Control of spiral wave location

V. Zykov,

H. Brandtstaedter, G. Bordiougov, J. Breuer,
H. Engel

Institute of Theoretical Physics, Technical University Berlin, Germany

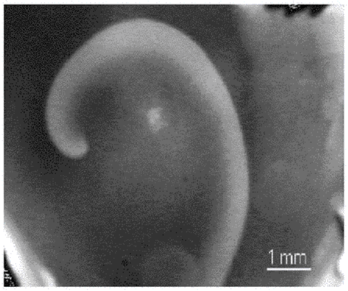
S. Zykov, V. Davydov

Moscow Institute of Radioengeneering, Electronics and Automation, Russia

Contents

- Introduction
- Resonant drift of a spiral wave
- Discrete feedback control
- Continuous feedback control
- Traveling-wave modulation of excitable media
- Summary
- Outlook

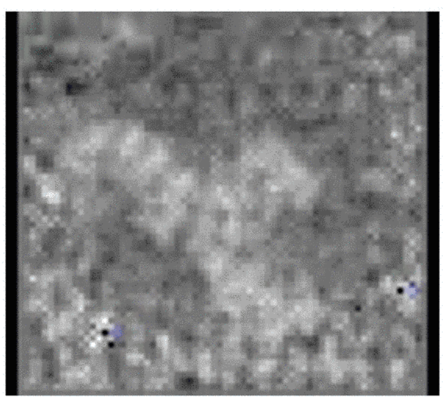
Spiral waves in biology



Spiral-shaped propagation
of "Spreading Depression"
in neuronal tissue,
here through the chicken retina.

Spiral geometry of a signal transmitter in an amoeba population (*Dictyostelium discoideum*) leads to chemotactic movements of cells in direction toward the spiral core.

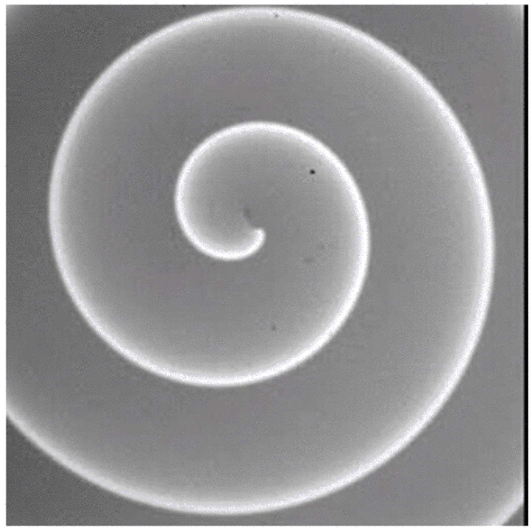
Spiral waves in medicine



Cardiac cells, when grown as a sheet of tissue, often exhibit spiral waves of electrical activity after two days in culture. Such spiral waves, which have been associated with abnormal rhythms in human hearts, may be a precursor to fatal cardiac rhythms such as ventricular fibrillation.

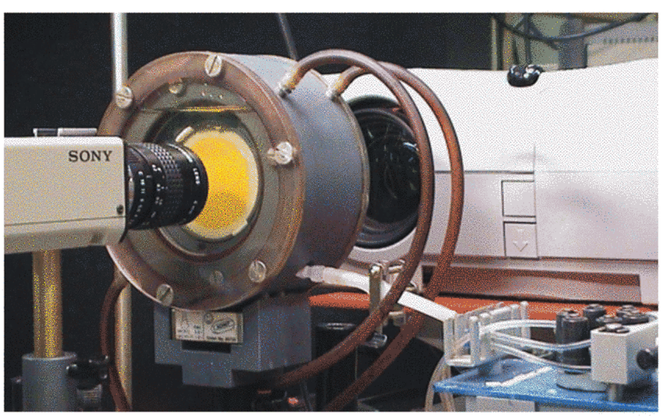
Gil Bub, Alvin Shrier, and Leon Glass

Spiral waves in the Belousov-Zhabotinsky reaction

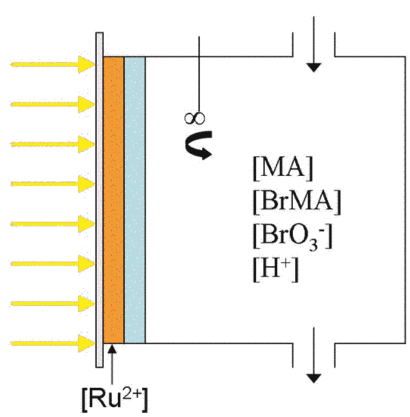


Open reactor for the light sensitive BZ medium

Experimental setup



Side view



The Oregonator model for the light-sensitive BZ reaction

$$\frac{\partial u}{\partial t} = D\nabla^2 u + \frac{1}{\varepsilon}\left[u - u^2 - (fv + I)\frac{u-q}{u+q}\right]$$

