EOS EFFECTS IN THE EARLY NS INSPIRAL

- Equation of State (EoS)- dependent tidal deformation
- Small but robust signature in the waveform
- Characterized by the tidal deformability λ

$$\lambda = -\frac{(\text{induced quadrupole})}{(\text{perturbing tidal field})}$$

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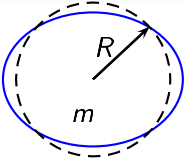
TIDAL DEFORMABILITY λ



- Equilibrium star placed in a tidal field $\mathcal{E}_{ij}$
- Linear response: $Q_{ij} = -\lambda\, \mathcal{E}_{ij}$
  (induced quadrupole)

$Q_{ij}, \mathcal{E}_{ij}$ Defined via the gravitational potential outside the star:

$$\frac{(1+g_{tt})}{2} \approx \frac{m_1+m_2}{r} + \frac{1}{2}\left[-\frac{3\,Q_{ij}}{r^5} + \mathcal{E}_{ij}\right] x^i x^j + \ldots$$

$$\approx \frac{m_1+m_2}{r} + \frac{1}{2}\left[2\,k_2\frac{R^5}{r^5} + 1\right] \mathcal{E}_{ij} x^i x^j + \ldots$$

$$\mathcal{E}_{ij} x^i x^j \sim \frac{m_2}{r^3} Y_{20}(\theta,\phi) r^2$$

$$\lambda = \frac{2}{3} k_2 R^5$$

dimensionless Love number　　　radius of the star

Computation of λ

Linear perturbations lead to a master equation for $\delta g_{tt} = H(r) Y_{20}(\theta, \phi)$

- Inside the NS: numerically integrate
- Outside: perturbed Schwarzschild with asymptotics

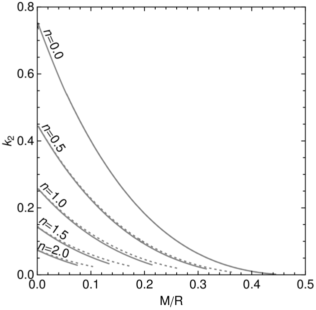
  $$H \sim (\lambda\, \mathcal{E})/r^3 \;\&\; \mathcal{E}\, r^2$$

- Match H and H' at $r = R$,

  Result:

$$\lambda = \frac{2}{3} k_2 \left(\frac{M}{R}, \; \frac{RH'}{H} \right) R^5$$

k_2 for polytropes
$(p = K \rho^{1+1/n})$



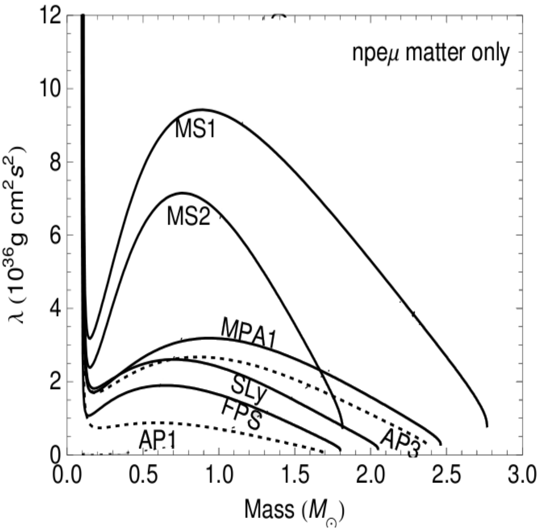
λ for various EoS models

Feature of NS EoS models:

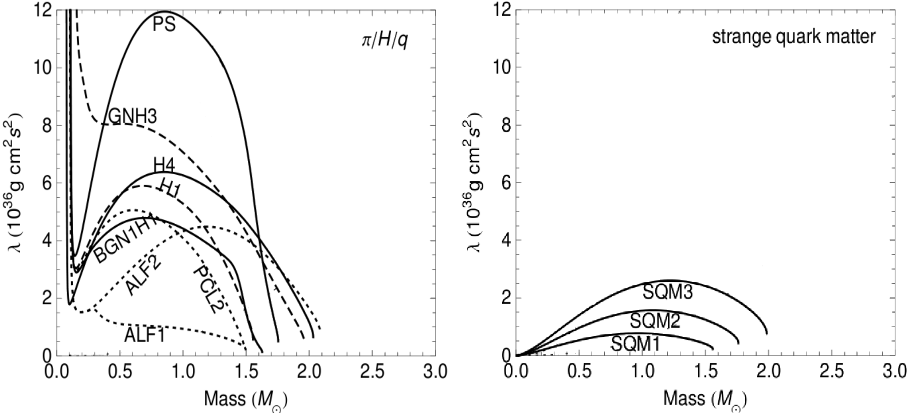
fixed relationship $R(m)$

$\lambda = \lambda(m)$ for a given EoS,

for a black hole: $\lambda = 0$



λ for various EoS models



MODELLING THE NS BINARY DYNAMICS

- post-Newtonian formalism (weak field, slow motion)
- internal quadrupolar degrees of freedom of each star parameterized by Q_{ij}
- coupling to the companion's $\ell = 2$ tidal field
- leading order GW dissipation terms

