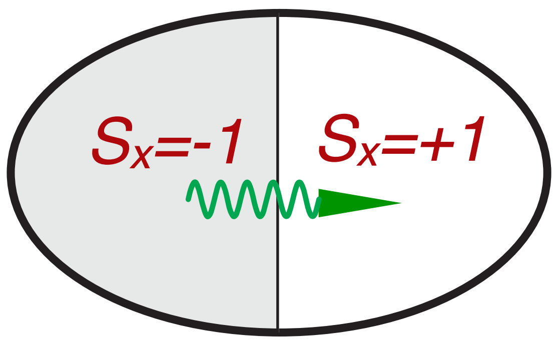
$$S_x = \sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}\sigma_x^{(4)}$$

$$H = h\sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}\sigma_x^{(4)}$$

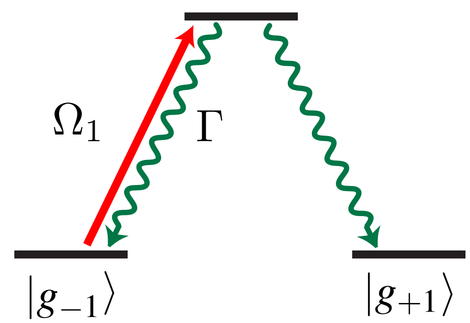
- Dissipative evolution: quantum jump operator

$$c = \sqrt{\gamma}\sigma_z^{(1)}\left(1 - \sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}\sigma_x^{(4)}\right)$$

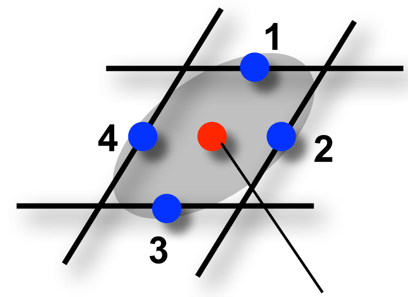
pumping of stabilizer



analogous to optical pumping



A Toy Model



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- Dissipative evolution: quantum jump operator

$$c = \sqrt{\gamma}\sigma_z^{(1)}\left(1 - \sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}\sigma_x^{(4)}\right)$$

Challenge:

How (efficiently) …
✓n-spin interactions
✓n-spin quantum jump operators

… with Rydberg atoms
& dipolar interactions
+ optical pumping

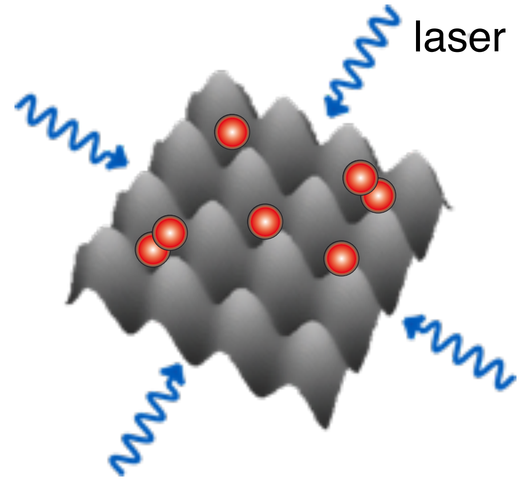
Remark 1: Quantum Simulation

	Hamiltonian	open system
always on (analog)		
stroboscopic (digital)		X

Coherent Quantum Dynamics

- **"analog" simulation**

optical lattice emulators



We "build" a quantum system with desired dynamics & controllable parameters,
e.g. Hubbard models of atoms in optical lattices

$$H = -J \sum_{\langle i,j \rangle} b_j^\dagger b_i + \frac{1}{2} U \sum_i b_i^{\dagger 2} b_i^2$$

example: bose (or fermi) Hubbard model

It is difficult to mimic n-body interactions & constraints

$$V^{(n)} \sim V^{(2)} \frac{1}{E-H} V^{(2)} \dots V^{(2)} \frac{1}{E-H} V^{(2)} \rightarrow \text{"0"}$$

n-body ↑ 2-body ↑ effective n-body interactions in perturbation theory ↑

extended Hubbard models

Coherent Quantum Dynamics

- **“stroboscopic” or “digital” simulation**

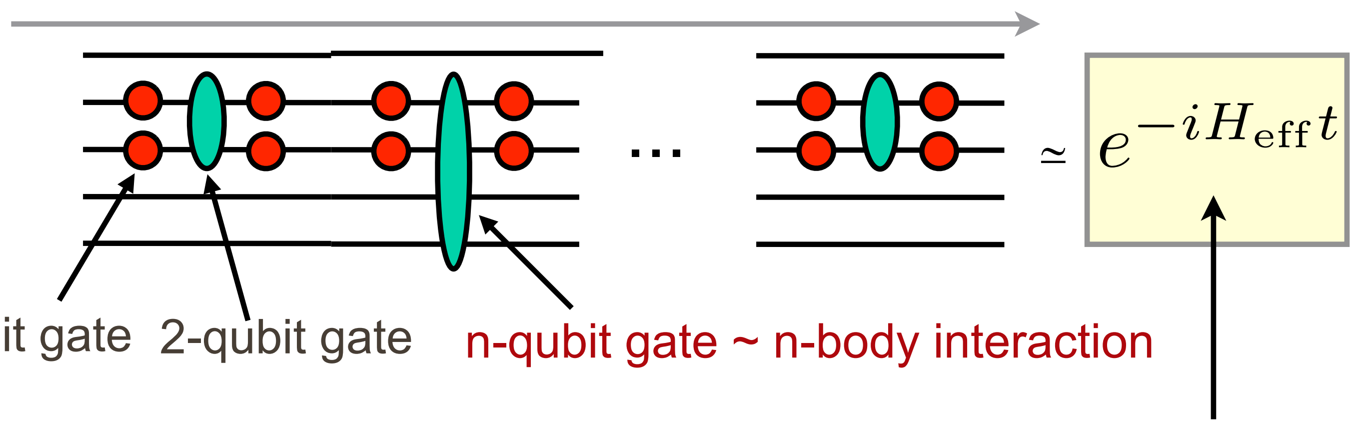
time evolution of many body spin Hamiltonian

$$U(t) \equiv e^{-iHt} = e^{-iH\Delta t_n} \ldots e^{-iH\Delta t_1} \qquad (t = \sum_i \Delta t_i)$$

$$H = \sum_\alpha h_\alpha \qquad e^{-iH\Delta t} = \prod_\alpha e^{-ih_\alpha \Delta t} + \text{errors}$$

Trotter formula

stroboscopic time evolution as sequence of quantum gates

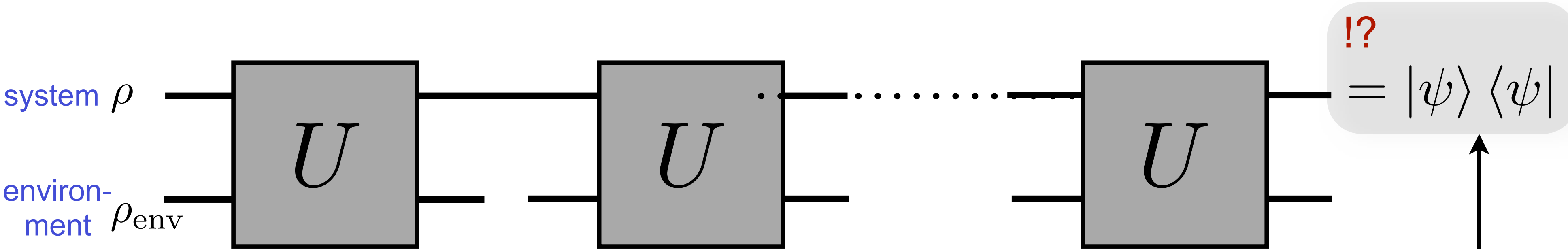


Lloyd, ...

desired many body Hamiltonian “on the average”