$$\frac{\partial v}{\partial t} = u - v$$

$$I = I(x,y,t)$$

Kinematics of spiral wave drift under periodic parameter modulation

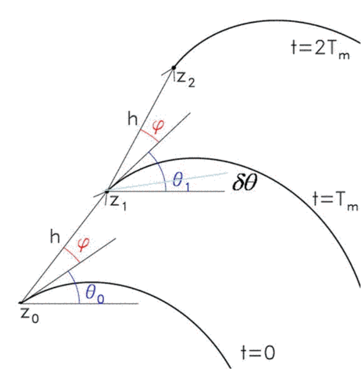
$$I(t) = A_I \sum_{l=0}^{k} \delta(t - lT_m),$$

$$z_k = z_{k-1} + h\exp(i\theta_{k-1} + i\varphi),$$

$$\theta_{k-1} = \theta_{k-2} + (\overline{\omega} - \omega_m)T_m, \quad \overline{\omega} = \omega_0 + \delta\theta / T_m$$

$$\omega_m = \overline{\omega} \quad \Rightarrow \quad z_k = z_0 + kh\exp(i\theta_0 + i\varphi)$$

$$I(t) = A_I \sum_{l=0}^{k} \delta(t - lT_m - t_0) \quad \Rightarrow \quad \gamma = \varphi + \theta_0 + \phi_{\text{mod}}$$

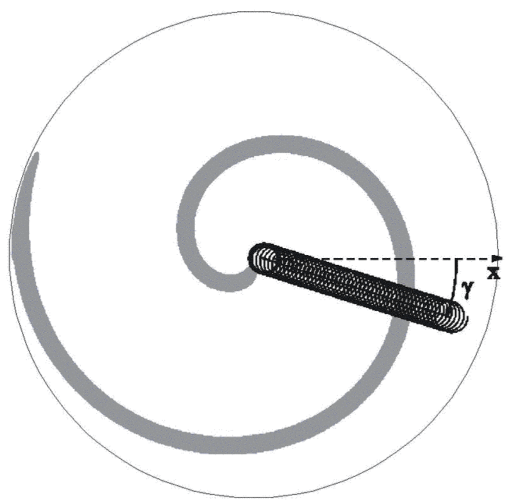


$$\omega_m = n\overline{\omega} \quad \Rightarrow \quad z_{k+n} = z_k$$

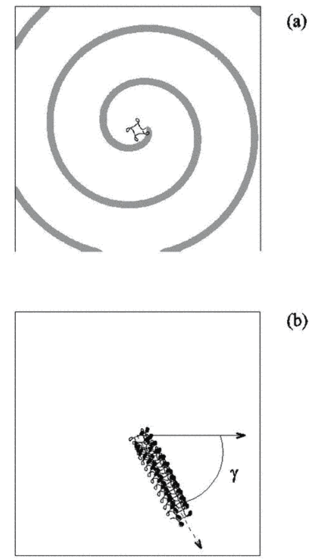
$$|\omega_m / \overline{\omega} - 1| << 1 \quad \Rightarrow \quad \dot{z} = \frac{h}{T_m}\exp\{i[(\overline{\omega} - \omega_m)t + \theta_0 + \varphi]\}$$

Resonant drift of spiral waves

Rigidly rotating spiral



Slightly meandering spirall



First attempts of a phase feedback control

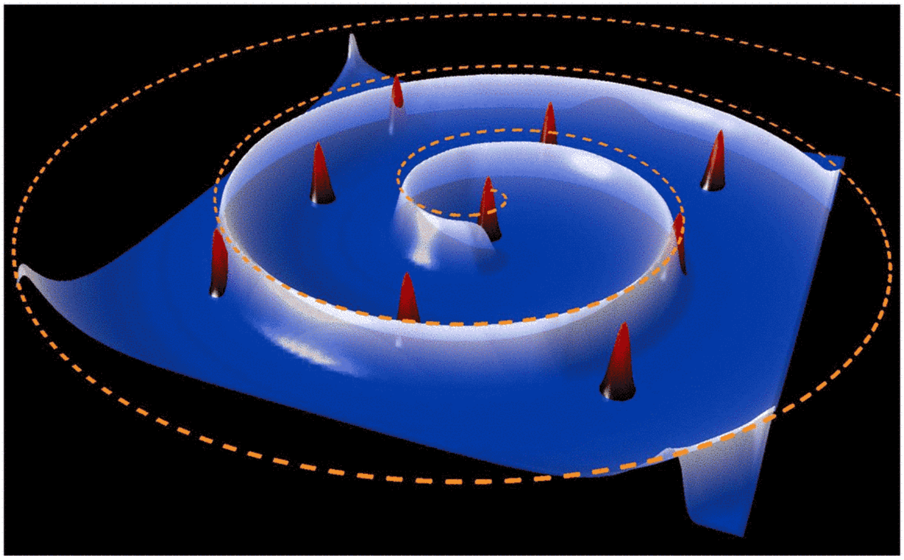
Agladze, Davydov, Mikhailov, JETP Lett., (1987)

Resonant drift or phase feedback control ?

