Analytical Modeling of Spinning Precessing Compact Binary Waveforms

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Rattle and Shine: Gravitational Wave and Electromagnetic Studies of Compact Binary Mergers

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Motivation

- Compact binary coalescence are the most promising gravitational-wave (GW) sources.
- Accurate waveforms crucial for their GW observation – templates for detection, parameter estimation, testing GR
- Astrophysical BHs possess spin and spins bring distinct features to waveforms
- Numerical-Relativity (NR) simulations are too expensive to generate sufficient number of waveforms covering the parameter space

We need accurate analytical waveforms of spinning precessing compact binaries

Motivation – NS and BH

NSNS

Low frequency inspiral relevant for data analysis, rely on PN

NSBH with BH mass < 20 Msun

Low frequency inspiral relevant for data analysis. Several PN approximants exist, with large systematic effects, especially for spin BH and precession

– We could miss the signal

– Systematics may lead to large bias in measurement and localization, we could miss it in EM sky

Improving precessing BHBH modeling would also improve NSBH modeling

[For finite size effects of NS, talks by Masaru Shibata and Jocelyn Read]

Goals

- Primary: a range of low-cost accurate analytical inspiral-merger-ringdown (IMR) waveform models for GW data analysis

Goals – Accuracy Requirement

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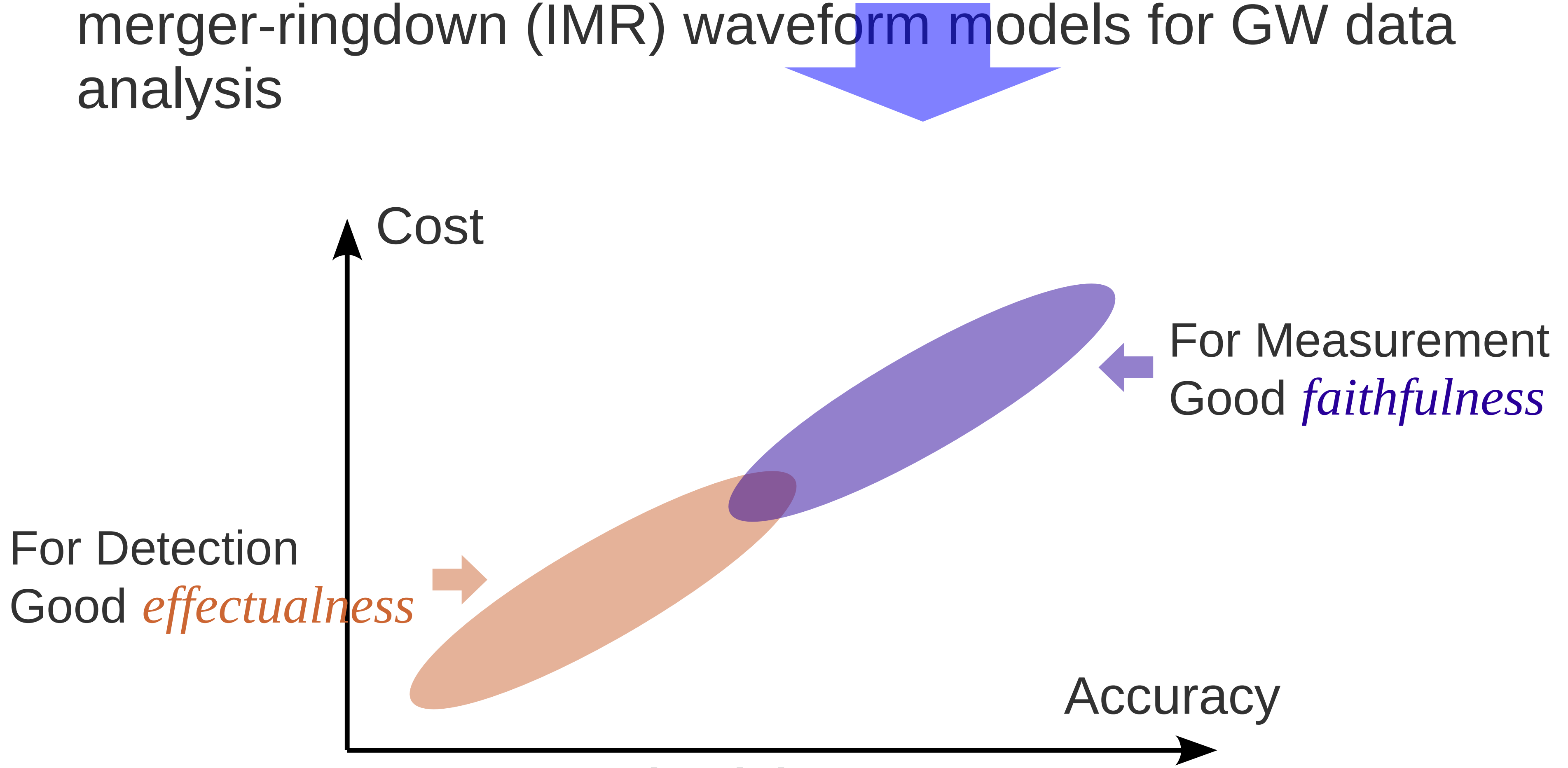
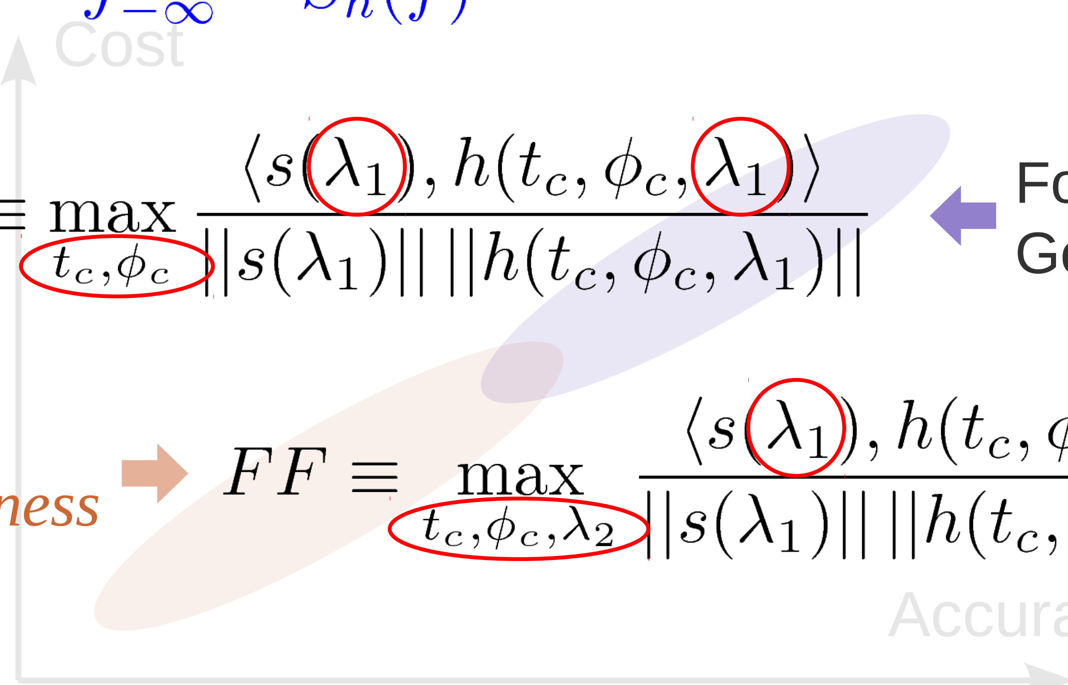


Goals – Effectualness and Faithfulness

- Primary: a range of low-cost accurate analytical inspiral-merger-ringdown (IMR) waveform models for GW data analysis

$$\langle s, h \rangle \equiv \int_{-\infty}^{\infty} \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_h(f)} df \qquad ||h|| \equiv \sqrt{\langle h, h \rangle}$$



$$\mathcal{O} \equiv \max_{t_c, \phi_c} \frac{\langle s(\lambda_1), h(t_c, \phi_c, \lambda_1) \rangle}{||s(\lambda_1)|| \, ||h(t_c, \phi_c, \lambda_1)||}$$

For Measurement
Good *faithfulness*

For Detection
Good *effectualness*

$$FF \equiv \max_{t_c, \phi_c, \lambda_2} \frac{\langle s(\lambda_1), h(t_c, \phi_c, \lambda_2) \rangle}{||s(\lambda_1)|| \, ||h(t_c, \phi_c, \lambda_2)||}$$

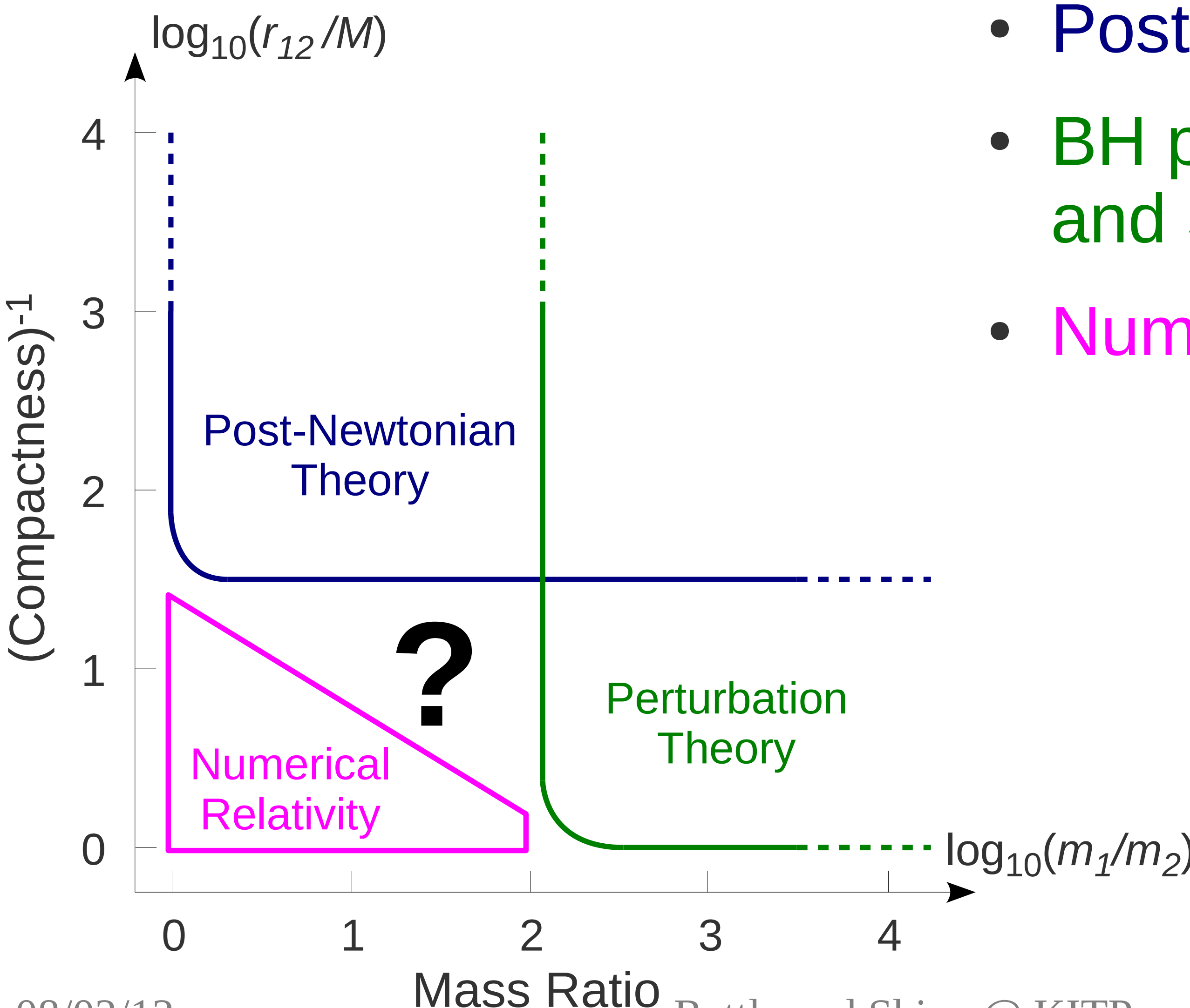
Goals

- Primary: a range of low-cost accurate analytical inspiral-merger-ringdown (IMR) waveform models for GW data analysis
- Byproduct: better understanding of the physics of strong, dynamical gravitational fields

