Machine Learning for Fluid Mechanics

(Video by Petros Vrellis)

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Machine Learning for Fluid Mechanics

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JC Loiseau

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Alan Kaptanoglu

Isabel Scherl

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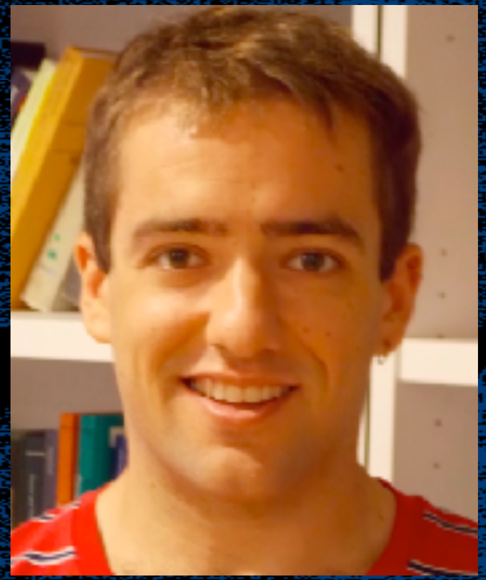

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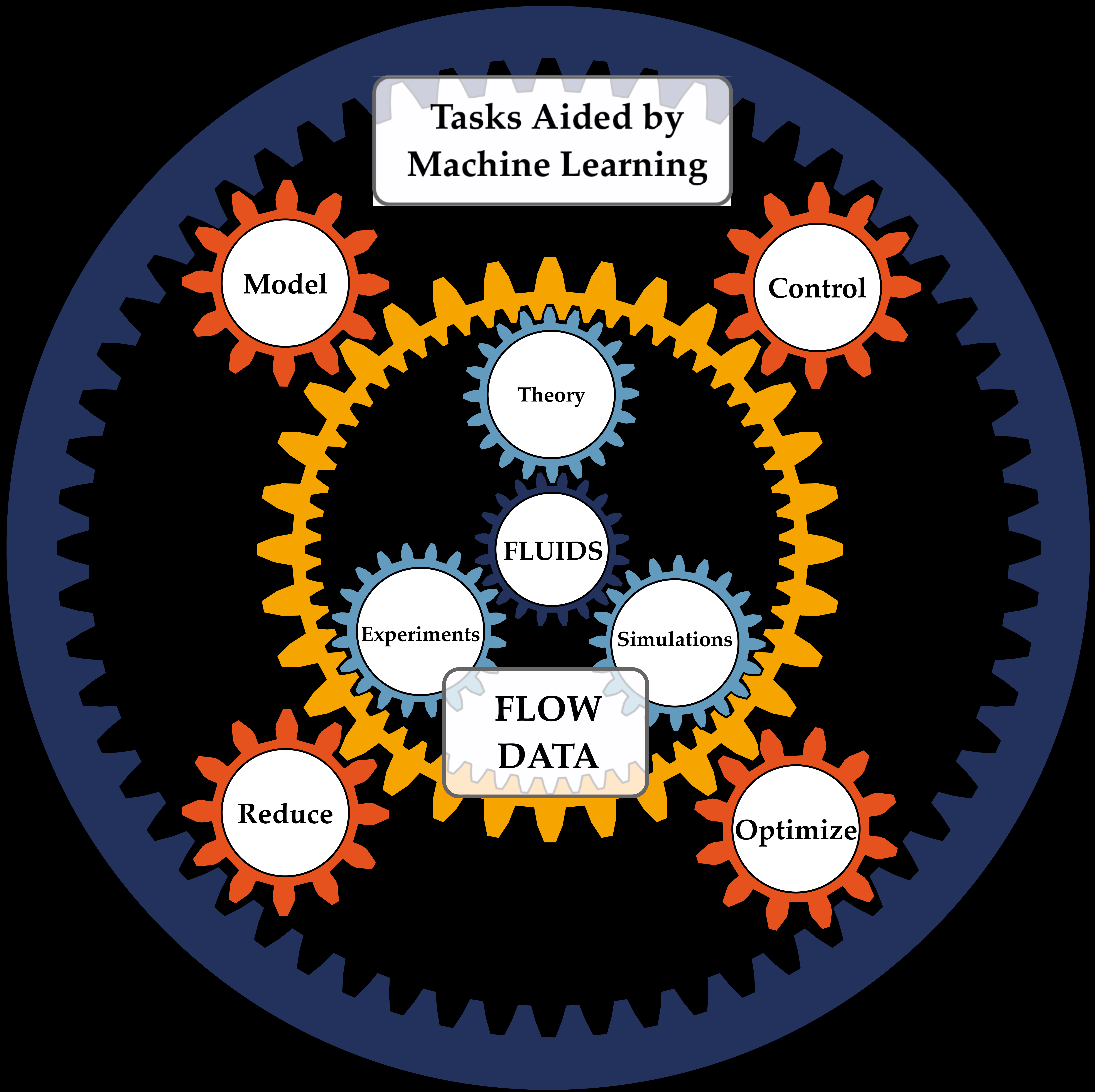
Benjamin Herrmann

Kathleen Champion



Machine Learning for Fluid Mechanics

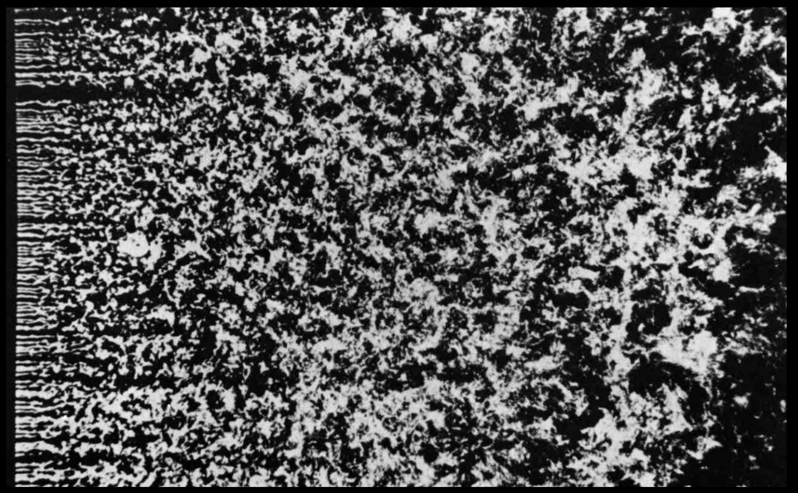
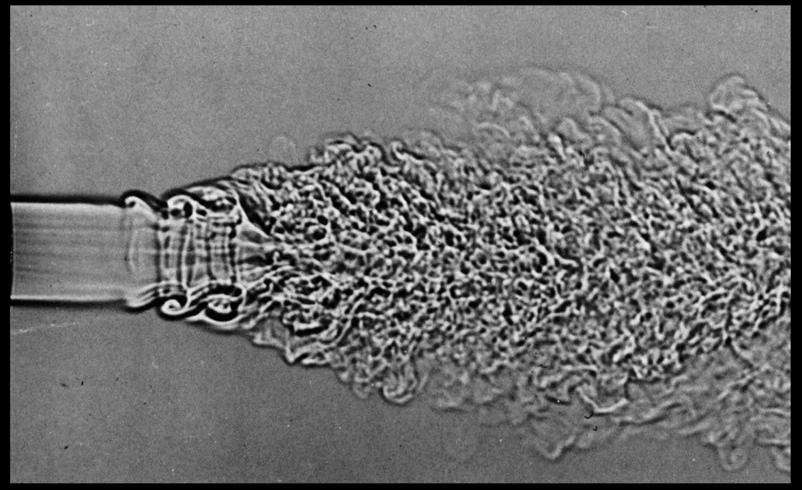
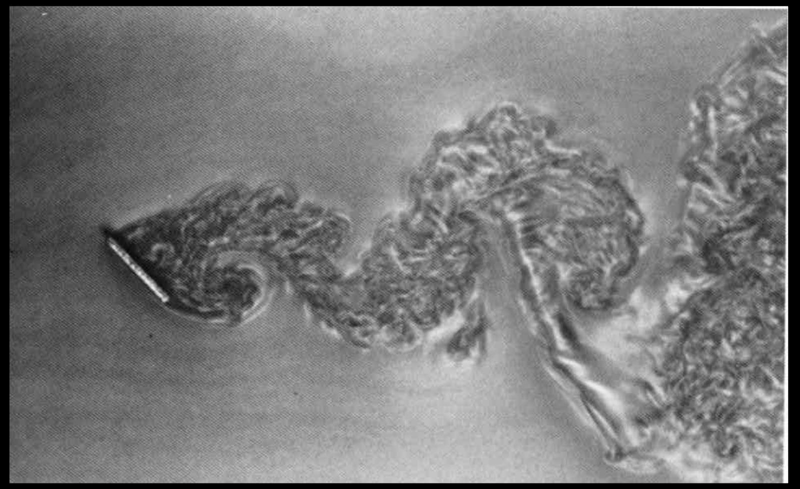
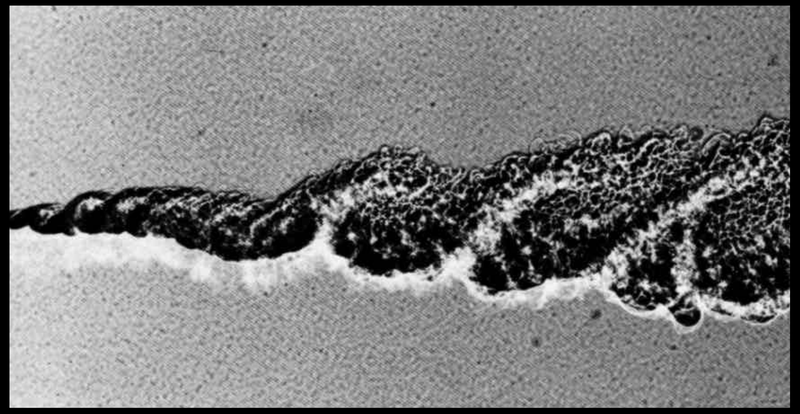
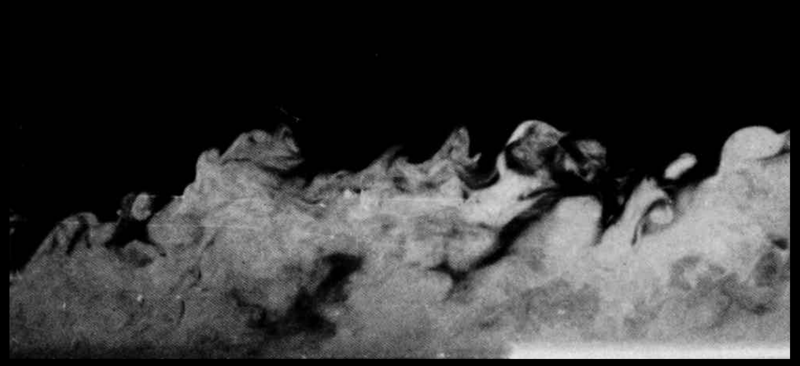
Steven L. Brunton,[1] Bernd R. Noack,[2] and Petros Koumoutsakos[3,4]

Keywords

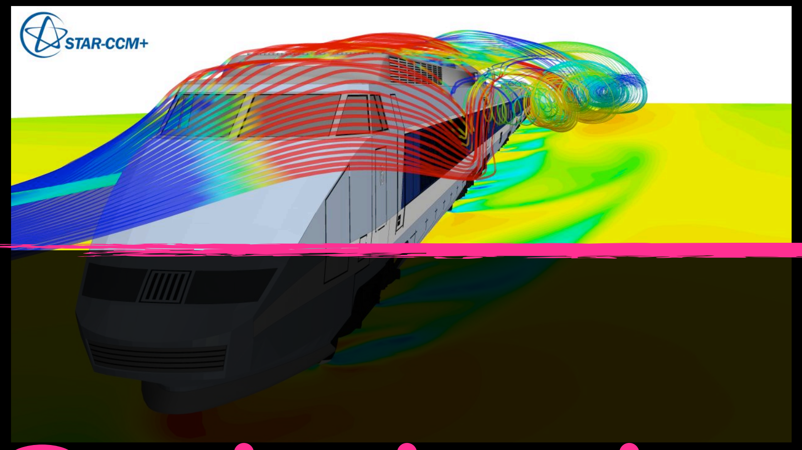
machine learning, data-driven modeling, optimization, control

Abstract

The field of fluid mechanics is rapidly advancing, driven by unprecedented volumes of data from experiments, field measurements, and large-scale simulations at multiple spatiotemporal scales. Machine learning presents us with a wealth of techniques to extract information from data that can be translated into knowledge about the underlying fluid mechanics. Moreover, machine learning algorithms can augment domain knowledge and automate tasks related to flow control and optimization. This article presents an overview of past history, current developments, and emerging opportunities of machine learning for fluid mechanics. We outline fundamental machine learning methodologies and discuss their uses for understanding, modeling, optimizing, and controlling fluid flows. The strengths and limitations of these methods are addressed from the perspective of scientific inquiry that links data with modeling, experiments, and simulations. Machine learning provides a powerful information processing framework that can augment, and possibly even transform, current lines of fluid mechanics research and industrial applications.

Machine Learning: Models from Data via Optimization

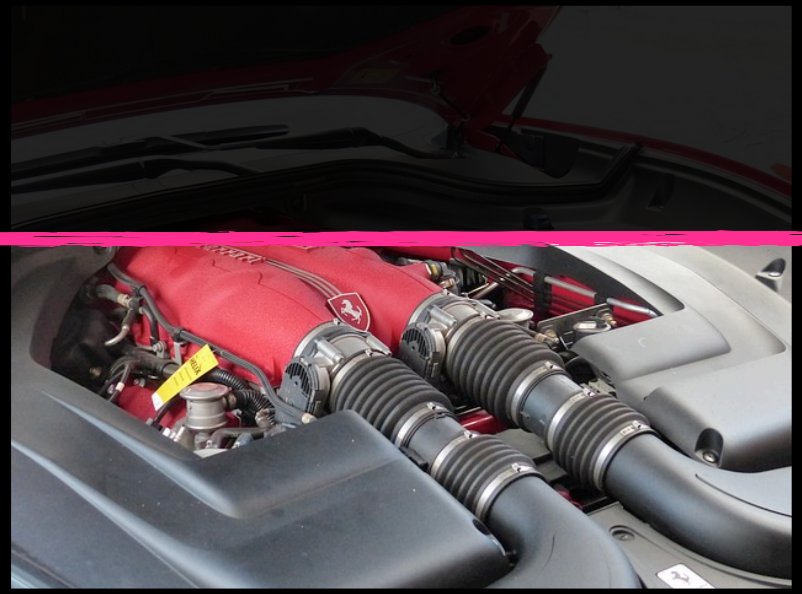


Optimization Problems:

- High-dimensional
- Nonlinear
- Non-convex
- Multiscale

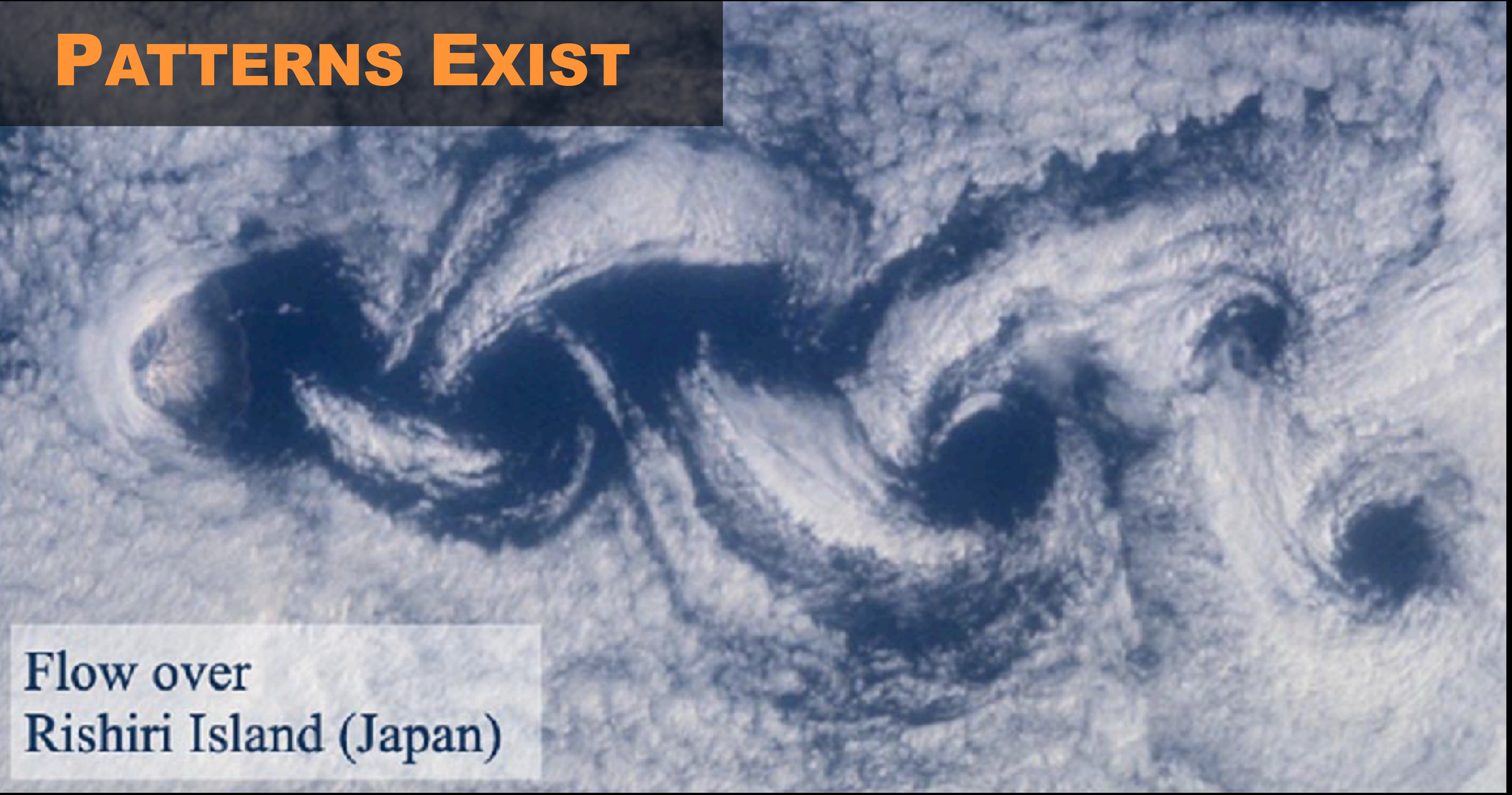
Fluid Dynamics Tasks:

- Dimension Reduction
- Reduced Order Modeling
- Sparse Sensing
- Control

Patterns Exist

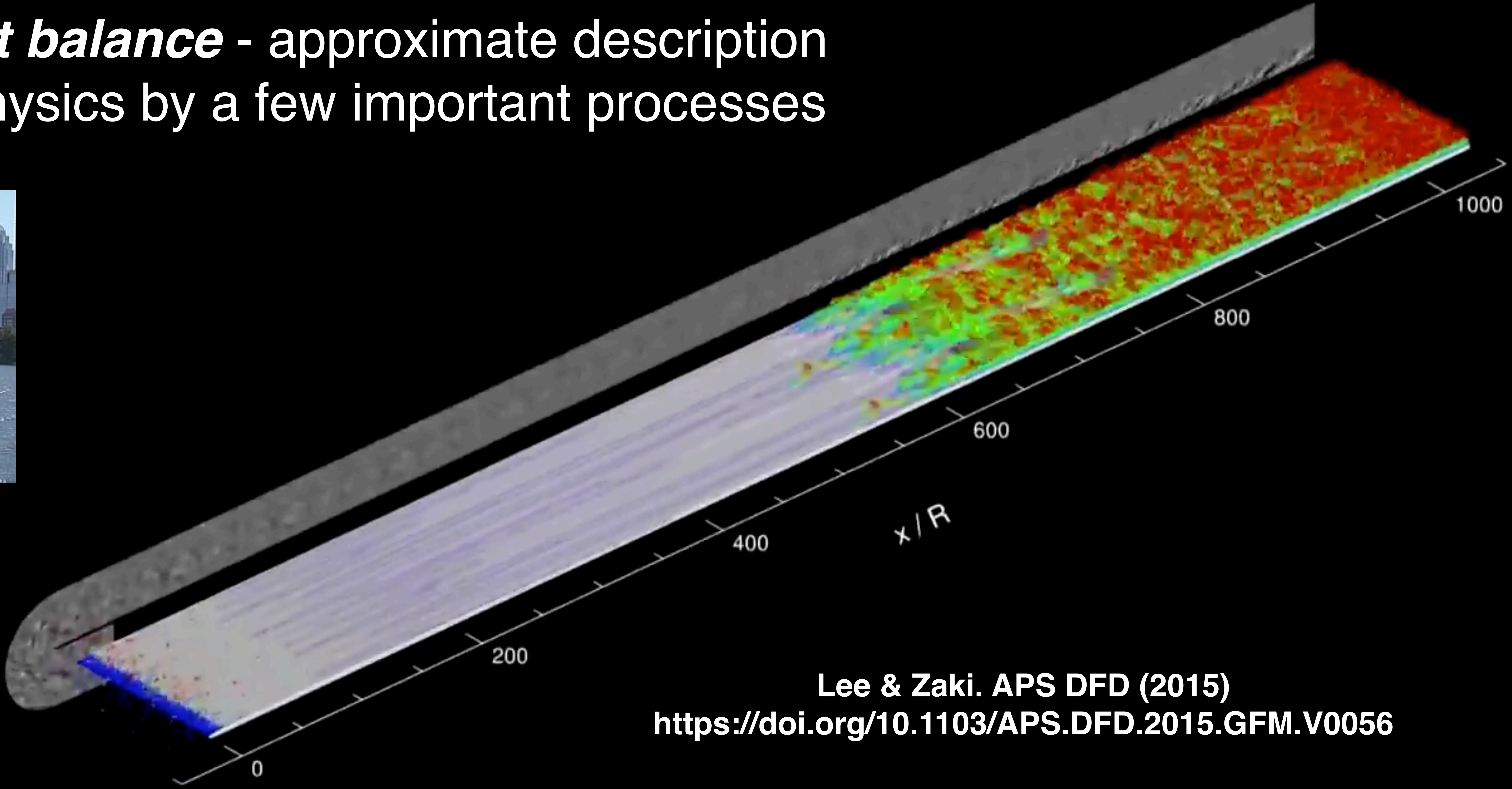
PATTERNS EXIST



Taira et al., AIAA J. 2017

DOMINANT BALANCE

Dominant balance - approximate description of local physics by a few important processes



