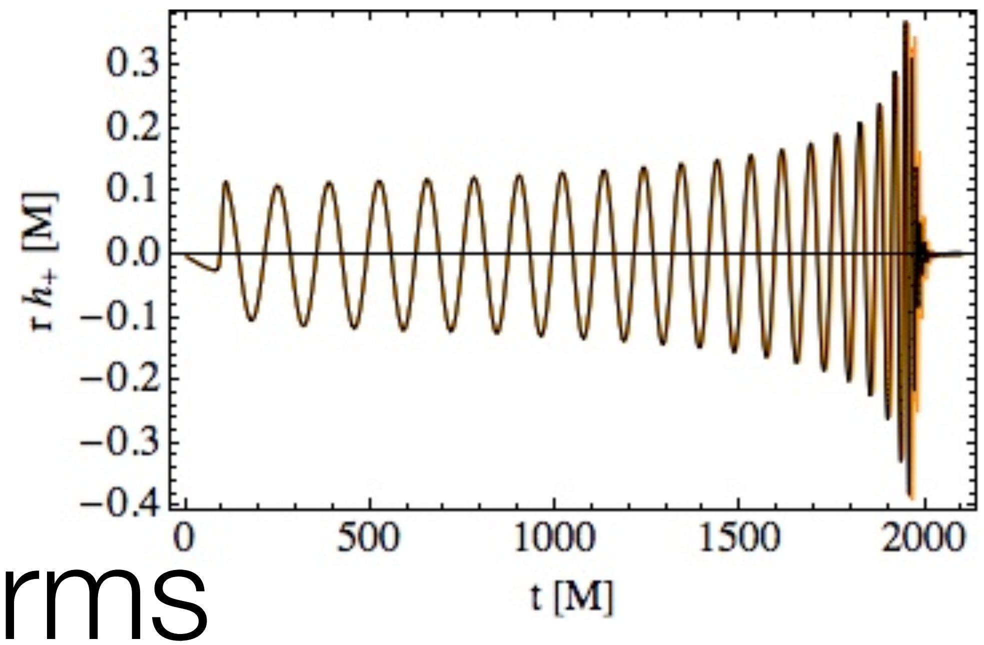
For a few digits more ...

Length and Accuracy of Numerical Relativity Waveforms



GW detectors cost $$$ - make sure we find as many signals & identify sources as accurately as the hardware allows!

S. Husa
Universitat de les Illes Balears

Chirps, Mergers and Explosions, KITP 2012

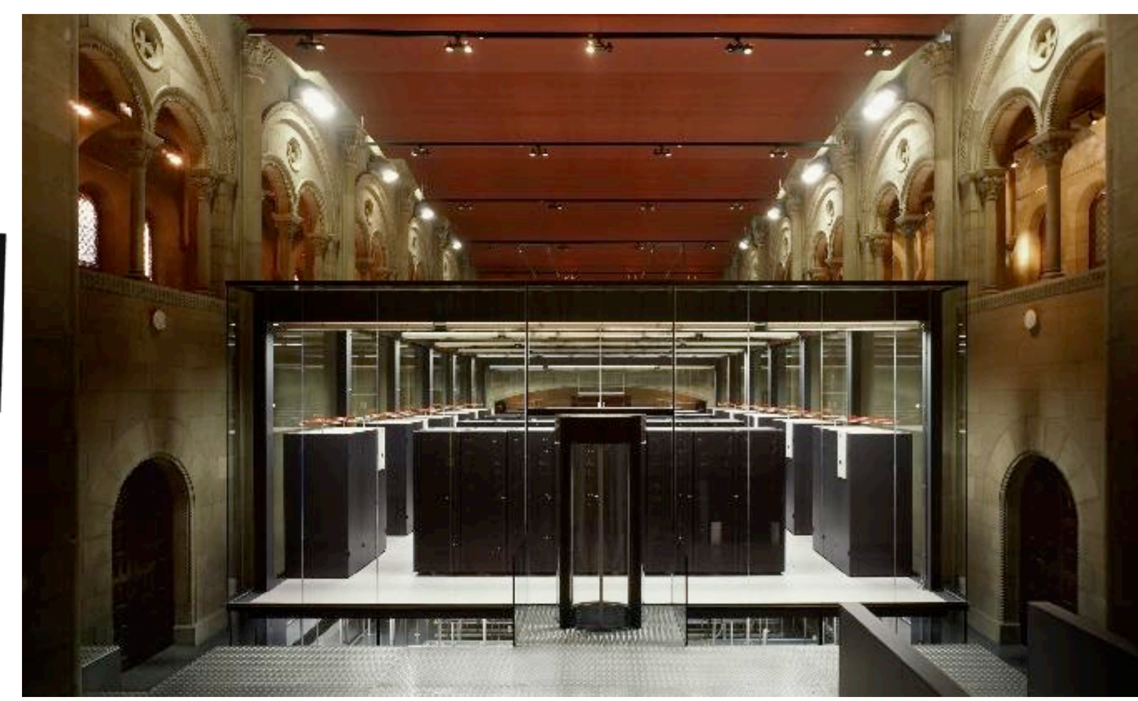
Advanced detectors put NR on a tight timeline to show its worth: ~ 2015/2018 for early/design sensitivity aLIGO!

Theorists may be cheap, computer time expensive

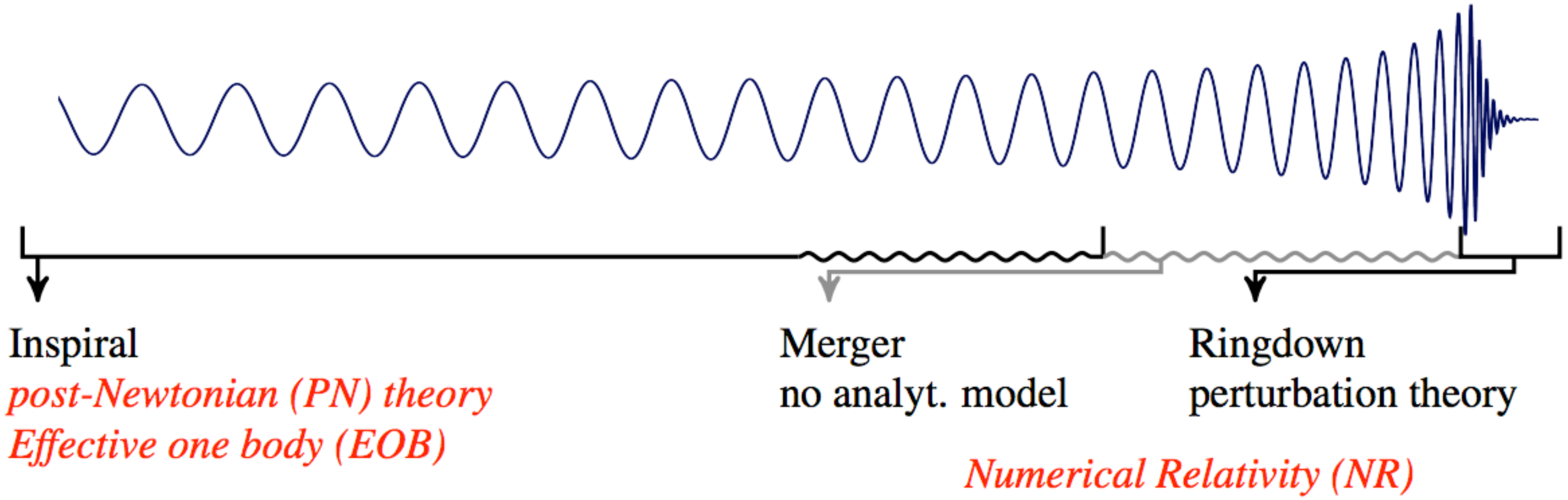
Acknowledgements

- Collaborators from: UIB, Cardiff, Jena, CalTech, AEI:
  - P Ajith, B Brügmann, M Hannam, Nathan Johnson-McDaniel, F Ohme, D Pollney, M Pürrer, A Vaño Viñuales, C Reisswig, M Ruiz, P Schmidt, M Thierfelder
- Discussions with members of SXS collaboration
- Computertime: PRACE [Hermit, Curie], BSC, LRZ

