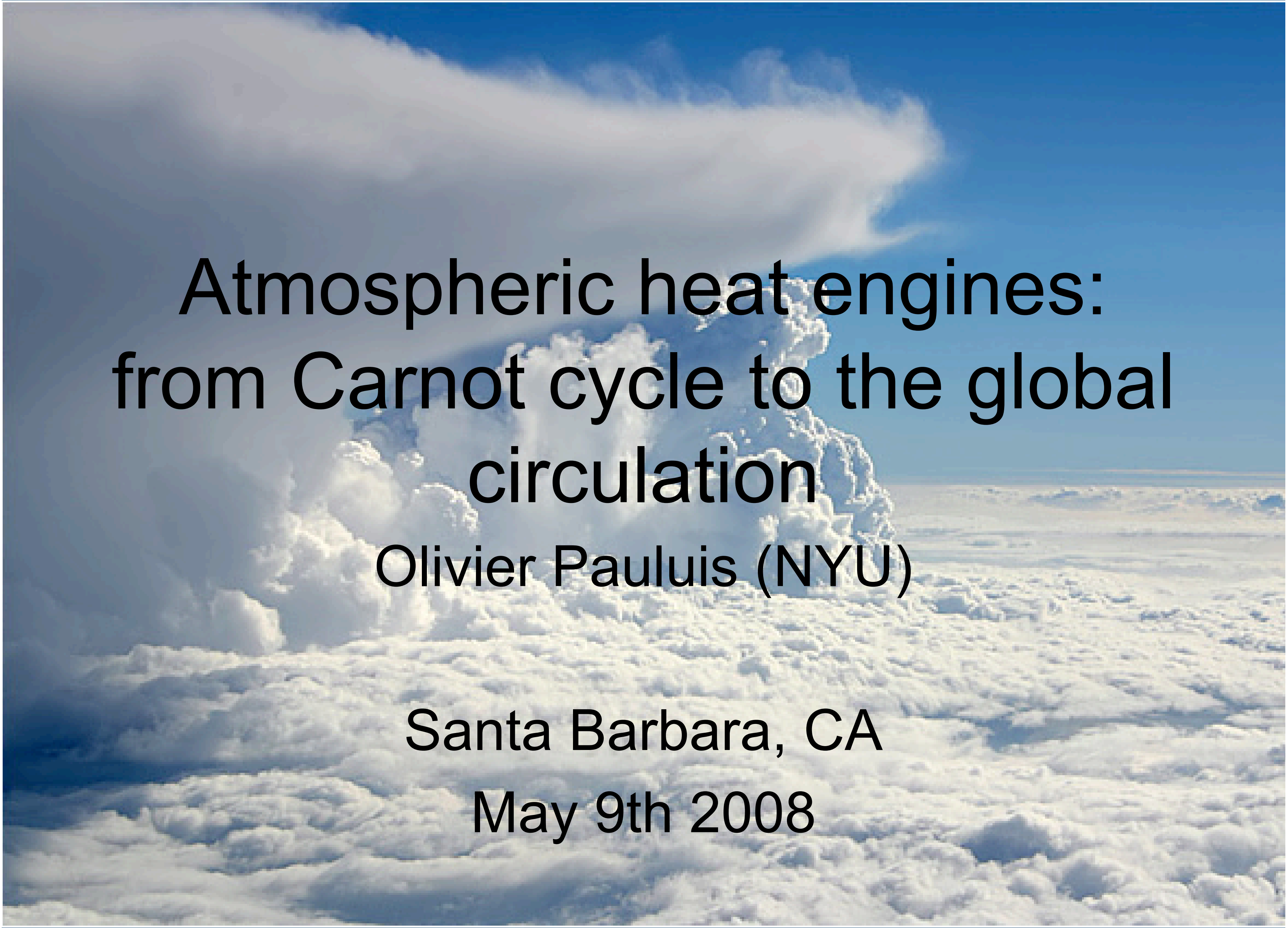

Atmospheric heat engines: from Carnot cycle to the global circulation

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Outline

- Introduction
- Carnot cycle and humidifier
- Irreversible thermodynamics and mechanical efficiency
- The global circulation as a combination of dehumidifier and humidifier

