Hierarchies in Decentralized Control and the Human Motor System

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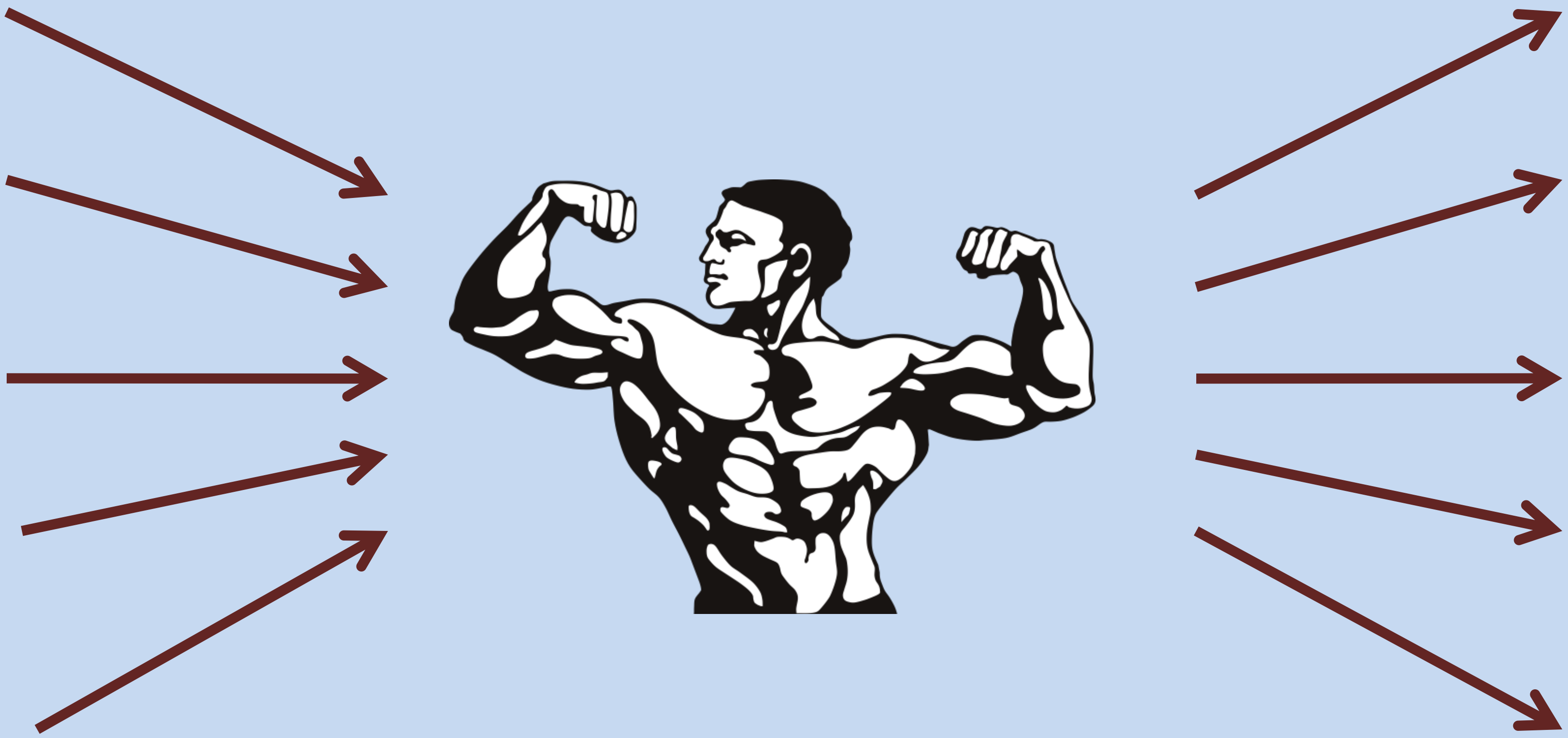
Author: NASA



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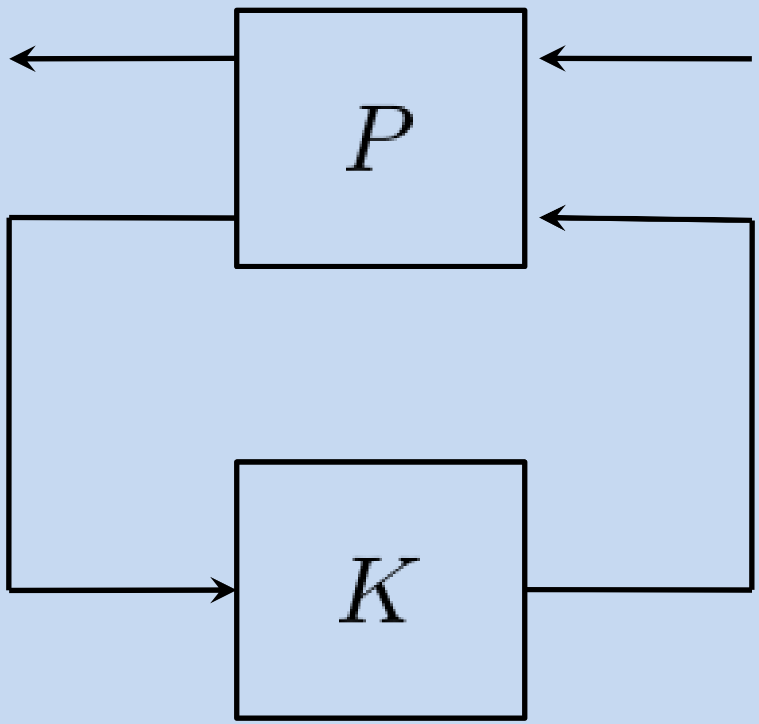
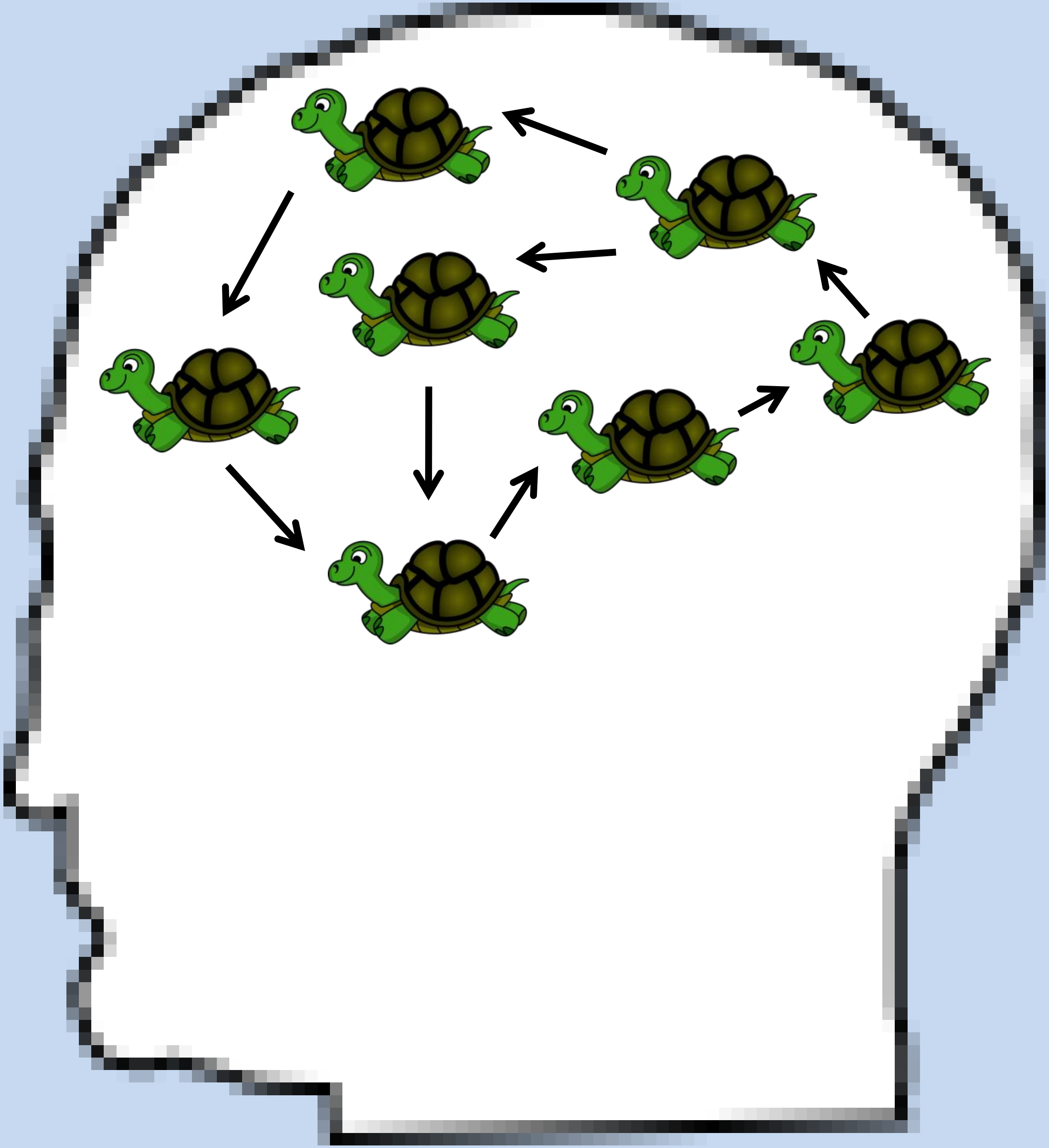
System or “Plant”