Partnership for Advanced Computing in Europe

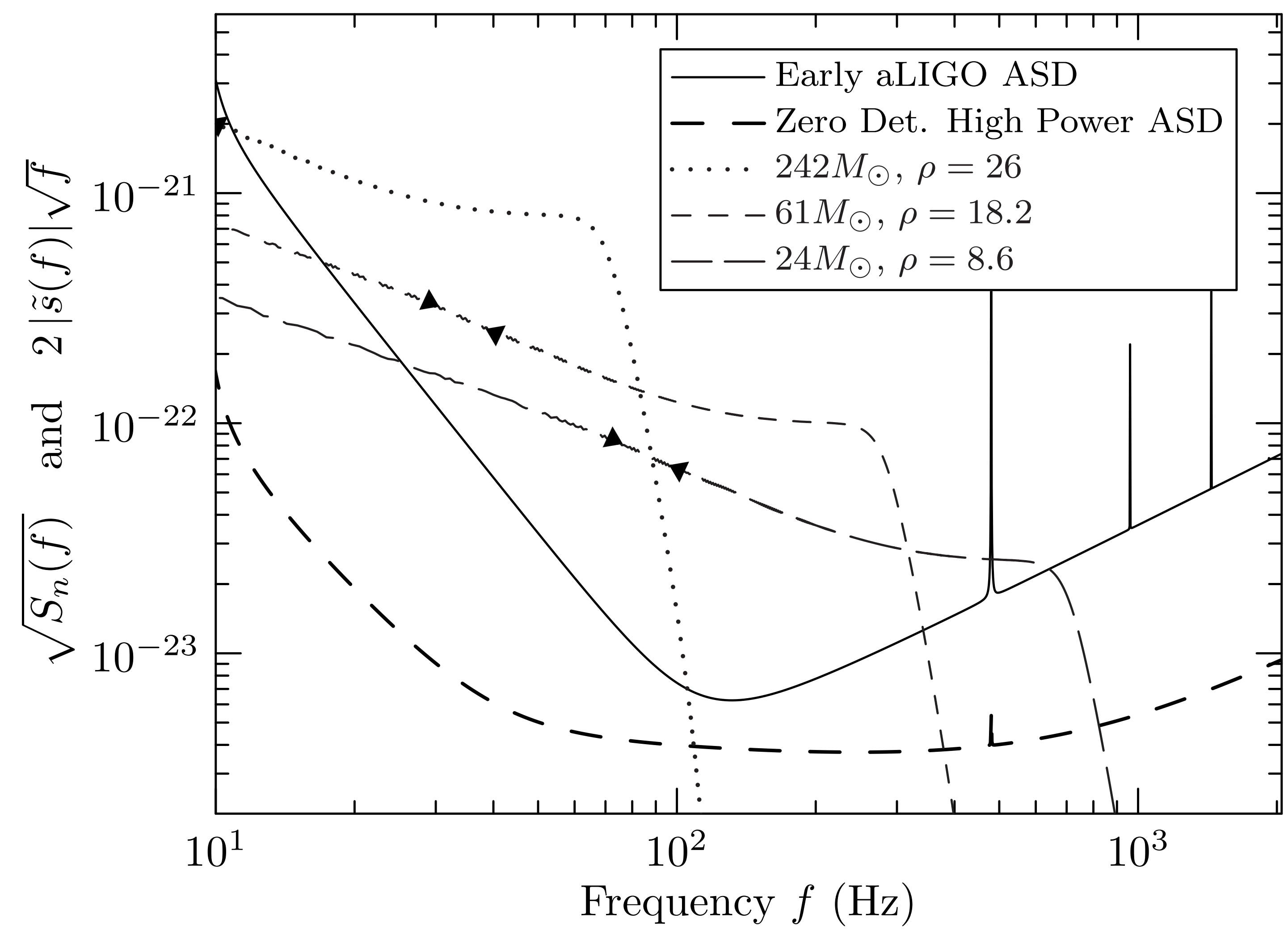


Entire WFs & end-to-end errors



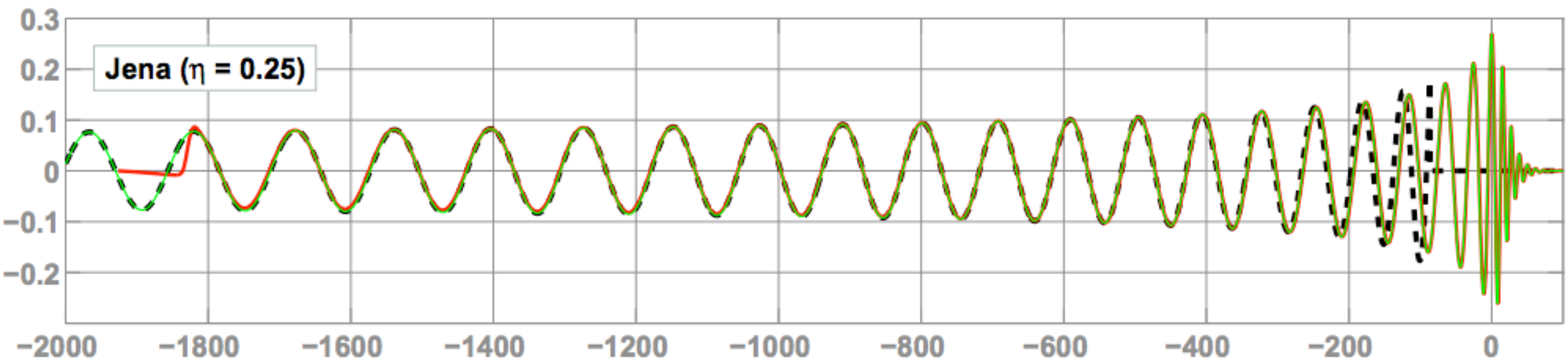
- Comparison of pN approximants [Buonanno+, PRD '09]: M ≥ 12M⊙ requires NR for construction of optimal detection templates.

- Unknown higher order PN terms ->
    - zoo of PN approximants: TaylorT*, TaylorF*, EOB*, ...
- Combined error in PN/NR/matching/parameter space modeling?
- Effect on detection & parameter estimation?



Hybrid PN-NR waveforms

Choose a frequency, least squares fit time and phase shift between NR and PN WFs in a suitable interval around that frequency.



Different hybridization methods appear to have comparable performance, e.g. Santamaría+ ’10 compares time and freq. domain methods.

Main influence on hybridization error from fitting window.
Tradeoff:
fit early: PN errors minimized
fit late: fitting problem is better conditioned (stronger WF variation)

Alternative approach: tuning of Effective-One-Body version of PN (EOBNR)

Waveform Overlap

- WF error in matched filter context is naturally defined in terms of overlap:

$$\langle h_1, h_2 \rangle = \max_{\phi_0, t_0} 4\Re \int_{f_1}^{f_2} \frac{\tilde{h}_1(f)\, \tilde{h}_2^*(f)}{S_n(f)}\, df$$

$$||\Delta h||^2 = \langle h_1 - h_2, h_1 - h_2 \rangle$$

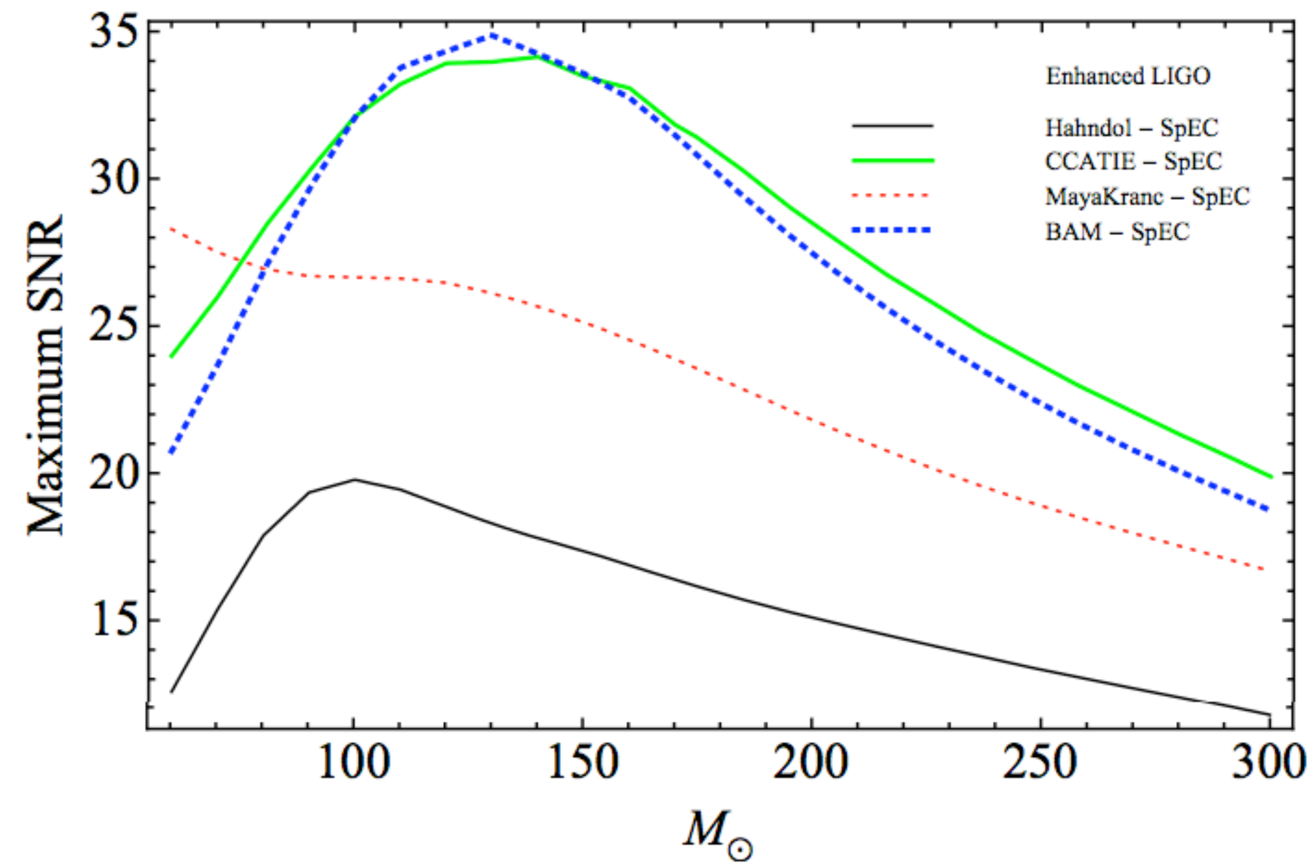
$$\mathcal{M} = 1 - \langle h_1, h_2 \rangle / (||h_1||\, ||h_2||)$$

SNR: $\rho = ||h||$

indistinguishable: $||\Delta h||^2 < 1$ Fairhurst, KITP 2008

$$||h_1|| \approx ||h_2|| \Rightarrow ||\Delta h||^2 = 2\rho^2 \mathcal{M}$$

Samurai: Hannam+ '09

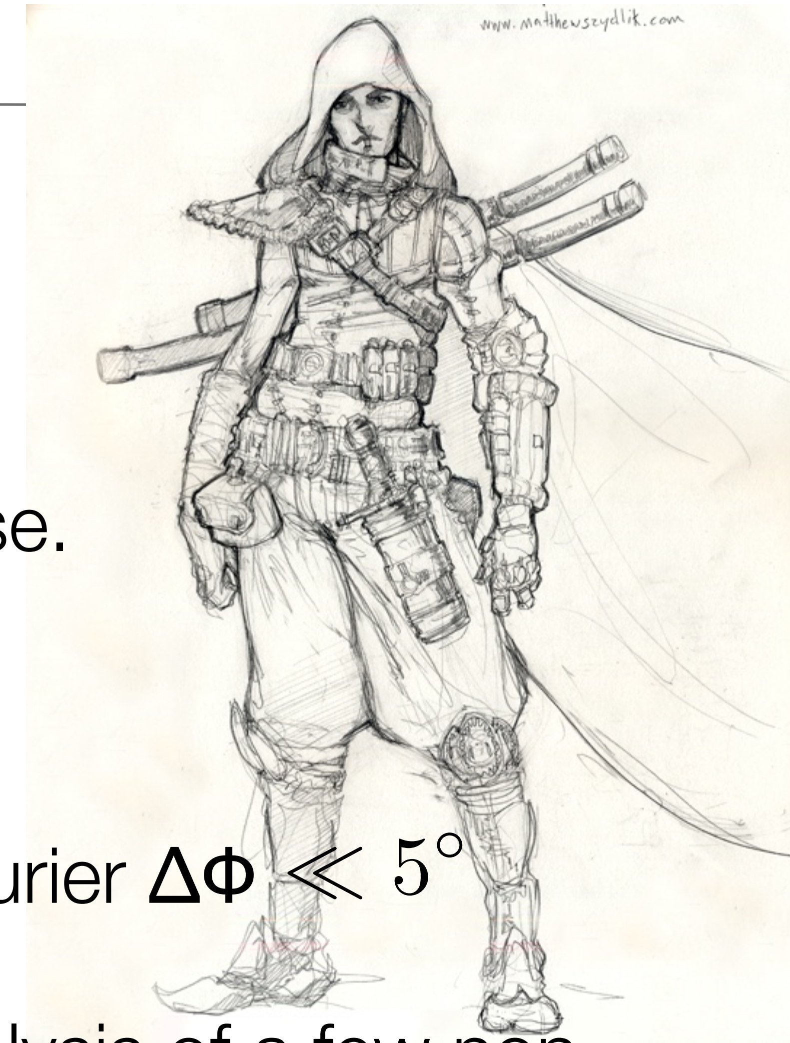


M=3% ≅ 10 % signal loss, **M=0.5 %, 0.2 %** undistinguishable @ **SNR 10,16**.