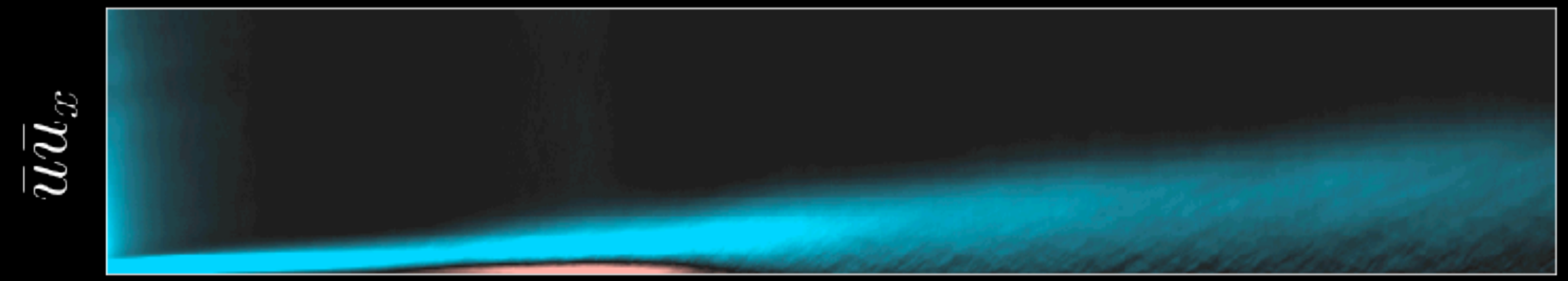
Lee & Zaki. APS DFD (2015)
https://doi.org/10.1103/APS.DFD.2015.GFM.V0056

Callaham, Koch, Brunton, Kutz, SLB, *Nat Comm 2021*

Reynolds-averaged Navier-Stokes equations

$$\bar{u}\bar{u}_x + \bar{v}\bar{u}_y + (\overline{u'^2})_x + (\overline{u'v'})_y = -\rho^{-1}\bar{p}_x + \nu\nabla^2\bar{u}$$

$$u(x,t) = \bar{u}(x) + u'(x,t)$$



Callaham, Koch, Brunton, Kutz, SLB, *Nat Comm 2021*

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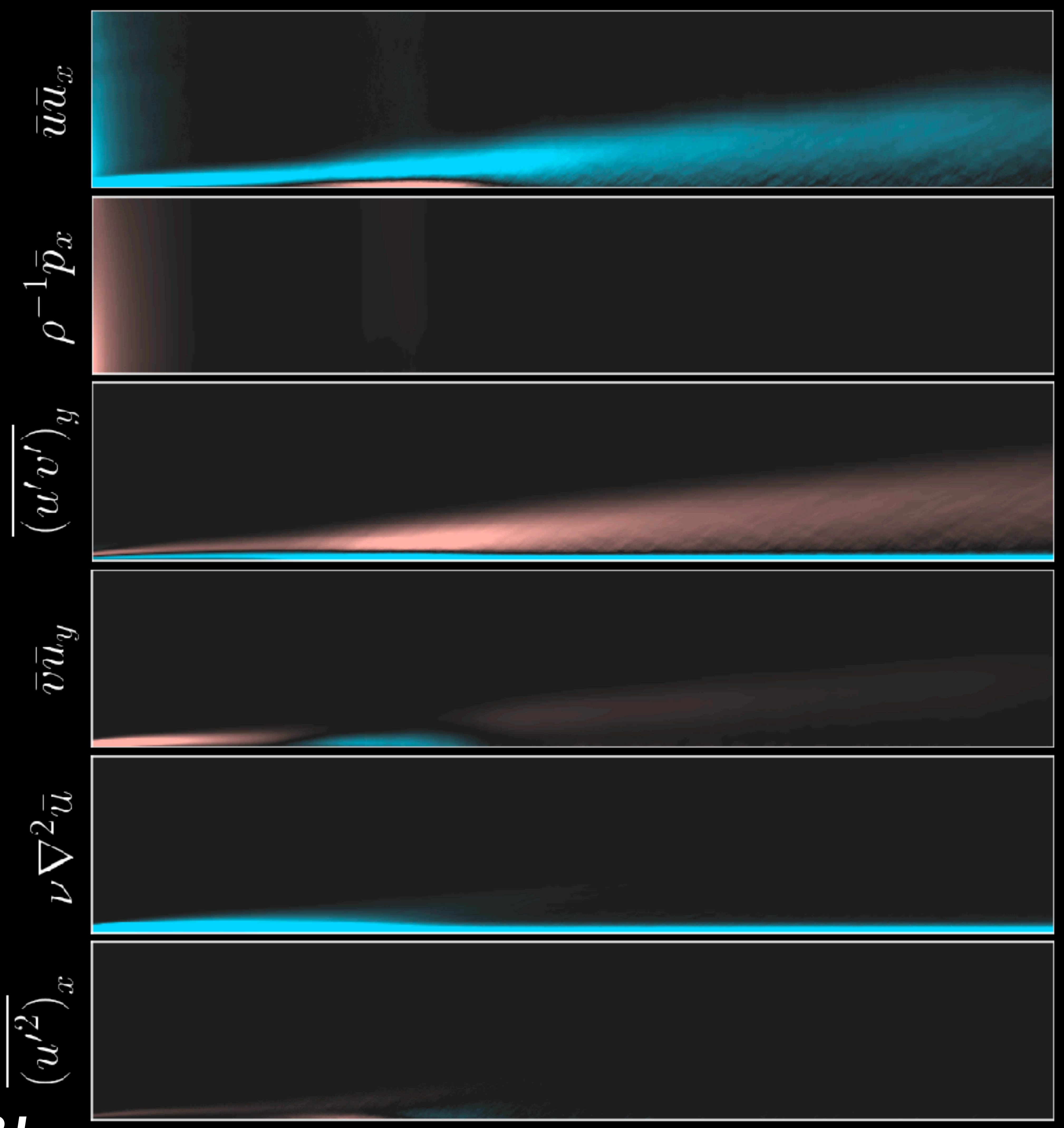
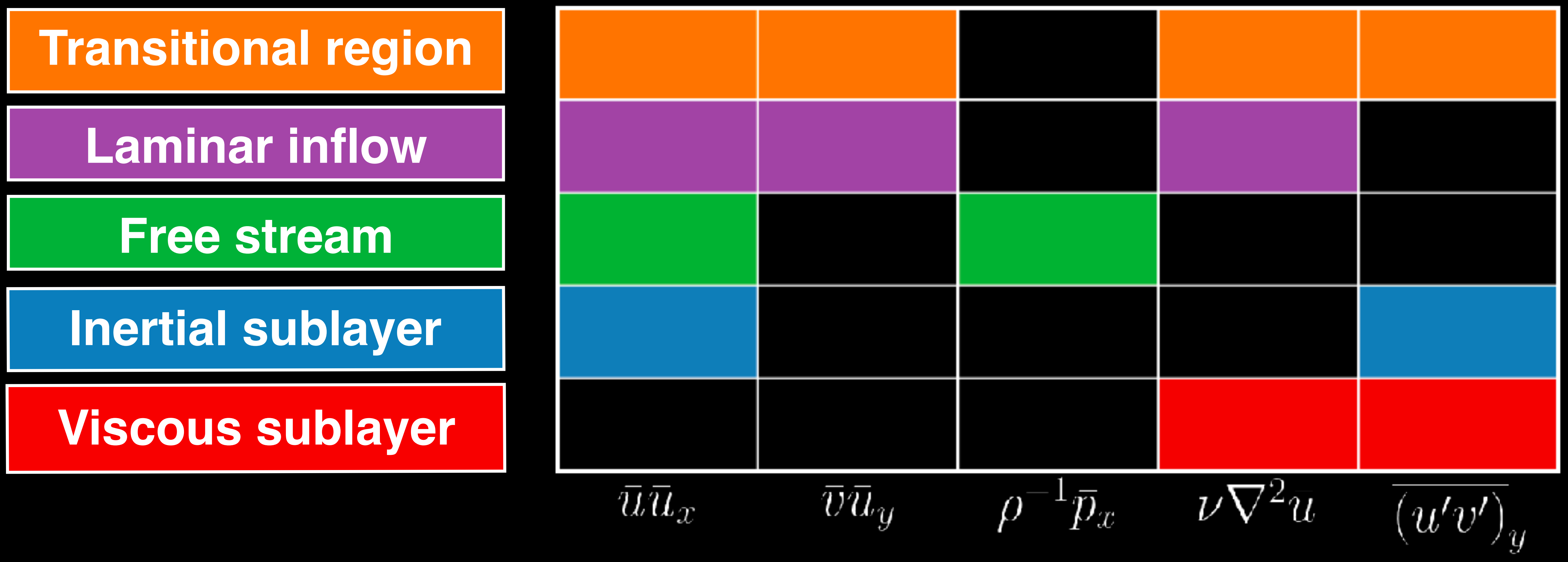
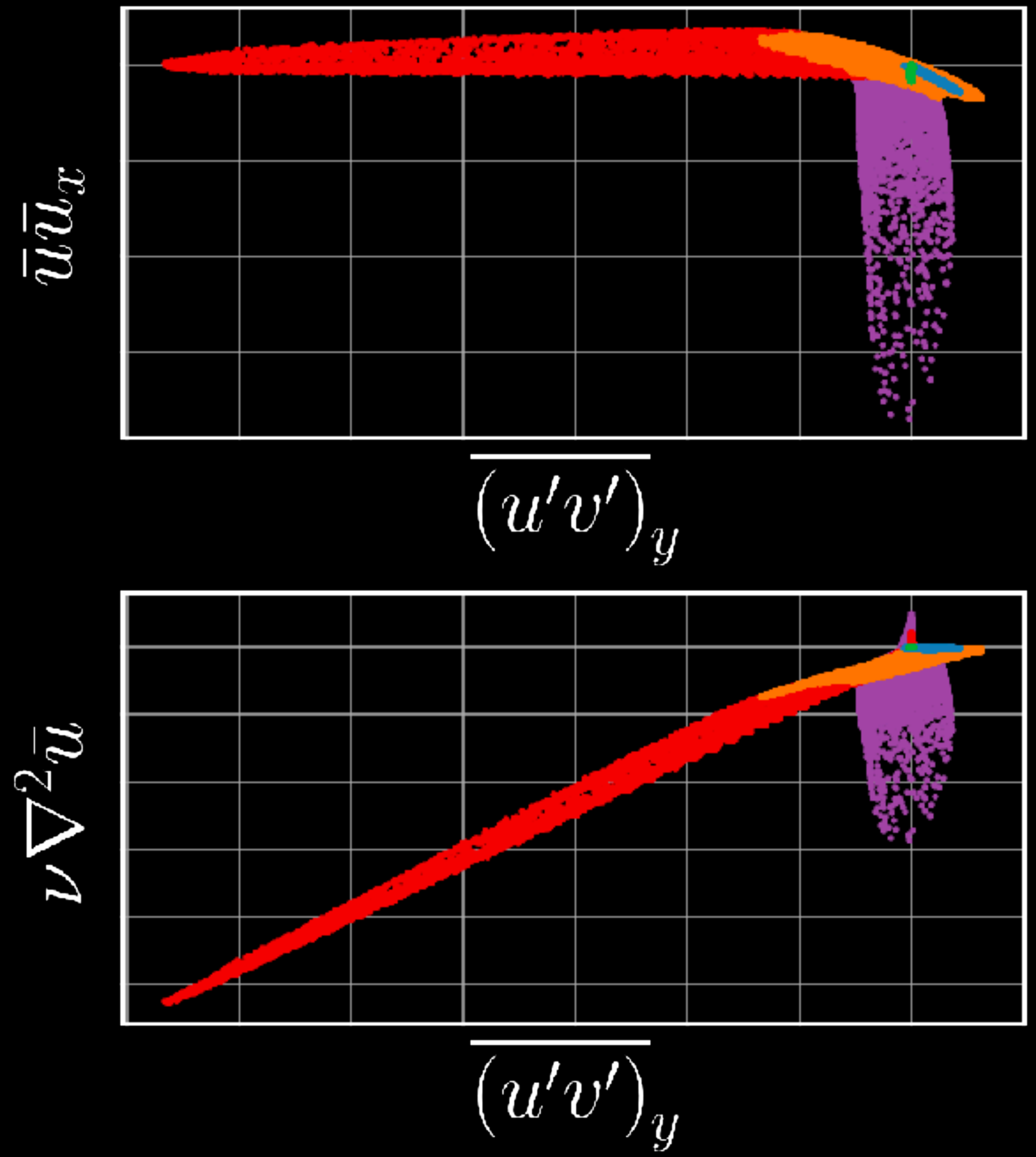


Callaham, Koch, Brunton, Kutz, SLB, *Nat Comm 2021*

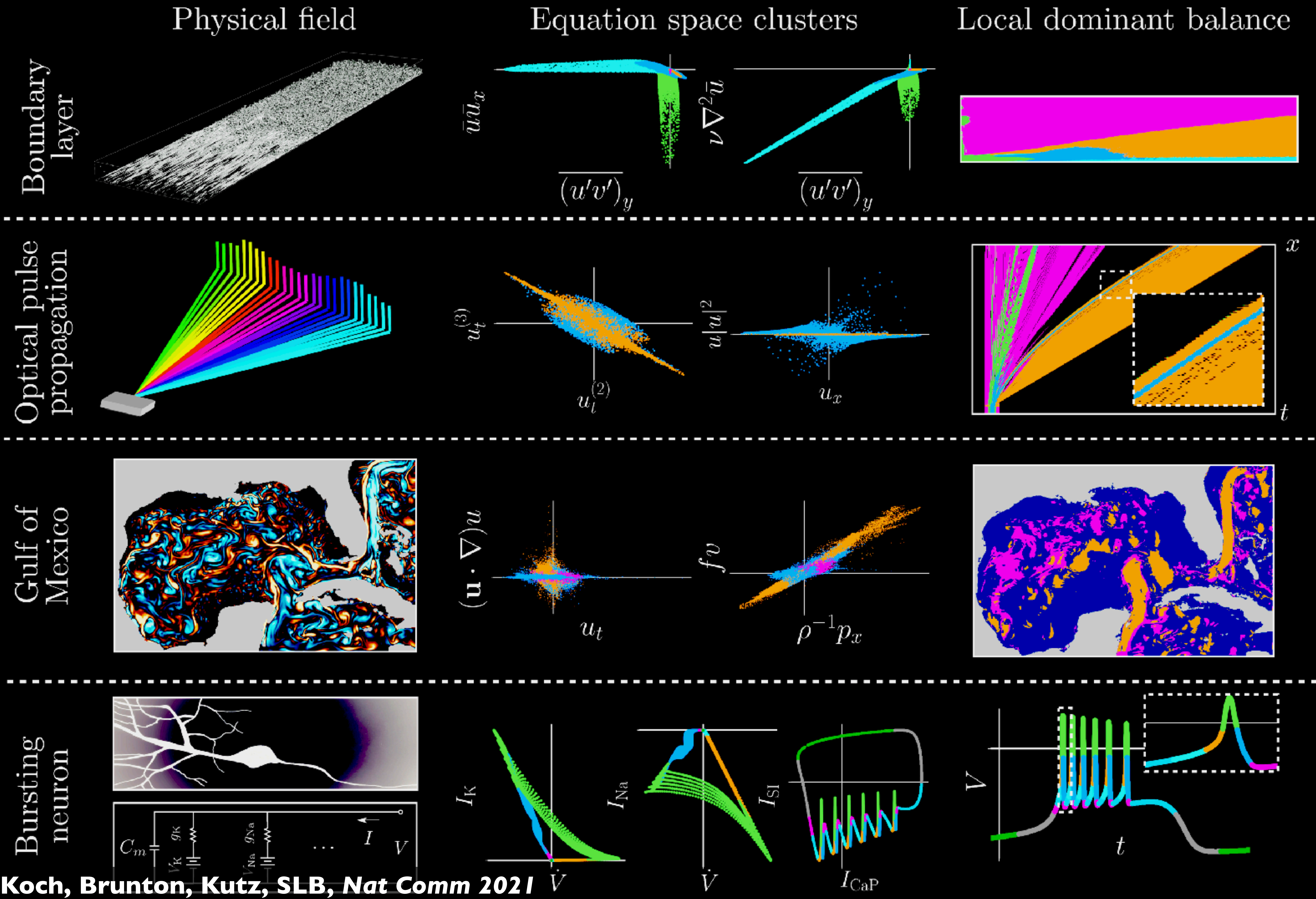
Equation space representation





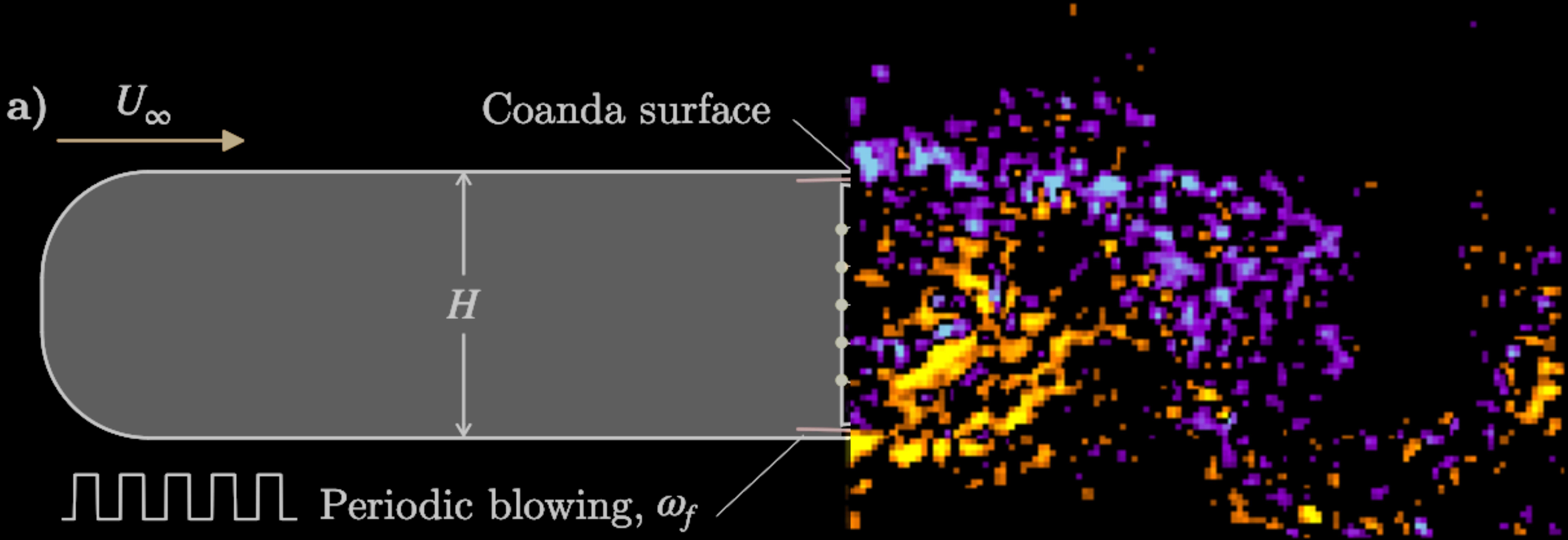
- Clustering (GMM)
- Subspace identification (Sparse PCA)
- Interpretable balance laws

Callaham, Koch, Brunton, Kutz, SLB, *Nat Comm 2021*



Callaham, Koch, Brunton, Kutz, SLB, *Nat Comm 2021*

Reduced Order Models



$$\frac{d}{dt}\mathbf{x} = \mathbf{f}(\mathbf{x})$$

There is a need for INTERPRETABLE and GENERALIZABLE Machine Learning

F = ma

There is a need for

INTERPRETABLE and GENERALIZABLE

Machine Learning

EVERYTHING SHOULD BE MADE

AS SIMPLE AS POSSIBLE,

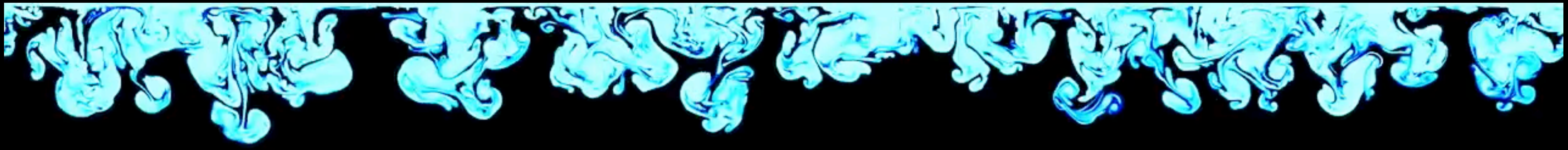
BUT NOT SIMPLER.