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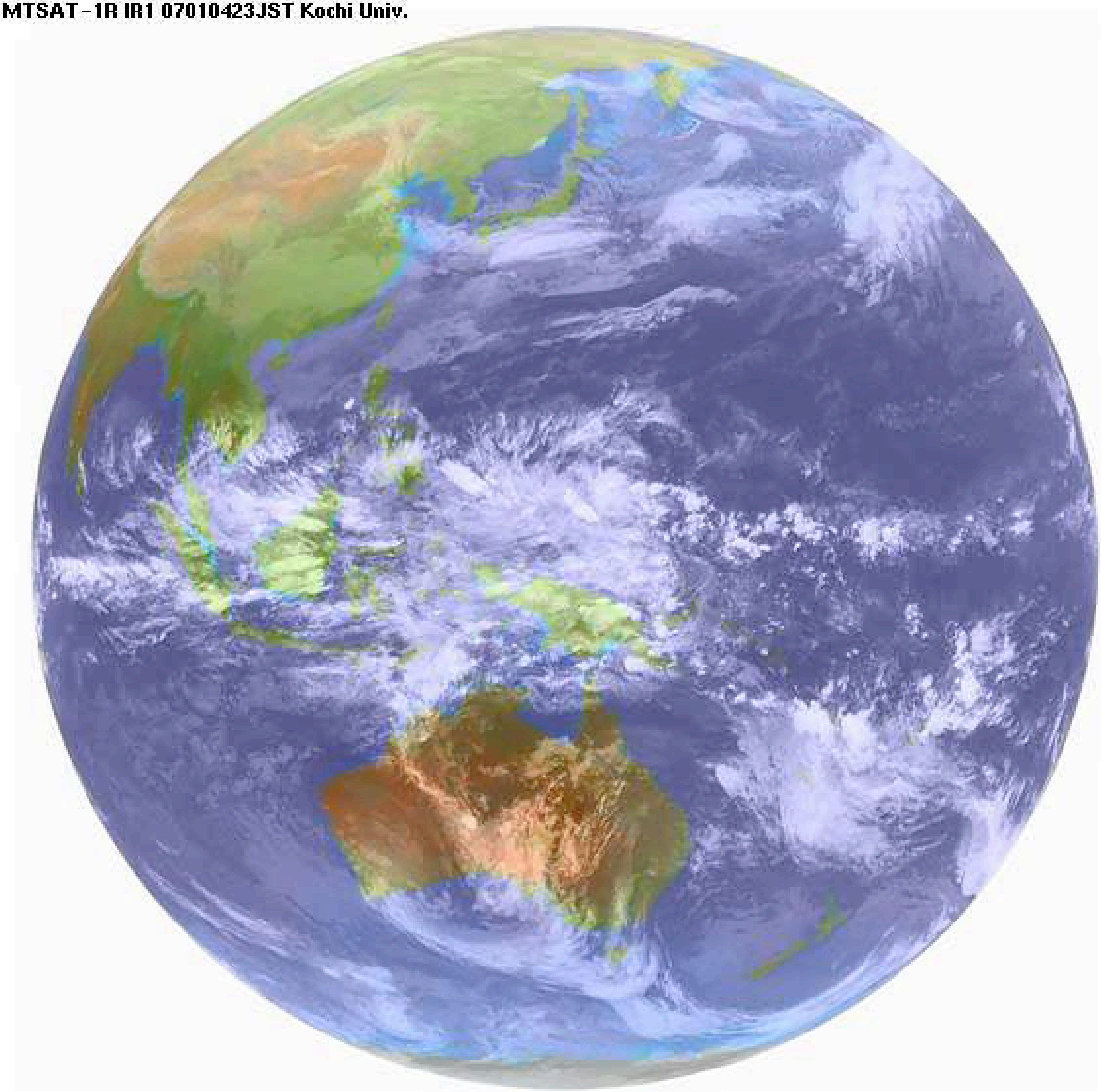
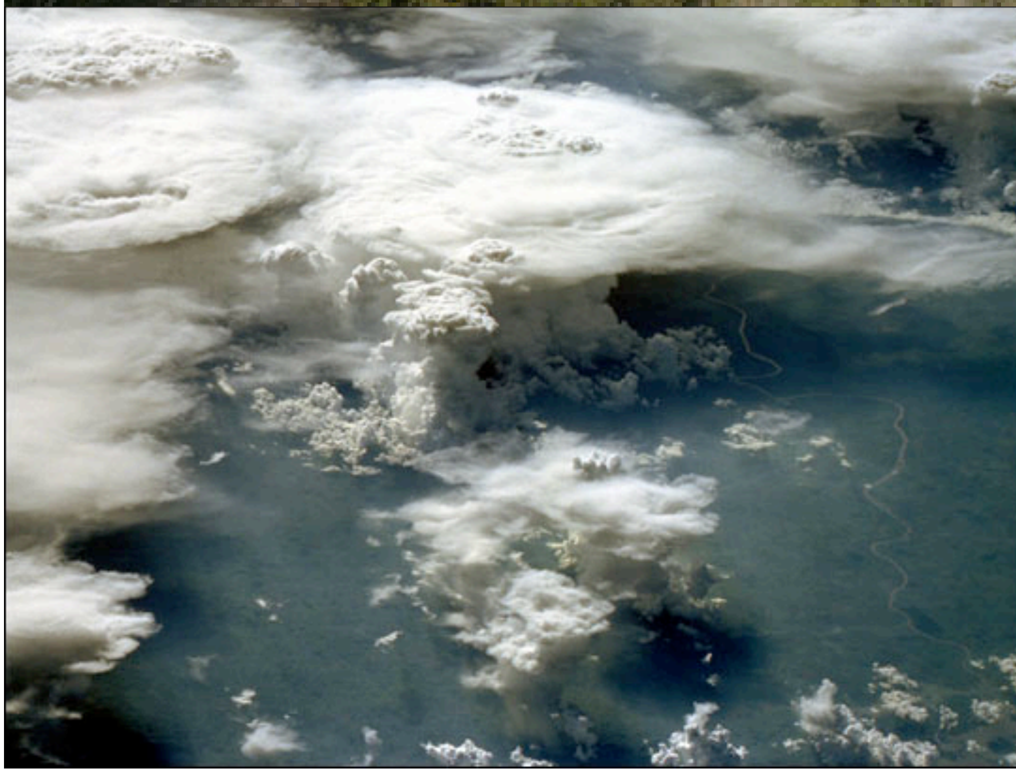



Entropy of moist air

- Moist air can be treated as an ideal mixture of dry air, water vapor and liquid water. The entropy per unit mass of dry air S is then:

$$S = s_d + r s_v + r_l s_l$$

With S: entropy per unit mass of dry air;

r: mixing ratio (water vapor concentration)

r_l: mixing ratio for condensed water

$r_T = r + r_l$: mixing ratio for total water

$s_d, s_v. s_l$: specific entropy for dry air, water vapor and liquid water

- The specific entropies are defined up to an additive constant:

$$s_d = C_{pd} \ln \frac{T}{T_o} - R_d \ln \frac{p_d}{p_o} + s_{d0}$$

$$s_v = C_{pv} \ln \frac{T}{T_o} - R_v \ln \frac{e}{e_o} + s_{v0}$$

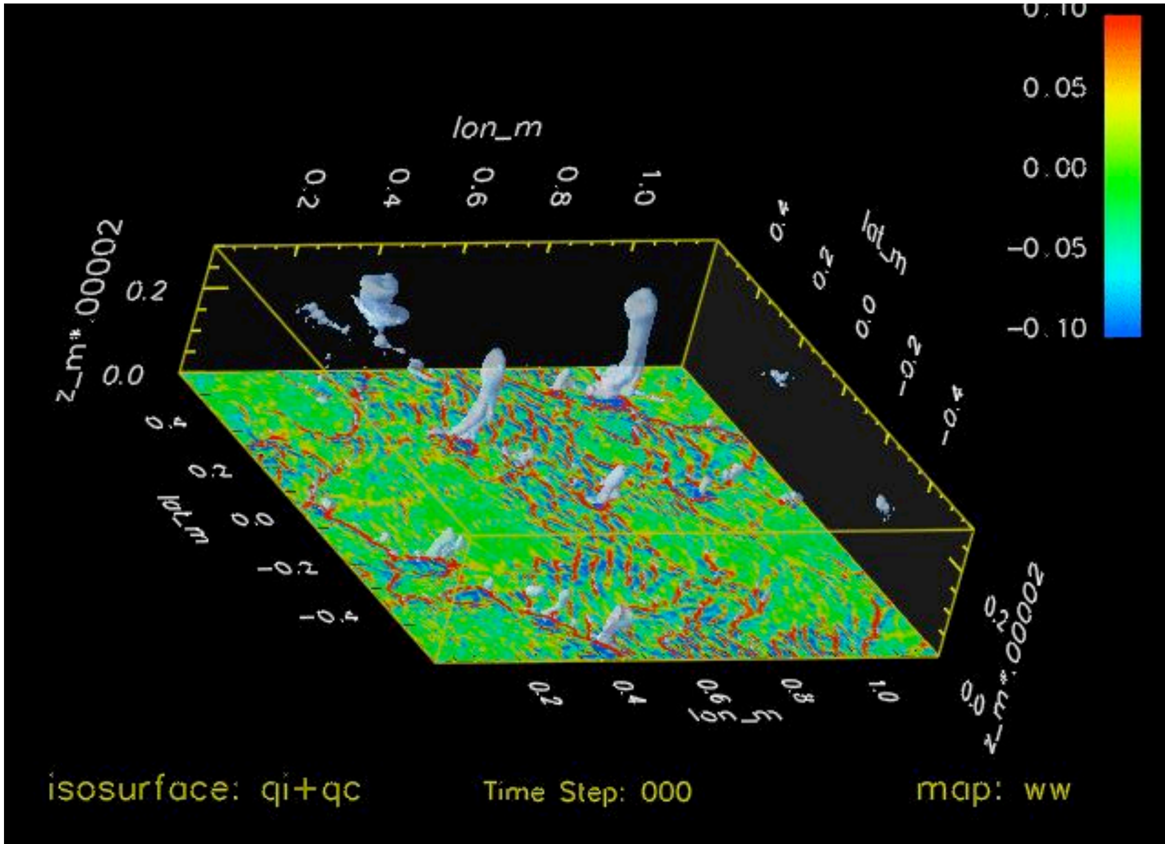
$$s_l = C_l \ln \frac{T}{T_o} + s_{l0}$$

- We cannot put all the integration constant to 0 because the entropy of water vapor and liquid water must be such that:

$$s_v - s_l = \frac{L_v}{T} \text{ at saturation } (e = e_s(T) \text{ or } H = 1)$$

- 'Moist entropy *S*': set $s_{l0} = s_{d0} = 0$

$$\rightarrow S = (C_{pd} + r_T C_l) \ln \frac{T}{T_0} + R_d \ln \frac{p_d}{p_0} + r\left(\frac{L_v}{T} - R_v \ln H\right)$$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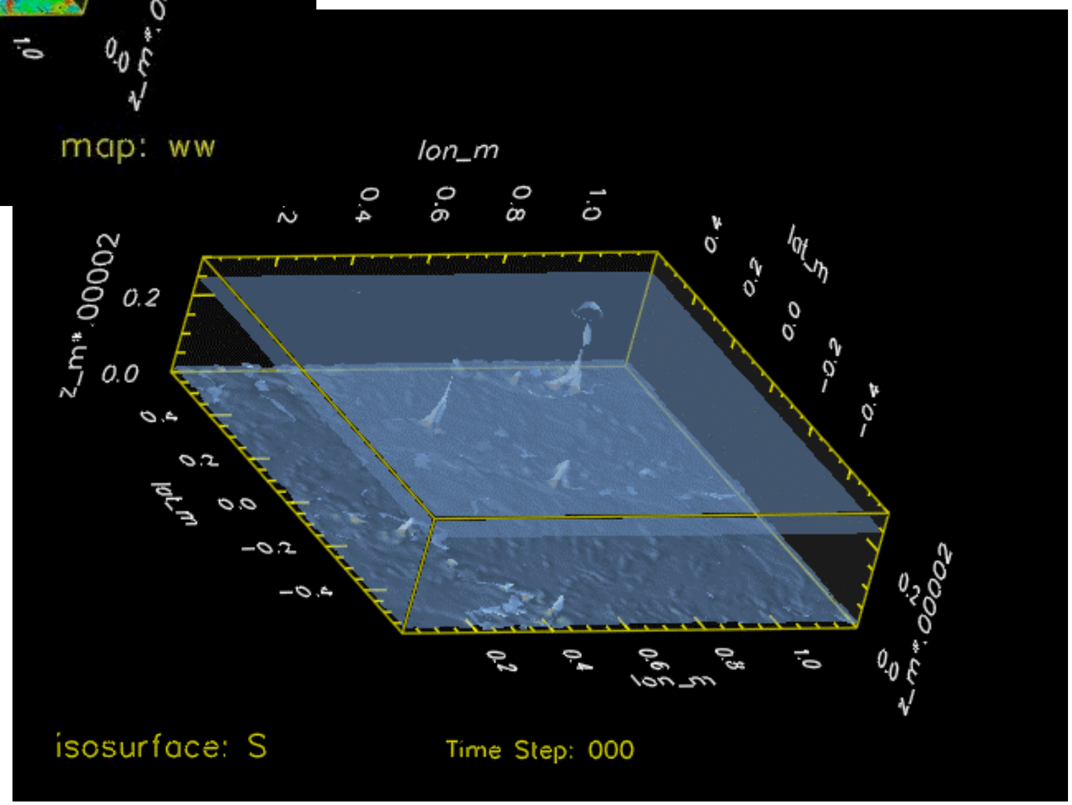
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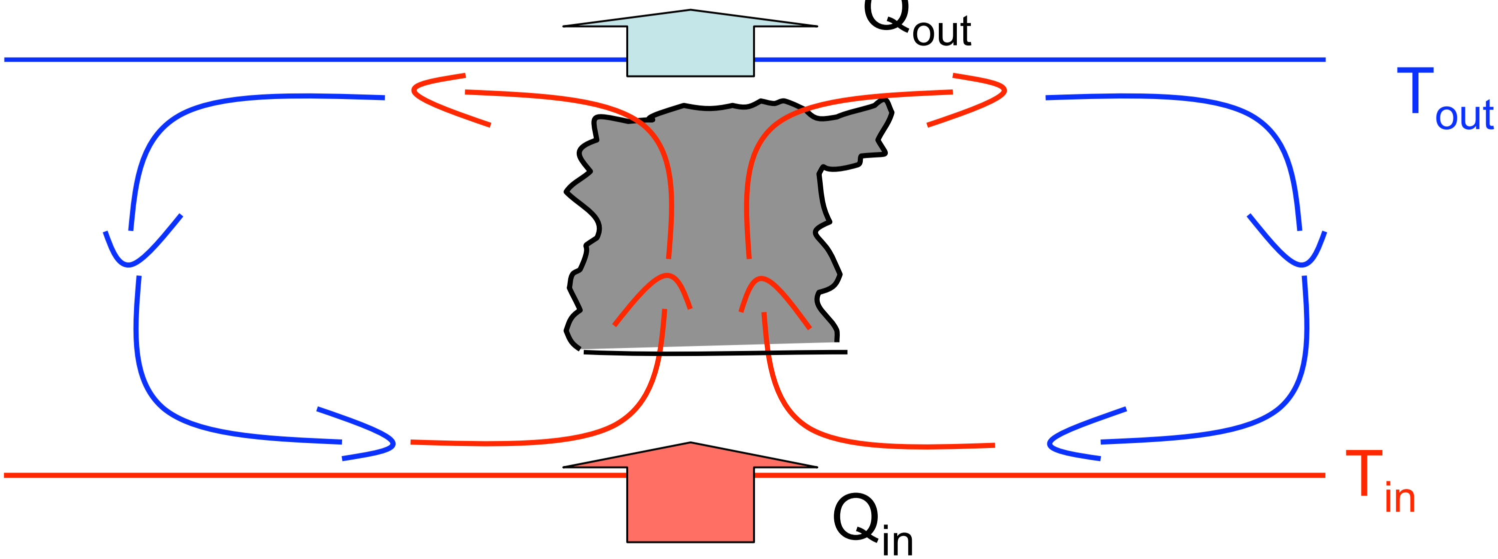
I. Carnot cycle and humidifier