Outline

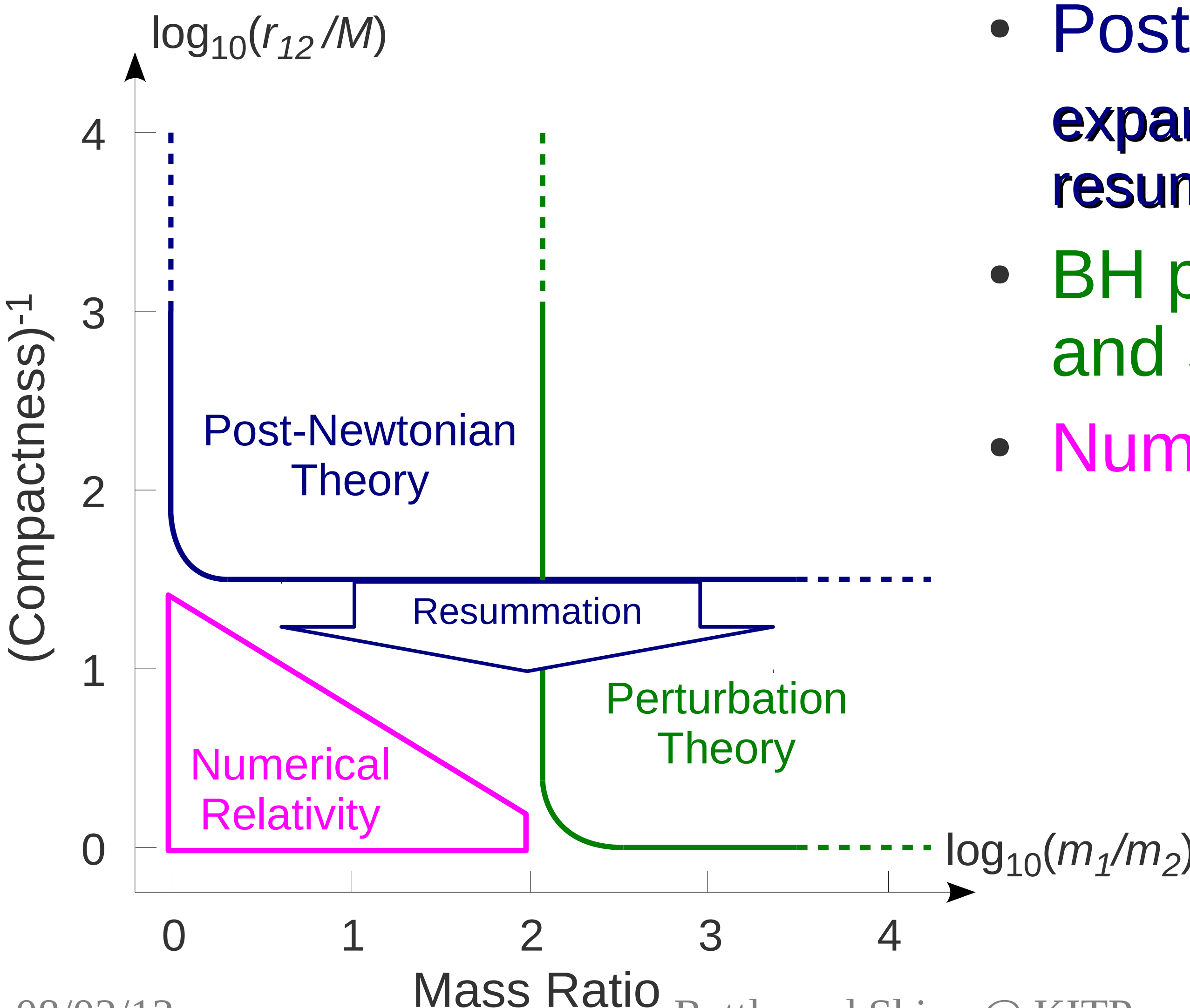
- **Tools we have**
- Past models – a chronicle
- Current models – those available for data analysis now
- Future models – road to precession

Tools we have



- Post-Newtonian Theory
- BH perturbation Theory and Self-force
- Numerical Relativity

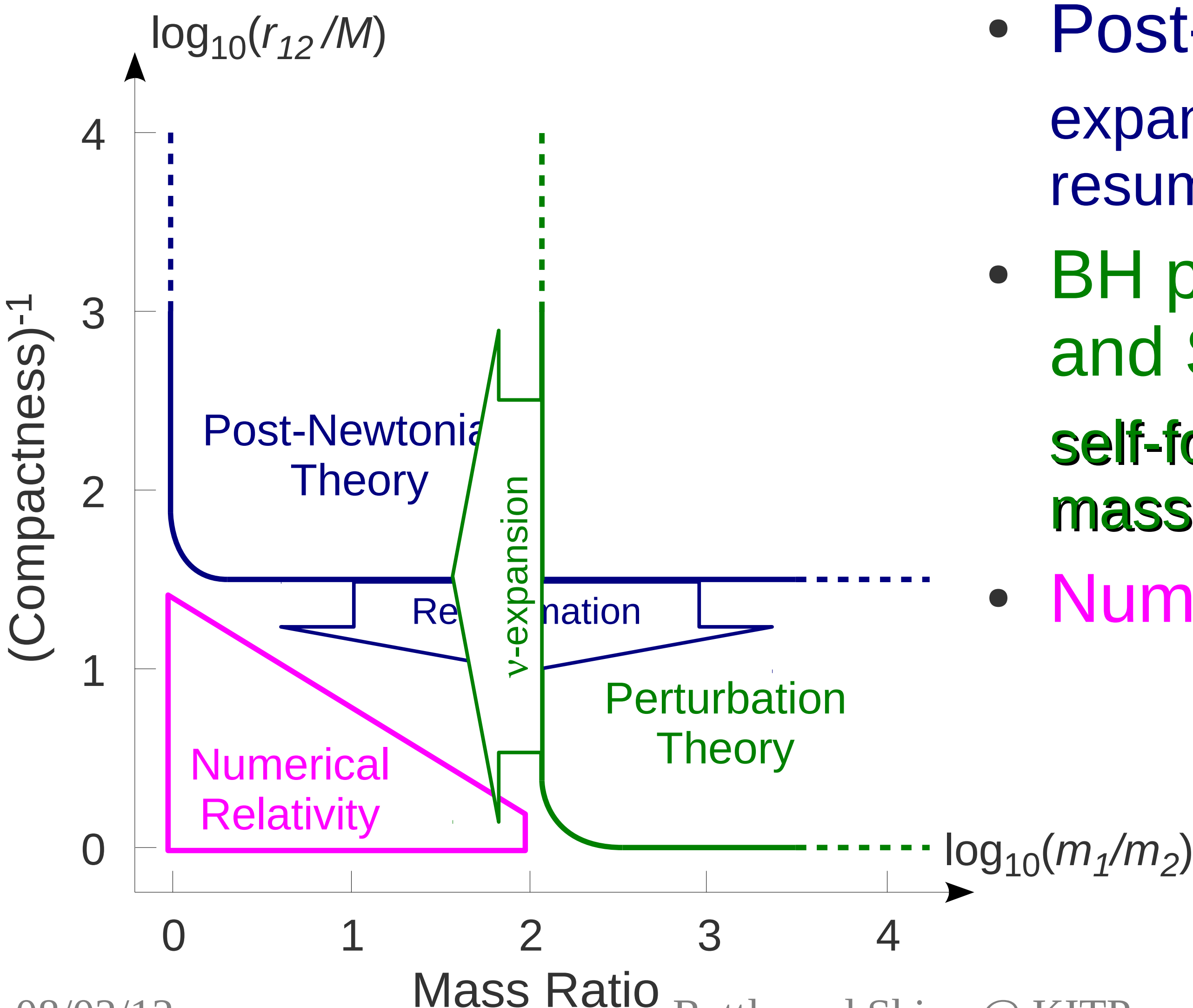
Expand PN to Close Separation



- Post-Newtonian Theory

  expand PN validity through resummation
- BH perturbation Theory and Self-force
- Numerical Relativity

BH Perturbation for Comparable-mass



- Post-Newtonian Theory

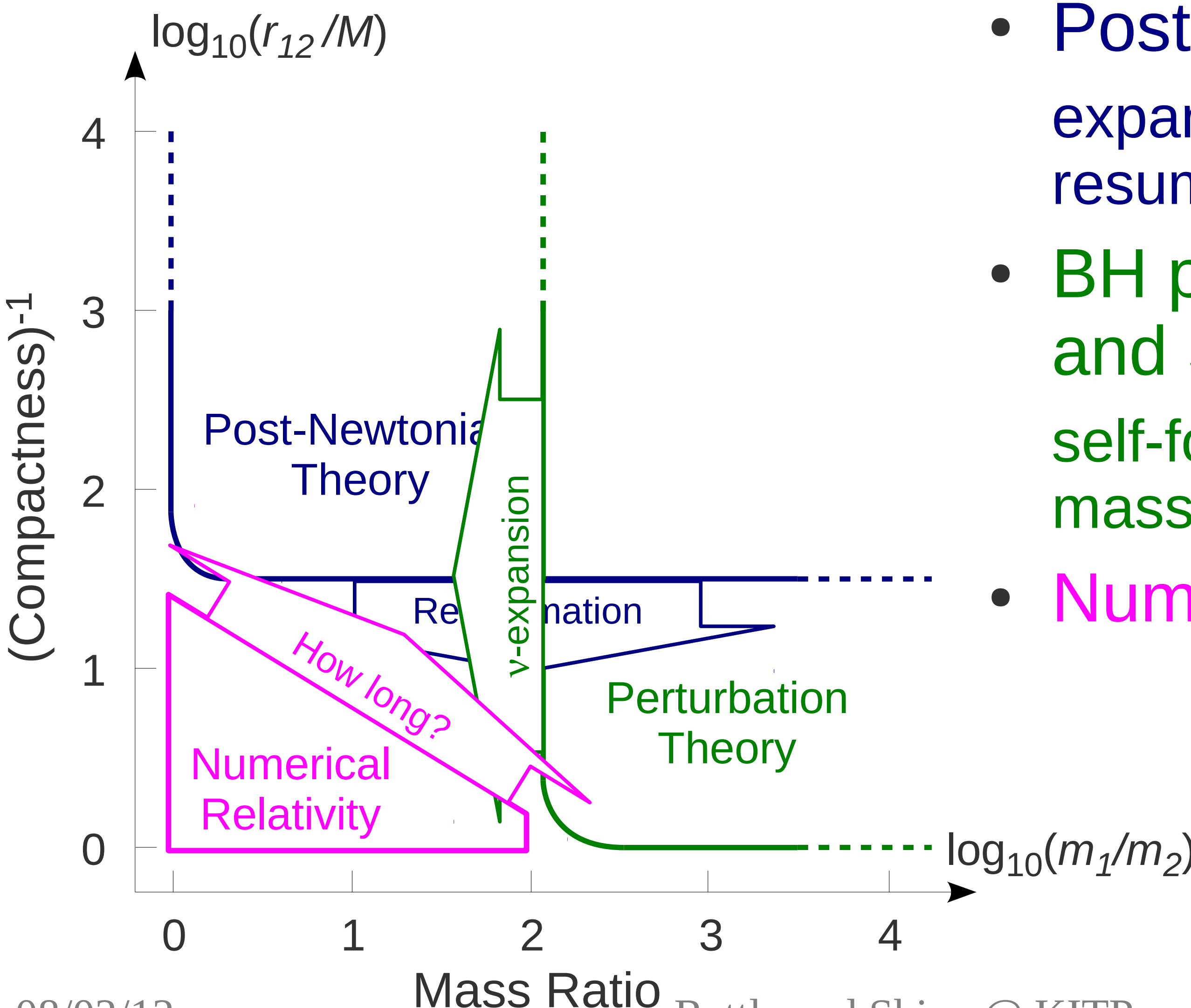
  expand PN validity through resummation

- BH perturbation Theory and Self-force

  self-force for comparable-mass binaries [Le Tiec et al., 11]

- Numerical Relativity

Tools we have



- Post-Newtonian Theory
  expand PN validity through resummation
- BH perturbation Theory and Self-force
  self-force for comparable-mass binaries
- Numerical Relativity
  length requirement (next talk by Sascha Husa)

Outline

- Tools we have
- **Past models – a chronicle**
- Current models – those available for data analysis now
- Future models – road to precession

PN Approximants

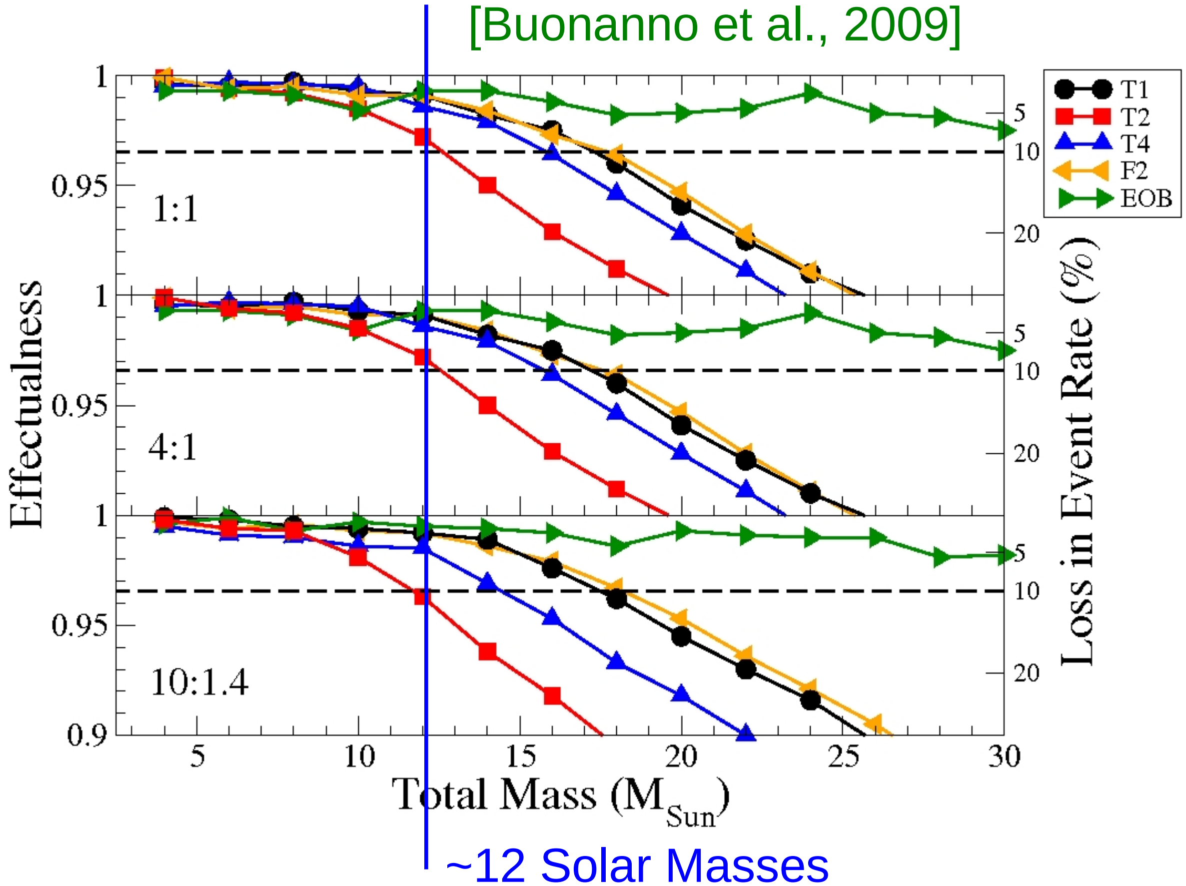
19xx	Several PN approximants [Damour, Iyer, Sathya, Blanchet & Will]

Features

- Adiabatic circular orbits
- Energy balance equation
- Stationary Phase Approx.