Open Quantum Systems

- **Q.: dissipative preparation of entangled states**



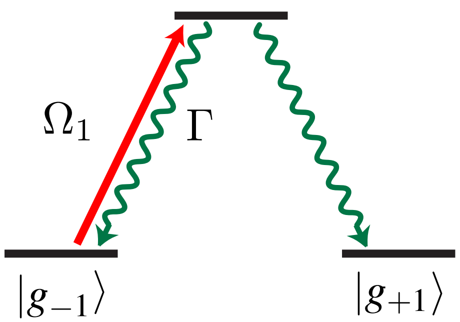
$$\rho \longrightarrow \mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

<u>n-body quantum jump operators</u> ☹

engineer coupling to a quantum reservoir

pumping into *pure* entangled state of interest

- **optical pumping (Kastler) or laser cooling**



$$\rho(t) \xrightarrow{t\to\infty} |g_+\rangle\langle g_+|$$

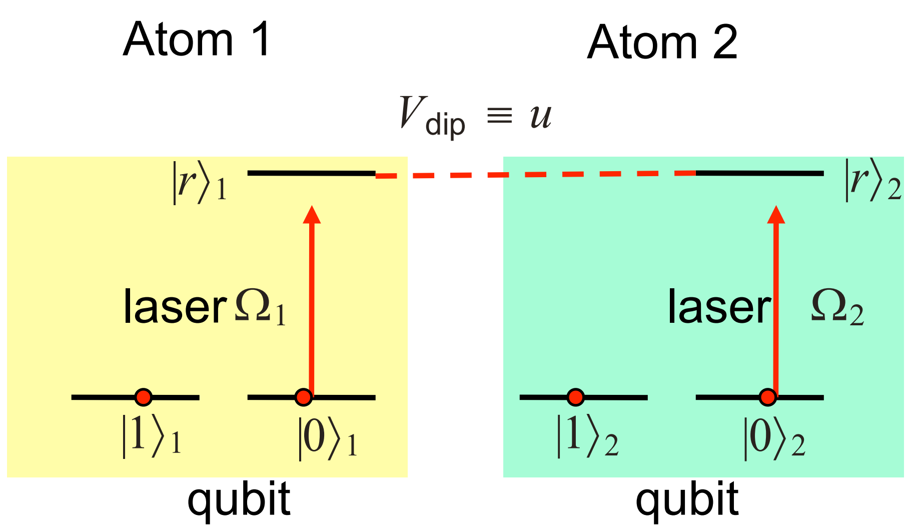
driven dissipative dynamics "purifies" the state

Remark 2: Rydberg Gates

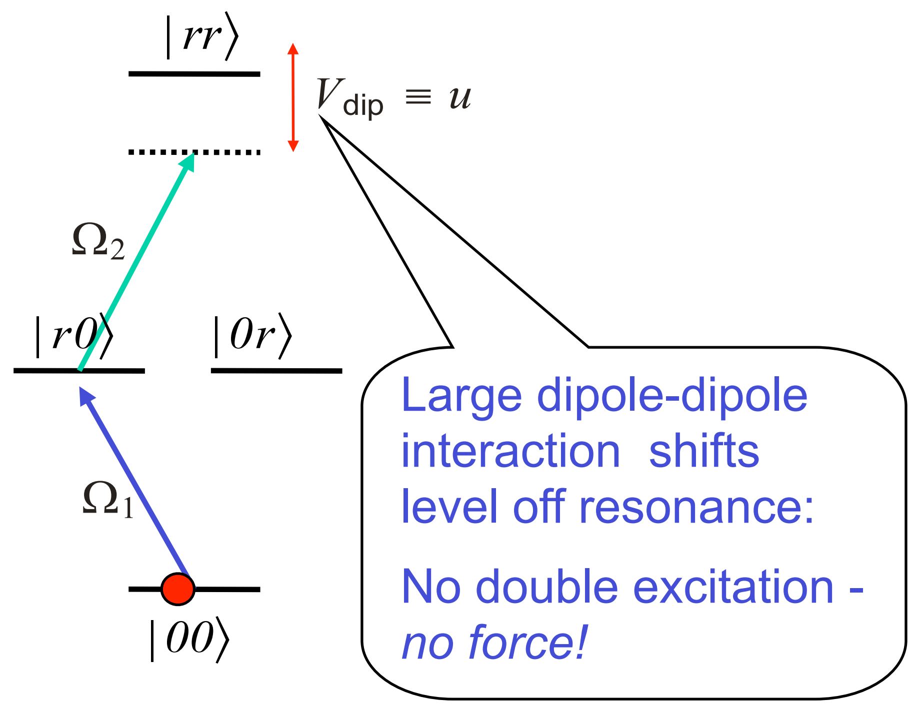
- efficient n-spin entangling gates

Two-Qubit Gate & Dipole blockade

- atomic configuration



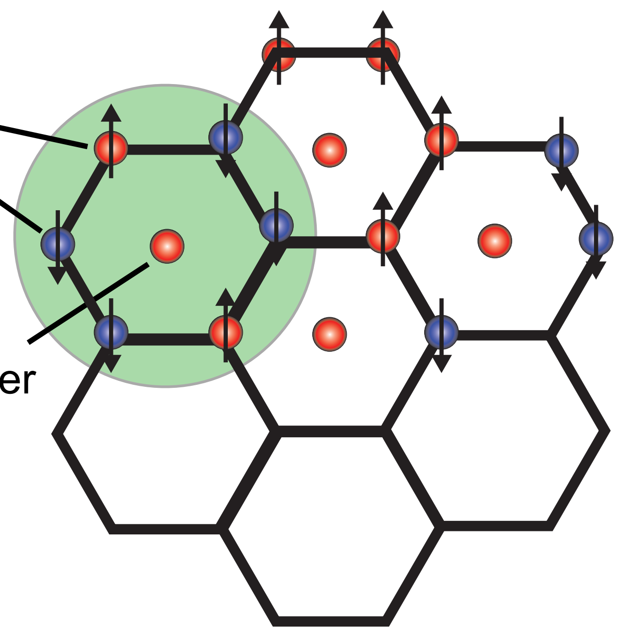
- dipole blockade



- theory: NIST, Orsay, Wisconsin, Aarhus, ...
- exp: Wisconsin (2009), Orsay (2009), ...

… and an “n-qubit CNOT” Rydberg Gate

- **gate: ingredients**
  - atoms in a large spacing optical lattice: addressability
  - Rydberg dipole-dipole



features:

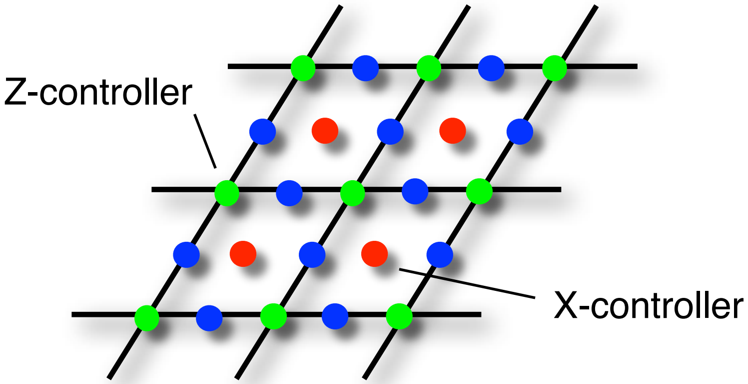
- ✓ High fidelity even for moderately large # qubits
- ✓ Fast 3 laser pulses
- ✓ Long-range interactions
- ✓ Robust with respect to
  - inhomogeneities in the interparticle distances
  - variations in the interaction strengths
  - no mechanical effects
- ✓ experimentally realistic parameters