Searches & parameter estimation use WF families - maximize over mass, spins, ...

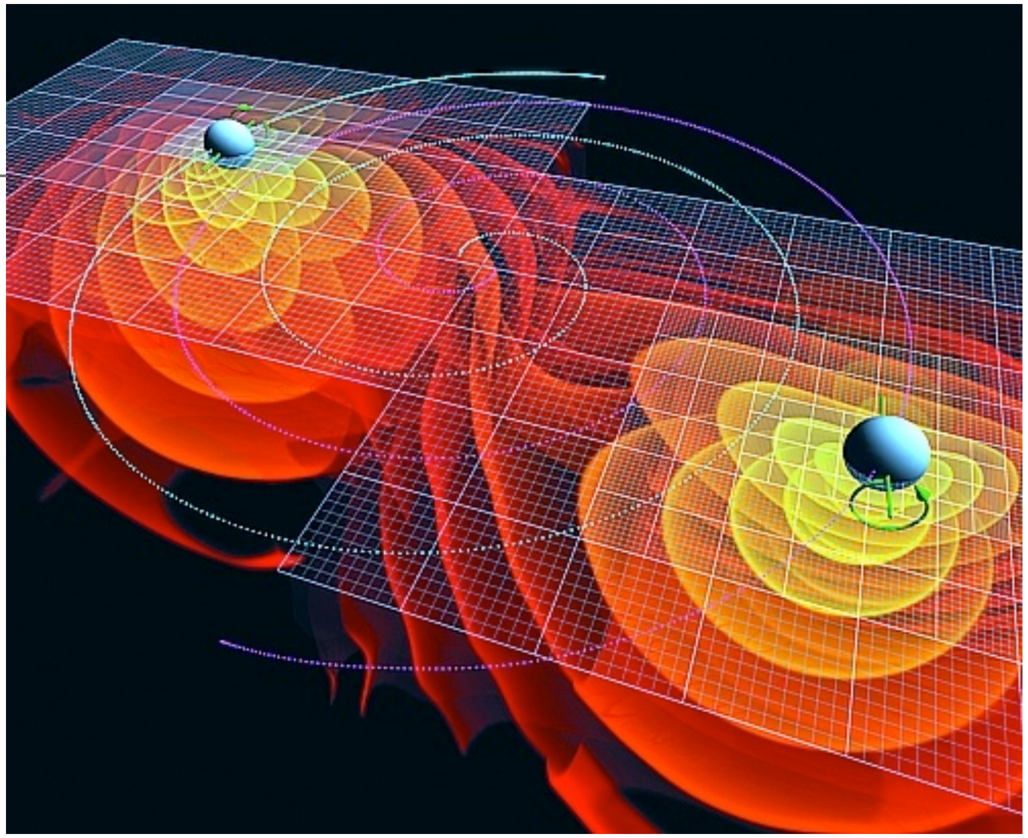
Computing M with fixed physical parameters can drastically overestimate accuracy requirements: small bias in physical parameters may have large effect on match.

Conclusions

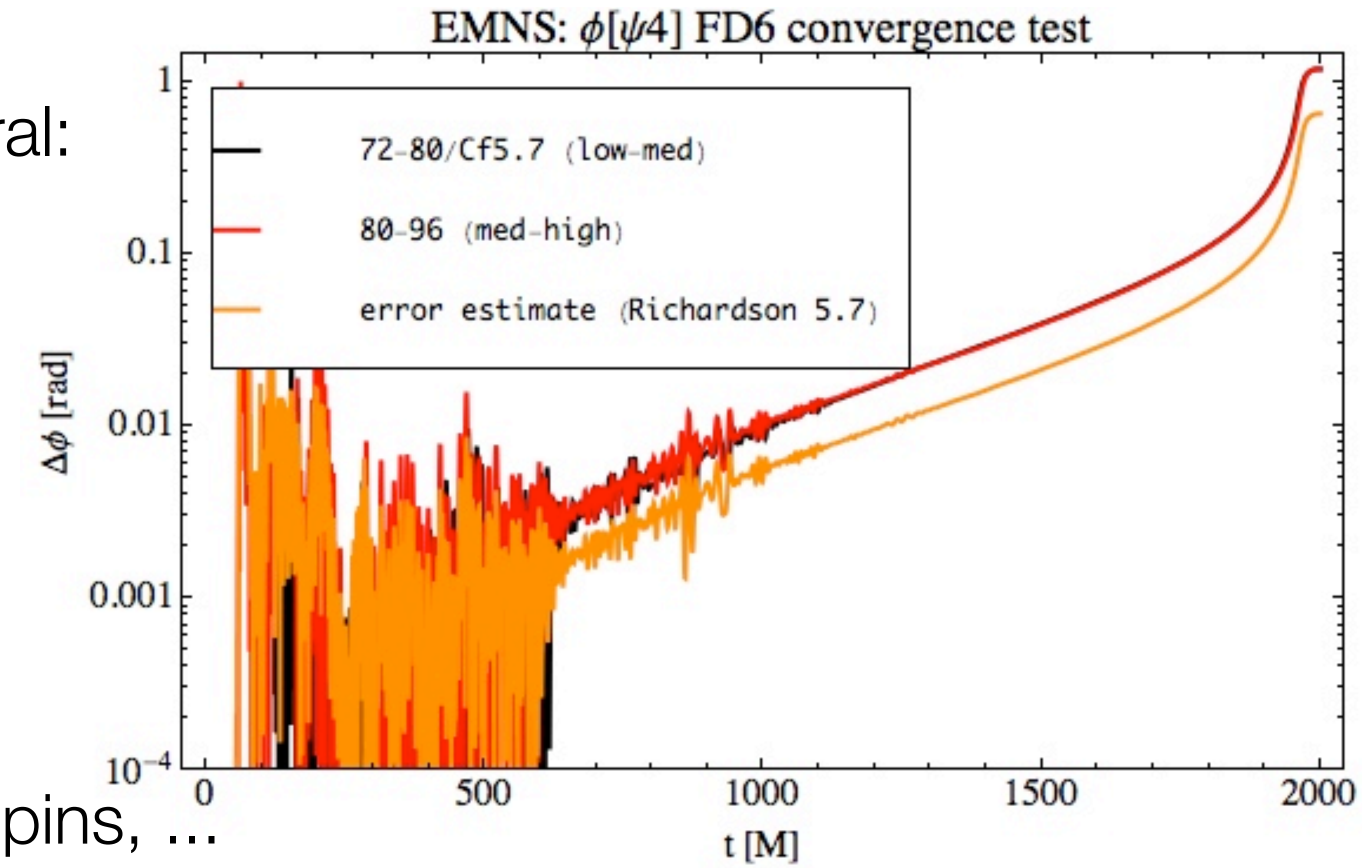


- Ultimately: want to understand WF errors in the context of actual LIGO/Virgo noise and GW searches.
  - **ninja-project.org**: NR+PN WFs -> S6/VSR23 noise.
  - Now: theoretical results, based on Gaussian noise.
  - May account for calibration error by demanding Fourier $\boldsymbol{\Delta\Phi} \ll 5^\circ$
  - So far our understanding is based on simplified analysis of a few non-precessing cases [**trust me and die**]:
    - NR & hybridization errors much smaller than PN uncertainty.
    - > 5-10 NR orbits help detection significantly, parameter bias ok < 2018?
    - “slow inspiral”: much better accuracy requires **much longer** WFs.
    - **A lot more work is required** [Perfect! More papers, students, ...]