Problems

- They are different
- Not reliable beyond ISCO
- > 11 parameters, > 1 billion templates

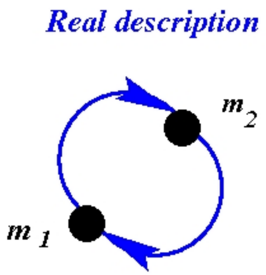


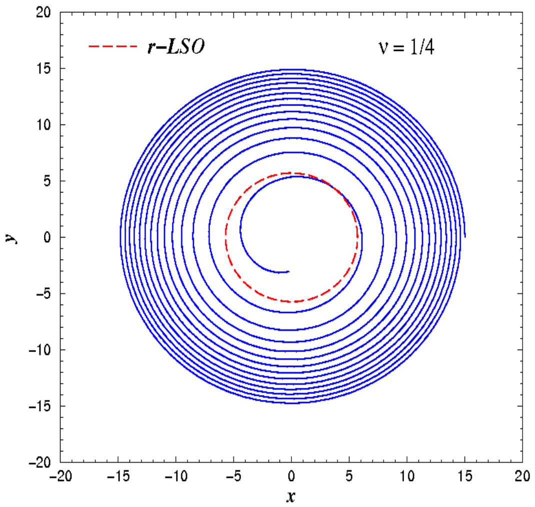
EOB – First Inspiral-Merger-Ringdown

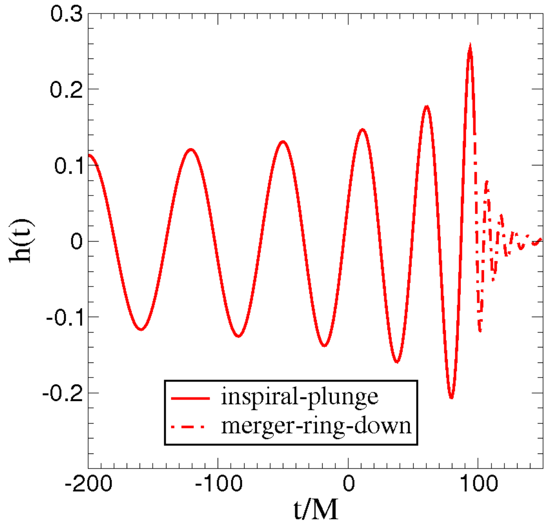
19xx	Several PN approximants
1999 – 2000	**EOB** [Buonanno and Damour]

Idea & Features

- Effective-One-Body
- Non-adiabatic
- PN beyond ISCO
- Instant trans. to ringdown
- Complete IMR waveform







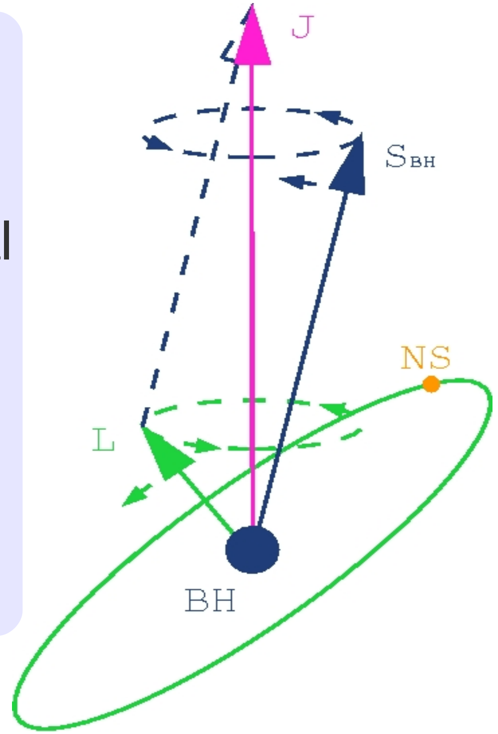
Generic Precessing Models

19xx	Several PN approximants
1999 – 2000	**EOB** [Buonanno and Damour]
2003 – 2004	Precessing Templates [BCV, YP]

PN Precession Picture [Apostolatos et al., 94]

- Simple and Transitional Precession
- Time scale: radiation reaction >> precession >> orbital
- Equal-mass equivalent to Single-spin
- Strong precession when S >> L, i.e. small q

NSBH can be strongly precessing, need better waveform models!



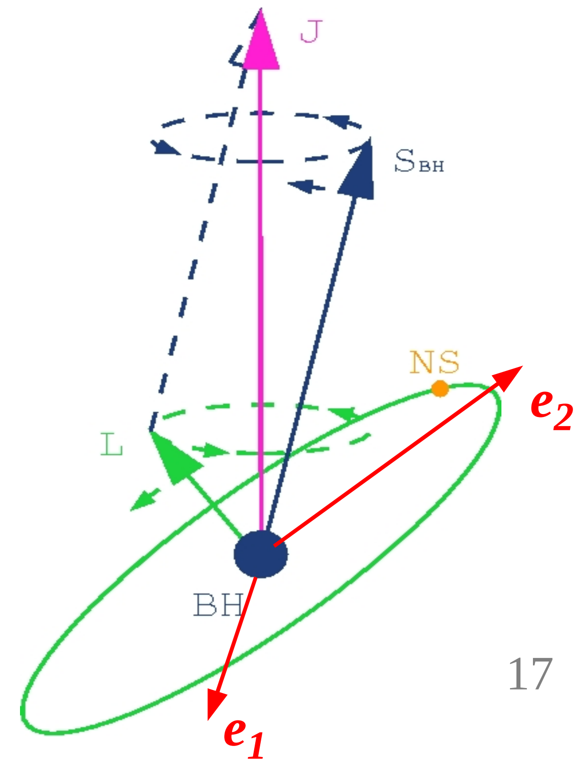
Precessing Frame

19xx	Several PN approximants
1999 – 2000	**EOB** [Buonanno and Damour]
2003 – 2004	Precessing Templates [BCV, YP]

Key ideas on precessing in these models

- Precessing frame simplifies waveform modeling

- Wave modulation mainly from precession of e_1-e_2-L frame, wave in the frame is nearly non-precessing
- Choice of e_1-e_2 – no rotation around L



Dimension Reduction

19xx	Several PN approximants
1999 – 2000	**EOB** [Buonanno and Damour]
2003 – 2004	Precessing Templates [BCV, YP]

Key ideas on precessing in these models

- Precessing frame simplifies waveform modeling
- Dimension reduction of the parameter space

Effectively single-spin:

q = 1 (ignore SS) and q << 1, number of intrinsic parameters from 7 to 4. Expect approximation to be worst for q ~ 1/2 and 1/3.

Matching generic BBH with single spin model, percentage with FF < 0.97

q = 1,	1%
q = 1/2,	10%
q = 1/3,	1%
q = 1/4,	0%

Key Ideas for Precessing Modeling

19xx	Several PN approximants
1999 – 2000	**EOB** [Buonanno and Damour]
2003 – 2004	Precessing Templates [BCV, YP]

Key ideas on precessing in these models

- Precessing frame simplifies waveform modeling
- Dimension reduction of the parameter space

Both concepts improved in recent works

Spin EOB

19xx	Several PN approximants
1999 – 2000	**EOB** [Buonanno and Damour]
2003 – 2004	Precessing Templates [BCV, YP]
2006	**Spin EOB** [Buonanno, Chen and Damour]

Numerical Relativity

19xx	Several PN approximants
1999 – 2000	**EOB** [Buonanno and Damour]
2003 – 2004	Precessing Templates [BCV, YP]
2006	**Spin EOB** [Buonanno, Chen and Damour]

NR

Current Models

19xx	Several PN approximants
1999 – 2000	**EOB** [Buonanno and Damour]
2003 – 2004	Precessing Templates [BCV, YP]
2006	**Spin EOB** [Buonanno, Chen and Damour]
NR	
2007	**EOBNRv1** [Buonanno et al.], **PhenomA** [Ajith et al.]
2009 – 2011	**EOBNRv2, SEOBNRv1** [Pan et al., Taracchini et al.] **PhenomB,C** [Ajith et al., Santamaria et al.] **PhenSpin** [Sturani et al.], Reduced-spin Template [Ajith]

Outline

- Tools we have
- Past models – a chronicle
- **Current models – those available for data analysis now**
- Future models – road to precession

Currently Available Models

- **Those you can use immediately** (with a c-compiler)
- **PhenSpin** – Generic spins, 24 short (4-6 cycles) NR waves
- **Phenom** – Aligned spins [-0.85, 0.85], 35 long (10-22 cycles) NR waves
- **EOBNR** – Aligned spins [-1, 0.7], 7 longer (21-43 cycles) NR waves, higher harmonics
- Common features:
  - Inspiral, PN, PN inspired, or PN resummed, may calibrate to NR
  - Merger, Fit to NR phase, and/or amplitude, around or at the peak
  - Ringdown, Quasinormal Modes (QNMs), or Lorentzian fit to NR

PhenSpin – PN & Phenom Merger

- Adiabatic PN inspiral until merger
- Phenomenological ansatz for merger, fit to NR
- Matched to a linear combination of QNMs

$$\sum_n A_n e^{-t/\tau_n} e^{i\omega_n t}$$

$$\omega_{orb}(t) = \frac{\omega_1}{1 - t/T_A} + \omega_0$$

$$\frac{d\alpha}{dt} = \frac{\dot{\alpha}_1}{1 - t/T_A} + \dot{\alpha}_0$$

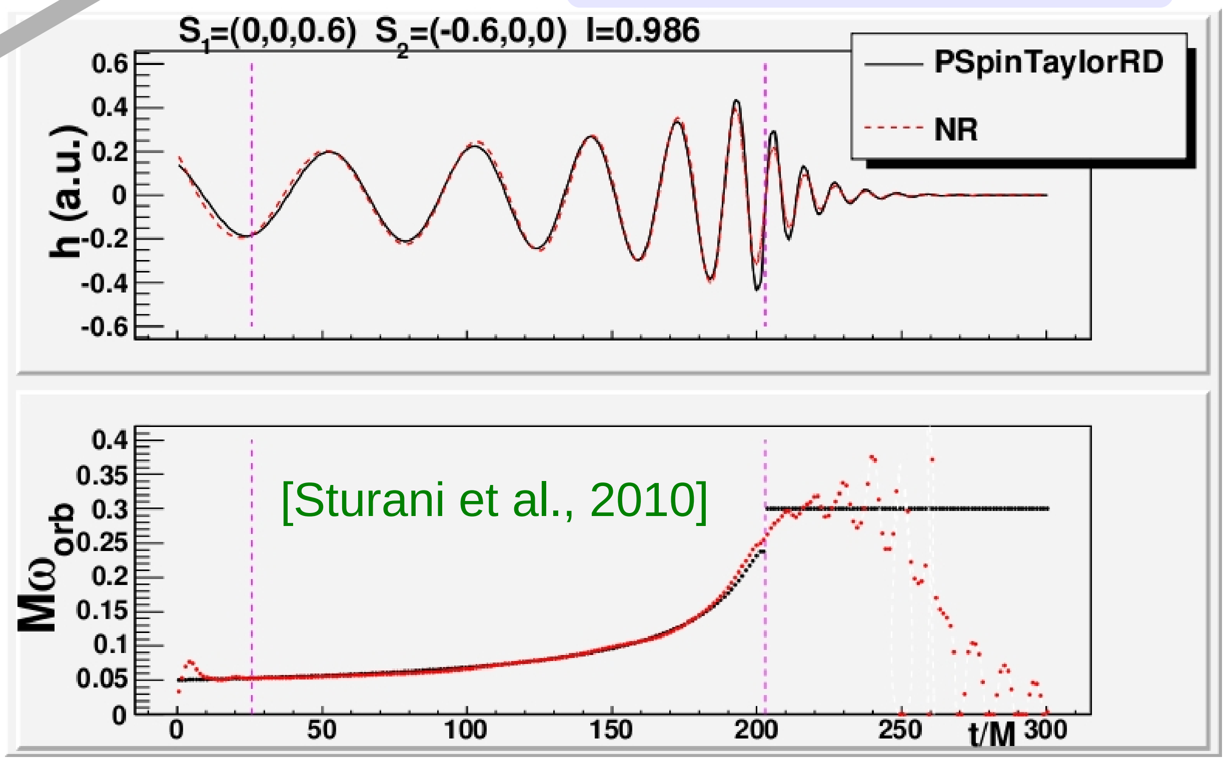
PhenSpin – Qualitative Agreement

- Adiabatic PN inspiral
- Phenomenological ansatz for merger, fit to NR
- Matched to a linear combination of QNMs
- Achieved: generic low-cost model emphasizing agreement around merger
- Requires hundreds of generic NR waveforms for accurate calibration, while only a few dozen available

$$\sum_n A_n e^{-t/\tau_n} e^{i\omega_n t}$$

$$\omega_{orb}(t) = \frac{\omega_1}{1 - t/T_A} + \omega_0$$

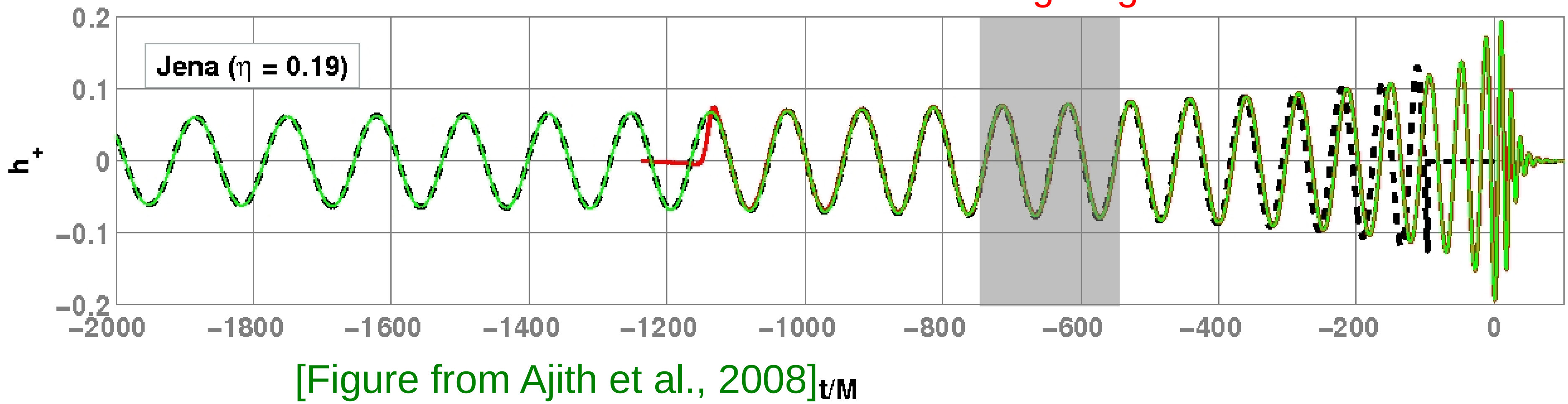
$$\frac{d\alpha}{dt} = \frac{\dot{\alpha}_1}{1 - t/T_A} + \dot{\alpha}_0$$