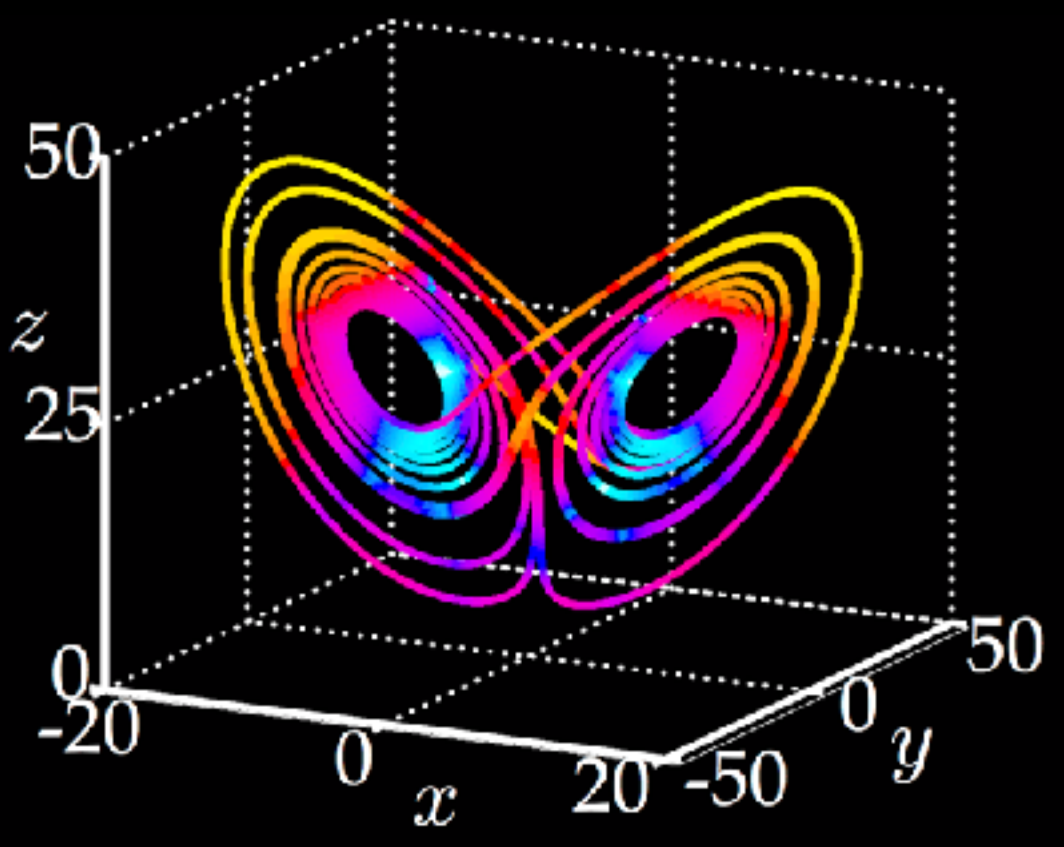
Albert Einstein

There is a need for INTERPRETABLE and GENERALIZABLE Machine Learning

- LOW-DIMENSIONAL
- SPARSE

Chaotic Thermal Convection



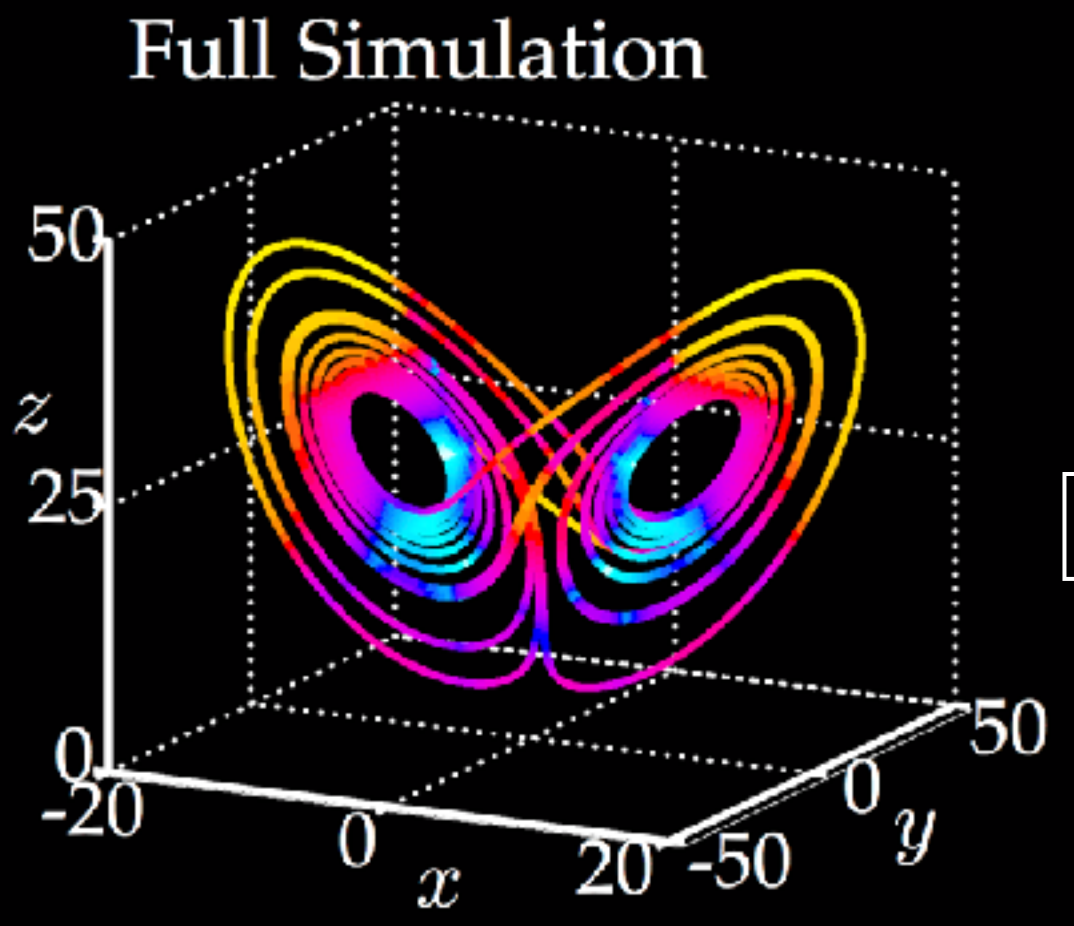


$$
\begin{aligned}
\dot{x} &= \sigma(y - x) \\
\dot{y} &= x(\rho - z) - y \\
\dot{z} &= xy - \beta z.
\end{aligned}
$$

YouTube: turbulenceteam

Lorenz, Fetter, Hamilton, 1963

Sparse Identification of Nonlinear Dynamics (SINDy)



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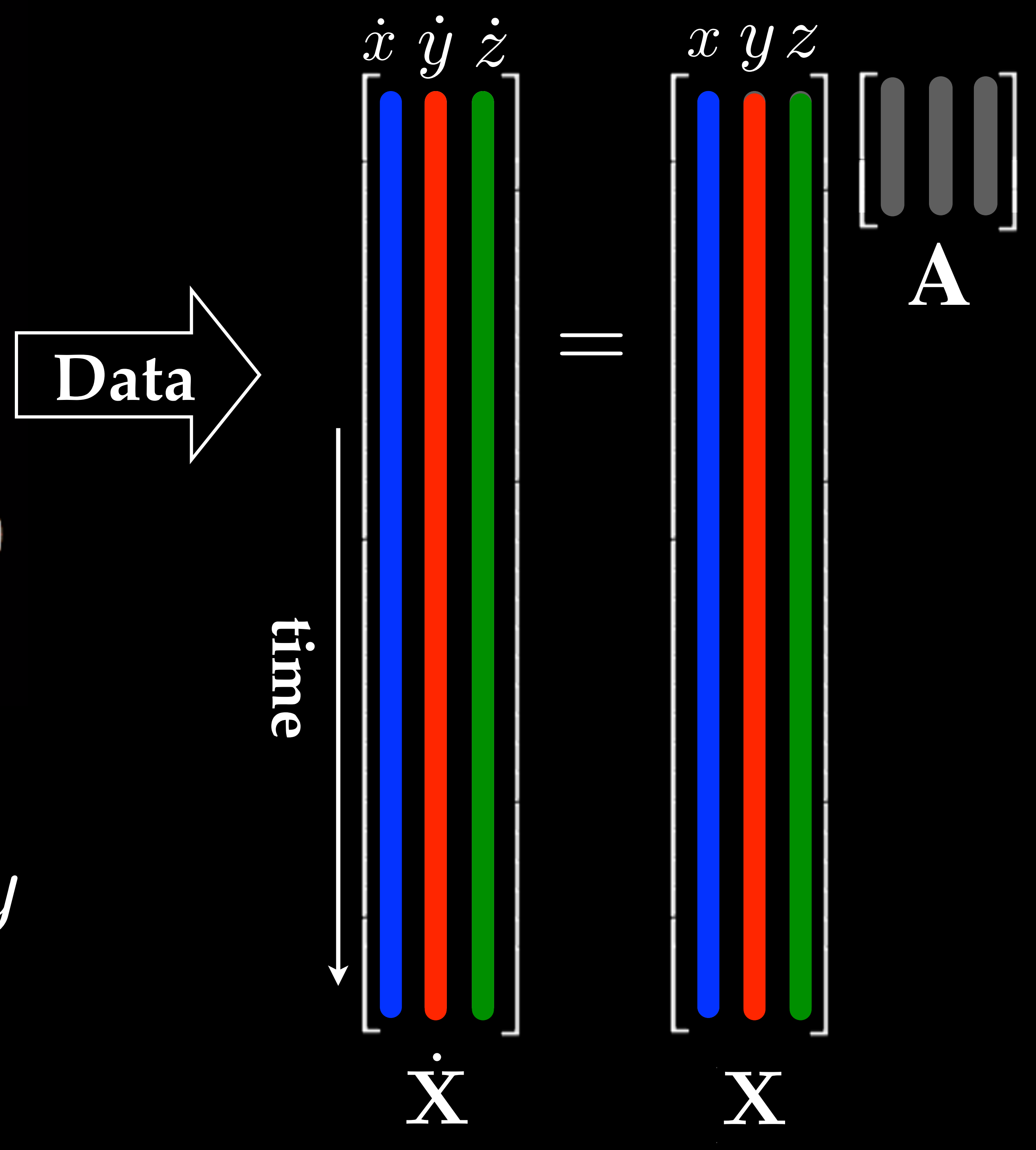
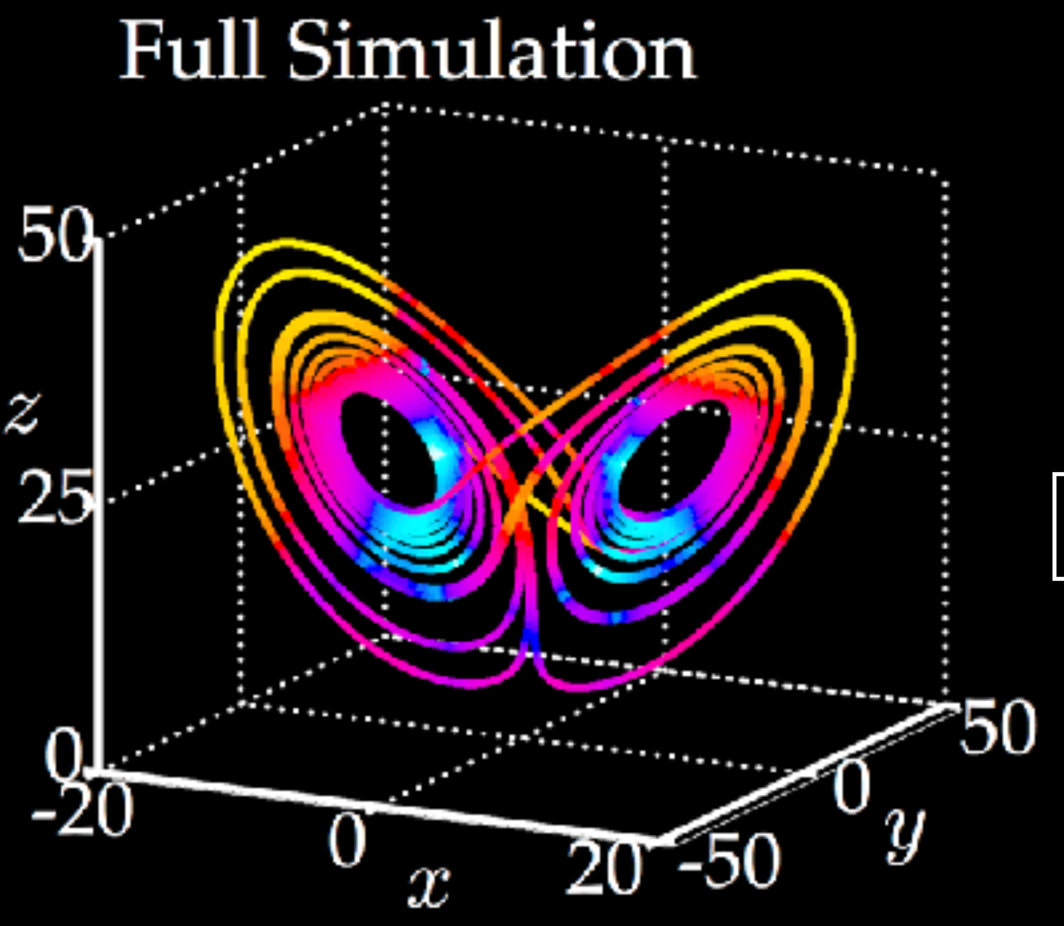


DMD

SLB, Proctor, Kutz, *PNAS* 2016.

Sparse Identification of Nonlinear Dynamics (SINDy)



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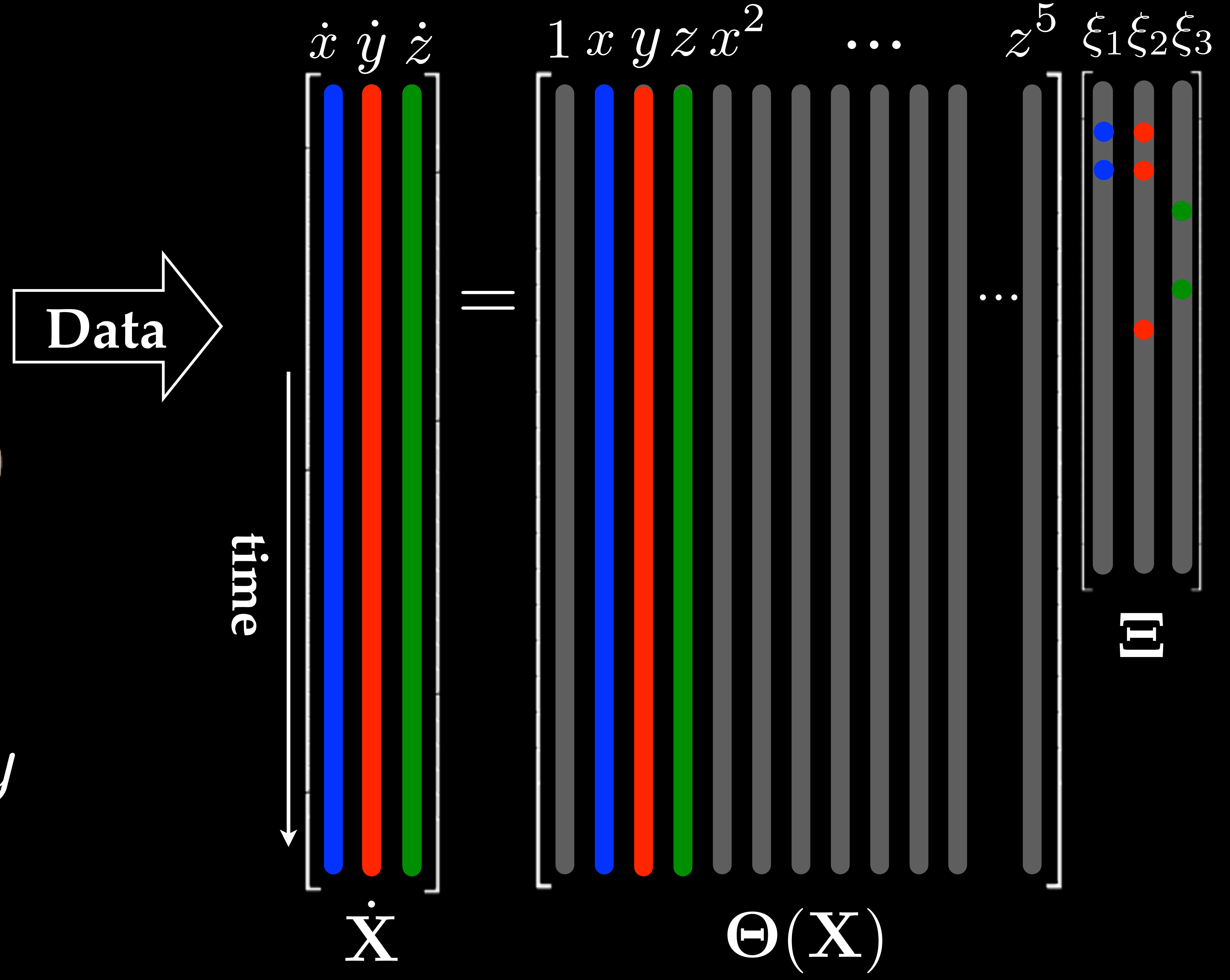

SLB, Proctor, Kutz, *PNAS* 2016.

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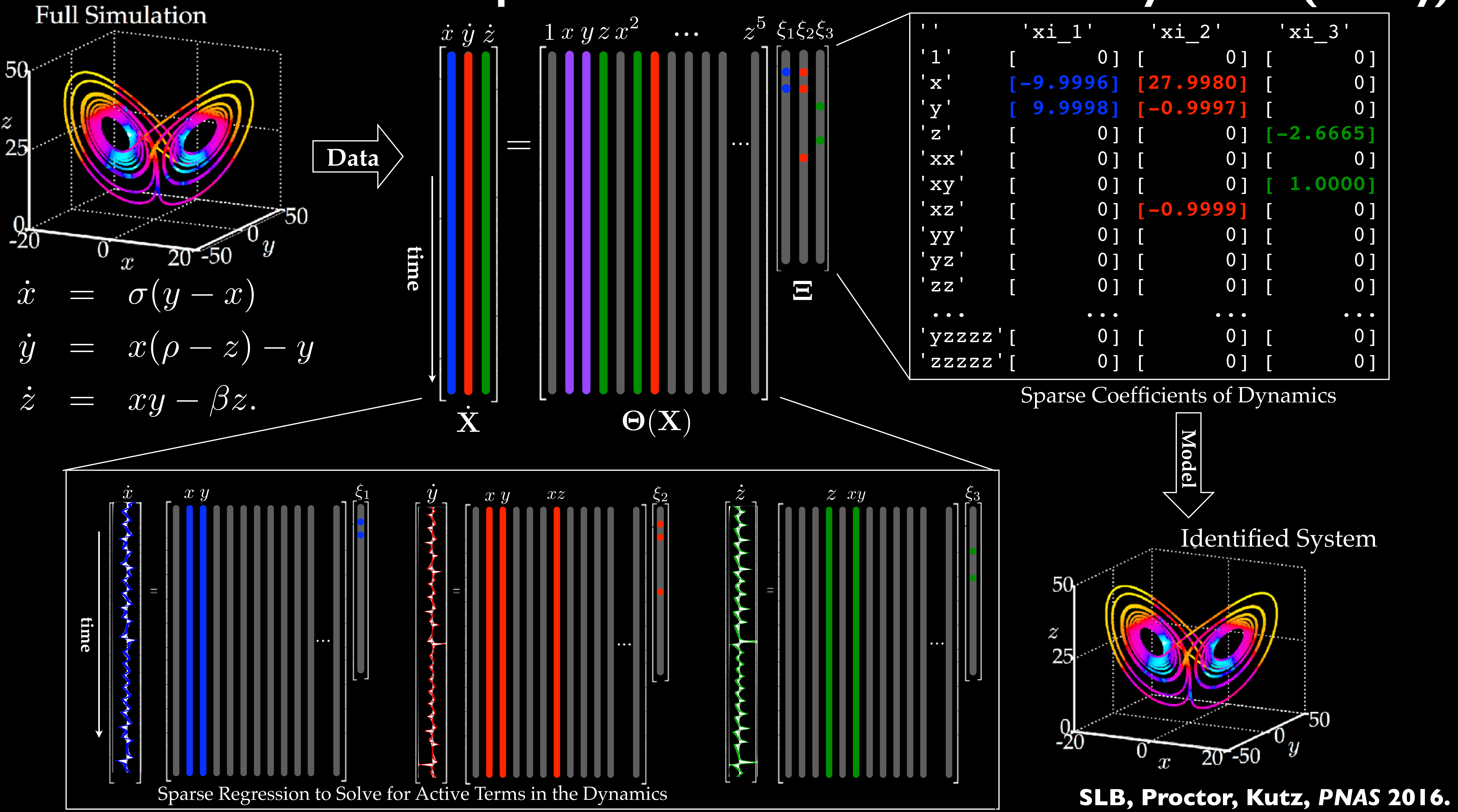
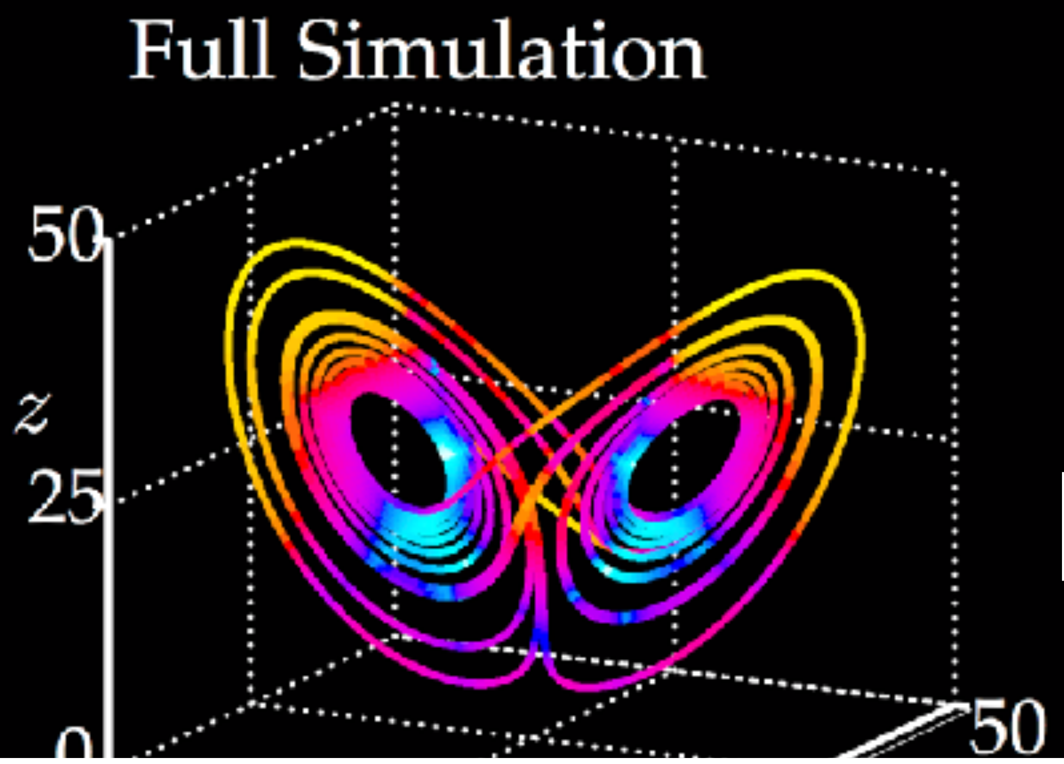
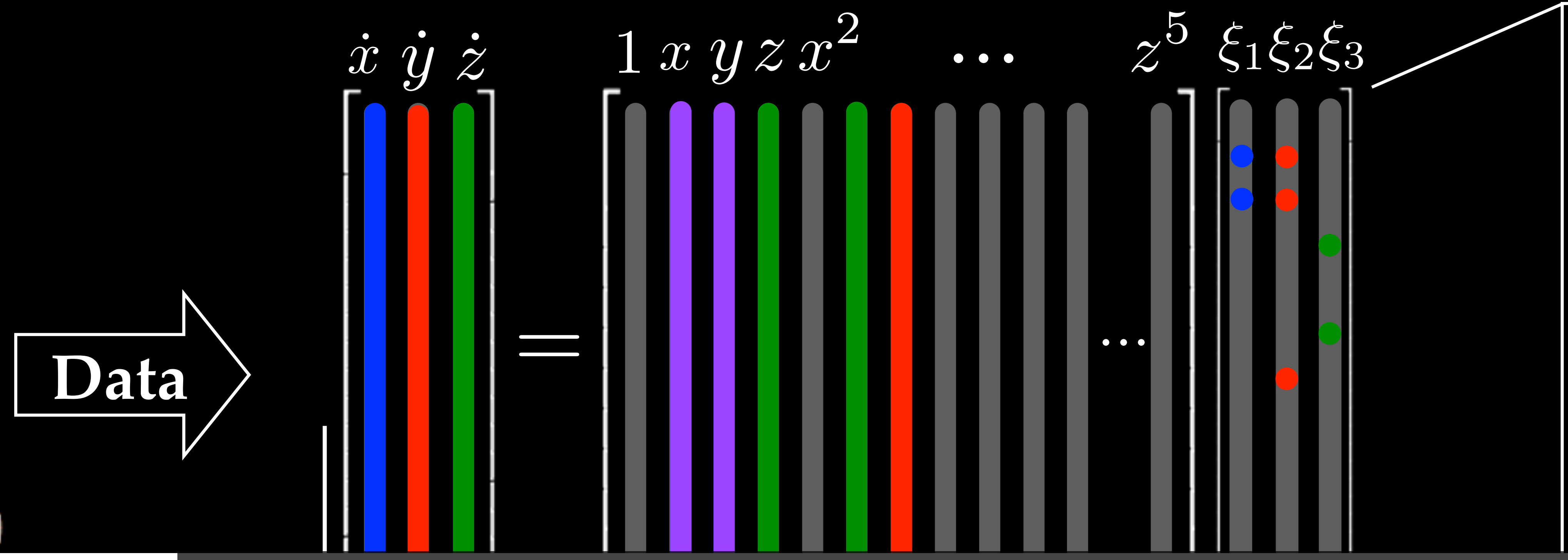


SLB, Proctor, Kutz, *PNAS* 2016.