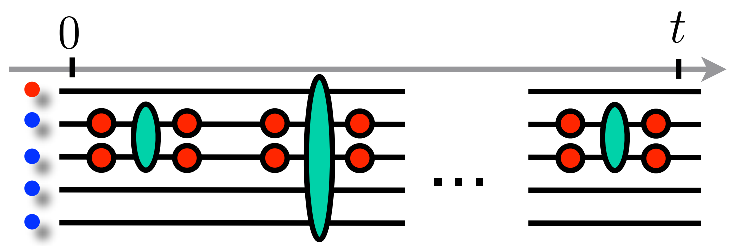
$$G = |0\rangle_c\langle 0| \otimes 1 + |1\rangle_c\langle 1| \otimes \sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}\sigma_x^{(4)} \dots$$

Rydberg controller

n-qubits

Digital simulation: a single time step





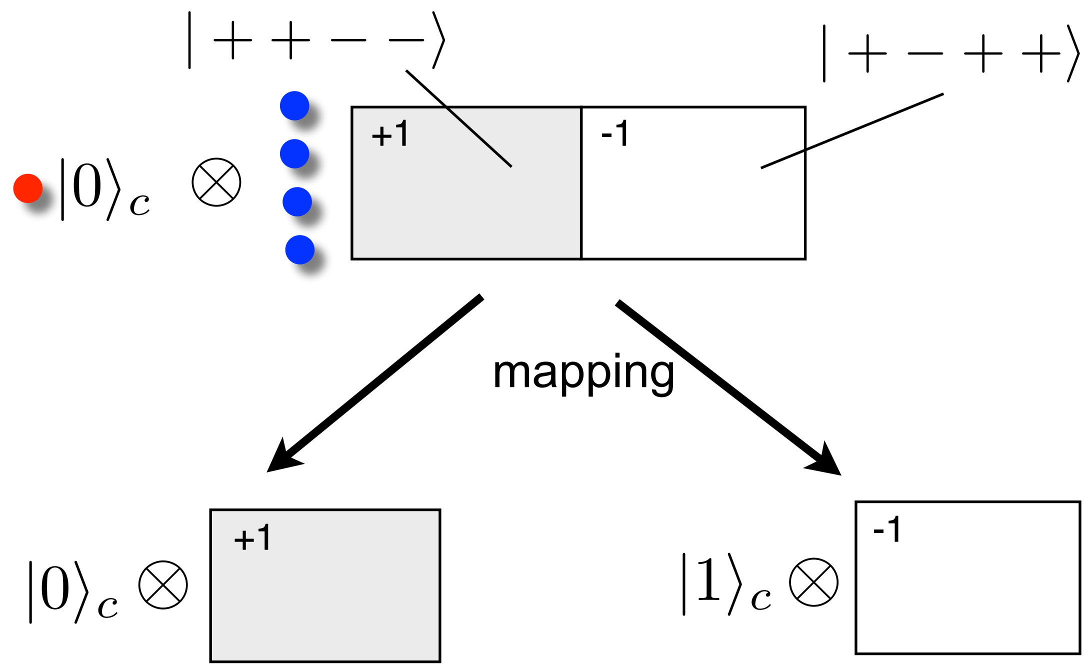
general approach:

a) **map** the information on the four spins onto the auxiliary qubit

b) **manipulation** of the auxiliary atom

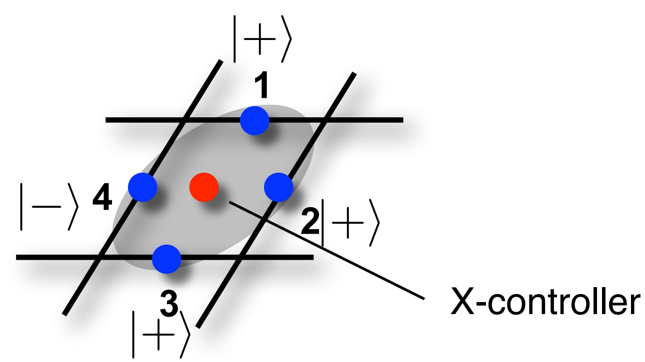
c) undo the mapping

$$S_x = \sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}\sigma_x^{(4)}$$



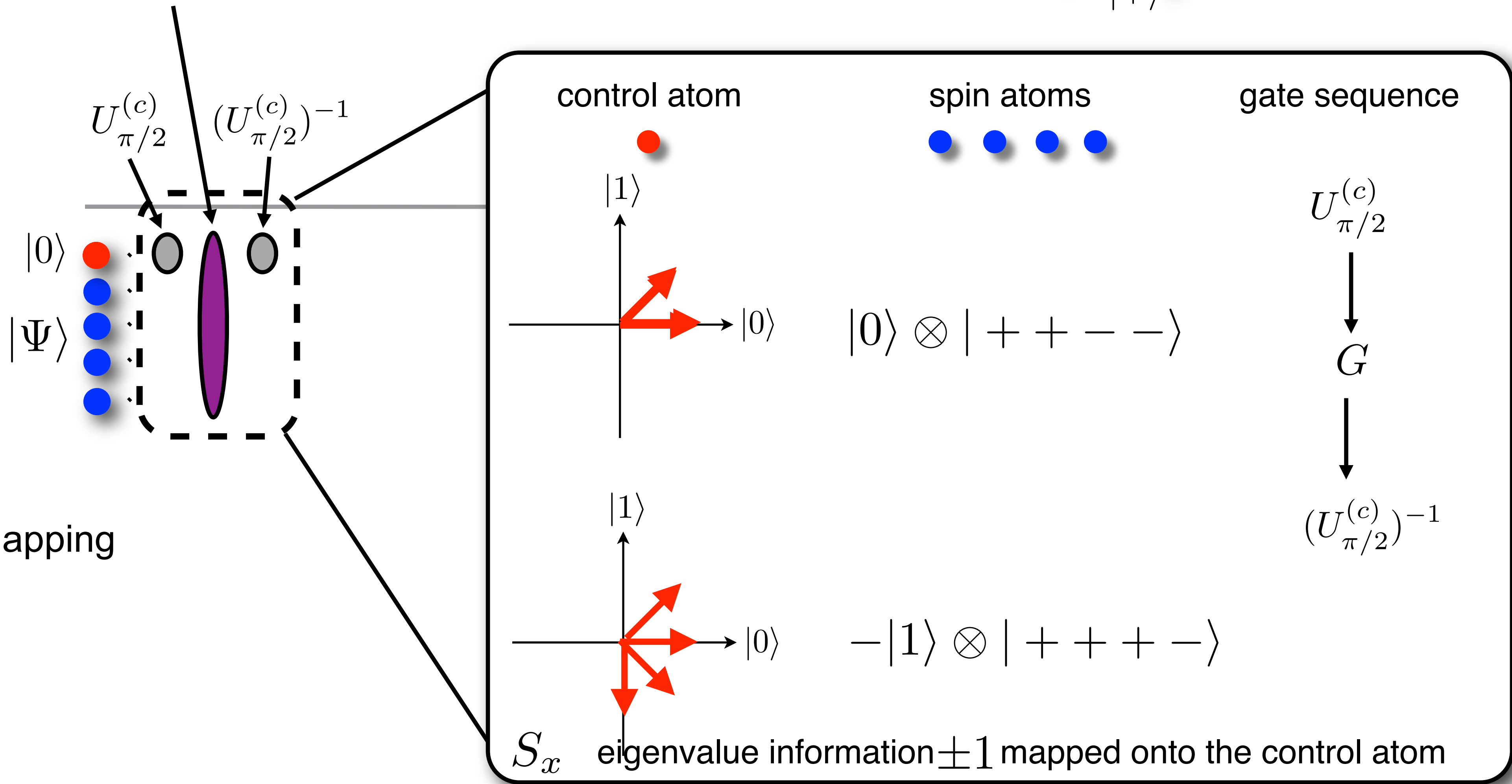
Implementation of a single time step

example: four-body interactions $\sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}\sigma_x^{(4)}$
in Kitaev's toric code



