Properties of dense matter that might influence neutron star mergers dynamics.

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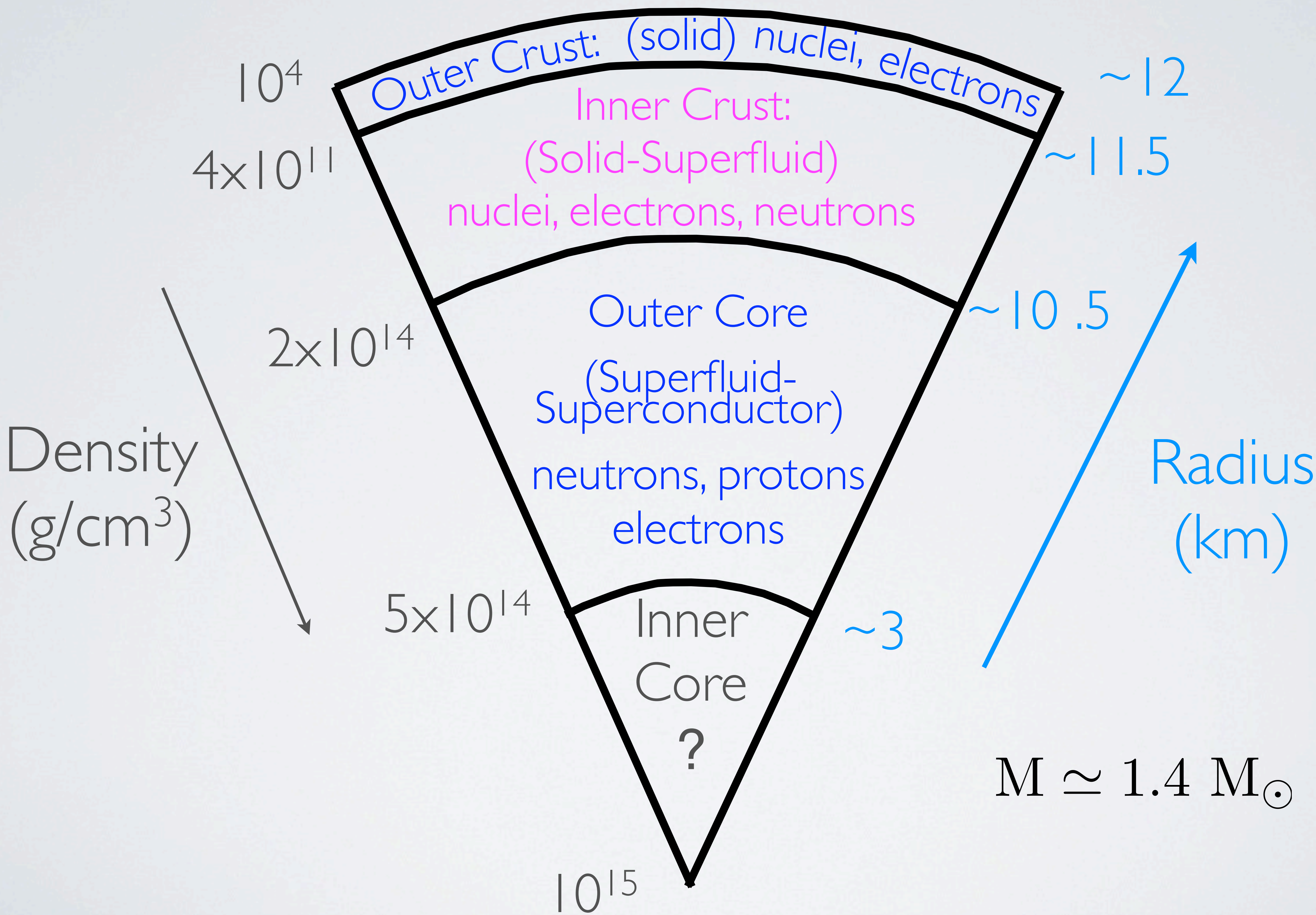
INT, Univ. of Washington

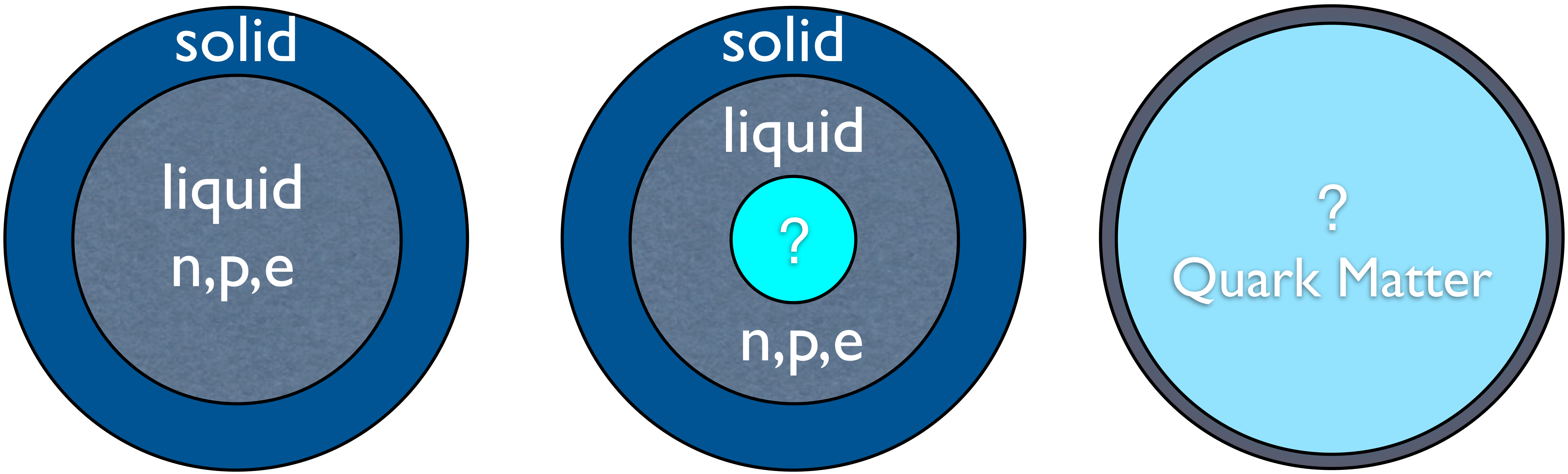
- Cold Equation of state (Pre-Merger)
- Crust Physics (Precursors)
- Hot and Dense Matter (Post-Merger)

Cold and Complex



$M \simeq 1.4\ M_\odot$

Interior Structure Still Uncertain



Nucleon Stars	Hybrid Stars	Strange Stars
R ≅ 10-15 km	R ≅ 6-13 (?) km	R ≅ ? -12 (?) km
M ≅ 1-2.5 $M_\odot$	M ≅ 1-2 (?) $M_\odot$	M ≅ ?-2.5 (?) $M_\odot$

BUT, equation of state at T=0 and up to ~ 2 ρ_0 is constrained by nuclear physics. Constraints will improve in the short-term.

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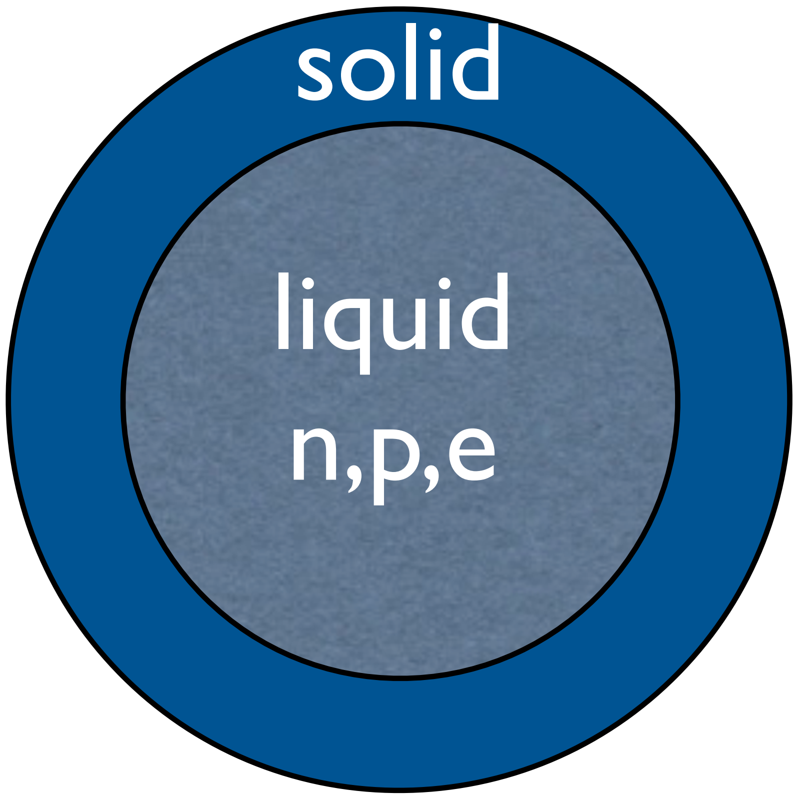
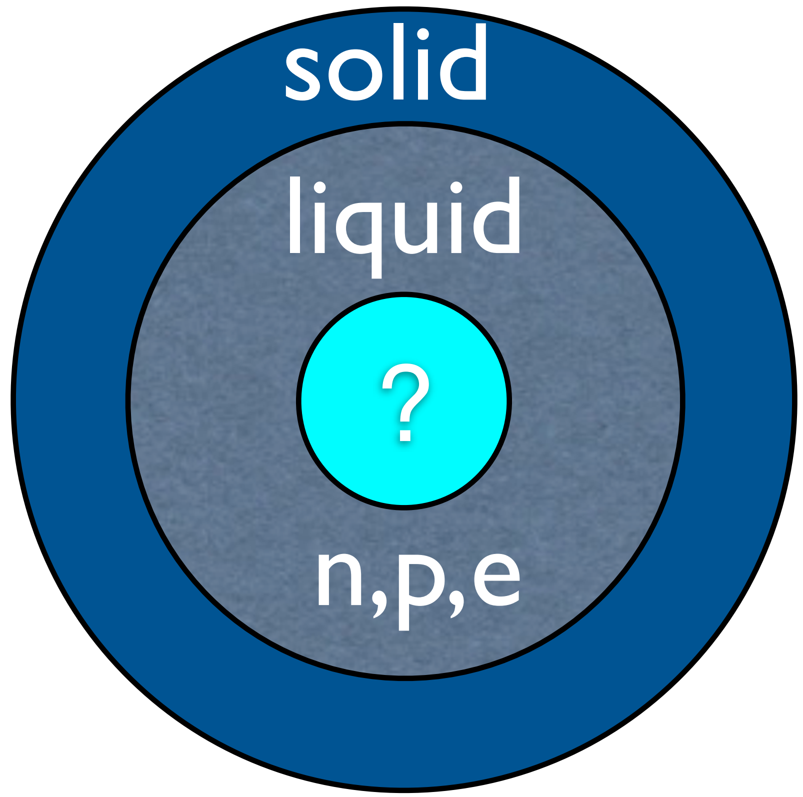
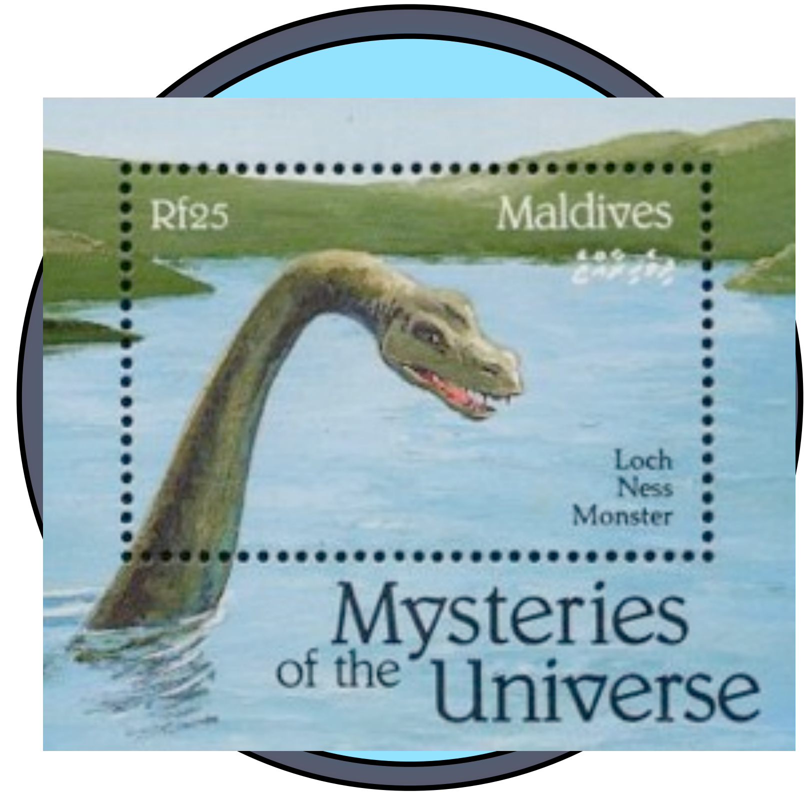






Nucleon Stars

$R \cong 10\text{-}15$ km

$M \cong 1\text{-}2.5\ M_\odot$

Hybrid Stars

$R \cong 6\text{-}13$ (?) km

$M \cong 1\text{-}2$ (?) $M_\odot$

Strange Stars

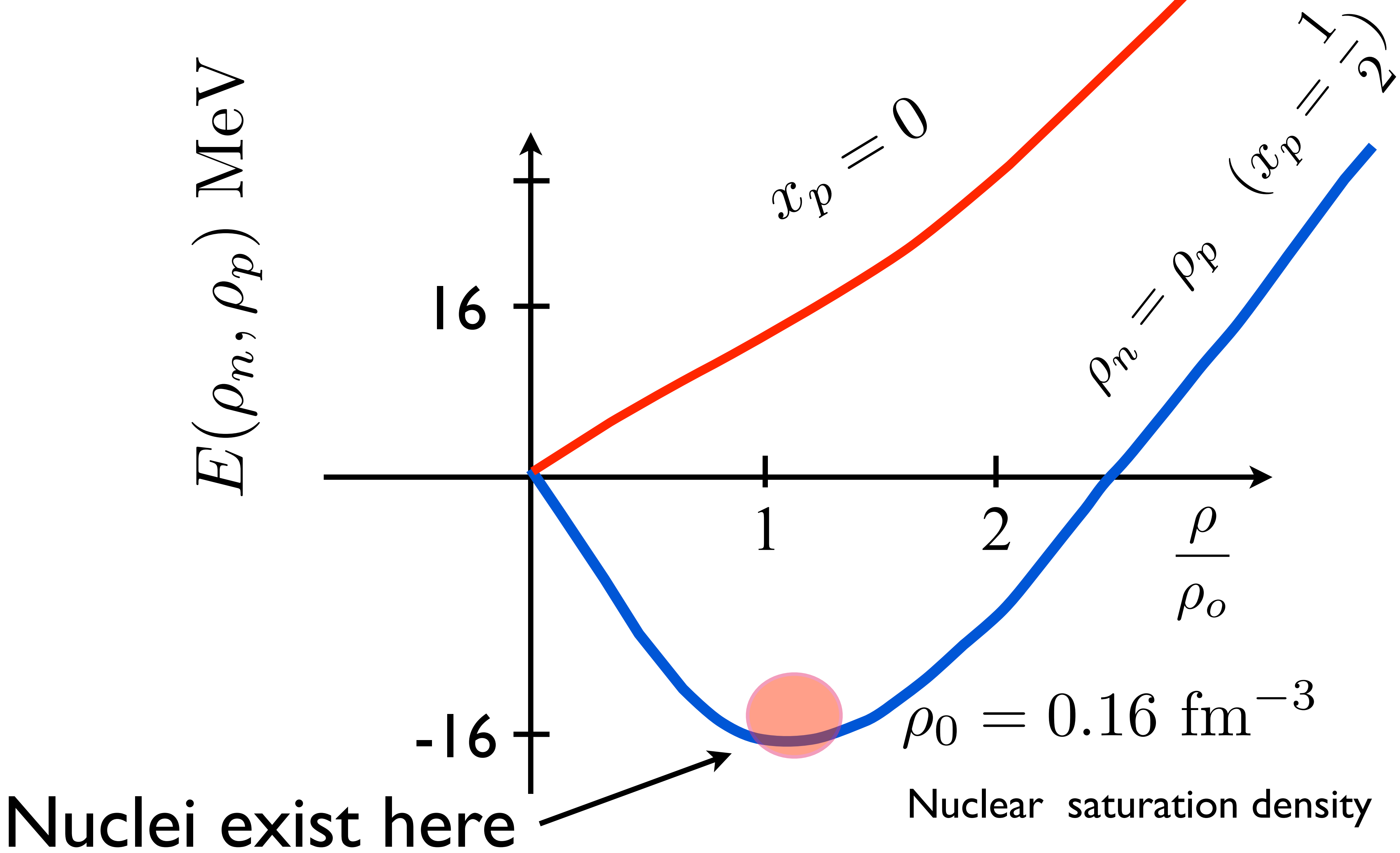
$R \cong$? -12 (?) km

$M \cong$?-2.5 (?) $M_\odot$

BUT, equation of state at T=0 and up to ~ 2 ρ_0 is constrained by nuclear physics. Constraints will improve in the short-term.

The Nuclear Equation of State

$$\begin{cases} \epsilon = \rho \; E(\rho) \\ P(\epsilon) = \rho^2 \dfrac{\partial E(\rho)}{\partial \rho} \end{cases}$$



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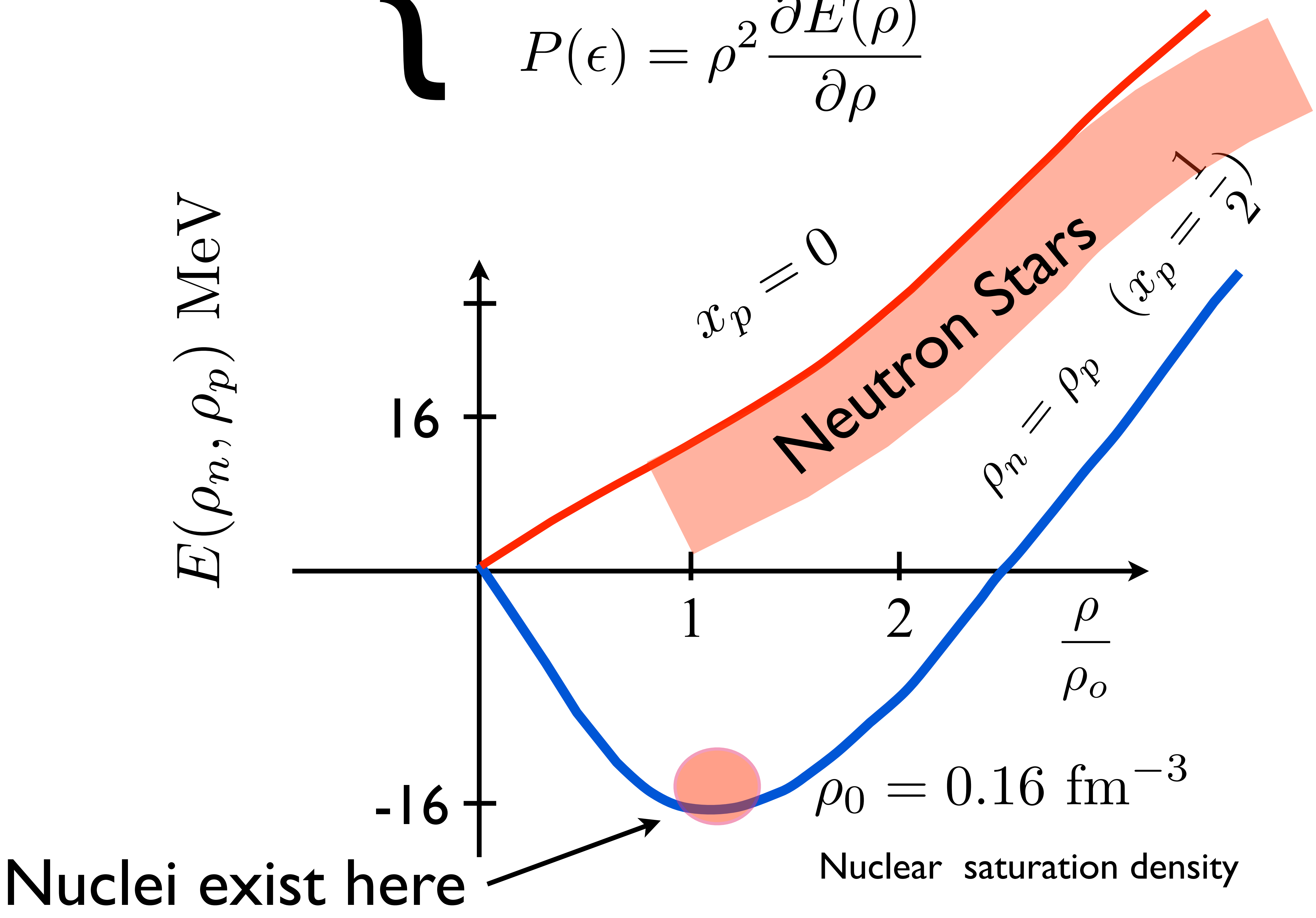

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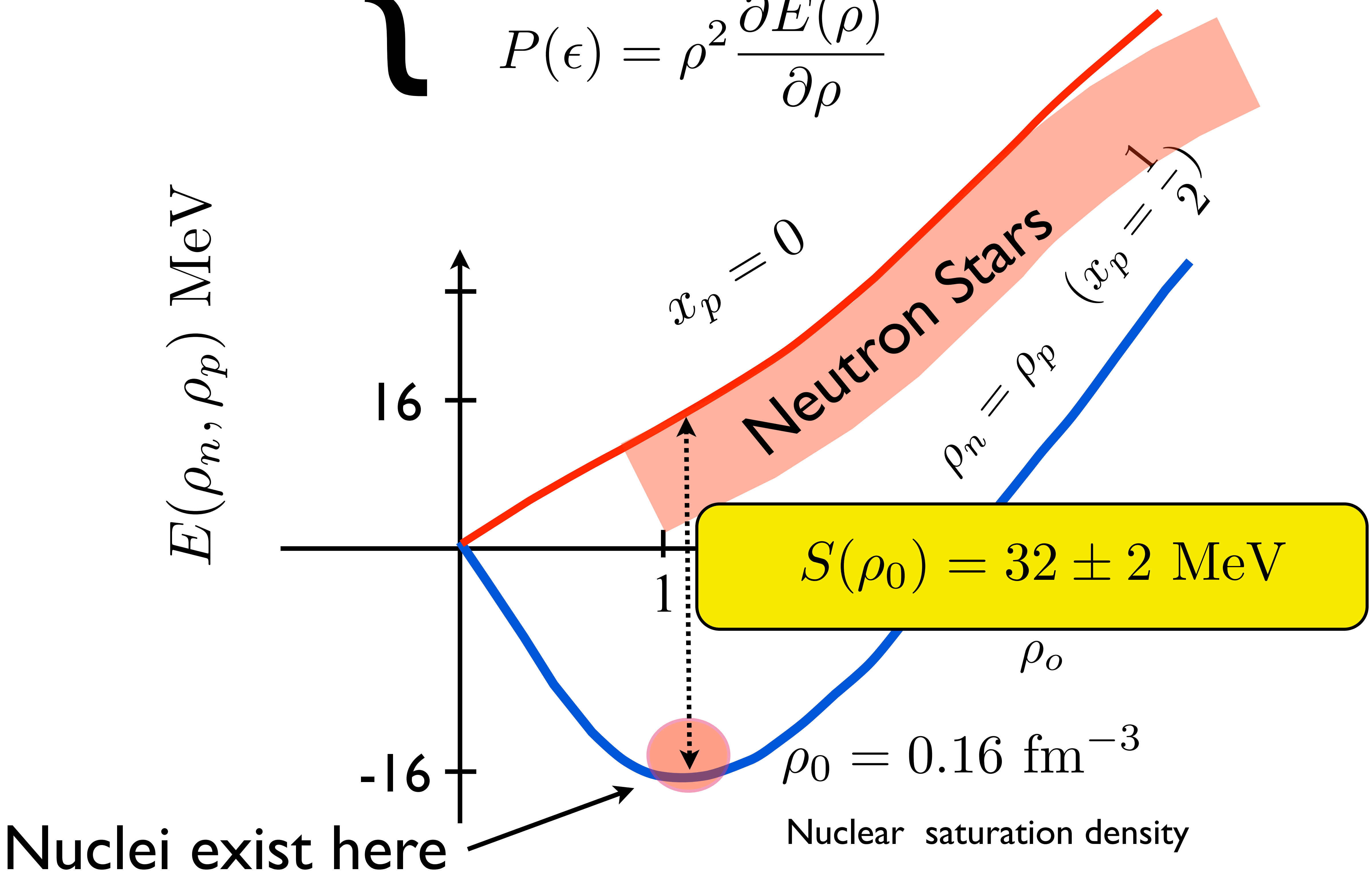
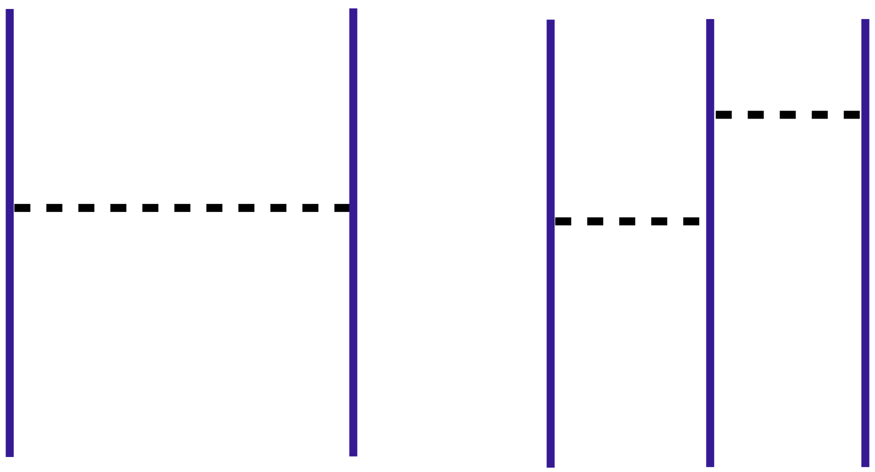
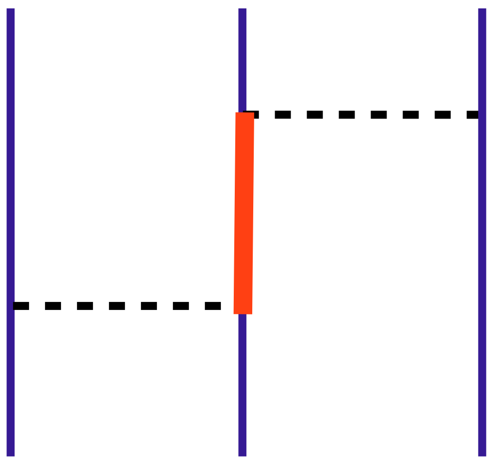


Nuclear Many Body Theory

$$H_{\text{nuclear}} = \frac{\nabla^2}{2M} + V_{\text{NN}} + V_{\text{NNN}} + \cdots$$





Phenomenological potentials (Argonne etc) tuned to fit scattering and light nuclei.