Sparse Identification of Nonlinear Dynamics (SINDy)





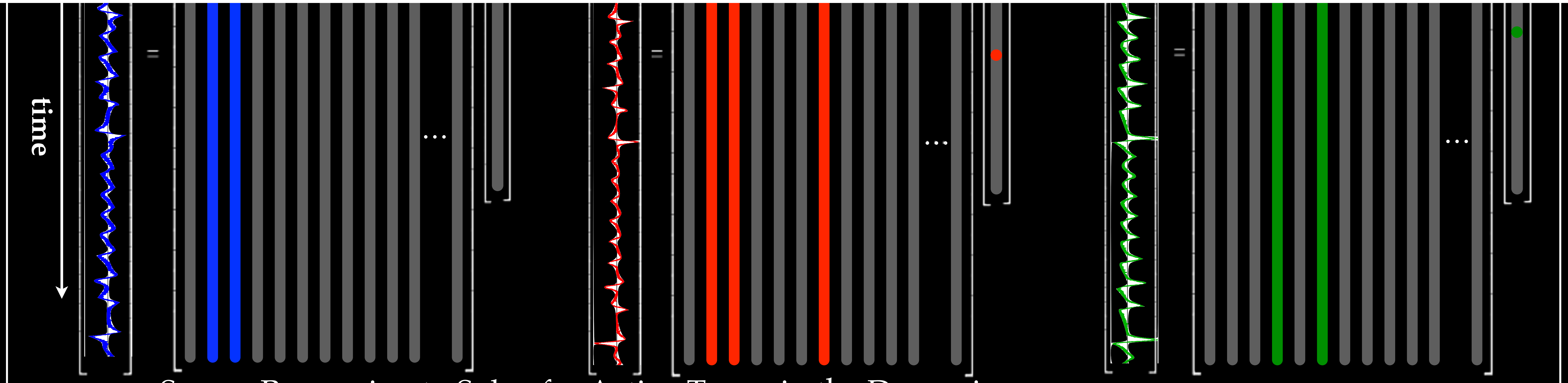
''	'xi_1'	'xi_2'	'xi_3'
'1'	[0]	[0]	[0]
'x'	[-9.9996]	[27.9980]	[0]
'y'	[9.9998]	[-0.9997]	[0]
'z'	[0]	[0]	[-2.6665]
'xx'	[0]	[0]	[0]
'xy'	[0]	[0]	[1.0000]
'xz'	[0]	[-0.9999]	[0]

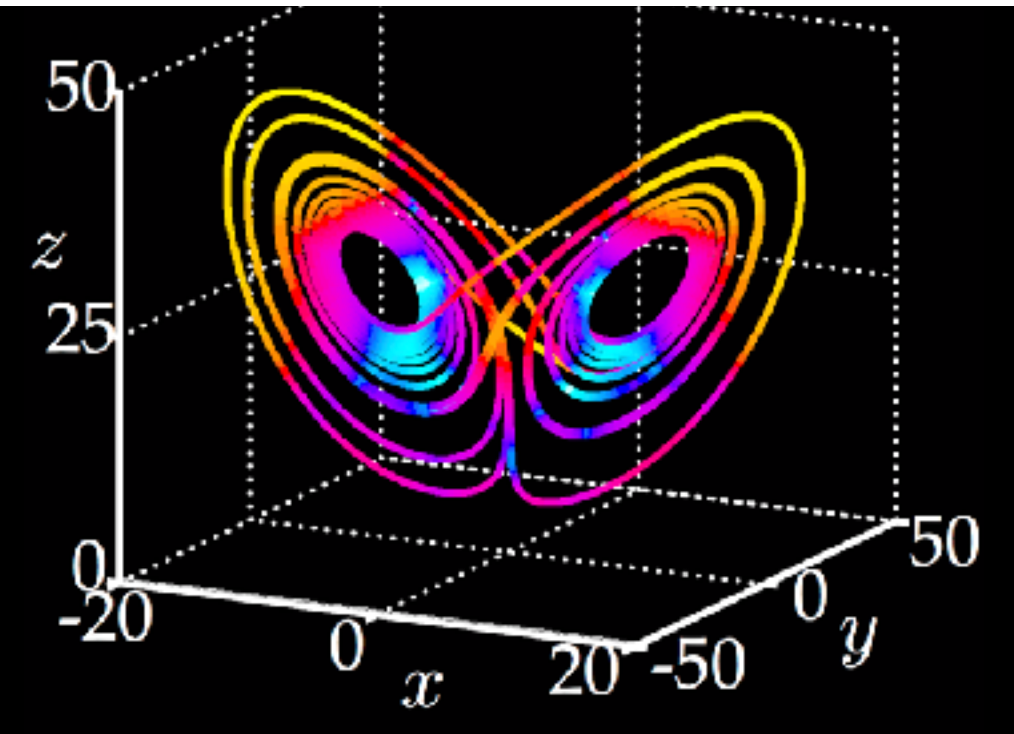
PySINDy

Open-Source Software

Build CI passing | docs passing | pypi package 1.0.0 | codecov 95% | JOSS 10.21105/joss.02104 | DOI 10.5281/zenodo.3832319

PySINDy is a sparse regression package with several implementations for the Sparse Identification of Nonlinear Dynamical systems (SINDy) method introduced in Brunton et al. (2016a), including the unified optimization approach of Champion et al. (2019) and SINDy with control from Brunton et al. (2016b). A comprehensive literature review is given in de Silva et al. (2020).



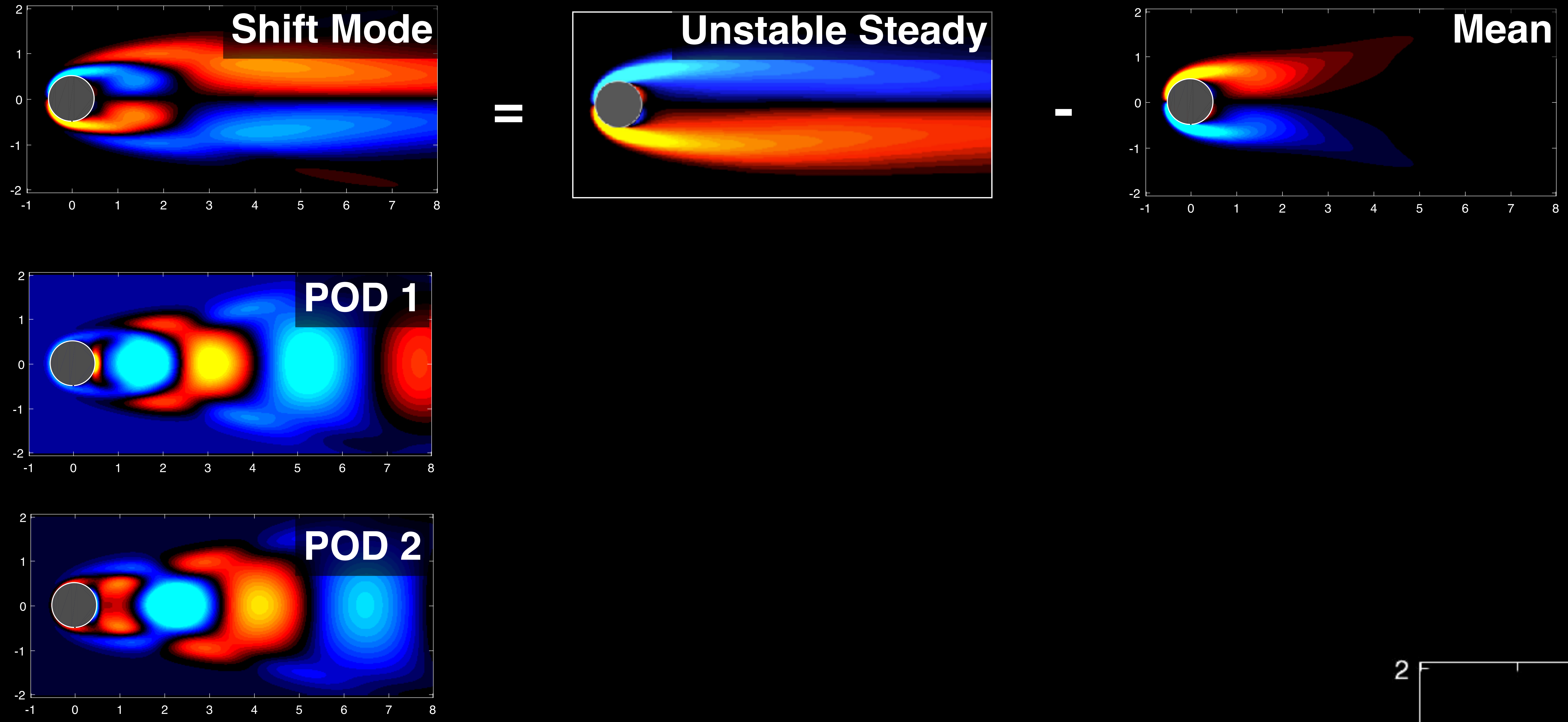


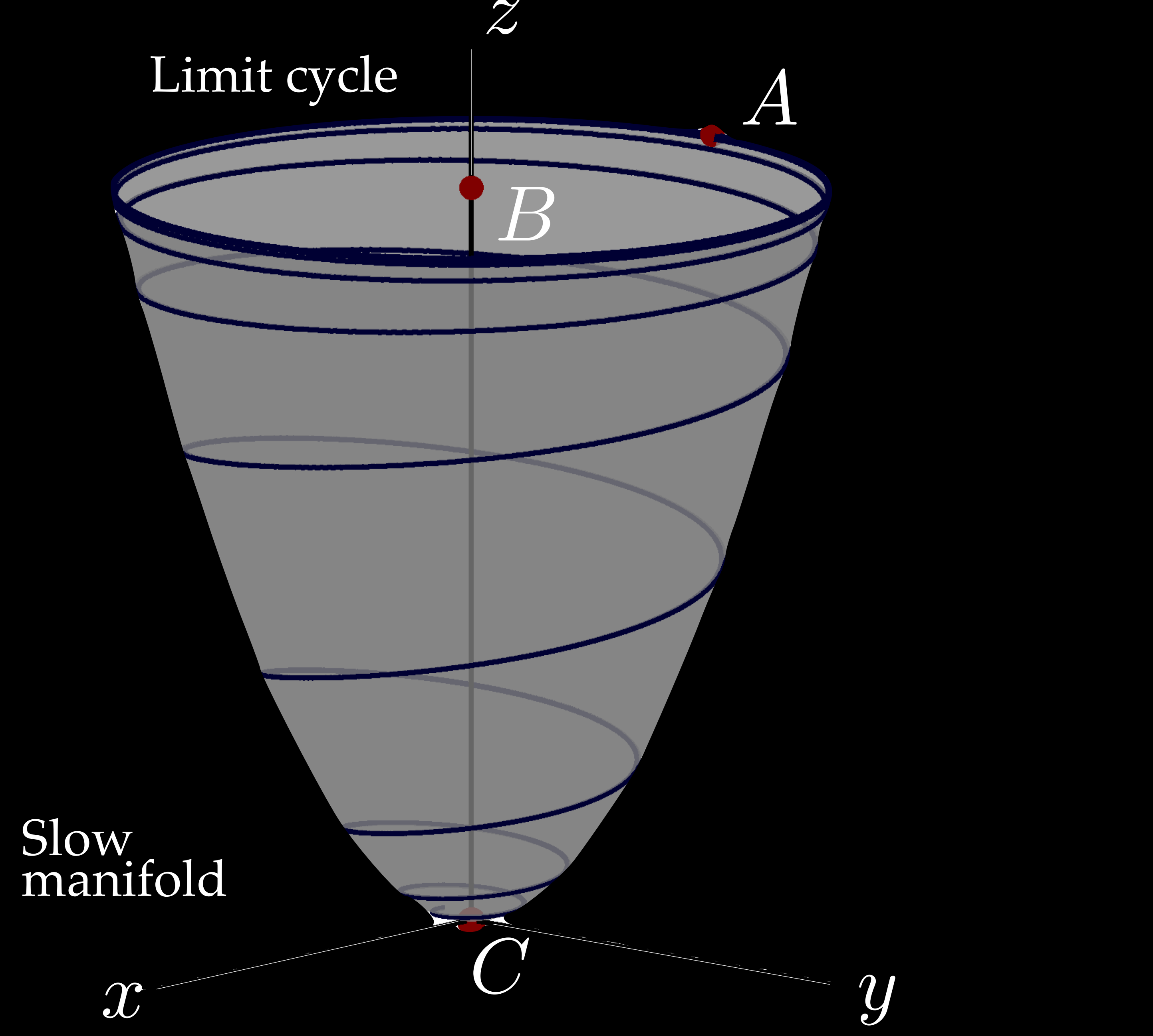
SLB, Proctor, Kutz, *PNAS* 2016.

PDEs

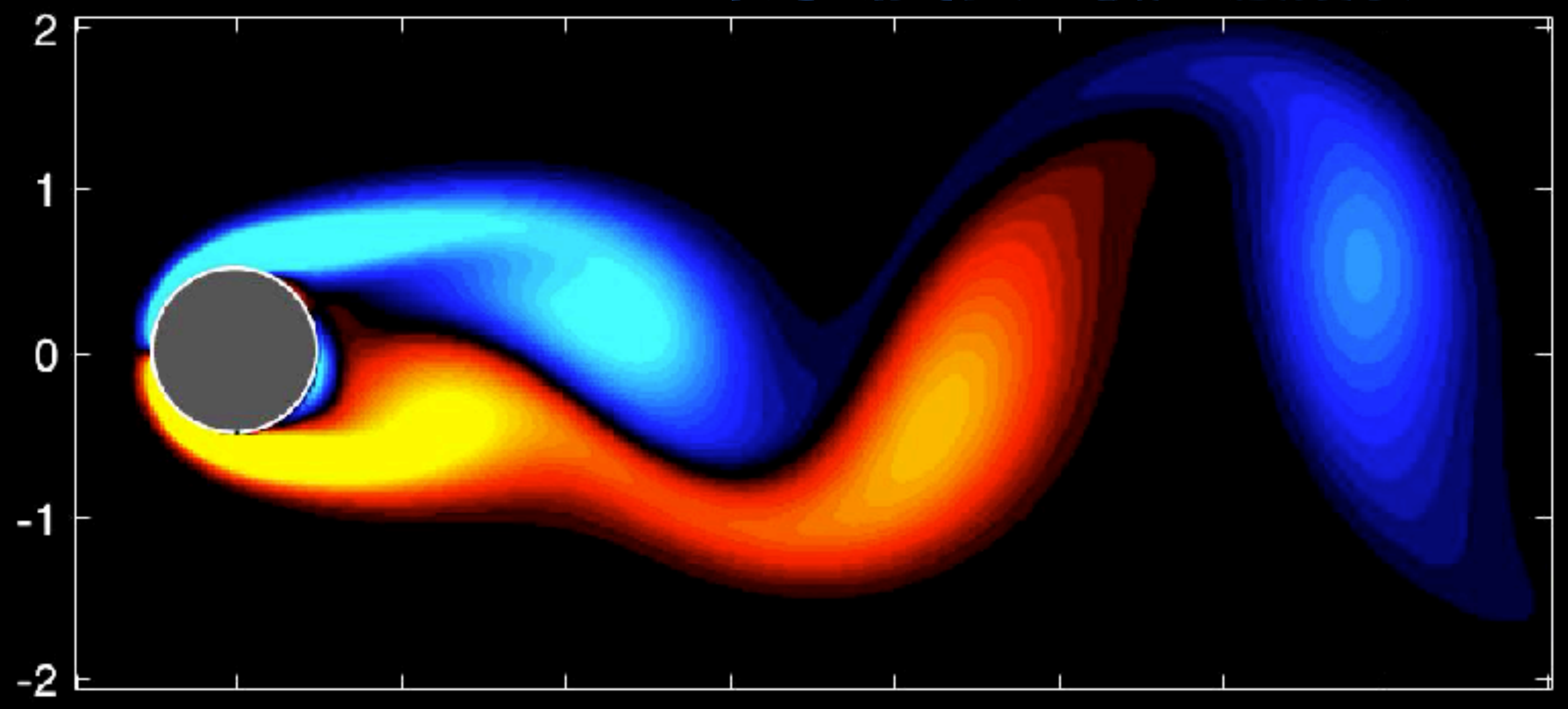
Rudy, SLB, Proctor, Kutz
Science Advances, 2017

REDUCED ORDER MODELS





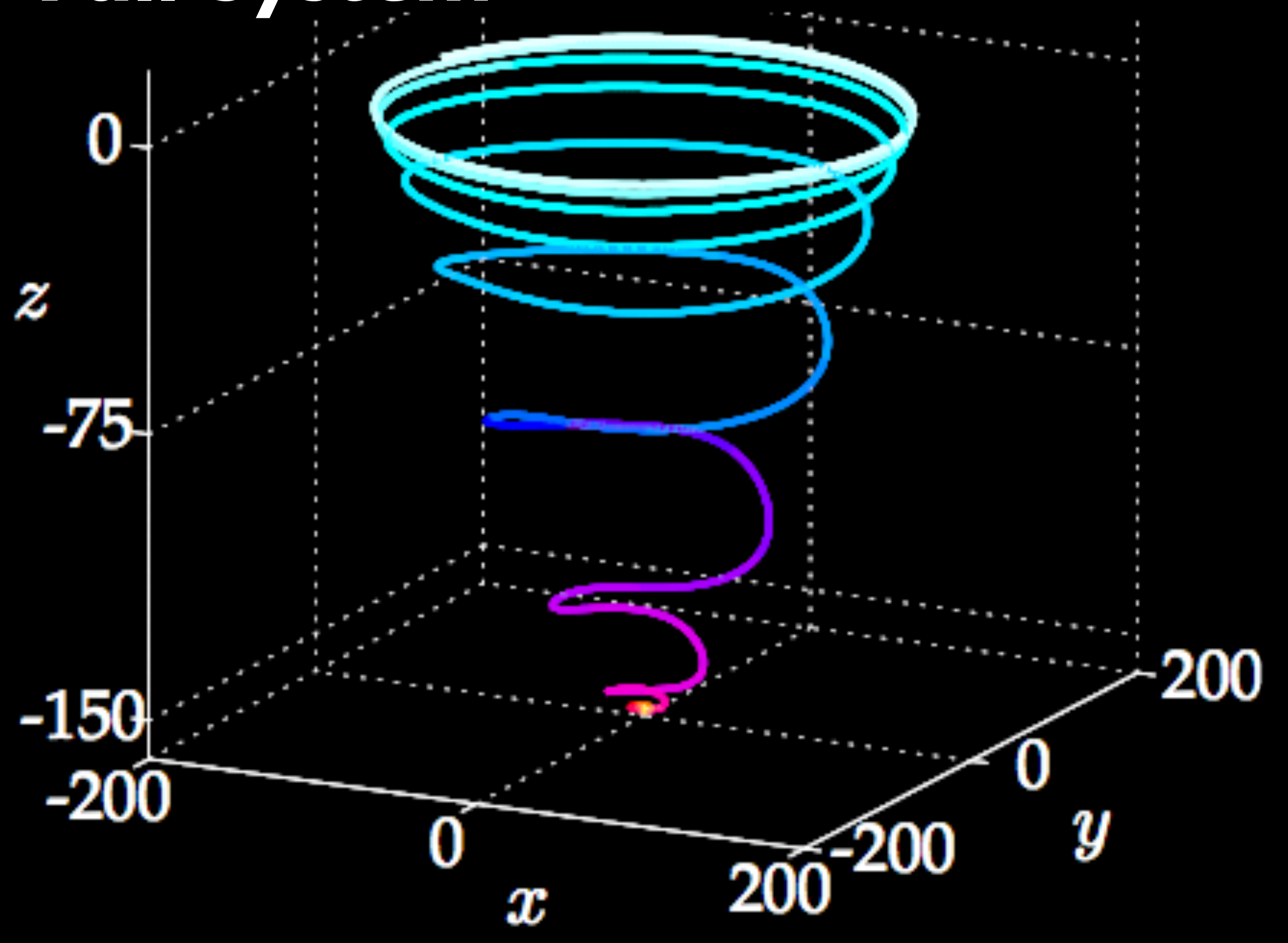
$$\begin{aligned}\dot{x} &= \mu x - \omega y + Axz\\ \dot{y} &= \omega x + \mu y + Ayz\\ \dot{z} &= -\lambda(z - x^2 - y^2).\end{aligned}$$