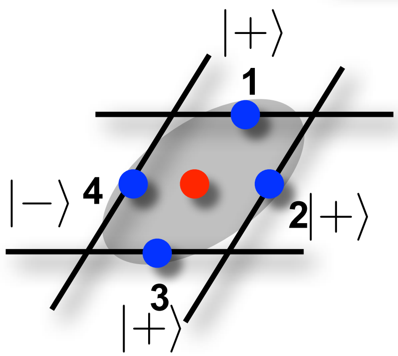
our multi-qubit CNOT-gate

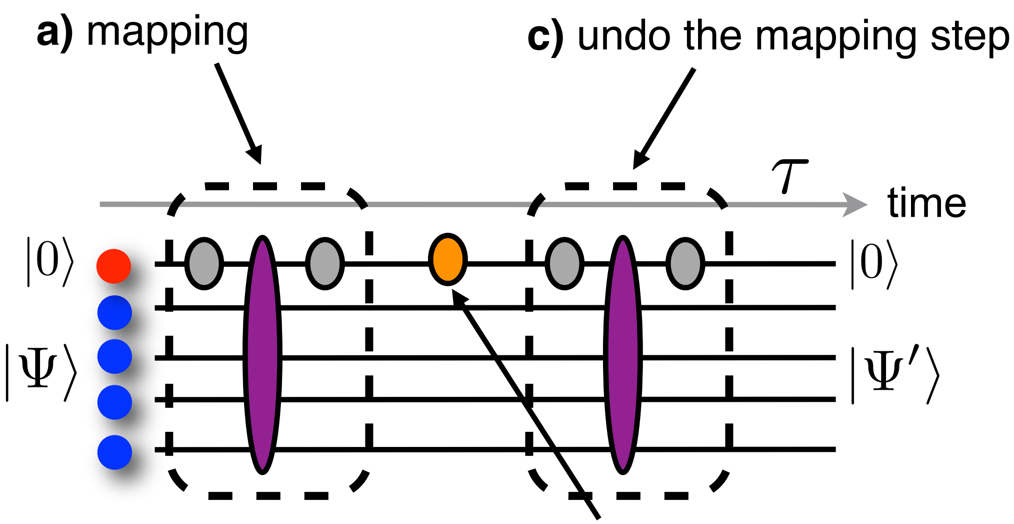
$$G = |0\rangle_c\langle 0| \otimes 1 + |1\rangle_c\langle 1| \otimes \sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}\sigma_x^{(4)}$$



a) mapping

Implementation of a single time step





$$R = \exp(i\alpha\sigma_z^{(c)})$$

b) manipulation: small local rotation of the control atom $\alpha \ll 1$

$$|0\rangle|++--\rangle \rightarrow e^{i\alpha}|0\rangle|++--\rangle$$

$$|1\rangle|+++-\rangle \rightarrow e^{-i\alpha}|1\rangle|+++-\rangle$$

- auxiliary atom factorizes out
- all +1 eigenstates have picked up a phase $+\alpha$
- all -1 eigenstates have picked up a phase $-\alpha$

- composed evolution $|\Psi'\rangle = U|\Psi\rangle$

  $$U \equiv \exp(-iH\tau/\hbar)$$

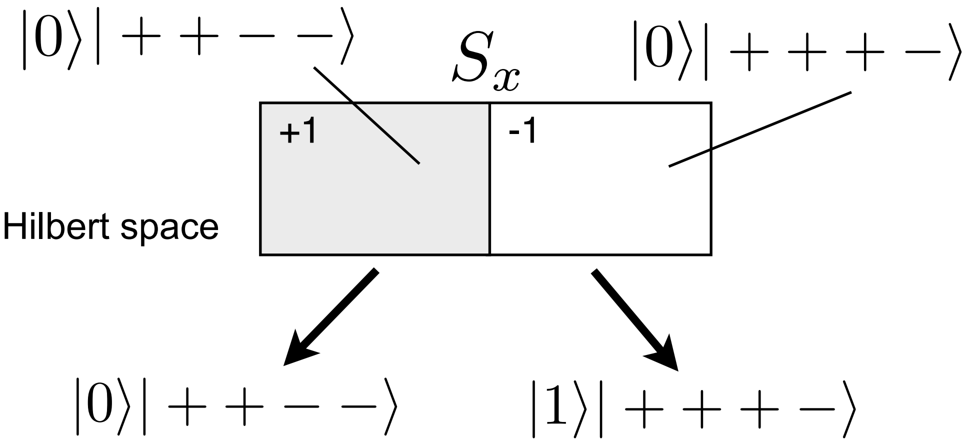
  with

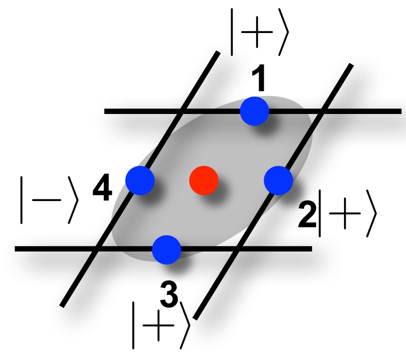
  $$H = -\frac{\hbar\alpha}{\tau}\sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}\sigma_x^{(4)}$$

- stroboscopic simulation
- energy scale set by rotation angle α and gate duration τ can be on the order 100 kHz

Cooling: one dissipative time step

- goal: prepare the spin system in +1 eigenstates
- a) mapping





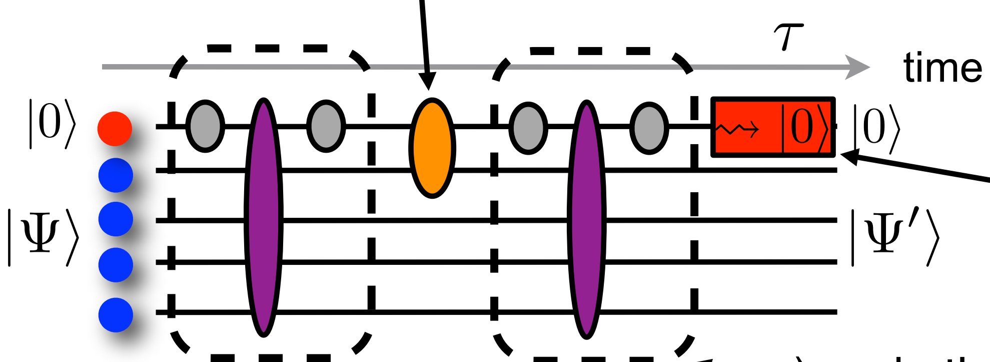
- b) conditional spin flip of one qubit

$$C = |0\rangle_c\langle 0| \otimes 1 + |1\rangle_c\langle 1| \otimes \exp(i\phi\sigma_z^{(1)})$$

$$|0\rangle|++--\rangle \rightarrow |0\rangle|++--\rangle$$

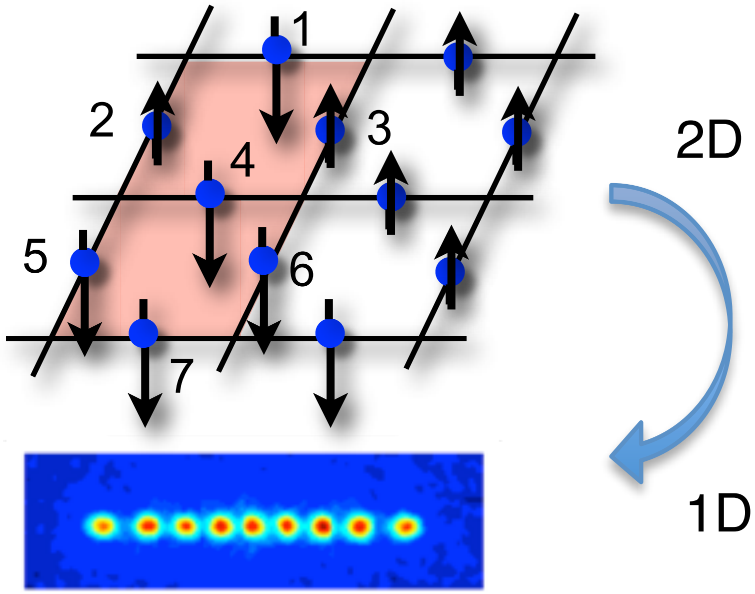
$$|1\rangle|+++-\rangle \rightarrow |1\rangle|-++-\rangle$$