Controller



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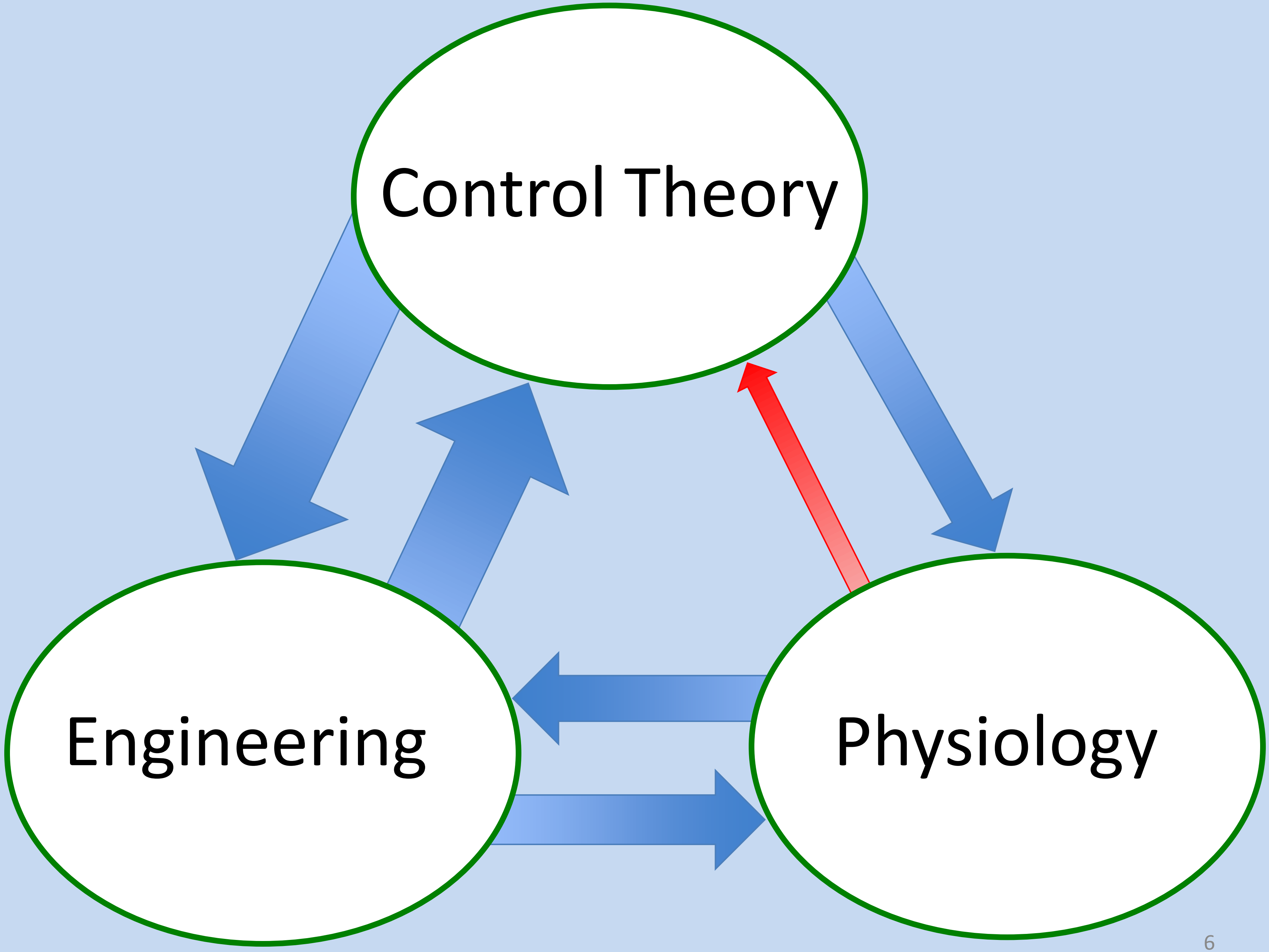
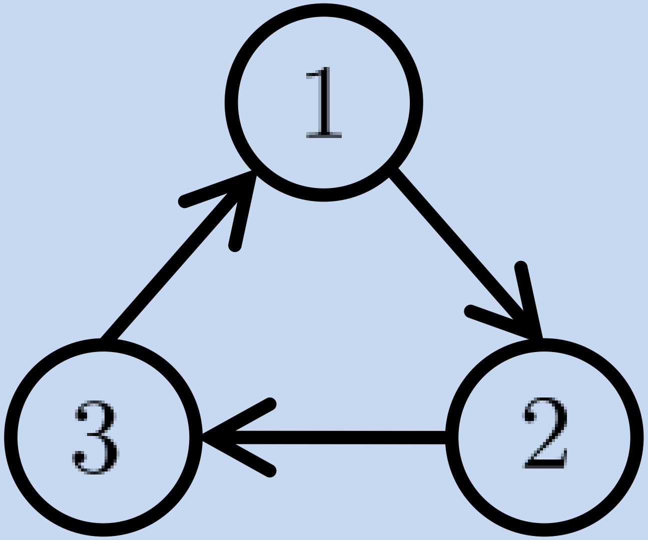
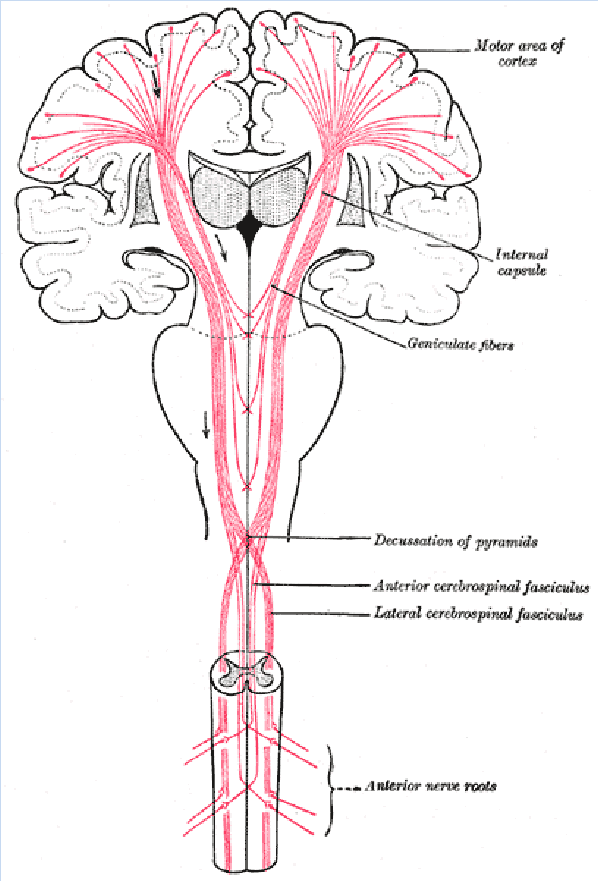

Control Theory
Engineering
Physiology

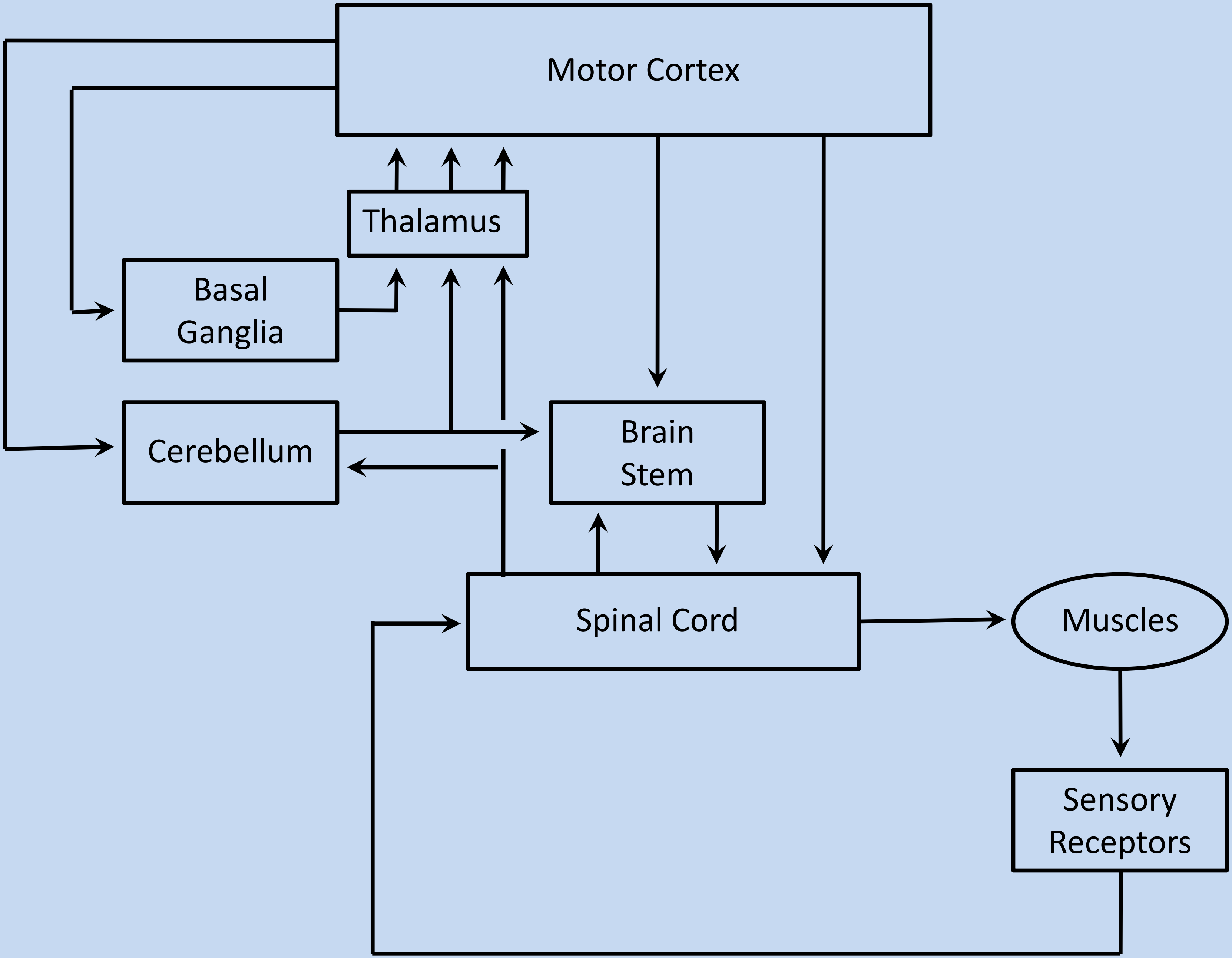
Decentralized Optimal Control and the Motor System