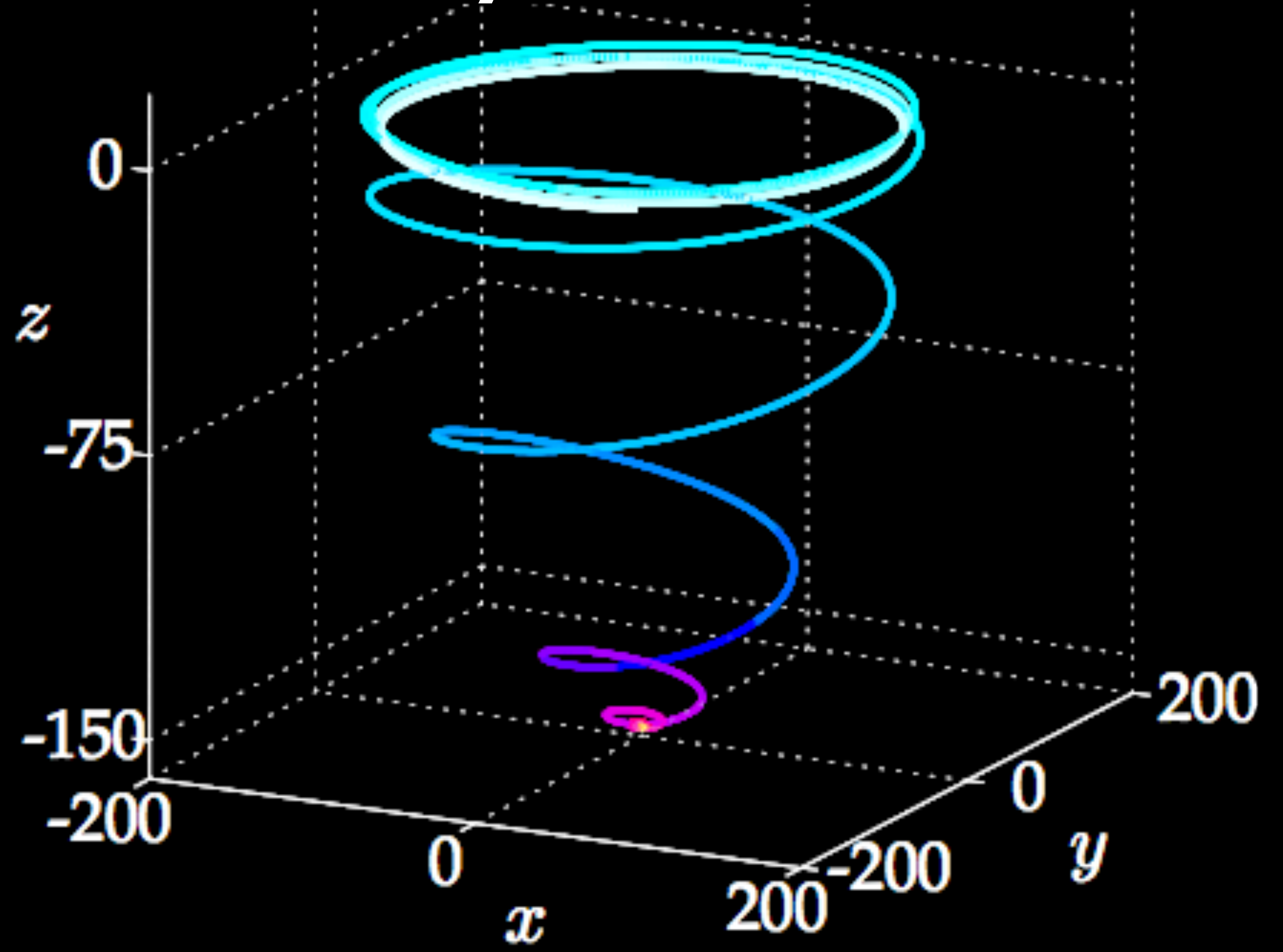
Noack *et al.*, JFM 2003.

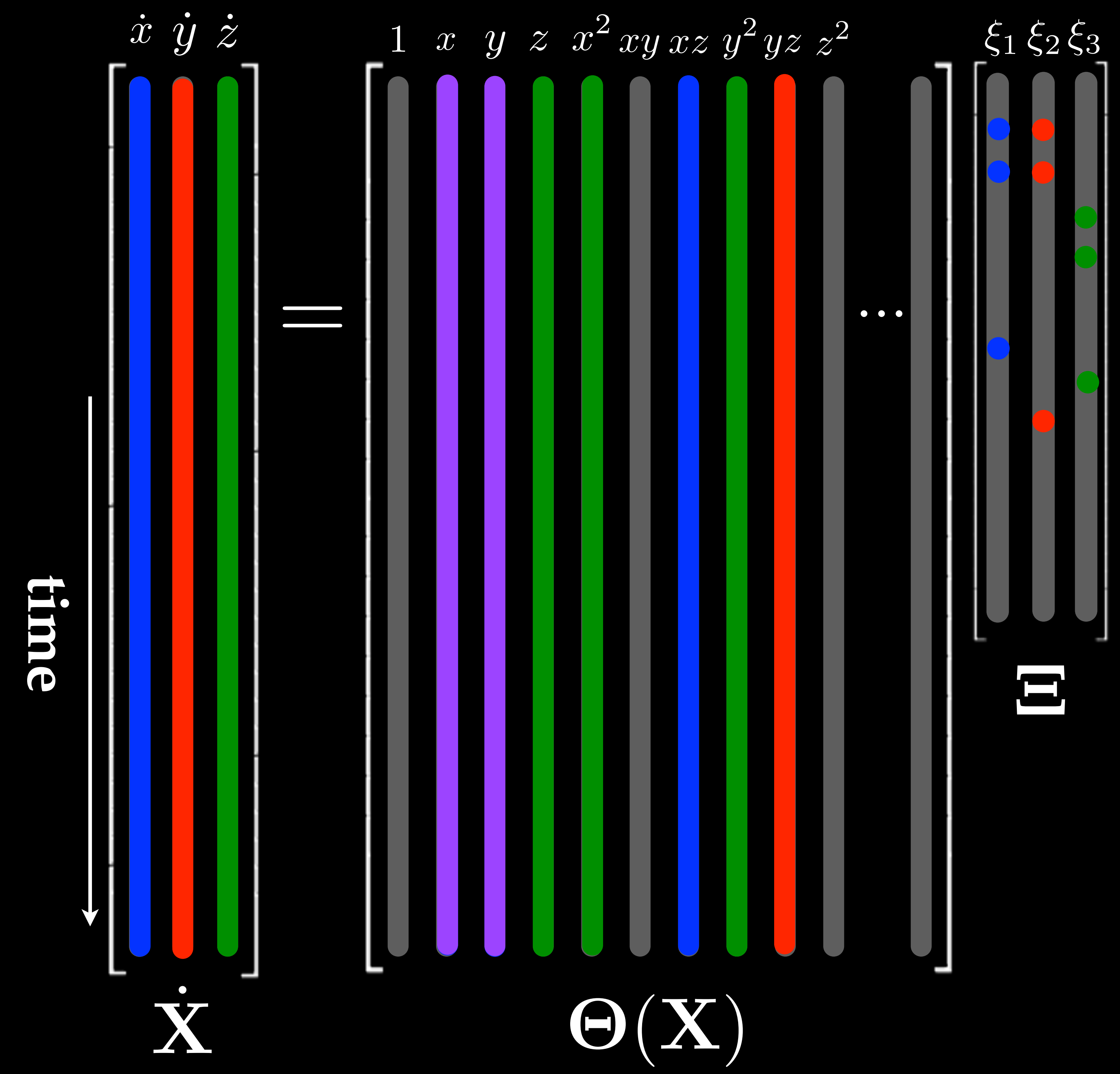
Sparse Identification of Nonlinear Dynamics (SINDy)

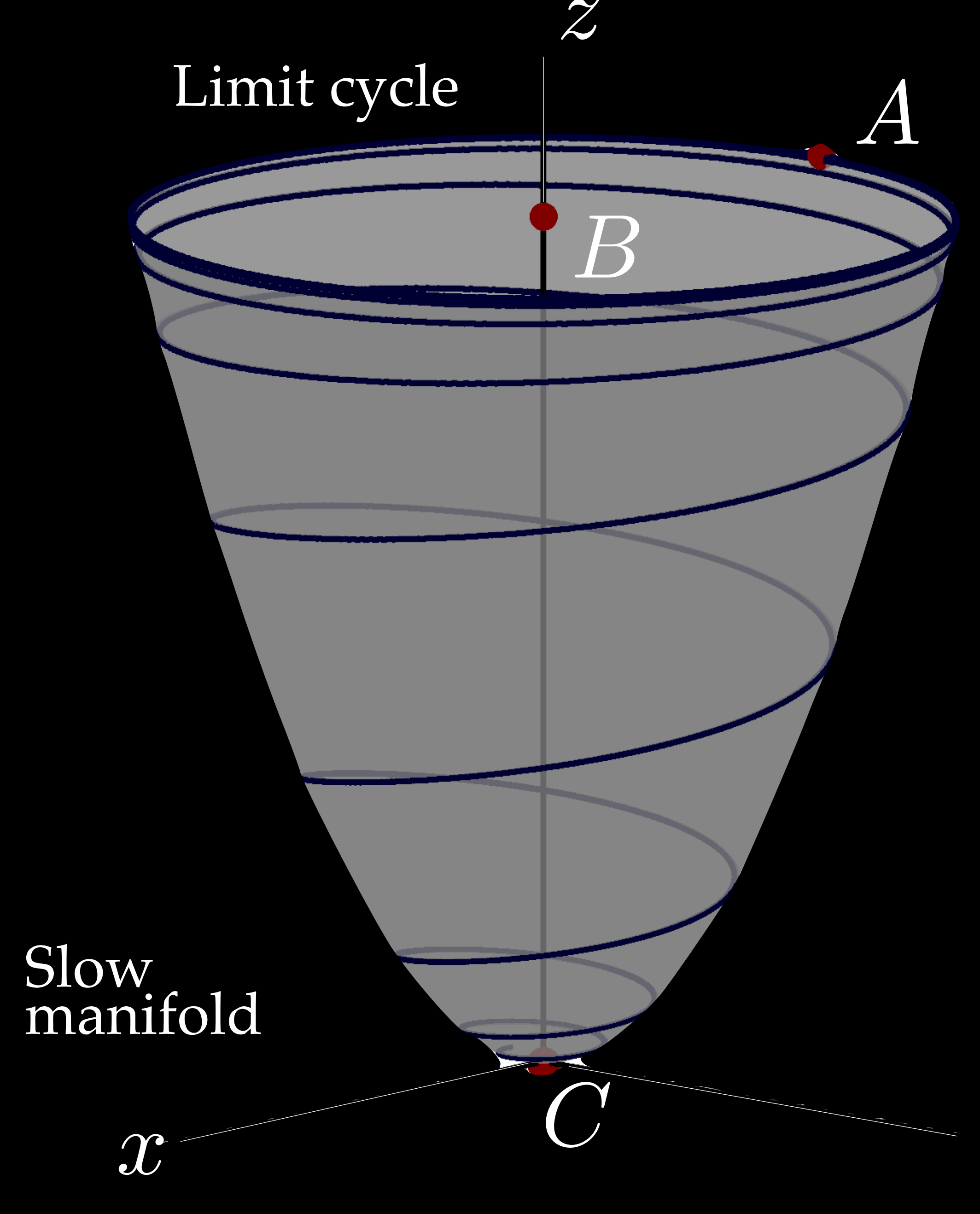
Full System



Identified System







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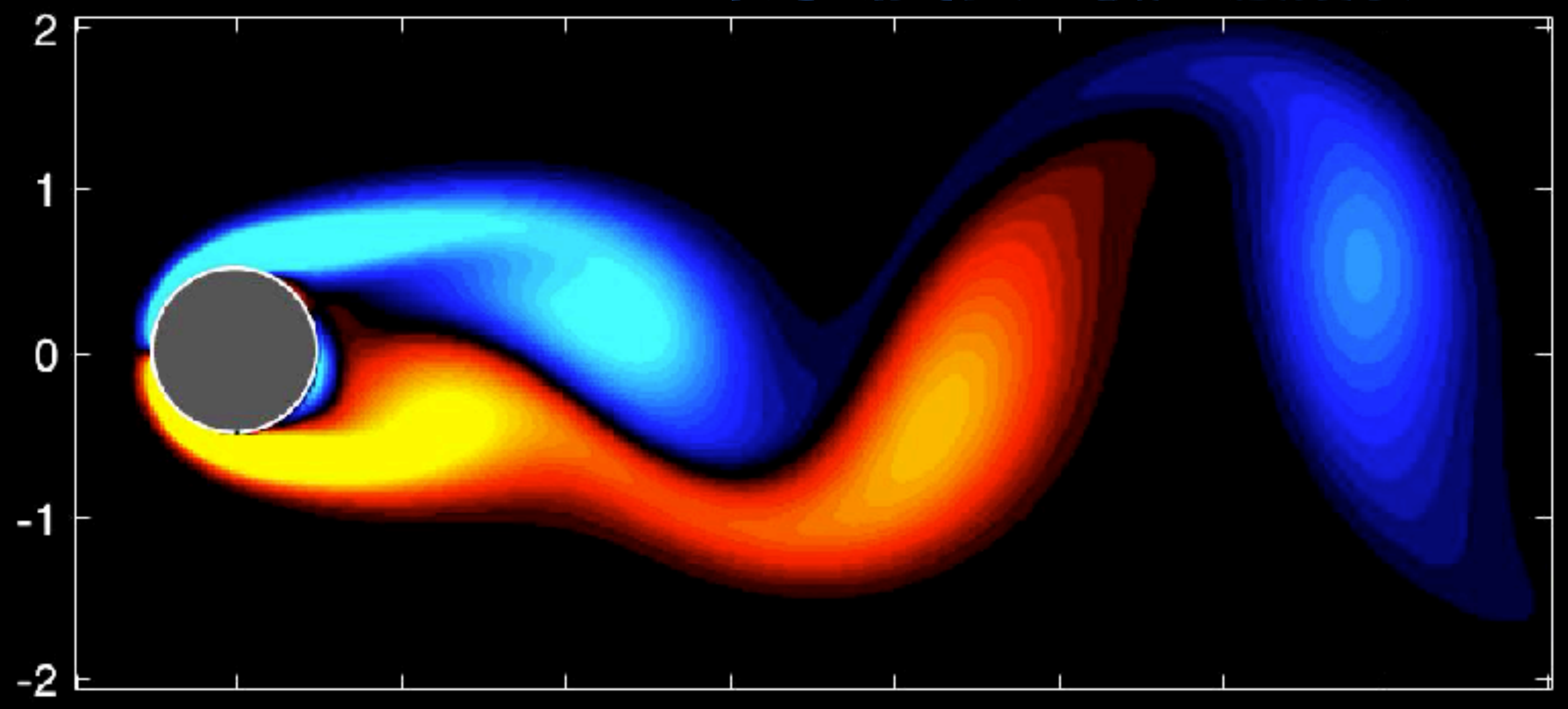
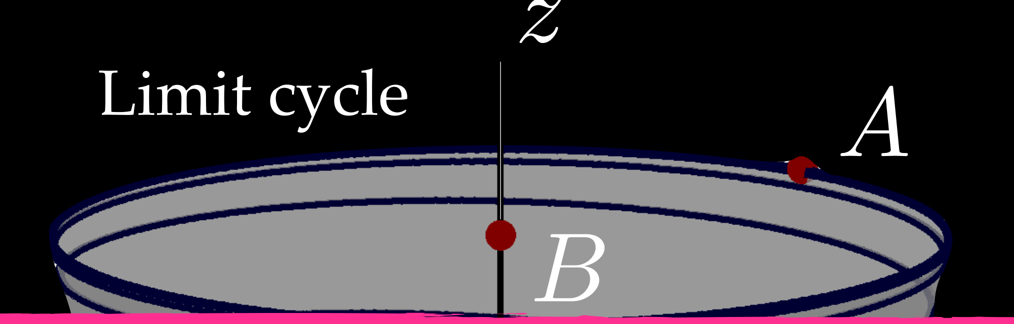
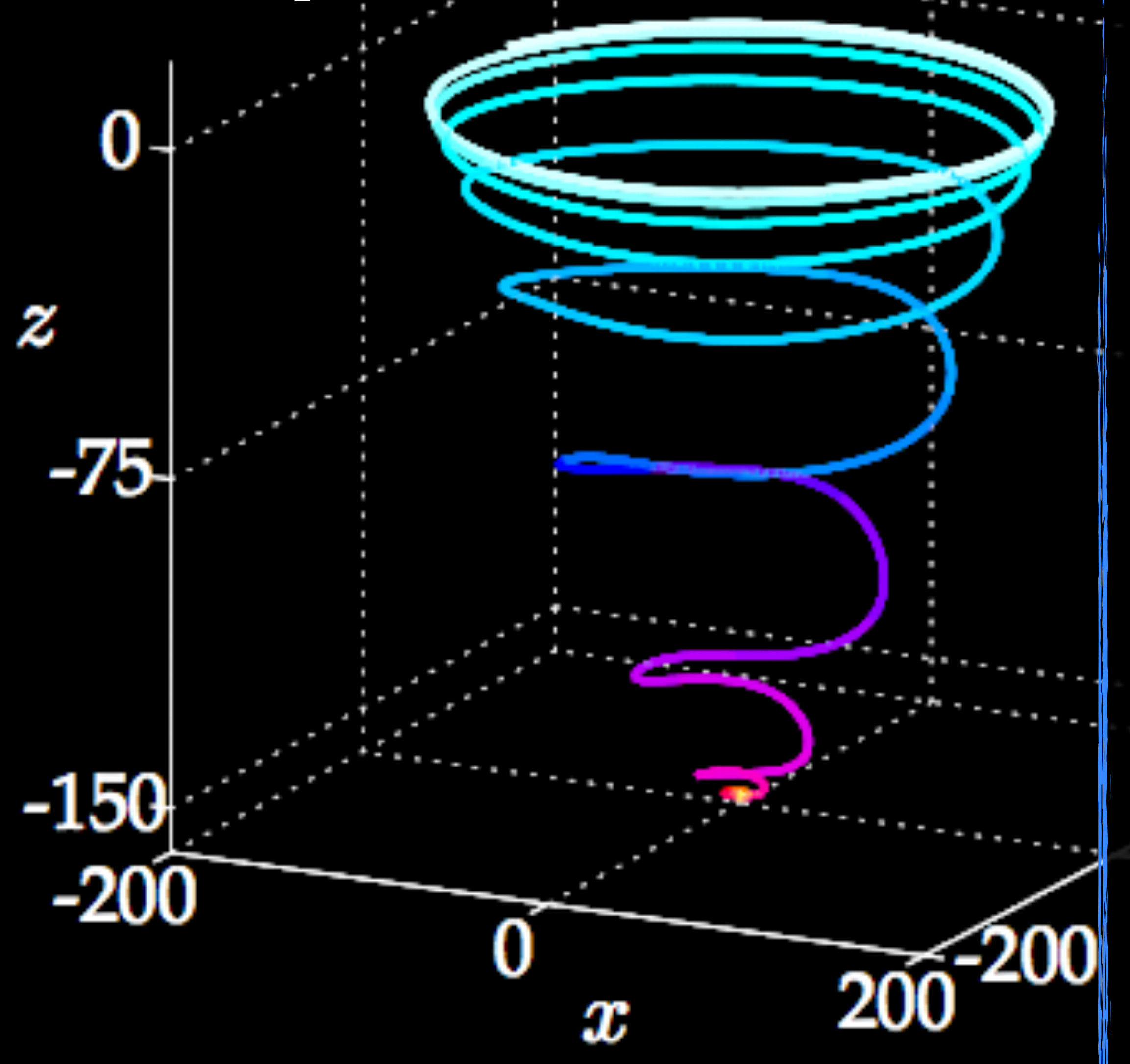