Chiral potentials and softer low energy potentials obtained using RG.

Computational Methods: Quantum Monte Carlo

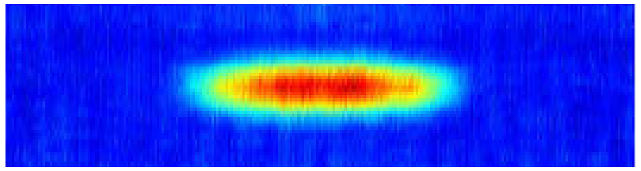
Diagrammatic Methods

$E(\rho_n, \rho_p)$: Energy per particle

Validation and Benchmarks

- Nuclear properties.
- Empirical nuclear matter properties.
- Cold atom experiments.

Cold Gas of Fermion Atoms (^{6}Li):



Short-range interaction with tunable scattering length.
Only one interaction scale in the problem = a

Unitary Gas $a = \infty$

$$E = \xi \, E_F \qquad \Delta = \beta \, E_F$$

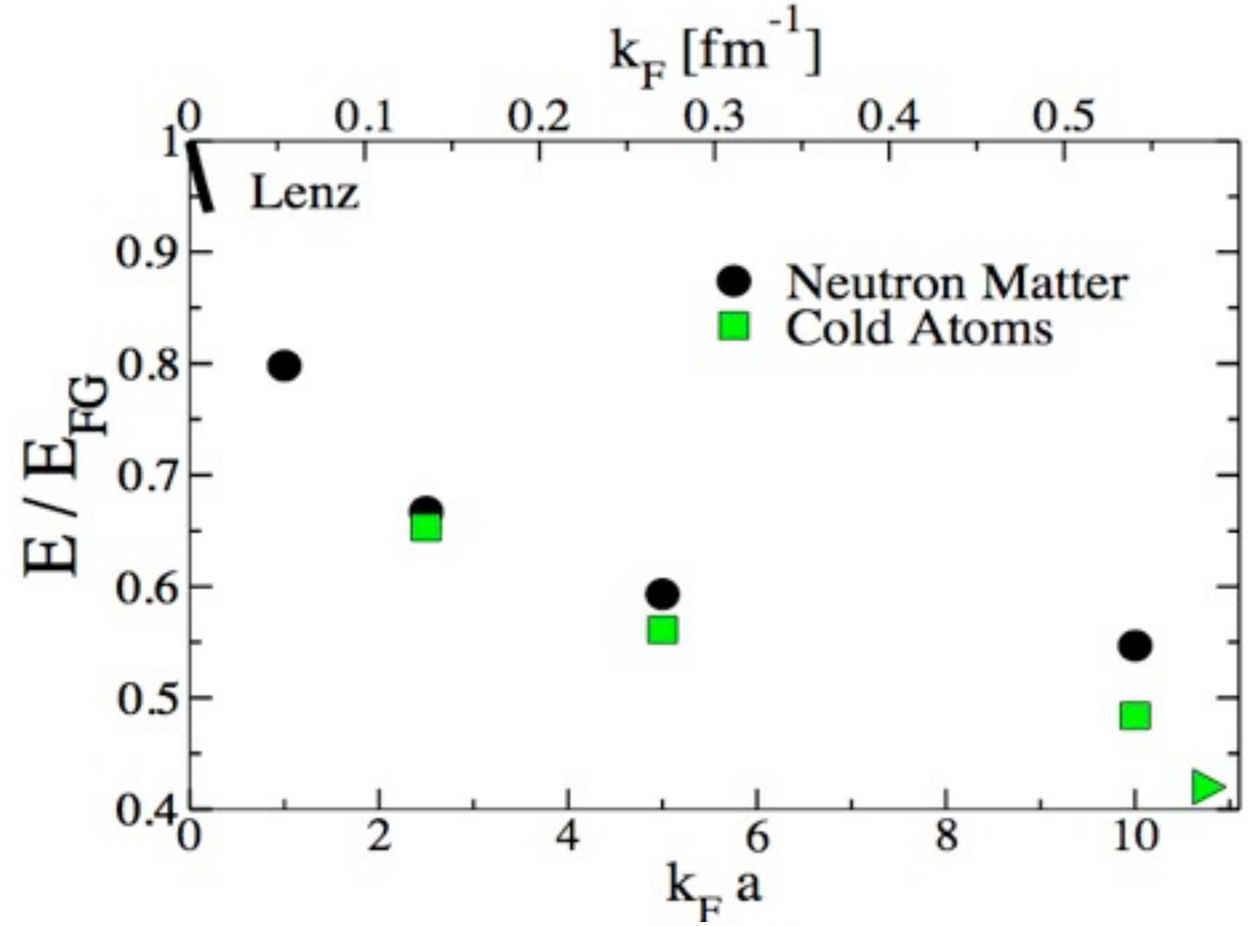
$\xrightarrow{\text{Expt.}}$

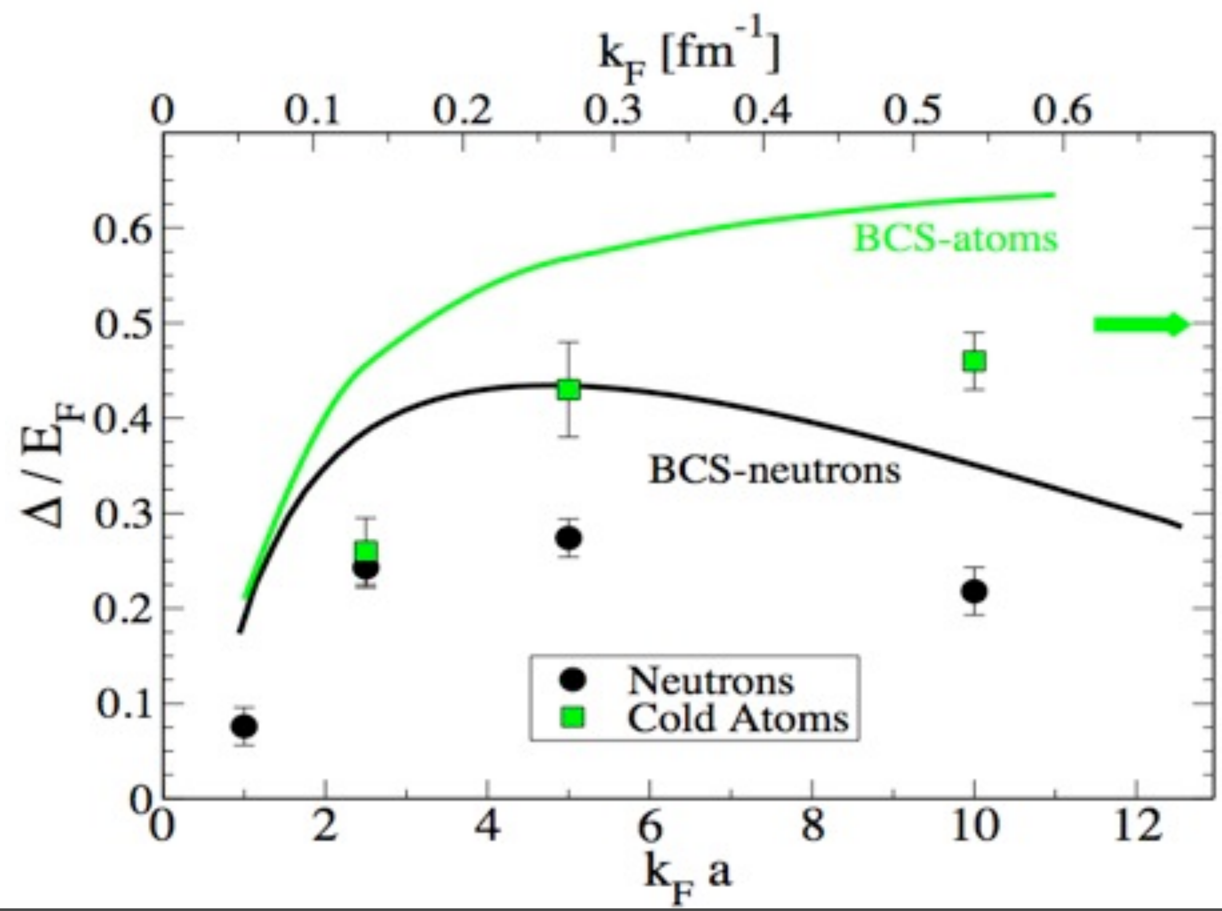
$$\xi = 0.41 \pm 0.1 \qquad \beta = 0.45 \pm 0.05$$

Cold Atoms & Neutron Matter

QMC predicted EoS and pairing gap.
(few percent)

Diagrammatic methods and mean field provide a qualitatively correct description.

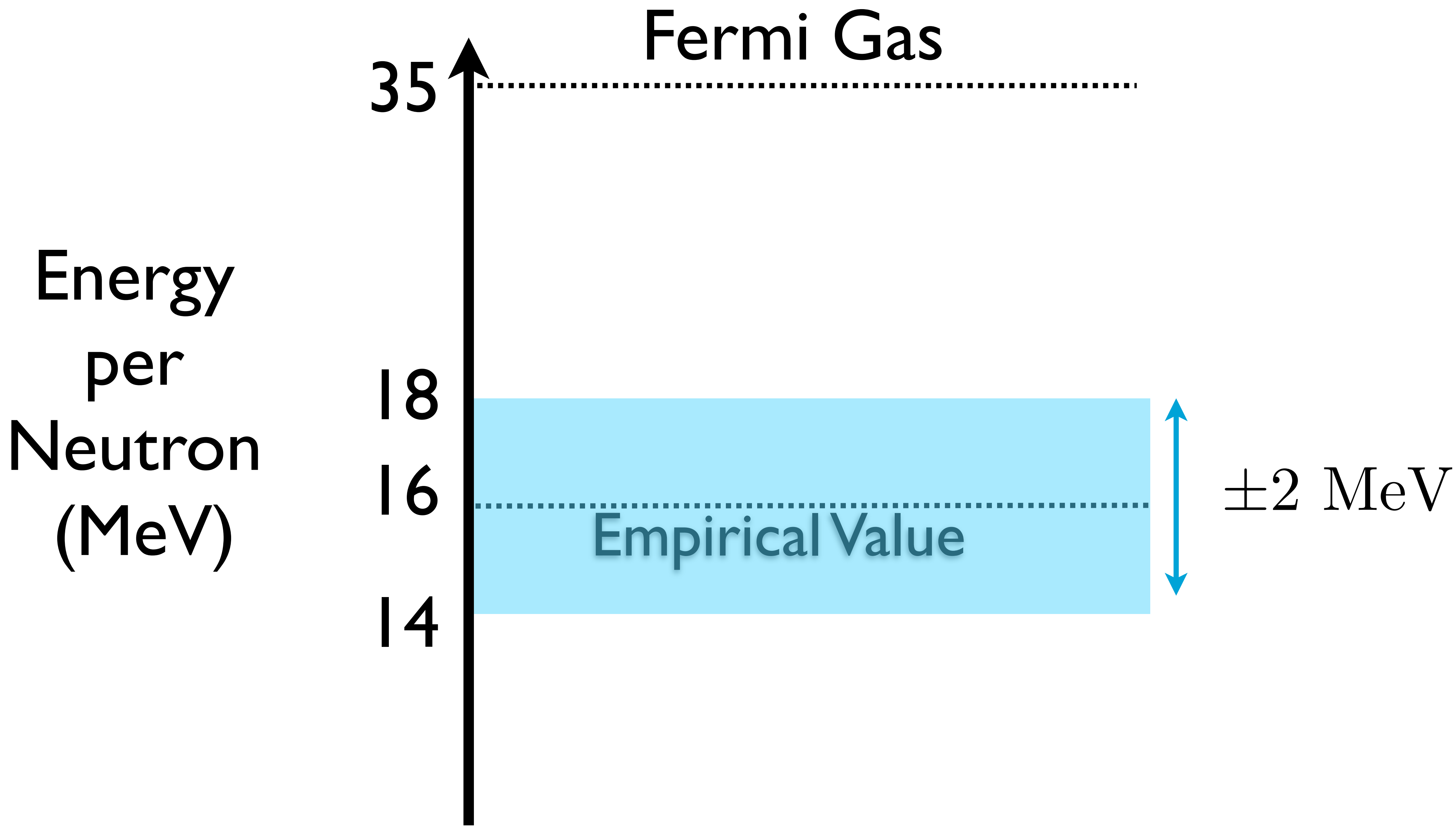




Carlson, Fantoni, Gandolfi, Gezerlis, Pethick, Reddy
Schwenk, Schmidt ..

3N Forces in Neutron Matter

(at nuclear saturation density)



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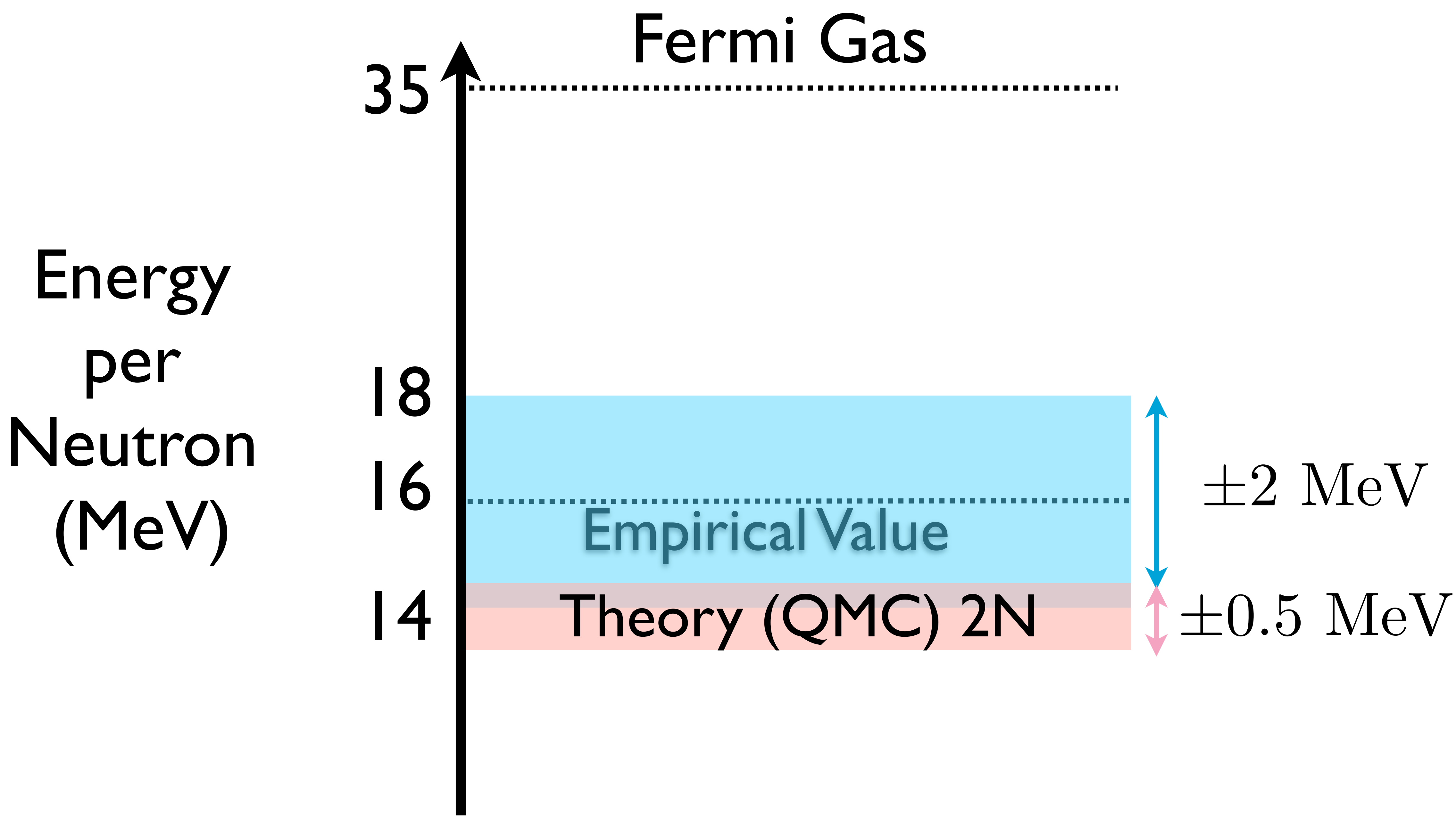

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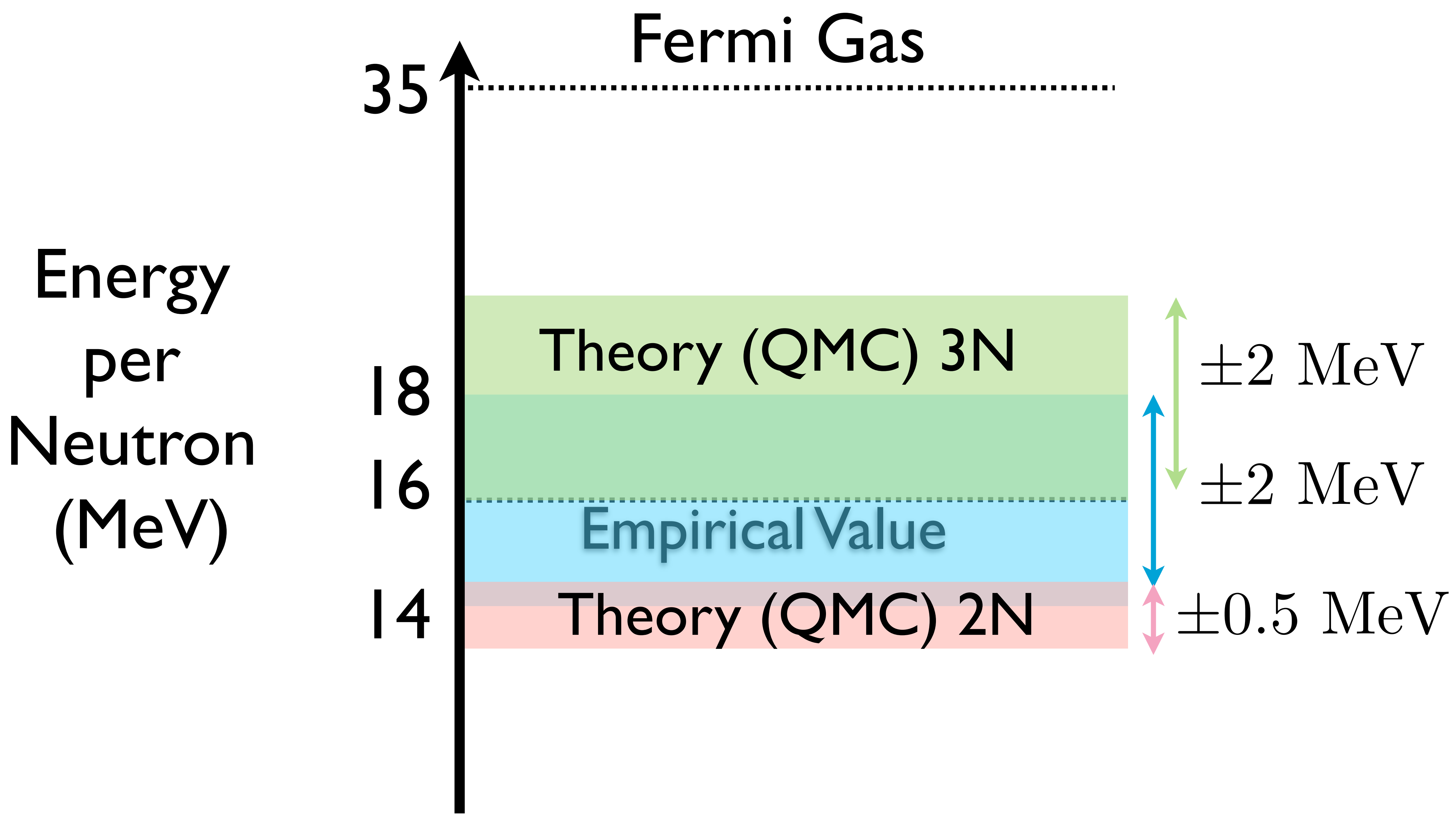
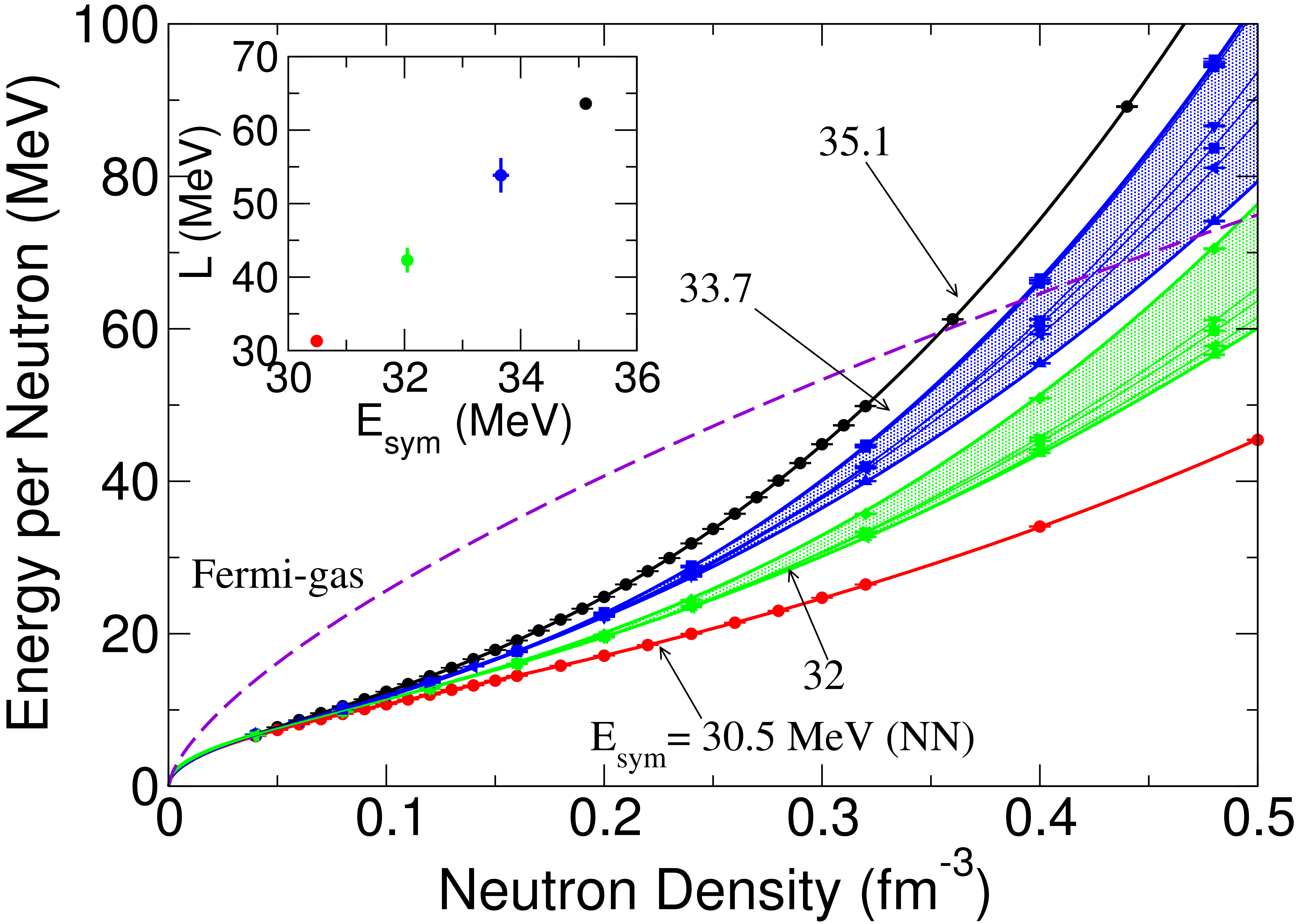