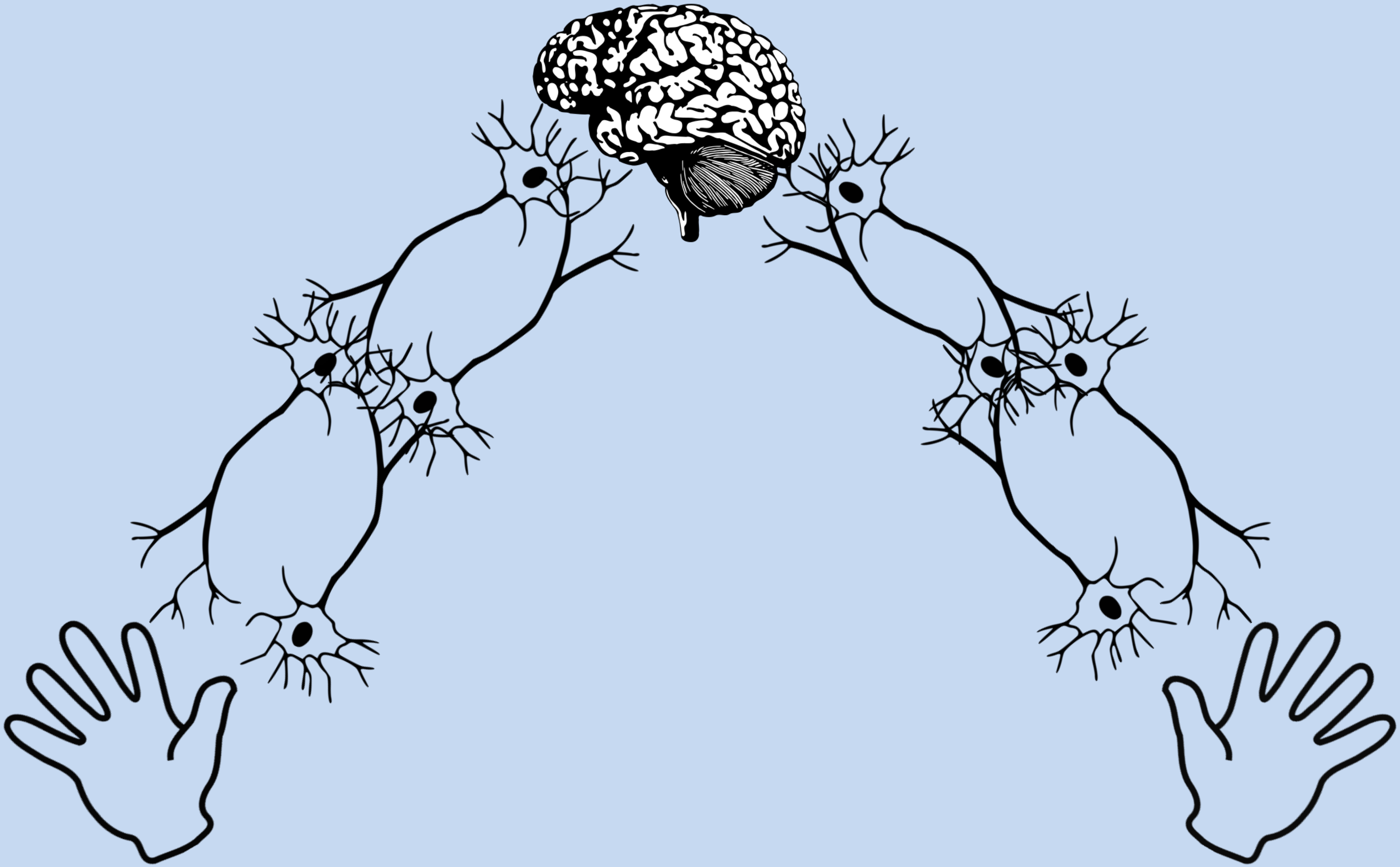


Optimal Feedback Control in the Motor System

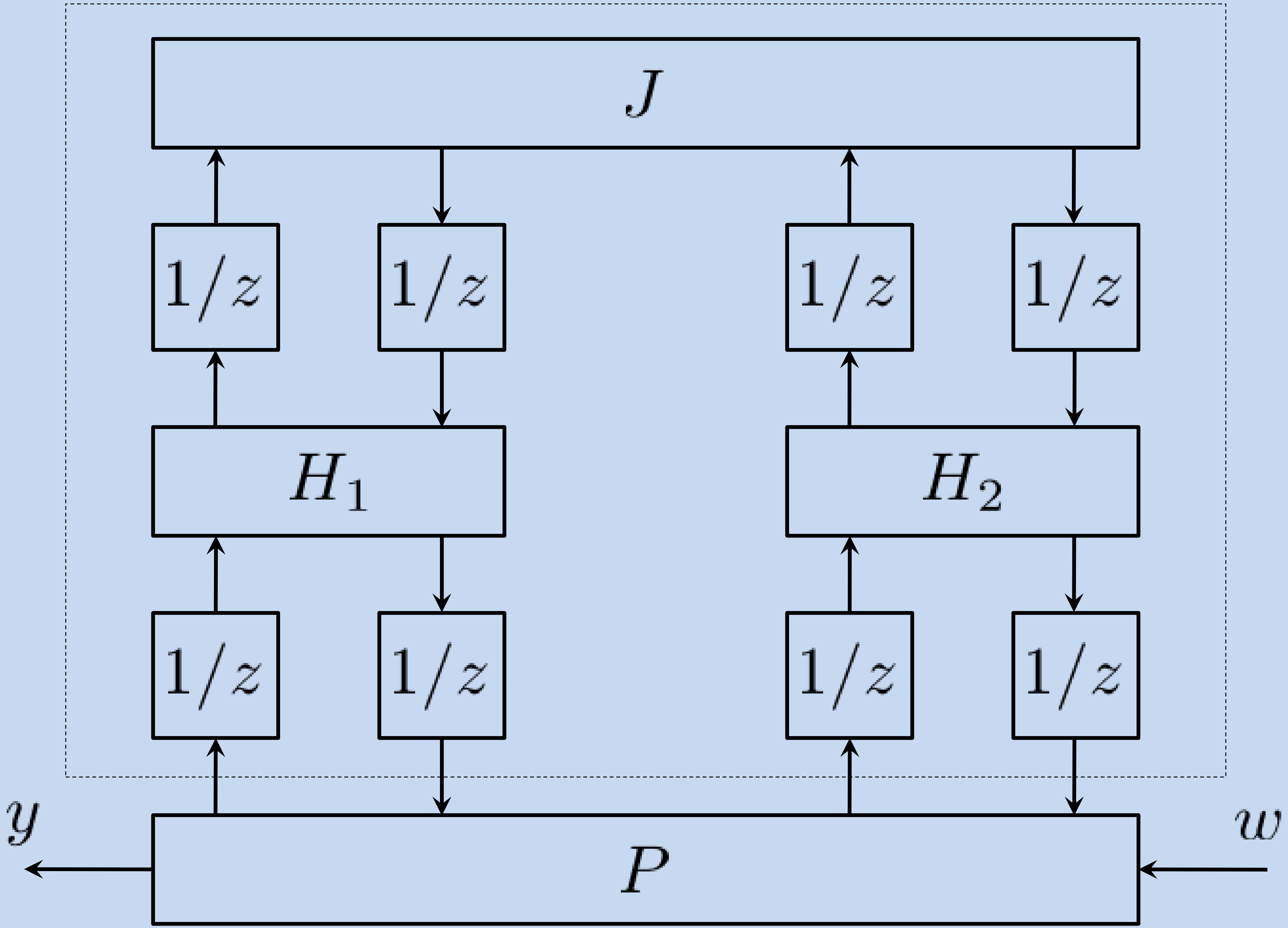
- 1990s: LQR gains for disturbance rejection
- Early 2000s: Optimal Trajectory Planning
- Early/Mid 2000s: Optimal Feedback Control
- Recent: Brain Regions for Controllers
- Recent: Sophisticated Cost Functions
- Recent: Hierarchical Control
- Not Well Studied: Distributed Processing



Adapted from Kandel, Schwartz, and Jessell



Hand: MTchemik; Neuron: Notjim; Brain: Pendethan, Wikimedia Commons

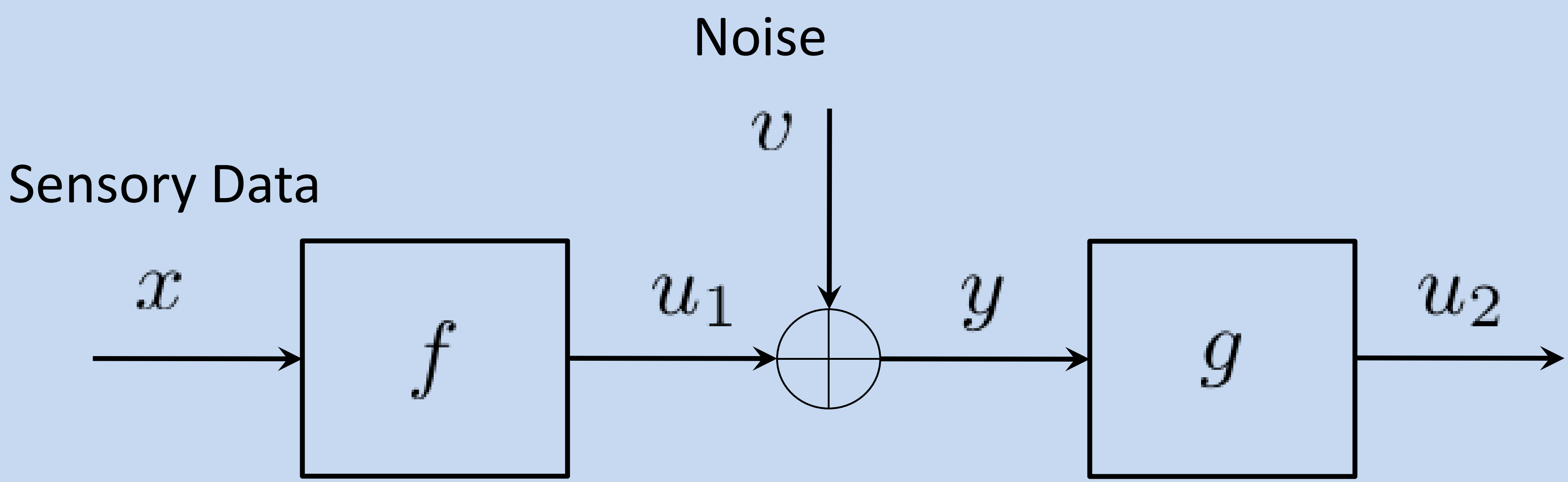

J
1/z
1/z
1/z
1/z
H_1
H_2
1/z
1/z
1/z
1/z
y
P
w

Decentralized Optimal Control

- 1960s: Optimal Controller May Be Nonlinear
- 1970s: Sufficient Conditions for Linearity
- Early/Mid 2000s: Convex Parameterizations
- Recent: Convex Optimization
- Recent: Explicit Controller Structures

$$\begin{aligned} \min_{Q} \quad & \|T_1 + T_2 Q T_3\| \\ \text{s.t.} \quad & Q \in S \end{aligned}$$

Witsenhausen's Counterexample (1968)