SLB, Proctor, Kutz, *PNAS* 2016.
Loiseau & SLB, *JFM* 838, 2018

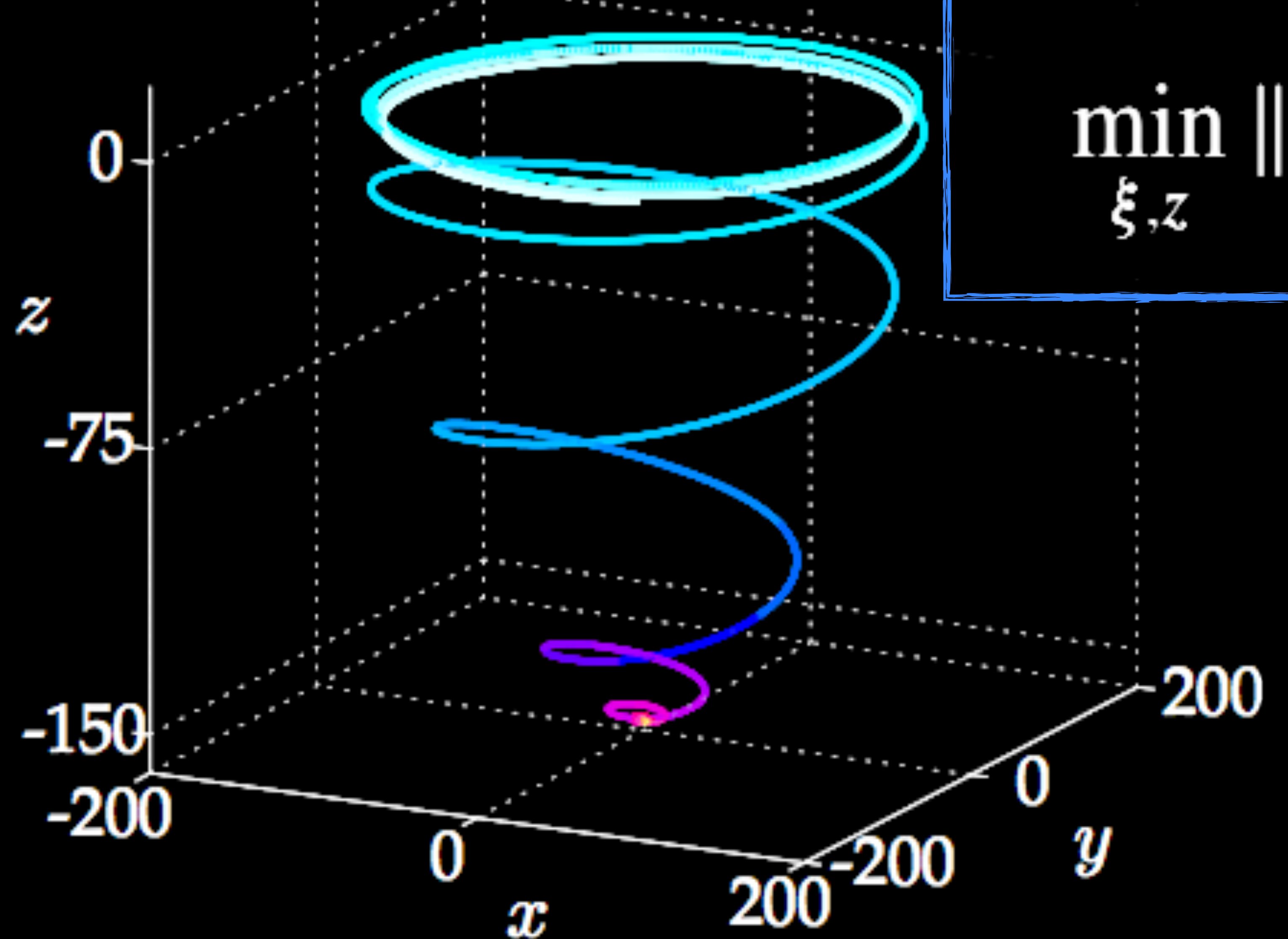
Sparse Identification of Nonlinear Dynamics (SINDy)



Full System



Identified System



Innovation 1: Enforcing known constraints

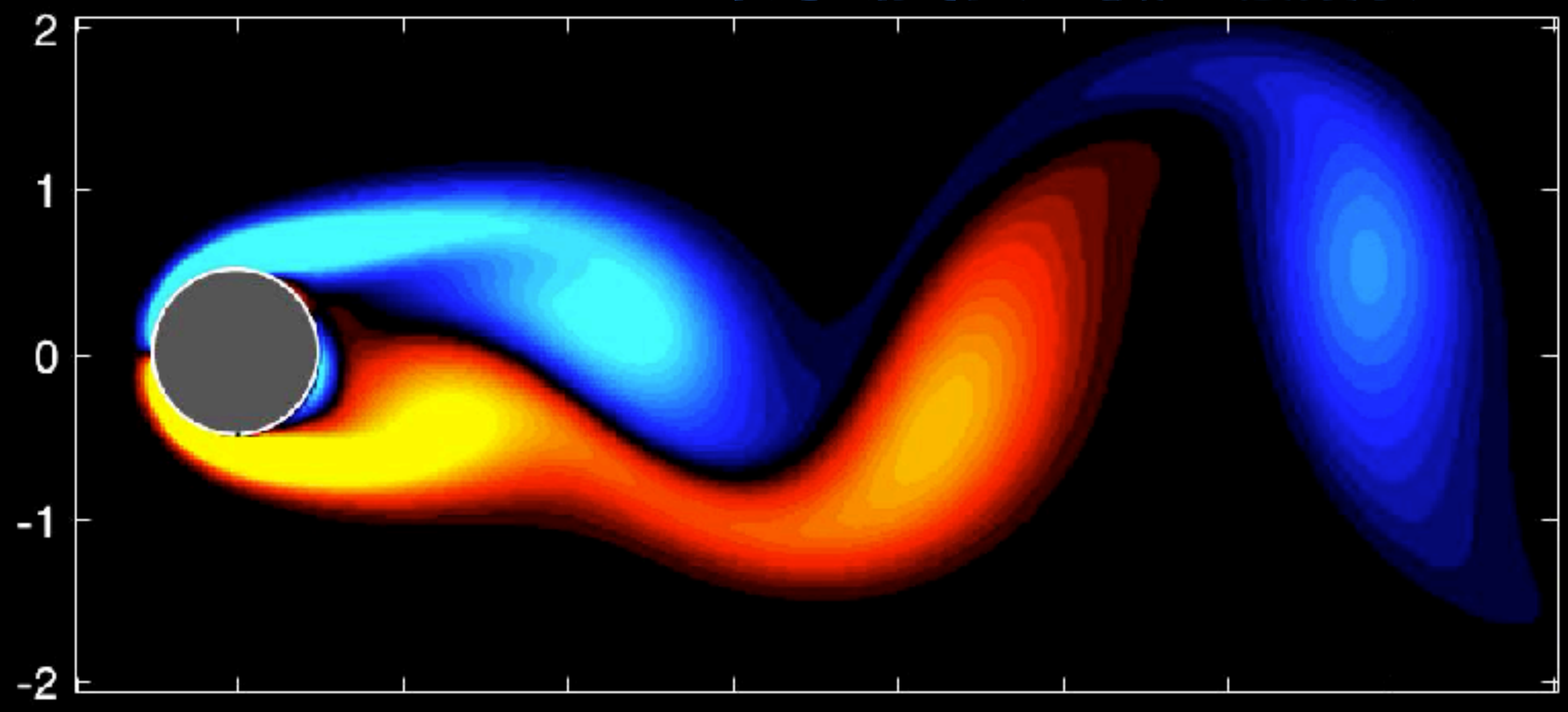
- Skew-symmetric quadratic nonlinearities to enforce energy conservation
- Improved stability

$$\min_{\xi, z} \|\Theta(X)\Xi - \dot{X}\|_2^2 + z^T(C\xi - d)$$

Innovation 2: Higher-order Nonlinearities

- Cubic, Quintic, Septic terms approximate truncated terms in Galerkin expansion

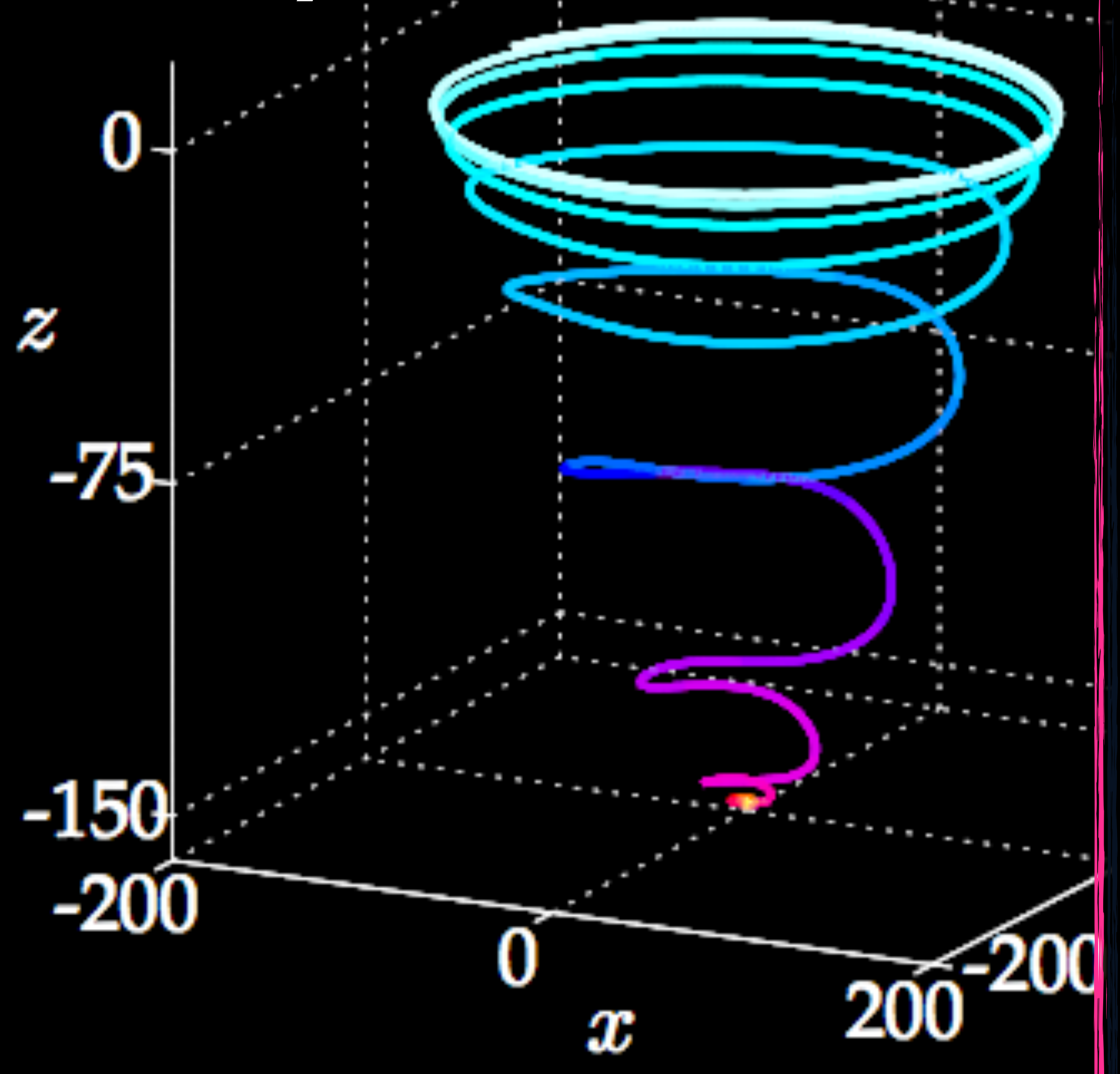
$$\begin{aligned} \dot{x} &= \mu x - \omega y + Axz \\ \dot{y} &= \omega x + \mu y + Ayz \\ \dot{z} &= -\lambda(z - x^2 - y^2). \end{aligned}$$



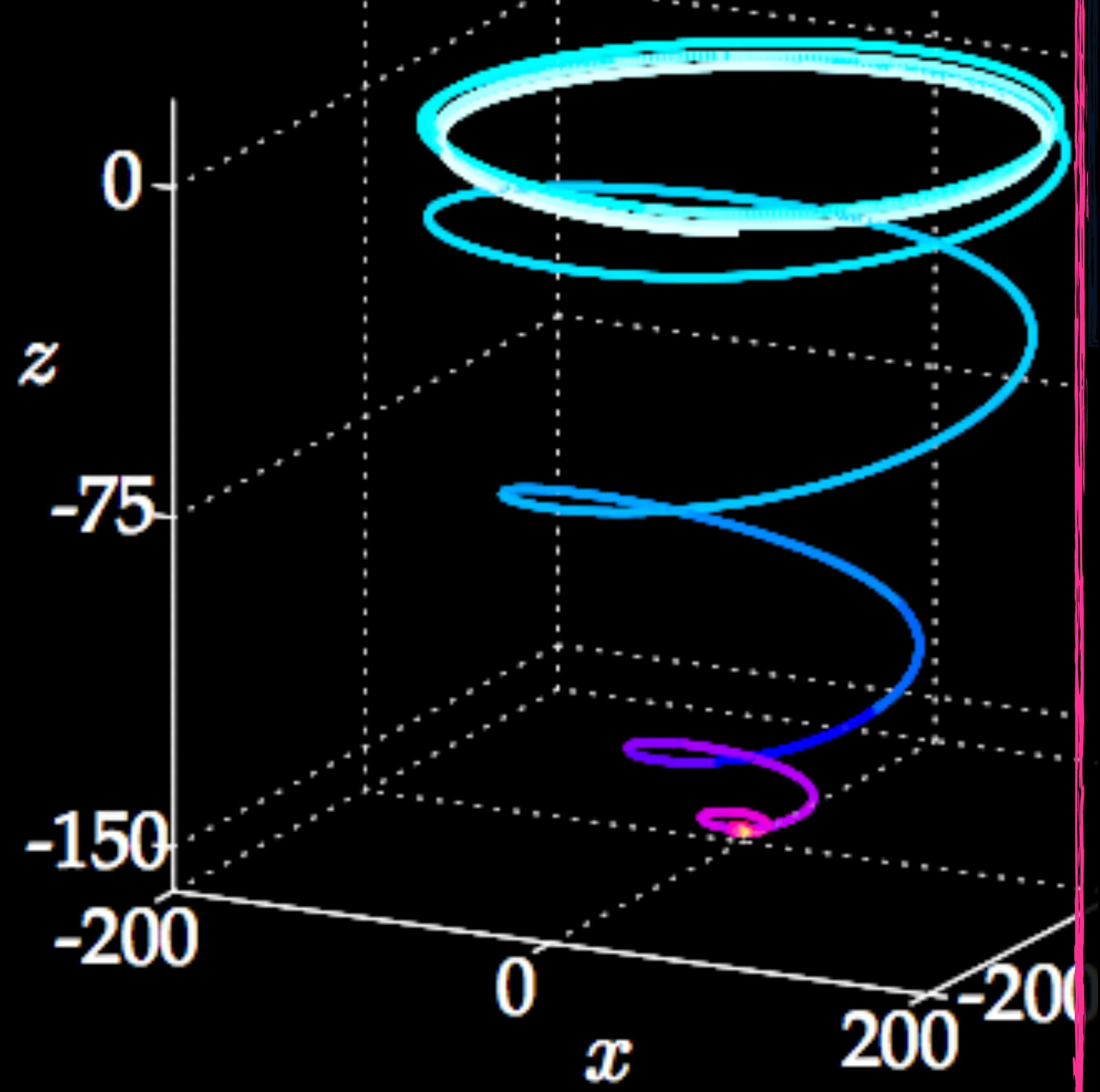
SLB, Proctor, Kutz, *PNAS* 2016.
Loiseau & SLB, *JFM* 838, 2018

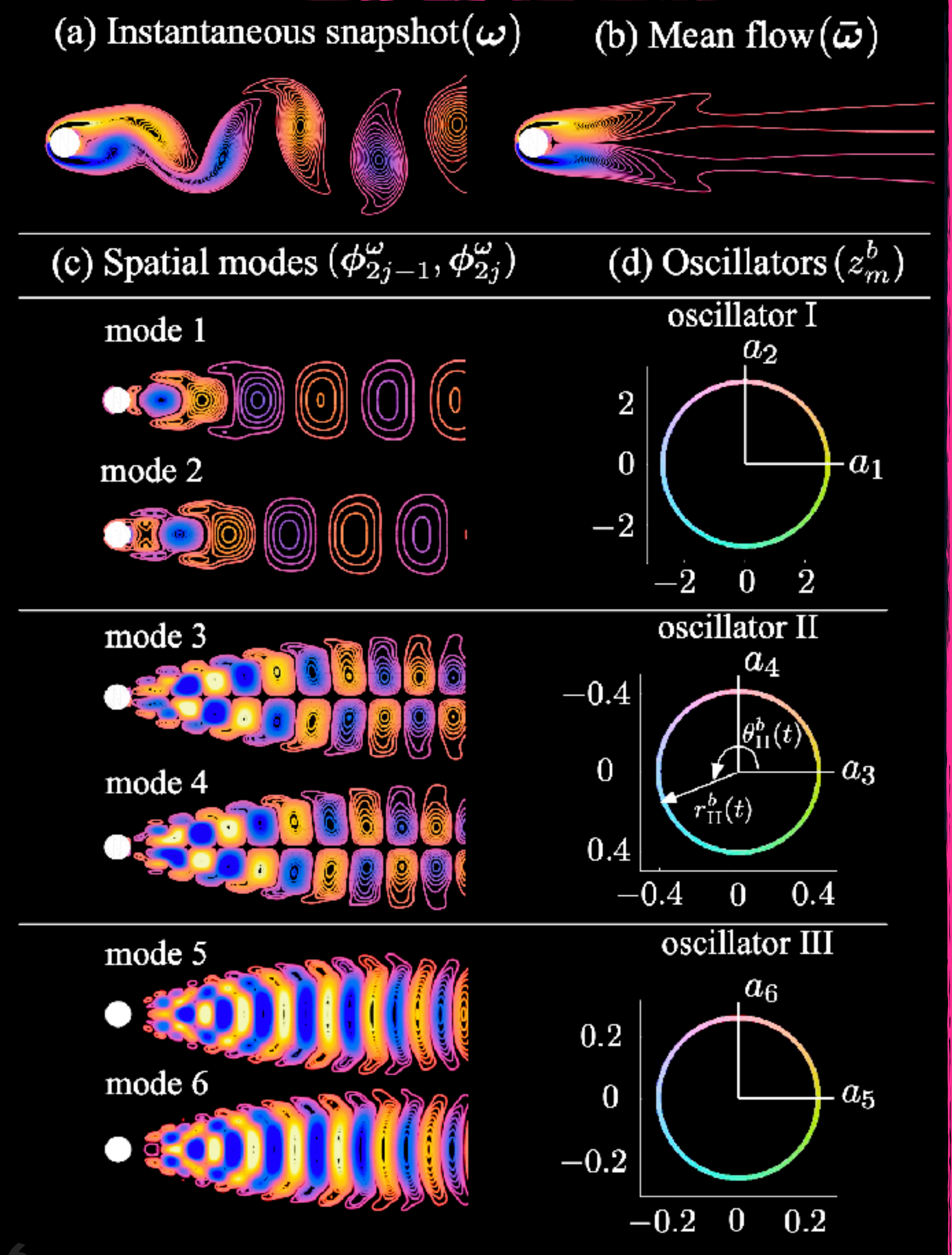
Sparse Identification of Nonlinear Dynamics (SINDy)