Phenom – Hybrid PN+NR Waveforms

- PN+NR Hybrid waveforms (time or freq domain)
- At a small number of points in BBH parameter space

Matching Region

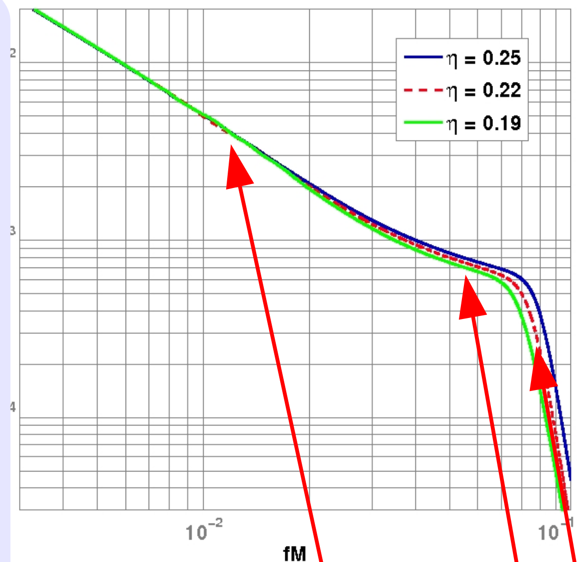


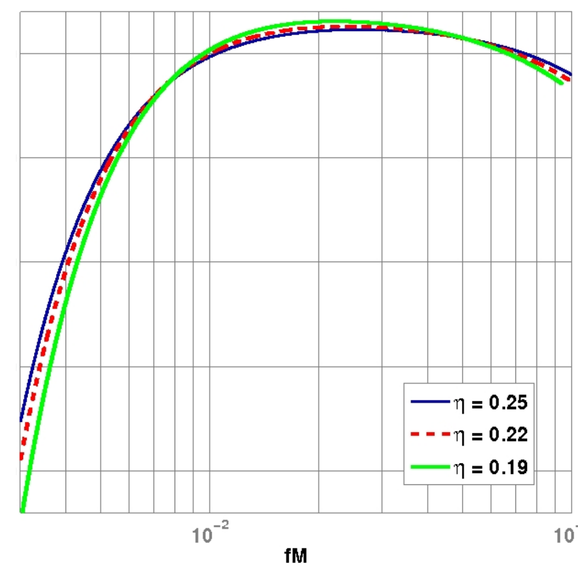
[Figure from Ajith et al., 2008]

PN
NR
Hybrid

Phenom – Fit to Hybrid Wave

- PN+NR Hybrid waveforms (time or freq domain)
- At a small number of points in BBH parameter space
- Interpolate hybrid waves using the freq domain phenomenological model
- Achieved: spin-aligned low-cost model with effectualness > 0.98 faithfulness > 0.965
- Effectual even for mildly precessing binaries effectualness > 0.965





$$A(f) \propto \begin{cases} f'^{-7/6}\,(1 + \sum_{i=2}^{3} \alpha_i\, v^i) & \text{inspiral} \\ w_m\, f'^{-2/3}\,(1 + \sum_{i=1}^{2} \epsilon_i\, v^i) & \text{merger} \\ w_r\, \mathcal{L}(f, f_2, \sigma) & \text{ringdown} \end{cases}$$

$$\Psi(f) \equiv 2\pi f t_0 + \varphi_0 + \frac{3}{128\,\eta\, v^5}\,(1 + \sum_{k=2}^{7} v^k\, \psi_k)$$

EOBNR - Dynamics

- Nonadiabatic EOB dynamics + IMR wave

H_{eff} describes a test particle moving here

$$ds^2_{\text{eff}} = -A_\eta(r)\,dt^2 + \frac{D_\eta(r)}{A_\eta(r)}\,dr^2 + r^2\,d\Omega^2$$

$$A_\eta(r) = 1 - \frac{2}{r} + \frac{2\eta}{r^3} + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\frac{\eta}{r^4} + a_5\frac{\eta}{r^5}$$

Real two-body Hamiltonian $\mathcal{H}^{\text{EOB}} = \sqrt{1 + 2\eta\,(\mathcal{H}^\eta_{\text{eff}} - 1)}$

EOBNR – Dynamics

- Nonadiabatic EOB dynamics + IMR wave

Real two-body Hamiltonian $\mathcal{H}^{\rm EOB} = \sqrt{1 + 2\eta\,(\mathcal{H}^{\eta}_{\rm eff} - 1)}$

Hamilton Equations with R.R. Force

$$\frac{d\boldsymbol{x}}{dt} = \frac{\partial \mathcal{H}^{\rm EOB}}{\partial \boldsymbol{p}} \qquad \frac{d\boldsymbol{p}}{dt} = -\frac{\partial \mathcal{H}^{\rm EOB}}{\partial \boldsymbol{q}} + \boldsymbol{\mathcal{F}} \qquad \frac{d\boldsymbol{S_i}}{dt} = \frac{\partial \mathcal{H}^{\rm EOB}}{\partial \boldsymbol{S_i}} \times \boldsymbol{S_i}$$

EOBNR – Calibration

- Nonadiabatic EOB dynamics + IMR wave
- Calibrate to a few long & accurate NR [SpEC] & Teukolsky waveforms [Hughes, Khanna]

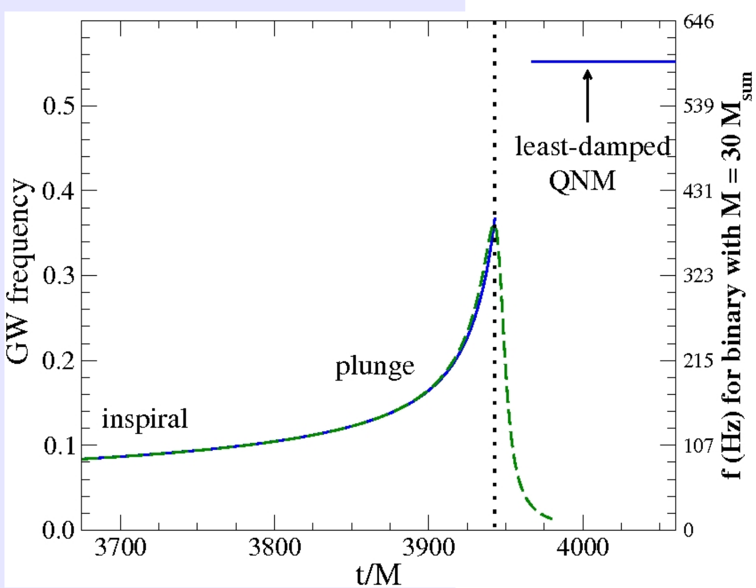
EOBNR – Attach Ringdown

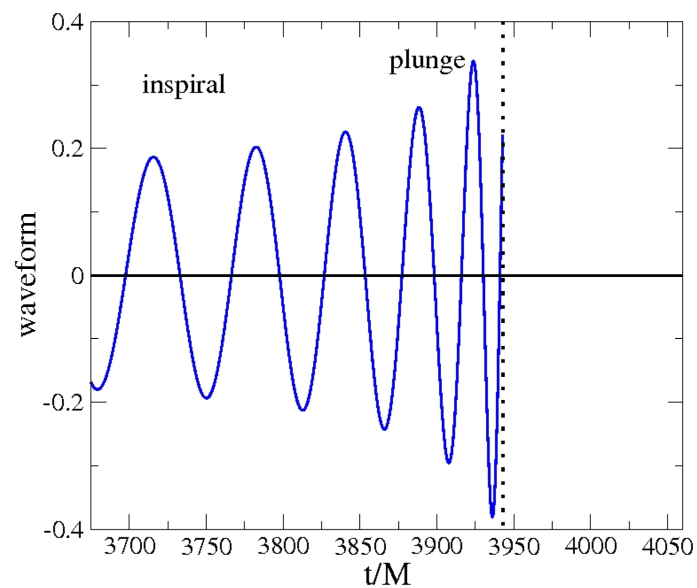
- Nonadiabatic EOB dynamics + IMR wave
- Calibrate to a few long & accurate NR [SpEC] & Teukolsky waveforms [Hughes, Khanna]
- Attach Ringdown QNMs

Linear combination of QNMs

Impose continuity & smoothness conditinos

How are QNMs exited?





EOBNR – Attach Ringdown

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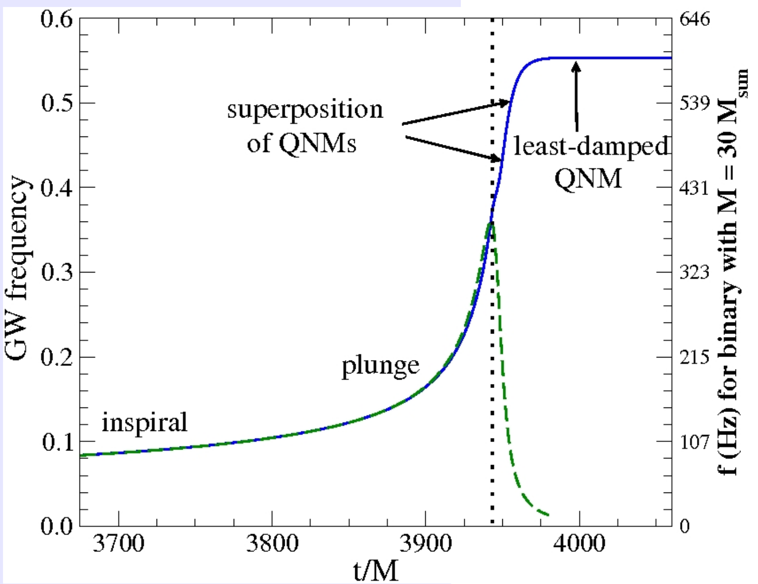
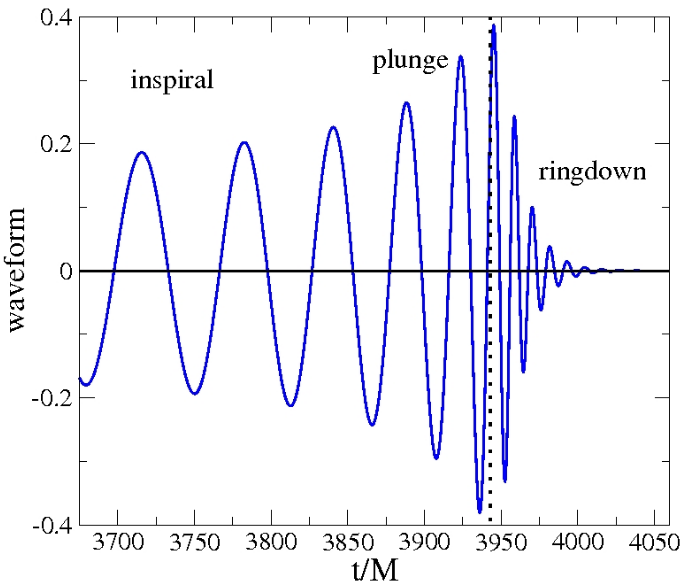
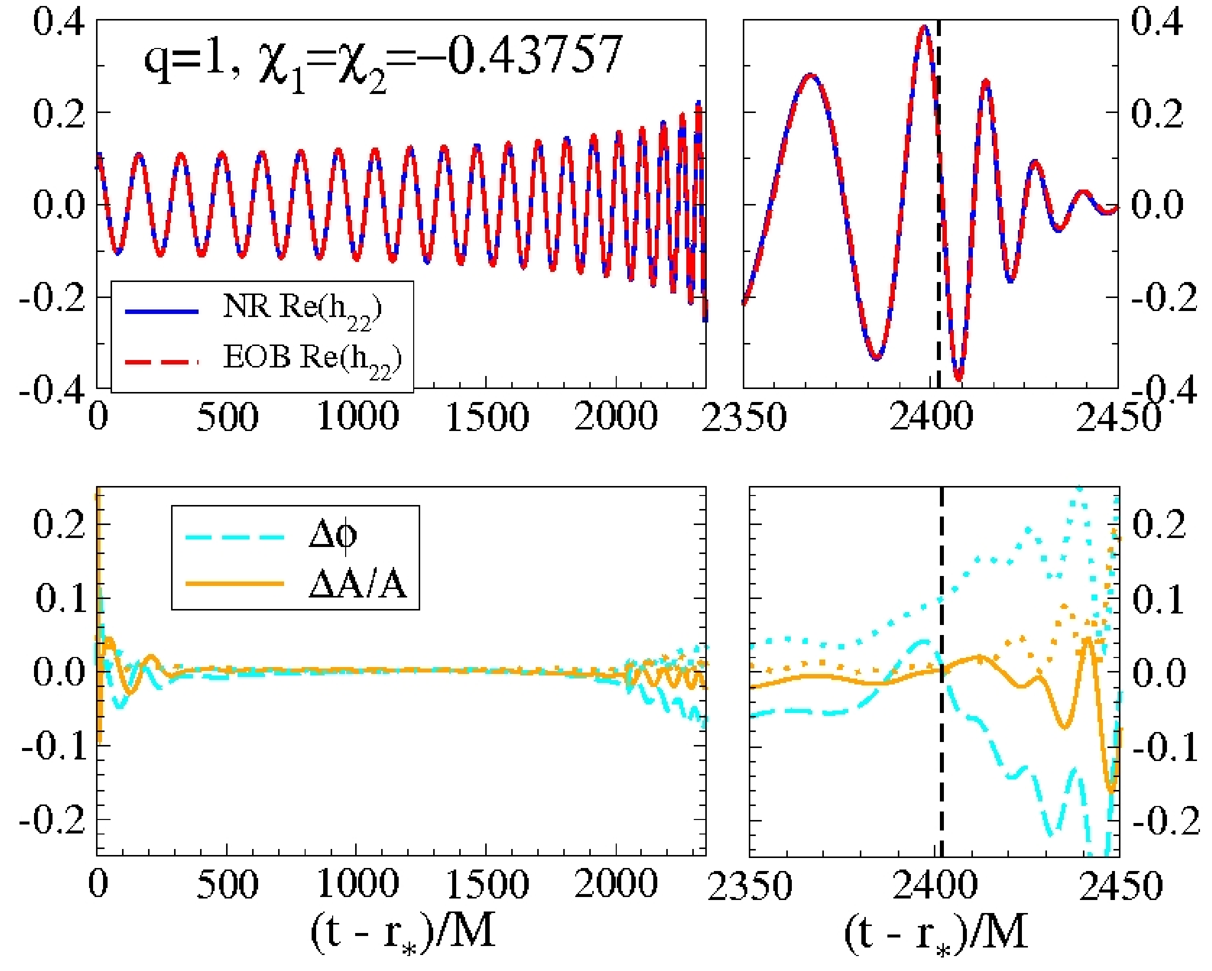