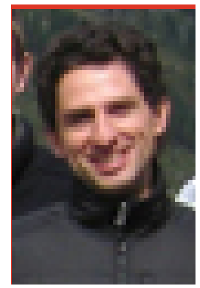
... with a small probability $\phi^2 \ll 1$



d) dissipative element: optical pumping of the control atom

c) undo the mapping step





Julio Barreiro
(exp)

Markus Müller
(theory)

R. Blatt's
ion trap lab

Experiment with Trapped Ions

- 2+1 ions: Bell state cooling
- **4+1 ions "one plaquette": stabilizer cooling and 4-body interactions**
- QND measurement of 4-qubit stabilizers

Julio Barreiro (exp: R. Blatt), M. Müller (theory)

Non-Equilibrium Dynamics on Optical Lattices:
Heating & Decoherence due Spontaneous Emission

$$\dot{\rho} = -i\left[H_{BH/FB} + \ldots, \rho\right] + \mathcal{L}\rho$$

strongly correlated
Hubbard dynamics

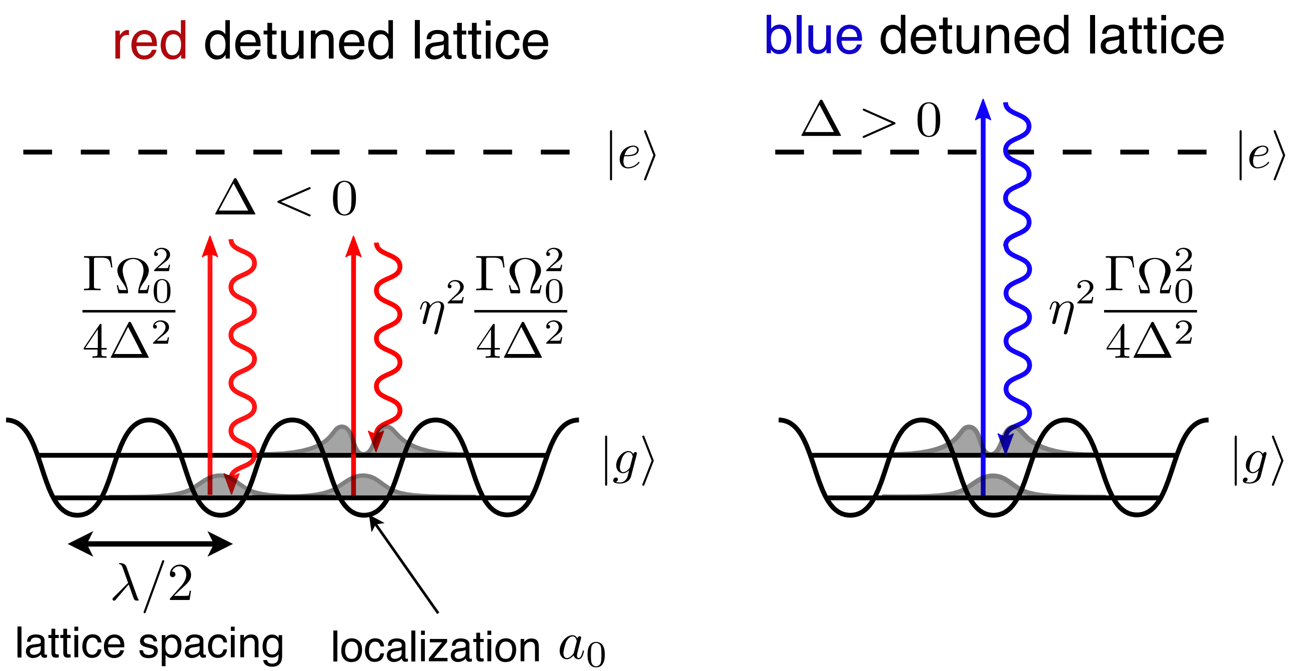
decoherence,
e.g. spontaneous emission

- **Q.: interplay decoherence & many body**

H. Pichler, A. Daley, and PZ, arXiv:1009.0194

Heating in optical lattices: single particle

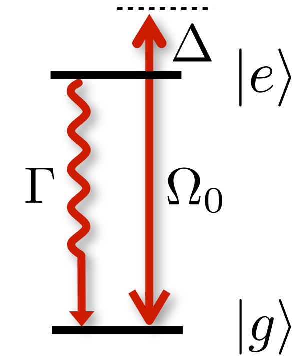
- **Spontaneous emission**



Lamb-Dicke parameter: $\eta = 2\pi a_0/\lambda << 1$

- Red detuning gives rise to more spontaneous emission events in deep lattices
- Processes returning atoms to the lowest band are strongly suppressed for blue detuning

N-boson master equation



- **master equation for atoms in ground state:**

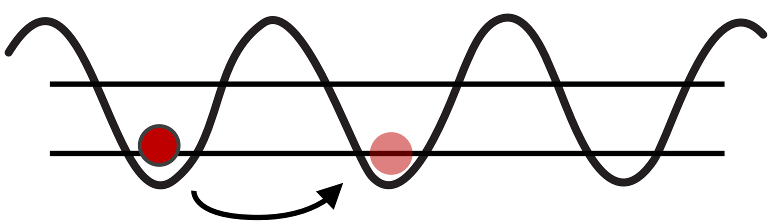
$$\dot{\rho} \quad = \quad -i\left(H_{\text{eff}}\rho - \rho H_{\text{eff}}^{\dagger}\right) + \mathcal{J}_{\text{rad}}\rho$$

- effective non-Hermitian Hamiltonian:

$$H_{\text{eff}} \quad = \quad H_0 + H_{\text{eff}}^{\text{rad}} + H_{\text{eff}}^{\text{coll}}.$$

- **single particle motion in optical lattice**

$$H_0 = \int d^3x \hat{\psi}^{\dagger}(\boldsymbol{x}) \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{opt}}(\boldsymbol{x})\right) \hat{\psi}(\boldsymbol{x})$$



