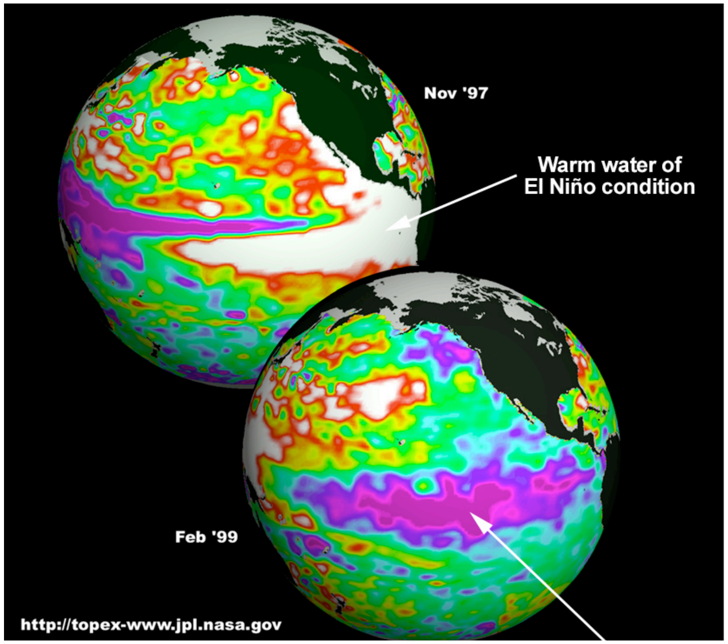
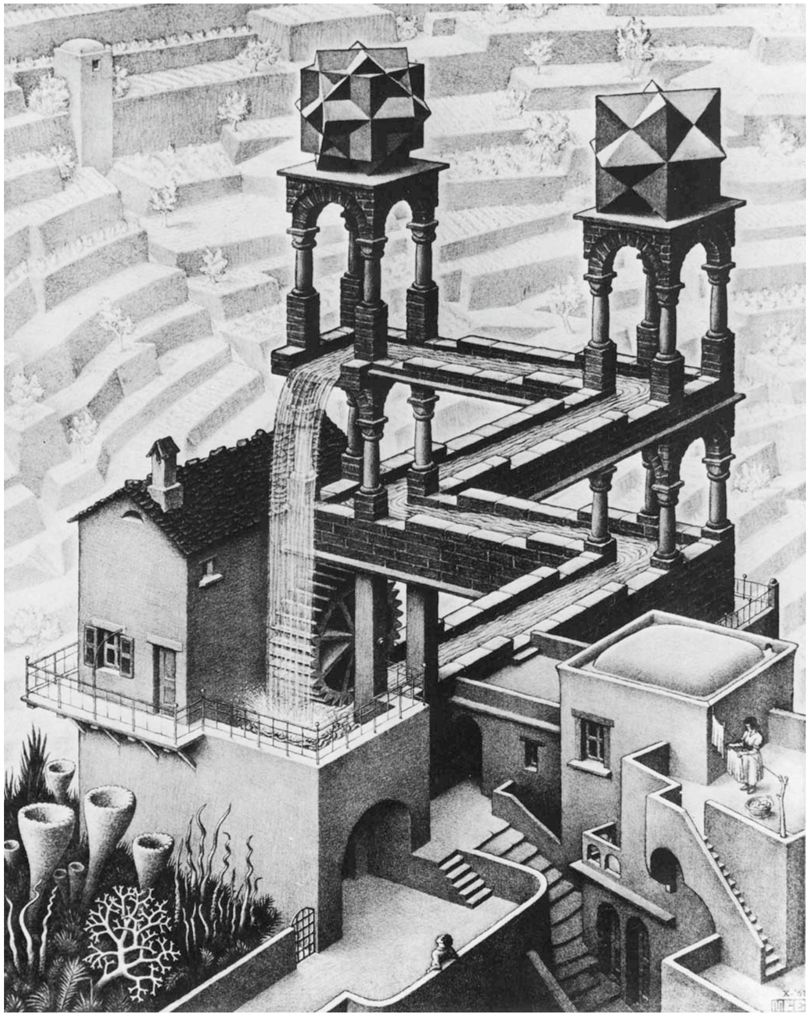
Nonequilibrium Statistical Mechanics of Climate Variability





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Outline

- what do I mean by climate variability?
- stochastic models of climate fluctuations
- detailed balance and climate *(Weiss, 2003)*
- nonequilibrium statistical mechanics
- structure of climate variability *(Weiss, 2007)*