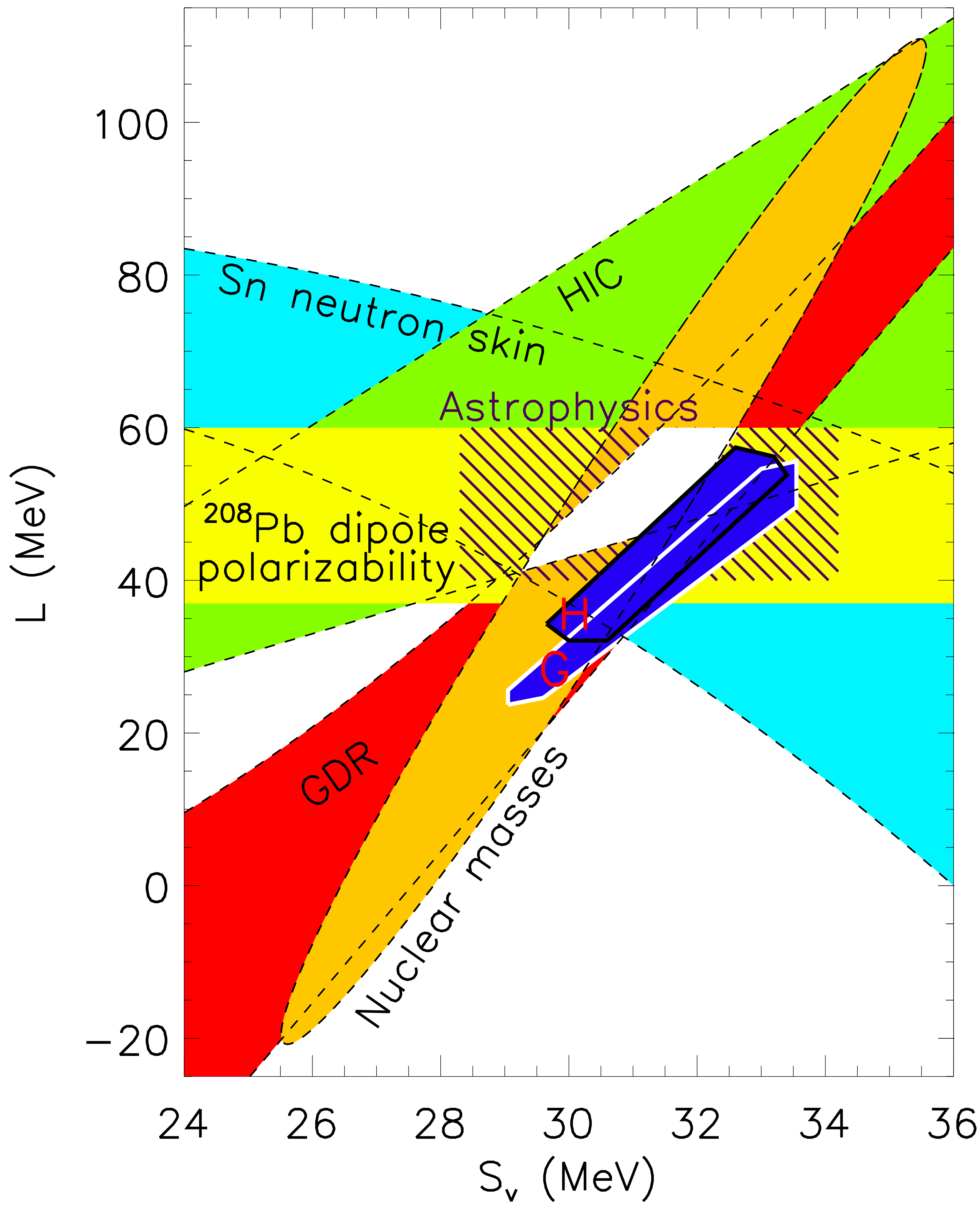
- Phenomenology suggests repulsive contribution from 3N forces in neutron matter. (its attractive in nuclei)

3N Forces & Neutron Matter



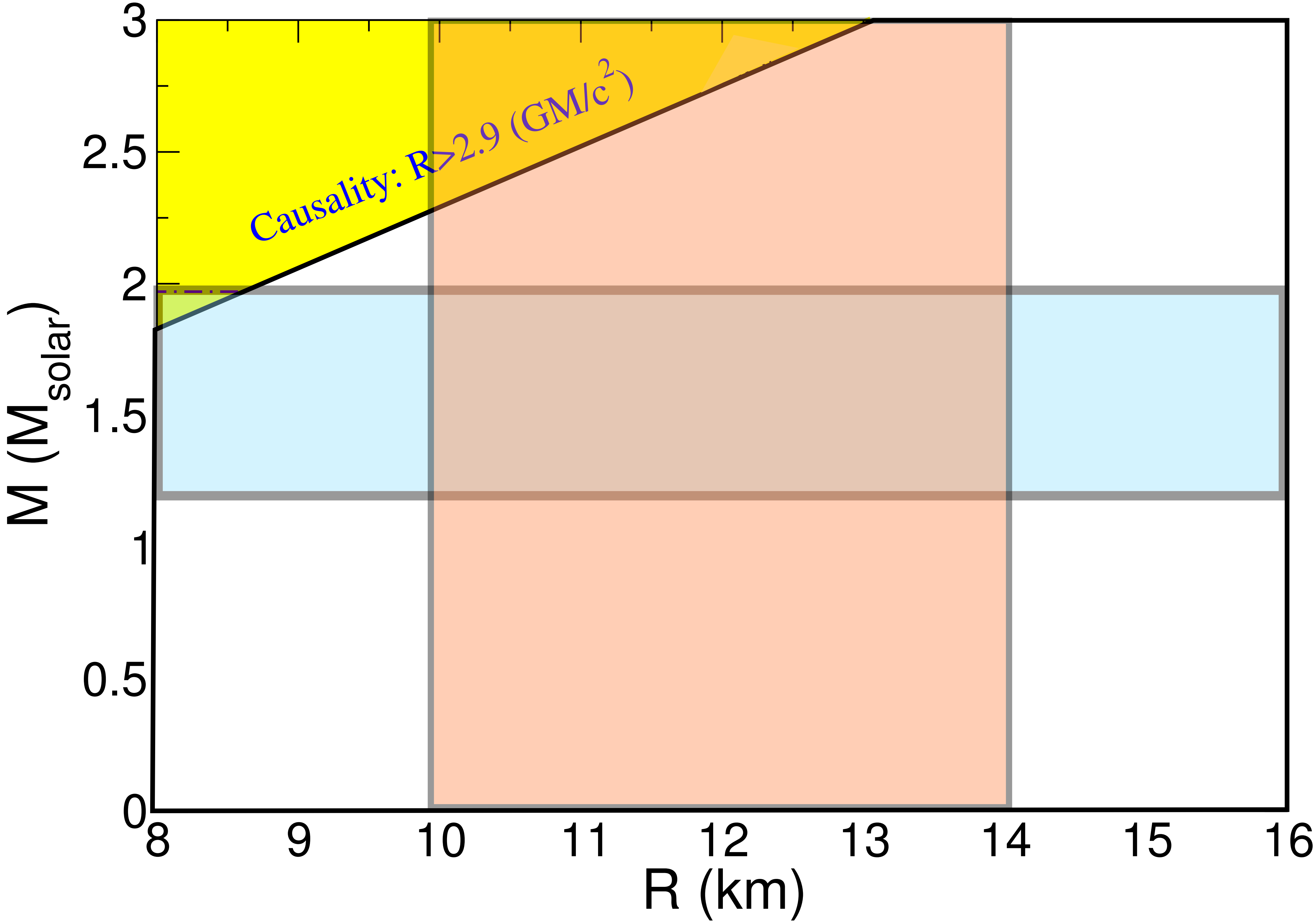
Gandolfi, Carlson, Reddy (2011)

Nuclear experiments are trying to pin down L and S (E_{sym})



Lattimer & Yuan (2102)

Mass and Radius



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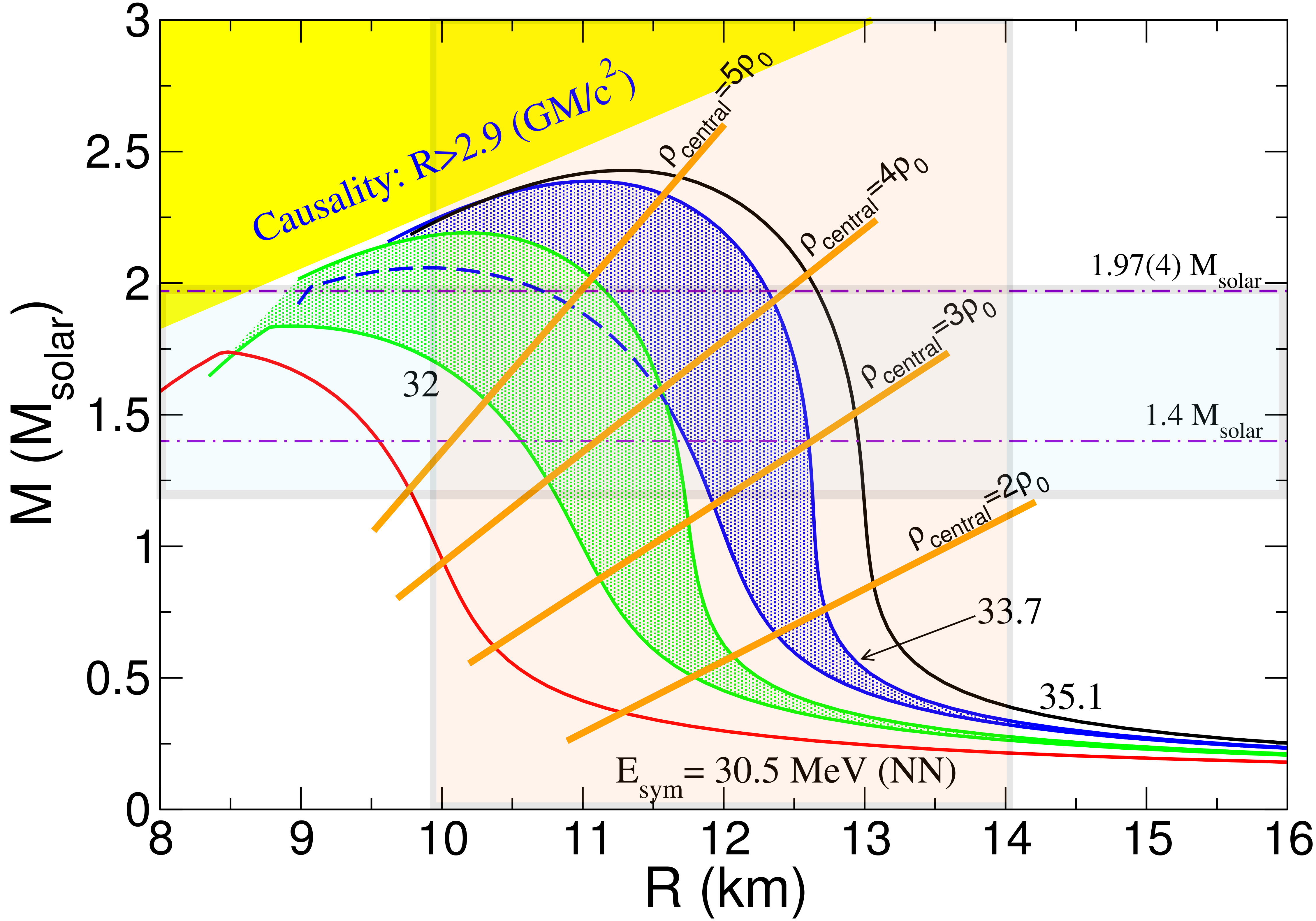

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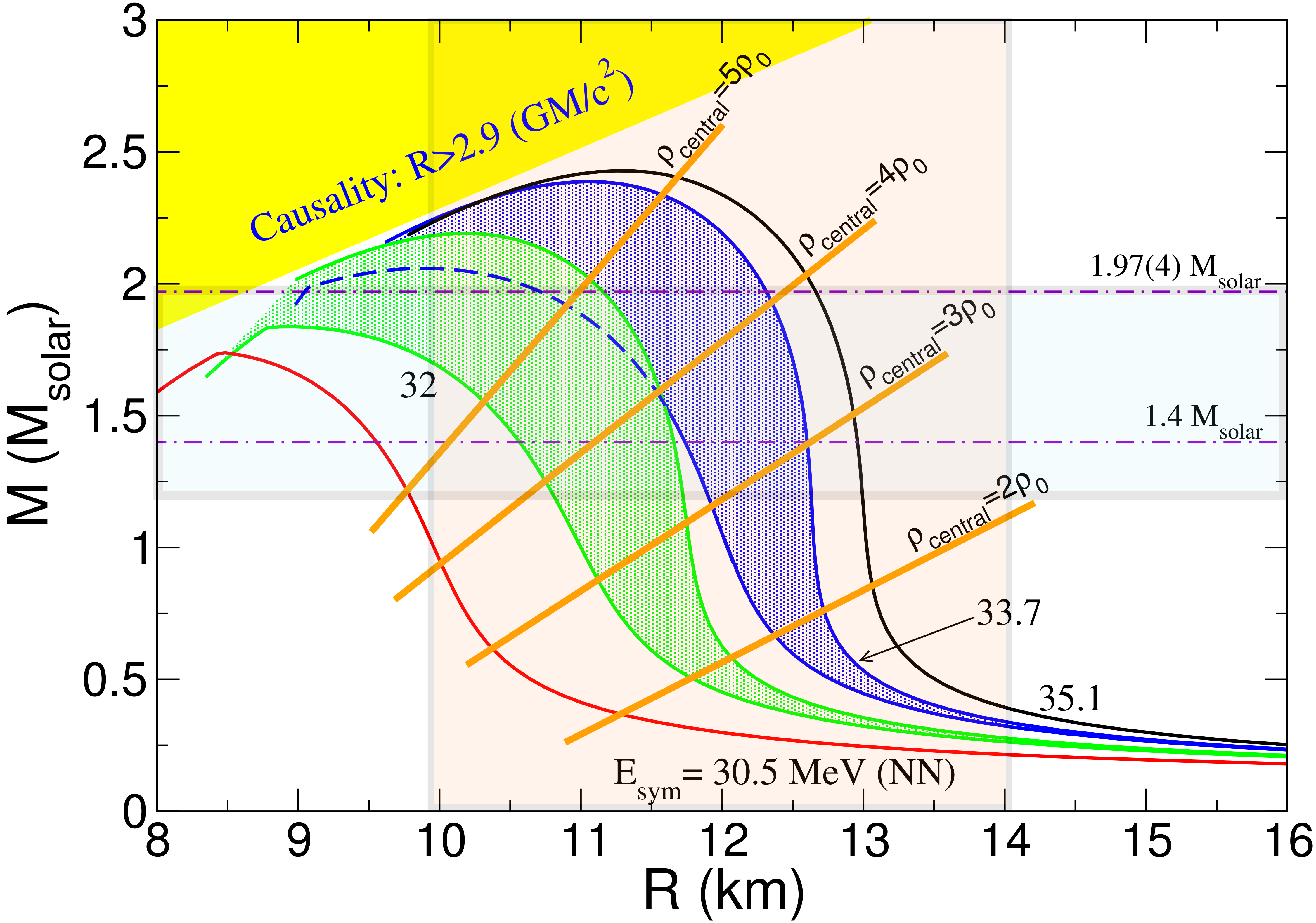
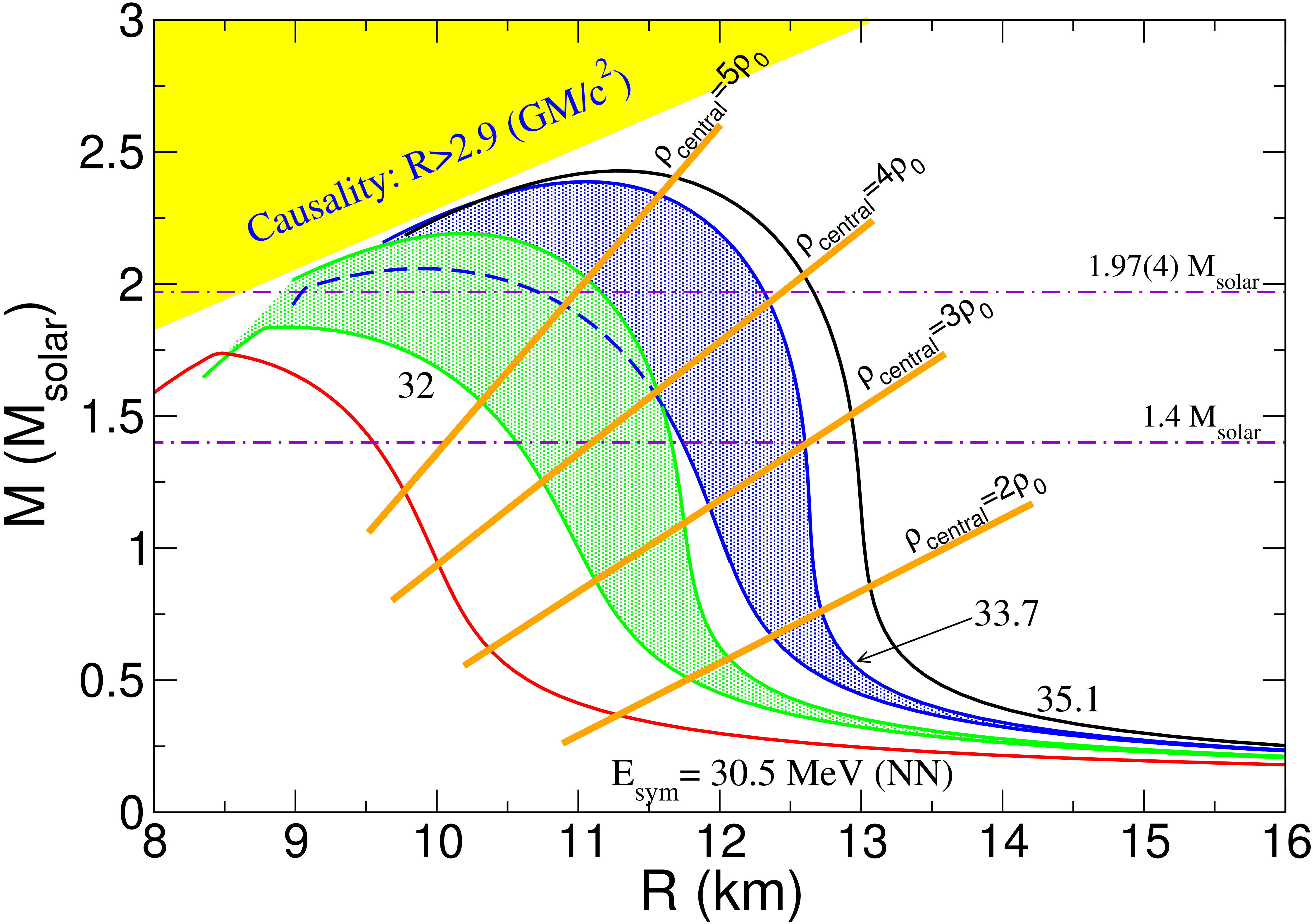


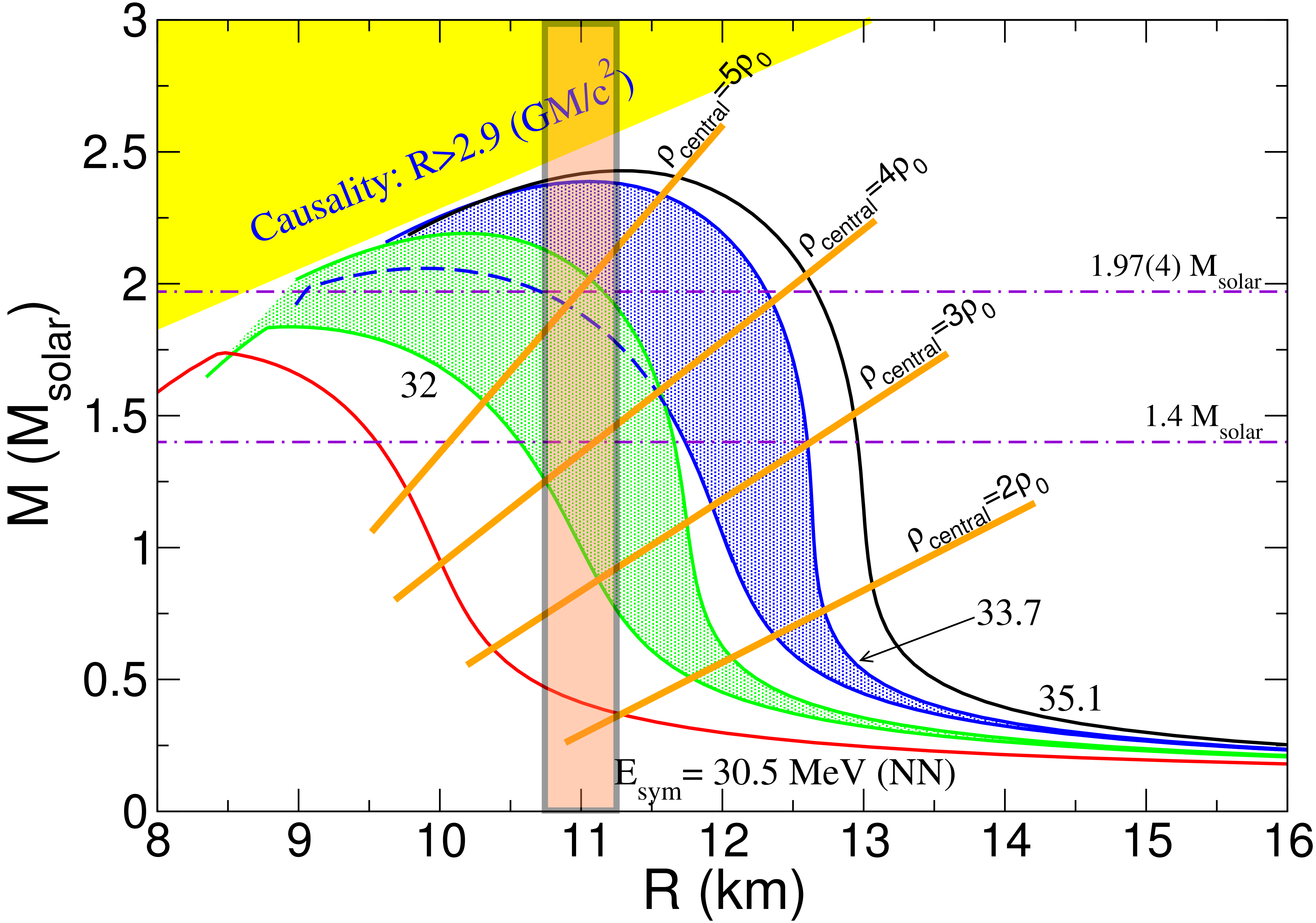
P(ε) determines the mass and radius of neutron stars.