EOBNR – Exact Agreement

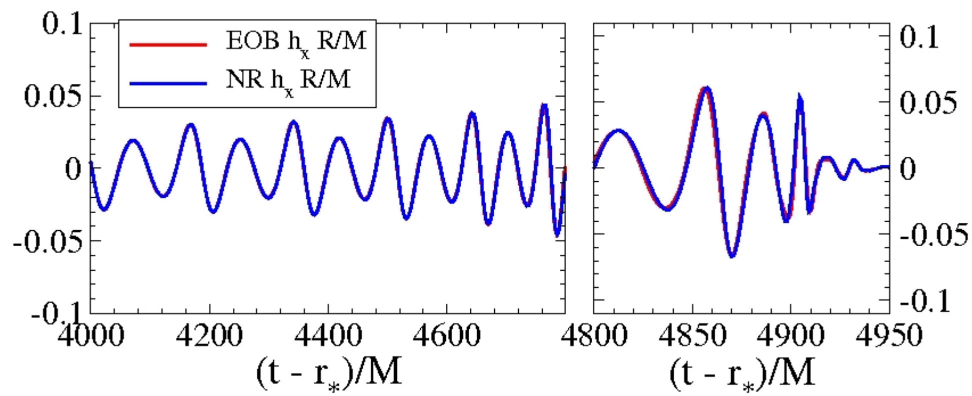
- Nonadiabatic EOB dynamics + IMR wave
- Calibrate to a few long & accurate NR [SpEC] & Teukolsky waveforms [Hughes, Khanna]
- Attach Ringdown QNMs
- Achieved: modeling errors ~ numerical errors
- Spin-aligned model with faithfulness > 0.99

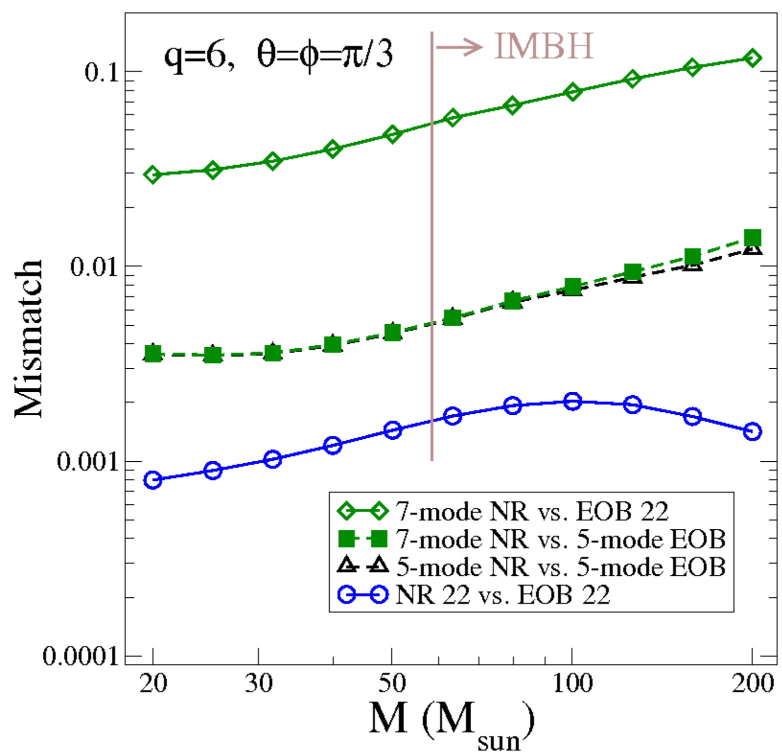


EOBNR – Higher Harmonics I

- Nonadiabatic EOB dynamics + IMR wave
- Calibrate to a few long & accurate NR [SpEC] & Teukolsky waveforms [Hughes, Khanna]
- Attach Ringdown QNMs
- Achieved: modeling errors ~ numerical errors
- Spin-aligned model with faithfulness > 0.99
- **Higher harmonics, new challenges**
  - Effectualness

[Poster by P. Kumar]





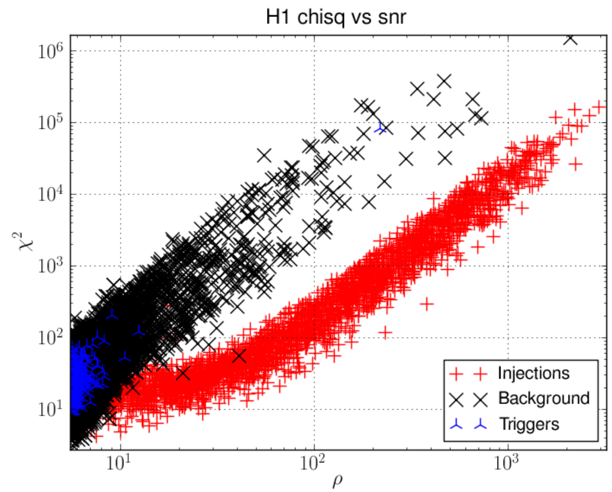
$q = 6, \theta = \phi = \pi/3$

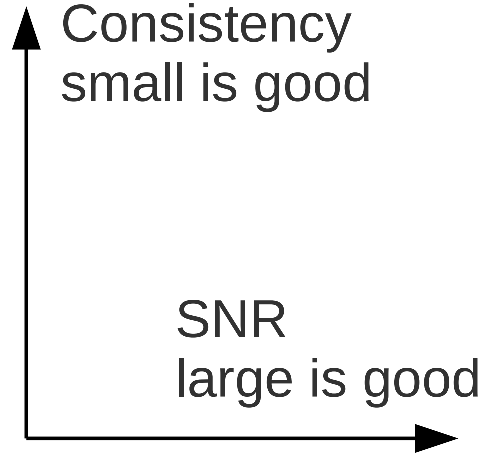
Mismatch ≡ 1 - effectualness

EOBNR – Higher Harmonics II

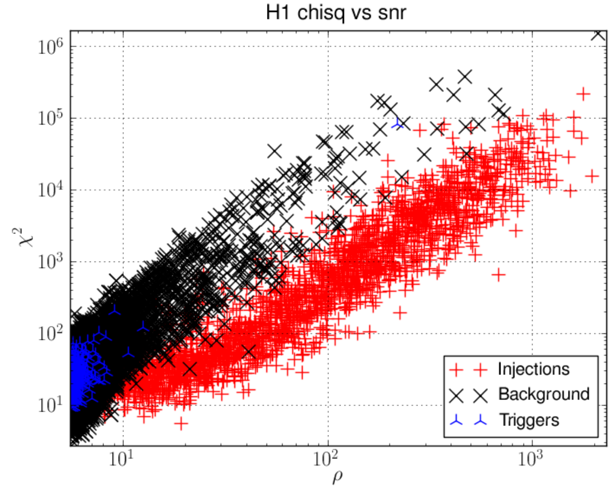
- Nonadiabatic EOB dynamics + IMR wave
- Calibrate to a few long & accurate NR [SpEC] & Teukolsky waveforms [Hughes, Khanna]
- Attach Ringdown QNMs
- Achieved: modeling errors ~ numerical errors
- Spin-aligned model with faithfulness > 0.99
- **Higher harmonics, new challenges**
  - Effectualness
  - Consistency check

q=1

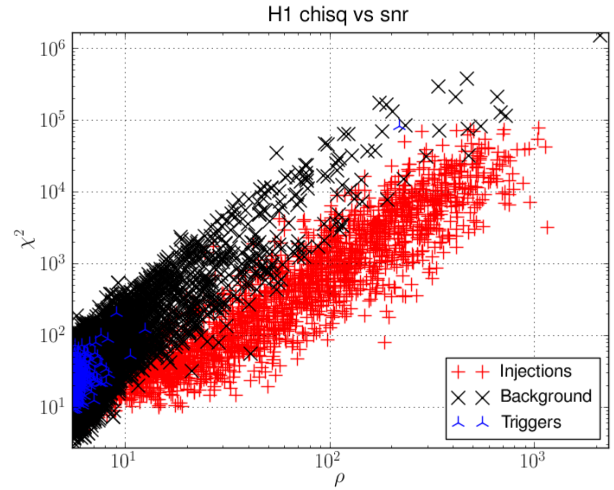




q=3



q=6



[Robinson, 2011]

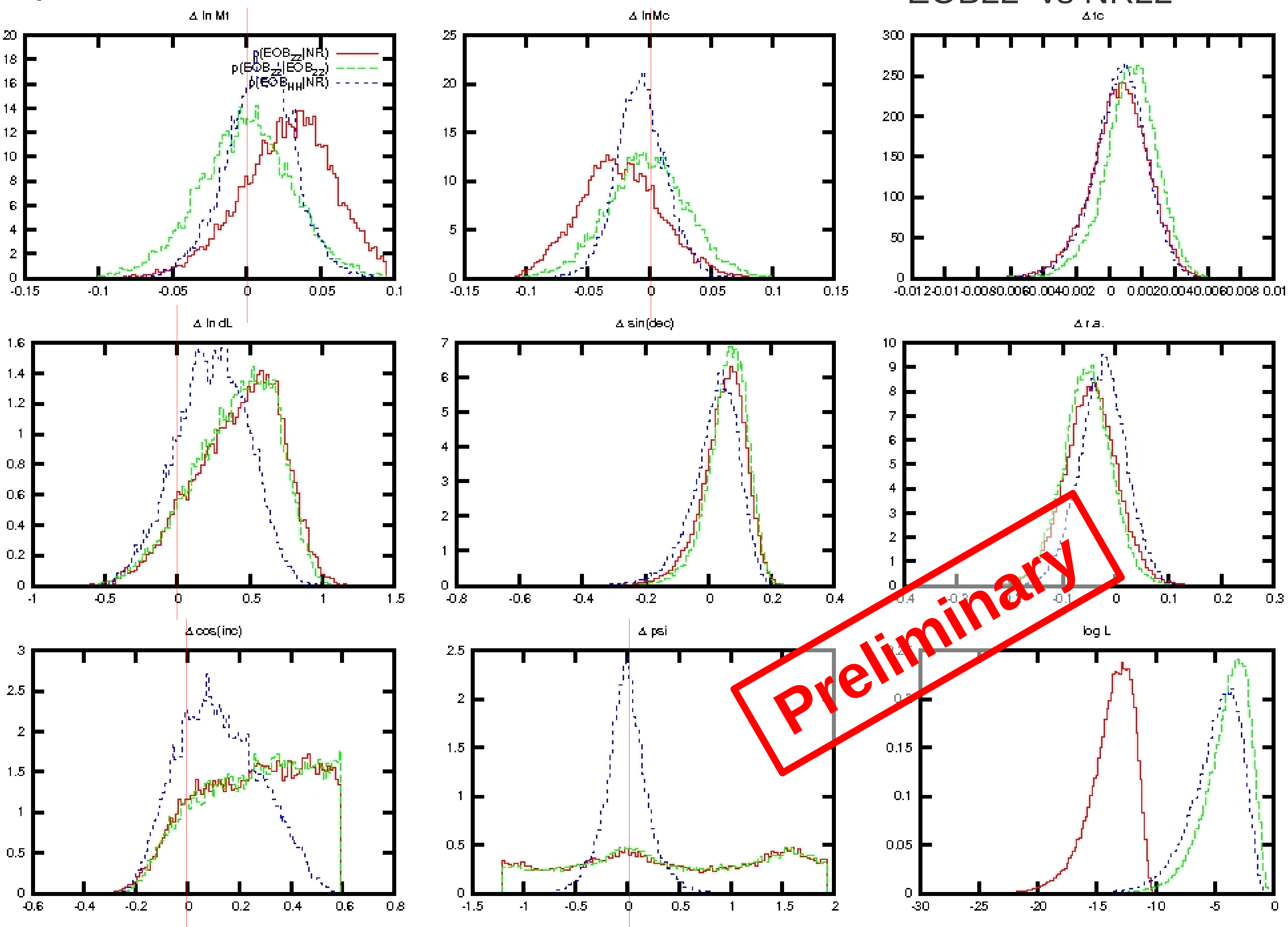
EOBNR – Higher Harmonics III

- Nonadiabatic EOB dynamics + IMR wave
- Calibrate to a few long & accurate NR [SpEC] & Teukolsky waveforms [Hughes, Khanna]
- Attach Ringdown QNMs
- Achieved: modeling errors ~ numerical errors
- Spin-aligned model with faithfulness > 0.99
- **Higher harmonics, new challenges**
  - Effectualness
  - Consistency check
  - Parameter Estimation