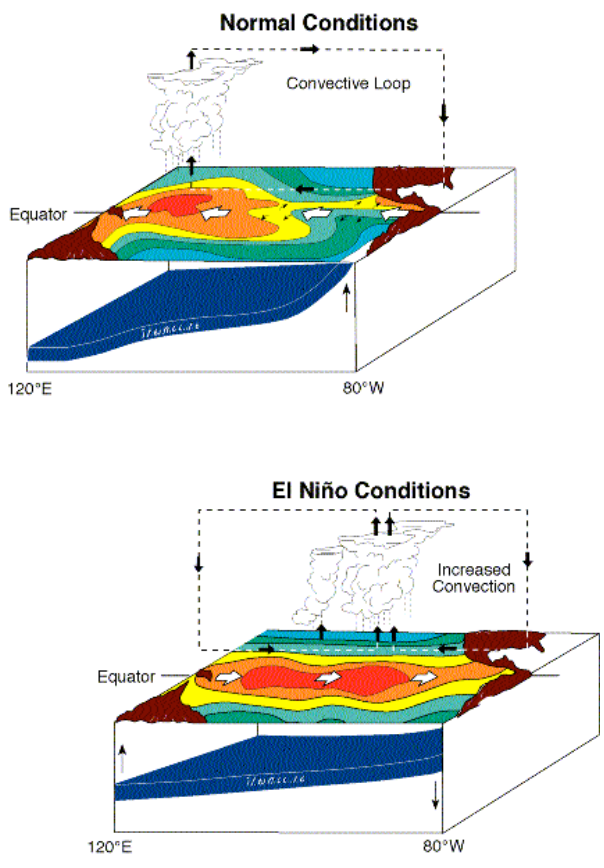
Climate Variability

- Climate system has many identifiable variations on many timescales
- El-Niño
  - coupled tropical atmosphere-ocean
  - timescales: 9 months, 4-5 years
- Gulf Stream
  - internal ocean variability
  - timescales: months to very long

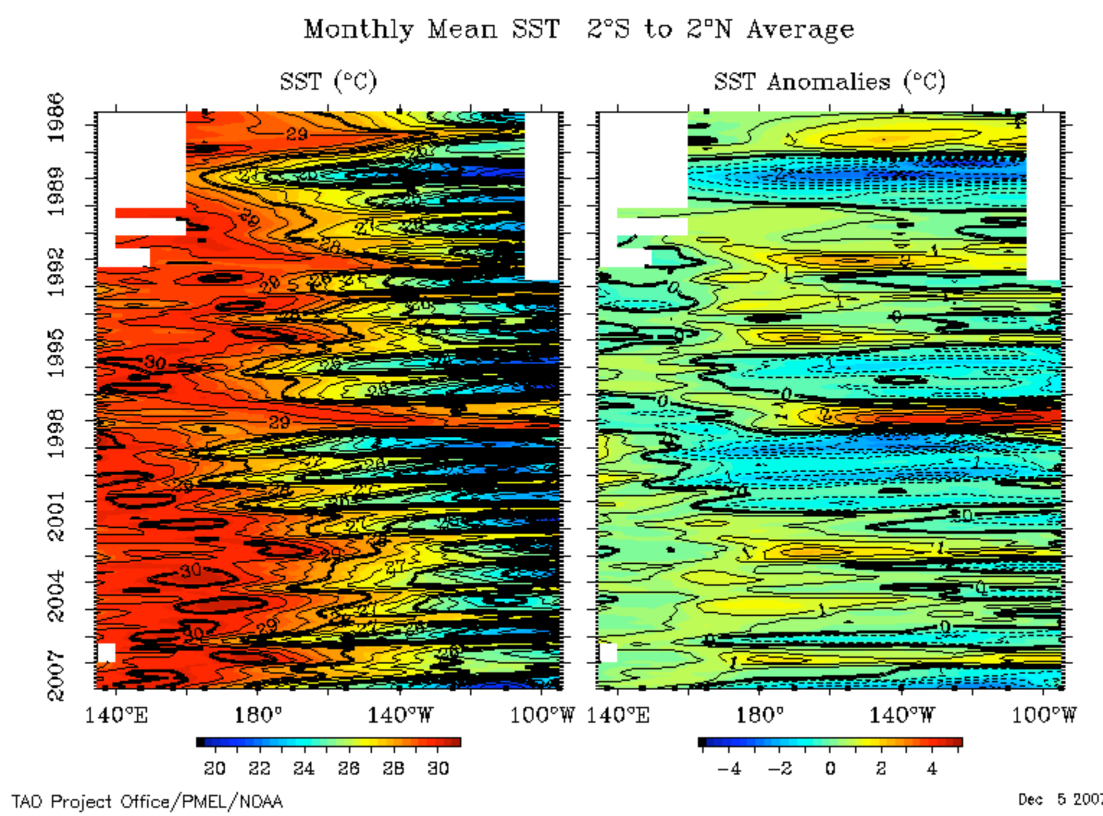
- Storm Tracks
  - variations in weather due to SST
  - timescales: seasonal to decadal
- North Atlantic Oscillation
  - atmospheric mass variations across North Atlantic
  - timescales: years to decades
- Fluctuations have large impact and are difficult to predict under climate change
- Will longer records reveal new phenomena with longer timescales?