Tung, Chan, PRL, (2002)

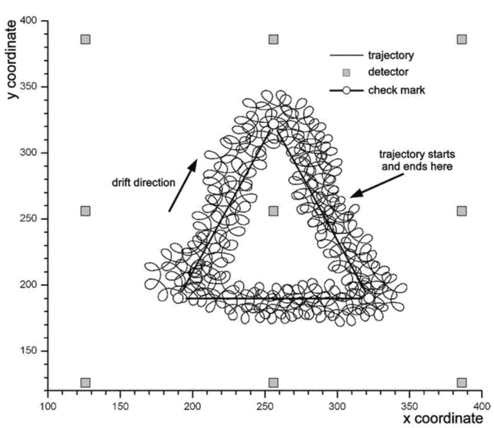
Works for rigidly rotating spiral. Does not work for meandering spiral.

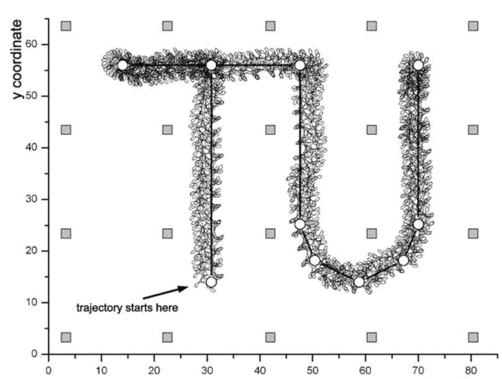
Identification of spiral wave parameters



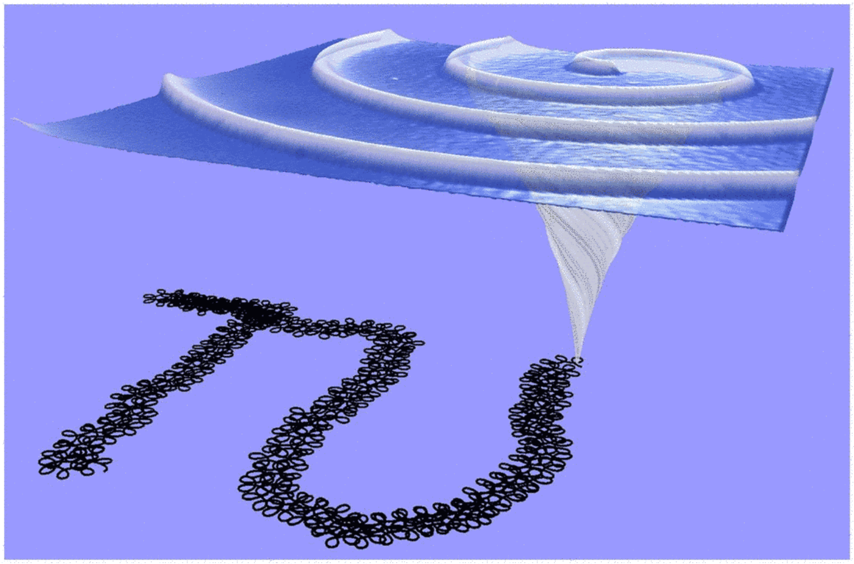
$$\theta(r,t) = \theta_0 - 2\pi r / \lambda + \omega t \quad \Leftrightarrow \quad \theta_0, \lambda, \omega, x_0, y_0$$

Phase feedback control in the Oregonator model





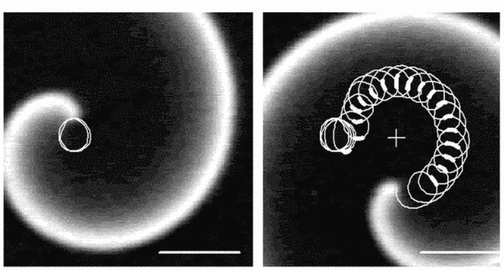
Phase feedback control in the BZ reaction