Cost & error

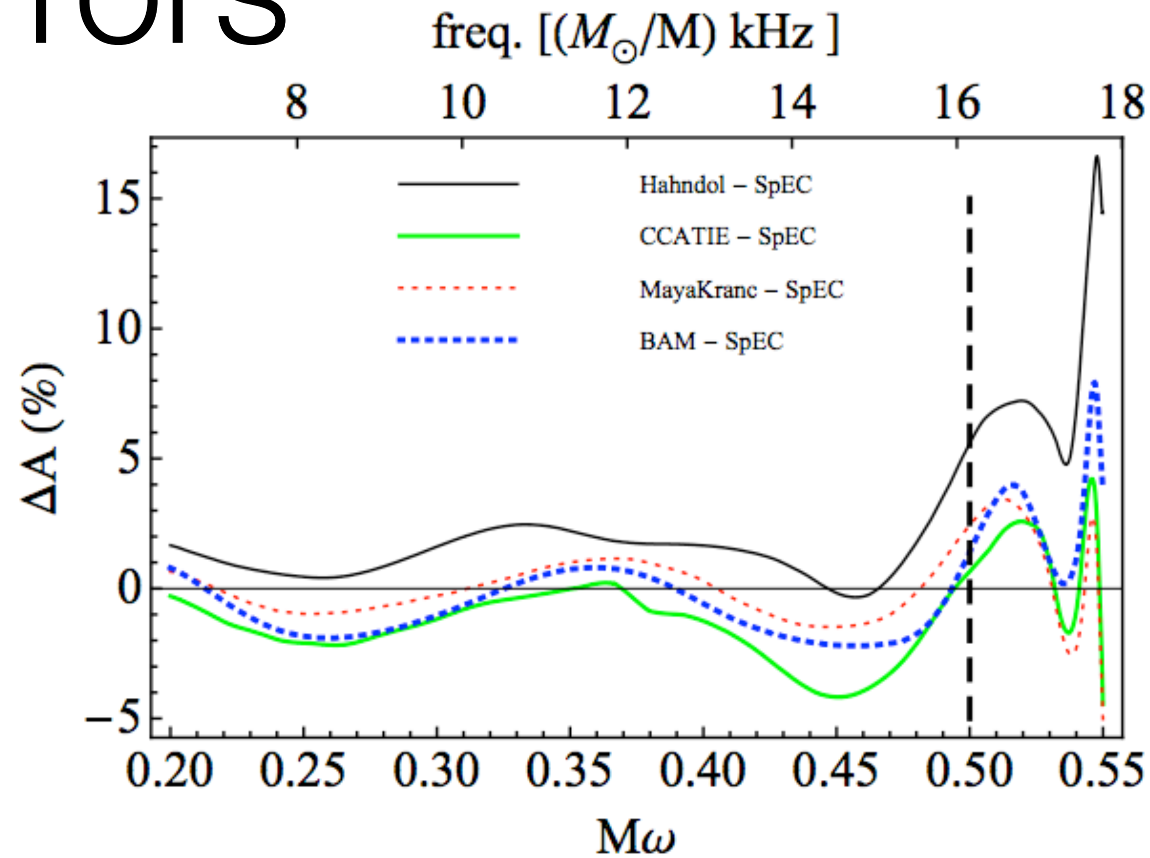


- computational cost in 3+1 D: $\propto \Delta x^{-3} \Delta t^{-1}$
  - x 2 resolution -> x 16 computational cost
- convergence: $X(\Delta x) = X_0 + e\Delta x^n + O(\Delta x^{n+1})$
  - 3 resolutions determine X_0, e, n
    - typical n: 6-10; spectral code: exponential convergence (SpEC/SXS)
  - dominant error at least for BBH inspiral:
      - dephasing
  - neglected in this talk:
    - WF extraction error, higher modes
    - systematics: initial data, defining spins, ...



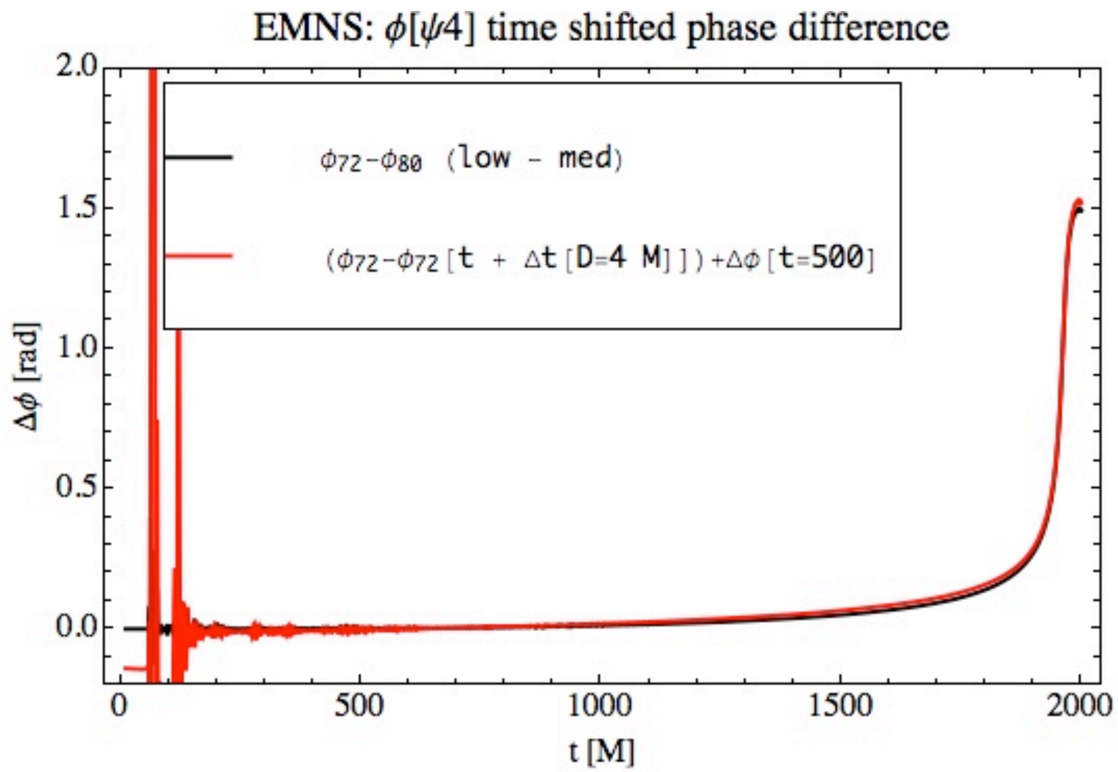
Numerical relativity errors

- consistency check: compare different codes
- Samurai, Ninja-2 [hybrids] projects
- SAMURAI [Hannam+ ‘09]: q=1 nonspinning NR WFs from different codes: **M < 0.1%.**



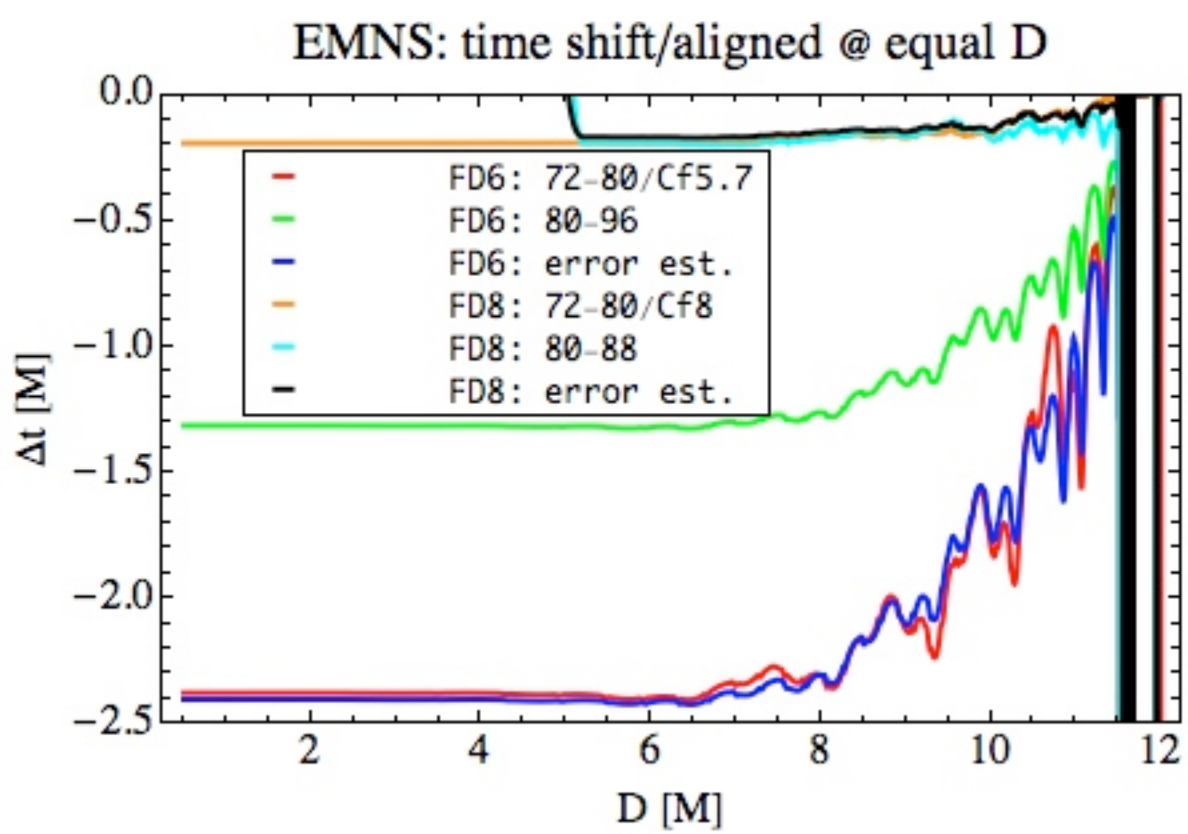
- [Santamaría+ ‘10]: nonspinning, q = 2 hybrids for Llama/BAM: **M < 0.2%**.
- [MacDonald+ ‘11]: Effect of low vs. high resolution (SpEC), q=1nonspinning: **M < 0.1%**.
- [Pan+ ‘11]: EOBNR-model agrees with NR data for q = 6 with **M 0.5%**.
- Want to quantify errors independent of match/noise/mass, see how error develops as a function of time/frequency/separation, ..