El-Niño

- El-Niño theme page: http://www.pmel.noaa.gov/tao/elnino/nino-home.html



NOAA/PMEL/TAO



Stochastic Model of El-Niño

- SST evolves on slow ocean timescale
  - months
- forced by chaotic weather
  - Lyapunov time of a few days
- on monthly timescale, weather is random noise

Stochastic Climate Models

- Stochastic prediction models go back decades

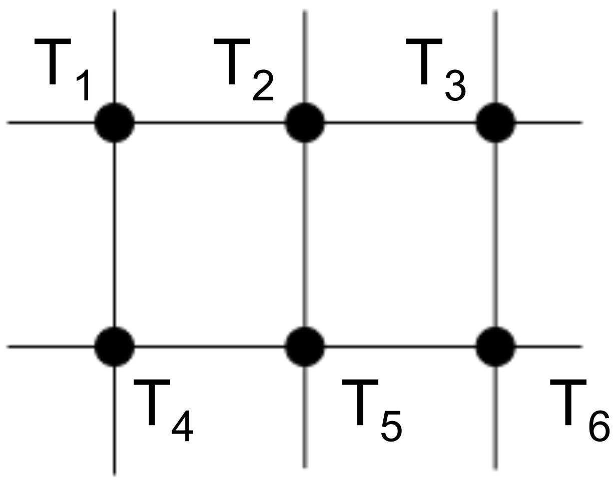
  (Epstein, 1969; Fleming, 1971; Leith, 1974; Hasselman, 1976; North and Cahalan 1981)

- Dynamical GCMs and chaos intervened
- Renewed interest in last decade

  (e.g. Penland and Magorian, 1993; Farrell and Ioannou, 1993; Moore and Farrell, 1993)

Stochastic Dynamics

- Truncate state to finite state vector $\vec{x}$
  - e.g. put SST T(x,y) on a grid



- state evolves via nonlinear deterministic operator

$$\frac{d\vec{x}}{dt} = \vec{\mathcal{N}}(\vec{x})$$

- linearize about some mean state $\vec{x}_0$
- parameterize everything ignored as random noise

$$\frac{d\vec{x}}{dt} = \mathbf{A}\vec{x} + \text{noise}$$

- $\vec{x}$ is an N dim vector, departure from mean
- $\mathbf{A}$ is an N x N matrix

- simplest noise process: additive Gaussian white noise

$$\frac{d\vec{x}(t)}{dt} = \mathbf{A}\vec{x}(t) + \mathbf{F}\vec{\xi}(t)$$

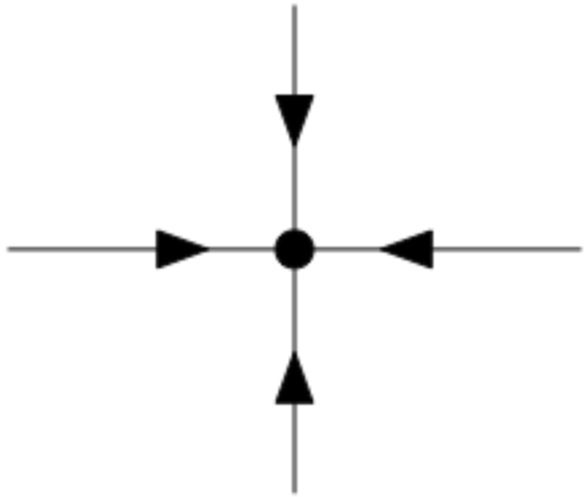
$$\left\langle \vec{\xi}(t) \right\rangle = 0$$

$$\left\langle \vec{\xi}(t)\vec{\xi}^T(s) \right\rangle = \mathbf{I}\delta(t-s)$$

- N x N diffusion matrix

$$\mathbf{D} = \mathbf{F}\mathbf{F}^T/2$$

- For this to make sense, deterministic part must be a stable fixed point



- Adding noise results in a statistically steady distribution governed by
  - deterministic dynamics $\mathbf{A}$
  - diffusion $\mathbf{D}$

- Traditionally in statistical mechanics, noise parameterizes fast molecular motion
- Here, separation of timescales assumed
  - slow motion regarded as deterministic
  - fast motion parameterized as noise
- Some rigorous results, but work still needed *(e.g. Majda, et al.)*

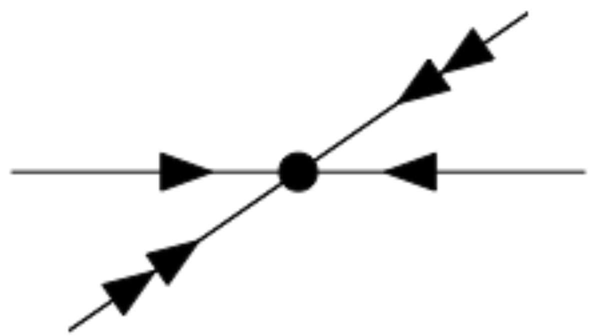
Constructing stochastic models

Two common approaches:

1. Start with known nonlinear equations
   - linearize about fixed point
   - add unknown random noise
   - typically assume $\mathbf{D} = \mathbf{I}$ from ignorance
   - rigorous reduction would give noise
2. Create matrices from data
   - fit time average correlations

Non-normality

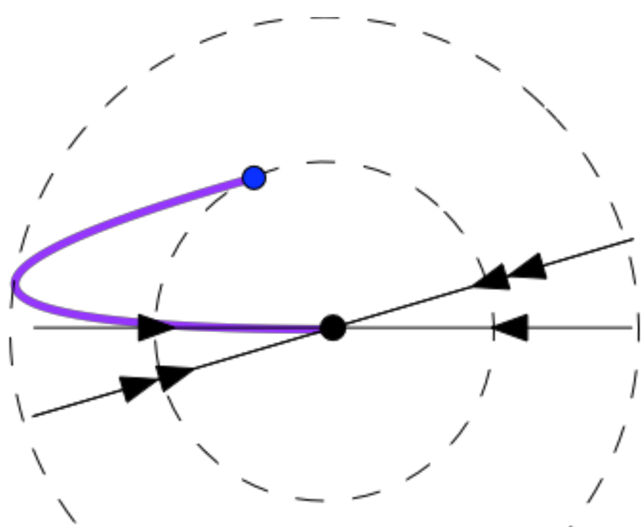
- Many papers focus on non-normality of deterministic dynamics
  - $\mathbf{A}\mathbf{A}^T - \mathbf{A}^T\mathbf{A} \neq 0$
  - eigenvectors not orthogonal