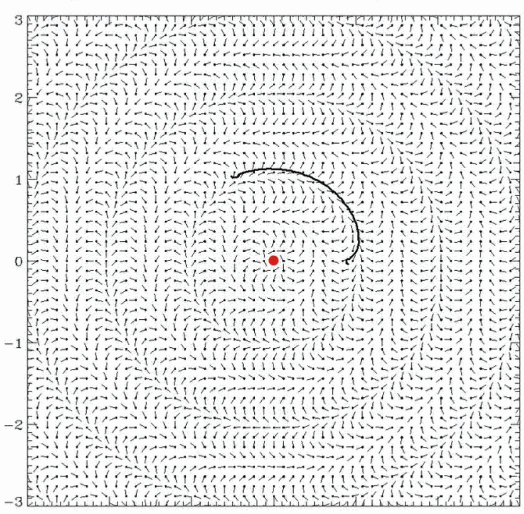


Breuer, Zykov, Engel, Schöll, in preparation

One-channel feedback

Biktashev, Holden, J. Theor. Biol. (1994); Grill, Zykov, Müller, PRL (1995)





$$\overline{\omega}\, t_k + \theta_0 - \frac{2\pi}{\lambda} |z| = \pi + \arg(z) + 2\pi n$$

$$I(t) = A_I \sum_{l=0}^{\infty} \delta(t - lT_m - \phi_{\mathrm{mod}} / \omega_m),$$

$$\phi_{\mathrm{mod}} = \pi + \arg(z) - \theta_0 + \frac{2\pi}{\lambda} |z| + \overline{\omega}\,\tau$$

$$\gamma = \varphi + \theta_0 + \phi_{\mathrm{mod}}$$

$$\gamma = \varphi + \pi + \arg(z) + \frac{2\pi}{\lambda} |z| + \overline{\omega}\,\tau$$

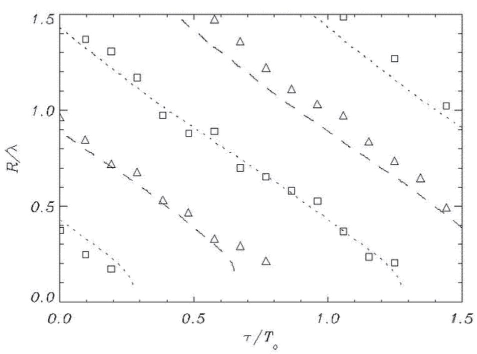
$$\dot{z} = \frac{h}{T_m} \exp(i\,\gamma)$$

Radius of a resonant attractor

$$\gamma = \arg(z) + \pi/2 + \pi n$$

$$\frac{R}{\lambda} = m - \frac{1}{4} - \frac{\varphi}{2\pi} - \frac{\tau\overline{\omega}}{2\pi}, \quad n = 2m$$

$$\frac{R}{\lambda} = m + \frac{1}{4} - \frac{\varphi}{2\pi} - \frac{\tau\overline{\omega}}{2\pi}, \quad n = 2m+1$$



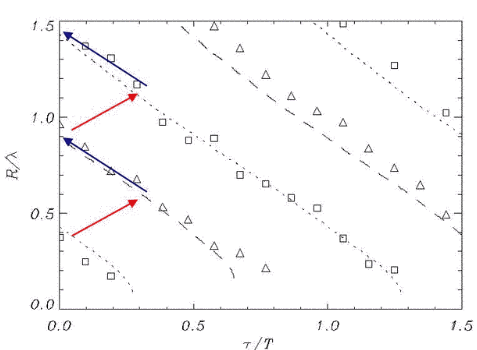
Karma, Zykov, PRL (1999)

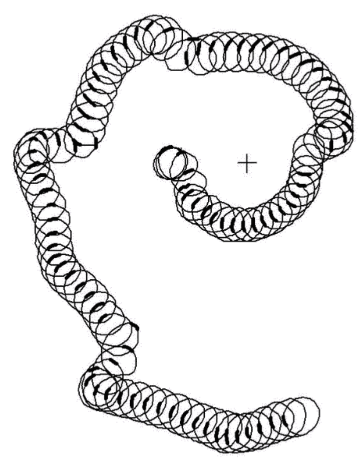
Snail-shaped drift

$$\gamma = \arg(z) + \pi/2 + \pi n$$

$$\frac{R}{\lambda} = m - \frac{1}{4} - \frac{\varphi}{2\pi} - \frac{\tau\overline{\omega}}{2\pi}, \quad n = 2m$$

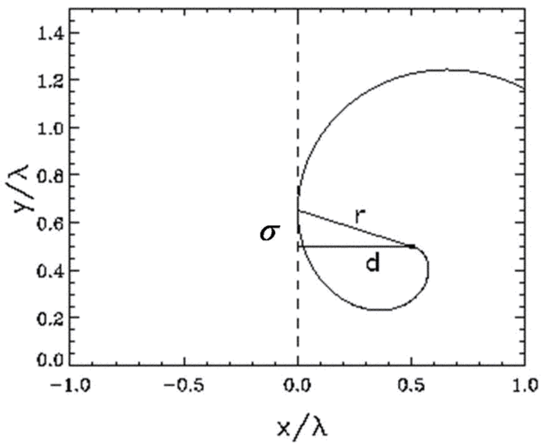
$$\frac{R}{\lambda} = m + \frac{1}{4} - \frac{\varphi}{2\pi} - \frac{\tau\overline{\omega}}{2\pi}, \quad n = 2m+1$$