- Idealized problem: convection transport water vapor and energy upward from a warm/moist source to a dry/cold sink.
- Situation is analogous to shallow, non-precipitating convection.

Carnot cycle

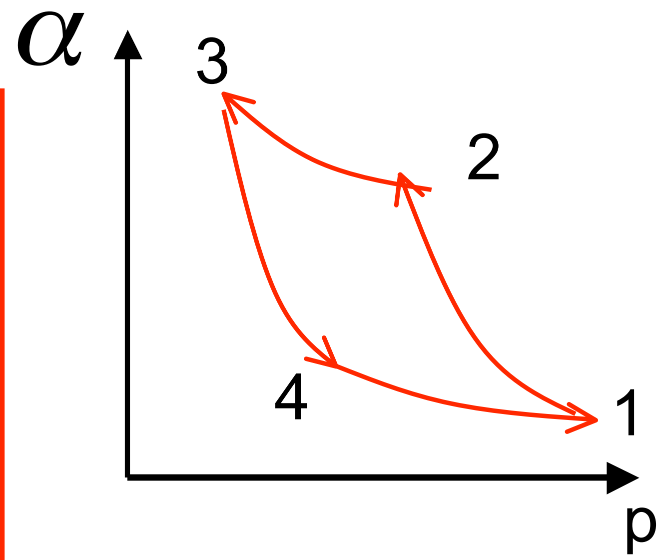
$1 \rightarrow 2$: isothermal expansion at T_{in}

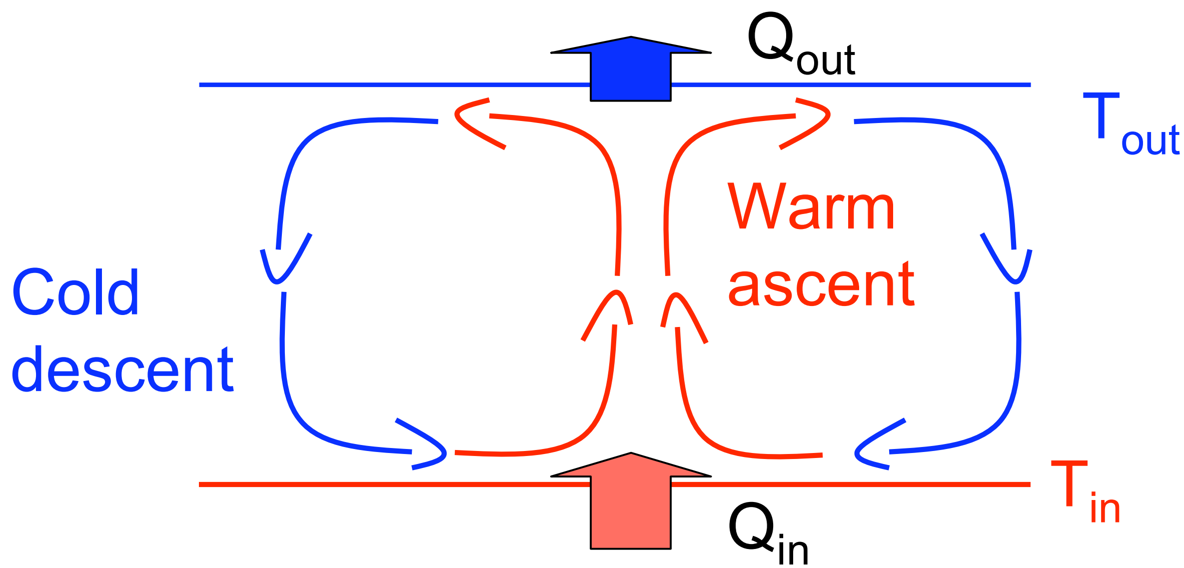
$2 \rightarrow 3$: adiabtic expansion with $S_2 = S_3$