Radius



A few% measurement of the radius (with different systematics) would be a valuable constraint.

Radius



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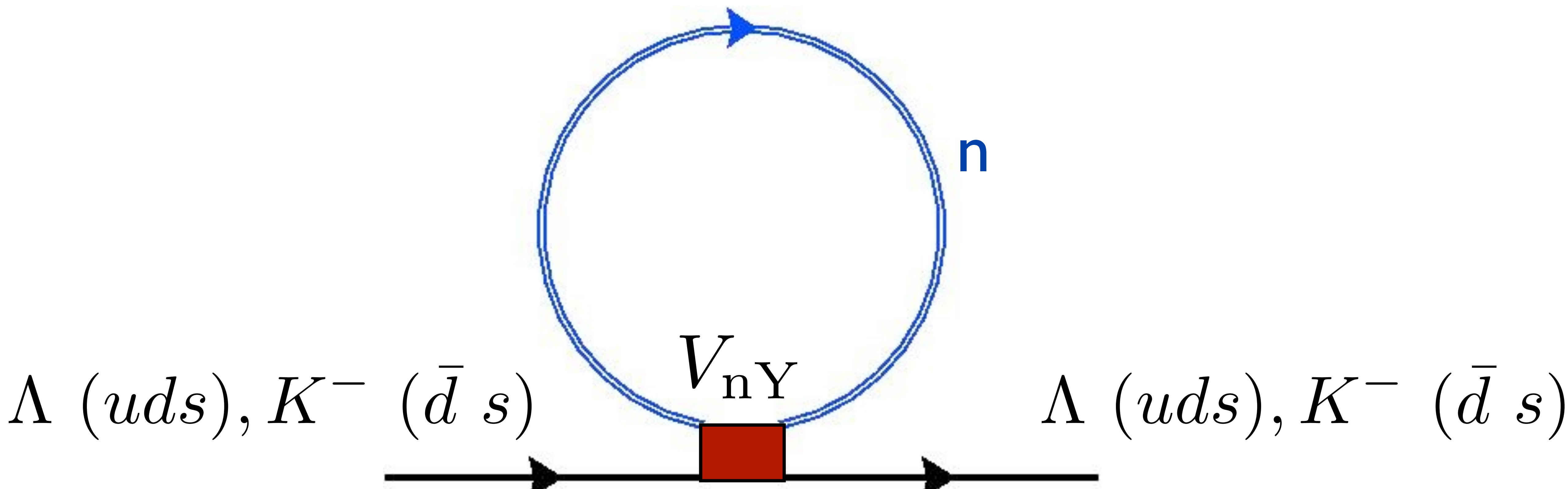
Neutron Matter - Too Many Down Quarks

Strangeness can alleviate this frustration

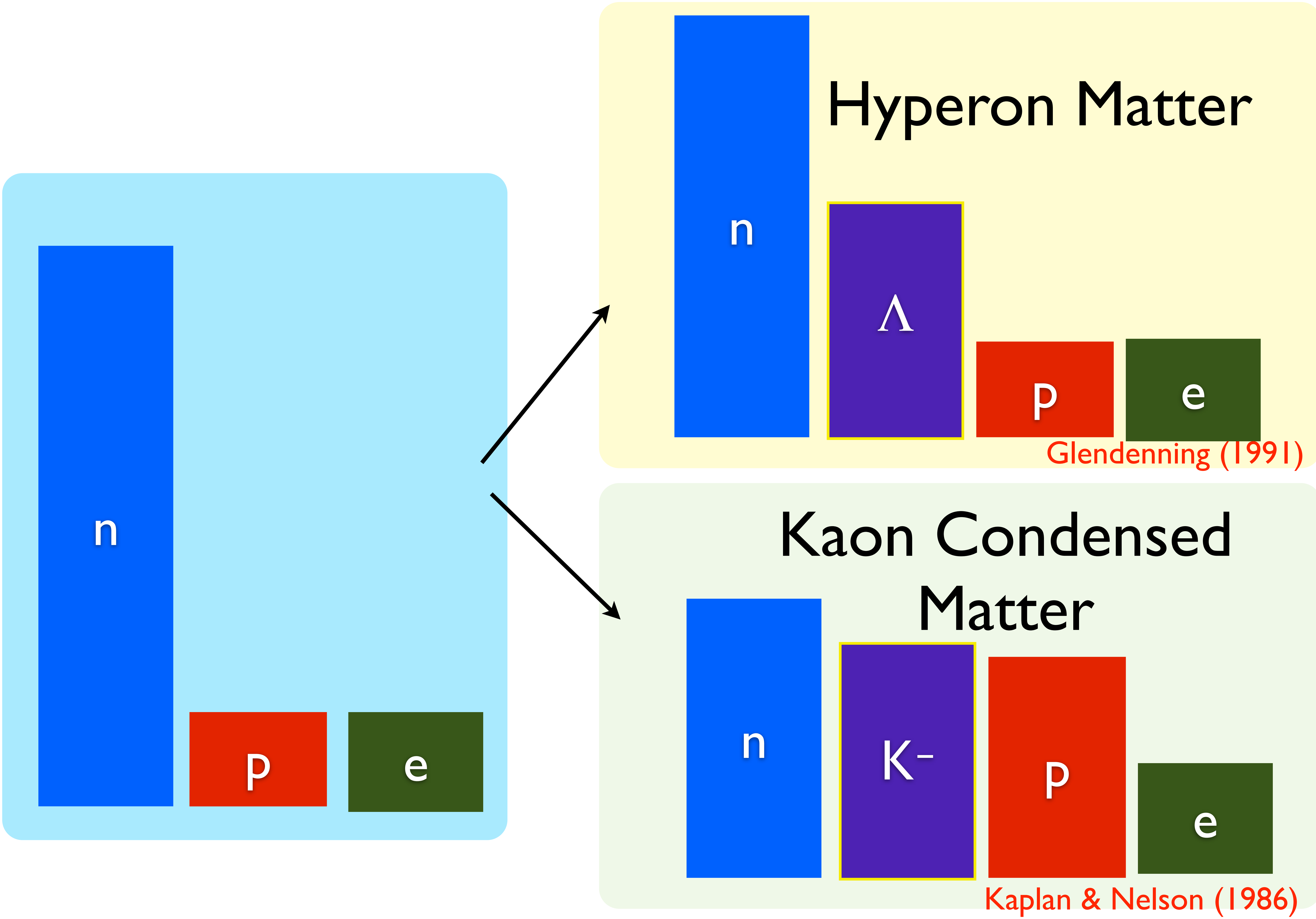
Three possible phases:

- Hyperons
- Kaons
- De-confined Quark Matter

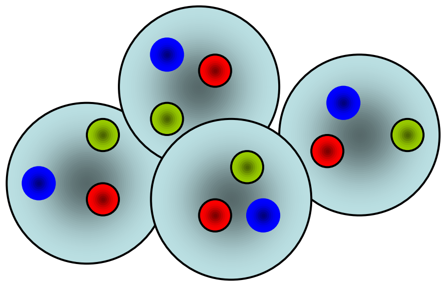
$$
\begin{aligned}
E_\Lambda(p=0) &= M_\Lambda + V_{\mathrm{n}\Lambda}(\rho) \leq \mu_B \\
E_{\mathrm{K}^-}(p=0) &= M_{\mathrm{K}^-} + V_{\mathrm{nK}^-}(\rho) \leq \mu_e
\end{aligned}
$$

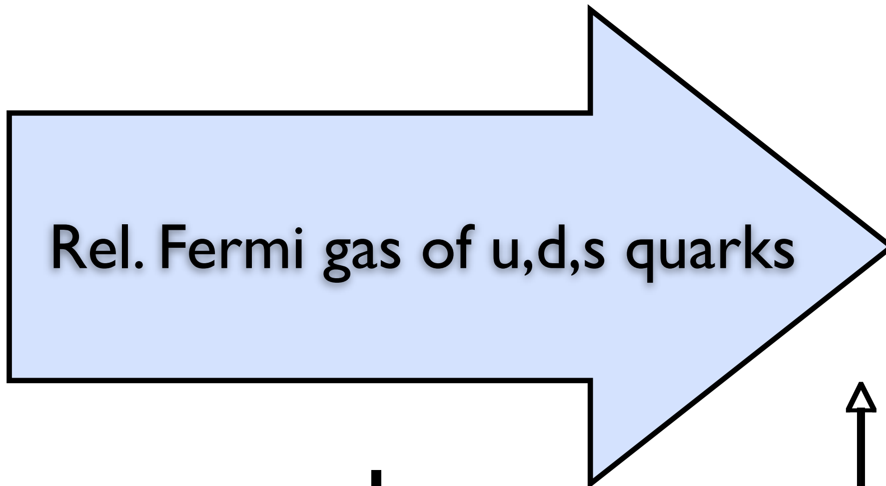


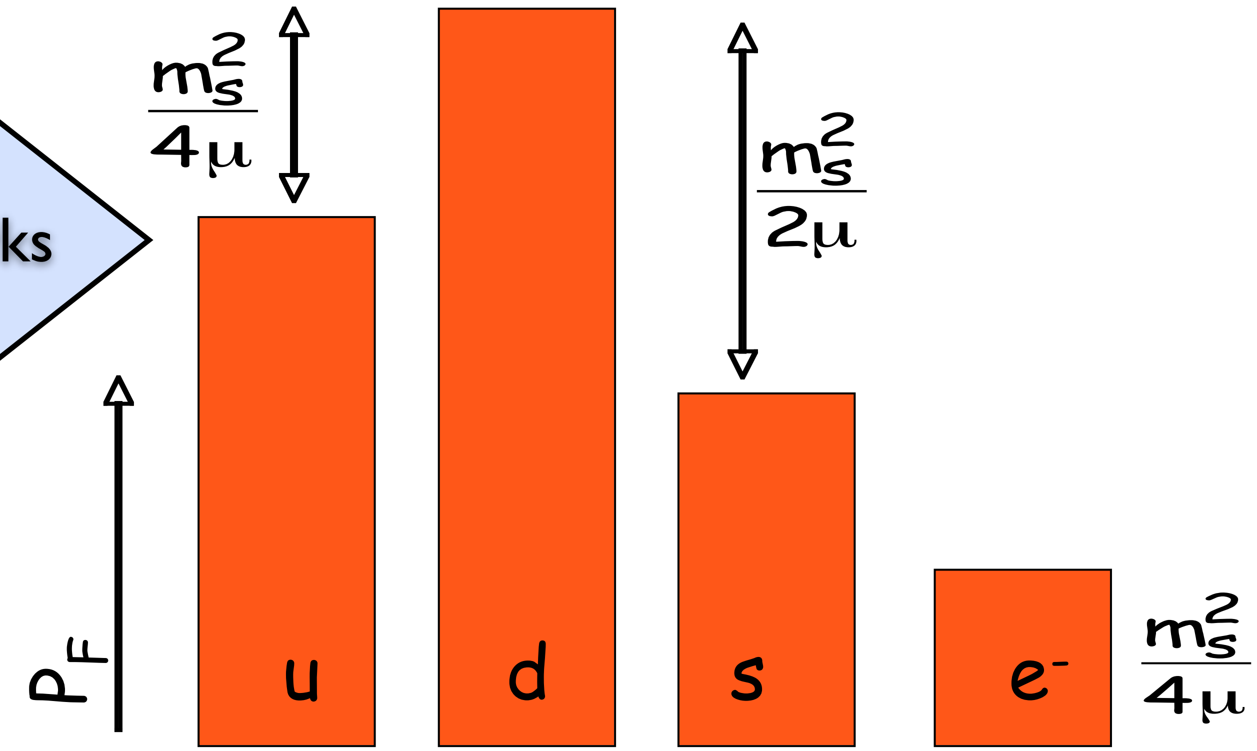
Hyperons & Kaon Condensation



Asymptotic Density







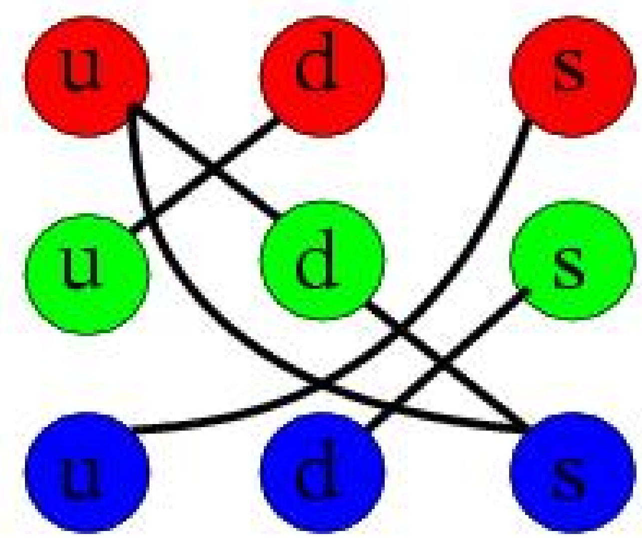
Interactions are nearly perturbative - calculable.

Interactions lead to pairing and color superconductivity

Strongest attraction in color-antisymmetric channel:

Color-Flavor-Locking

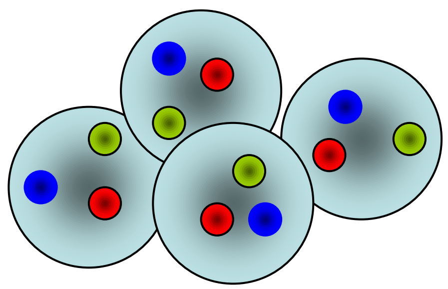
$$\Delta \gg \frac{m_s^2}{4\mu}$$

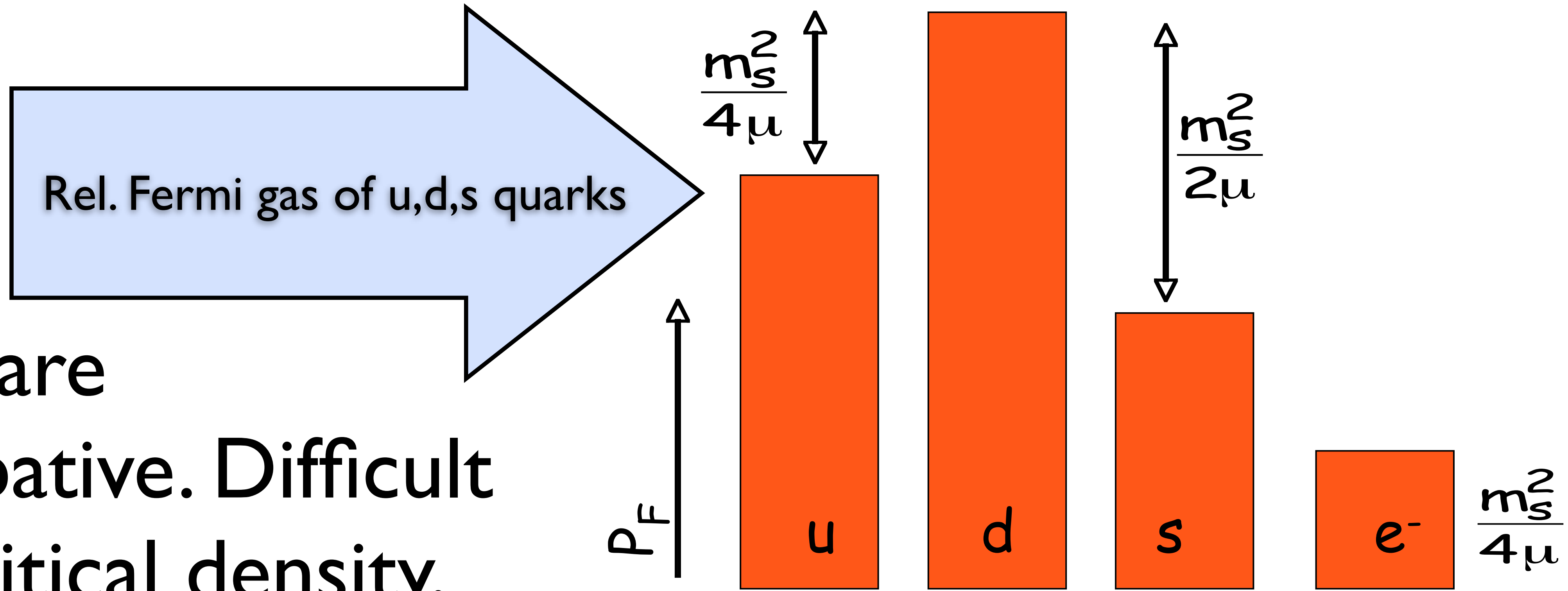


$$n_u = n_d = n_s$$

Alford, Rajagopal, Wilczek (1999)

Quark Matter in Neutron Stars





Interactions are non- perturbative. Difficult to predict critical density.

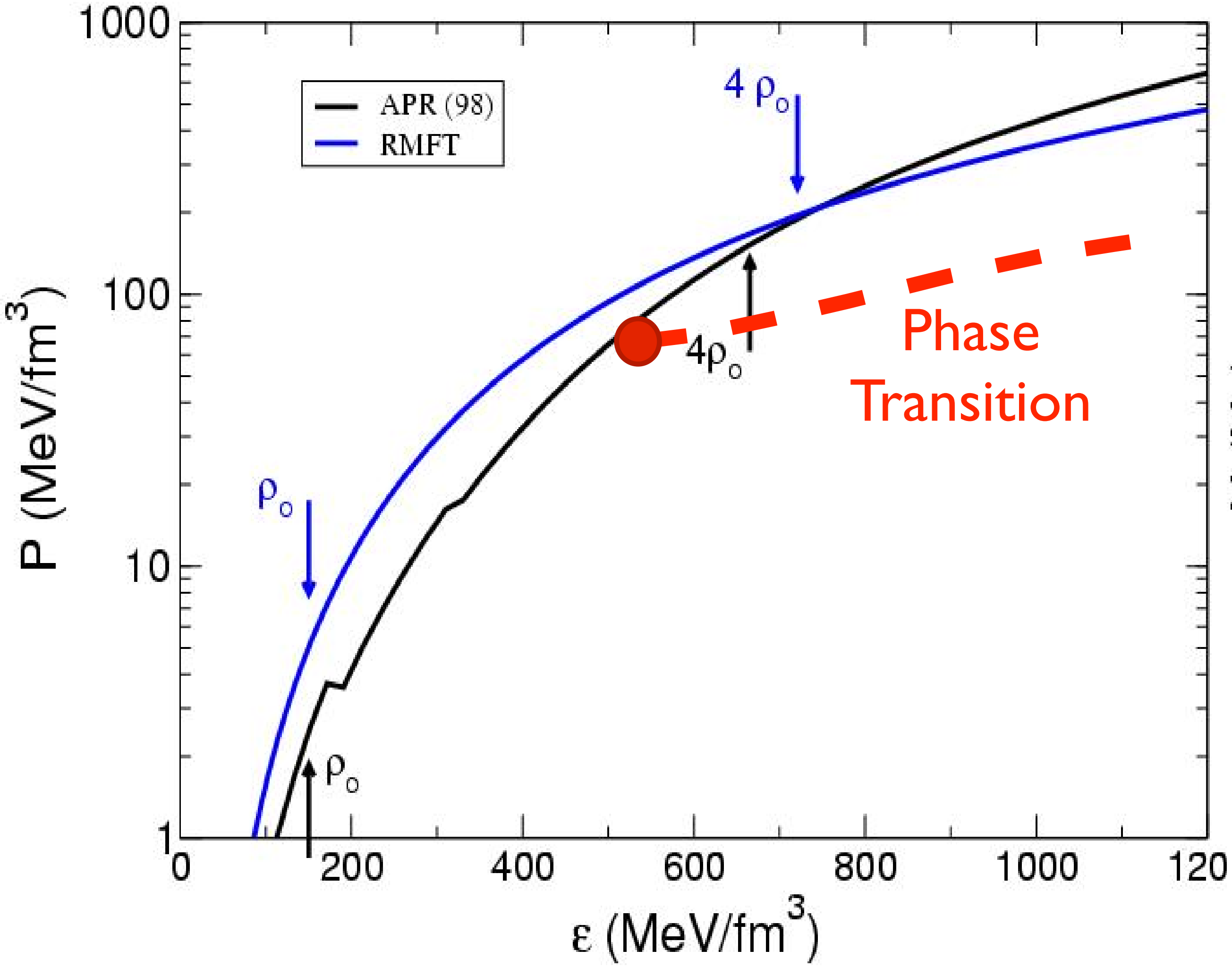
$$\Delta \simeq \frac{m_s^2}{4\mu}$$

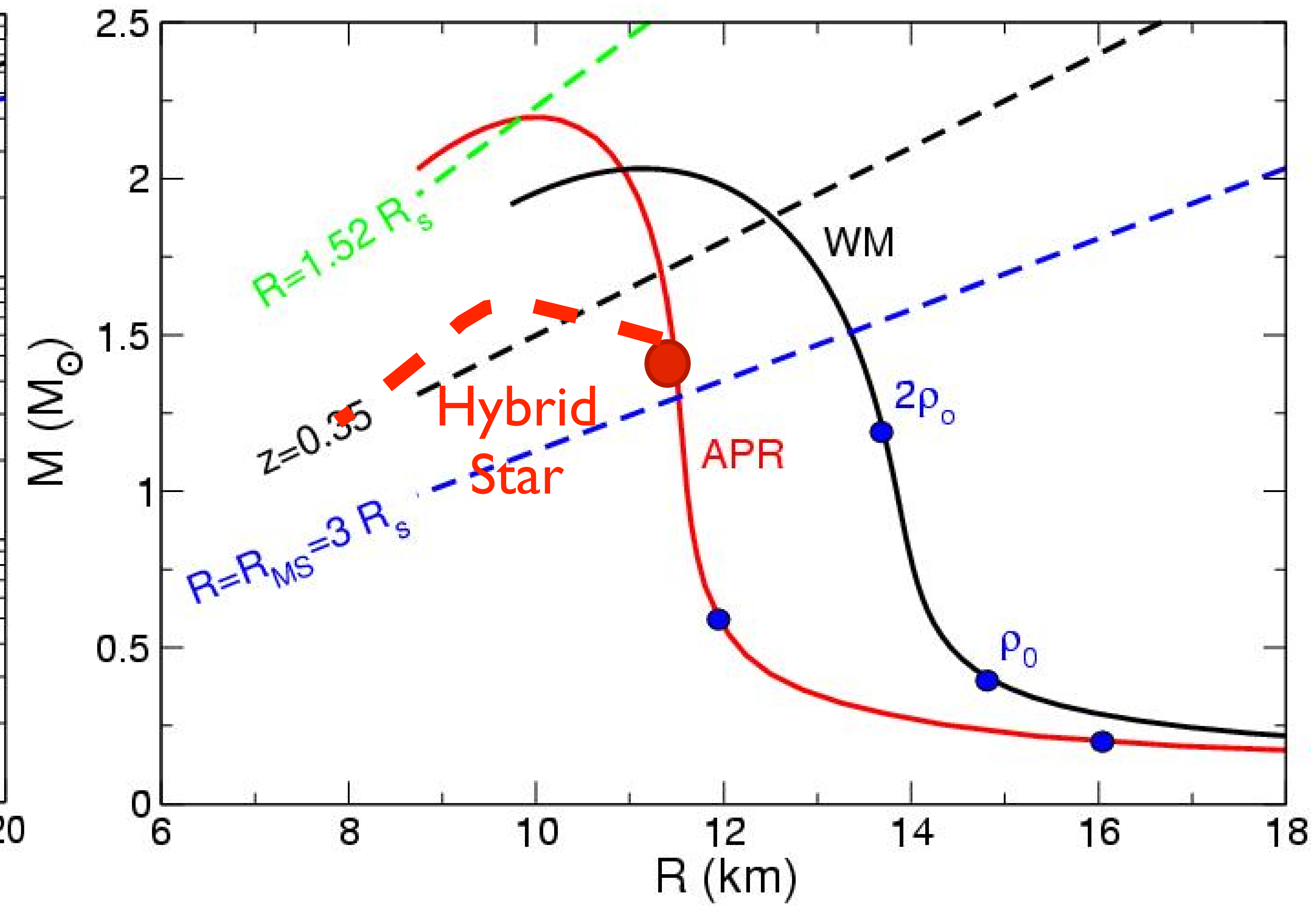
- Difficult to predict ground state.
- Complicated spectrum of excitations (Strongly coupled quasi-particles)

$$\Delta \geq \frac{m_s^2}{4\mu}$$

- Ground state is CFL.
- Low energy spectrum is simple (Goldstone modes - weakly coupled)

$M(R) \leftrightarrow P(\epsilon)$





Discovery of a 2 $M_\odot$ neutron star rules out a strong first-order transition at high density.