$$V_{\text{opt}} = \frac{|\Omega(\mathbf{x})|^2}{4\Delta}$$

N-boson master equation

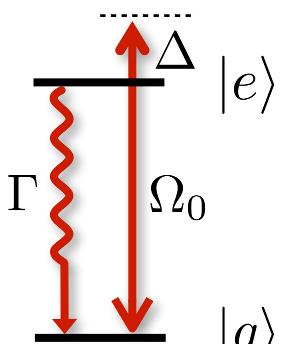
- **radiative processes**

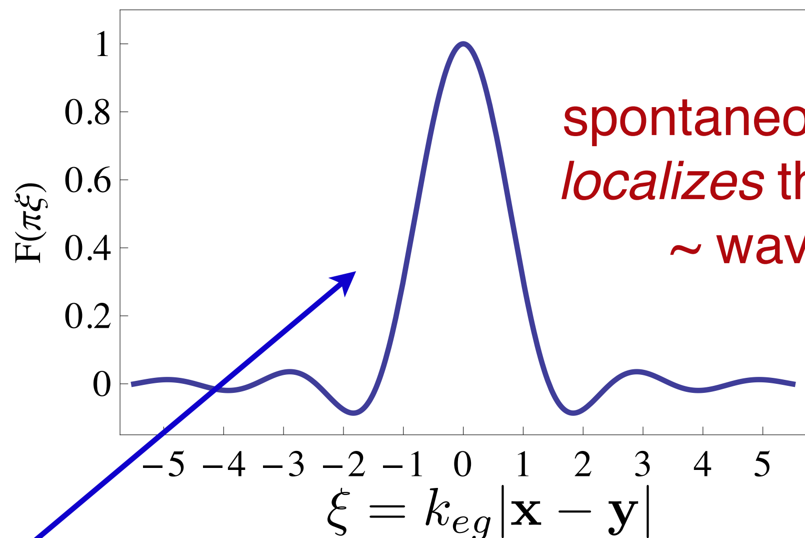
dipole-dipole interaction

$$H_{\text{eff}}^{\text{rad}} = \iint d^3x d^3y \frac{\Gamma\Omega(\boldsymbol{y})\Omega^*(\boldsymbol{x})}{4\Delta^2} G(k_{eg}(\boldsymbol{x}-\boldsymbol{y}))\hat{\psi}^\dagger(\boldsymbol{x})\hat{\psi}^\dagger(\boldsymbol{y})\hat{\psi}(\boldsymbol{y})\hat{\psi}(\boldsymbol{x})$$
$$- i\frac{1}{2}\int d^3x \frac{\Gamma|\Omega(\boldsymbol{x})|^2}{4\Delta^2}\hat{\psi}^\dagger(\boldsymbol{x})\hat{\psi}(\boldsymbol{x}) - i\frac{1}{2}\iint d^3x d^3y \frac{\Gamma\Omega(\boldsymbol{y})\Omega^*(\boldsymbol{x})}{4\Delta^2} F(k_{eg}(\boldsymbol{x}-\boldsymbol{y}))\hat{\psi}^\dagger(\boldsymbol{x})\hat{\psi}^\dagger(\boldsymbol{y})\hat{\psi}(\boldsymbol{y})\hat{\psi}(\boldsymbol{x})$$

single particle optical pumping

super- / subradiant effects





spontaneous emission *localizes* the particle to ~ wavelength

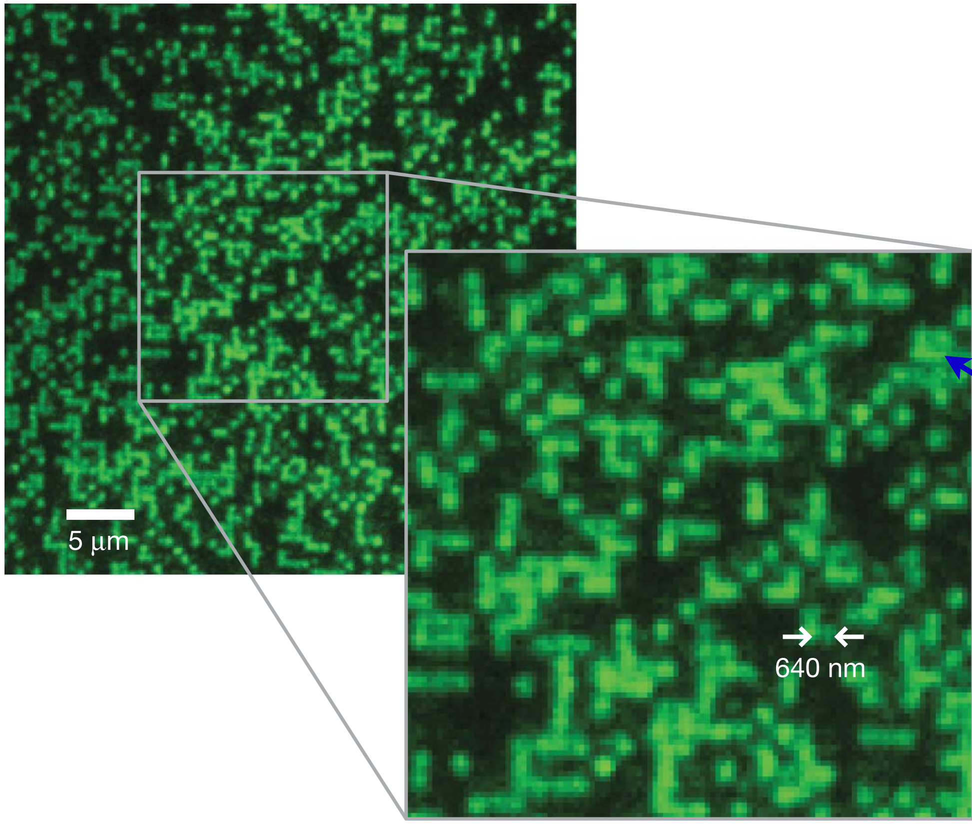
recycling term: return of electron to ground state after emission

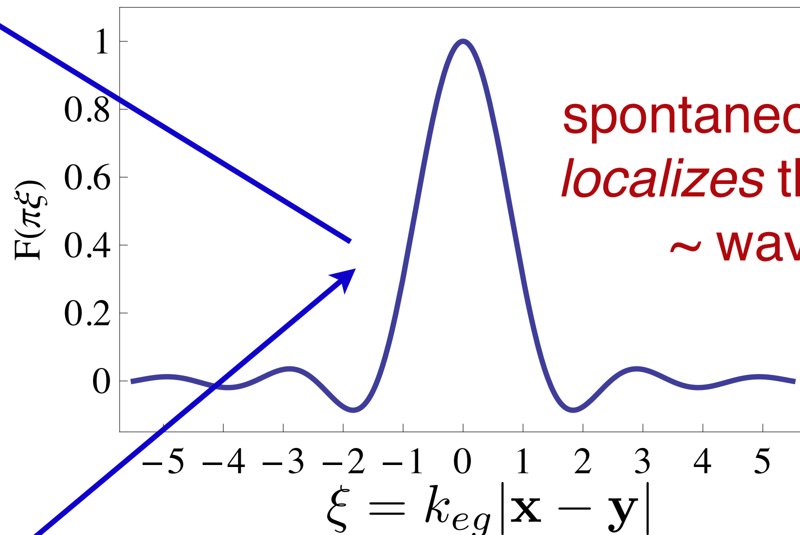
$$\mathcal{J}\rho = \iint d^3x d^3y \frac{\Gamma\Omega(\boldsymbol{x})\Omega(\boldsymbol{y})}{4\Delta^2} F(k_{eg}(\boldsymbol{x}-\boldsymbol{y}))\hat{\psi}^\dagger(\boldsymbol{x})\hat{\psi}(\boldsymbol{x})\rho\hat{\psi}^\dagger(\boldsymbol{y})\hat{\psi}(\boldsymbol{y})$$

density density

N-boson master equation

- **Imaging Hubbard models:**
  Harvard, Munich, Chicago, Penn State, ...





spontaneous emission *localizes* the particle to ~ wavelength

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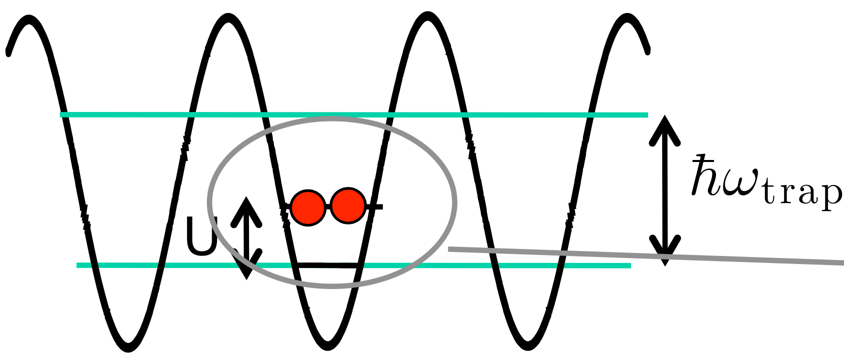
$$\mathcal{J}\rho = \iint d^3x d^3y \frac{\Gamma\Omega(\boldsymbol{x})\Omega(\boldsymbol{y})}{4\Delta^2} F(k_{eg}(\boldsymbol{x}-\boldsymbol{y}))\hat{\psi}^\dagger(\boldsymbol{x})\hat{\psi}(\boldsymbol{x})\rho\hat{\psi}^\dagger(\boldsymbol{y})\hat{\psi}(\boldsymbol{y})$$