Full System

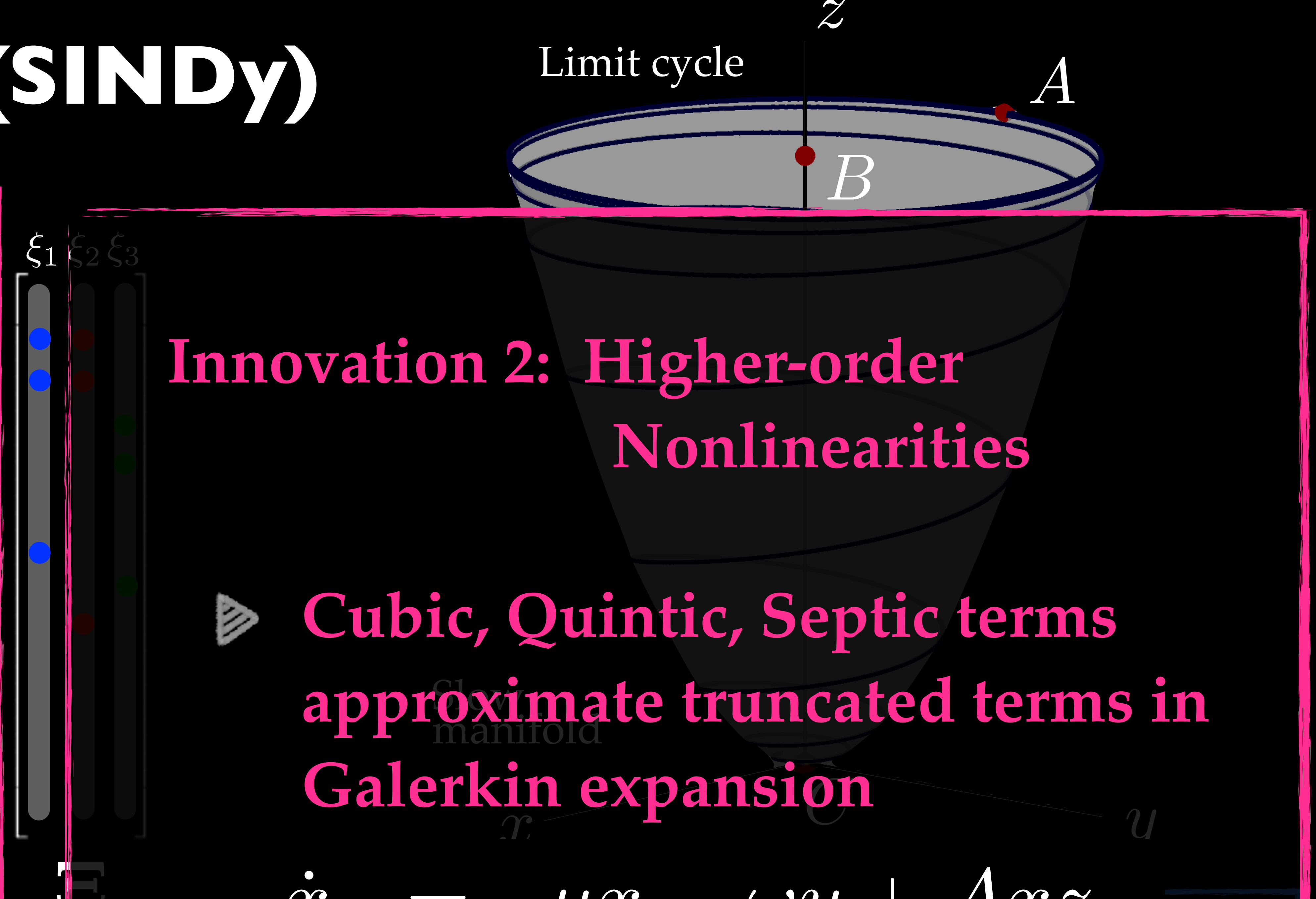


Identified System





Nair, SLB, Taira, PRE 2018



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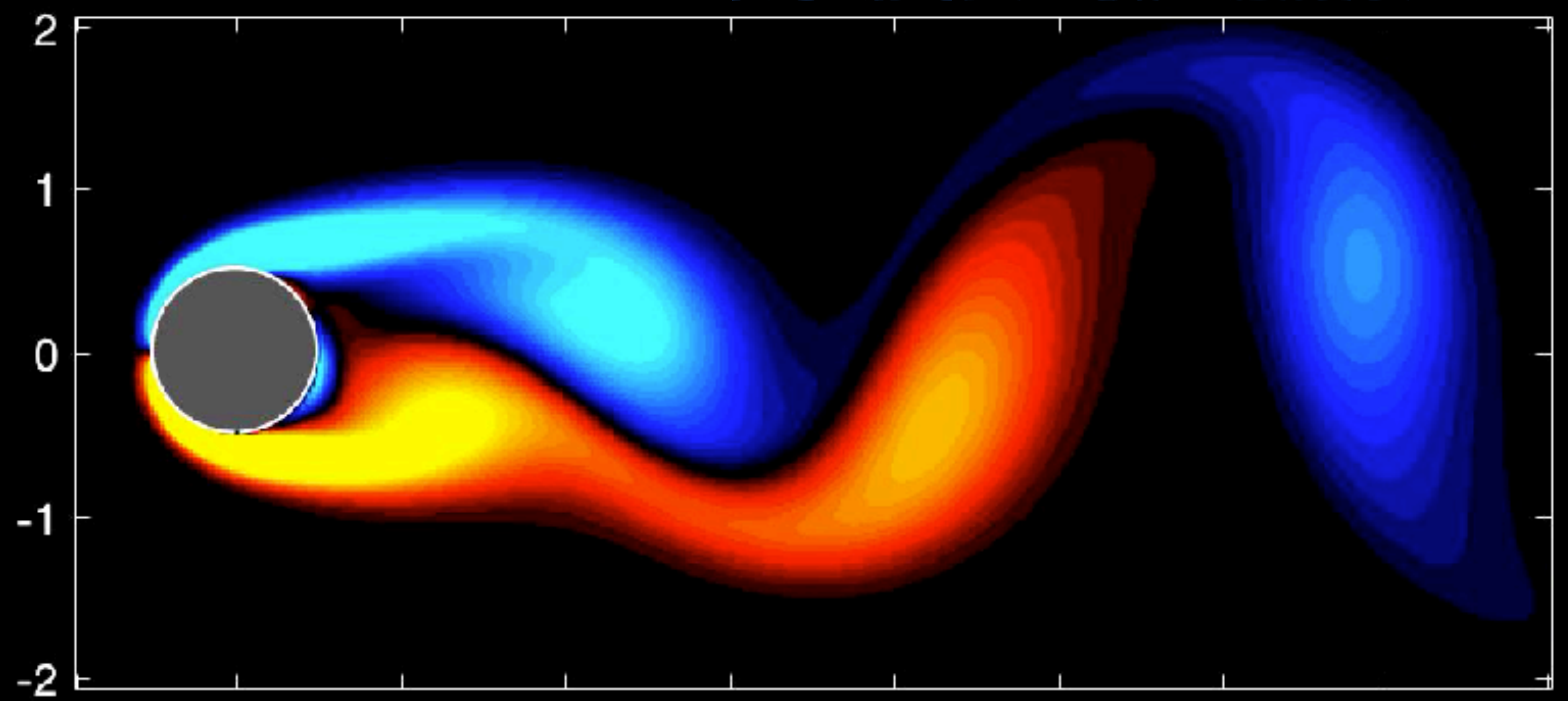
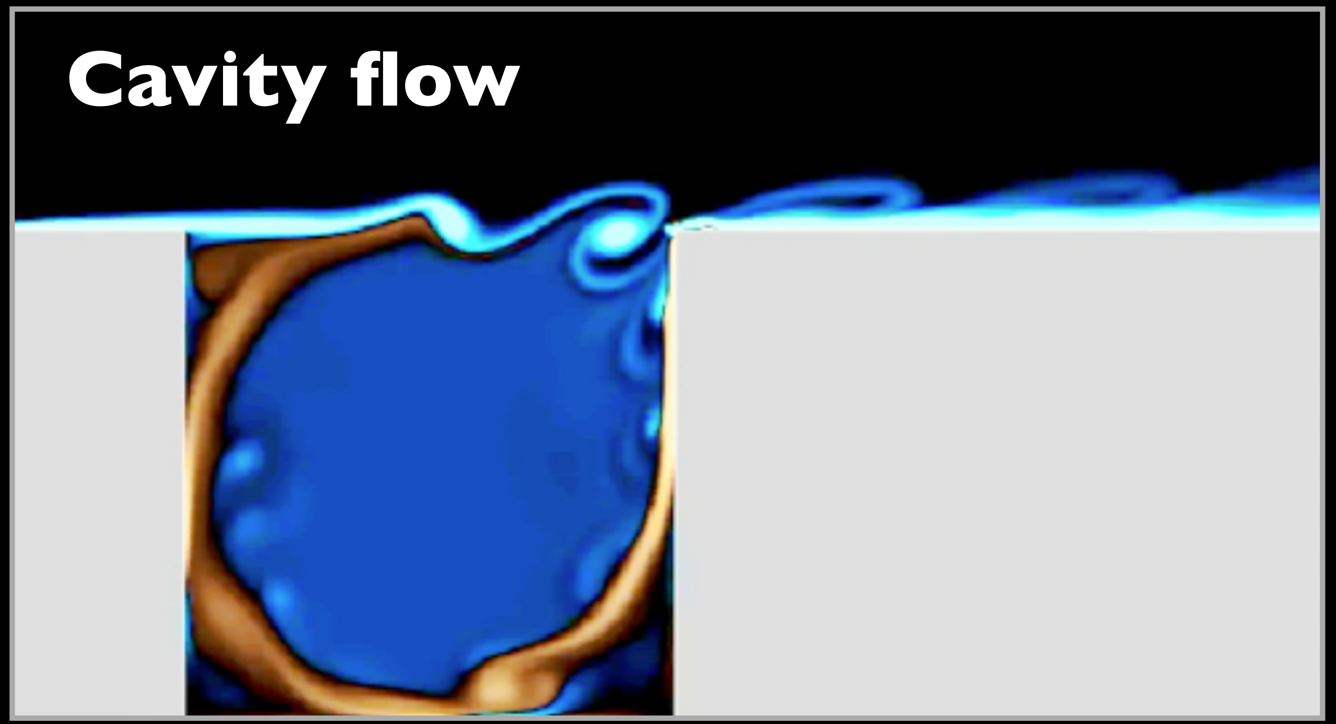
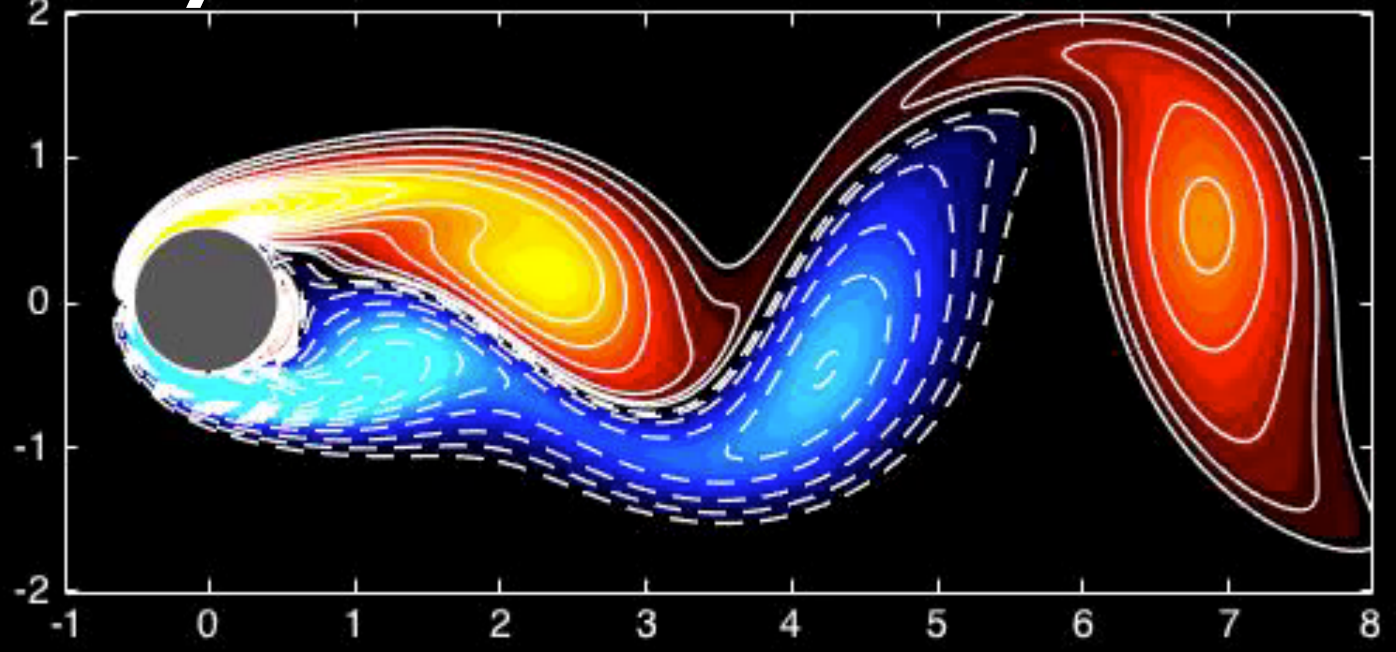
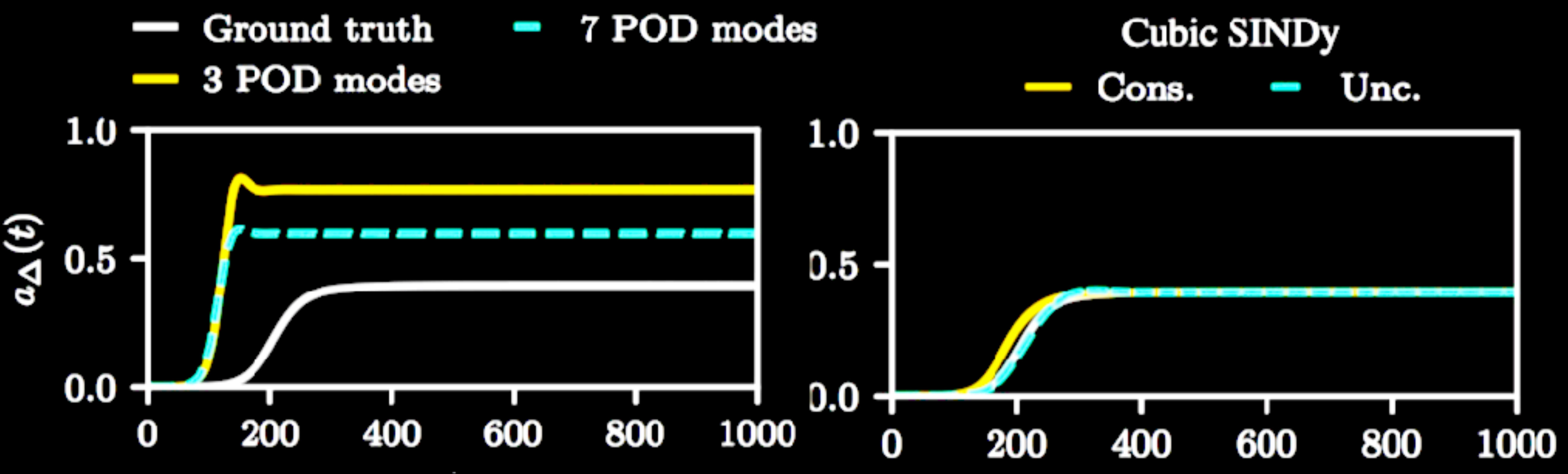


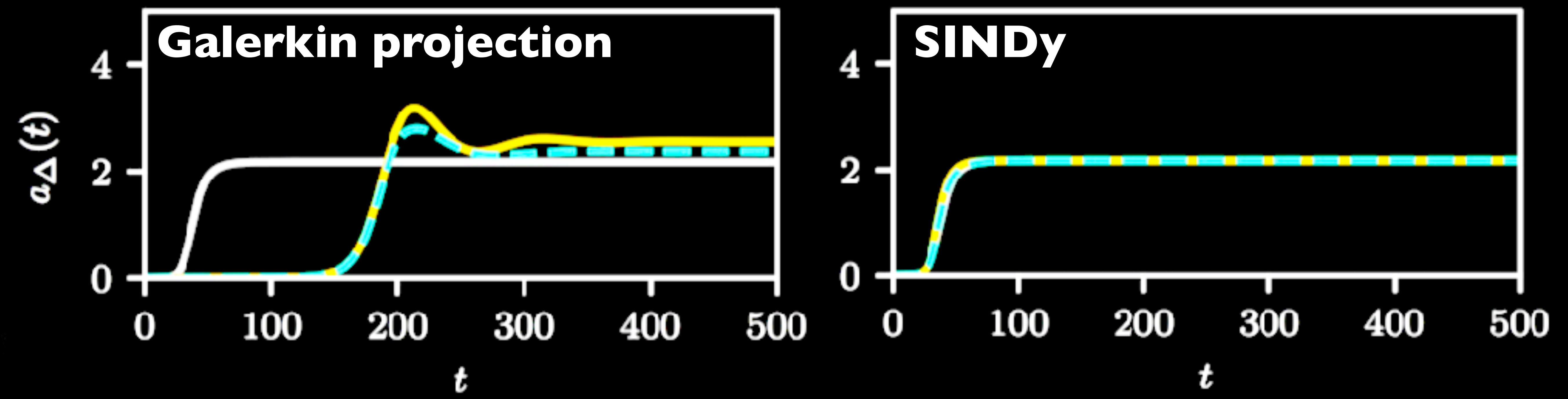
SLB, Proctor, Kutz, *PNAS* 2016.
Loiseau & SLB, *JFM* 838, 2018

Constrained Sparse Galerkin Regression







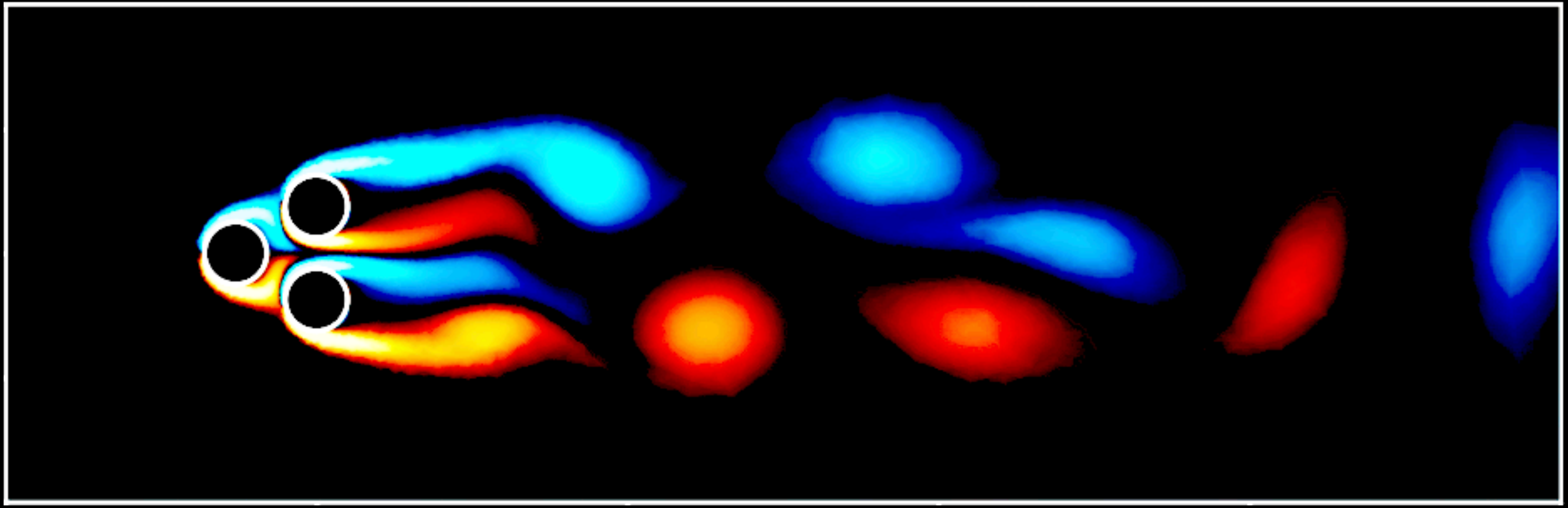


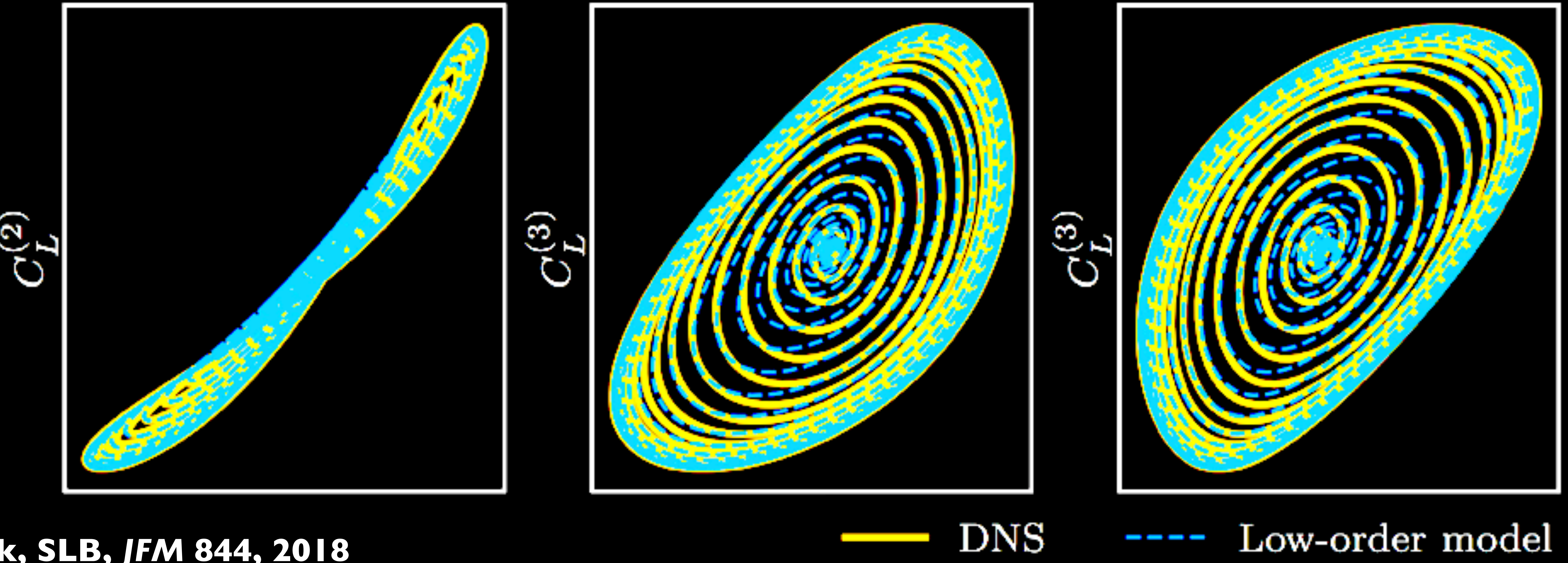
$$\ddot{x} - \underbrace{\left(0.2 - 0.24x^2 - 0.15\dot{x}^2\right)}_{k(x,\dot{x})}\dot{x} + 1.26x = 0$$

Spring-Mass Damper with Nonlinear Damping!