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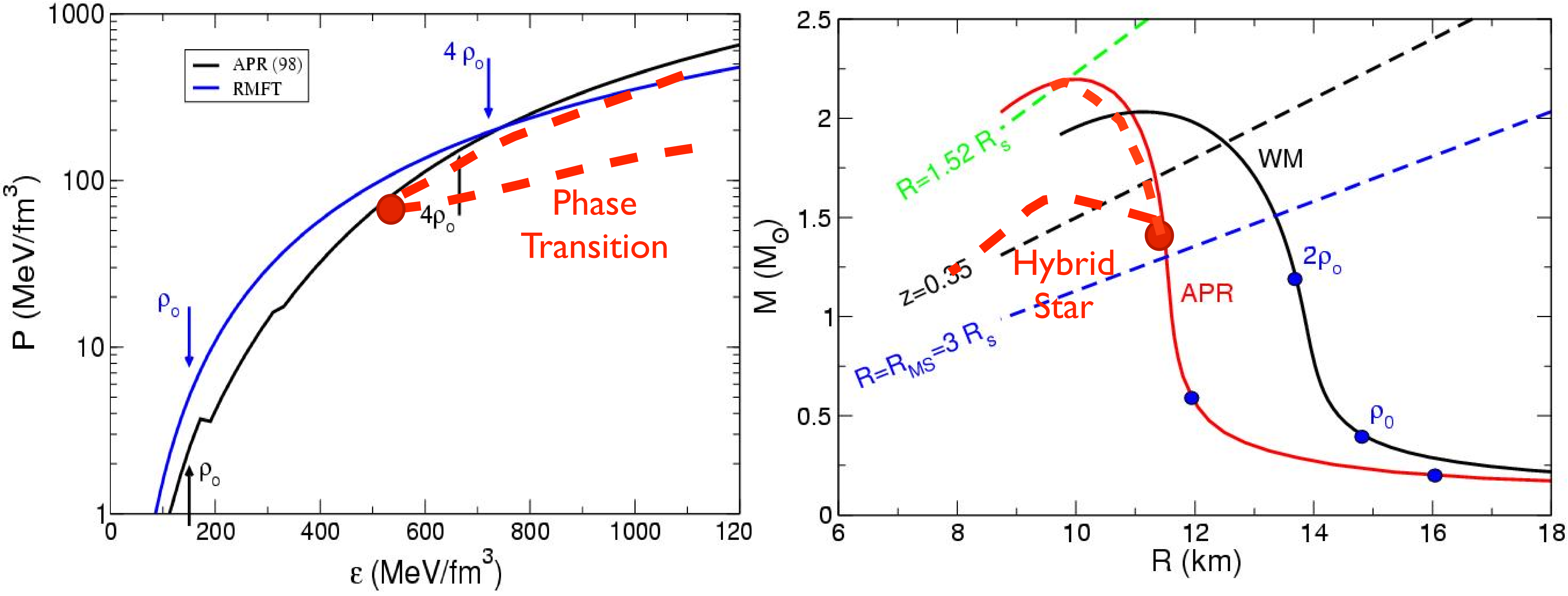


Discovery of a 2 $M_\odot$ neutron star rules out a strong first-order transition at high density.

Soft or Stiff ?

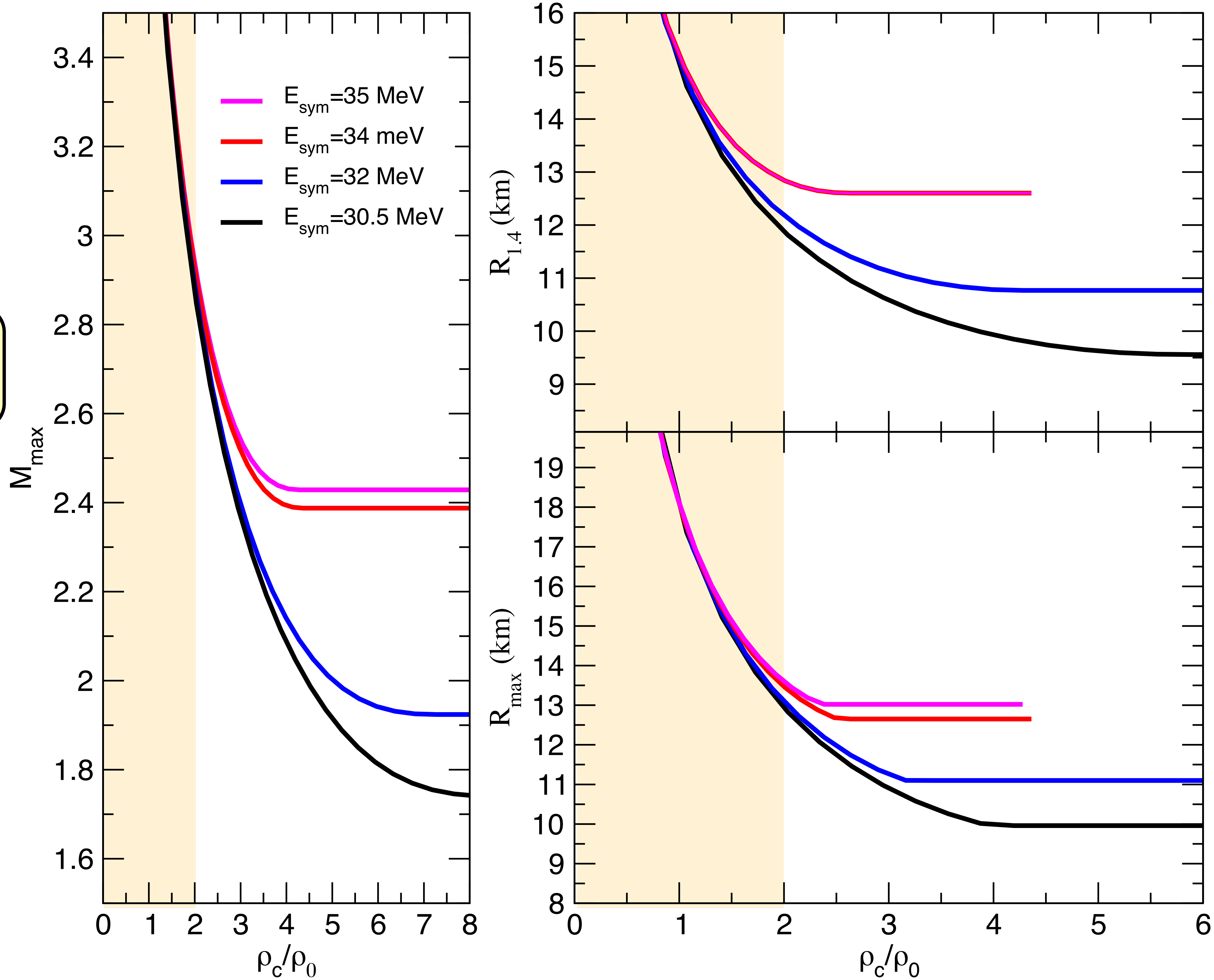
- At “low” density up to about 2 ρ_0 equation of state is soft.
- At intermediate (2-4 ρ_0) density equation of state is stiff.
- At higher density we do not know - could be driven soft by a phase transition !
- At asymptotic density (where QCD is perturbative) EoS is soft.

Upper Bounds on M & R

The maximally stiff EoS is the causal EoS:

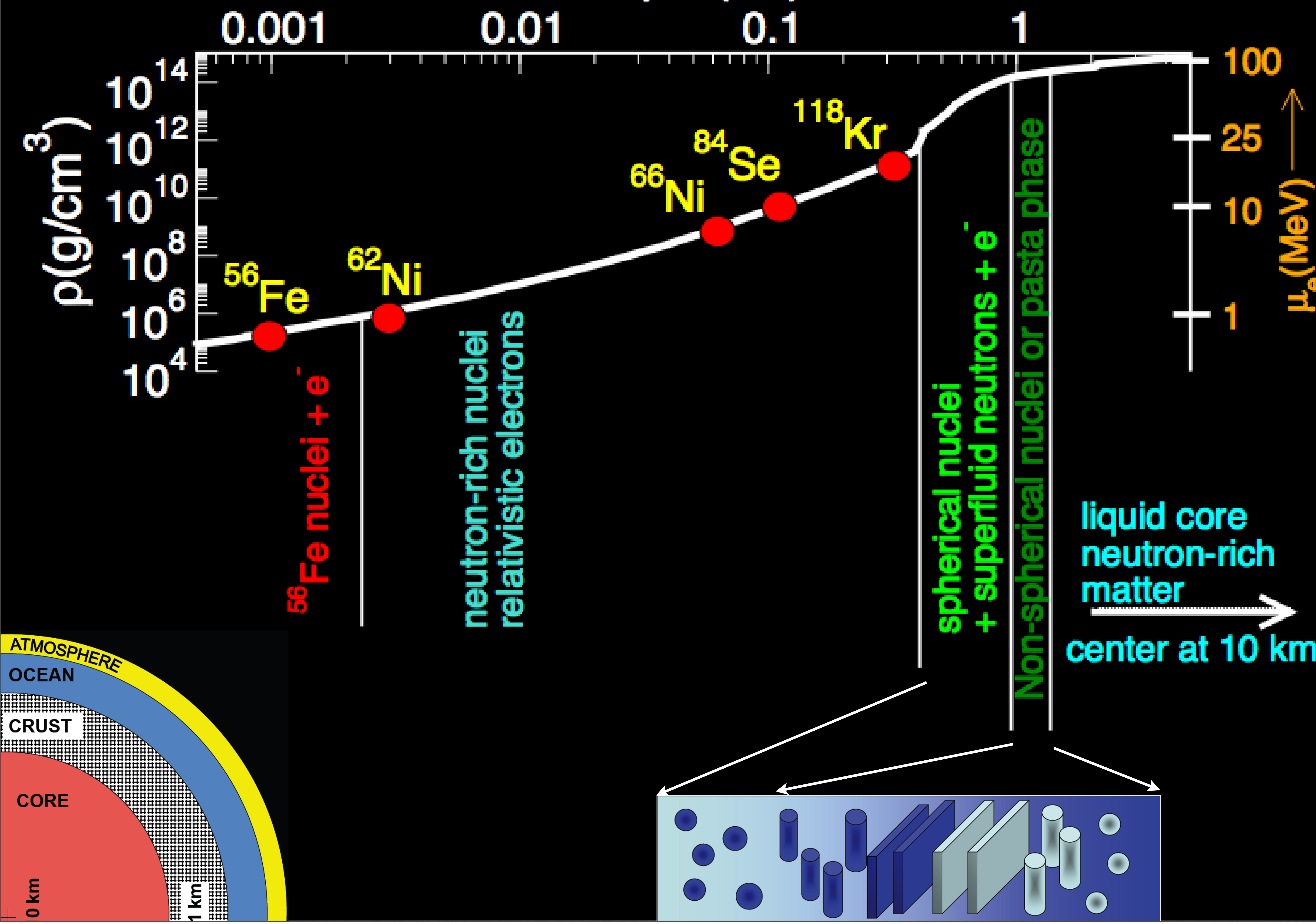
$$P = c\,\epsilon - \epsilon_0$$

Assume that EoS is known up to a critical density and is maximally stiff thereafter.

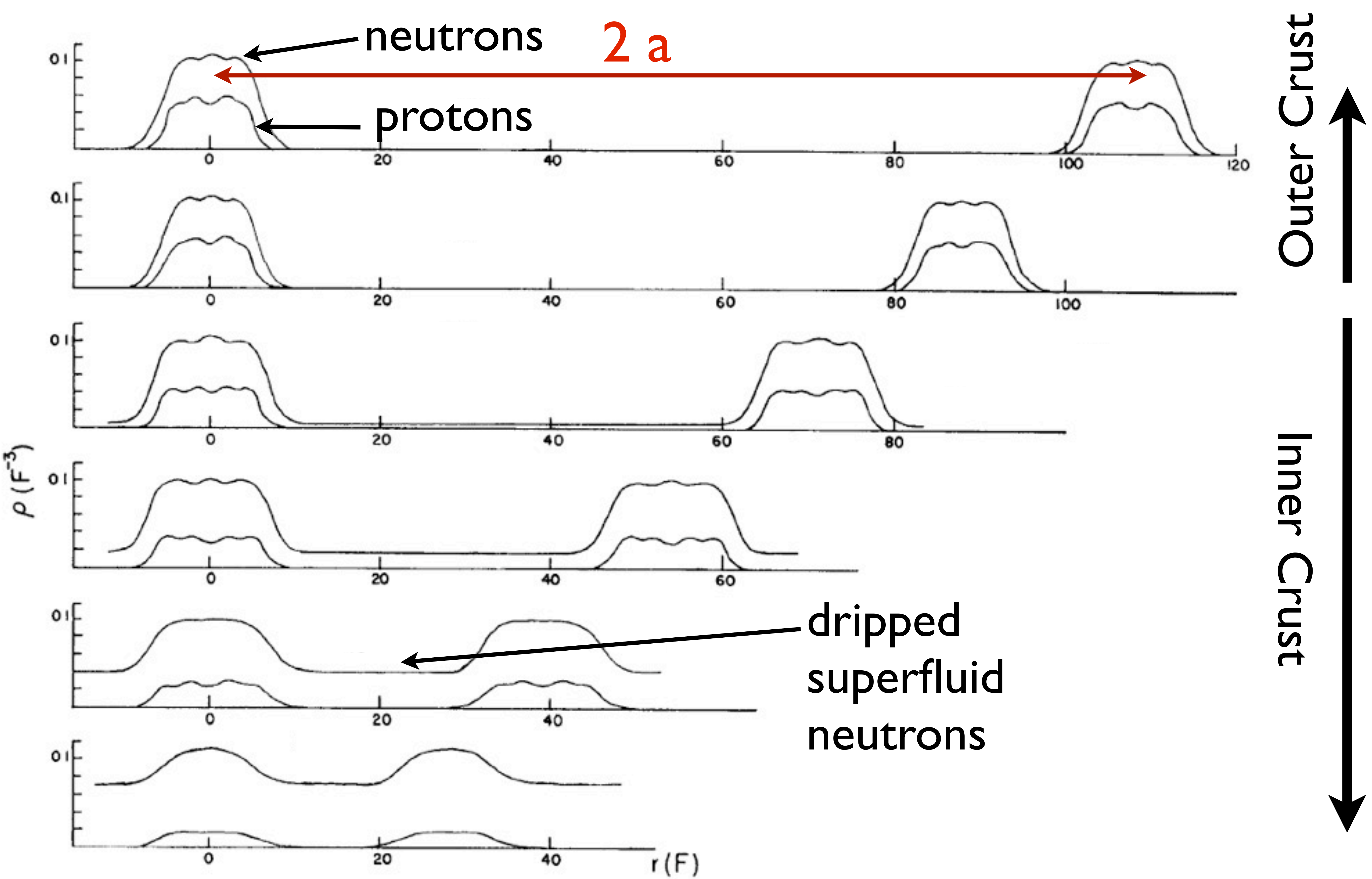


Crust Physics

Neutron Star in



Microscopic Structure of the Crust



Baym Pethick & Sutherland (1971) Negele & Vautherin (1973)

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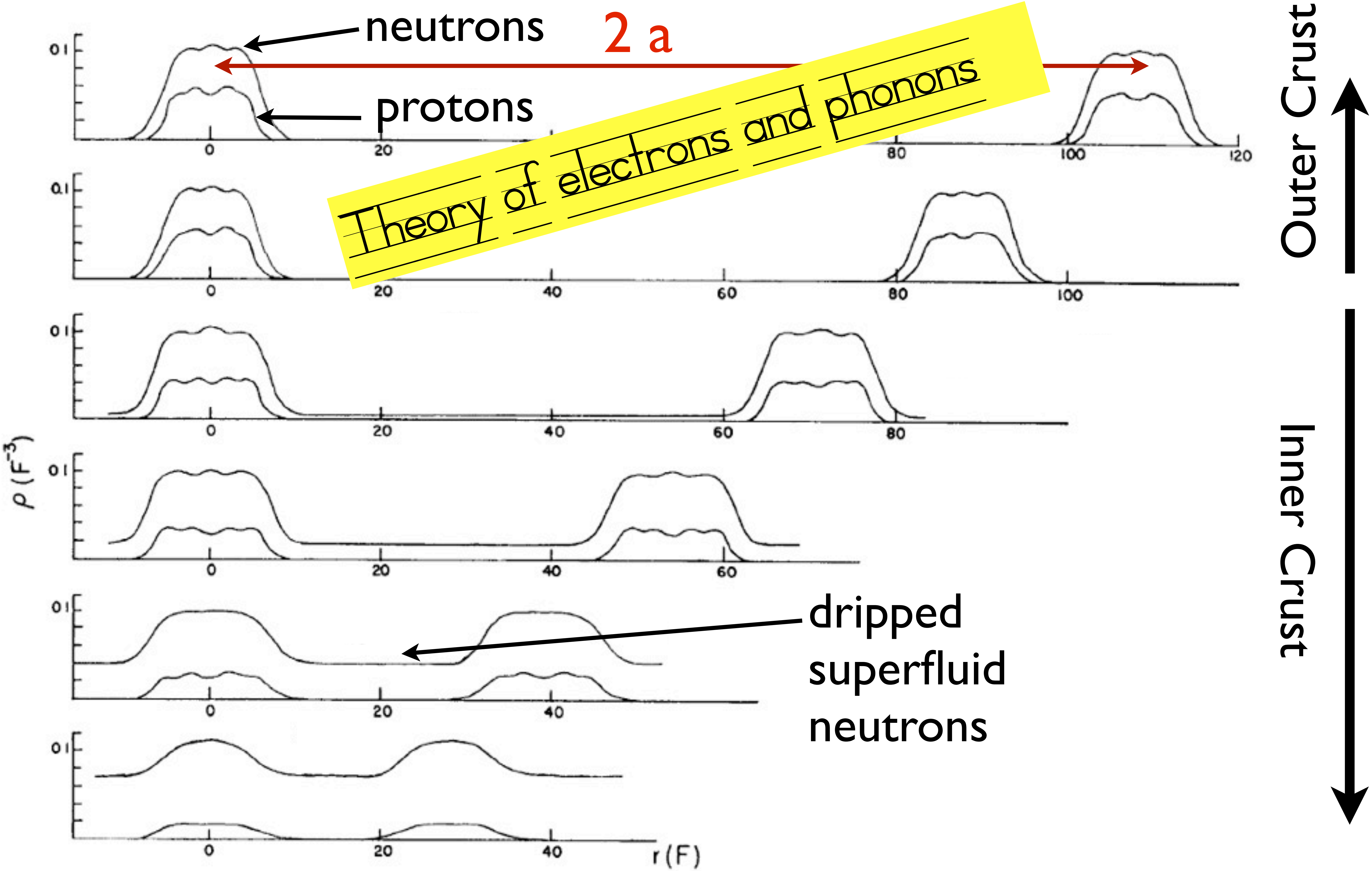

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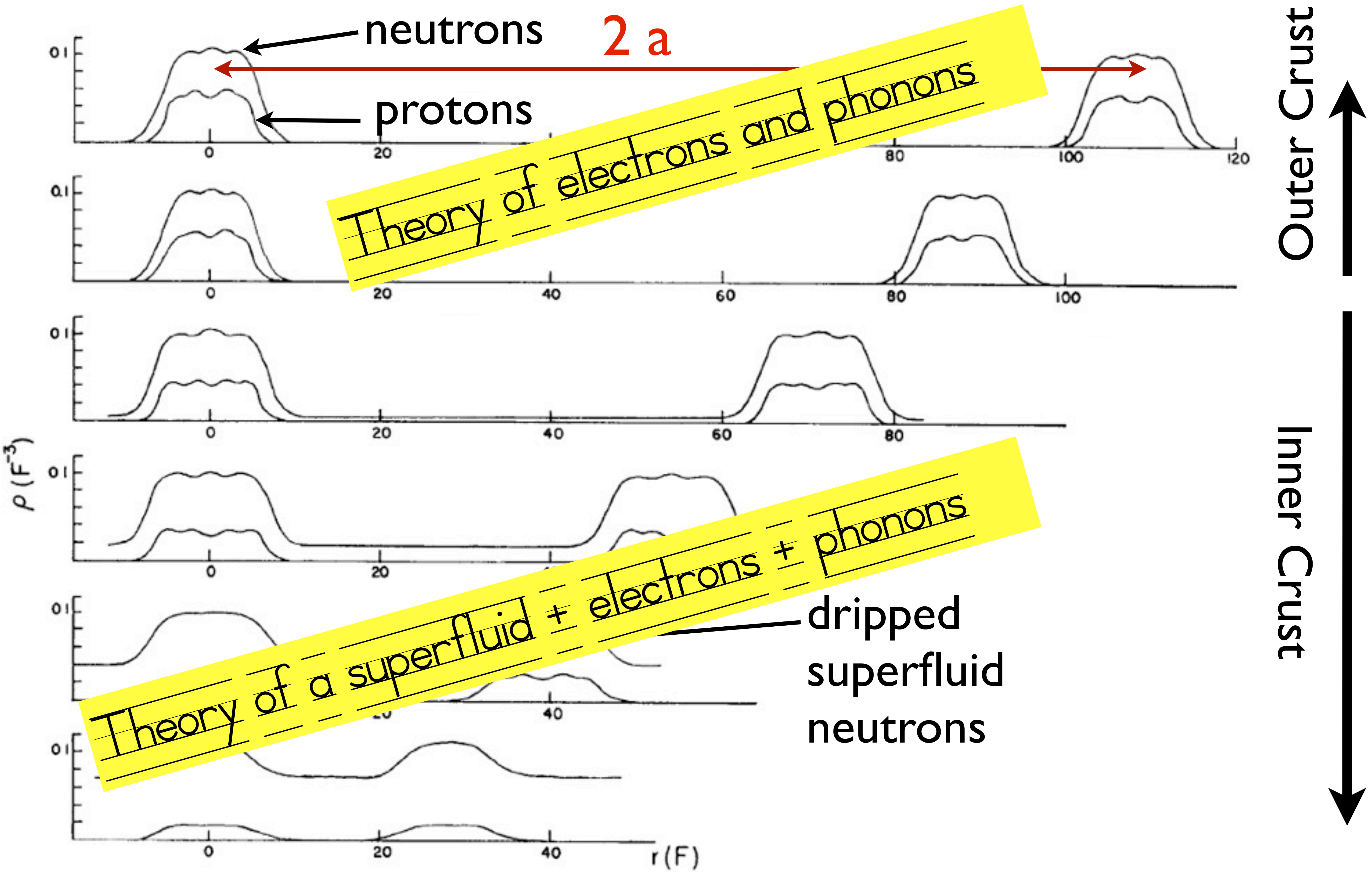

