Exotic vortices in Sr_2RuO_4, why and how

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- Overview
- Our starting point

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- How to detect fractional vortex - Hendrik Bluhm

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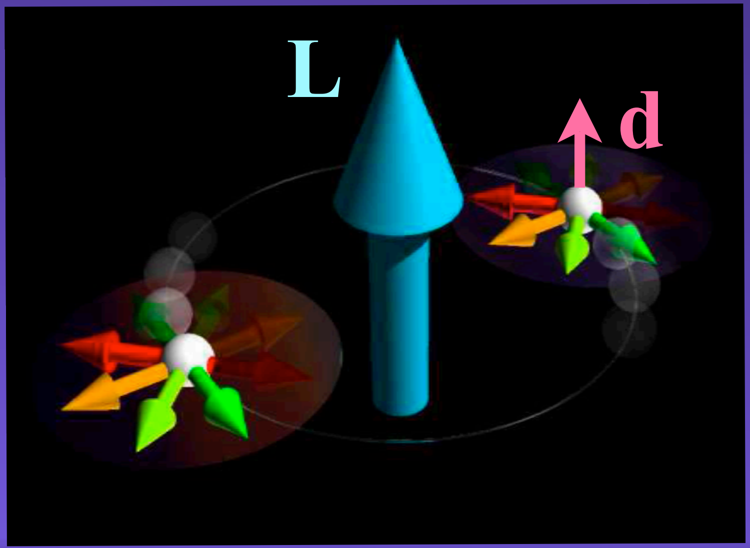
  - 1/2QV requires rotation of **d**



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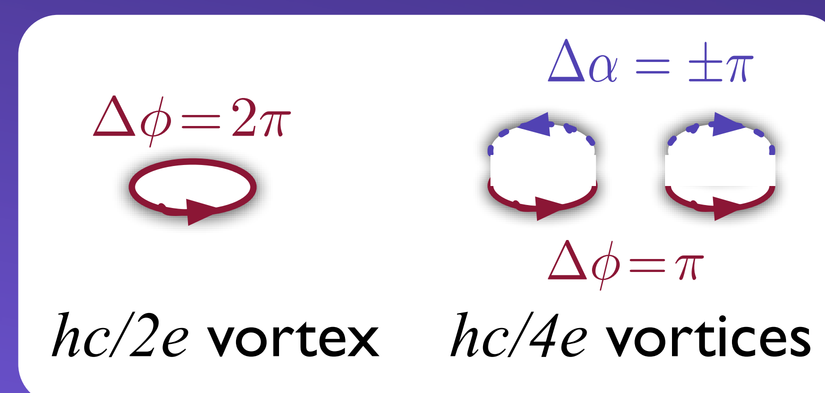
$\leftrightarrow$ π winding of order parameter phase ϕ
+ π rotation of $\mathbf{d}$ vector



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1/2 QV's: single Majorana zero mode

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  - $\mathbf{d}$ // ab: for $H_\perp$ [illegible] 200 G , Murakawa et al, PRL (04)

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Text

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- For large enough H, $\mathbf{d} \perp \mathbf{L}$!

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Sigrist &Ueda RMP, 63, 239 (1991)