Kheowan, Zykov, Rangsiman, Müller, PRL (2001)

Drift along a line detector



$$\bar{\omega}\, t_k + \theta_0 - \frac{2\pi}{\lambda} r = \pi - \arctan\left(\frac{\sigma}{x}\right) + 2\pi n,$$

$$r(x) = x\sqrt{0.5 + \sqrt{0.25 + \left(\frac{\lambda}{2\pi x}\right)^2}},$$

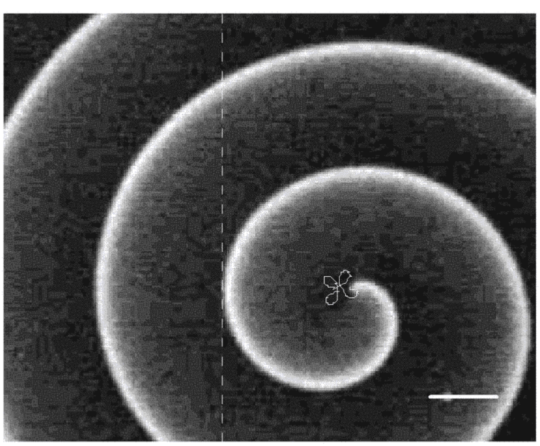
$$\sigma = \sqrt{r^2 - x^2} = \frac{\lambda x}{2\pi r}$$

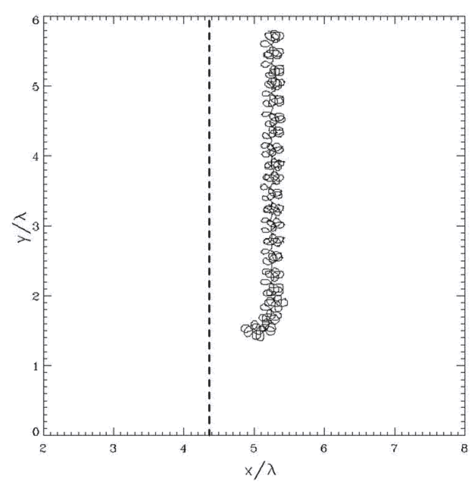
$$\phi_{\text{mod}} = \pi - \theta_0 + \frac{2\pi}{\lambda} r - \arctan\left(\frac{\sigma}{x}\right) + \bar{\omega} t$$

$$\gamma(x) = \varphi + \pi + \frac{2\pi}{\lambda} r - \arctan\left(\frac{\sigma}{x}\right) + \bar{\omega} t$$