▷ Equations of motion for relative distance $\mathbf{x}$, Q_{ij}:

$$\ddot{x}_i = a_i^{\rm Newt}\,[1 + ({\rm PN})] + a_i^{\rm tidal}\,[1 + ({\rm PN})] - x^j\ a_{ij}^{\rm RR},$$

$$\ddot{Q}_{ij} + \omega_0^2\,Q_{ij} = \boldsymbol{\lambda}\ \omega_0^2\ \mathcal{E}_{ij} - \boldsymbol{\lambda}\ \omega_0^2\ a_{ij}^{\rm RR}$$

▷ Total quadrupole: $Q_{ij}^{\rm system} = Q_{ij} + Q_{ij}^{\rm orbit}$

EFFECTIVE DESCRIPTION

$$\begin{array}{lll} \text{Timescales :} & {\omega_0}^{-1} & - \text{ internal f} - \text{modes} \\ & \omega^{-1} & - \text{ orbital} \\ & t_{\rm rr} & - \text{ gravitational radiation reaction} \end{array}$$

Adiabatic limit: $t_{\rm rr} \gg \omega^{-1},\ \omega_0^{-1}$, solutions for quasi-circular orbits:

$$E(\omega) = -\frac{\mu}{2}(M\omega)^{2/3}\left[\, 1 + \mathrm{PN} - \boldsymbol{\lambda}_1\ \delta E(1 + \mathrm{PN}) + 1 \leftrightarrow 2\right],$$

$$\dot{E}(\omega) = -\frac{32}{5}(\mu M^2\omega^5)^2\left[\, 1 + \mathrm{PN} + \boldsymbol{\lambda}_1\ \delta \dot{E}(1 + \mathrm{PN}) + 1 \leftrightarrow 2\right]$$

Fourier transform of the GW signal:

$$\tilde{h}(f) = \mathcal{A}e^{i\Psi}$$

Obtain the phase from energy balance:

$$d^2\Psi/d\omega^2 = 2(dE/d\omega)/\dot{E}$$

EFFECT ON THE GW SIGNAL

$$\tilde{h}(f) = \frac{Q(\alpha,\delta,\iota,\psi)}{D_L}\mathcal{M}^{5/6} f^{-7/6}(1+\mathrm{PN})\; e^{i\Psi}$$

Further adiabatic approx. : $\omega \ll \omega_0$, simple tidal phase contribution:

$$\begin{aligned}\Psi \quad \sim \quad & 2\pi f t_c - \phi_c + \frac{3}{4}\left(8\pi\mathcal{M} f\right)^{-5/3}\left\{1 + \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\eta\right) x + O(x^{3/2})\right. \\ & -\boldsymbol{\lambda}_1 \frac{x^5}{M^4 m_2}\left[24\left(1 + 11\frac{m_1}{M}\right)\right. \\ & \qquad \left.\left. + \frac{5x}{28}\left(3179 - 912\frac{m_2}{M} - 2286\frac{m_2^2}{M^2} + 260\frac{m_2^3}{M^3}\right)\right] + (1 \leftrightarrow 2)\right\}\end{aligned}$$

identical stars : $\quad \Psi^{\mathrm{tidal}} = \dfrac{3}{32 x^{5/2}}\left[-624\dfrac{x^5}{M^5}\boldsymbol{\lambda}\left(1 + \dfrac{3115}{1248}x\right)\right]$

$$M = m_1 + m_2, \quad x = (M\omega)^{2/3}, \quad \eta = m_1 m_2/M^2, \quad \mathcal{M} = (m_1 m_2)^{3/5} M^{-1/5}$$

ACCURACY OF THE PHASING MODEL

- Compute fractional corrections to the phasing model by relaxing the approximations made.
- At $f = 450\text{Hz} \ll f_{\rm isco}$, these are $< 10\%$:

$\triangleright$ Adiabatic approx : (i) $\omega^{-1} \ll t_{\rm rr}$: $\lesssim 0.01$

(ii) $\omega \ll \omega_0$: $\approx 0.012\,(f/450\mathrm{Hz})$

$\triangleright$ Higher multipoles: $\approx 0.007\,(f/450\mathrm{Hz})^{4/3}$

$\triangleright$ Nonlinear effects : (i) nonlin. response : $\approx 0.03\,(f/450\mathrm{Hz})^{5/3}$

(ii) nonlin. hydro : $\approx 0.002\,(f/450\mathrm{Hz})^{2}$

$\triangleright$ Spin: $\lesssim 0.003$, viscosity: negligible

$\triangleright$ Higher PN: 1PN was dominant : $\sim 18\%$, 1PN now included .