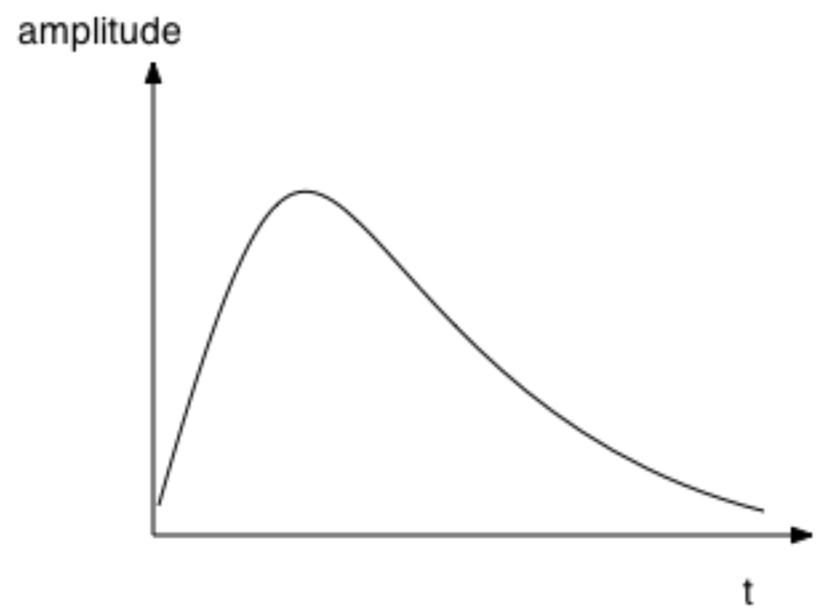


- first sign: multivariate linear system has complexity due to matrix non-commutativity

Transient amplification

- consider deterministic trajectories
- stable fixed point:
  - all perturbations eventually decay to zero
- non-orthogonal eigenvectors:
  - interference
  - transient growth