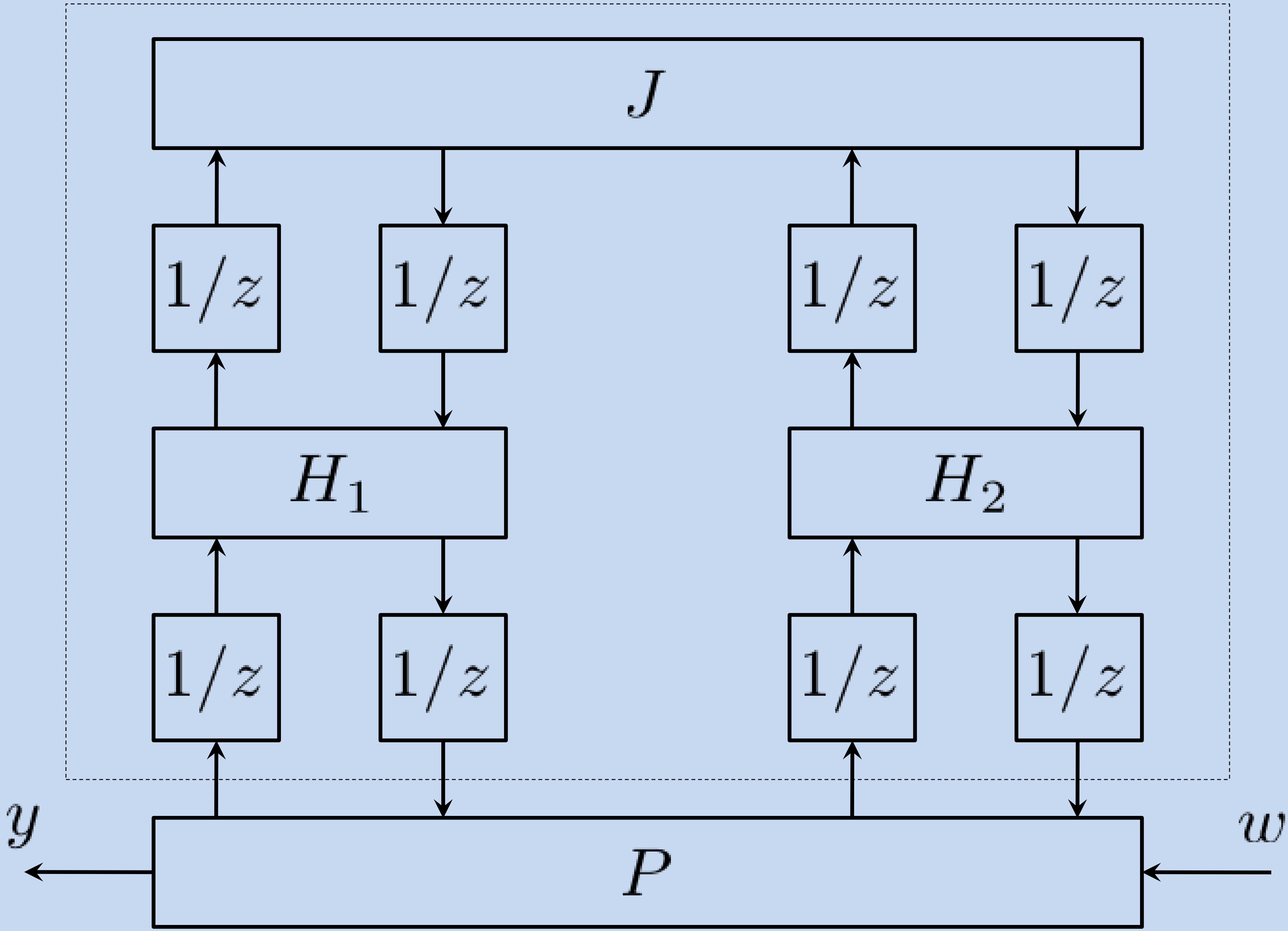


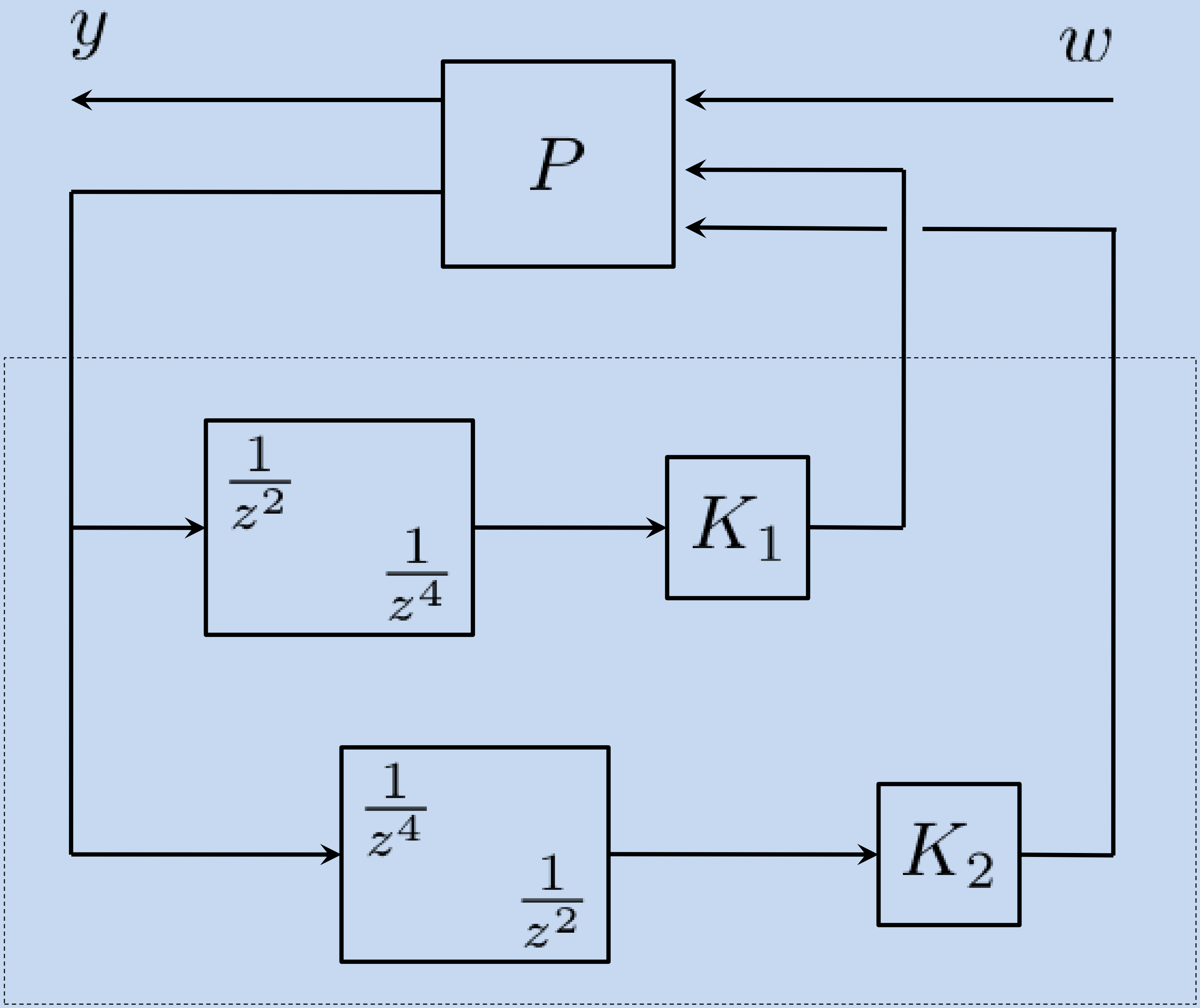
x and v are Gaussian random variables.

$$\min_{f,g} \mathbb{E}\left[k(x-u_1)^2 + (u_1-u_2)^2\right]$$

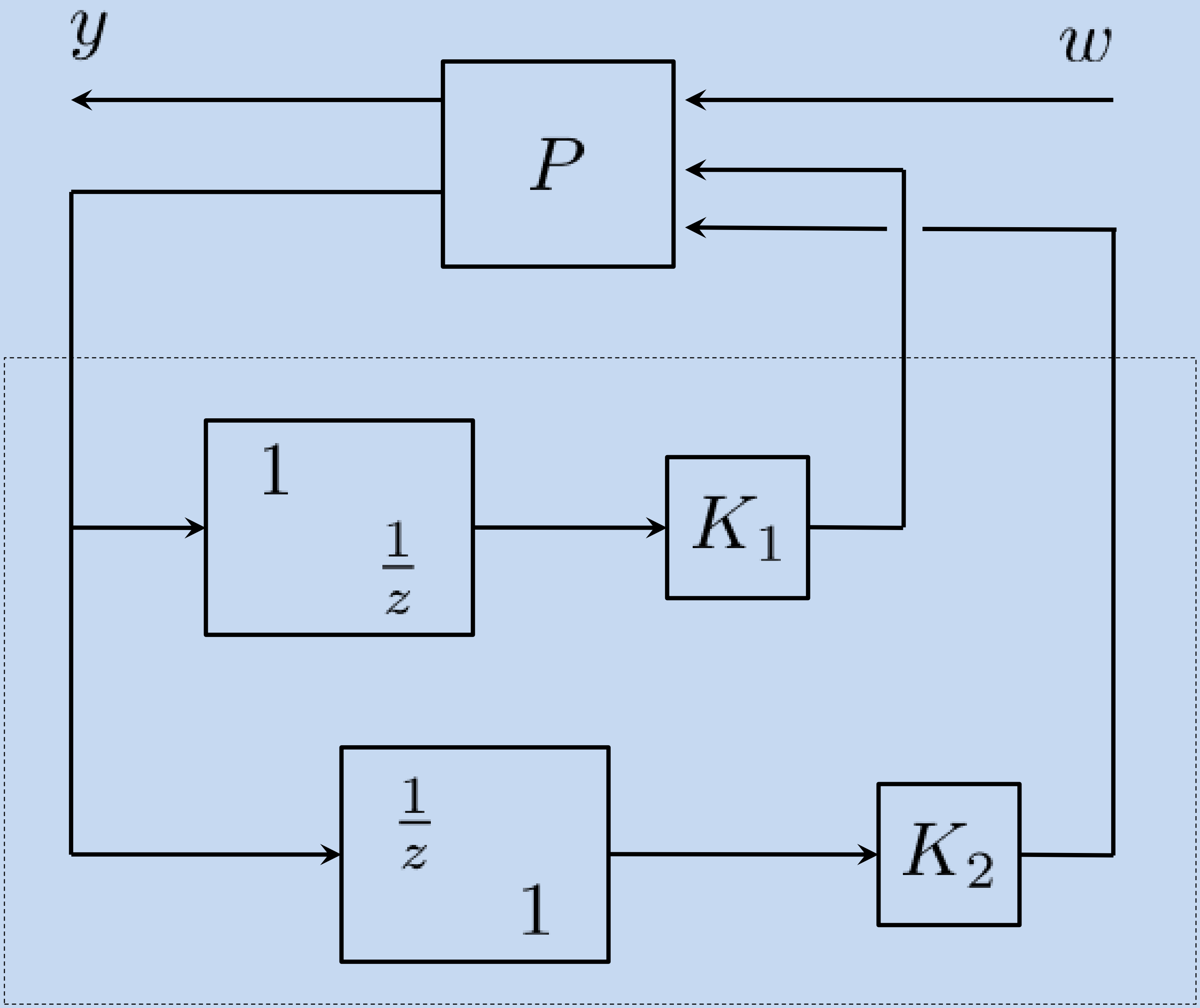
Babies Are Partially Nested

Wikimedia Commons
Author: Kyle Flood

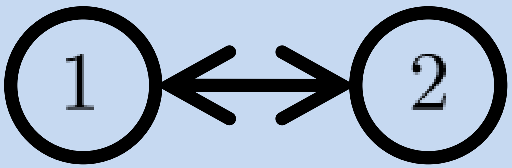

J
1/z
1/z
1/z
1/z
H1
H2
1/z
1/z
1/z
1/z
y
P
w



y
w
P
$\frac{1}{z^2}$
$\frac{1}{z^4}$
K_1
$\frac{1}{z^4}$
$\frac{1}{z^2}$
K_2


y
w
P
1
1/z
K1
1/z
1
K2

Simple Decentralized Control Problem



$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_{11} & 0 \\ 0 & B_{22} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} + \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$$

w_1, w_2 independent Gaussian white noise processes

$$\min_u \lim_{N\to\infty} \frac{1}{N} \sum_{t=0}^{N-1} \mathbb{E}\left[x(t)^T Q x(t) + u(t)^T R u(t)\right]$$

$$\begin{aligned} u_1(t) &= \gamma_{1,t}(x_1(0:t), x_2(0:t-1)) \\ u_2(t) &= \gamma_{2,t}(x_1(0:t-1), x_2(0:t)) \end{aligned}$$