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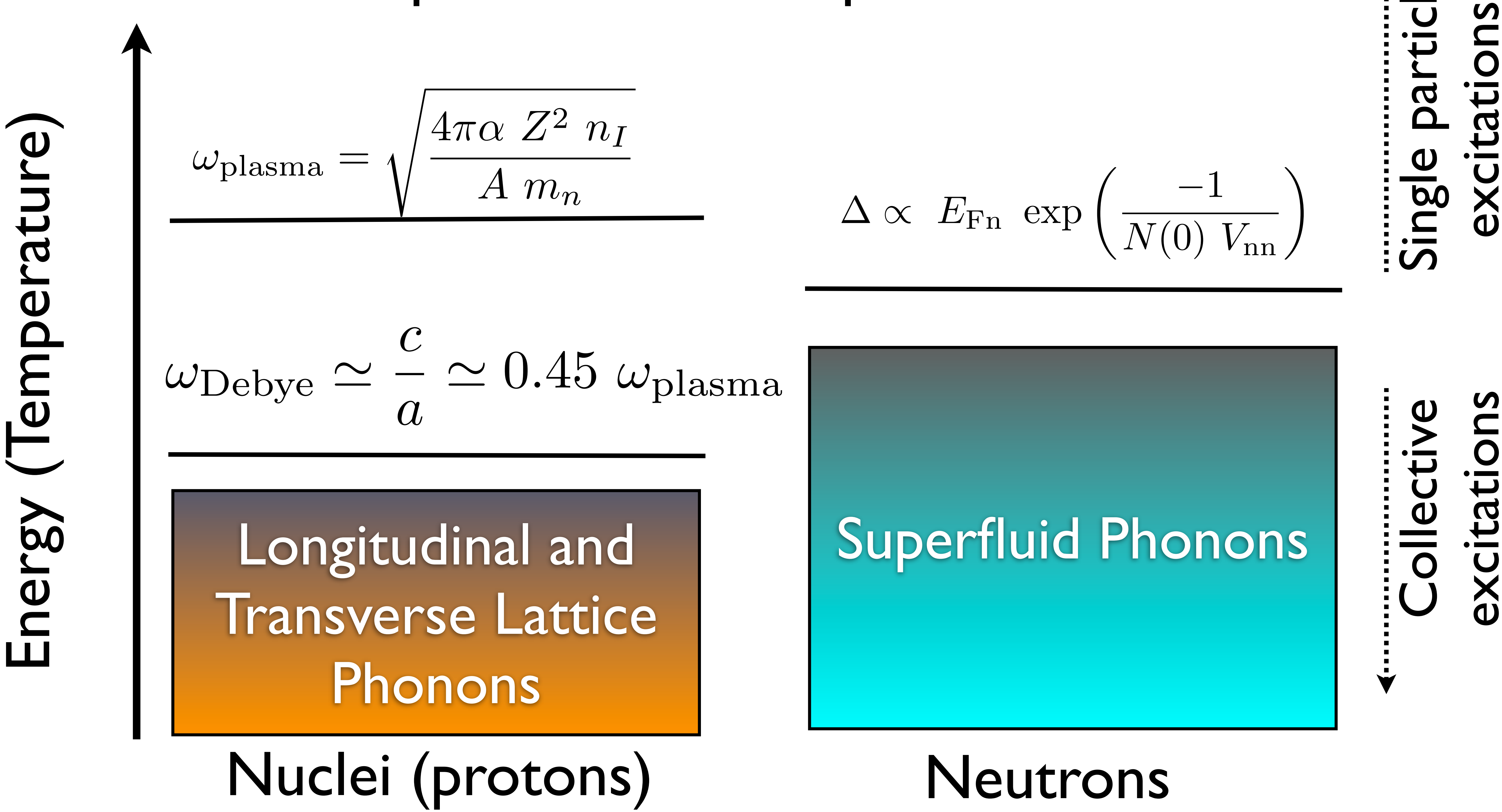
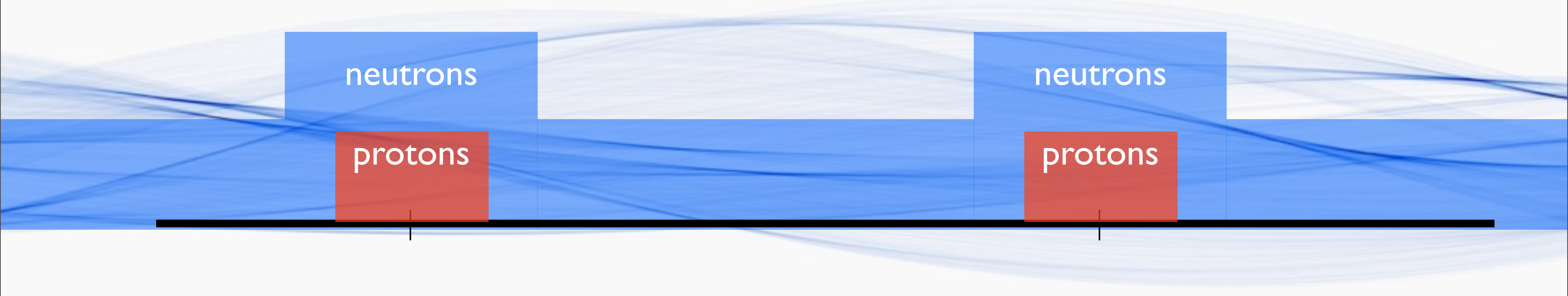
Separation of Scales

- Protons cluster (pairing + shell gaps)
- Proton clusters form a Coulomb lattice.
- Neutrons pair to form a superfluid.



Low Energy Theory of Phonons



Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the phase of the condensate.

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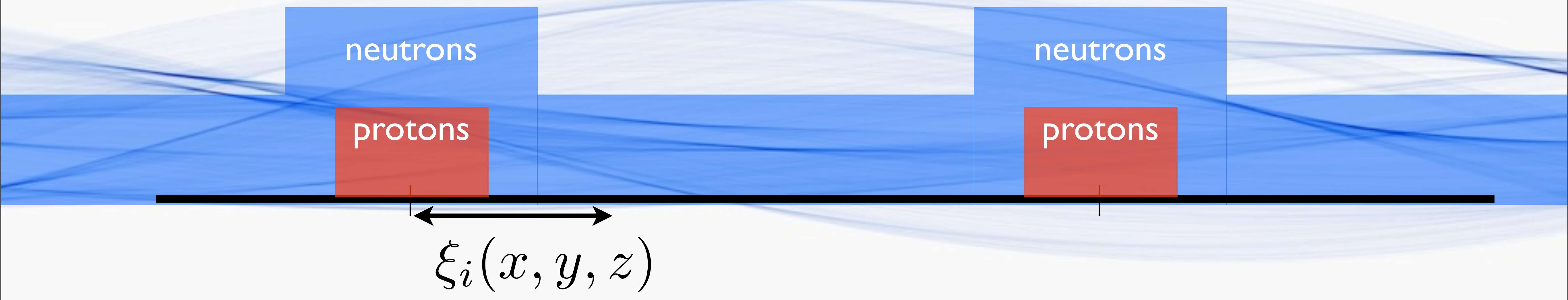

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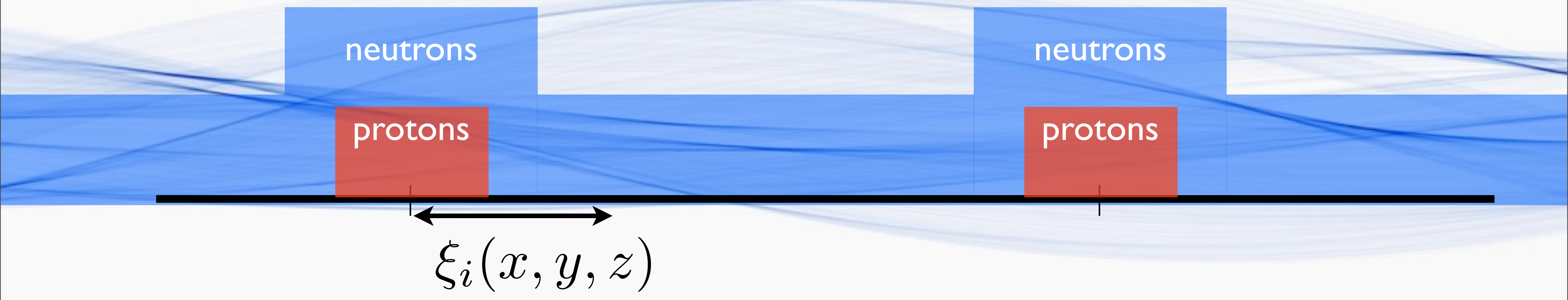

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Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the phase of the condensate.

$$\langle \psi_\uparrow(r) \psi_\downarrow(r) \rangle = |\Delta| \ \exp\left(-2i \ \theta\right)$$

"coarse-grain"

Collective coordinates:

Vector Field: $\xi_i(r, t)$
Scalar Field: $\phi(r, t)$

The Coupled System

Epstein 1988, Cirigliano, Reddy & Sharma (2011)

$$\mathcal{L}_{n+p} = \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2} v_s^2 \; (\partial_i \phi)^2 + \frac{1}{2}(\partial_t \xi_i)^2 - \frac{1}{2}(c_l^2 - g^2) \; (\partial_i \xi_i)^2$$
$$+ \, g \; \partial_t \phi \; \partial_i \xi_i + \tilde{\gamma} \; \partial_i \phi \; \partial_t \xi_i$$

Velocities : $v_s^2 = \dfrac{n_f}{m \chi_n}$ $\quad c_l^2 = \dfrac{K + 4\mu_s/3}{m(n_p + n_b)}$

Entrainment: protons drag neutrons.

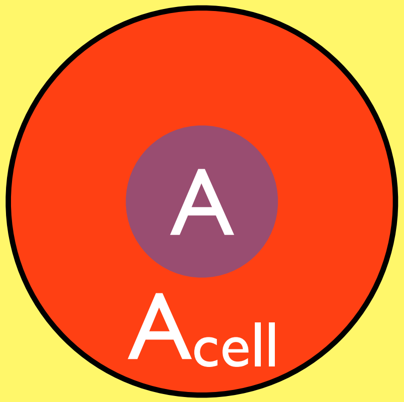
- Bound neutrons: $n_b = \gamma \; n_n$
- Free neutrons: $n_f = n_n \; (1 - \gamma)$

Entrainment:

$n_b \neq$ number of "bound" neutrons.

Bragg scattering off the lattice is important.

$$A^* = A + \left(\frac{m^* - m}{m} \right) \; (A_{\rm cell} - A)$$



A=N+Z

Chamel (2005)
Carter, Chamel & Haensel (2006)

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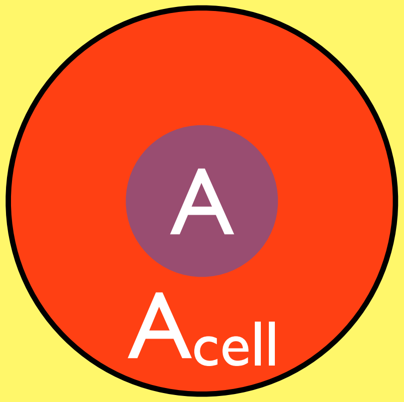

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Entrainment: protons drag neutrons.

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Longitudinal lattice phonons and superfluid phonons are coupled:

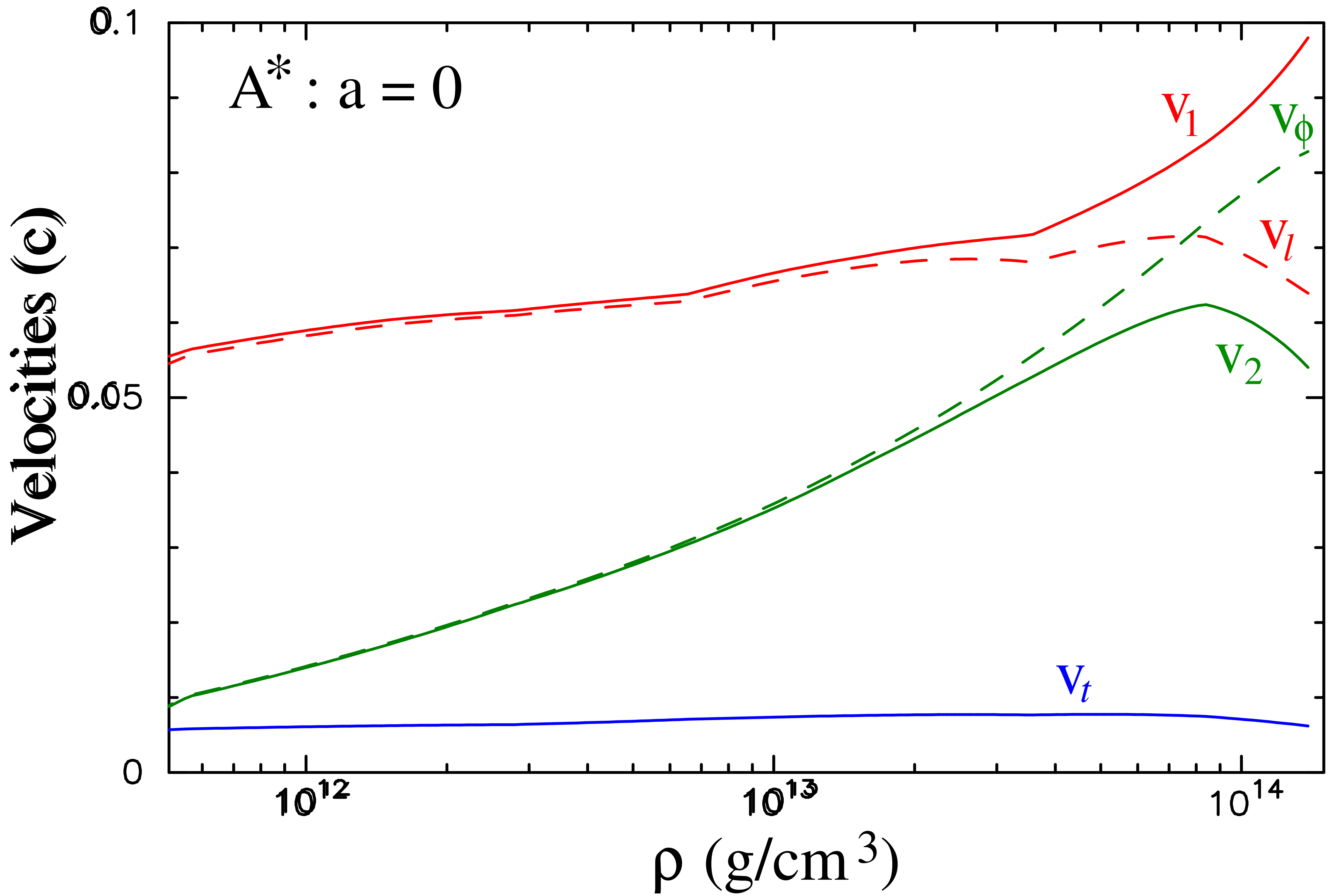
$$g = n_p \; E_{np} \sqrt{\frac{\chi_n}{m(n_p + n_b)}} \qquad \tilde{\gamma} = \frac{-n_b \; v_s}{\sqrt{(n_p + n_b) n_f}}$$

Transverse lattice phonons:

$$\mathcal{L}_t = \frac{1}{2}(\partial_t \xi_i)^2 - \frac{1}{2} c_t^2 (\partial_i \xi_j + \partial_j \xi_i)^2 \quad \Rightarrow \quad c_t^2 = \frac{\mu_s}{m(n_p + n_b)}$$

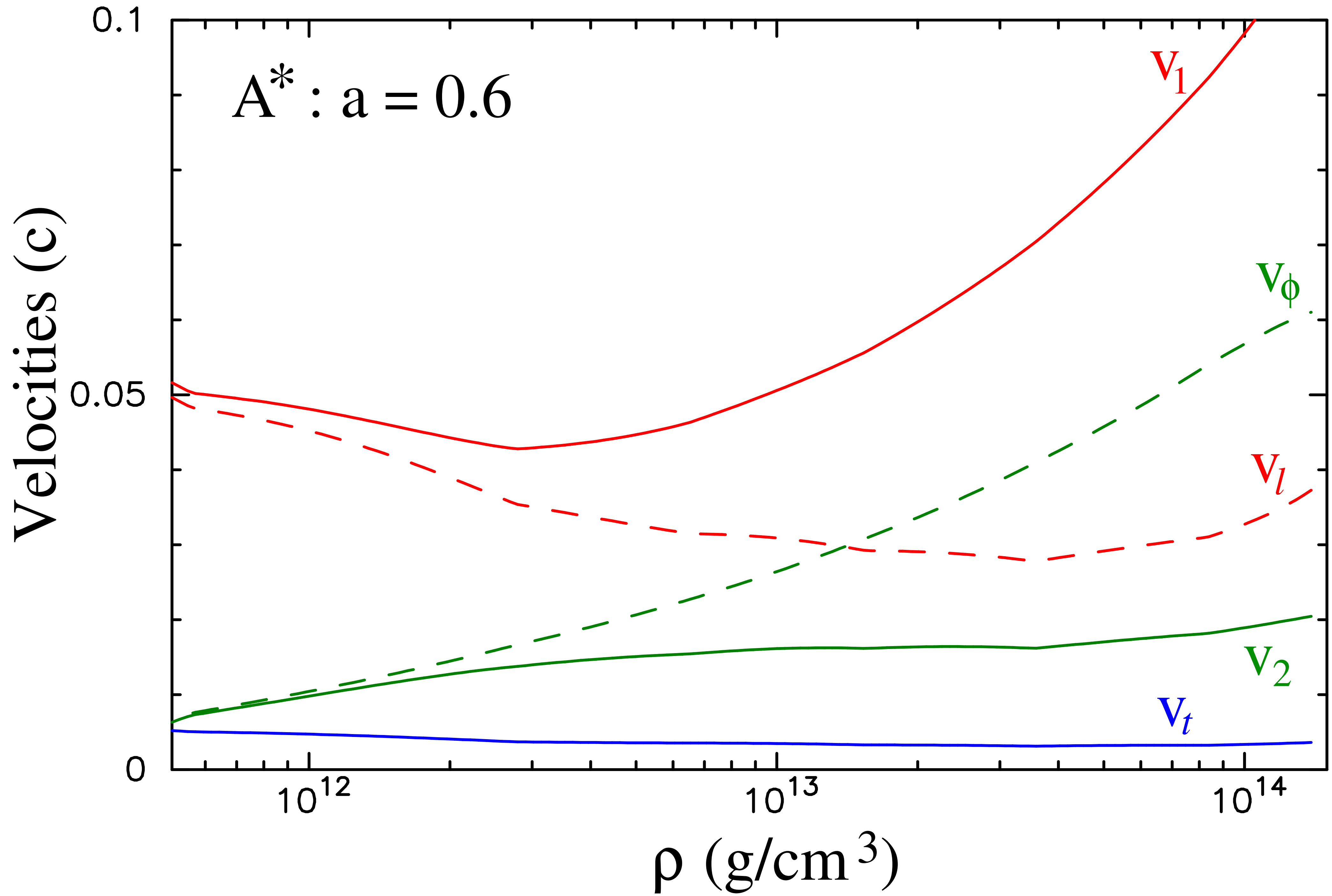
Acoustics Waves in the Crust

$$v_\ell = \sqrt{\frac{K + 4S/3}{\rho_I}} \qquad v_\phi = \sqrt{\frac{n_n^c}{m}\frac{\partial \mu_n}{\partial n_n}} \qquad v_t = \sqrt{\frac{S}{\rho_I}}$$



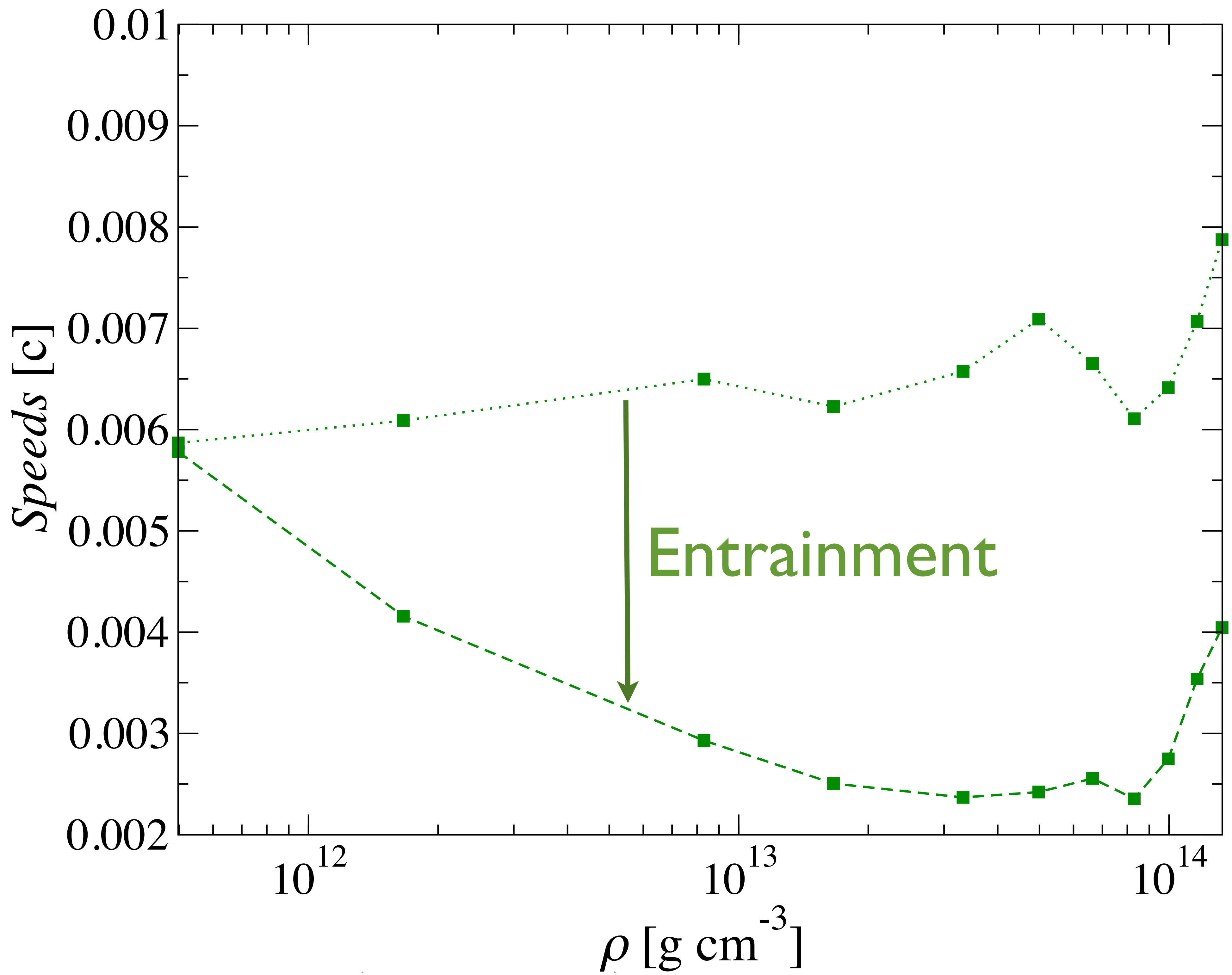
Acoustics Waves in the Crust

$$v_\ell = \sqrt{\frac{K + 4S/3}{\rho_{\rm I}}} \qquad v_\phi = \sqrt{\frac{n_n^{\rm c}}{m}\frac{\partial \mu_n}{\partial n_n}} \qquad v_t = \sqrt{\frac{S}{\rho_{\rm I}}}$$



Speed of Transverse (Shear) Modes

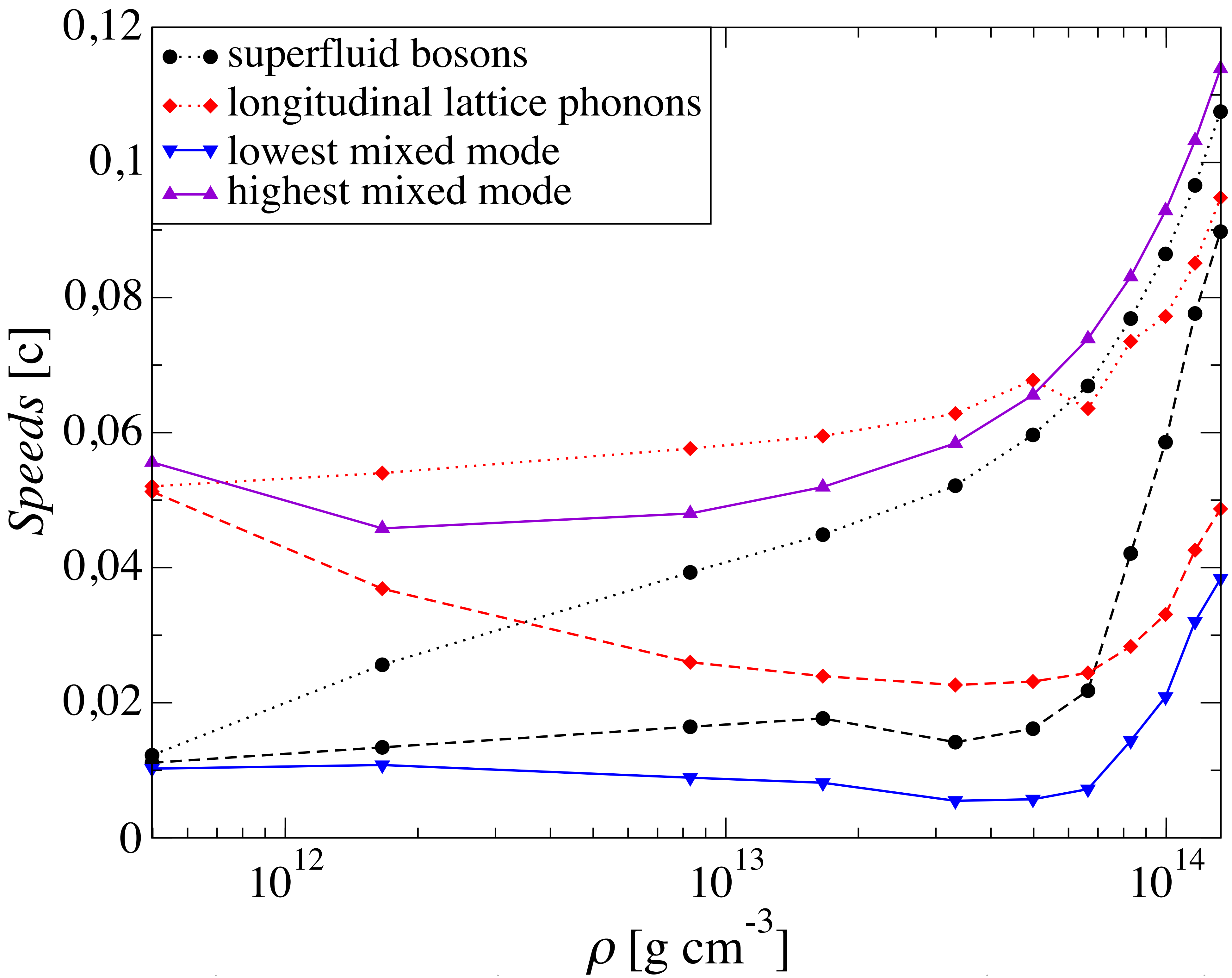
from a Microscopic Theory



Chamel, Page, Pethick, Reddy (2012)