Loiseau & SLB, *JFM* 838, 2018

More Complex Flow: Fluidic Pinball

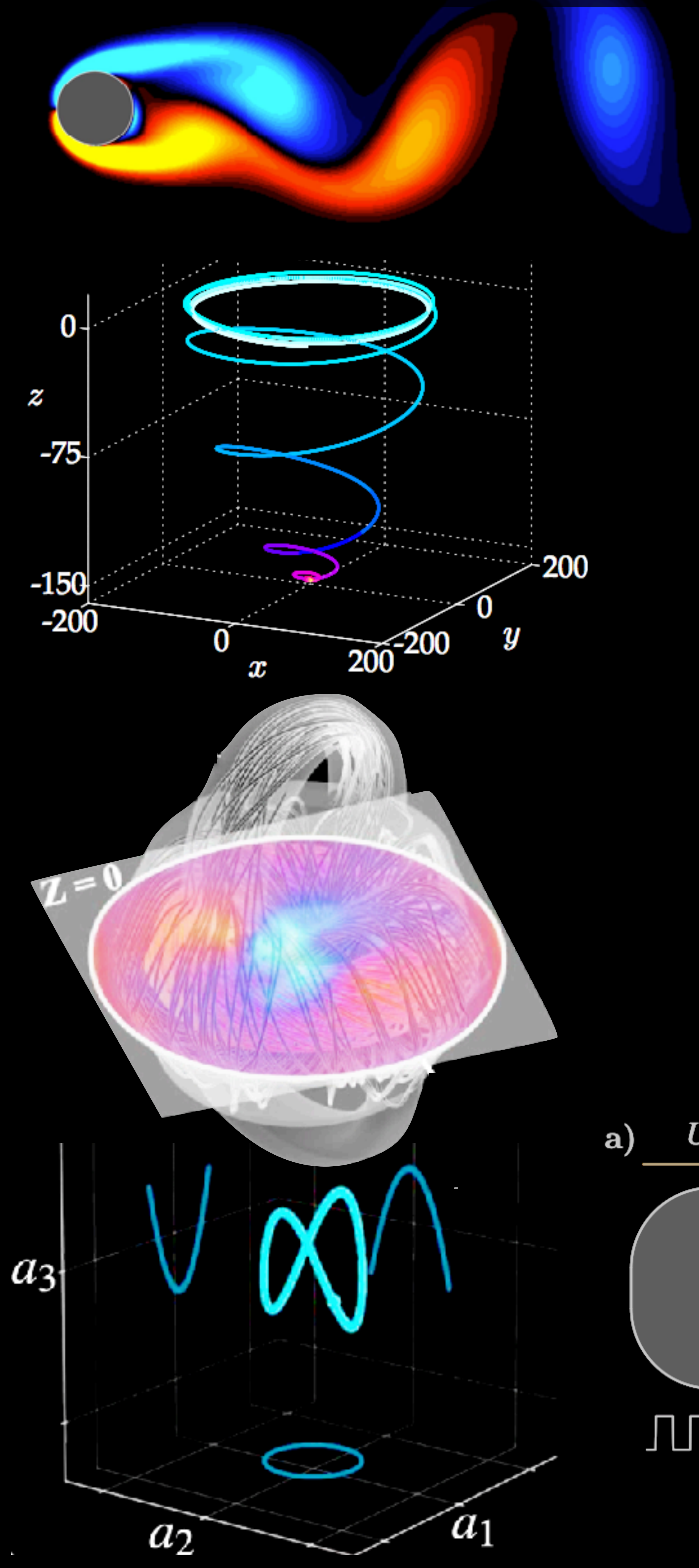


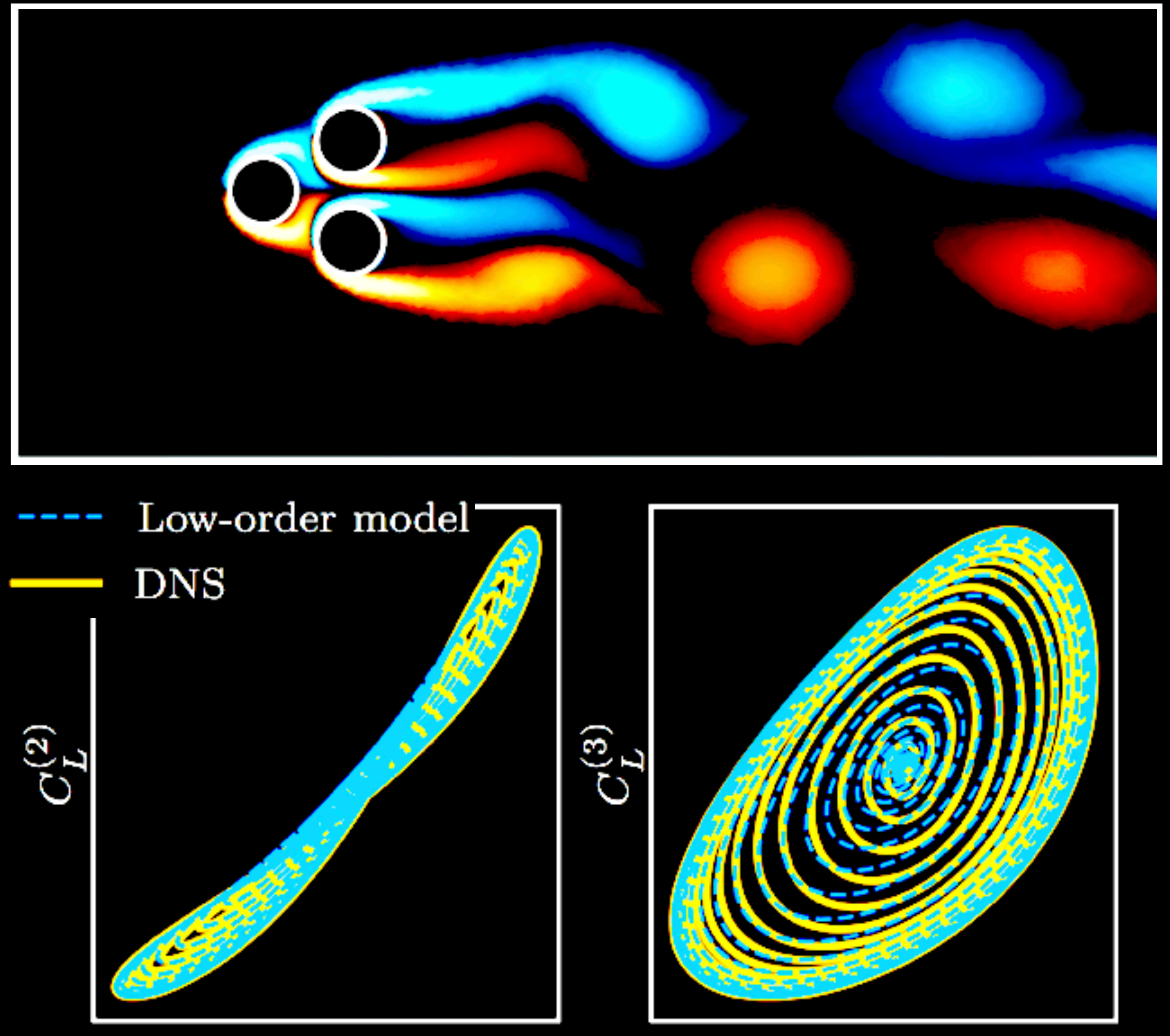


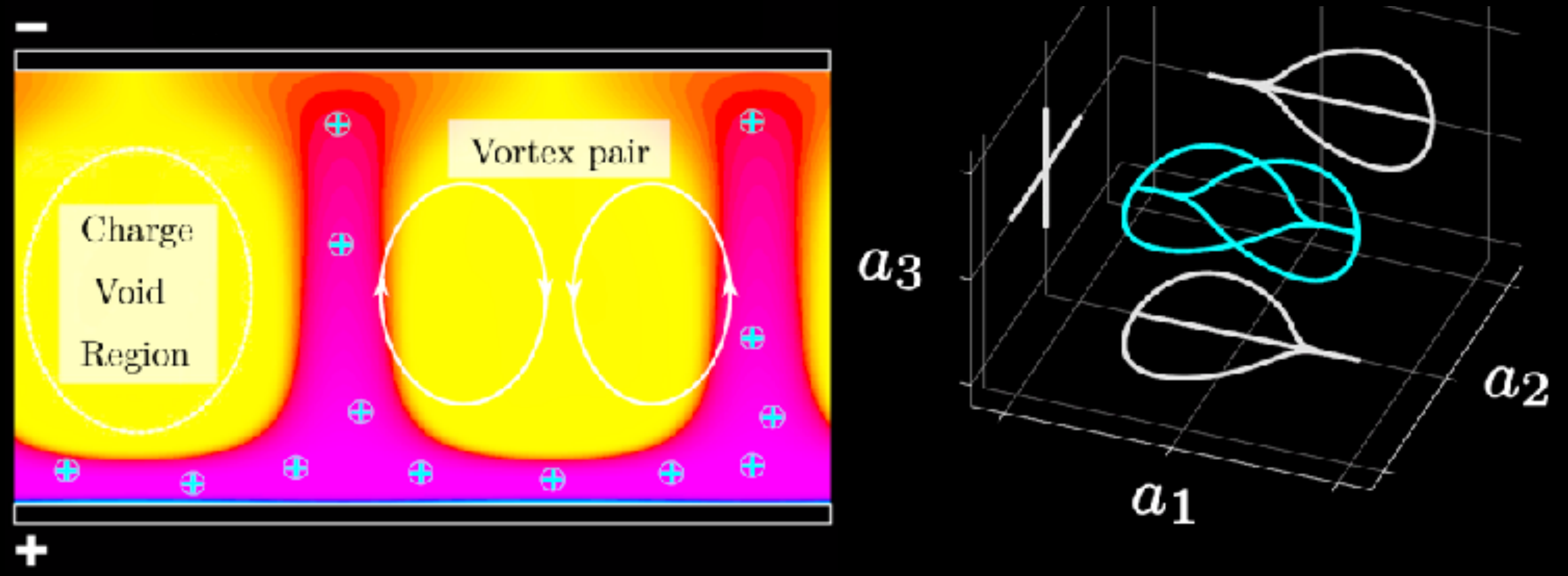
Loiseau, Noack, SLB, *JFM* 844, 2018

Brunton et al. *PNAS* 2016

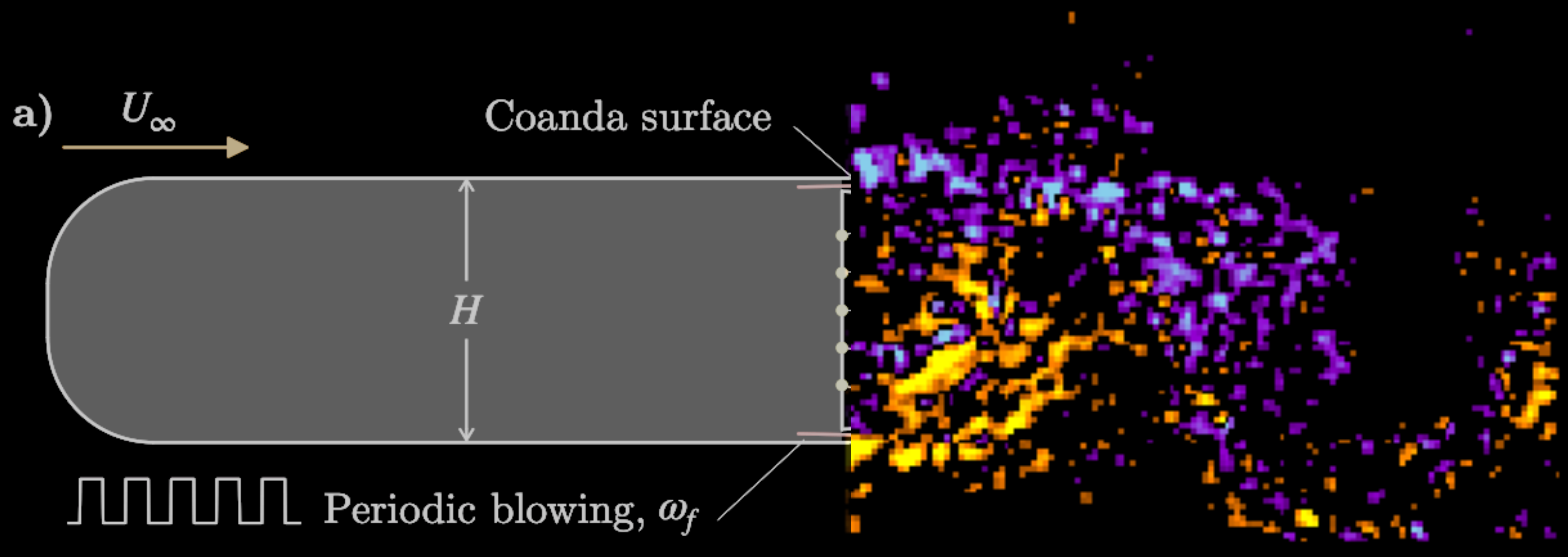
Sparse Nonlinear Models of Fluid Dynamics

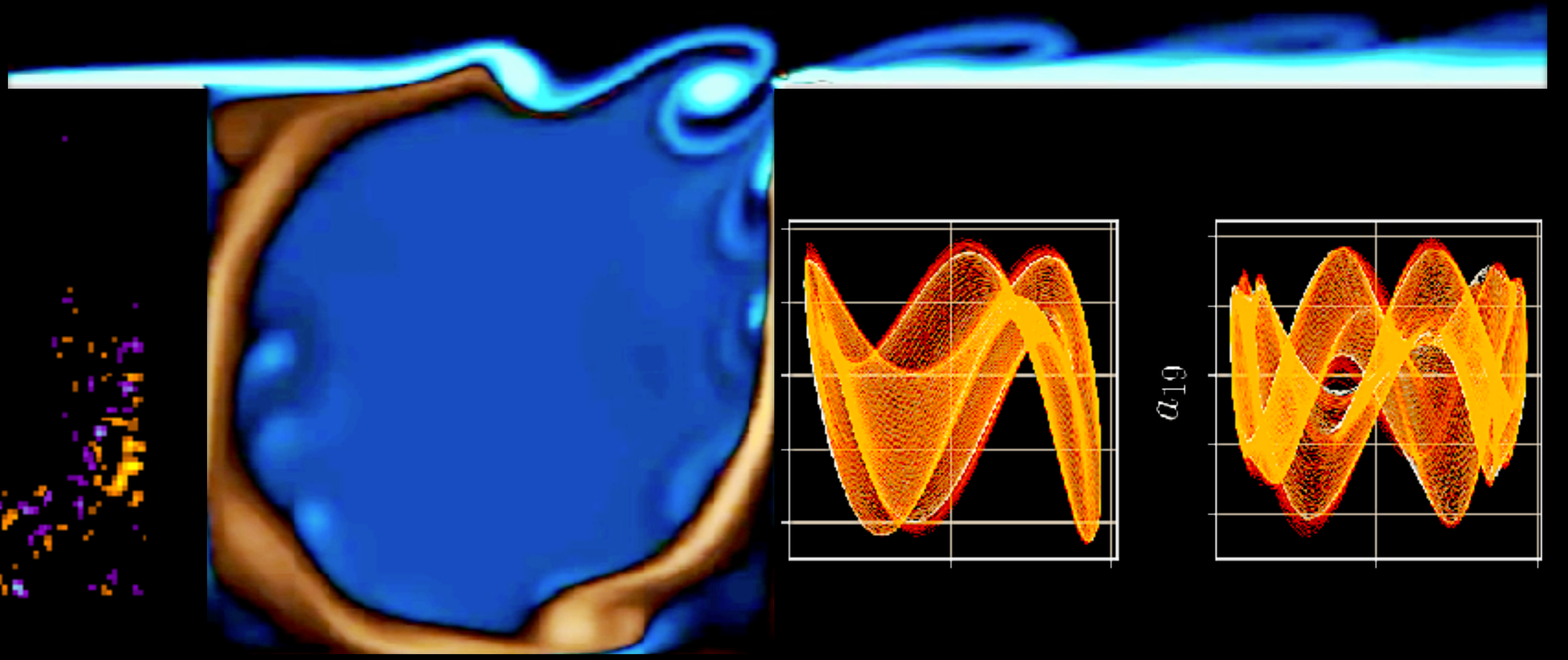






$$\frac{d}{dt}\mathbf{x} = \mathbf{f}(\mathbf{x})$$

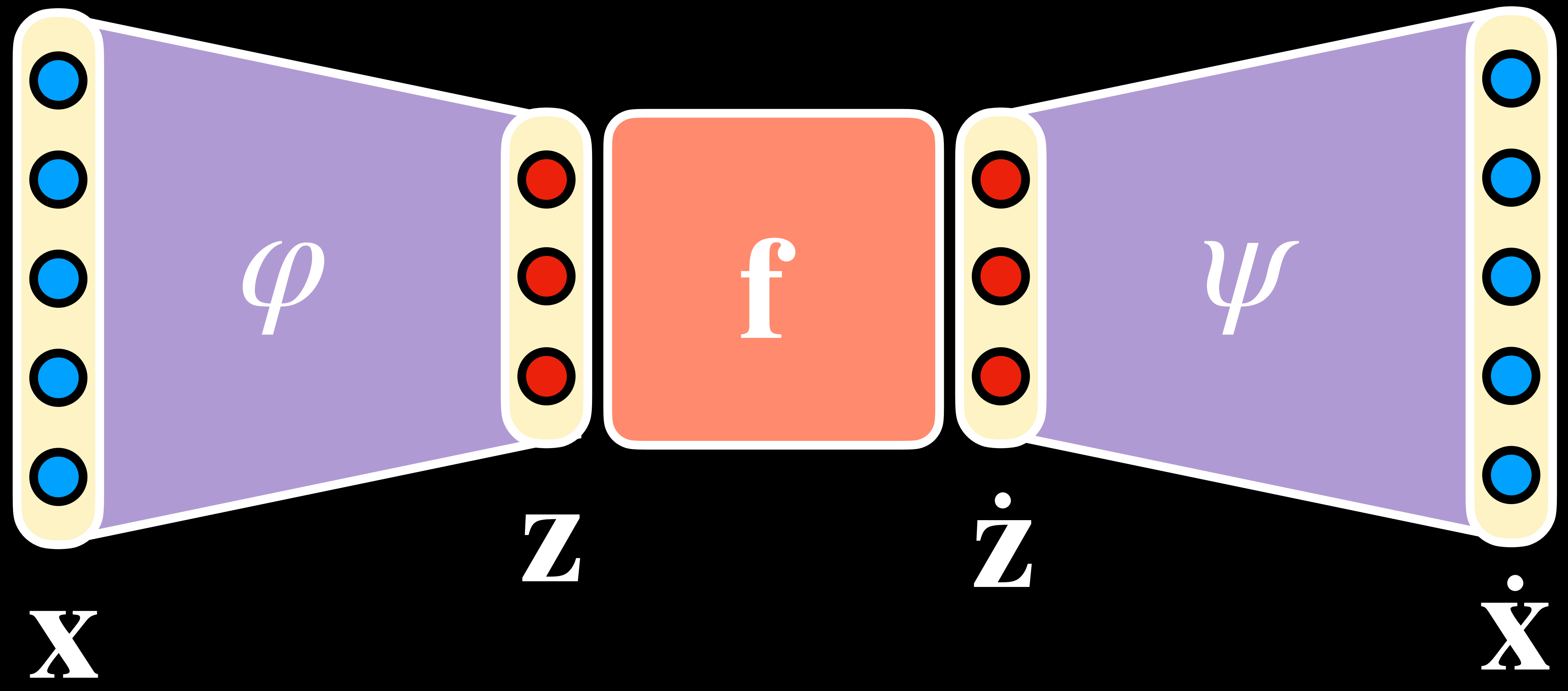




There is a need for INTERPRETABLE and GENERALIZABLE Machine Learning

- LOW-DIMENSIONAL
- SPARSE

There is a need for INTERPRETABLE and GENERALIZABLE Machine Learning



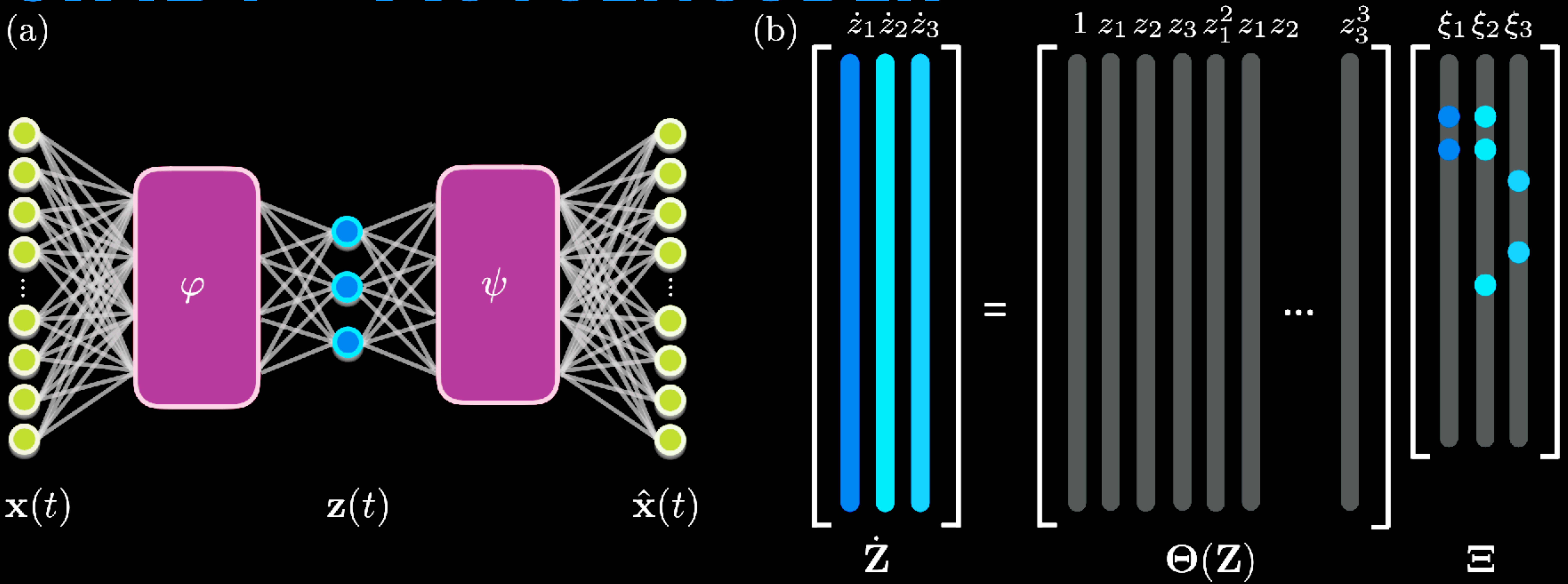
$$\frac{d}{dt}\mathbf{z} = \mathbf{f}(\mathbf{z})$$

Heliocentrism

Geocentrism

Malin Christersson

SINDy + Autoencoder



$$\dot{\mathbf{z}}_i = \nabla_{\mathbf{x}} \varphi(\mathbf{x}_i)\dot{\mathbf{x}}_i \qquad \boldsymbol{\Theta}(\mathbf{z}_i^T) = \boldsymbol{\Theta}(\varphi(\mathbf{x}_i)^T)$$

$$\underbrace{\|\mathbf{x} - \psi(\mathbf{z})\|_2^2}_{\text{reconstruction loss}} + \lambda_1 \underbrace{\left\|\dot{\mathbf{x}} - (\nabla_{\mathbf{z}}\psi(\mathbf{z}))\left(\boldsymbol{\Theta}(\mathbf{z}^T)\boldsymbol{\Xi}\right)\right\|_2^2}_{\text{SINDy loss in } \dot{\mathbf{x}}} + \lambda_2 \underbrace{\left\|(\nabla_{\mathbf{x}}\mathbf{z})\,\dot{\mathbf{x}} - \boldsymbol{\Theta}(\mathbf{z}^T)\boldsymbol{\Xi}\right\|_2^2}_{\text{SINDy loss in } \dot{\mathbf{z}}} + \lambda_3 \underbrace{\|\boldsymbol{\Xi}\|_1}_{\text{SINDy regularization}}$$

Champion, Lusch, Kutz, Brunton. *Proc. Nat. Acad. Sci*, 2019

QUESTIONS