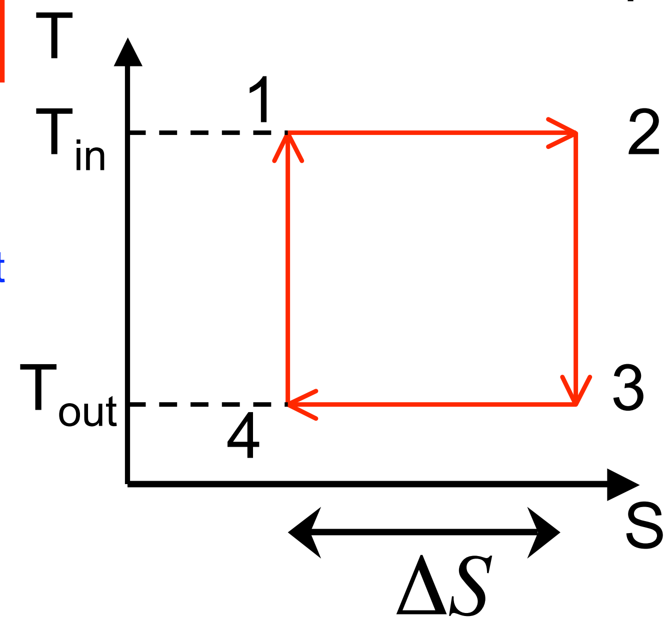
$3 \rightarrow 4$: isothermal compression at T_{out}

$4 \rightarrow 1$: adiabatic compression with $S_2 = S_3$

Total water content is constant through the entire cycle!







- Mechanical work is defined as

$$W = \oint -\alpha(S, r_T, p)dp$$

- Using the thermodynamic relationship

$$TdS = dh - \alpha dp - \mu dr_T$$

we get:

$$W = \oint TdS + \oint \mu dr_T = (T_{in} - T_{out})\Delta S$$

$dr_T=0$

- External heating

$$\delta Q = dh - \alpha dp = TdS + \mu_v dr_T$$

- Heating at the warm source:

$$Q_{in} = \oint \delta Q^+ = \int_1^2 TdS = T_{in}\Delta S$$

$$\text{Efficieny } \eta_C = \frac{W}{Q_{in}} = \frac{T_{in} - T_{out}}{T_{in}}$$

Humidifier cycle

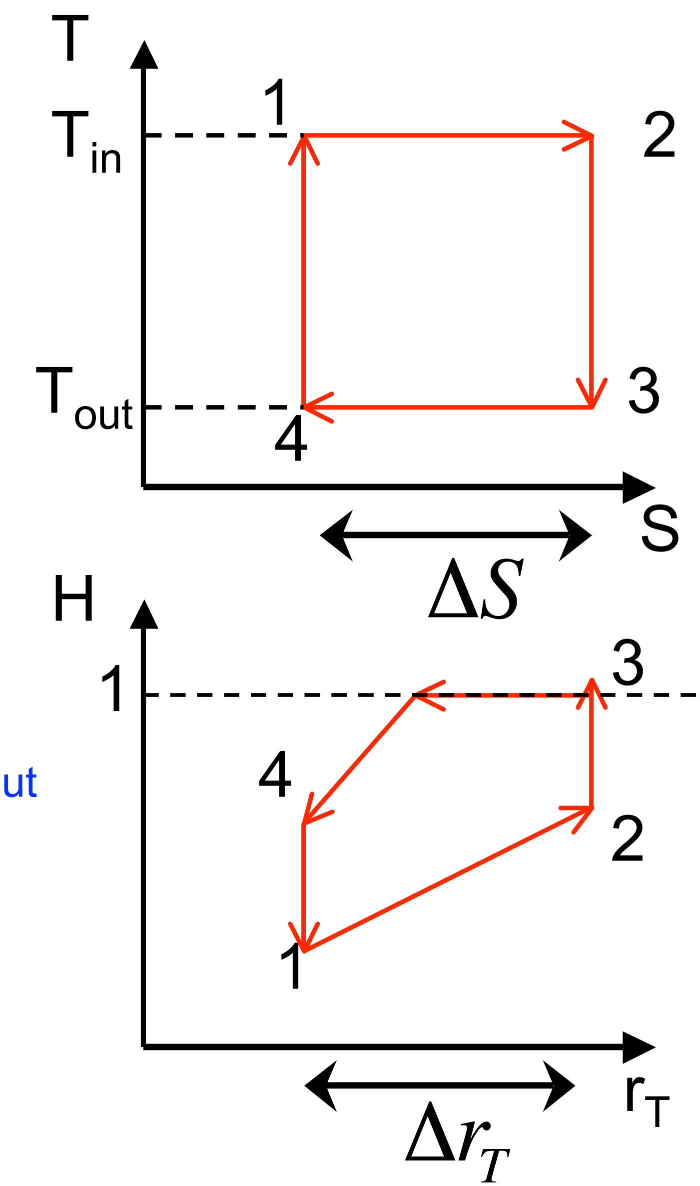
$1 \rightarrow 2$: isothermal moistening at T_{in}