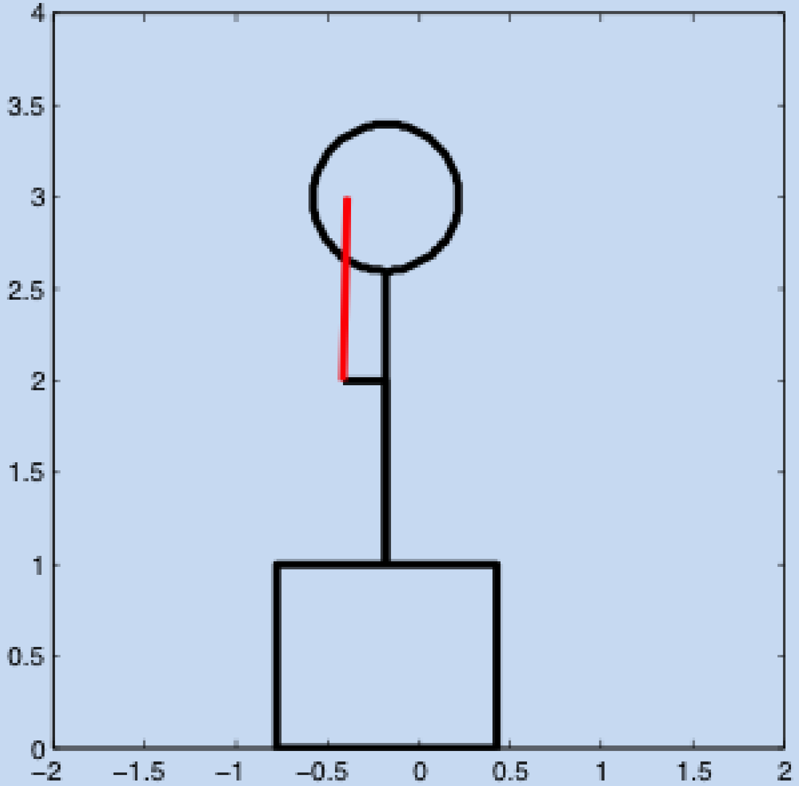
1) Centralized
$$\begin{cases} u_1(t) &= \gamma_{1,t}(x_1(0:t), x_2(0:t)) \\ u_2(t) &= \gamma_{2,t}(x_1(0:t), x_2(0:t)) \end{cases}$$

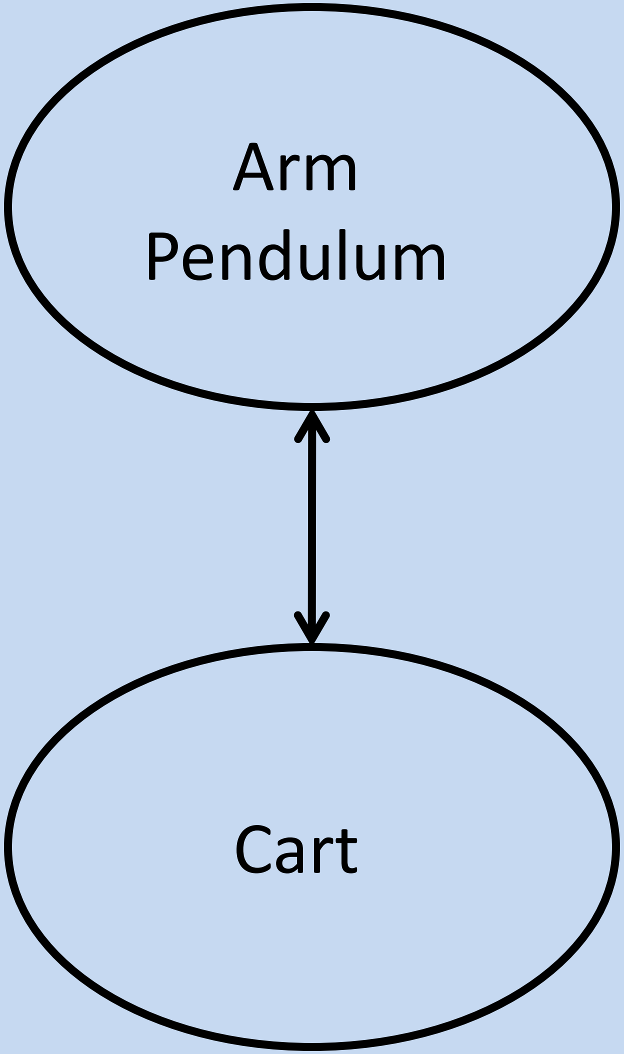
2) Decentralized
$$\begin{cases} u_1(t) &= \gamma_{1,t}(x_1(0:t), x_2(0:t-1)) \\ u_2(t) &= \gamma_{2,t}(x_1(0:t-1), x_2(0:t)) \end{cases}$$

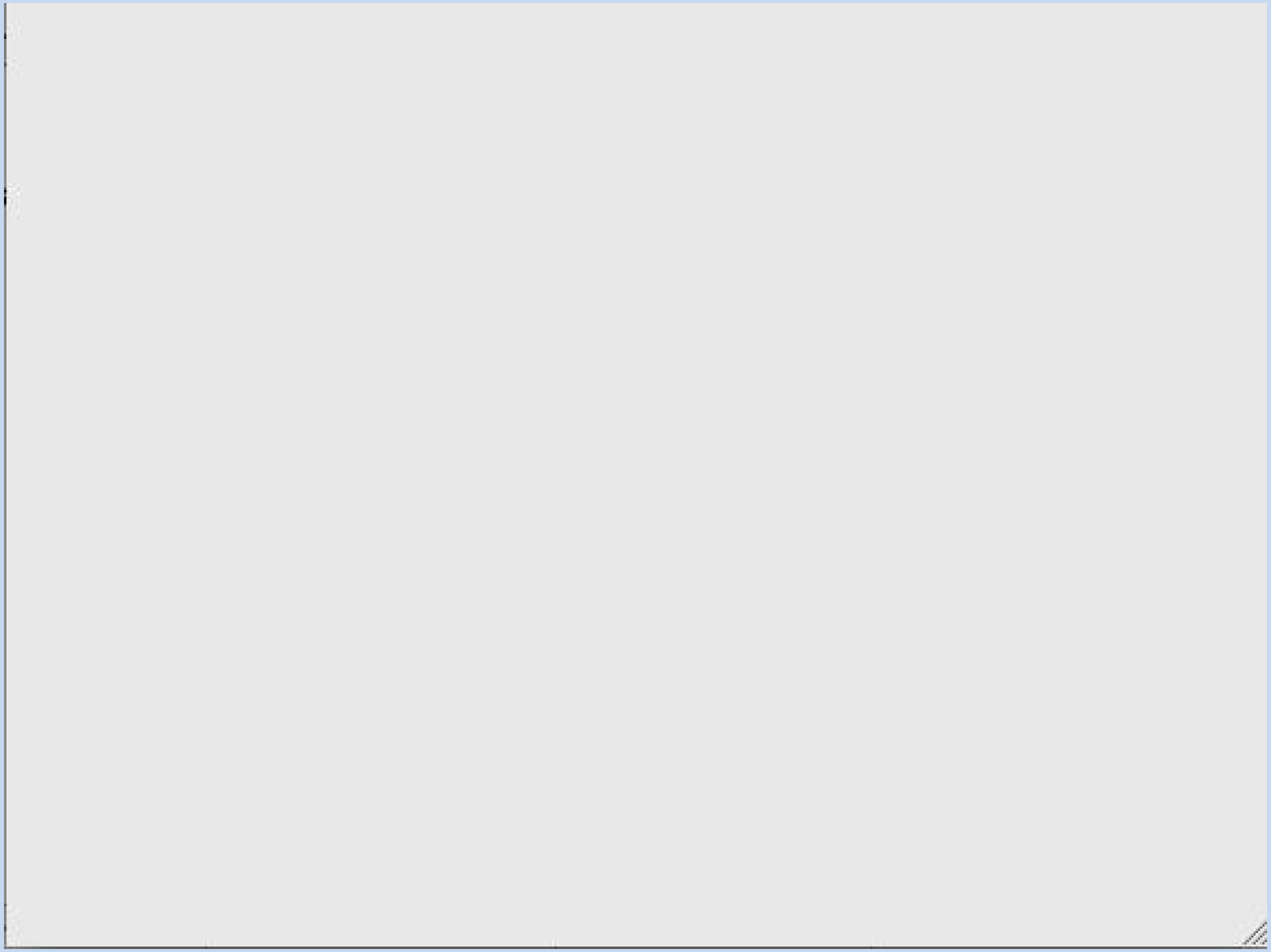
3) Delayed
$$\begin{cases} u_1(t) &= \gamma_{1,t}(x_1(0:t-1), x_2(0:t-1)) \\ u_2(t) &= \gamma_{2,t}(x_2(0:t-1), x_2(0:t-1)) \end{cases}$$

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_{11} & 0 \\ 0 & B_{22} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} + \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$$

$$\min_u \lim_{N\to\infty} \frac{1}{N} \sum_{t=0}^{N-1} \mathbb{E}\left[x(t)^T Q x(t) + u(t)^T R u(t)\right]$$


4
3.5
3
2.5
2
1.5
1
0.5
0
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2


Arm
Pendulum
Cart



1) Centralized

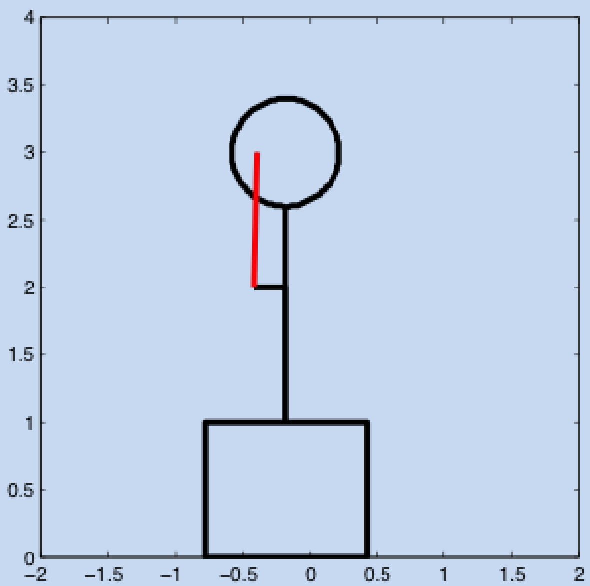
$$\begin{cases} u_1(t) = \gamma_{1,t}(x_1(0:t), x_2(0:t)) \\ u_2(t) = \gamma_{2,t}(x_1(0:t), x_2(0:t)) \end{cases}$$