[Littenberg et al., in preparation]

q = 1/6, SNR = 12

EOBHH vs NRHH
EOB22 vs NRHH
EOB22 vs NR22



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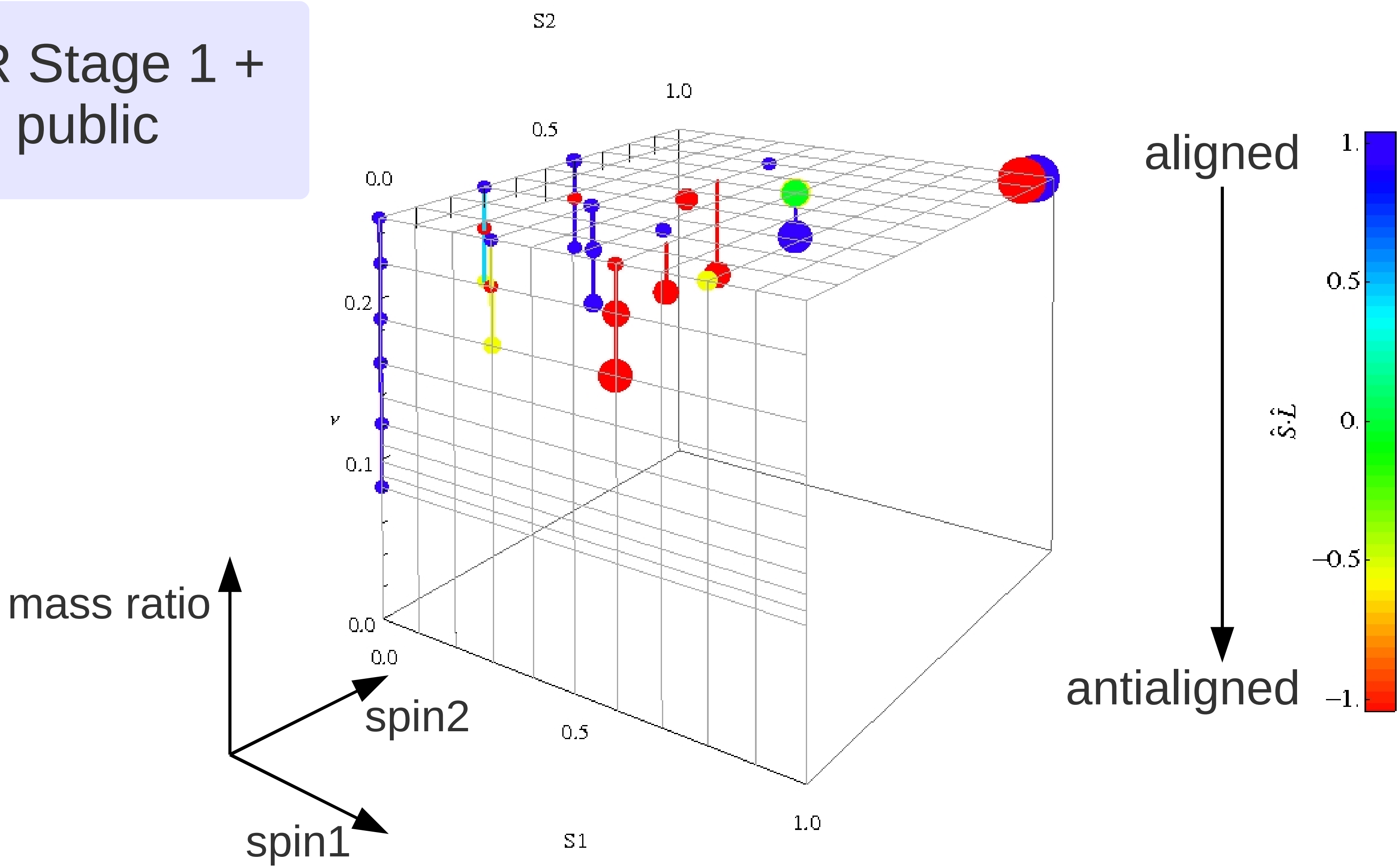
NRAR Collaboration

Numerical Relativity Analytical Relativity (NRAR) Collaboration coordinates 13 NR groups and used 11M CPU hours from NSF and their local resources to simulation new waveforms for developing analytical waveform models

- AEI
- University of Urbino
- University of Jena
- University of the Balearic Islands, Palma
- IHES
- Cardiff University
- Syracuse University
- Cornell University
- NASA Goddard Space Flight Center
- University of Maryland
- RIT
- CITA
- Florida Atlantic University
- Georgia Institute of Tech
- UIUC
- Louisiana State University
- Caltech

