Generating **entanglement** through **measurements**:

A time to **tear down** and a time to **build**

Zhu-Xi Luo
KITP, University of California, Santa Barbara
& Harvard University
Casual self–introduction

• Shanghai → Salt Lake City → SB → Cambridge

• Condensed matter theory; its intersections with high energy theory and quantum information
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Measurements have Observer effect which generates Long-range entanglement and makes the system Topological quantities help prevent errors in Quantum computation
Measurements

In daily life
How can measurements affect the system being measured?
Your see a police officer measuring your speed

And you slow down
Observer effect
Disturbance of an observed system by the act of observation

Observer effect is more dramatic in the quantum world.
Quantum bear in the woods

\[ |\text{bear}\rangle = c_1 |\text{left}\rangle + c_2 |\text{middle}\rangle + c_3 |\text{right}\rangle \quad \iff \quad |\text{bear}\rangle = |\text{left}\rangle \]
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• Describes how independent the subsystems are.
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\[ B \]

\[ A \]
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How measurement destroys entanglement

• Start from the maximally entangled twin state \( \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle_{AB} + |\downarrow\downarrow\rangle_{AB}) \)

• Measure A. Get either up or down, say it’s up. Then \( |\uparrow\uparrow\rangle_{AB} = |\uparrow\rangle_{A} \otimes |\uparrow\rangle_{B} \) and there is no entanglement left.

• There are four different types of “twin states”, all two-qubit states can be described by combinations of them.

| \( \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle_{AB} + |\downarrow\downarrow\rangle_{AB}) \) | \( \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle_{AB} + |\downarrow\uparrow\rangle_{AB}) \) |
| --- | --- |
| \( \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle_{AB} - |\downarrow\downarrow\rangle_{AB}) \) | \( \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle_{AB} - |\downarrow\uparrow\rangle_{AB}) \) |
There is ... a time to tear down and a time to build.  (Ecclesiastes 3:3)
How measurement *creates* entanglement

(i) Start from the unentangled state $|\uparrow\uparrow\rangle_{AB}$

(ii) Measure A and B to see whether they are *up or down*. System will remain unentangled.

(iii) Instead of doing (ii), measure A and B to see which *twin state* they are in. Whatever the result, AB will become entangled.

<table>
<thead>
<tr>
<th>Cheatsheet</th>
<th></th>
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<tbody>
<tr>
<td>$\frac{1}{\sqrt{2}}(</td>
<td>\uparrow\uparrow\rangle_{AB} +</td>
</tr>
<tr>
<td>$\frac{1}{\sqrt{2}}(</td>
<td>\uparrow\uparrow\rangle_{AB} -</td>
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Measurements can change entanglement

So what?
The logic

Quick review and preview

Measurements have Observer effect which generates Long-range entanglement and makes the system Topological quantities help prevent errors in Quantum computation
Quantum computation

$$|\text{bear}\rangle = c_1 |\text{left}\rangle + c_2 |\text{middle}\rangle + c_3 |\text{right}\rangle$$
Is it good?

- Quantum computers can solve **all** problems that classical computers can solve.
- Quantum computers tackle certain problems **much faster** than classical computers do.
- Quantum computation do **not always** significantly outperform classical computation, even theoretically.

<table>
<thead>
<tr>
<th># of qubits</th>
<th>Computation Power</th>
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<tbody>
<tr>
<td></td>
<td>OK</td>
</tr>
<tr>
<td></td>
<td>Exponentially better</td>
</tr>
<tr>
<td></td>
<td>Polynomially better</td>
</tr>
<tr>
<td>$e^x$</td>
<td>OK</td>
</tr>
<tr>
<td>$t^2$</td>
<td>OK</td>
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• Quantum computers can solve all problems that classical computers can solve.
• Quantum computers tackle certain problems much faster than classical computers do.
• Quantum computation do not always significantly outperform classical computation, even theoretically.
• In reality, quantum states are more fragile than classical counterparts.
  ○ Biggest challenge: Error and decoherence.
Using Topology