density density

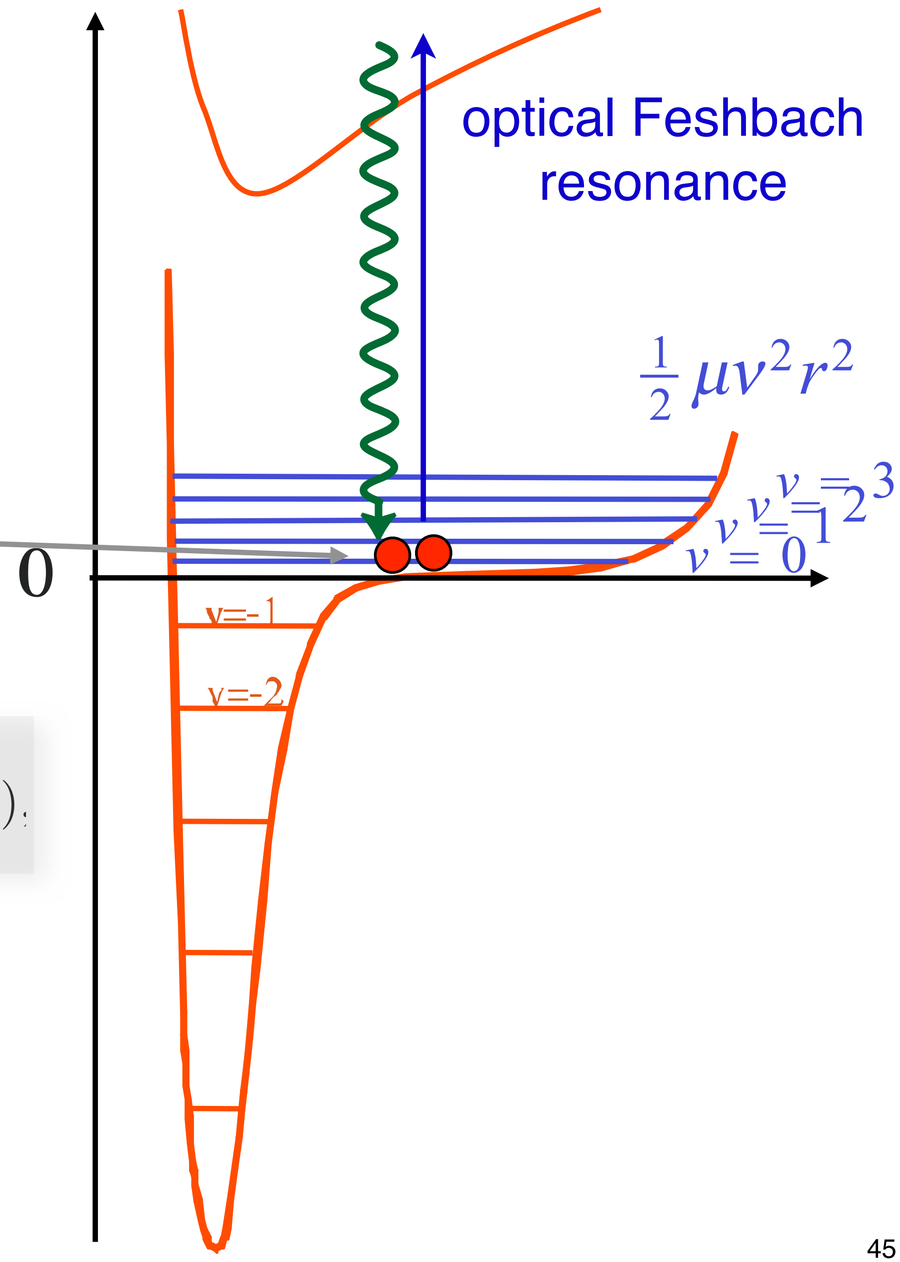
N-boson master equation

- **short range collisions & light induced collisions**

two atoms on one lattice site



Born-Oppenheimer potential



$$H_{\text{eff}}^{\text{coll}} = \int d^3x \left(g(\boldsymbol{x}) - i\frac{1}{2}\gamma_2(\boldsymbol{x}) \right) \hat{\psi}^\dagger(\boldsymbol{x})\hat{\psi}^\dagger(\boldsymbol{x})\hat{\psi}(\boldsymbol{x})\hat{\psi}(\boldsymbol{x}),$$

contact potential:
real & light induced imaginary scattering length

Hubbard Master Equation

- projecting of Wannier functions …

$$\dot{\rho} = -i\left[\hat{H}, \rho\right] + \sum_{\boldsymbol{n},\mu} \gamma_{\boldsymbol{n},\mu} \left(b^\dagger_{\boldsymbol{n},\boldsymbol{i}} b_{0,\boldsymbol{i}} \rho b^\dagger_{0,\boldsymbol{i}} b_{\boldsymbol{n},\boldsymbol{i}} - \frac{1}{2} b^\dagger_{0,\boldsymbol{i}} b_{\boldsymbol{n},\boldsymbol{i}} b^\dagger_{\boldsymbol{n},\boldsymbol{i}} b_{0,\boldsymbol{i}} \rho - \rho \frac{1}{2} b^\dagger_{0,\boldsymbol{i}} b_{\boldsymbol{n},\boldsymbol{i}} b^\dagger_{\boldsymbol{n},\boldsymbol{i}} b_{0,\boldsymbol{i}} \right)$$

band site

Hubbard decoherence

- Solution
  - time dependent perturbation theory in spontaneous emission
  - tDMRG + quantum trajectories

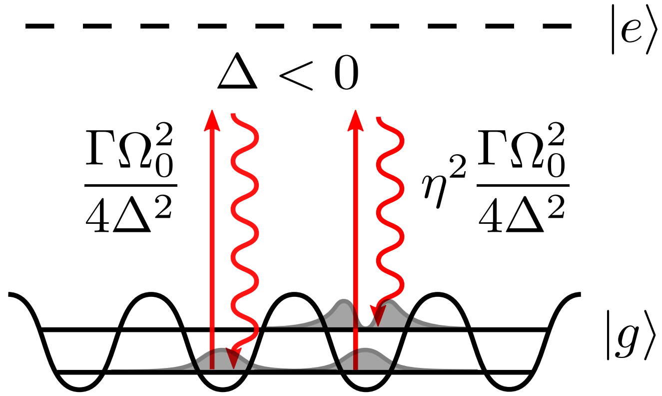
$$\dot{\rho} = -i[H_{BH}, \rho] + \sum_i \gamma(\hat{n}_i \rho \hat{n}_i - 1/2 \hat{n}_i \hat{n}_i \rho - 1/2 \rho \hat{n}_i \hat{n}_i)$$

  - mean field Gutzwiller:

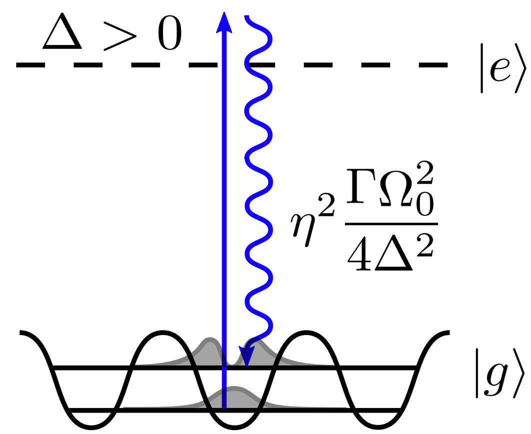
Heating in optical lattices: single particle