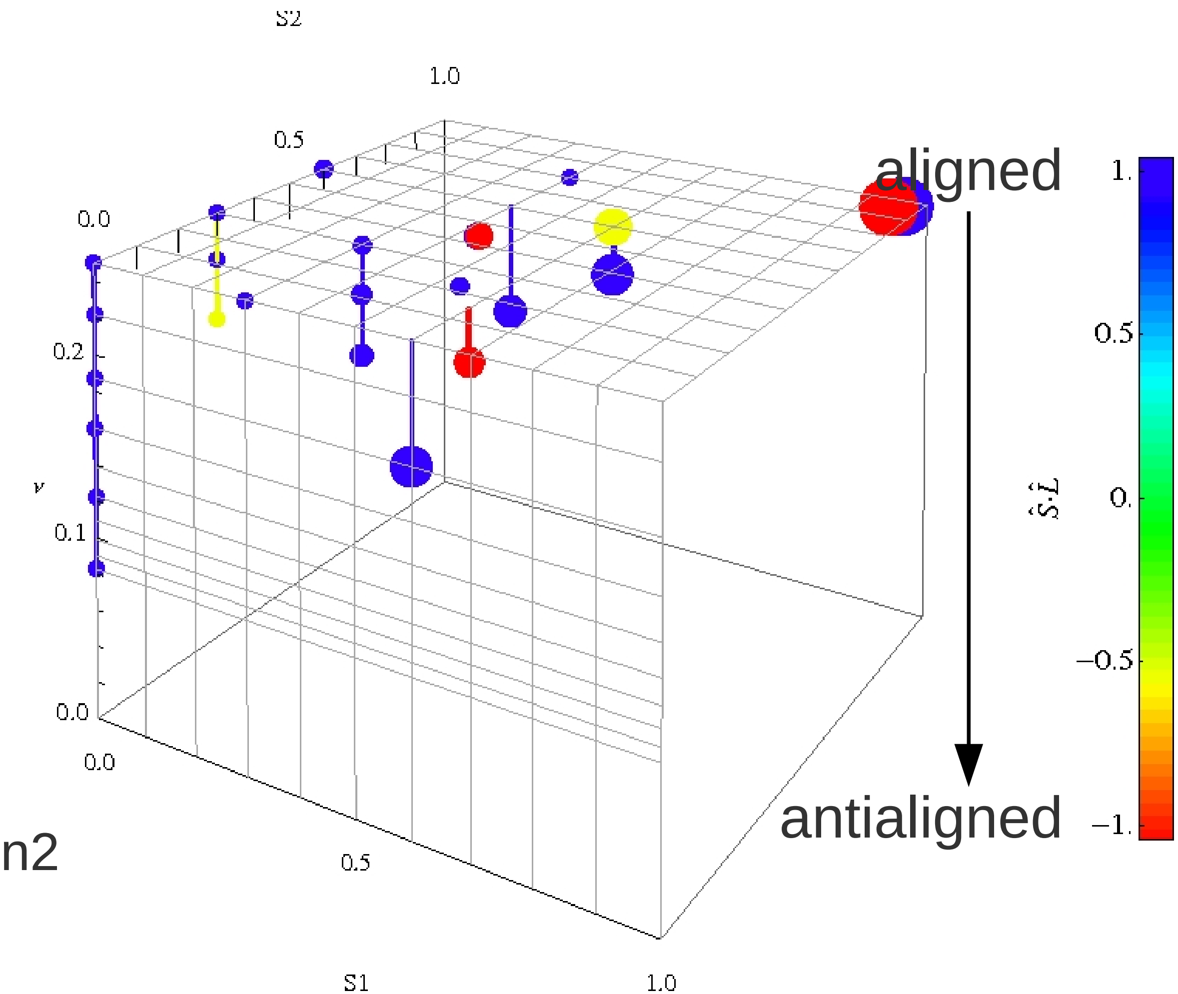
NRAR Stage 1 Plan

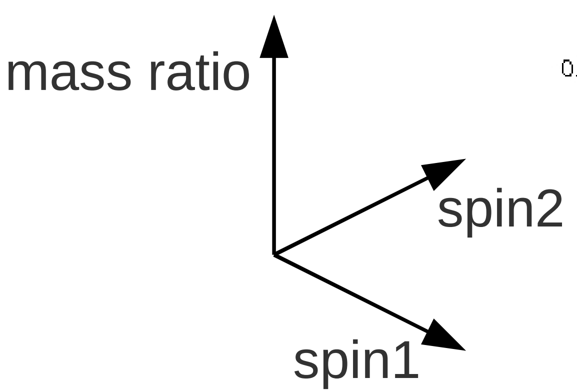
NRAR Stage 1 +
SpEC public



NRAR Stage 1 Available

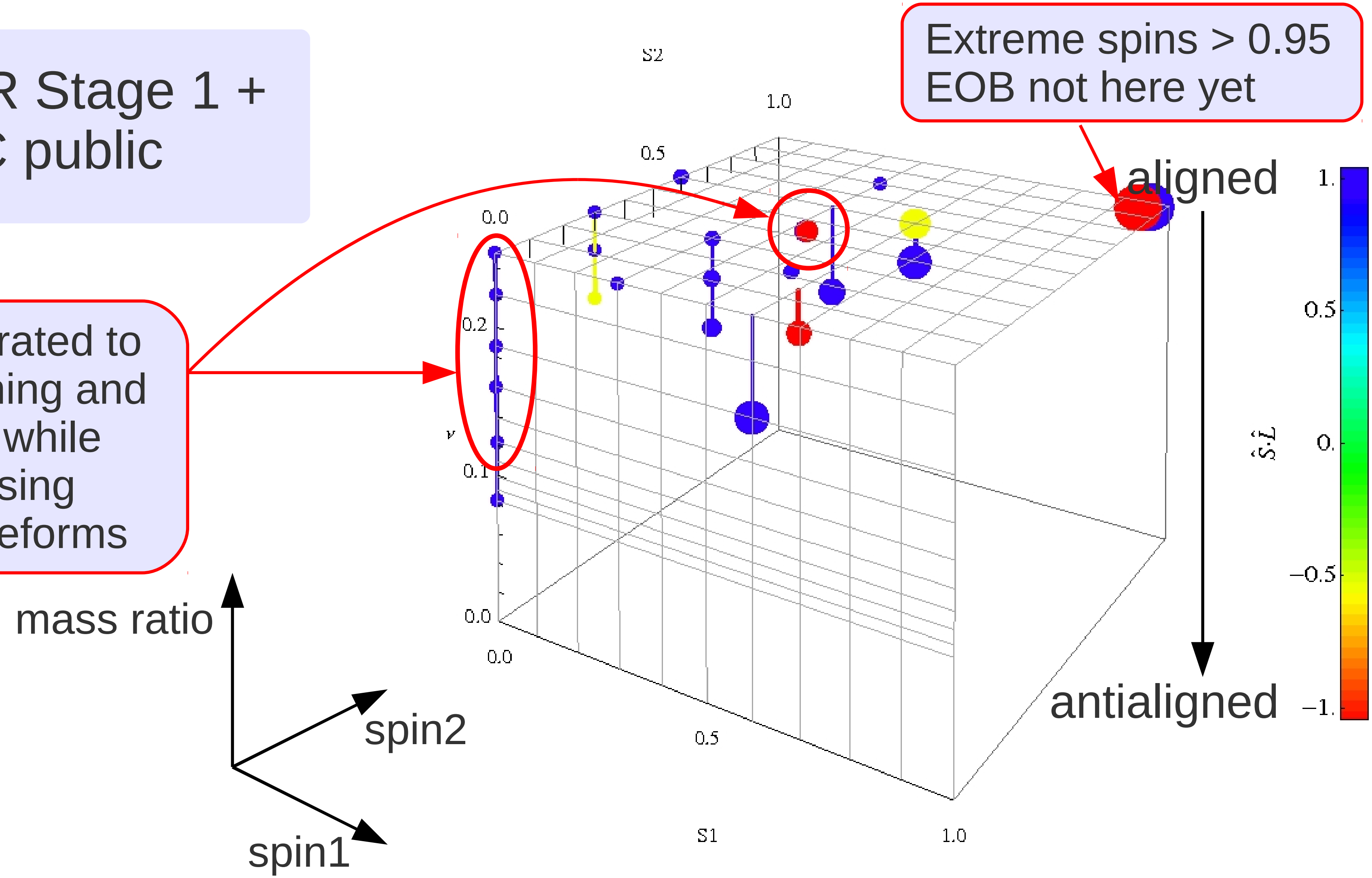
NRAR Stage 1 + SpEC public





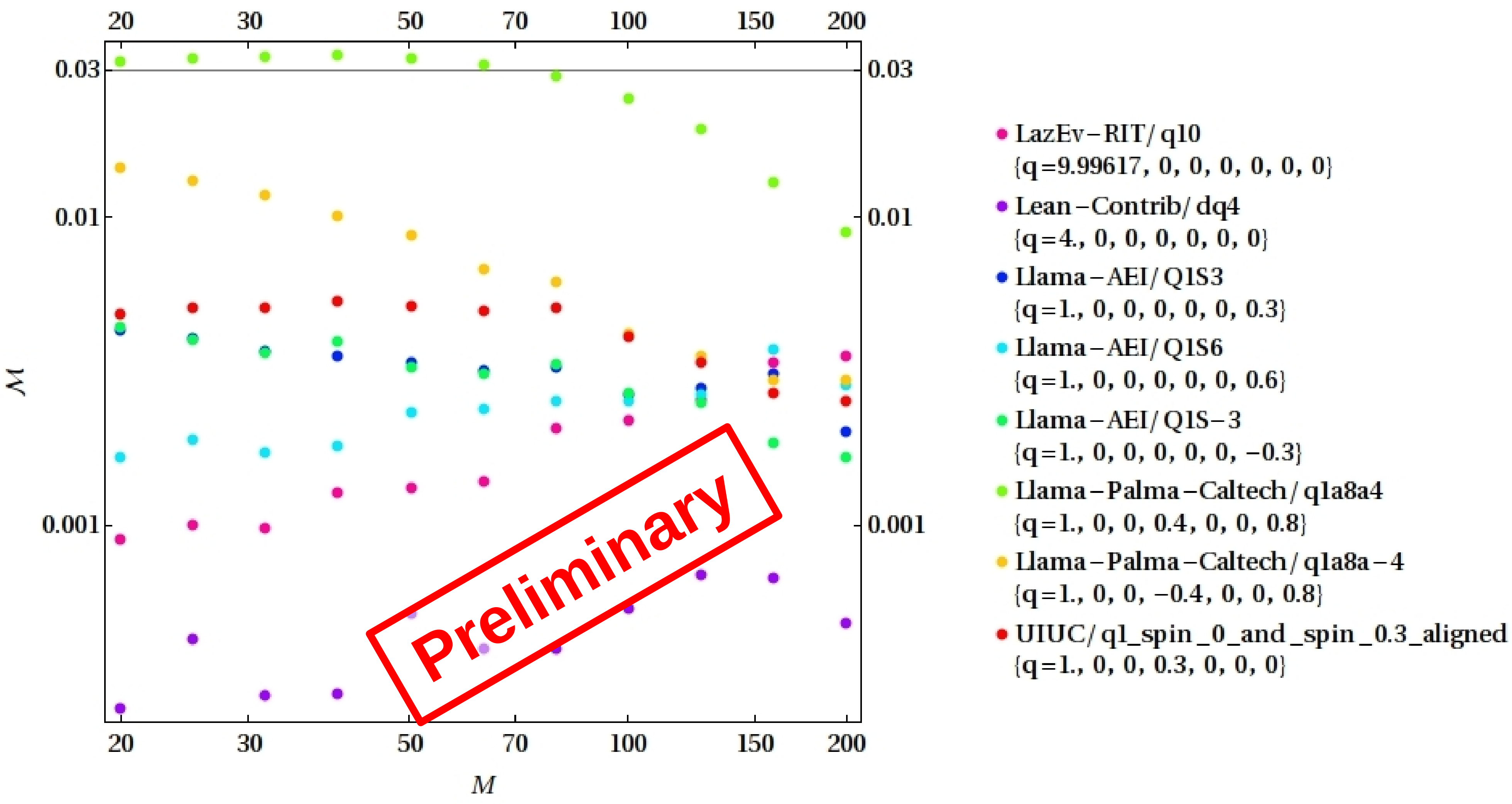
EOBNR Calibration

NRAR Stage 1 +
SpEC public



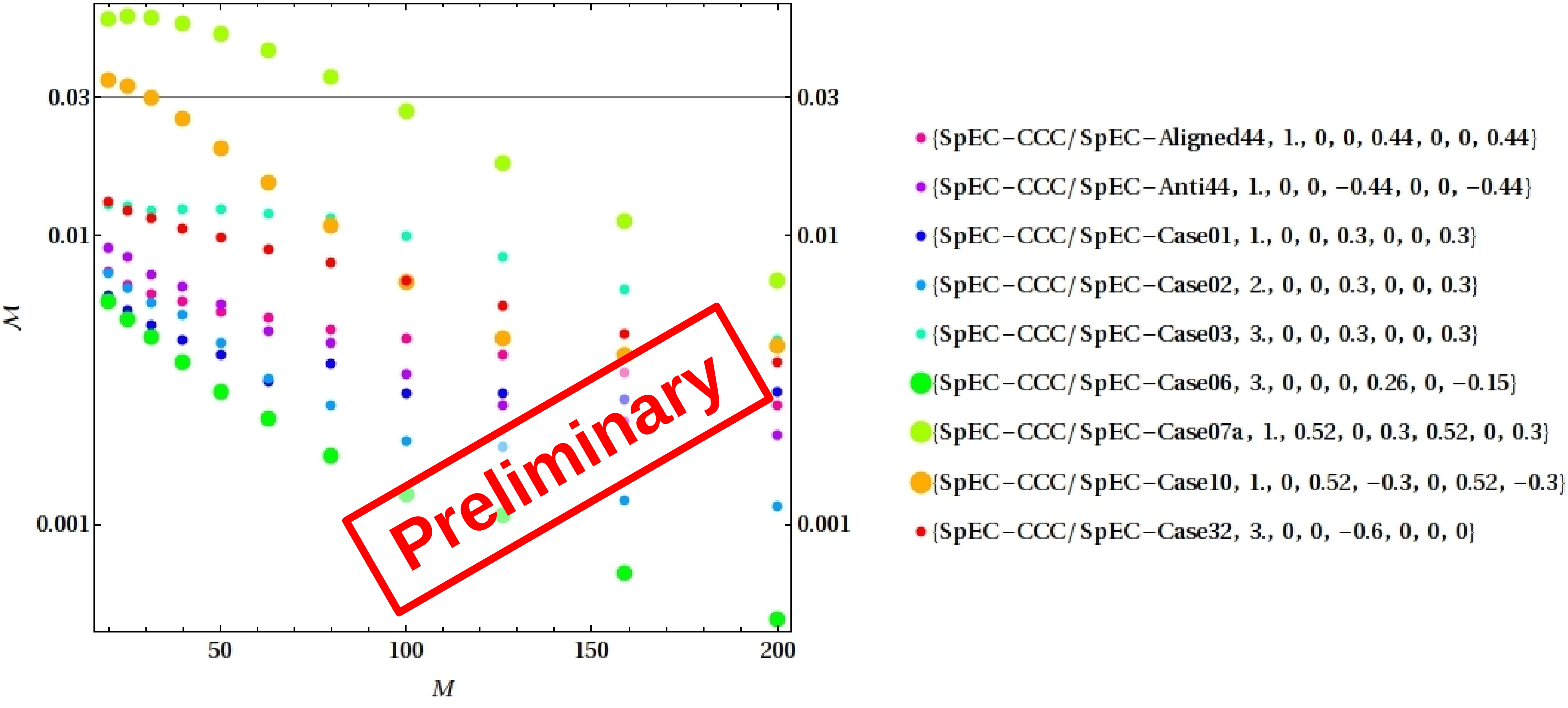
NRAR Stage 1 – Compare with EOBNR

Mismatch between NR and EOB Waveforms



NRAR Stage 1 – Compare with EOBNR

Mismatch between NR and EOB Waveforms



Road to Precession – Dimension Reduction

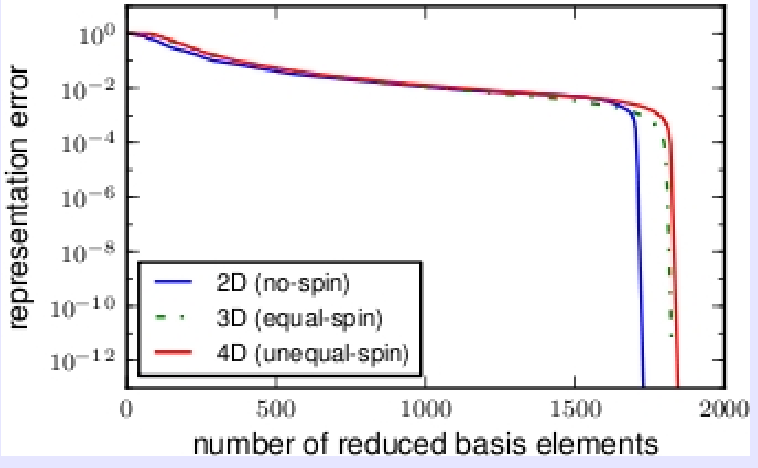
Degeneracy in signal space

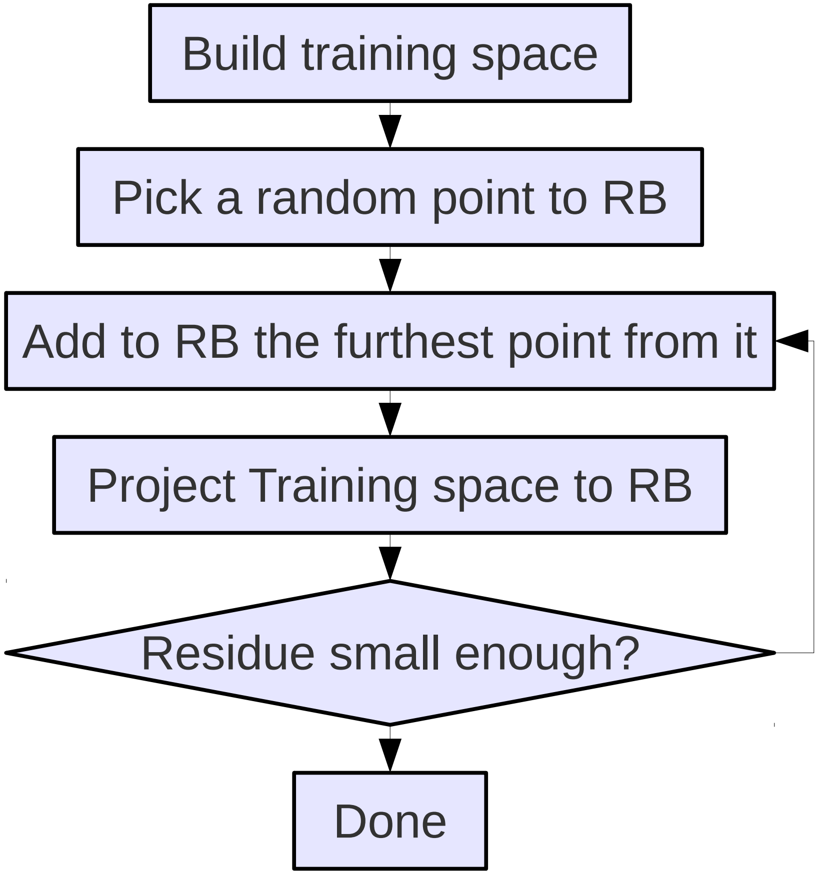
Singular Value Decomposition

[Cannon, Hanna & Keppel, 11]

Reduced Basis

[Field et al., 11; Herrmann et al., 12]





Road to Precession – Dimension Reduction

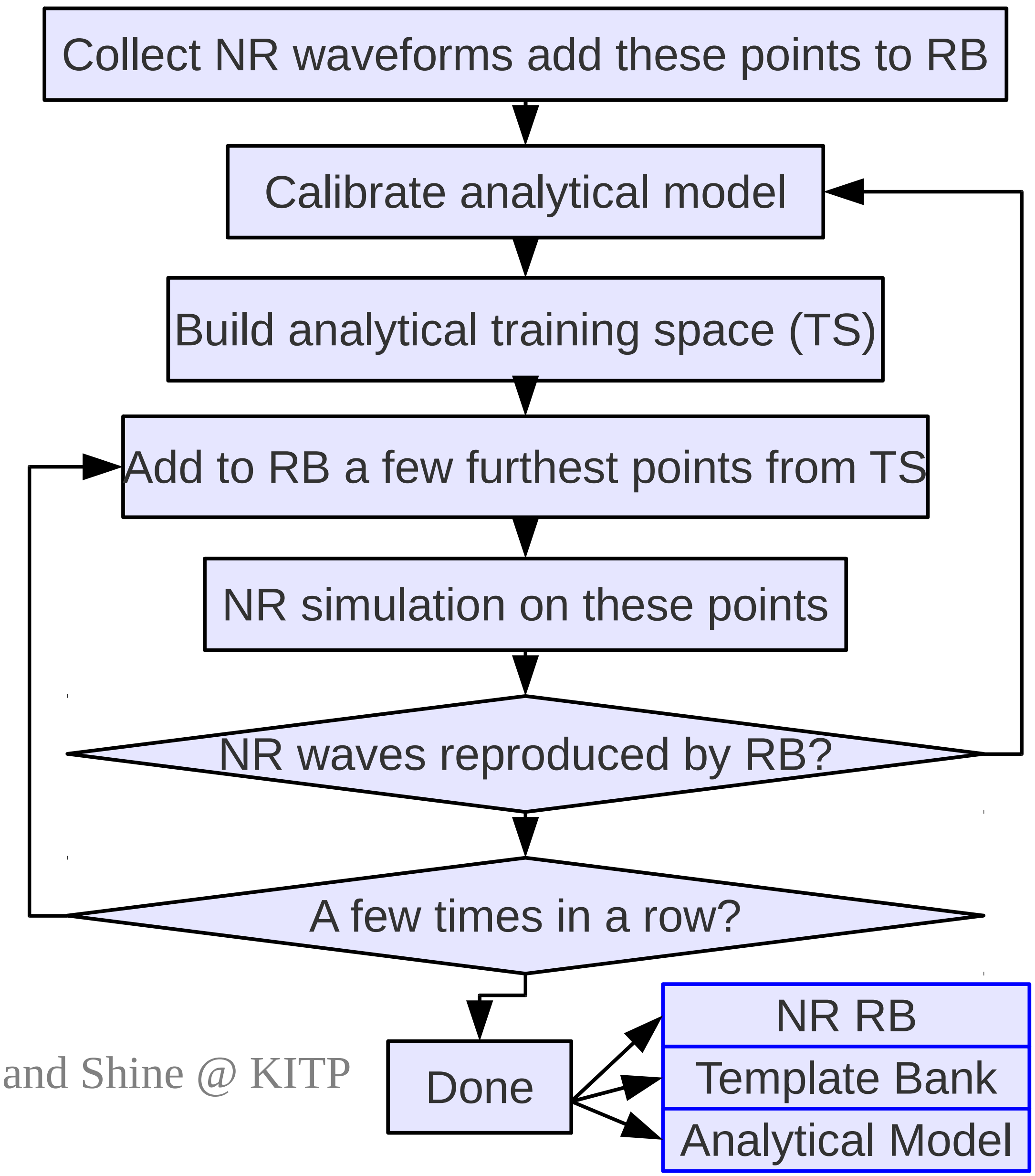
Degeneracy in signal space

Singular Value Decomposition
[Cannon, Hanna & Keppel, 11]

Reduced Basis
[Field et al., 11; Herrmann et al., 12]

Choose next NR Simulation?

NR-RB bank?



Road to Precession – Degeneracy

Degeneracy in signal space

Singular Value Decomposition
[Cannon, Hanna & Keppel, 11]

Reduced Basis
[Field et al., 11; Herrmann et al., 12]
Choose next NR Simulation?
NR-RB bank?