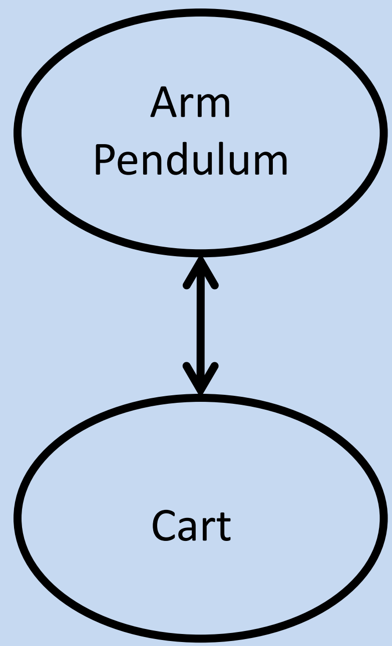
2) Decentralized

$$\begin{cases} u_1(t) = \gamma_{1,t}(x_1(0:t), x_2(0:t-1)) \\ u_2(t) = \gamma_{2,t}(x_1(0:t-1), x_2(0:t)) \end{cases}$$

3) Delayed

$$\begin{cases} u_1(t) = \gamma_{1,t}(x_1(0:t-1), x_2(0:t-1)) \\ u_2(t) = \gamma_{2,t}(x_2(0:t-1), x_2(0:t-1)) \end{cases}$$





Steady State Dynamic Programming

$$\mathbb{E}\left[V(x)\right] + c = \min_u \mathbb{E}\left[x^T Q x + u^T R u + V(x')\right]$$

- At steady state, $\mathbb{E}[V(x)] = \mathbb{E}[V(x')]$ and c is the average "cost-per-step"

$$c = \mathbb{E}\left[x^T Q x + u^T R u\right]$$

- Goal is to find V and c
- Assume that $V(x) = x^T S x$, then $\mathbb{E}[V(x)] = \mathbb{E}[V(x')]$ implies that

$$\begin{aligned} \mathbb{E}\left[x^T S x\right] &= \mathbb{E}\left[(Ax + Bu + w)^T S (Ax + Bu + w)\right] \\ &= \mathbb{E}\left[(Ax + Bu)^T S (Ax + Bu)\right] + \mathrm{Tr}(WS) \end{aligned}$$

- The Bellman equation becomes

$$\mathbb{E}\left[x^T S x\right] + c = \\ \min_u \mathbb{E}\left[x^T Q x + u^T R u + (Ax + Bu)^T S (Ax + Bu)\right] + \mathrm{Tr}(WS)$$

- Choose S to satisfy the discrete-time algebraic Riccati Equation:

$$S = Q + A^T S A - A^T S B (R + B^T S B)^{-1} B S A$$

- Let $\Omega = (R + B^T S B)$ and let $K = \Omega^{-1} B^T S A$ be the optimal gain.
- Completing the square shows that

$$x^T Q x + u^T R u + (Ax + Bu)^T S (Ax + Bu) = x^T S x + (Kx + u)^T \Omega (Kx + u)$$

- Combining the previous arguments shows that

$$\begin{aligned} &\mathbb{E}\left[x^T S x\right] + c \\ &= \mathbb{E}\left[x^T Q x + u^T R u + (Ax+Bu)^T S (Ax+Bu)\right] + \mathrm{Tr}(WS) \\ &= \mathbb{E}\left[x^T S x\right] + \mathbb{E}\left[(Kx+u)^T \Omega (Kx+u)\right] + \mathrm{Tr}(WS) \end{aligned}$$

- The cost is thus given by

$$c = \mathrm{Tr}(WS) + \mathbb{E}\left[(Kx+u)^T \Omega (Kx+u)\right]$$

- In the case of state feedback, the optimal control is seen to be $u = -Kx$ with optimal cost

$$c_{cen} = \mathrm{Tr}(WS)$$

The Delay Controller

- Start from the cost expression

$$c = \mathrm{Tr}(WS) + \mathbb{E}\left[(Kx+u)^T \Omega (Kx+u)\right]$$

- Let $\hat{x}(t) = \mathbb{E}[x(t)|x(0:t-1)]$ be the optimal state estimate, calculated as

$$\mathbb{E}[x(t)|x(0:t-1)] = Ax(t-1) + Bu(t-1)$$

- Note that $x(t) - \hat{x}(t) = w(t-1)$
- Decompose $Kx(t) + u(t)$ into independent terms as

$$Kx(t)+u(t) = K(x(t)-\hat{x}(t))+(K\hat{x}(t)+u(t)) = Kw(t-1)+(K\hat{x}(t)+u(t))$$

- The decomposition $Kx(t) + u(t) = Kw(t-1) + (K\hat{x}(t) + u(t))$ implies that

$$\begin{aligned} c &= \mathrm{Tr}(WS) + \mathbb{E}\left[(Kx(t)+u(t))^T\Omega(Kx(t)+u(t))\right] \\ &= \mathrm{Tr}(WS) + \mathrm{Tr}(WK^T\Omega K) + \mathrm{E}\left[(K\hat{x}(t)+u(t))^T\Omega(K\hat{x}(t)+u(t))\right] \end{aligned}$$

- Thus, the optimal delay controller is $u(t) = -K\hat{x}(t)$ with optimal cost

$$c_{del} = \mathrm{Tr}(WS) + \mathrm{Tr}(WK^T\Omega K)$$

- Immediate that $c_{del} \geq c_{cen}$:

$$c_{del} = \mathrm{Tr}(WS) + \mathrm{Tr}(WK^T\Omega K) \geq c_{cen} = \mathrm{Tr}(WS)$$

The Decentralized Controller

$$
\begin{aligned}
u_1(t) &= \gamma_{1,t}(x_1(0:t), x_2(0:t-1)) \\
u_2(t) &= \gamma_{2,t}(x_1(0:t-1), x_2(0:t))
\end{aligned}
$$

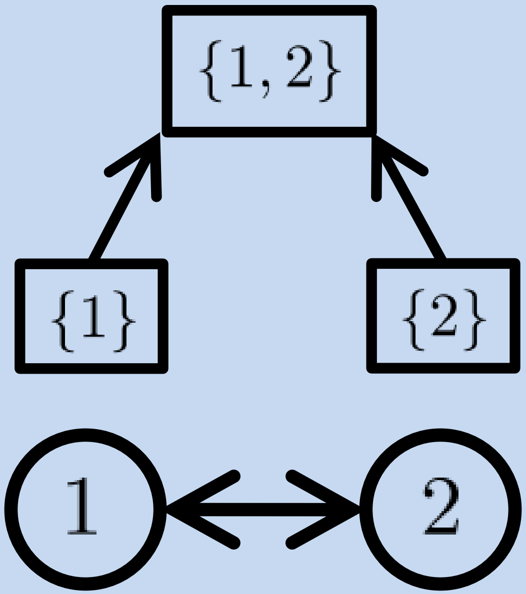
- Method needs to be modified
- $x(t)$ does not completely specify the state
- Need to describe "who knows what"
- Hierarchy arises from state decomposition

State Decomposition

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} (x_1(t) - \mathbb{E}[x_1(t)|x(0:t-1)]) + \mathbb{E}[x_1(t)|x(0:t-1)] \\ (x_2(t) - \mathbb{E}[x_2(t)|x(0:t-1)]) + \mathbb{E}[x_2(t)|x(0:t-1)] \end{bmatrix}$$

$$= \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} + \hat{x}(t)$$

ζ_1, ζ_2, and $\hat{x}$ specify the *state* of the dynamic system



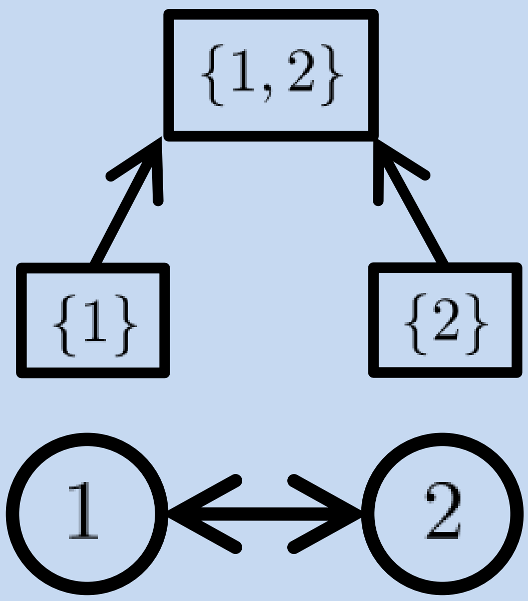
Control Structure Theorem

The optimal decentralized controller has the form

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = -\begin{bmatrix} H_1\zeta_1(t) \\ H_2\zeta_2(t) \end{bmatrix} - K\hat{x}(t).$$

Where K is the standard LQR gain.

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} + \hat{x}(t)$$



Augmented State Dynamics

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} + \hat{x}(t) \qquad \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} \varphi_1(t) \\ \varphi_2(t) \end{bmatrix} + \hat{u}(t)$$

- Recall that

$$\hat{x}(t+1) = Ax(t) + Bu(t), \quad x(t+1) - \hat{x}(t+1) = w(t)$$

- It follows that the augmented state dynamics have the form

$$\begin{aligned} \hat{x}(t+1) &= A\left(\begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} + \hat{x}(t)\right) + B\left(\begin{bmatrix} \varphi_1(t) \\ \varphi_2(t) \end{bmatrix} + \hat{u}(t)\right) \\ \zeta_1(t+1) &= w_1(t) \\ \zeta_2(t+1) &= w_2(t) \end{aligned}$$

Cost Decoupling

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} + \hat{x}(t) \qquad \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} \varphi_1(t) \\ \varphi_2(t) \end{bmatrix} + \hat{u}(t)$$

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \qquad R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$$

$$\begin{aligned} \mathbb{E}\left[x(t)^T Q x(t) + u(t)^T R u(t)\right] \quad = \quad & \mathbb{E}\left[\hat{x}(t)^T Q \hat{x}(t) + \hat{u}(t)^T R \hat{u}(t)\right] + \\ & \mathbb{E}\left[\zeta_1(t)^T Q_{11} \zeta_1(t) + \varphi_1(t)^T R_{11} \varphi_1(t)\right] + \\ & \mathbb{E}\left[\zeta_2(t)^T Q_{22} \zeta_2(t) + \varphi_2(t)^T R_{22} \varphi_2(t)\right] \end{aligned}$$

Augmented Dynamic Programming

$$\mathbb{E}[V(\hat{x},\zeta_1,\zeta_2)] + c = \min_{\hat{u},\varphi_1,\varphi_2} \mathbb{E}\left[L(\hat{x},\zeta_1,\zeta_2,\hat{u},\varphi_1,\varphi_2) + V(\hat{x}',\zeta_1',\zeta_2')\right]$$

$$\begin{aligned} L(\hat{x},\zeta_1,\zeta_2,\hat{u},\varphi_1,\varphi_2) &= \hat{x}^T Q\hat{x} + \hat{u}^T R\hat{u} + \\ &\quad \zeta_1^T Q_{11}\zeta_1 + \varphi_1^T R_{11}\varphi_1 + \\ &\quad \zeta_2^T Q_{22}\zeta_2 + \varphi_2^T R_{22}\varphi_2 + \end{aligned}$$

$$\begin{aligned} \hat{x}' &= A\left(\begin{bmatrix}\zeta_1\\ \zeta_2\end{bmatrix} + \hat{x}\right) + B\left(\begin{bmatrix}\varphi_1\\ \varphi_2\end{bmatrix} + \hat{u}\right) \\ \zeta_1' &= w_1 \\ \zeta_2' &= w_2 \end{aligned}$$

$$\mathbb{E}[V(\hat{x},\zeta_1,\zeta_2)]+c = \min_{\hat{u},\varphi_1,\varphi_2} \mathbb{E}\left[L(\hat{x},\zeta_1,\zeta_2,\hat{u},\varphi_1,\varphi_2) + V(\hat{x}',\zeta_1',\zeta_2')\right]$$