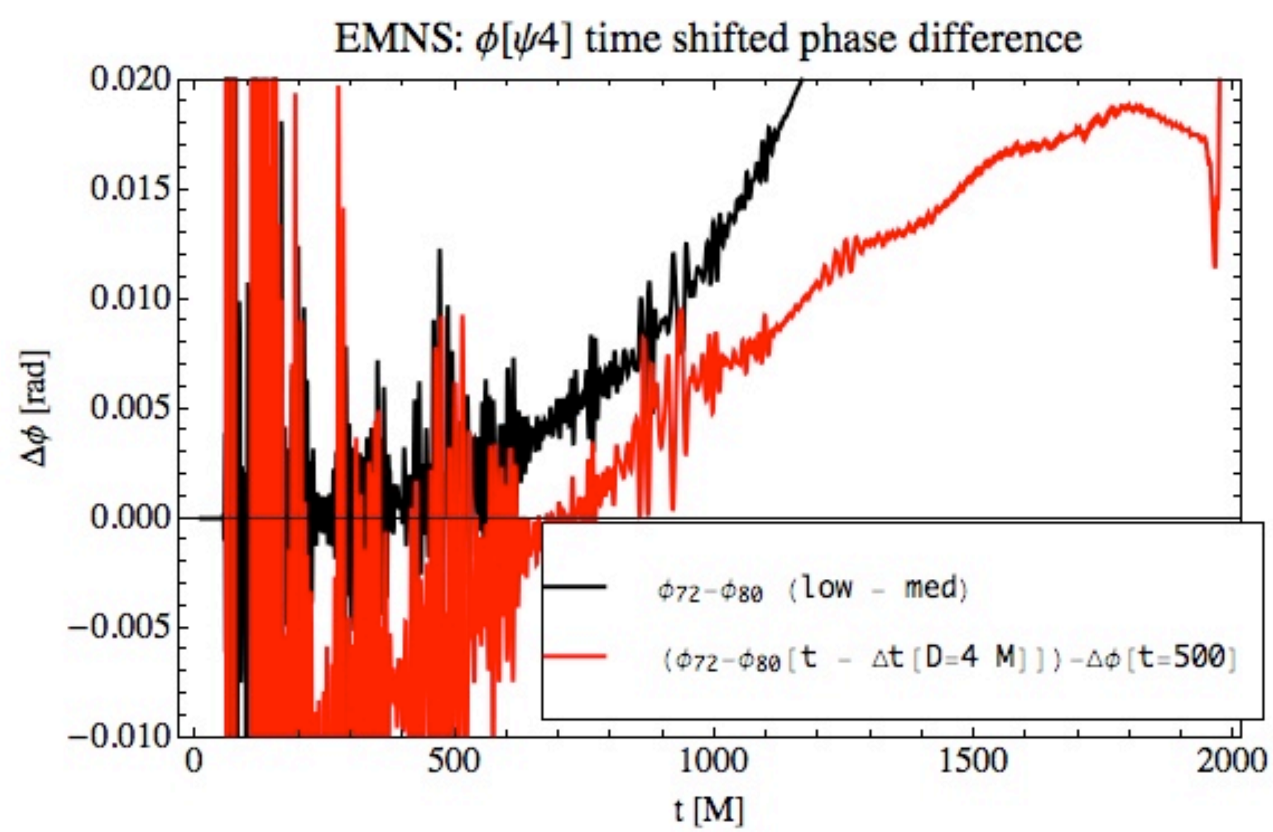
time shift vs. phase shift



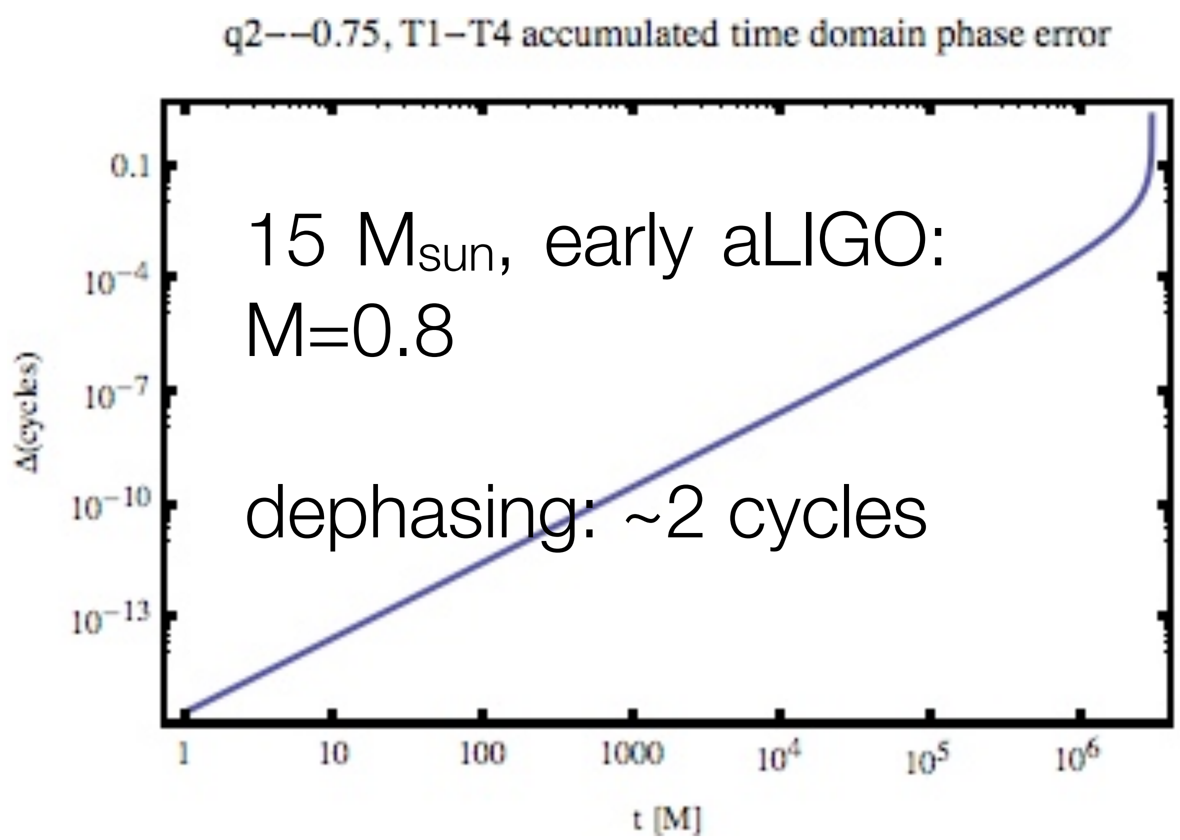
- Idea: express “dephasing” in terms of time-difference between different resolutions (compare e.g. events of same separation, frequency, ...)

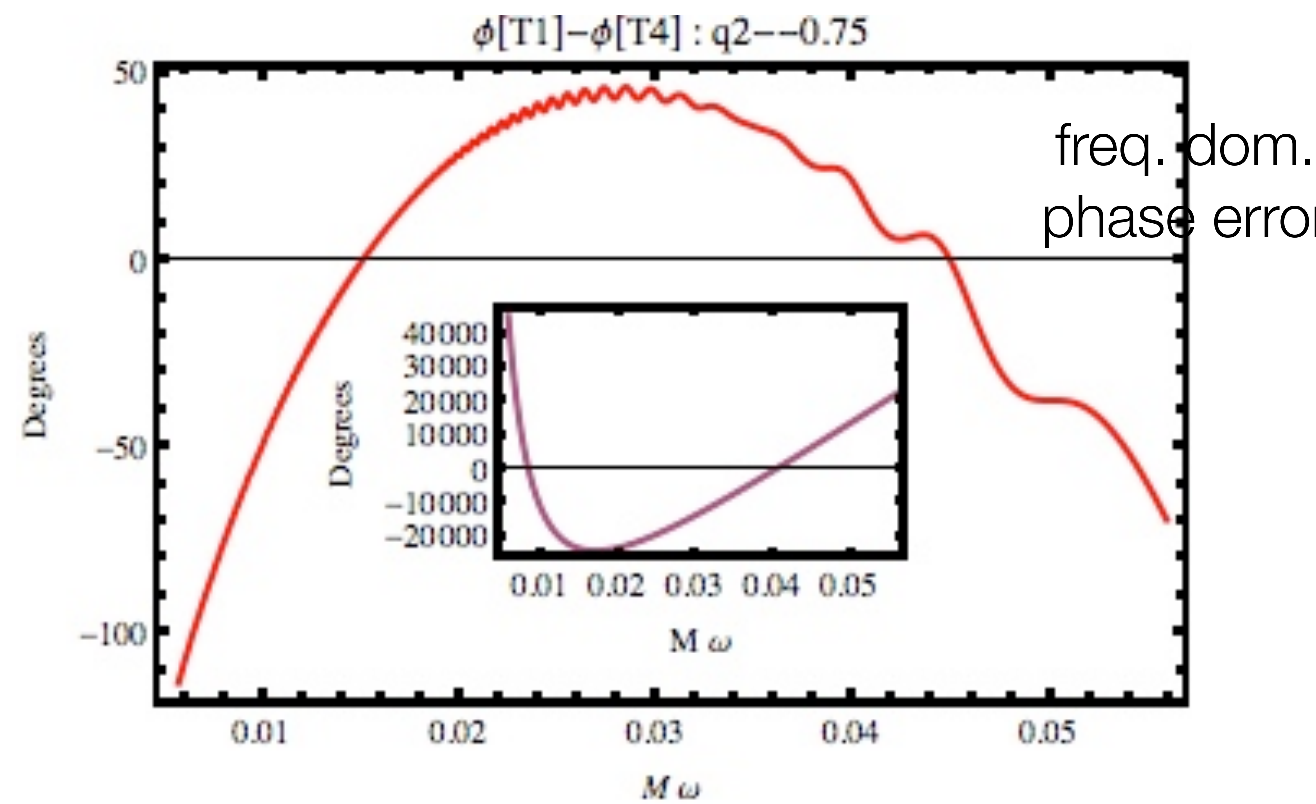
- No significant error in last orbits, can estimate phase error at merger a few orbits early.