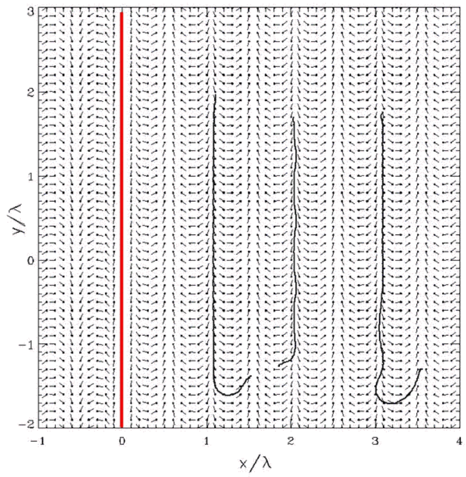
$$\gamma(x_a) = \pi / 2 + \pi n$$

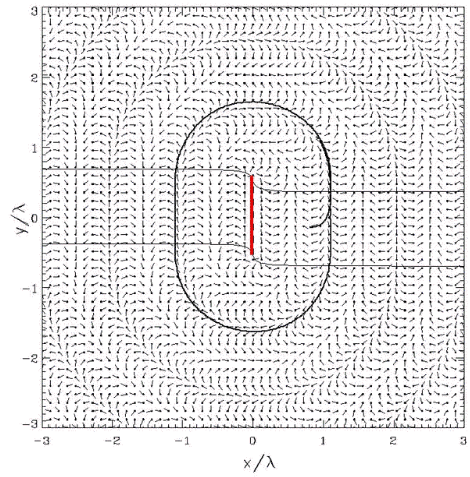
Line detector in the BZ medium



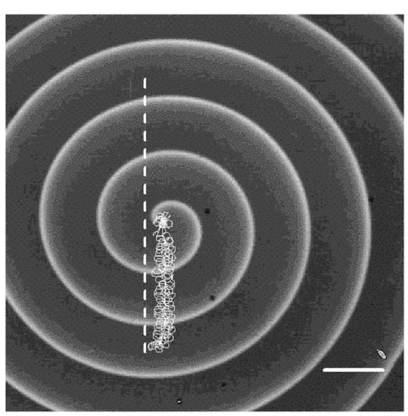


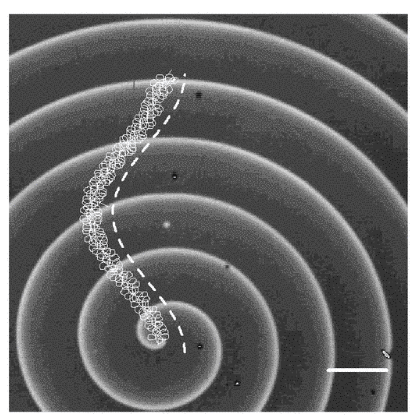
Drift velocity fields mediated by a line detector





A defect within the medium





Continuous feedback control

$$I(t) = k_{fb}[B(t-\tau) - B_0], \quad B(t) = \frac{1}{s}\int_S v(x', y', t)\,dx'\,dy'$$

Zykov, Mikhailov, Müller, PRL (1997)

$$I_1(t \mid z) = k_{fb} A(z)\cos[\bar{\omega} t - \bar{\omega}\tau - \phi(z)], \quad A(z)e^{i\phi(z)} = \frac{2}{T_\infty}\int_0^{T_\infty} B(t \mid z)\exp(i\bar{\omega} t)\,dt.$$

$$v_1(x', y', t \mid z) = v_{\max}\cos[\omega t - \arg(z' - z) - 2\pi \mid z' - z \mid / \lambda],$$

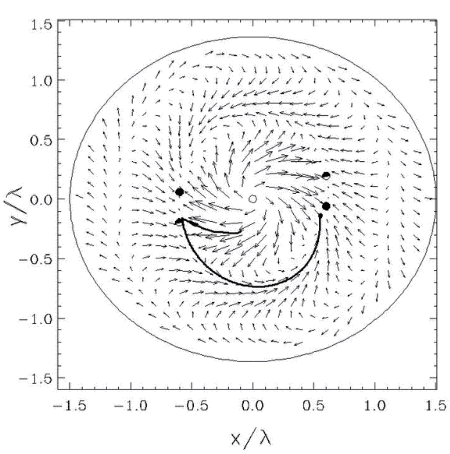
$$A(z)e^{i\phi(z)} = \frac{v_{\max}}{s}\int_S \exp[\Phi(z' \mid z)]\,dx'\,dy', \quad \Phi(z' \mid z) = \arg(z' - z) + 2\pi \mid z' - z \mid / \lambda$$

$$\dot{z} = A(z)\exp[i\gamma(z)], \quad \gamma(z) = \varphi + \bar{\omega}\tau + \phi(z)$$

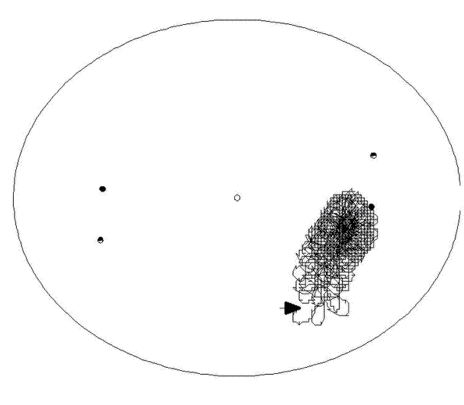
Zykov, Engel, Physica D (2004)