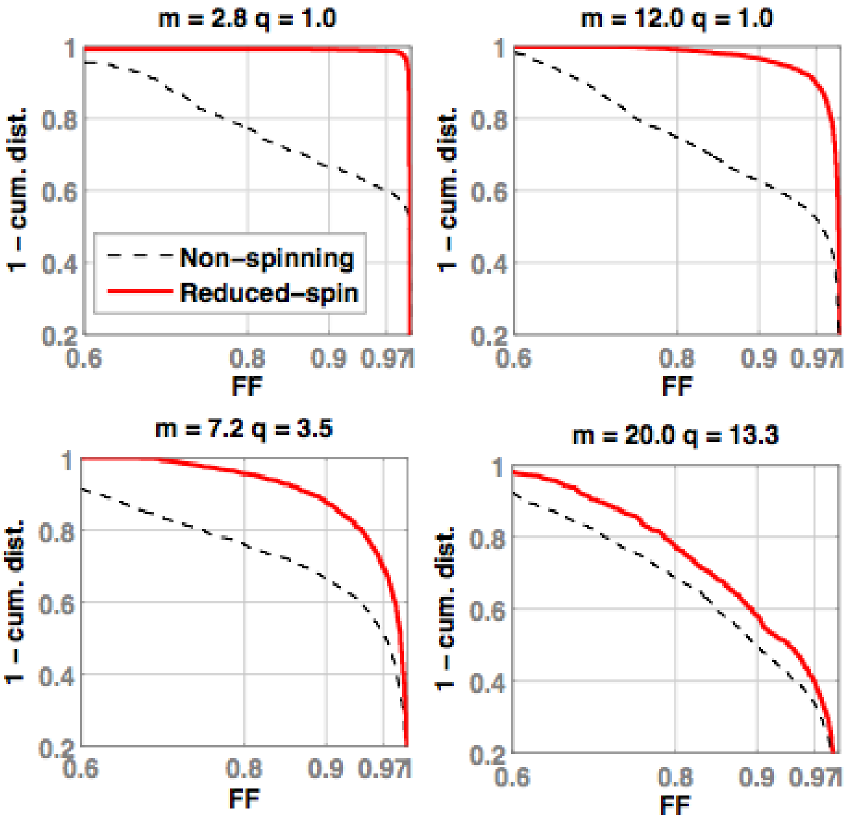
Degeneracy in parameter space

Effective spin in precession
[Damour, 01; Buonanno et al., 04]

Effective spin in phase
[Ajith, 11, Schmidt et al., 12]

$$\boldsymbol{S}_{\text{eff}} = \left(1 + \frac{3}{4}\frac{m_2}{m_1}\right)\boldsymbol{S}_1 + \left(1 + \frac{3}{4}\frac{m_1}{m_2}\right)\boldsymbol{S}_2$$

$$\boldsymbol{S}_{\text{eff}} = \left(1 + \frac{75}{113}\frac{m_2}{m_1}\right)\boldsymbol{S}_1 + \left(1 + \frac{75}{113}\frac{m_1}{m_2}\right)\boldsymbol{S}_2$$



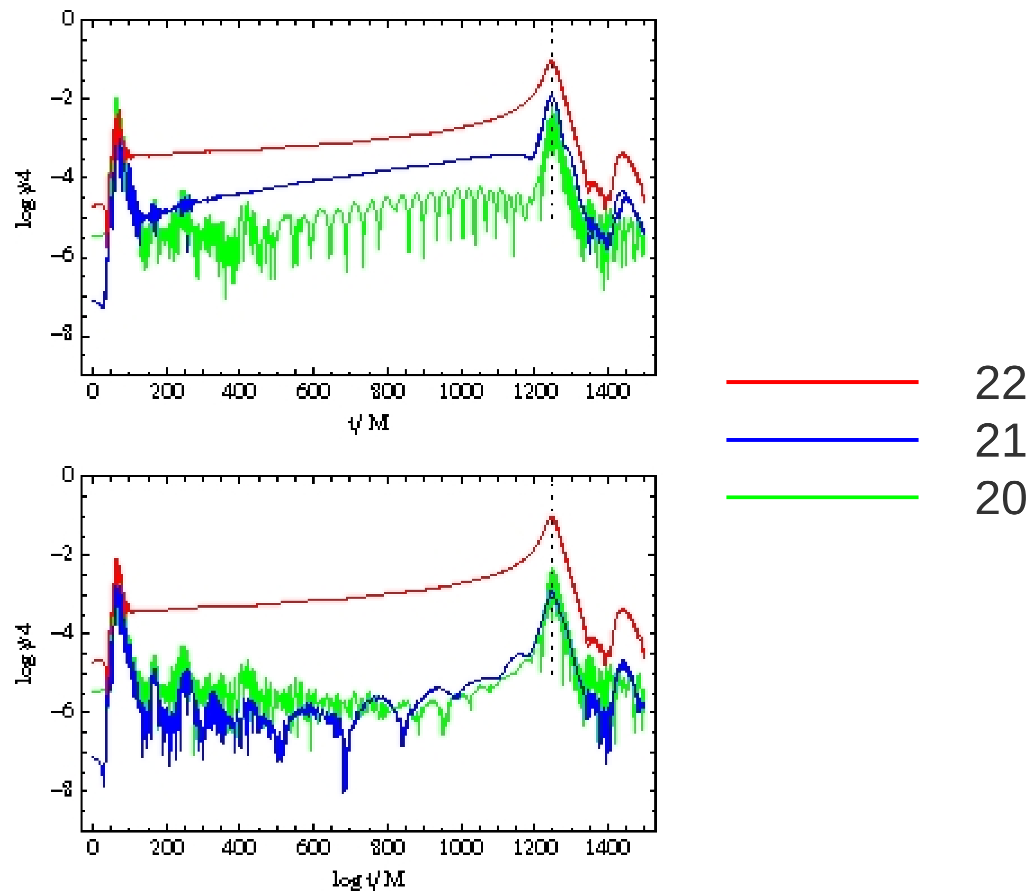
Road to Precession – Precessing Frame

Precessing Convention
[Buonanno, Chen & Vallisneri, 04]

Quadrupole aligned frame
[Schmidt, Hannam, Husa & Ajith, 11]
Max l = 2, m = ± 2 modes

Dominant emission axis
[O'Shaughnessy et al., 11]
Algebraic calculation of the dominant direction of the averaged emisison

Minimum rotation condition
[Boyle, Owen & Pfeiffer et al., 11]
No frame rotation along the preferred axis, equivalent to precessing convention



[O'Shaughnessy et al., 11]

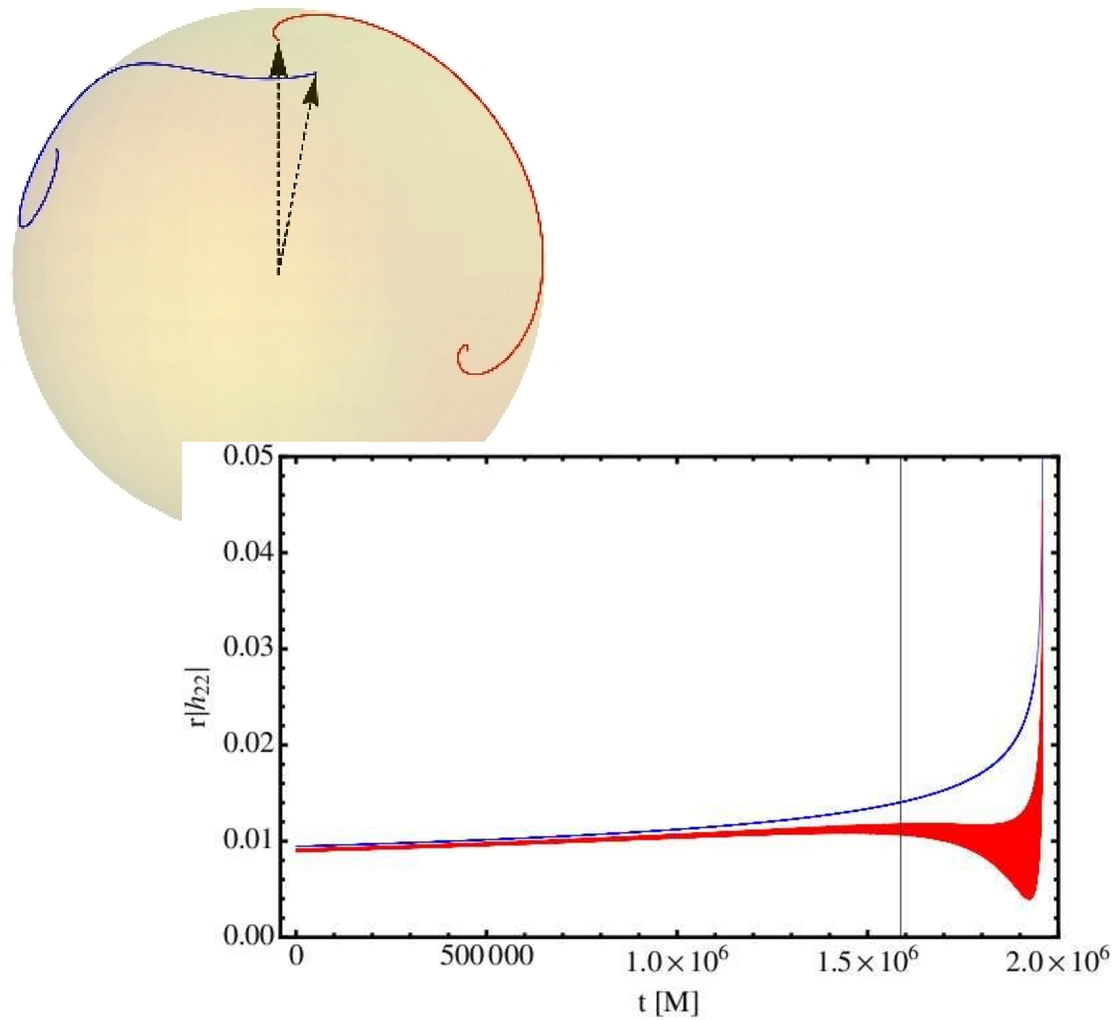
Road to Precession – Precessing Frame

Quadrupole aligned frame
 + Min rotation condition
[Schmidt, Hannam & Husa, 2012]
[Poster by P. Schmidt]

Generic precessing waveforms rotated to precessing frames become ~ non-precessing

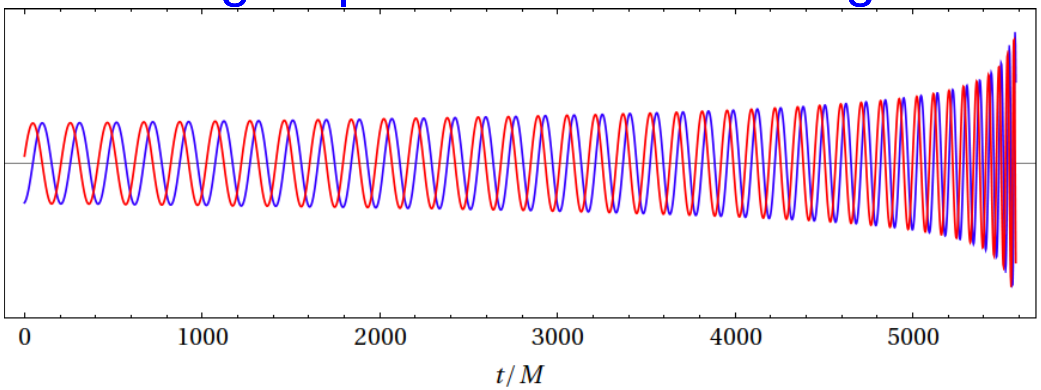
They are mapped to non-precessing waves parameterised by one effective spin parameter

Mapping is extremely accurate
Effectualness > 0.99
bias in effective spin < 0.04
… even transitional precession

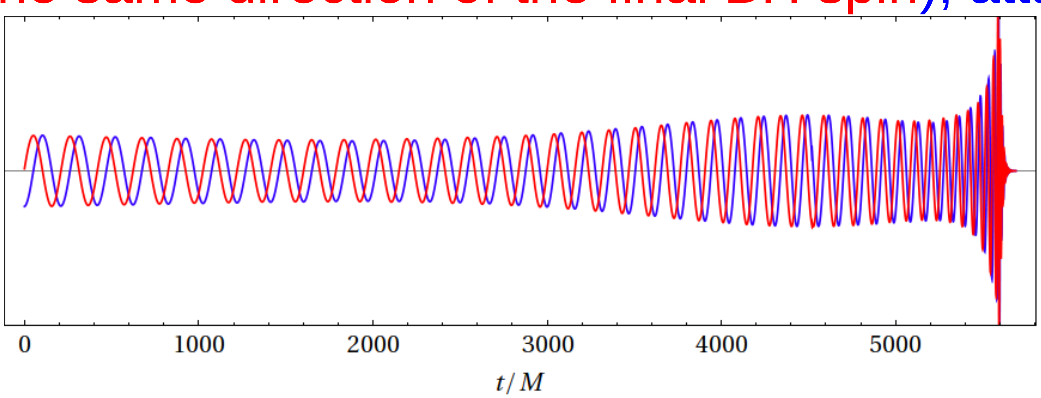


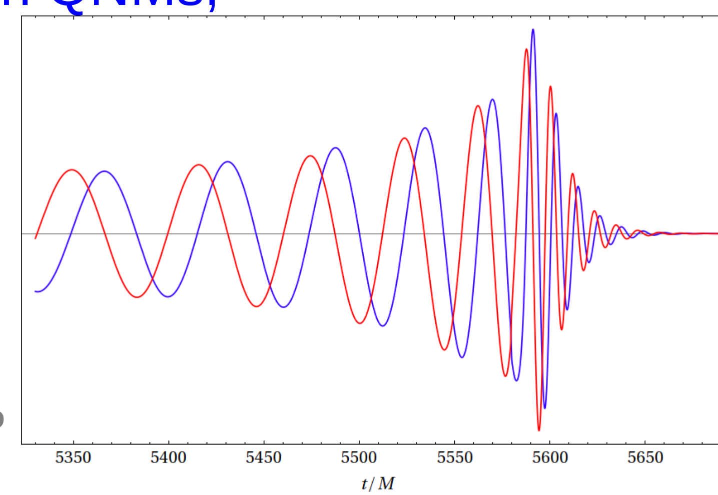
First EOBNR Model

Evolve precessing trajectory in arbitrary inertial frame; solve precessing frame (along $\boldsymbol{L}_N$) evolution; rotate all to precessing frames; generate non-precessing inspiral waveforms using existing model



Rotate inspiral waveforms to merger-$\boldsymbol{J}$-frame (assuming the merger $\boldsymbol{J}$ is in the same direction of the final BH spin); attach QNMs;





Summary

- We have some accurate waveform models, need more and better!
- Status of analytical models
  - Finish: non-spinning $q > 0.1$, a few harmonics;
  - Improve: non-precessing spin < 0.8, higher harmonics;
  - Attempt: extreme parameters, small q or large spin, precession.
- Questions
  - How to choose future NR simulations?
  - How do merger, spin, higher harmonics affect data analysis?
  - Can we understand merger-ringdown transition analytically?