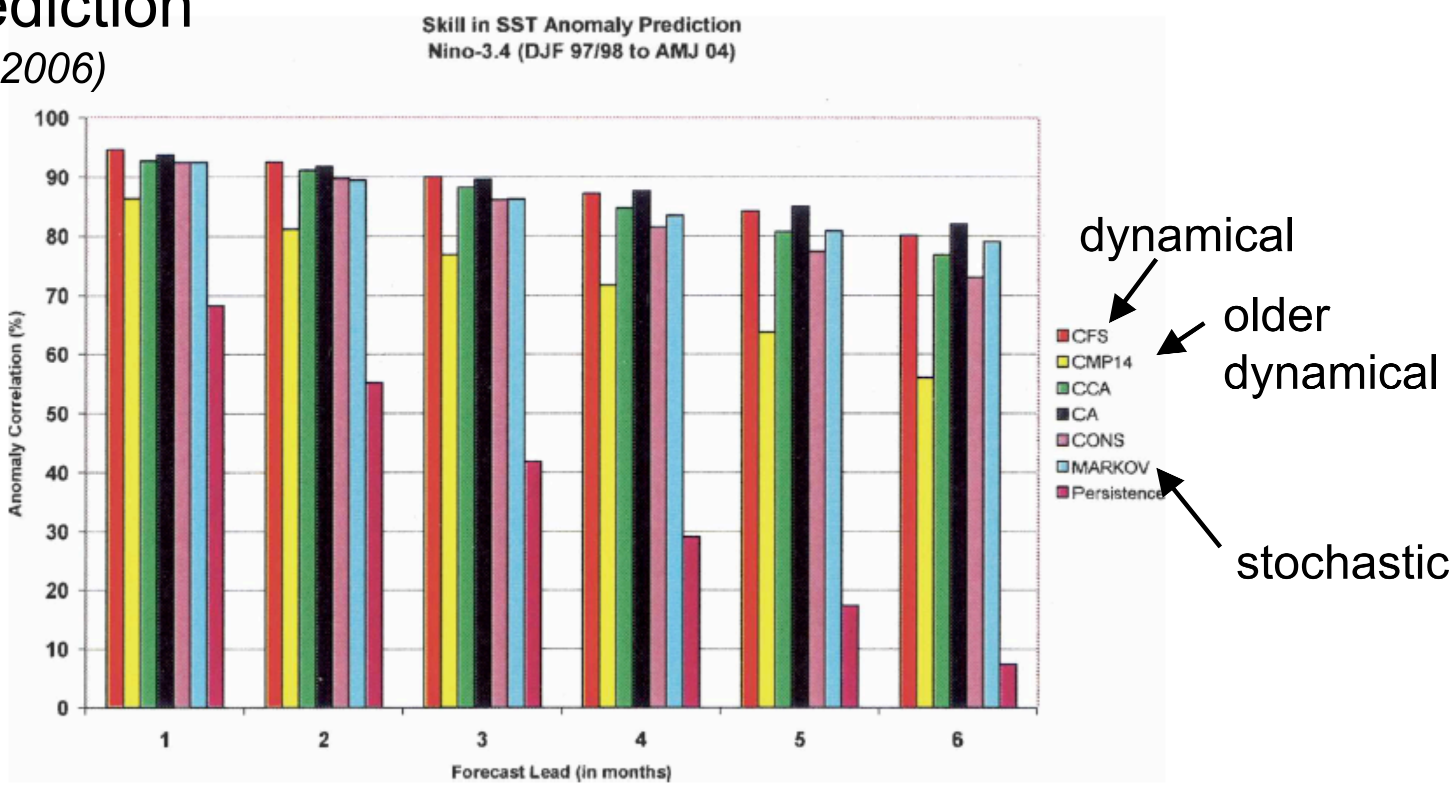
Two views of climate fluctuations

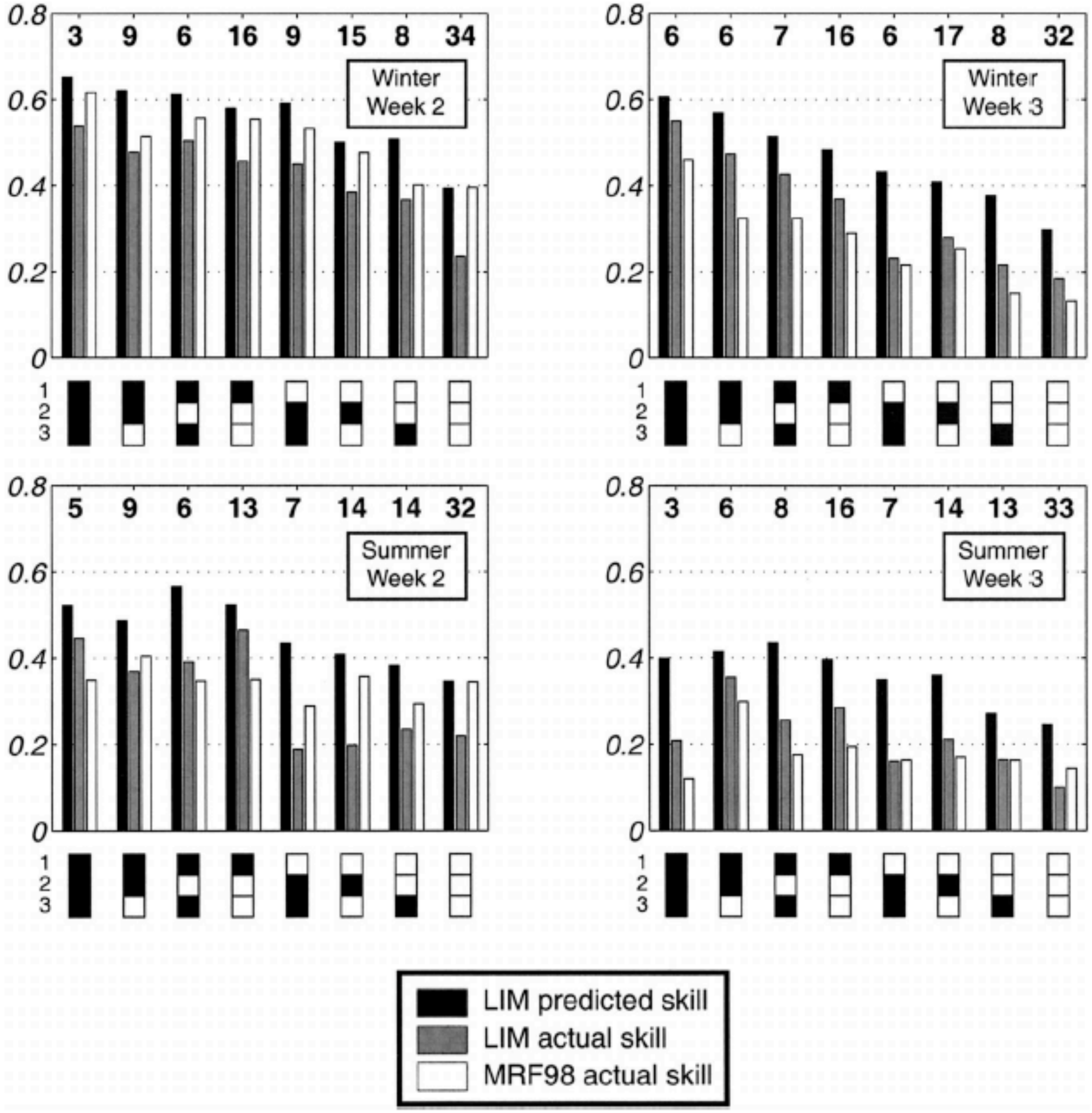
- Instabilities and nonlinear dynamics
- Transient amplification of small noise
- lots of fireworks
- reconciliation, at least for El-Niño:
  - near a bifurcation, indistinguishable
  - bifurcation parameter: Atmosphere-ocean coupling
- mystery:
  - why is El-Niño near a Hopf bifurcation?