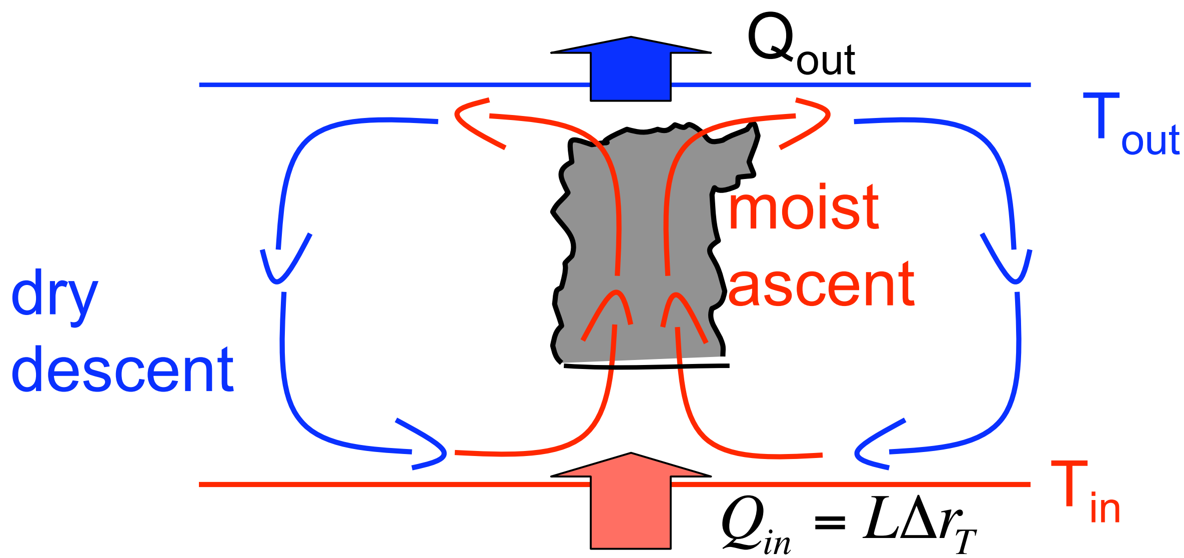
$2 \rightarrow 3$: adiabtic expansion

$3 \rightarrow 4$: isothermal drying at T_{out}

$4 \rightarrow 1$: adiabatic compression

Heating is due now solely to evaporation!





- Mechanical work:

$$W = \oint T dS + \oint \mu dr_T$$

$$= (T_{in} - T_{out})\Delta S + (\mu_{in} - \mu_{out})\Delta r_T$$

- Surface heating: $Q_{in} = T_{in}\Delta S + \mu_{in}\Delta r_T = L\Delta r_T$

- Entropy change: $\Delta S = \left(\frac{L - \mu_{in}}{T_{in}}\right)\Delta r_T$

$\mu = R_v T \ln H$: chemical potenital for water vapor

(aka Gibbs energy per unit of mass)

$$\mu_{in} = R_v T_{in} \ln H_{in}$$

$$\mu_{out} = R_v T_{out} \ln H_{out}$$

$$\text{Efficieny } \eta_H = \frac{W}{Q_{in}} = \frac{T_{in} - T_{out}}{T_{in}} + \frac{R_v T_{out}}{L} \ln \frac{H_{in}}{H_{out}}$$

$$\text{Efficieny } \eta_H = \frac{T_{in} - T_{out}}{T_{in}} + \frac{R_v T_{out}}{L} \ln \frac{H_{in}}{H_{out}}$$

additional term depending on saturation

Carnot efficiency

- The efficiency depends on the state of the system!!!
- Saturated case: *H=1*

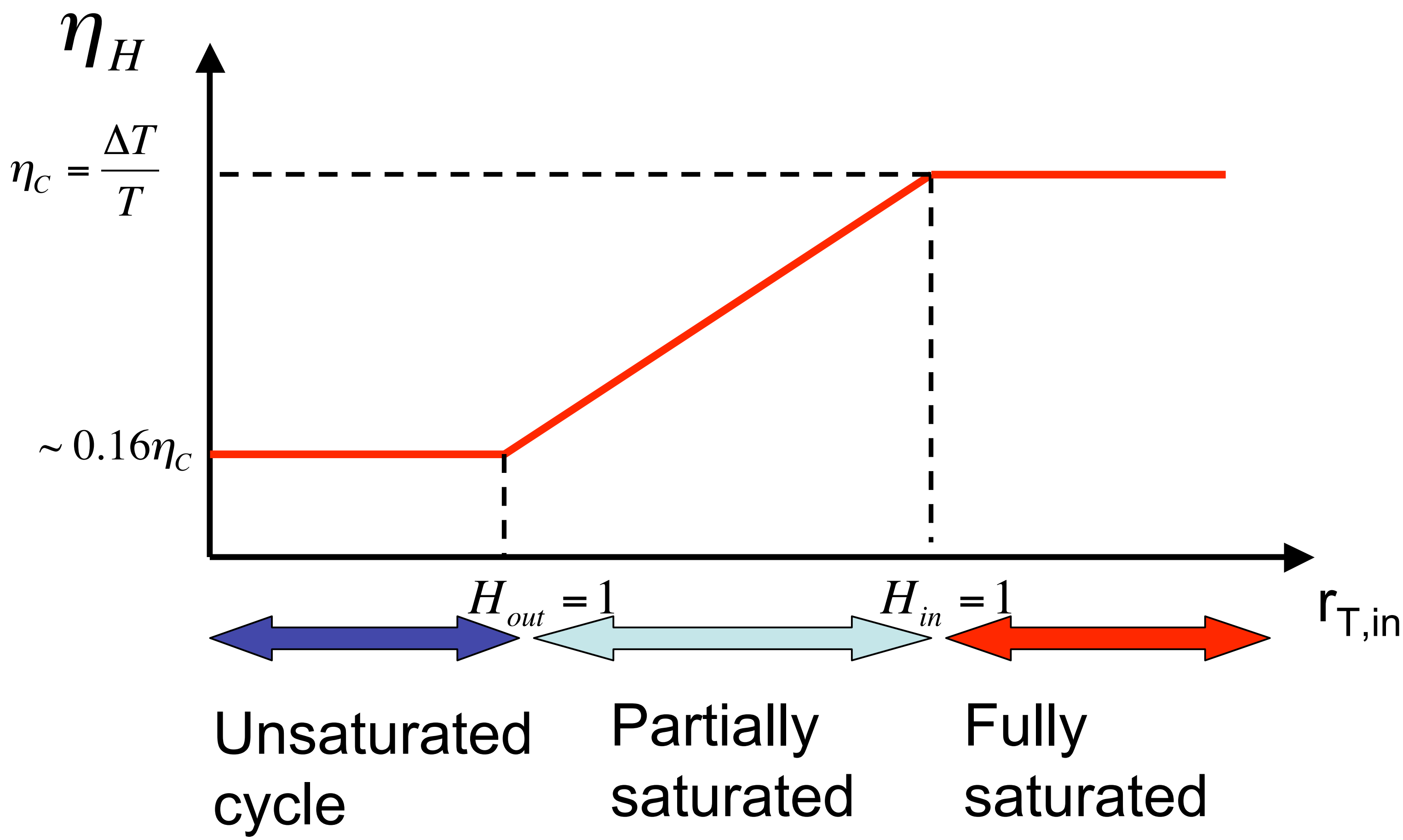
$$\text{Efficieny } \eta_{H,sat} = \frac{W}{Q_{in}} = \frac{T_{in} - T_{out}}{T_{in}}$$