Speed of Longitudinal Sound(s)

from a Microscopic Theory



Chamel, Page, Pethick, Reddy (2012)

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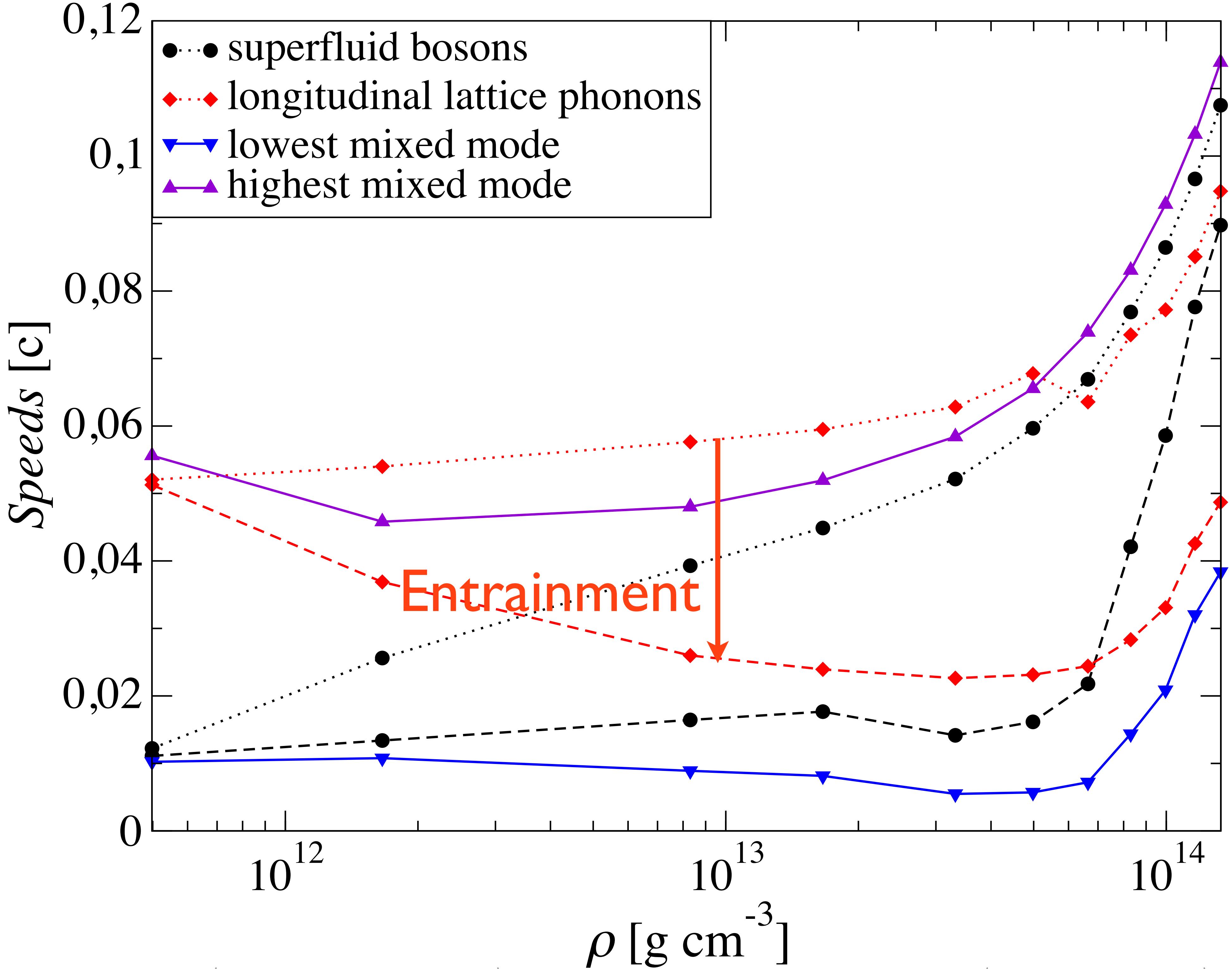

Chamel, Page, Pethick, Reddy (2012)

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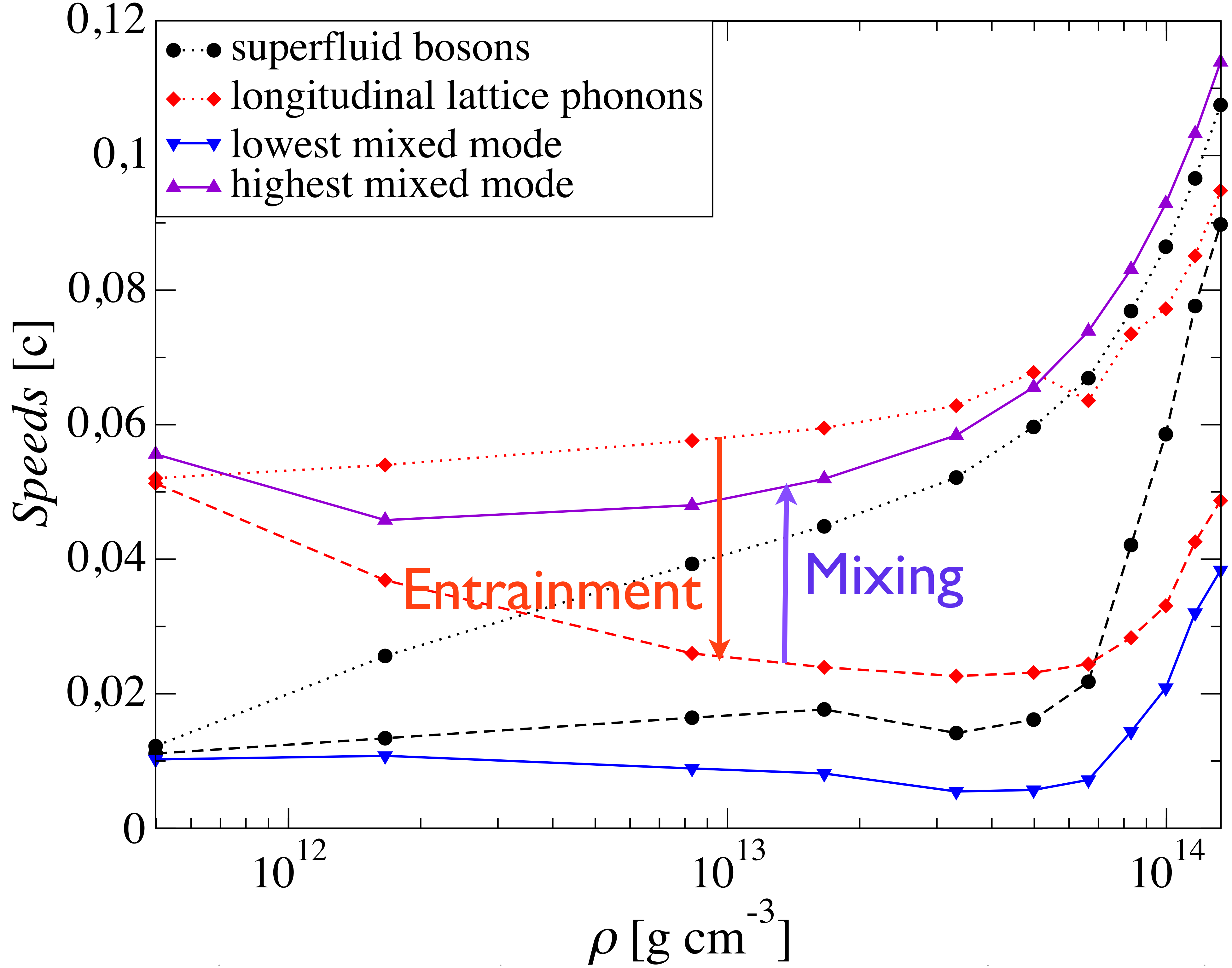

Chamel, Page, Pethick, Reddy (2012)

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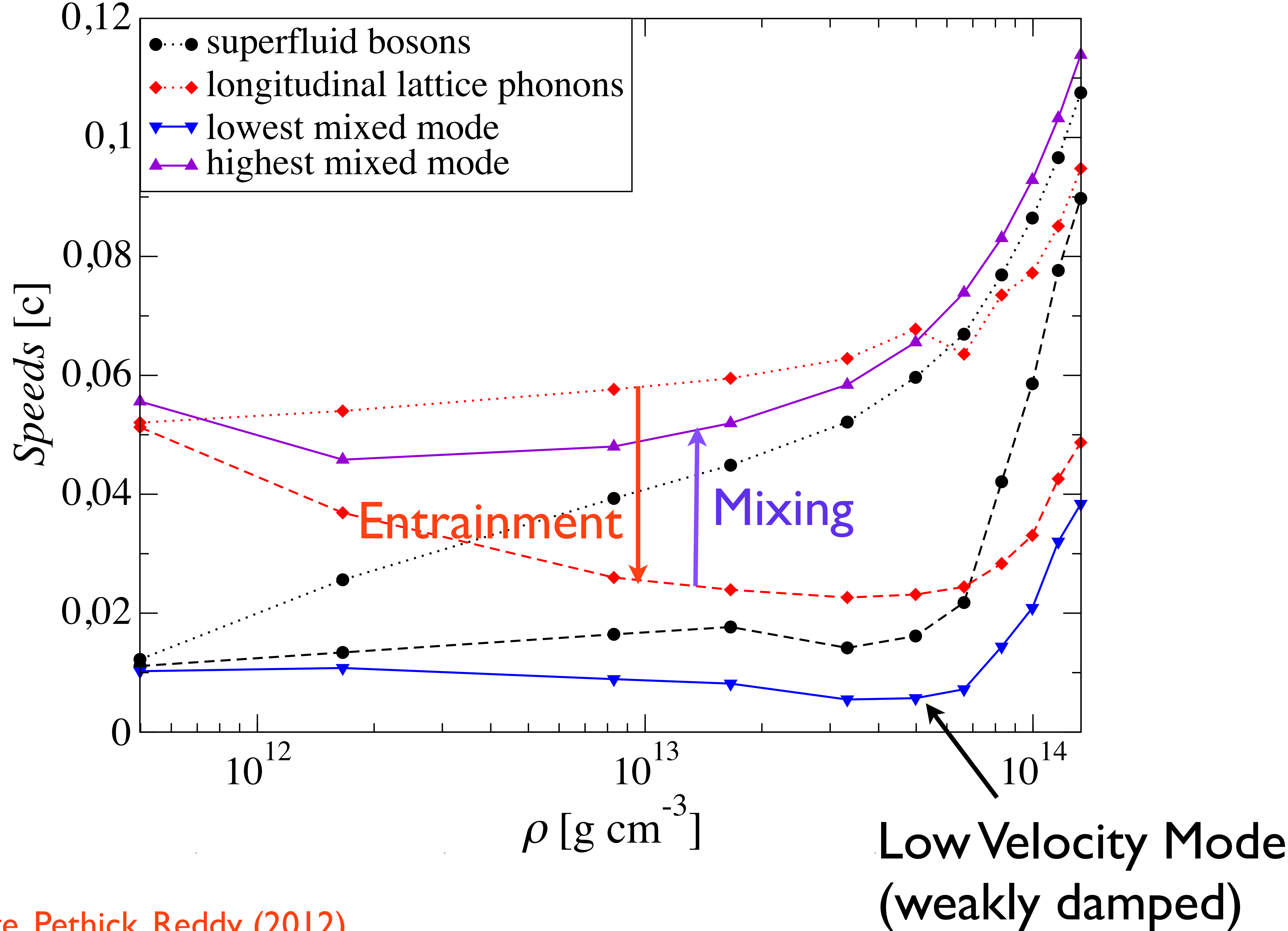


Chamel, Page, Pethick, Reddy (2012)

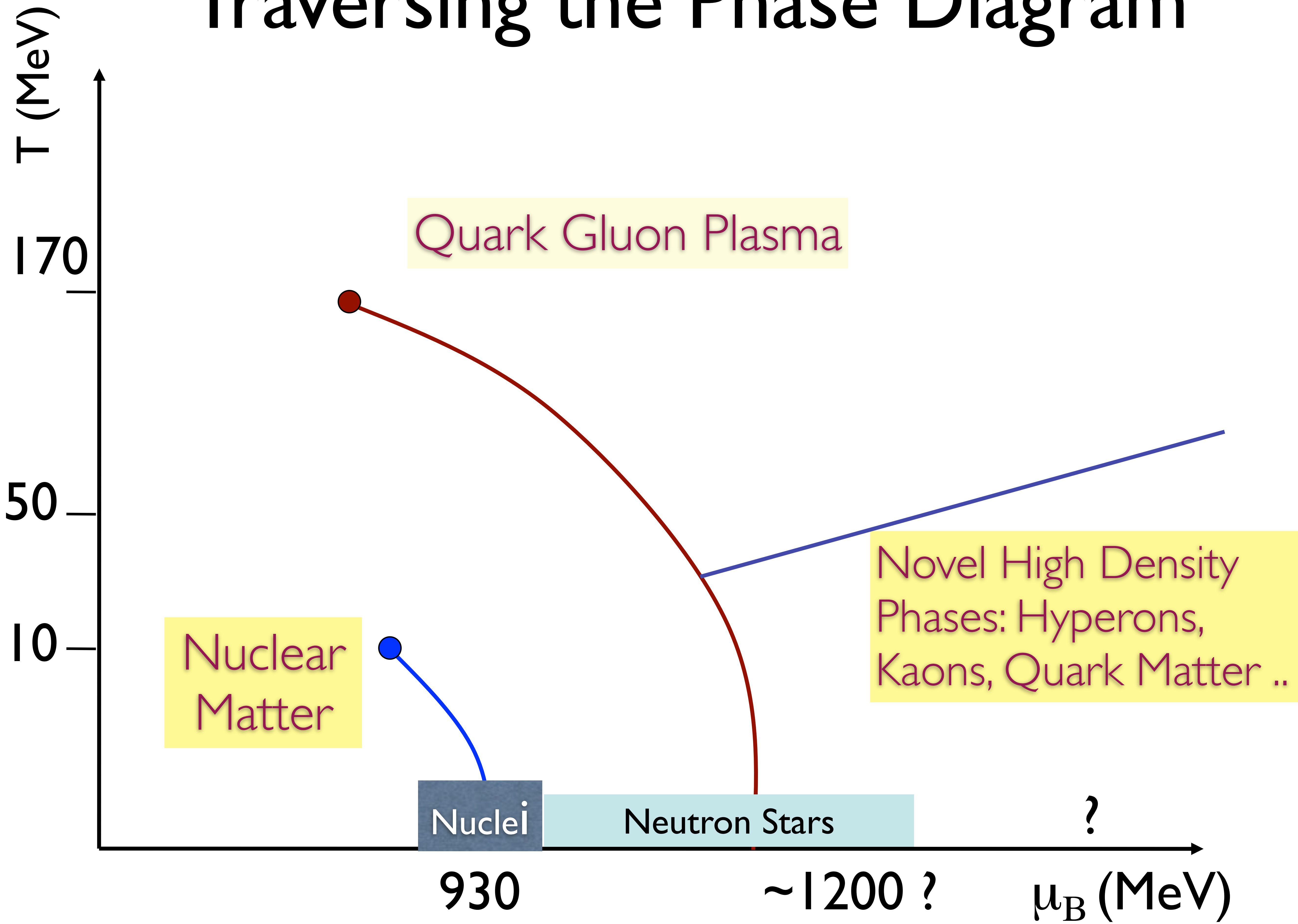
Post Merger Physics

- Hot equation of state: New models with neutron matter constraints at T=0.

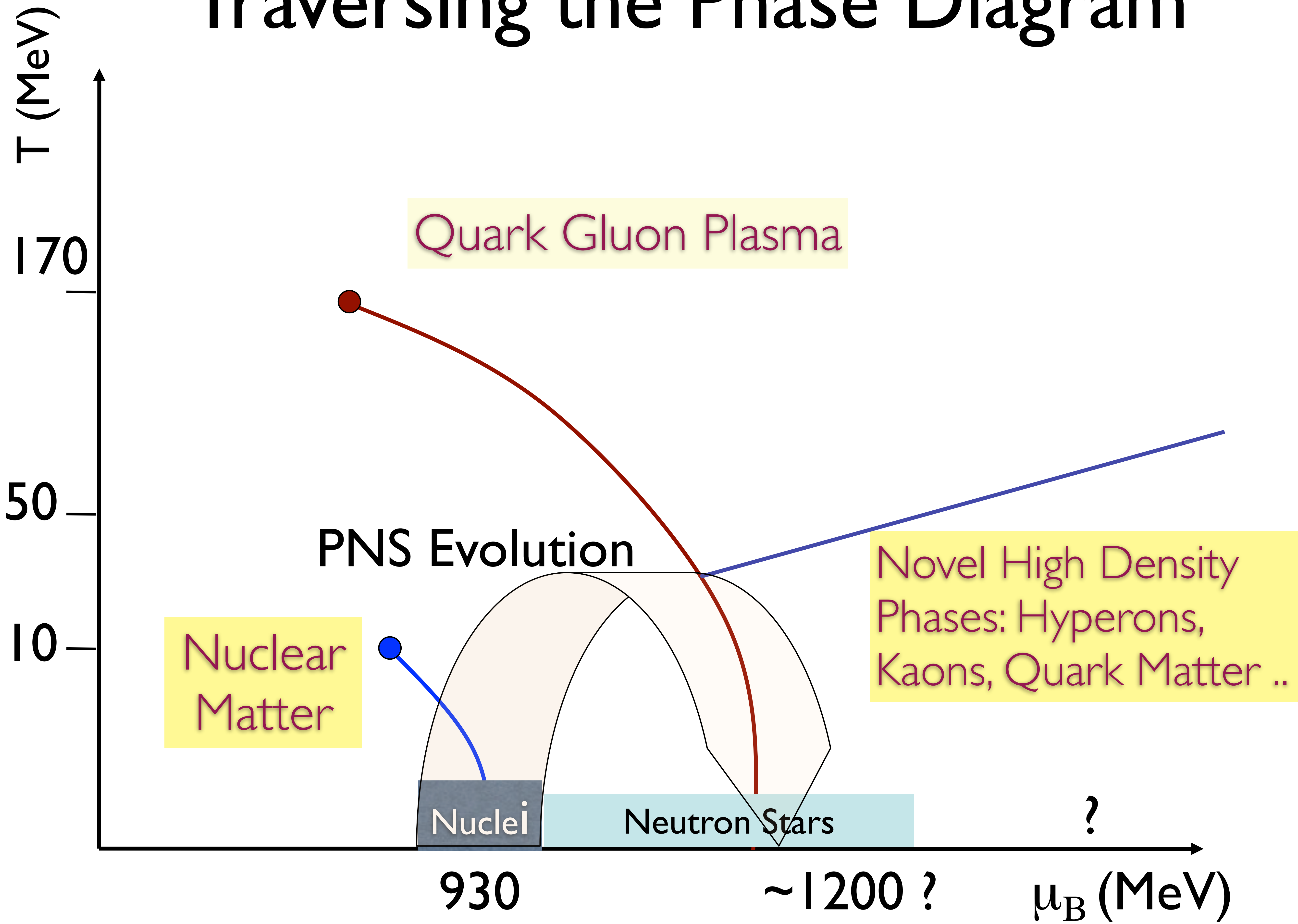
- Neutrino interactions: Correlations between nucleons are strong and can alter the opacity and associated transport timescales.

- Phase transitions or new degrees of freedom (pions, kaons, hyperons, quarks) are likely. Will impact the dynamics and lifetime of the hyper-massive neutron star phase.