Drift velocity field in an elliptical domain

The Oregonator model



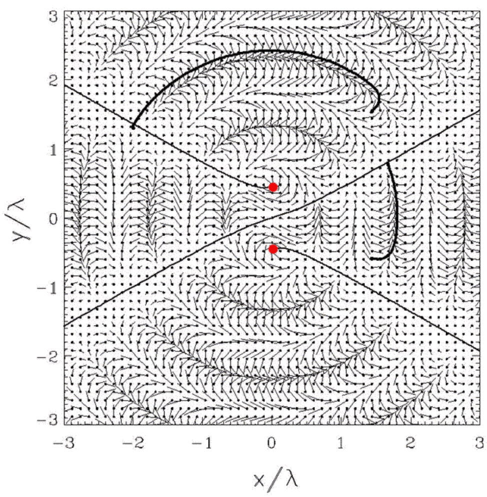
BZ experiment



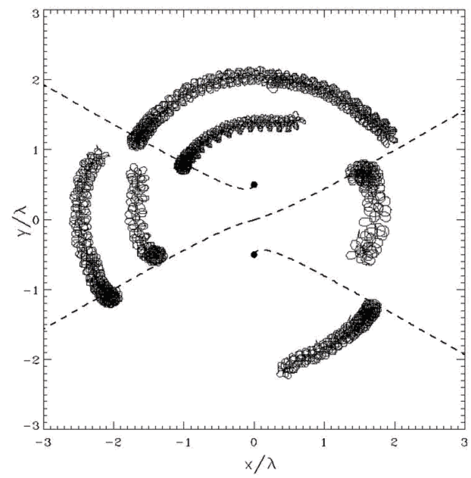
Zykov, Bordiougov, Brandtstädter, Gerdes, Engel, PRL (2004)

Drift velocity field as an interference pattern

The Oregonator model

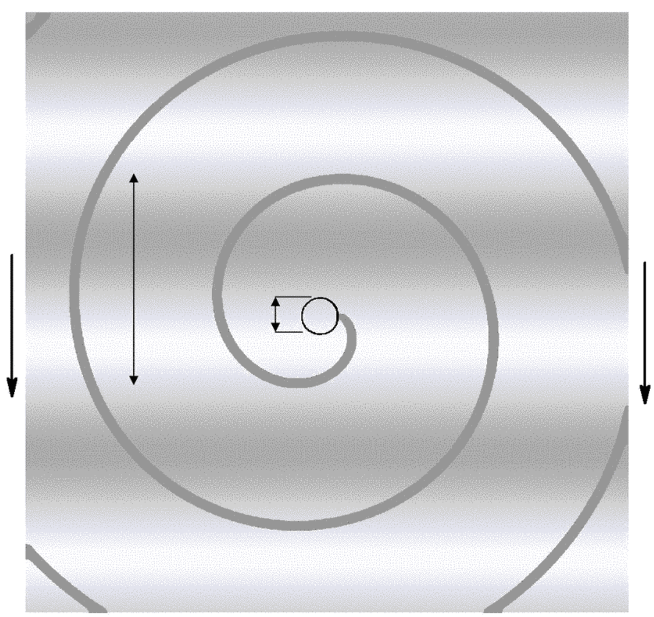


BZ experiments



Zykov, Brandtstädter, Bordiougov, Engel, PRE (2005)

Spatio-temporal modulation



Moving stripes

S. Zykov, V. Zykov, V. Davydov, Europhys. Lett. (2006)

Reaction-diffusion model

$$\frac{\partial u}{\partial t} = D\nabla^2 u + \frac{1}{\varepsilon}\left[u - u^2 - (fv + I)\frac{u-q}{u+q}\right]$$

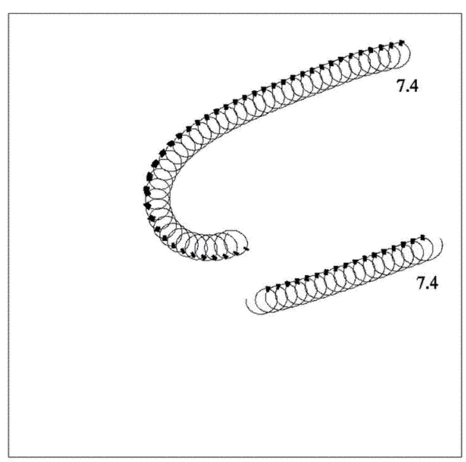
$$\frac{\partial v}{\partial t} = u - v$$

$$I(x,y,t) = A_I \cos(\omega_m t + k_m y) + I_0$$

$$\lambda_m = \frac{2\pi}{k_m} \qquad T_m = \frac{2\pi}{\omega_m} \qquad V_b = \frac{\omega_m}{k_m}$$

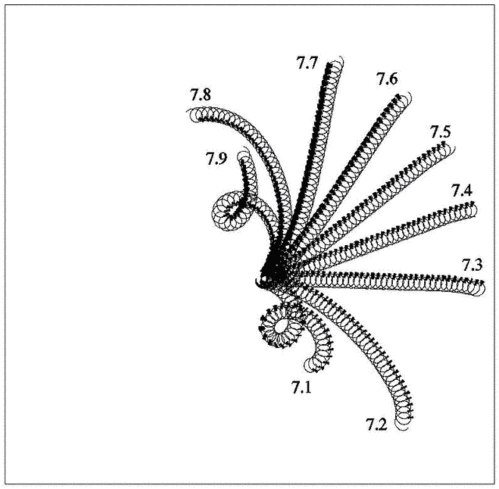
Drift induced by traveling-wave modulation

Drift without synchronization $T_m > T_0$



Synchronized spiral tip motion $T_m = T_0$

Drift direction vs. modulation period



Spiral tip motion for different T_m

Resonant drift under parametric modulation

$$I = I_0 + A_I \cos(\omega_m t + \phi_{\text{mod}})$$

$$\frac{dx_c}{dt} = V_d \cos(\overline{\omega} t - \omega_m t - \phi_{\text{mod}} - \varphi)$$

$$\frac{dy_c}{dt} = V_d \sin(\overline{\omega} t - \omega_m t - \phi_{\text{mod}} - \varphi)$$

Davydov, Zykov, Mikhailov (1991); Mantel, Barkley (1996); Hakim, Karma (1999).