- General case: the relative humidity decreases with pressure, I.e

$$H_{out} \geq H_{in} \rightarrow \eta_H \leq \frac{T_{in} - T_{out}}{T_{in}}$$

- Hence, the efficiency of a humidifier is equal or less than that of a Carnot cycle

- Three regimes:
  - The cycle is unsaturated at all time: efficiency is minimum.
  - The cycle is partially saturated: efficiency increases with amount of water in the cycle.
  - The cycle is saturated at all time: efficiency is maximum and given by the Carnot efficiency



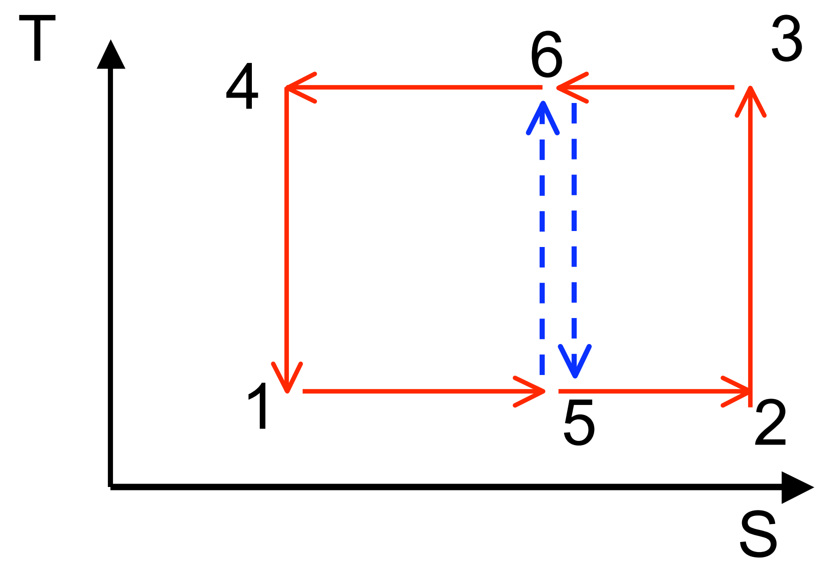
Mixed Carnot-humidifier cycle

$1 \rightarrow 2:$ isothermal heating and moistening at T_{in}

$2 \rightarrow 3:$ adiabtic expansion

$3 \rightarrow 4:$ isothermal cooling and drying at T_{out}

$4 \rightarrow 1:$ adiabatic compression



- Intermediary steps 5 and 6 such that cycle 1-5-6-4 is a humidifier and 5-2-3-6 is a Carnot cycle.

- Latent and sensible heat flux:

$$Q_{lat} = L\Delta r_T$$

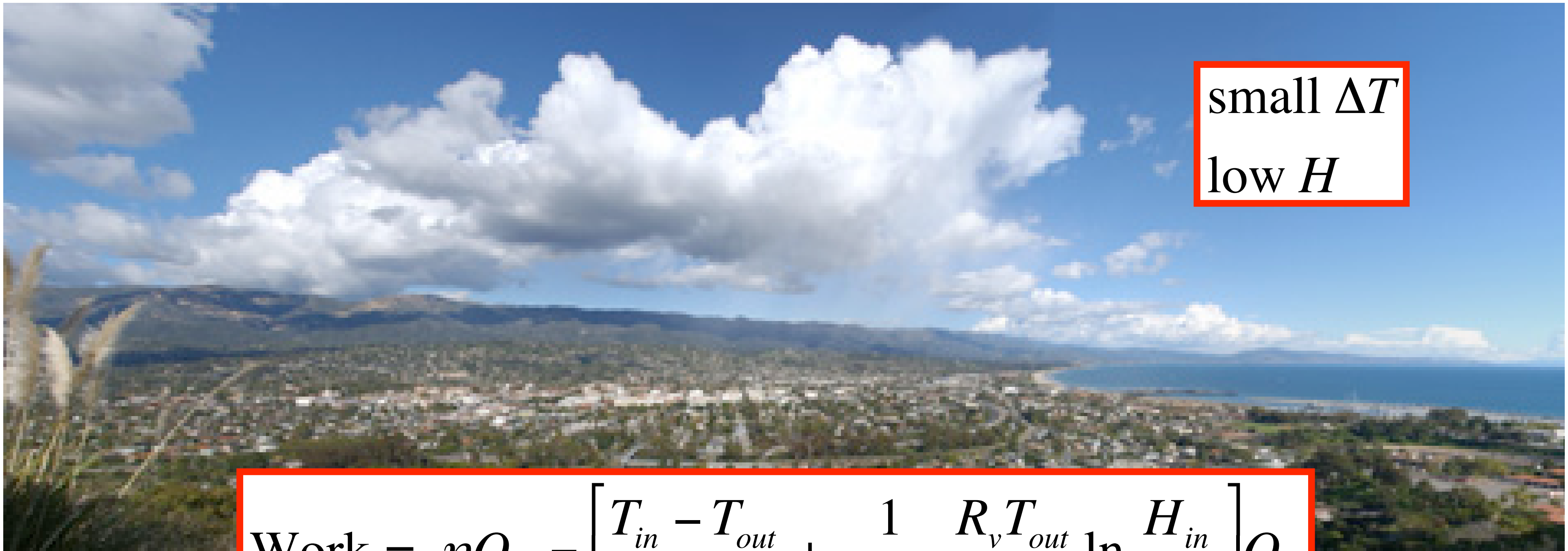
$$Q_{sen} = T_{in}\Delta S + (\mu_{in} - L)\Delta r_T$$