Model evaluation

- simple stochastic models do surprisingly well

SST Prediction
(Saha, et al 2006)





250 hPa pattern

(Newman, et al 2003)

theoretical objection

- appropriate coordinate transformation
  - renders eigenvectors orthogonal
  - removes transient amplification
- real physical phenomena should not depend on arbitrary coordinate choice
- climate fluctuations do not “look” random
  - have well-defined life-cycles

Detailed Balance and Climate

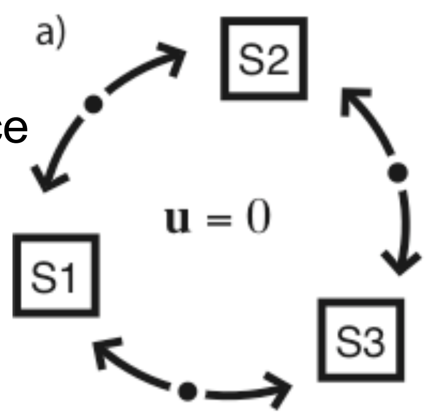
(Weiss, 2003)

- non-normality not a fundamental property
- fundamental coordinate-invariant property is detailed balance
- two kinds of steady states

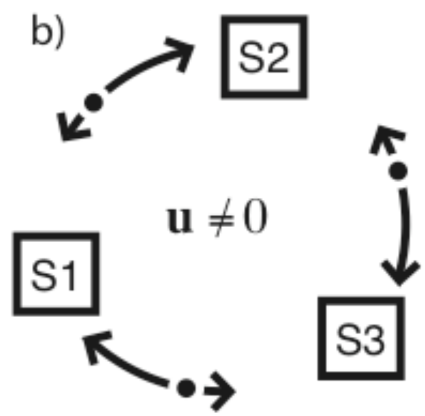
thermodynamic equilibrium

detailed balance satisfied

no current



thermodynamic non-equilibrium



detailed balance violated

nonzero current

- For systems

$$\frac{d\vec{x}(t)}{dt} = \mathbf{A}\vec{x}(t) + \mathbf{F}\vec{\xi}(t)$$

- equivalent statements:
  - detailed balance is violated
  - $\mathbf{AD} - \mathbf{DA}^T \neq 0$
  - cannot mutually diagonalize $\mathbf{A}$ and $\mathbf{D}$
    - essential multi-dimensional character
  - *nonequilibrium* steady-state

Probability Current

- nonequilibirum steady-state maintained by a probability current

$$\frac{\partial p}{\partial t} + \nabla \cdot (\vec{u}p) = 0$$

- phase space velocity

$$\vec{u} = \mathbf{\Omega}\vec{x}$$

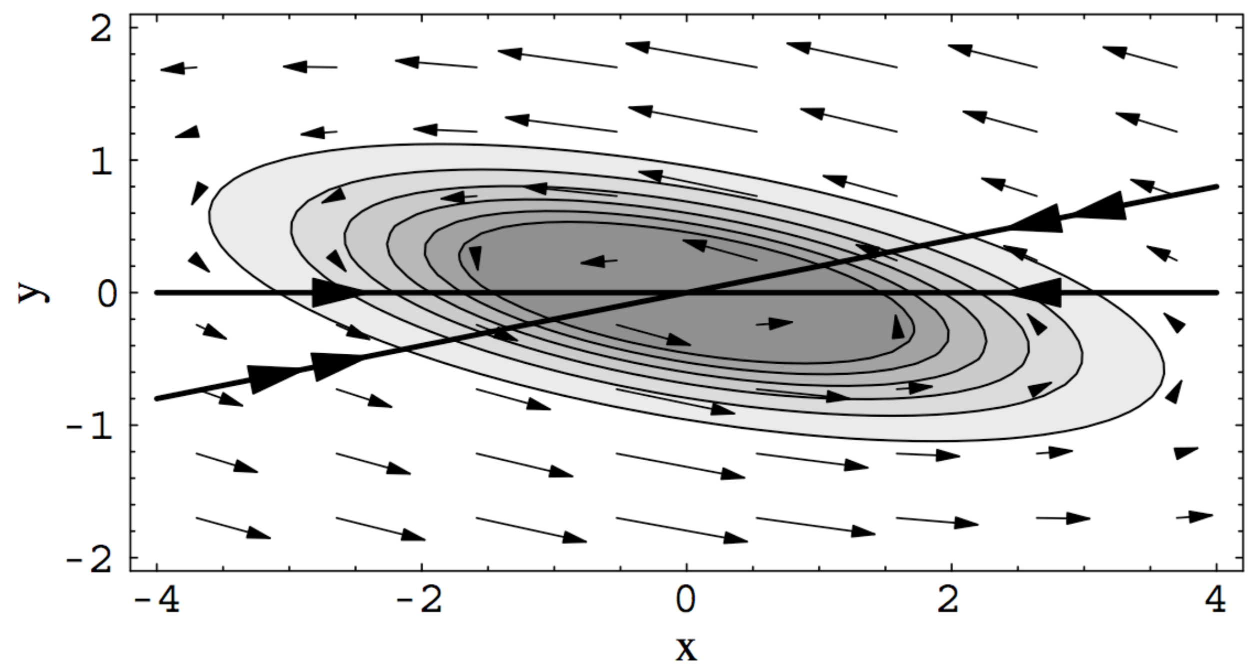
- Ω = matrix of frequencies
  - reminiscent of Landau’s theory of turbulence