- Assume that $V(\hat{x},\zeta_1,\zeta_2)$ has the form

$$V(\hat{x},\zeta_1,\zeta_2) = \hat{x}^T S\hat{x} + \zeta_1^T X_1\zeta_1 + \zeta_2^T X_2\zeta_2$$

- Decompose A and B as

$$A = \begin{bmatrix} A_1 & A_2 \end{bmatrix} \qquad B = \begin{bmatrix} B_1 & B_2 \end{bmatrix}$$

- By independence $\mathbb{E}\left[\hat{x}'^T S\hat{x}'\right]$ expands as

$$\begin{aligned}
&\mathbb{E}\left[\hat{x}'^T S\hat{x}'\right] \\
&= \mathrm{E}\left[\left(A\left(\begin{bmatrix}\zeta_1\\ \zeta_2\end{bmatrix} + \hat{x}\right) + B\left(\begin{bmatrix}\varphi_1\\ \varphi_2\end{bmatrix} + \hat{u}\right)\right)^T S\left(A\left(\begin{bmatrix}\zeta_1\\ \zeta_2\end{bmatrix} + \hat{x}\right) + B\left(\begin{bmatrix}\varphi_1\\ \varphi_2\end{bmatrix} + \hat{u}\right)\right)\right] \\
&= \mathbb{E}\left[(A\hat{x} + B\hat{u})^T S(A\hat{x} + B\hat{u})\right] + \\
&\quad \mathbb{E}\left[(A_1\zeta_1 + B_1\varphi_1)^T S(A_1\zeta_1 + B_1\varphi_1)\right] + \\
&\quad \mathbb{E}\left[(A_2\zeta_2 + B_2\varphi_2)^T S(A_2\zeta_2 + B_2\varphi_2)\right]
\end{aligned}$$

$$\mathbb{E}[V(\hat{x},\zeta_1,\zeta_2)] + c = \min_{\hat{u},\varphi_1,\varphi_2} \mathbb{E}\left[L(\hat{x},\zeta_1,\zeta_2,\hat{u},\varphi_1,\varphi_2) + V(\hat{x}',\zeta_1',\zeta_2')\right]$$

- Recall that V has the form

$$V(\hat{x},\zeta_1,\zeta_2) = \hat{x}^T S\hat{x} + \zeta_1^T X_1\zeta_1 + \zeta_2^T X_2\zeta_2$$

- $\zeta_1' = w_1$ and $\zeta_2' = w_2$ implies that

$$\mathbb{E}\left[\zeta_1'^T X_1\zeta_1'\right] = \mathbb{E}\left[w_1^T X_1 w_1\right] = \mathrm{Tr}(W_1X_1)$$
$$\mathbb{E}\left[\zeta_2'^T X_2\zeta_2'\right] = \mathbb{E}\left[w_2^T X_2 w_2\right] = \mathrm{Tr}(W_2X_2)$$

- Bellman equation decomposes as

$$\begin{aligned}
&\mathbb{E}\left[\hat{x}^T S\hat{x} + \zeta_1^T X_1\zeta_1 + \zeta_2^T X_2\zeta_2\right] + c = \\
&\quad \min_{\hat{u}} \mathbb{E}\left[\hat{x}^T Q\hat{x} + \hat{u}^T R\hat{u} + (A\hat{x}+B\hat{u})^T S(A\hat{x}+B\hat{u})\right] + \\
&\quad \min_{\varphi_1} \mathbb{E}\left[\zeta_1^T Q_{11}\zeta_1 + \varphi_1^T R_{11}\varphi_1 + (A_1\zeta_1+B_1\varphi_1)^T S(A_1\zeta_1+B_1\varphi_1)\right] + \\
&\quad \min_{\varphi_2} \mathbb{E}\left[\zeta_2^T Q_{22}\zeta_2 + \varphi_2^T R_{22}\varphi_2 + (A_2\zeta_1+B_2\varphi_2)^T S(A_2\zeta_2+B_2\varphi_2)\right] + \\
&\quad \mathrm{Tr}(W_1X_1) + \mathrm{Tr}(W_2X_2)
\end{aligned}$$

$$
\begin{aligned}
&\mathbb{E}\left[\hat{x}^T S\hat{x} + \zeta_1^T X_1 \zeta_1 + \zeta_2^T X_2 \zeta_2\right] + c = \\
&\quad \min_{\hat{u}} \mathbb{E}\left[\hat{x}^T Q\hat{x} + \hat{u}^T R\hat{u} + (A\hat{x} + B\hat{u})^T S(A\hat{x} + B\hat{u})\right] + \\
&\quad \min_{\varphi_1} \mathbb{E}\left[\zeta_1^T Q_{11}\zeta_1 + \varphi_1^T R_{11}\varphi_1 + (A_1\zeta_1 + B_1\varphi_1)^T S(A_1\zeta_1 + B_1\varphi_1)\right] + \\
&\quad \min_{\varphi_2} \mathbb{E}\left[\zeta_2^T Q_{22}\zeta_2 + \varphi_2^T R_{22}\varphi_2 + (A_2\zeta_1 + B_2\varphi_2)^T S(A_2\zeta_2 + B_2\varphi_2)\right] + \\
&\quad \mathrm{Tr}(W_1 X_1) + \mathrm{Tr}(W_2 X_2)
\end{aligned}
$$

- Optimal inputs seen to be

$$
\begin{aligned}
\hat{u} &= -(R + B^T SB)^{-1} B^T SA\hat{x} \\
\varphi_1 &= -(R_{11} + B_1^T SB_1)^{-1} B_1^T SA_1\zeta_1 \\
\varphi_2 &= -(R_{22} + B_2^T SB_2)^{-1} B_2^T SA_2\zeta_2
\end{aligned}
$$

- Choose X_1 and X_2 as

$$
\begin{aligned}
X_1 &= Q_{11} + A_1^T SA_1 - A_1^T SB_1(R_{11} + B_1^T SB_1)^{-1} B_1^T SA_1 \\
X_2 &= Q_{22} + A_2^T SA_2 - A_2^T SB_2(R_{22} + B_2^T SB_2)^{-1} B_2^T SA_2
\end{aligned}
$$

- Average cost-per-step becomes

$$
c_{dec} = \mathrm{Tr}(W_1 X_1) + \mathrm{Tr}(W_2 X_2)
$$

Cost Comparisons

- By independence of w_1 and w_2 the covariance is given by

$$W = \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix}$$

- In general $c_{cen} \leq c_{dec} \leq c_{del}$:

$$\mathrm{Tr}(WS) \leq \mathrm{Tr}(W_1 X_1) + \mathrm{Tr}(W_2 X_2) \leq \mathrm{Tr}(WS) + \mathrm{Tr}(WK^T \Omega K)$$

- After a bit of algebra

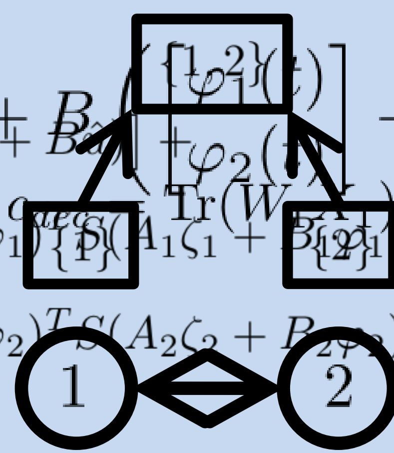
$$\begin{aligned} \begin{array}{c} \mathrm{Tr}(W(Q + A^T SA)) \\ -\mathrm{Tr}(B^T SA_1 W_1 A_1^T SB\Omega^{-1}) \\ -\mathrm{Tr}(B^T SA_2 W_2 A_2^T SB\Omega^{-1}) \end{array} &\leq \begin{array}{c} \mathrm{Tr}(W(Q + A^T SA)) \\ -\mathrm{Tr}\left(B^T SA_1 W_1 A_1^T SB \begin{bmatrix} \Omega_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix}\right) \\ -\mathrm{Tr}\left(B^T SA_2 W_2 A_2^T SB \begin{bmatrix} 0 & 0 \\ 0 & \Omega_{22}^{-1} \end{bmatrix}\right) \end{array} \\ &\leq \mathrm{Tr}(W(Q + A^T SA)) \end{aligned}$$

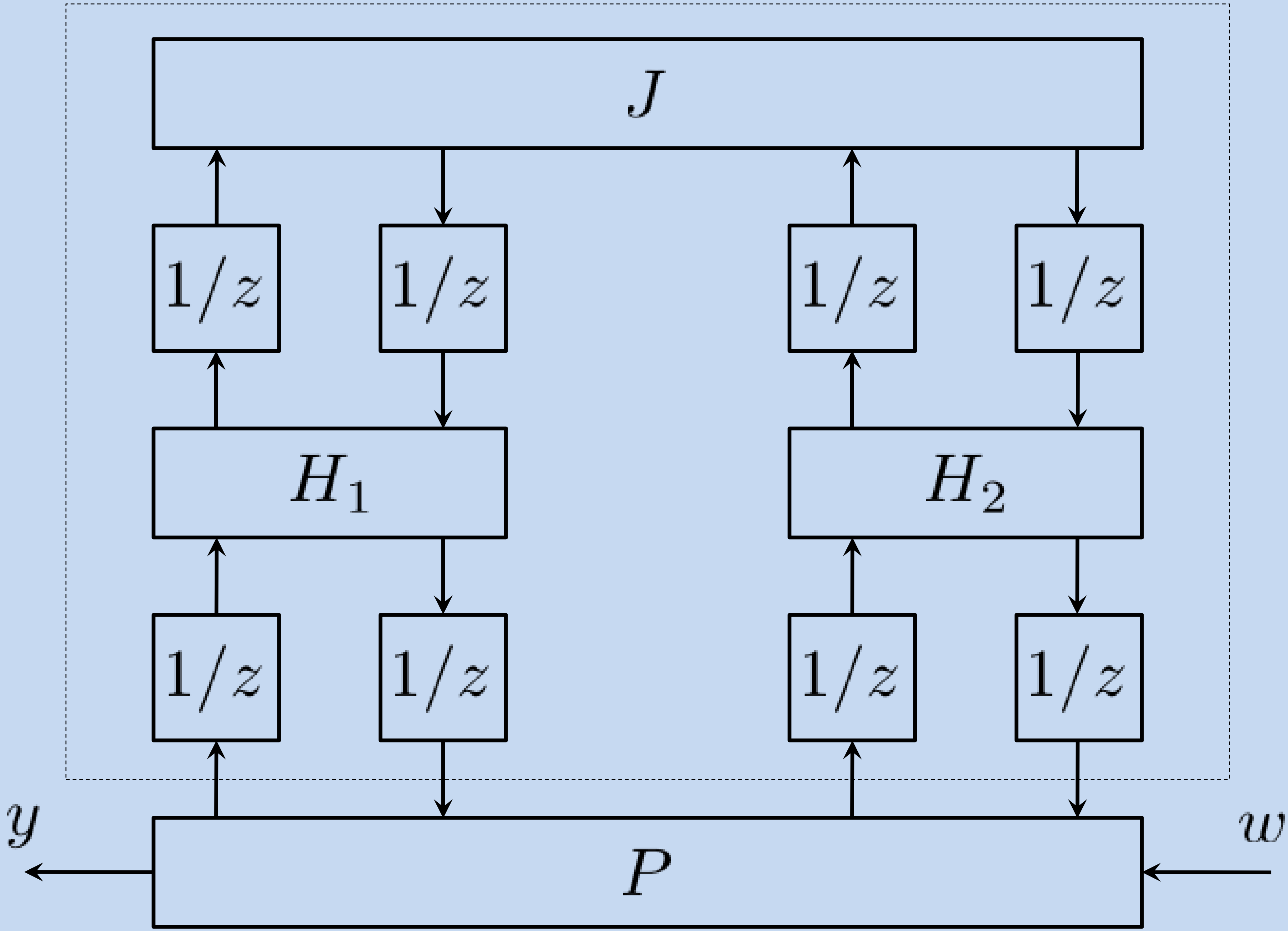
- First inequality from inverses of positive definite matrices:

$$\begin{bmatrix} \Omega_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \preceq \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}^{-1} \qquad \begin{bmatrix} 0 & 0 \\ 0 & \Omega_{22}^{-1} \end{bmatrix} \preceq \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}^{-1}$$

Review of Method

- Hierarchically decompose state and input
- Compute augmented state dynamics
- Decouple Bellman equation using independence
- Compute inputs and cost by dynamic programming





J
1/z
1/z
1/z
1/z
H_1
H_2
1/z
1/z
1/z
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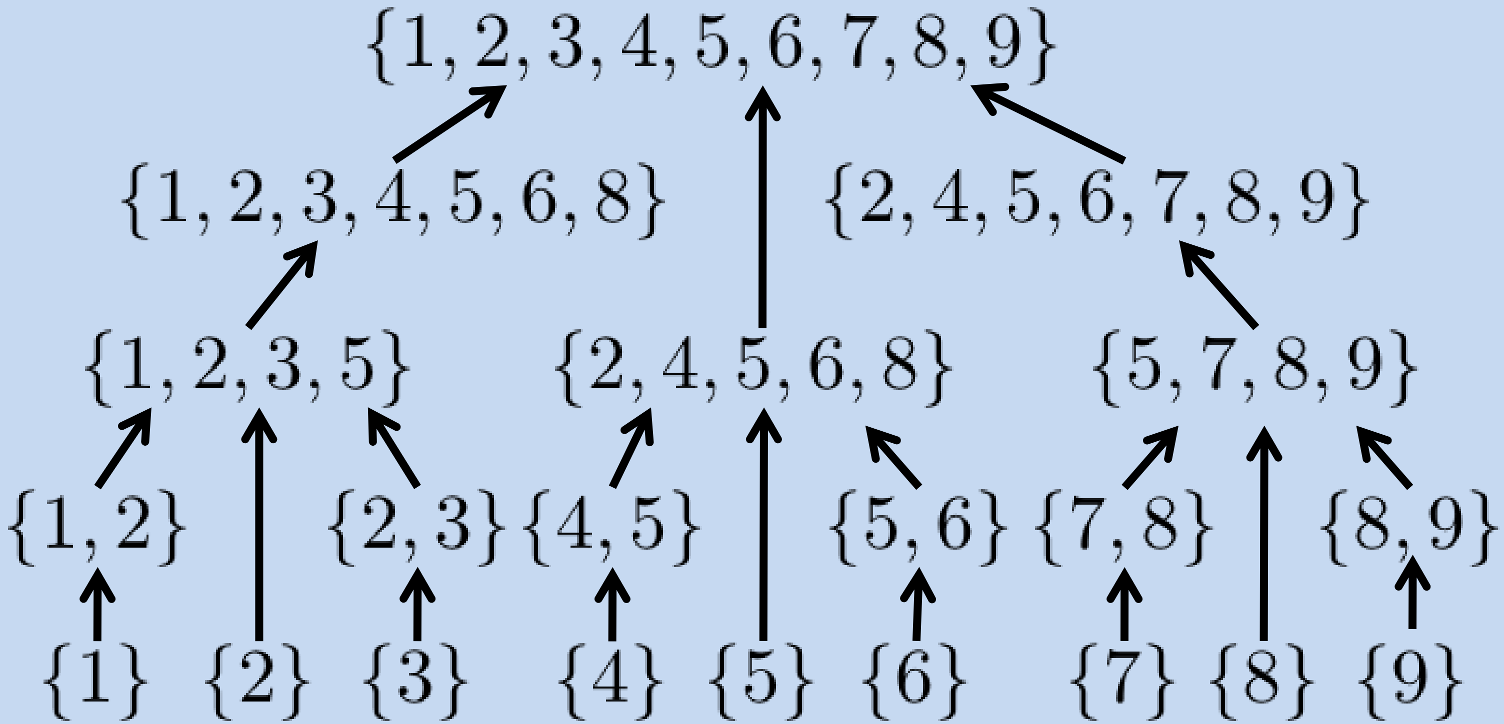
y
P
w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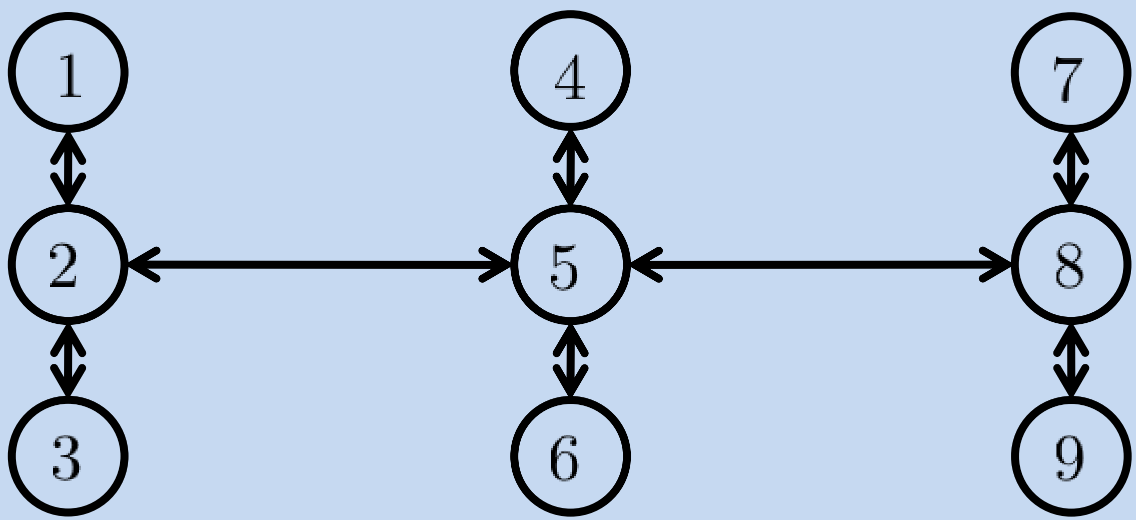
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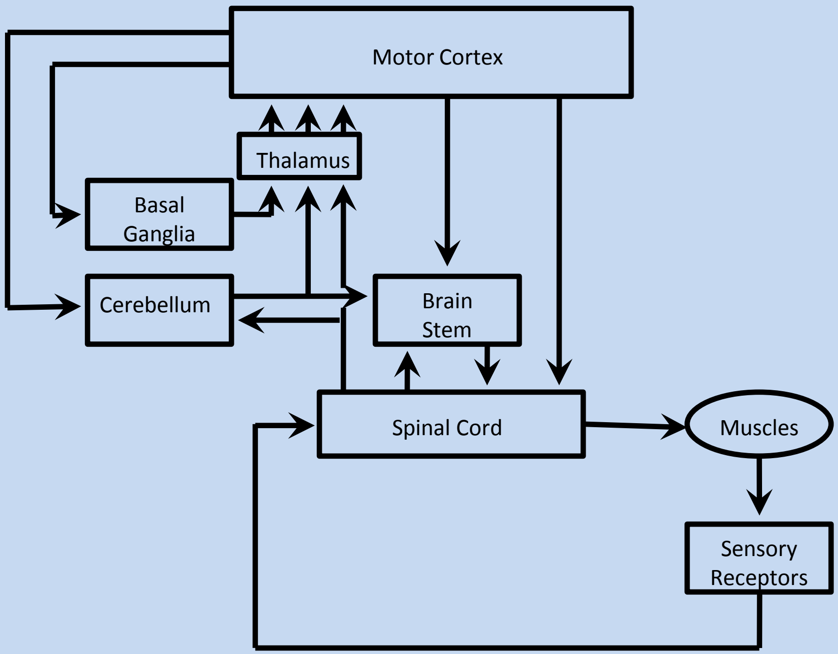
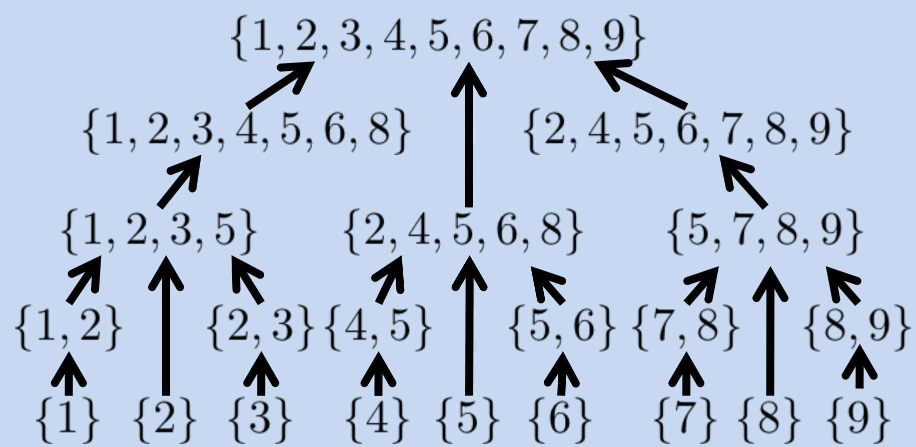


Relations to the Motor System?



- Hierarchy of feedback loops
- Information used as soon as possible

What Next?

- Output Feedback
- Risk-Sensitive Control
- Biologically Motivated Cost Functions
- Strengthen Connection to Biology