To protect quantum computation from errors

• Topological quantities won’t change unless you use scissors and glue.
• A classification coarser than shapes.
• Use topological quantities to encode information.  
  [Kitaev (97’), Freedman, Larsen, Wang (00’)]
• Local errors won’t change the information encoded.
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Topological systems are interesting by themselves

- Topological phases of matter are quantum and weird. They now comprise a huge field.
  - Host emergent quasiparticles with **fractionalized** charge and statistics
  - Exhibit **long-range entanglement**
Short-range entanglement

- Short-range entangled state are not separable but can become separable upon local transformations.

\[ |\Psi\rangle_{AB} \neq |\phi\rangle_A |\varphi\rangle_B \Rightarrow |\Psi'\rangle_{AB} = |\phi\rangle_A |\varphi\rangle_B \]

- If entanglement is short-ranged, it typically has this form

\[ S_A = \alpha l + \cdots \]
Long-range entanglement

Feature of topological phases of matter

- Long-range entangled states: neither separable nor separable after local transformations.

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- One measure of long-range entanglement is the topological entanglement entropy.

[Kitaev, Preskill (2005), Levin, Wen (2005)]
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\[ S_{\text{topo}} \equiv S_A + S_B + S_C - S_{AB} - S_{BC} - S_{AC} + S_{ABC} \]

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\[ l_{A,B} + l_{A,C} + l_{A,D} \quad \text{All short-range entanglement cancel.} \]

[Kitaev, Preskill (2005), Levin, Wen (2005)]
Our setup

• Honeycomb lattice, qubits live on vertices.
• Three types of links. Green: x-type, blue: y-type, red: z-type.
• Start from a state where each qubit is a twin with an external qubit.
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Random measurements

• At each time step, near a vertex, choose to measure one of the three bonds with probabilities $p_x$, $p_y$, $p_z$

• After measurement, the qubit in the system disentangle with the external qubit and entangle with an internal neighboring qubit.

• Entanglement among qubits on the honeycomb lattice increases.
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Competing measurements

• These measurements do not commute with each other: $\mathcal{M}_x \circ \mathcal{M}_z \neq \mathcal{M}_z \circ \mathcal{M}_x$

• They compete.

• With different combinations of $p_x$, $p_y$, $p_z$, the system will exhibit different behaviors.

Honeycomb
Main results

• A phase with two dynamically generated logical qubits.
• C phase with logarithmic violation of the short-range entangled form.
• Both have nontrivial topological entanglement entropy, i.e. are long-range entangled.
Summary

Measurements have Observer effect, which generates Long-range entanglement and makes the system Topological quantities help prevent errors in Quantum computation.
Thank you!
Additional slides
Entropy evolution in time

Long-cycle qubits protected until exponential time

(i) Early time: Plaquette qubits
(ii) Late time: Nontrivial-cycle qubits (2)
(iii) Intermediate: Bond qubits
Loop representation

• Evolution described by a loop model on a 3d lattice.

• Measurements reconnect the loops.

• The loop model has two phases:
  
  ○ Short loops, $Q(r) \sim \exp(-r/\xi)$, A phase.
  
  ○ Loops take Brownian walk, $Q(r) \sim r^{-2}$. C phase.
Mutual information

\[ I_2(A : C) = S_A + S_C - S_{AC} \]

A phase:

C phase:

\[ \int_{-L/2}^{L/2} dy \int_0^{L/4} dx' \int_{-L/2}^{L/2} dy \int_{x'+L/4}^{x'+L/2} dx \frac{1}{(x^2 + y^2)^{3/2}} \]
Tripartite information

\[ I_3(A : B : C) = I_2(A : B) + I_2(A : C) - I_2(A : BC) \]

- One in the A phase, and minus one in the C phase
- Different from TEE, which is always 1.