Example

$$\mathbf{A} = \begin{pmatrix} -1 & -\cot\theta \\ 0 & -2 \end{pmatrix}, \quad \mathbf{D} = \mathbf{I}/2$$

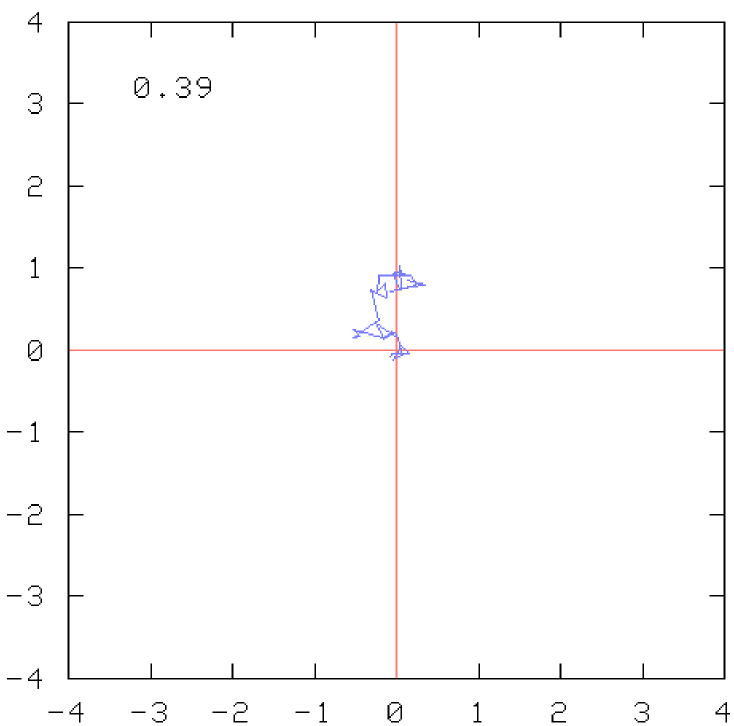
- detailed balance: $\theta = \pi/2$

steady-state pdf
and
probability current

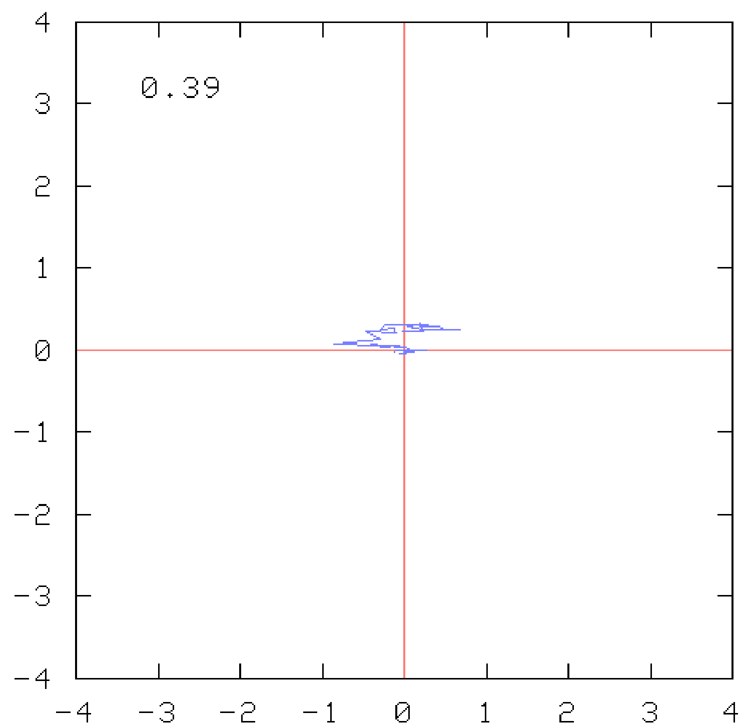


- equilibrium vs. nonequilibrium
- identical steady-state pdf's:
  - isotropic Gaussian; identity covariance