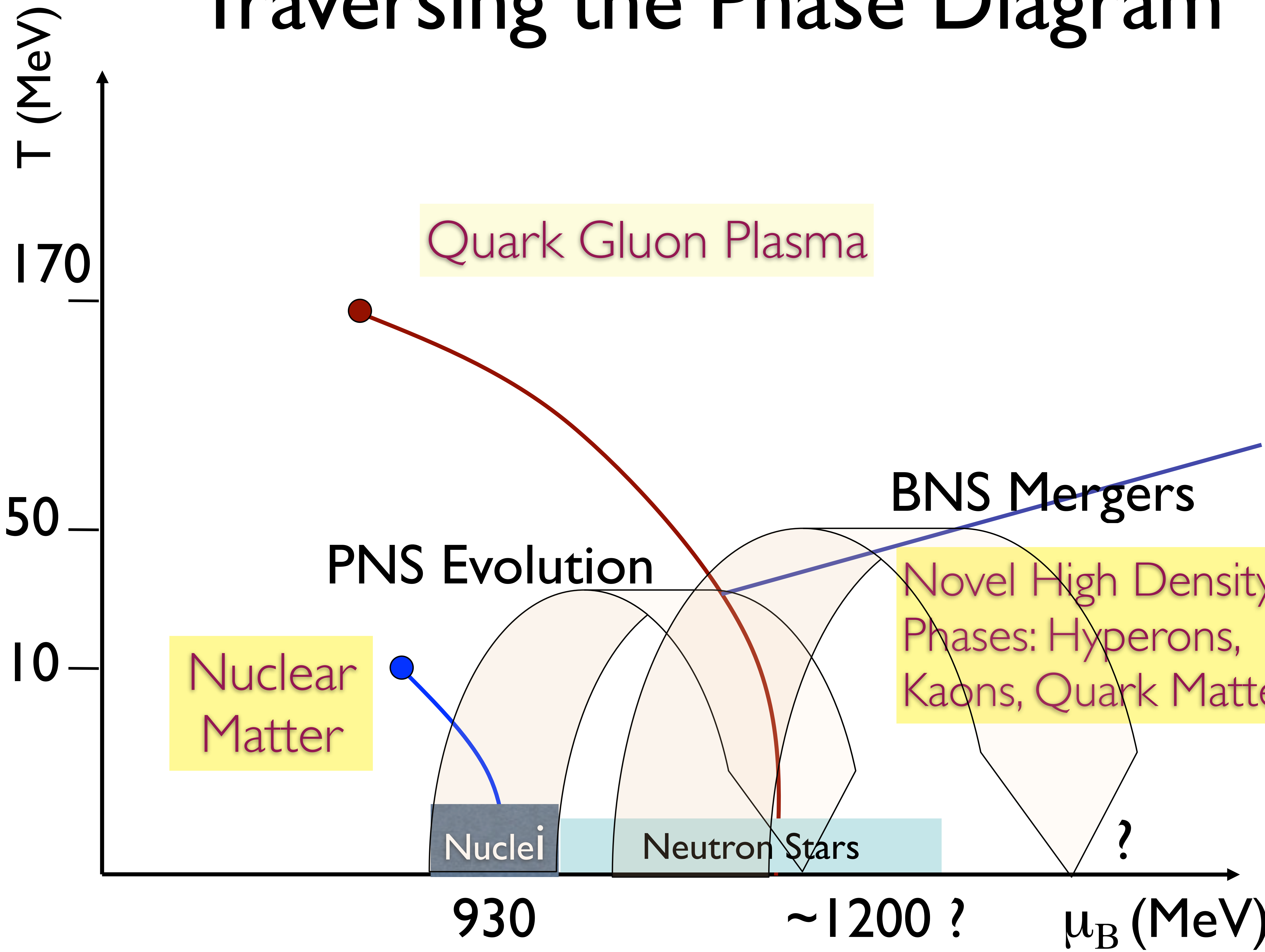
Traversing the Phase Diagram



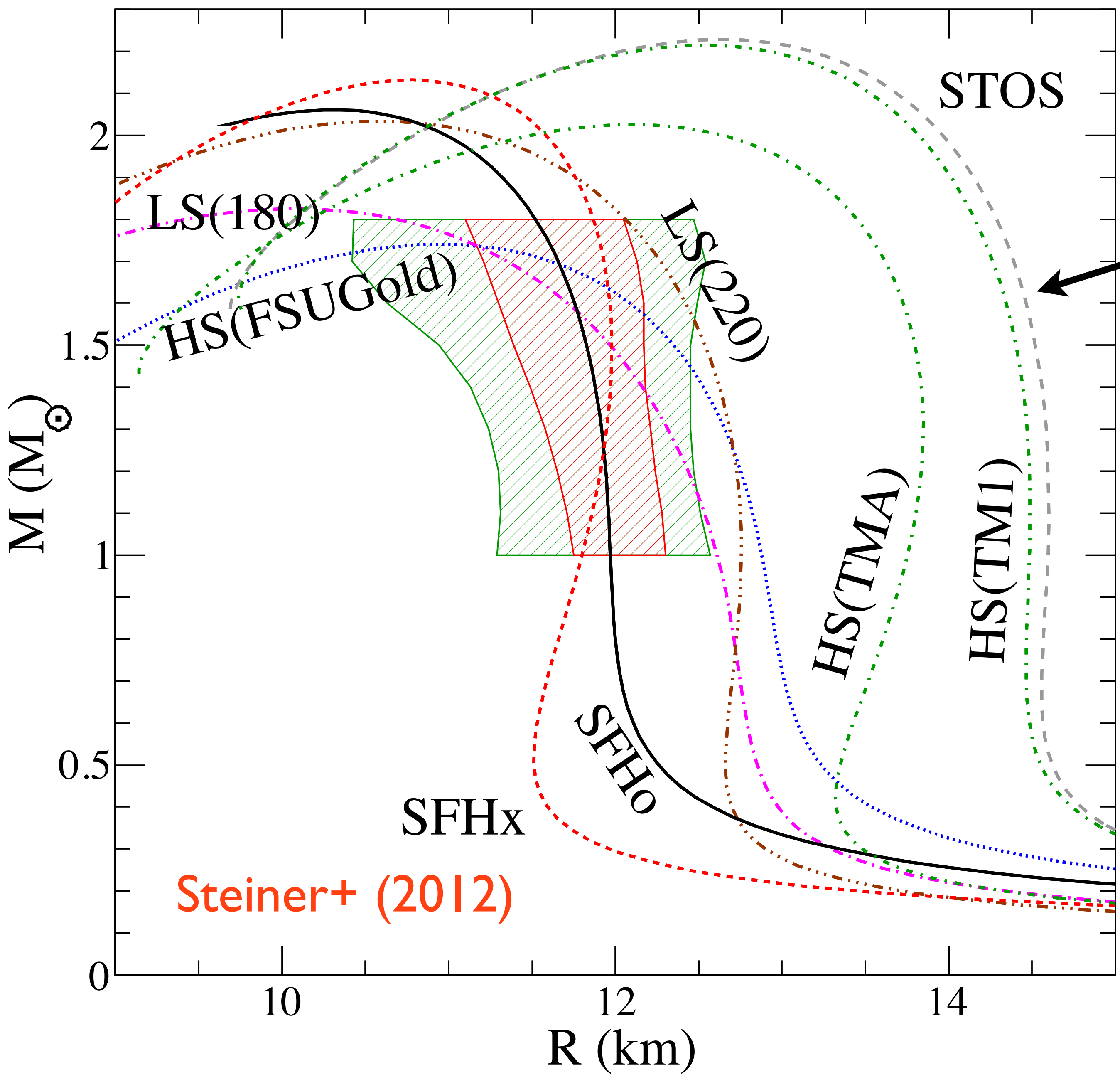
Traversing the Phase Diagram



Traversing the Phase Diagram



New EoS for Simulations



Violate Neutron Matter Constraints

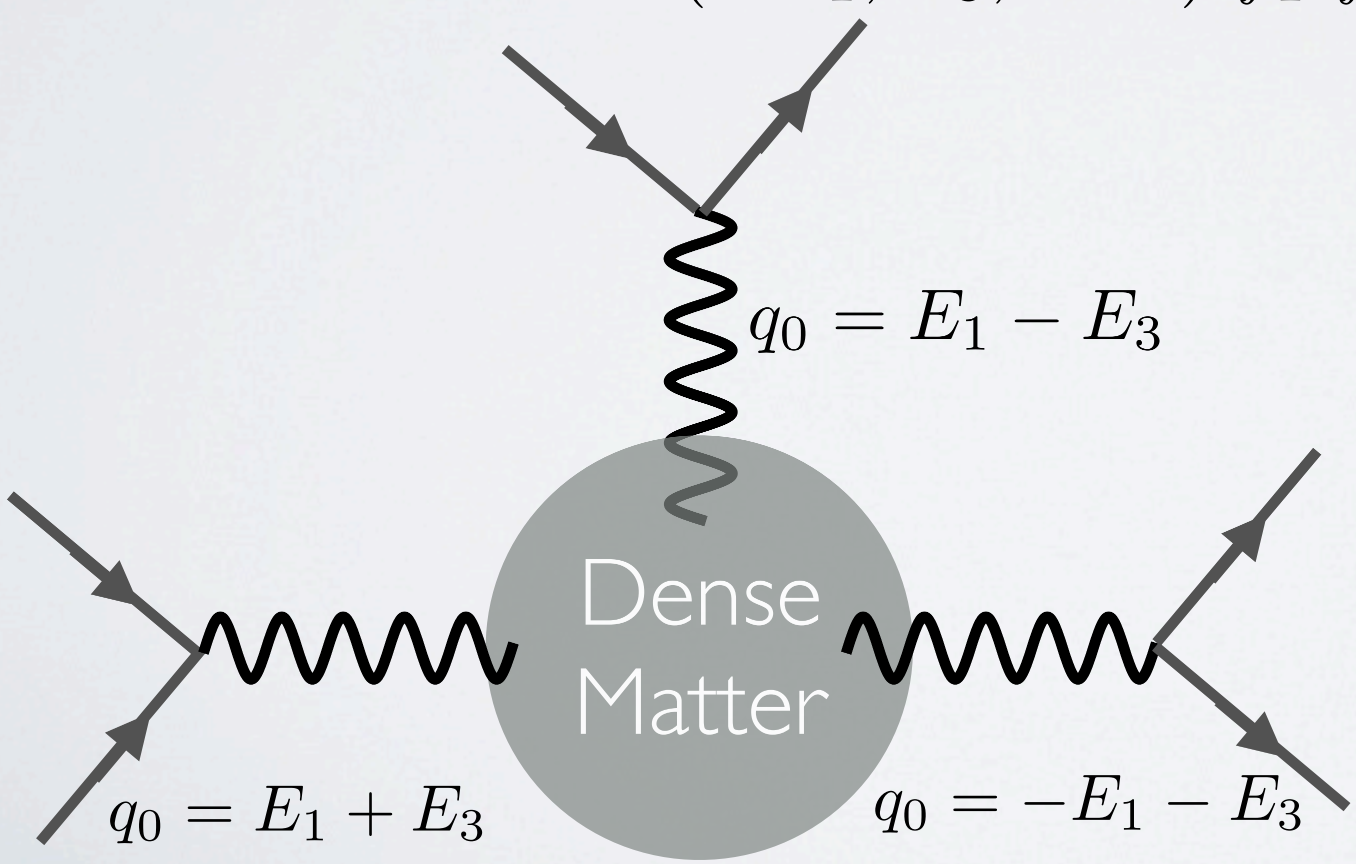
- Mean field models constructed to mimic T=0 behavior predicted by microscopic theories are being developed.

Hempel+ (2012), G. Shen+ (2011), Steiner+ (2012)

NEUTRINO TRANSPORT

- RHS of the Boltzmann Equation.

$$\frac{\partial f(E_1)}{\partial t} = \int \frac{d^3k_3}{(2\pi)^3}\ R(E_1, E_3, \cos\theta)\ f_3(1-f_1)$$
$$-R(E_3, E_1, \cos\theta)\ f_1(1-f_3)$$
$$+R(E_1, -E_3, \cos\theta)\ (1-f_1)(1-f_3)$$
$$-R(-E_1, E_3, \cos\theta)\ f_1\ f_3$$



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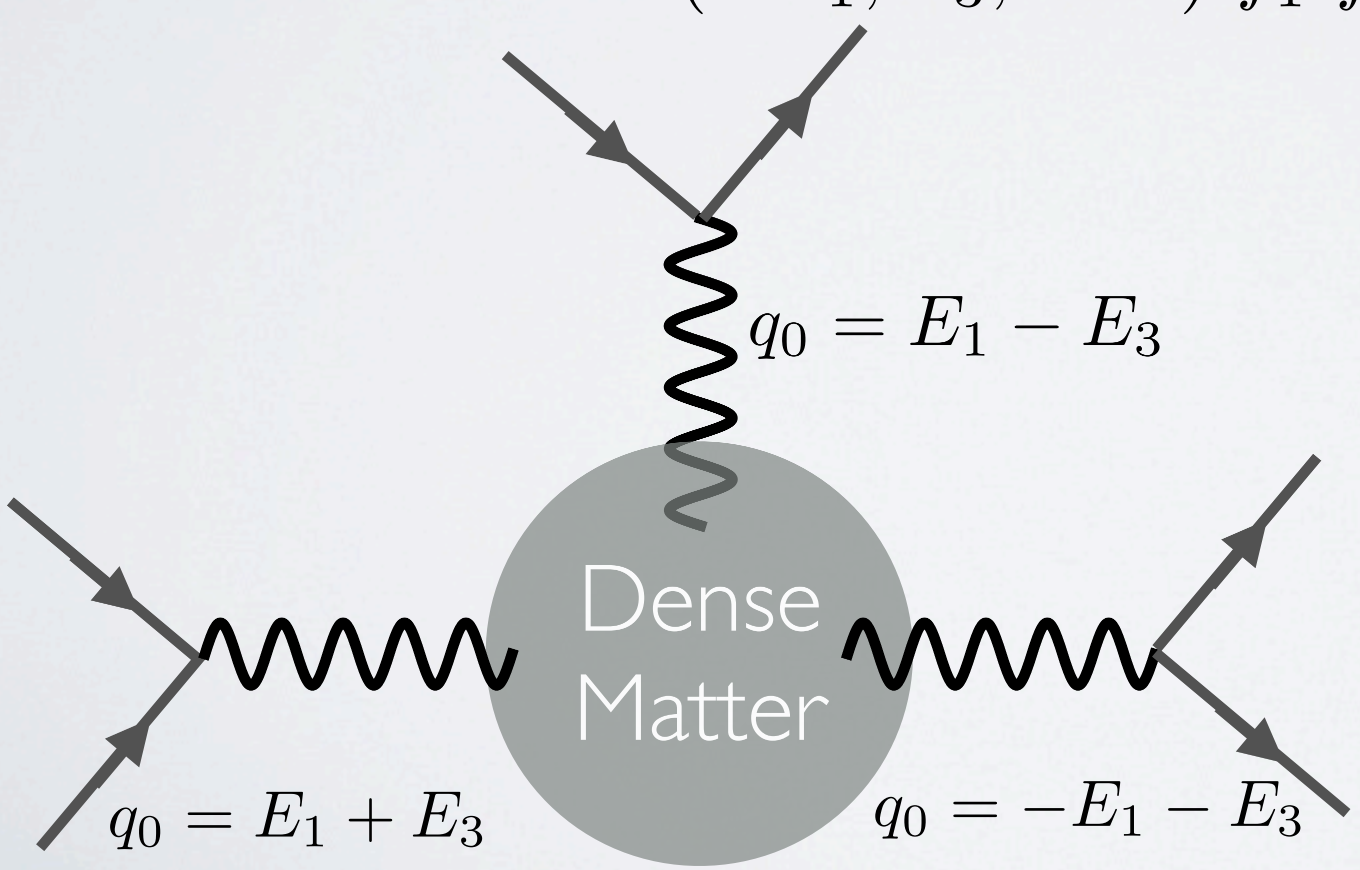
$$-R(E_3, E_1, \cos\theta) \; f_1(1-f_3) \longrightarrow \text{scattering-out}$$

$$+R(E_1, -E_3, \cos\theta) \; (1-f_1)(1-f_3) \longrightarrow \text{pair-production}$$

$$-R(-E_1, E_3, \cos\theta) \; f_1 \; f_3 \longrightarrow \text{pair-annihilation}$$



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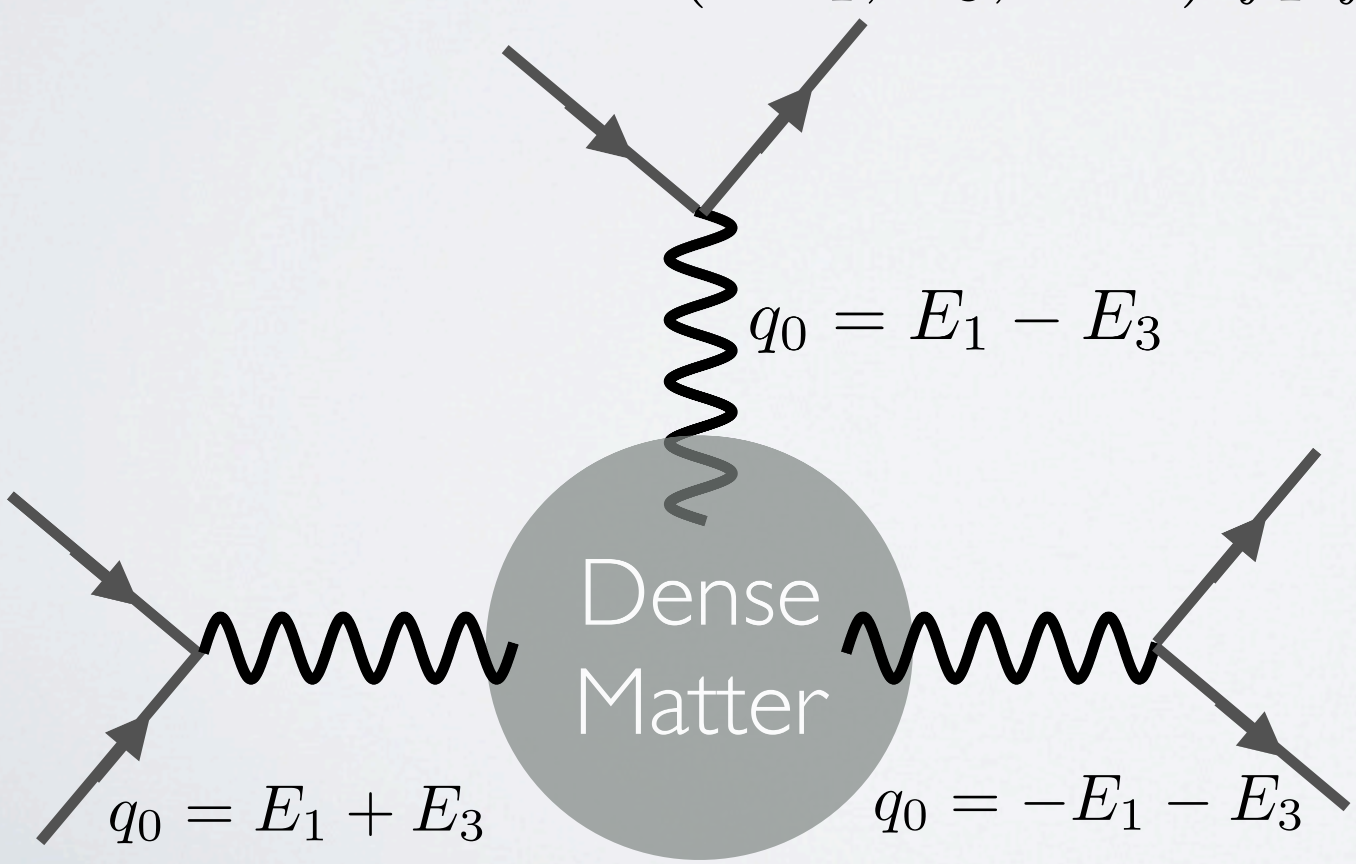
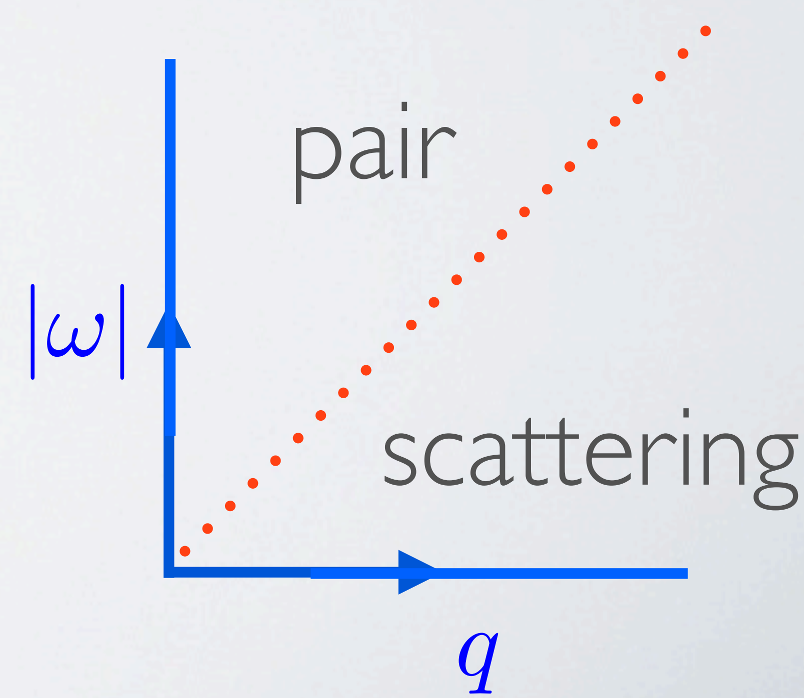
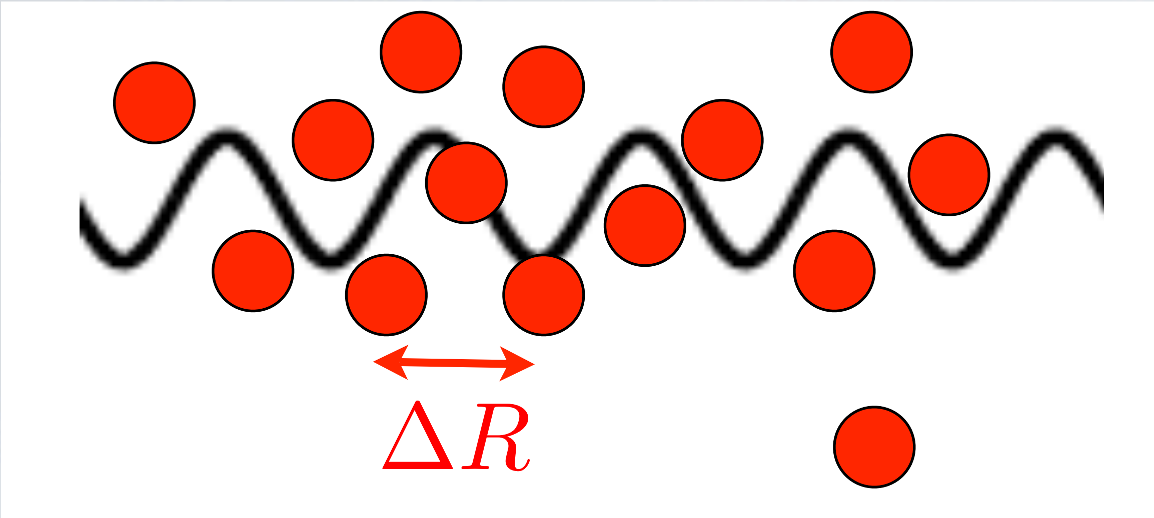




MANY-PARTICLE DYNAMICS



At small energy and momentum transfer, neutrinos cannot resolve a single nucleon.

- Neutrinos "see" more than one particle in the medium.
- Nature of spatial and temporal correlations between nuclei, nucleons and electrons affect the scattering rate.
- Nucleon dispersion relation is altered. Energy shifts and lifetimes play a role. are important.

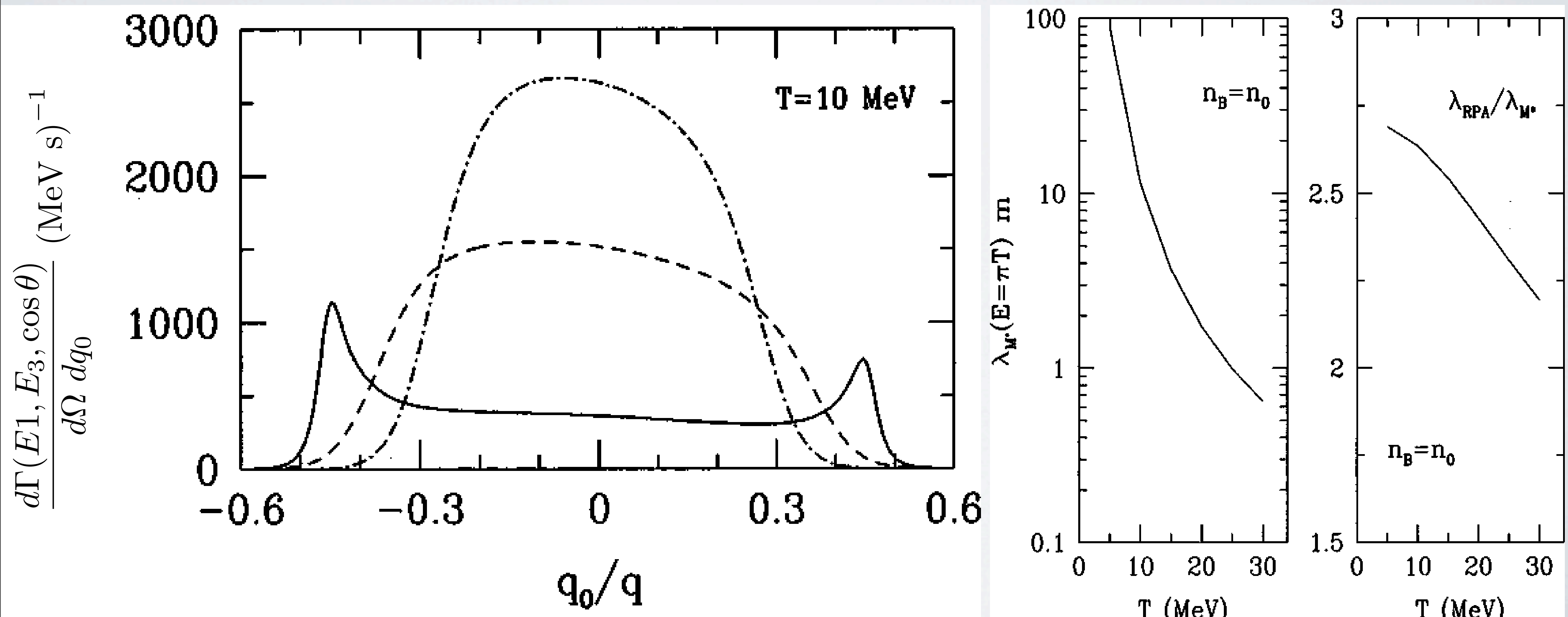
Sawyer (1975, 1989)
Iwamoto & Pethick (1982)
Horowitz & Wherberger (1991)
Raffelt & Seckel (1995)
Reddy, Prakash & Lattimer (1998, 1999)

NEUTRINO CROSS SECTIONS

Differential Scattering/Absorption Rate:

$$\frac{d\Gamma(E_1)}{d\cos\theta\ dq_0} = \frac{G_F^2}{4\pi^2}\ (E_1 - q_0)^2\ \left[(1+\cos\theta)\ S_{\rm V}^{\rm RPA}(q_0,q) + (3-\cos\theta)\ S_{\rm A}^{\rm RPA}(q_0,q)\right]$$

response function of the medium



SUMMARY

- Equation of state up to about 2 $\boldsymbol{\rho}_0$ is constrained by nuclear theory. Transition from soft to stiff is generic and is driven by the three nucleon interaction.

- Normal modes and transport properties in the crust are influenced by its solid and superfluid character. Entrainment and mixing are important. New longitudinal mode with small damping.

- Equations of state at finite T with neutron matter constraints are being developed. G. Shen et al, Steiner et al, Hempel et al. - Better suited for BNS mergers.

- Neutrino transport in the dense core is similar to that encountered in PNSs. Diffusion time scales of ~ 1 s. Nuclear correlations are important.