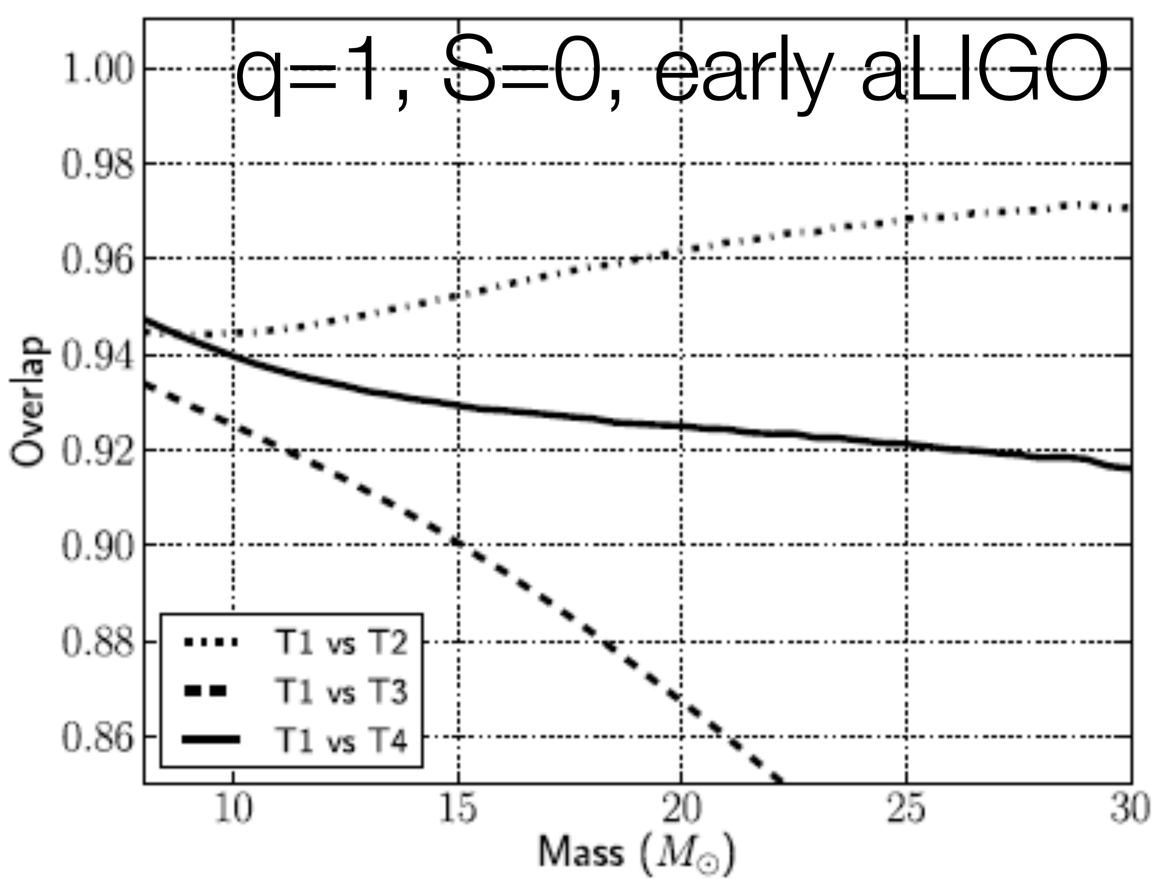


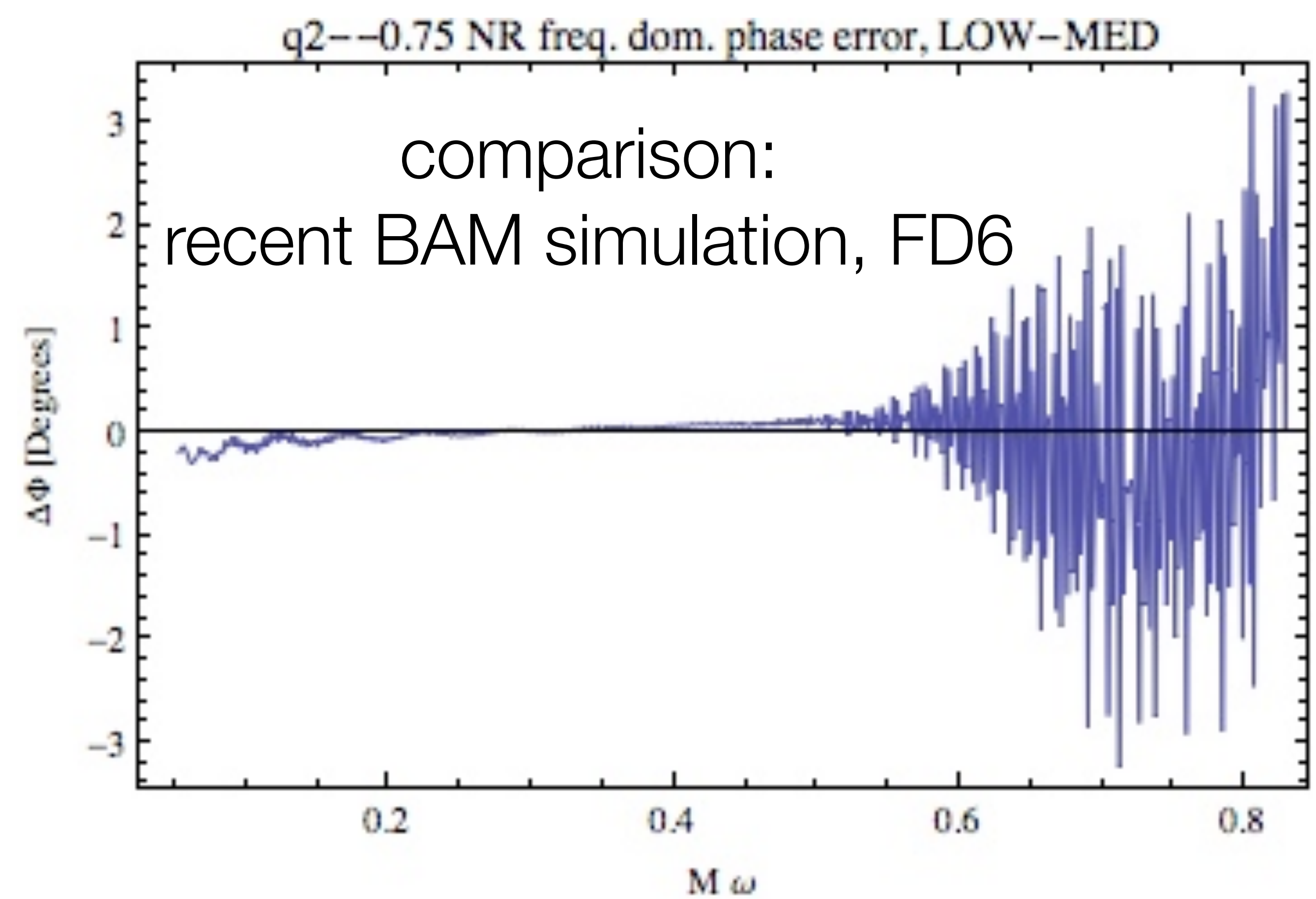


post-Newtonian errors: examples









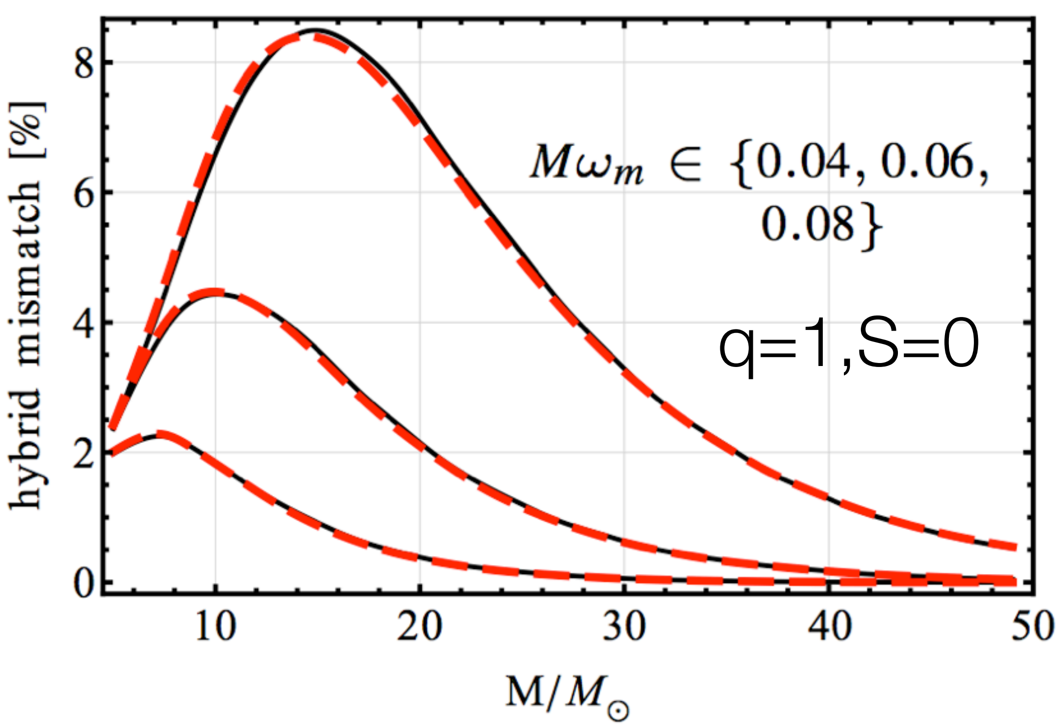
Total error: PN+NR+Hybridisation

- Boyle '11, Ohme+ '11: NR phase error is small -> estimate of NR amplitude is sufficient to compute approximate matches of hybrids with different PN versions:

$$\langle h_1, h_2\rangle = \max_{\phi_0,t_0}\left[4\Re\int_{f_1}^{f_2}\frac{\tilde h_1(f)\,\tilde h_2^*(f)}{S_n(f)}\frac{df}{\|h_1\|\|h_2\|}\right]$$

$$= \max_{\phi_0,t_0}\left[4\,\mathrm{Re}\int_{f_1}^{f_2}\frac{|A_1A_2|}{S_n}e^{i(\phi_1-\phi_2)}\,e^{i(2\pi f\,t_0+\phi_0)}\frac{df}{\|h_1\|\|h_2\|}\right]$$

$$\phi_1-\phi_2=\begin{cases}\phi_{\rm PN1}-\phi_{\rm PN2} & ,f<f_m\\ 0 & ,f\geq f_m\,.\end{cases}$$



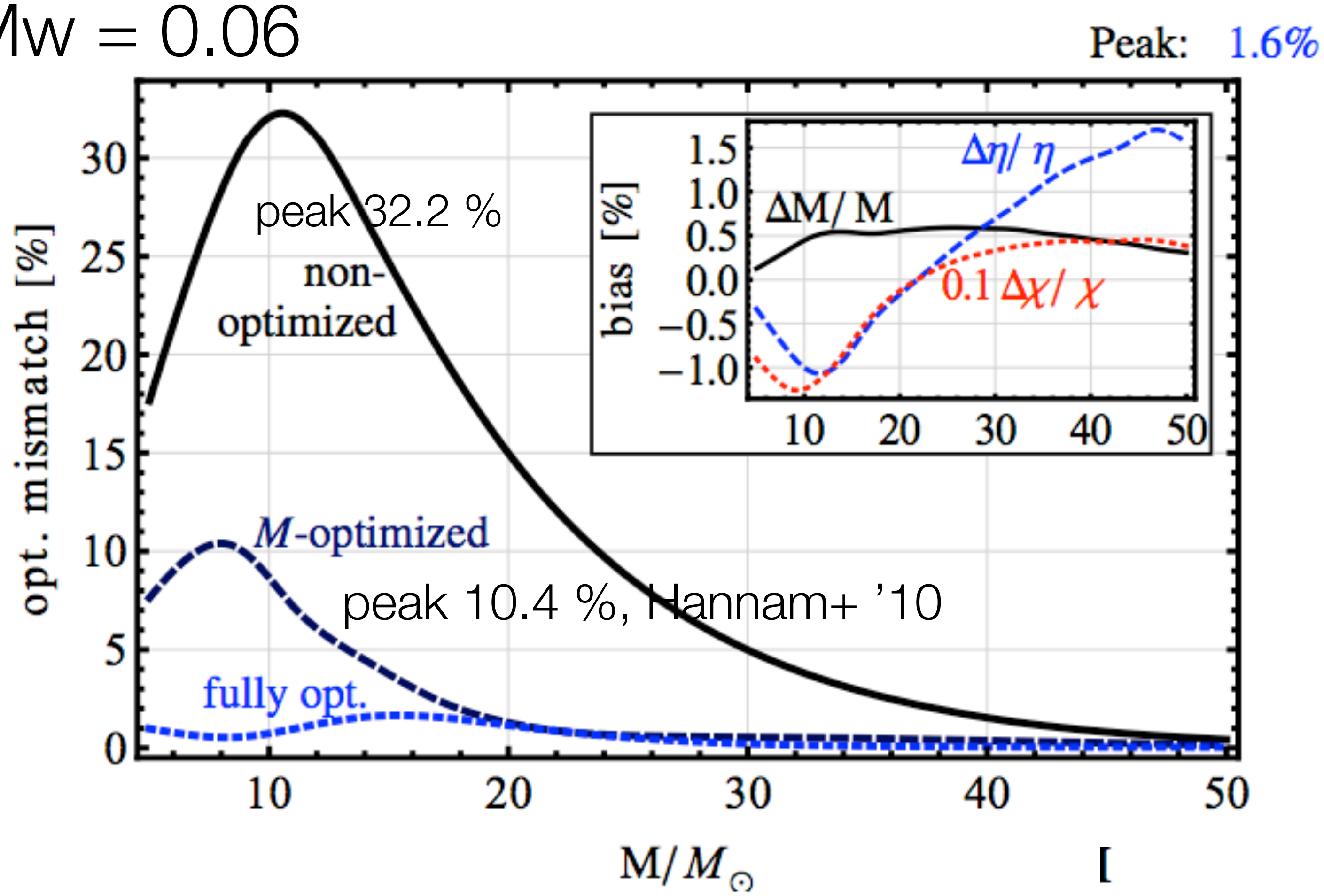
Ohme+ '11: Results consistent with previous studies: many more NR orbits [O(10^3)] are needed so that PN disagreement is indistinguishable!