equilibrium



nonequilibrium



Nonequilibrium Statistical Mechanics

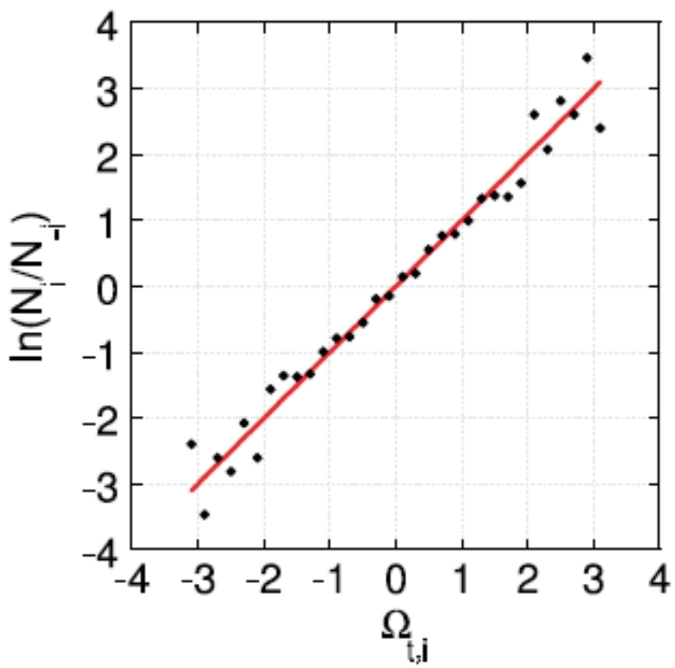
- stat mech has made significant progress in past decade
  - review articles: Evans and Searles, 2002; Seifert, 2007; Gallavotti, 2007
- Fluctuation Theorems:
  - many kinds related in various ways
  - many scenarios
    - steady-state, transient, forced, …
  - different classes of dynamics
    - Langevin, discrete processes, chaotic nonlinear, Hamiltonian, quantum, …
  - confused and confusing literature

Steady-state Fluctuation Thm

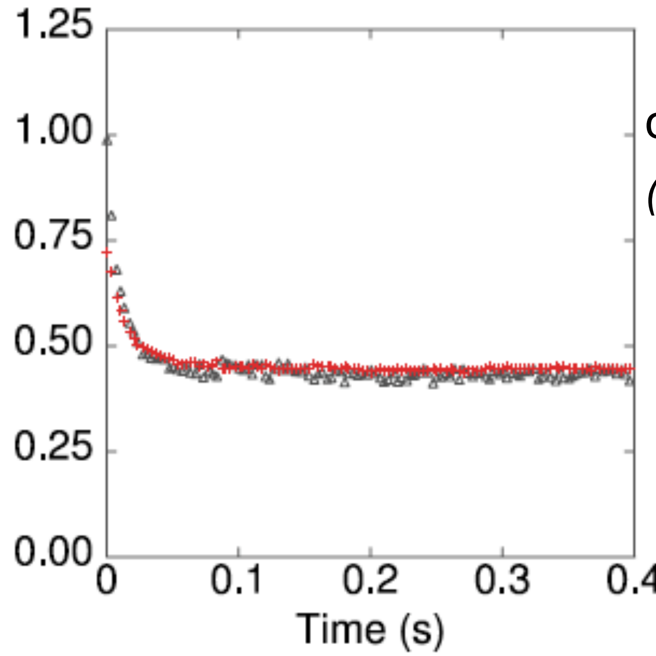
- FT describe probability of finding "violations" of 2nd Law of Thermodynamics
- non-equilibrium steady state maintained by dissipating heat => entropy production
- consider trajectory segment with time t
- trajectory has entropy change Δs
- $p(\Delta s)$ prob of finding trajectory with Δs

$$\frac{p(-\Delta s)}{p(\Delta s)} = \exp(-\Delta s)$$

- previously applied to mircoscopic systems
  - molecular motors
  - chemical reactions
  - molecular dynamics of fluids
- numerous experimental verifications



(a)



(b)

colloid in optical trap

(Carberry, et al, 2004)

Surprising Conclusion

- stochastic climate models are a good approximation to reality
- stochastic climate models are in class of models covered by noneq. stat mech.
- recent advances in nonequilibrium stat mech apply to global scale climate phenomena

Structure of climate fluctuations

(Weiss, 2007)

- Fluctuation Theorems provide constraint on probability distributions
- For microscopic systems, interested in averages over many fluctuations
- For climate, care about individual fluctuations as well as averages
- Look at properties not constrained by Fluctuation Theorem

Irreversibility

- trajectory probability $p(x_0,x_1,t)$
  - different than traditional probabilities $p(x)$, $p(x_1,t \mid x_0)$
- straightforward operational definition based on a timeseries
- irreversibility

$$r(x_1, x_0, t) = \ln\left(\frac{p(x_0, x_1, t)}{p(x_1, x_0, t)}\right)$$

irreversibility pdf p(r)

- Fluctuation Theorem

$$\frac{p(-r)}{p(r)} = \exp(-r)$$

- want details of p(r) as a function of parameters of stochastic model: $\mathbf{A}, \mathbf{D}$
- linear model:
  - can write solutions formally
  - multi-variate => must solve matrix equations numerically

results

- despite linearity, p(r) is non-Gaussian
- average irreversibility $\langle r \rangle \geq 0$

  for small t: $\langle r \rangle (t) \sim \sigma\, t$

  large t: $\langle r \rangle (t) \to 0$

- hence <r> has a max at some time
  - defines a timescale for fluctuations
- σ is the average entropy production, and is related to the noise amplification
- dimensional reduction:
  - fast modes do not affect p(r)

work in progress

- analyze irreversibility of El-Niño data
- data is three-month averages of SST
- entropy production gives correct Lyapunov timescale of chaotic atmosphere: few days
- quantitative comparisons in progress
- analyze other climate fluctuations next
- model climate change as forced change in steady state

Broad Conclusions

- Climate variability poorly understood but crucial for current and future climate impacts
- Can be modeled as fluctuations about a nonequilibrium steady-state
- Recent and future progress in statistical mechanics has implications for climate
- Stat mech may have something to say about climate change