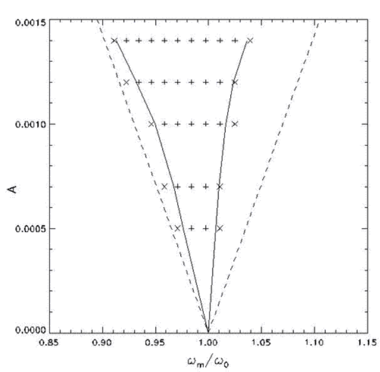
Synchronized drift in the kinematical model

$$I = I_0 + A_I \cos(\omega_m t + k_m y)$$

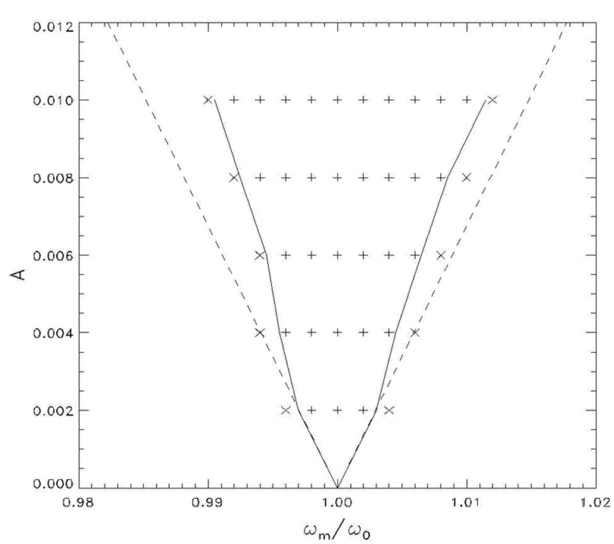
$$\frac{dx_c}{dt} = V_d \cos(\overline{\omega} t - \omega_m t - k_m y_c - \varphi)$$

$$\frac{dy_c}{dt} = V_d \sin(\overline{\omega} t - \omega_m t - k_m y_c - \varphi)$$

Synchronization bands



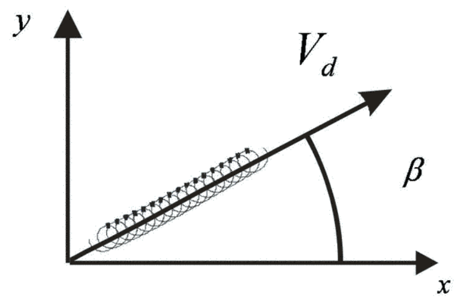
R-D Model



Kinematical model

+ - synchronization
X - no synchronization

Conditions for synchronized drift



$$y_c = y_0 + tV_d \sin\beta$$

$$\frac{dy_c}{dt} = V_d \sin(\overline{\omega} t - \omega_m t - k_m y_c - \varphi)$$

$$\begin{cases} \overline{\omega} - \omega_m - k_m V_d \sin\beta = 0 \\ \beta = -k_m y_0 - \varphi \end{cases}$$

$$1 - k_m \frac{V_d}{\omega_0} < \frac{\omega_m}{\omega_0} < 1 + k_m \frac{V_d}{\omega_0}$$

Doppler-shift of the modulation frequency

$$\overline{\omega} - \omega_m - k_m V_d \sin\beta = 0$$

$$V_b = \frac{\omega_m}{k_m}$$

$$\overline{\omega} = \omega_m \frac{V_b + V_d \sin\beta}{V_b}$$

Summary

- There are many different methods to control spiral wave location, which can be considered in framework of a unified theoretical approach
- Descrete and continuous feedbacks create opportunities for a robust and precise control
- Under the traveling-wave modulation the synchronized drift occurs within a certain range of the modulation frequency ω_m near ω_0
- Doppler-shift of the modulation frequency plays crucial role in the observed phase synchronization
- Theoretical predictions are in quantitative agreement with experimental data and numerical results

Outlook

- Drift instabilities under strong feedback strength
- Suppression of the drift instabilities
- From a single spiral to many co-existing spirals
- Control of scroll waves in 3-D (Wu, et al., PRE, 73, 2006)
- Nonhomogeneous and anizotropic media
- Relevance to cardiology