Are current PN+NR WFs useless?

No - we need to compute matches optimized over physical parameters and check parameter bias!

Examples: Mismatch optimized over physical parameters

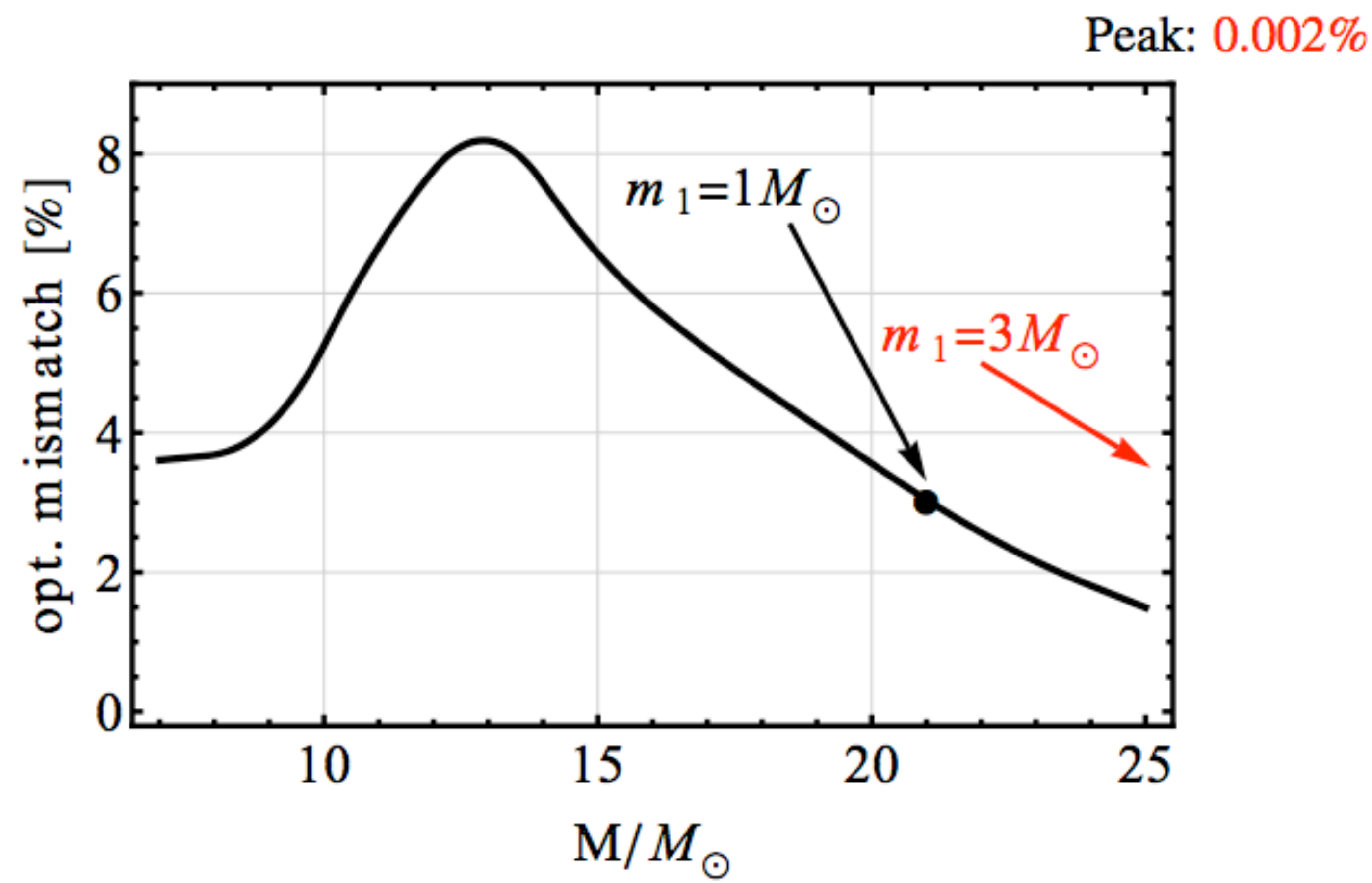
Mw = 0.06



$q=4$, $\chi_1=\chi_2=0.5$

Comparison of T1 vs F2 approx.

$q=20$, $\chi_1=\chi_2=0$



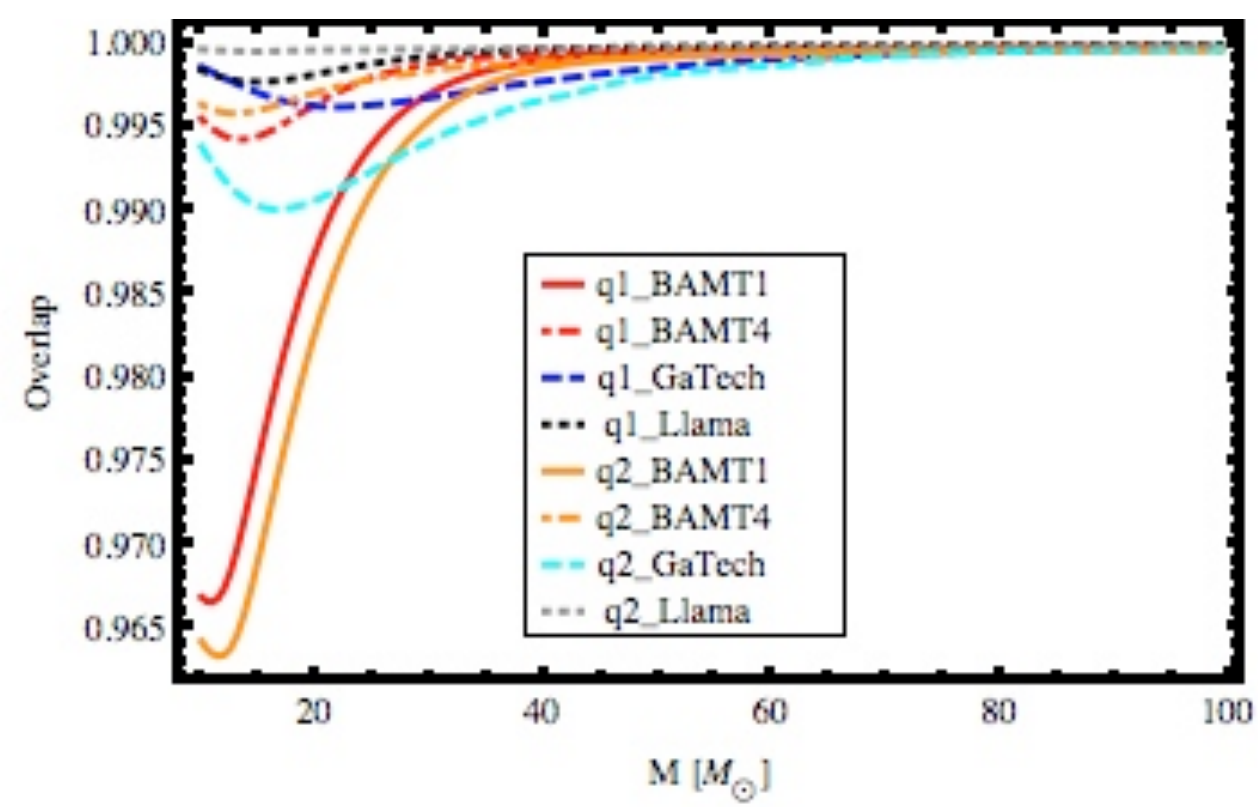
Boyle '11, Ohme+ '11: With astrophysical bias, larger mass ratios can be easier to model!