- **Spontaneous emission**

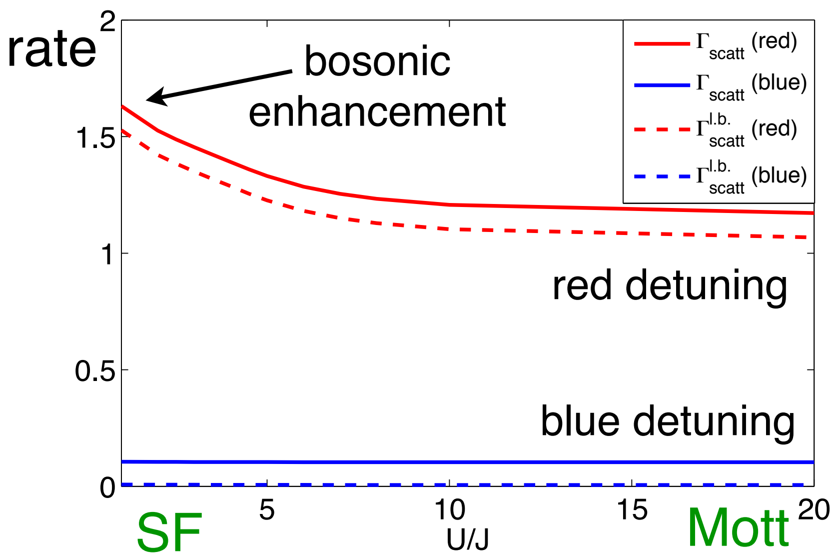
red detuned lattice



blue detuned lattice



- **total light scattering rate**



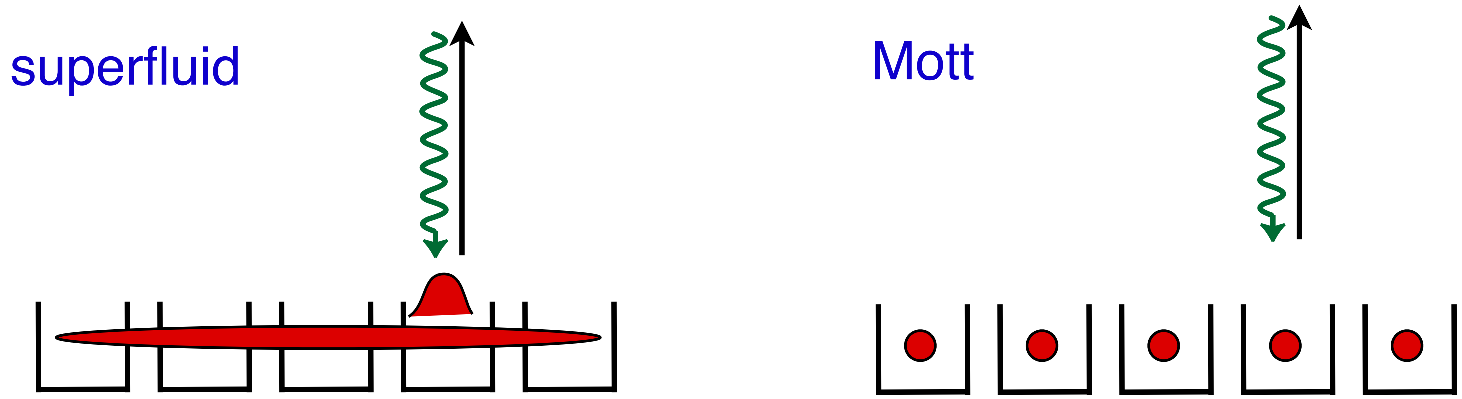
- **energy heating rate / atom**

$$\dot{E} = \frac{\Gamma|\Omega_0|^2}{4\Delta^2} E_R$$

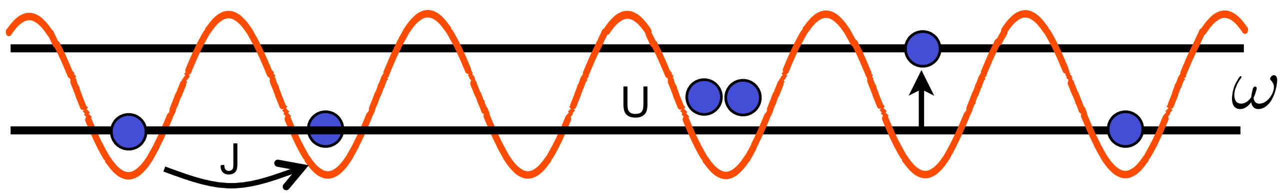
heating is the same in red and blue lattice

Destruction of Coherence, Collisions etc.

- spontaneous emission = localization



- collisions

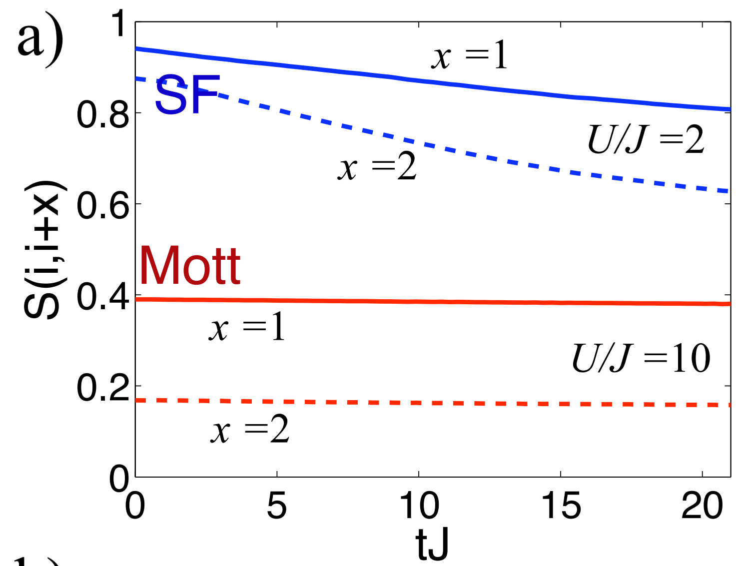


collisions do not necessarily thermalize on experimental time scales; this is particularly true for excited bands

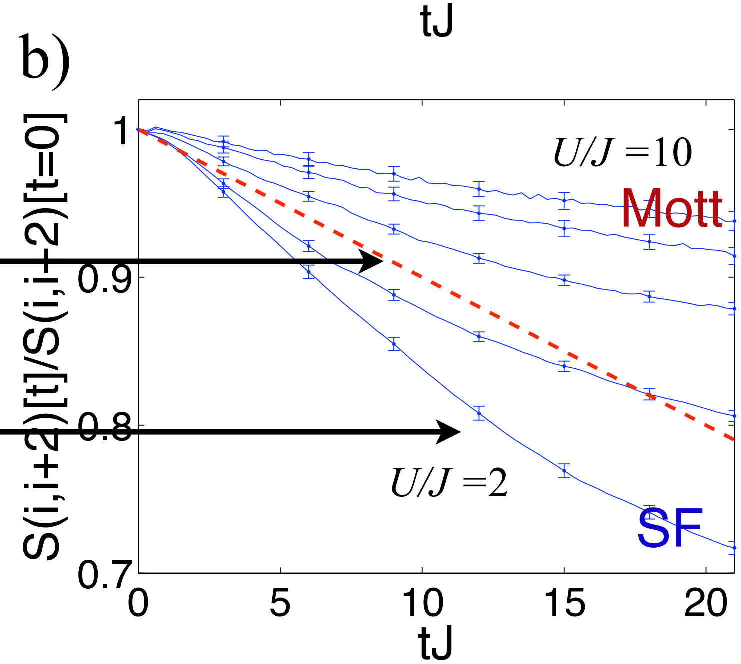
Destruction of Coherence, Collisions etc.

- decay of off-diagonal single particle correlation functions

$$S(i,j) = \langle b_i^\dagger b_j \rangle$$



- decay of initial correlations



perturbation theory

exact: t-DMRG
\+ Monte Carlo wave functions

Summary

- **Driven Dissipative State Preparation**
  - Example 1: d-wave pairing by dissipation
  - [Example 2: driven BEC & quantum phase transition]
- **Rydberg Quantum Simulator for Open System Dynamics**
  - Example: Kitaev toric code & stabilizer pumping
  - [an ion trap experiment: “single plaquette”]
- **Decoherence / heating: spontaneous emission in opt. lattices**