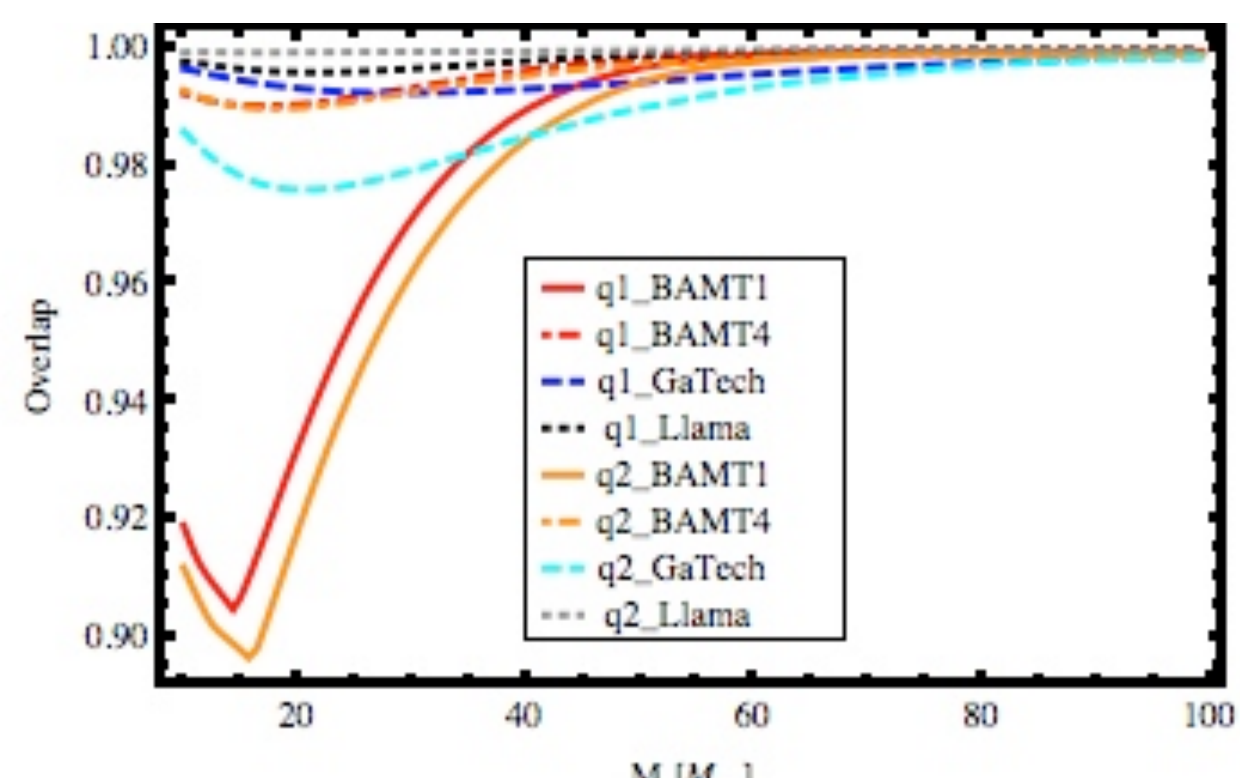
orbits	mass-ratio		$q = 20$
5	*3.0%:*	$q < 8.9$	$\max_M \mathscr{M}_{\rm FF} \approx 15\%$ $(19M_\odot)$
	1.5%:	$q < 6.8$	$21M_\odot : 12\%$, $63M_\odot : 0.3\%$
10	*3.0%:*	$q < 11.4$	$\max_M \mathscr{M}_{\rm FF} \approx 8.2\%$ $(13M_\odot)$
	1.5%:	$q < 8.6$	$21M_\odot : 3.0\%$, $63M_\odot : 1.6 \times 10^{-5}$
20	*3.0%:*	$q < 14.8$	$\max_M \mathscr{M}_{\rm FF} \approx 5.7\%$ $(11M_\odot)$
	1.5%:	$q < 10.7$	$21M_\odot : 0.8\%$, $63M_\odot : 6.4 \times 10^{-6}$

TABLE III. Accuracy of nonspinning hybrid waveforms, based on combining PN TaylorT4 or TaylorF2 data with NR waveforms of specified length (defined by the number of orbits before merge = number of GW cycles divided by 2). *Left column:* Range in mass-ratio where a given accuracy requirement ($\max_M \mathscr{M}_{\rm FF} < 3\%$ or 1.5%) is fulfilled. *Right column:* Mismatch error for $q = 20$, both at maximum of all masses (location indicated in parentheses) and at astrophysically motivated minimal values of the total mass (see text).

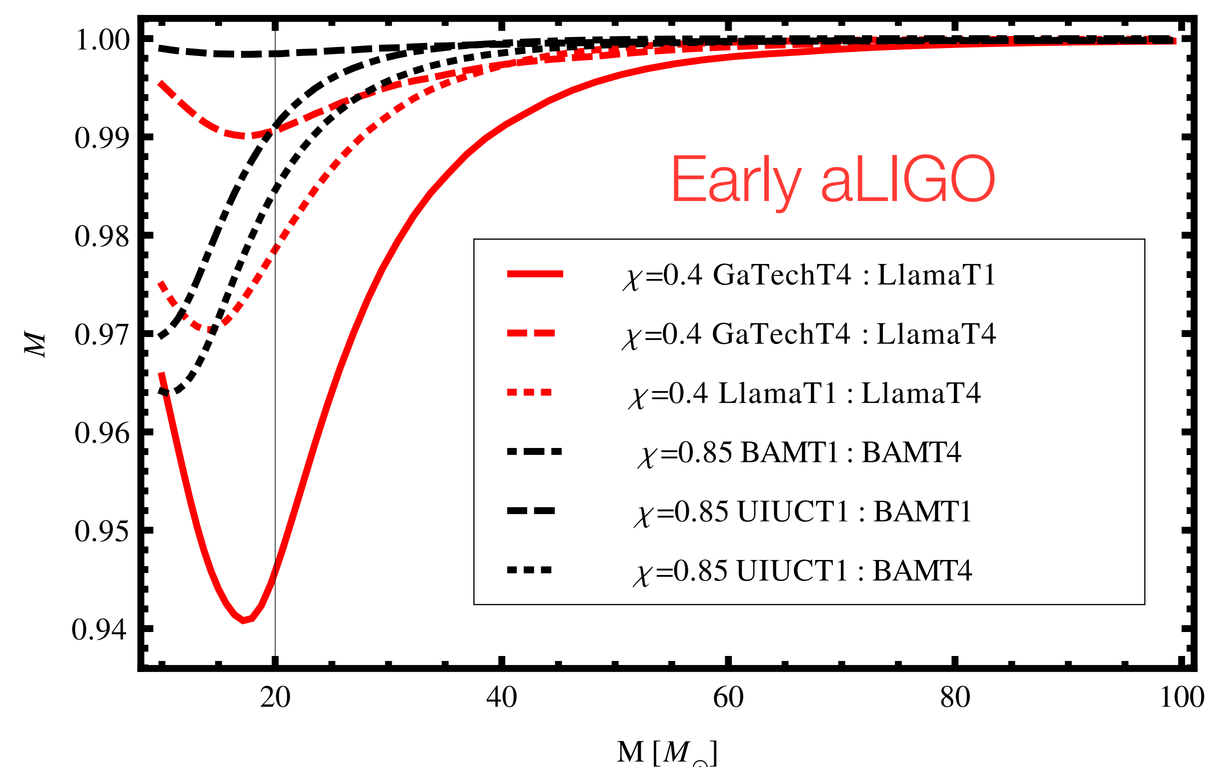
Ninja-2 catalog: Hybrid comparisons between NR groups

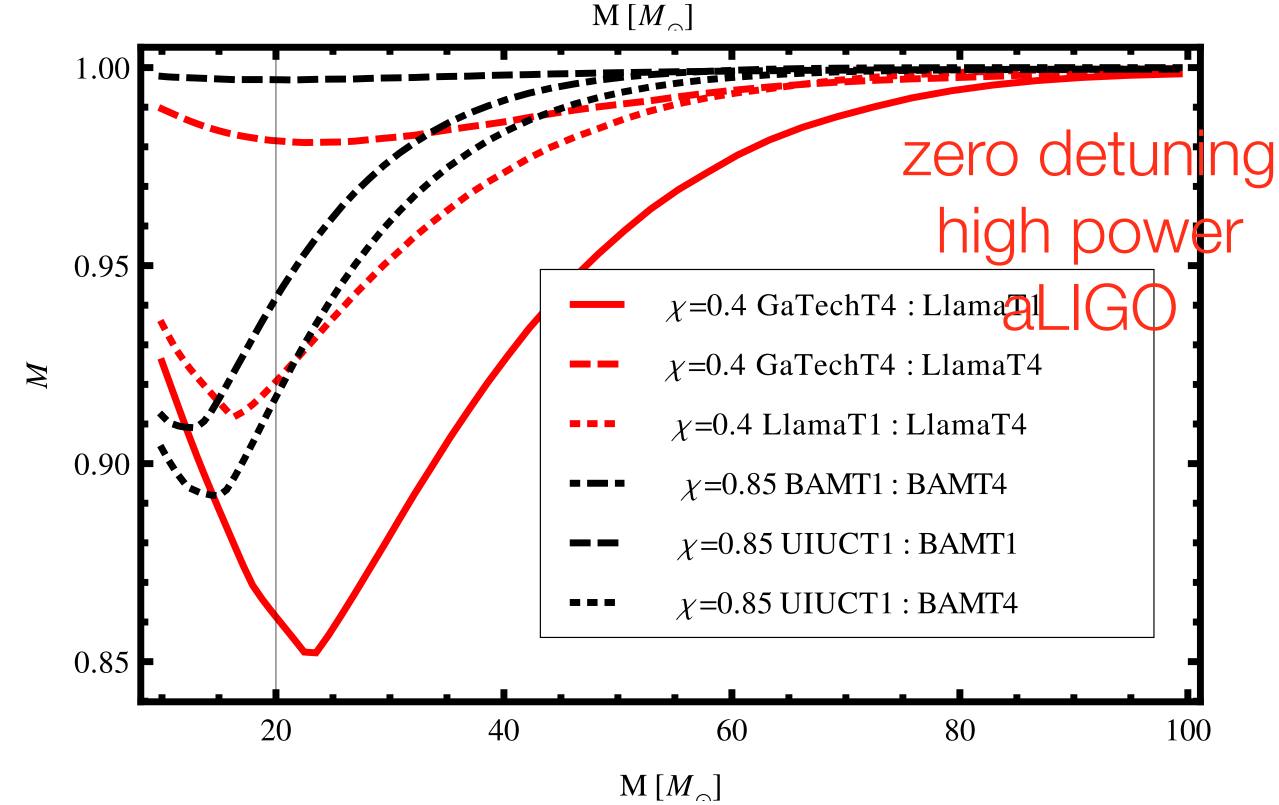
$q = 1,2 \quad \chi_i = 0$





$q = 1 \quad \chi_i = 0.4, 0.85$





Conclusions

- Waveform errors dominated by PN uncertainty/computational cost to produce far longer NR WFs. Modelling errors dominated by lack of NR waveforms.
- Current PN+NR combinations are good enough for detection (~10 NR orbits) for comparable masses or moderate spins.
- We are not able to make NR waveforms long enough to make PN uncertainty disappear.
- Parameter uncertainties may be small (~1% for M & η, 10 % for spin)
- For “design sensitivity” advanced detectors we will need more accurate/longer NR waveforms.
- NR should be able to prevent loss of detections due to waveform errors for most BBH systems, and contribute significantly to parameter estimation, but this will require major efforts: computational exploration of parameter space, analytic waveform models, error analysis, implementation in searches.