- Bowen ratio:

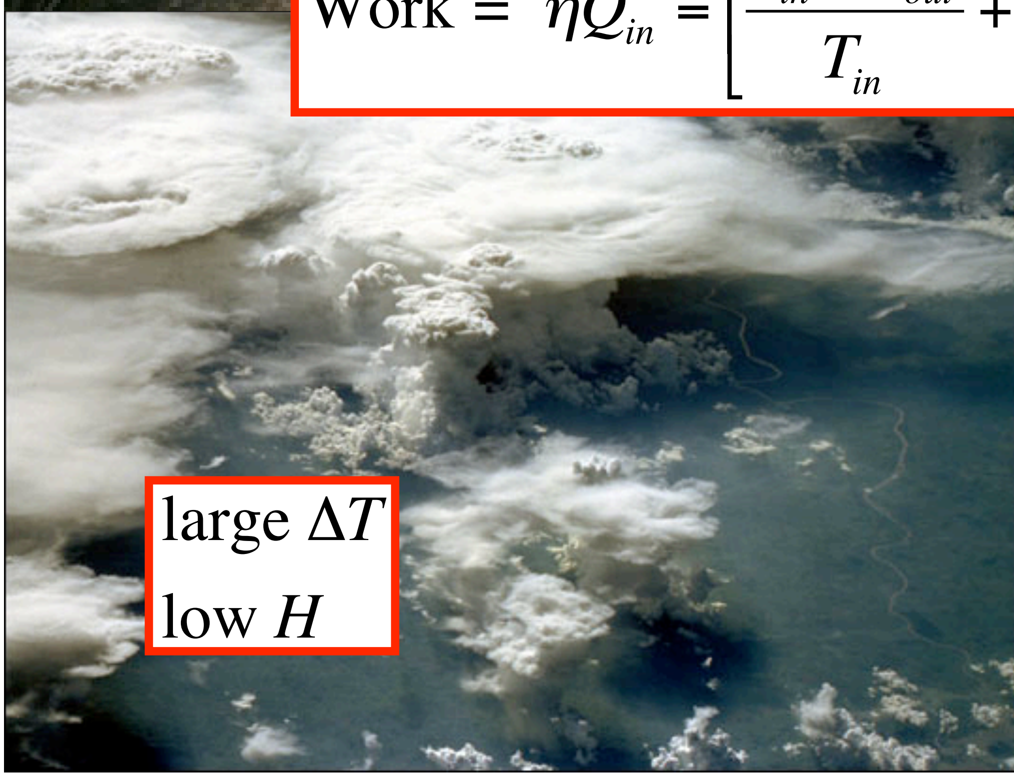
$$B = \frac{Q_{sen}}{Q_{lat}}$$

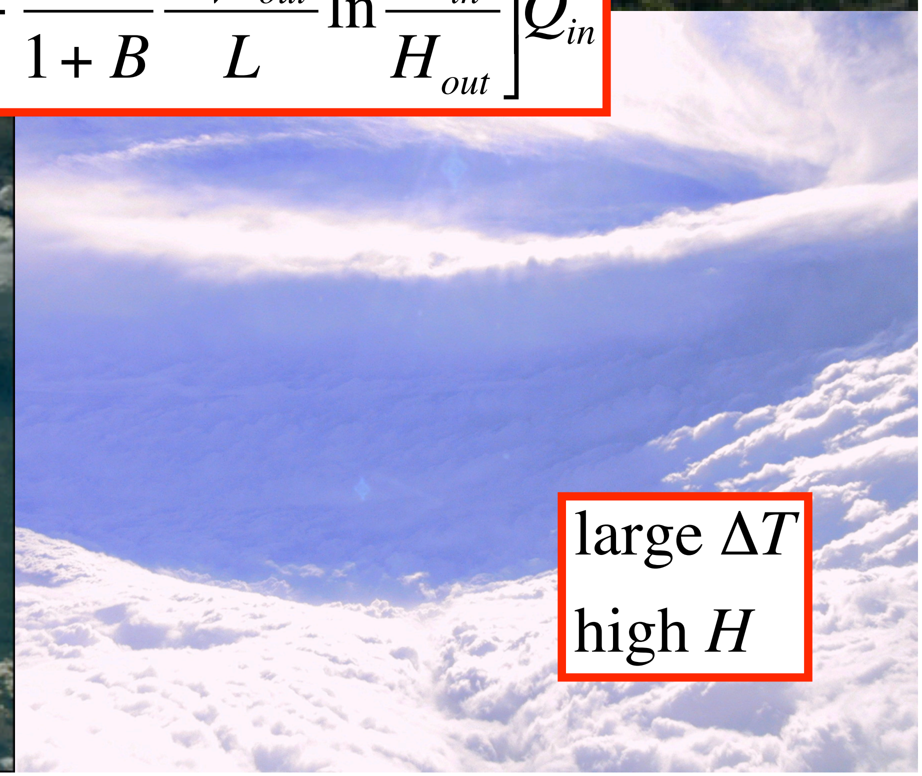
$$\text{Efficieny } \eta = \frac{B}{1+B}\eta_c + \frac{1}{1+B}\eta_H = \frac{T_{in} - T_{out}}{T_{in}} + \frac{1}{1+B}\frac{R_v T_{out}}{L}\ln\frac{H_{in}}{H_{out}}$$

The efficiency of an atmospheric heat engine depends on both its degree of saturation and on the Bowen ratio.



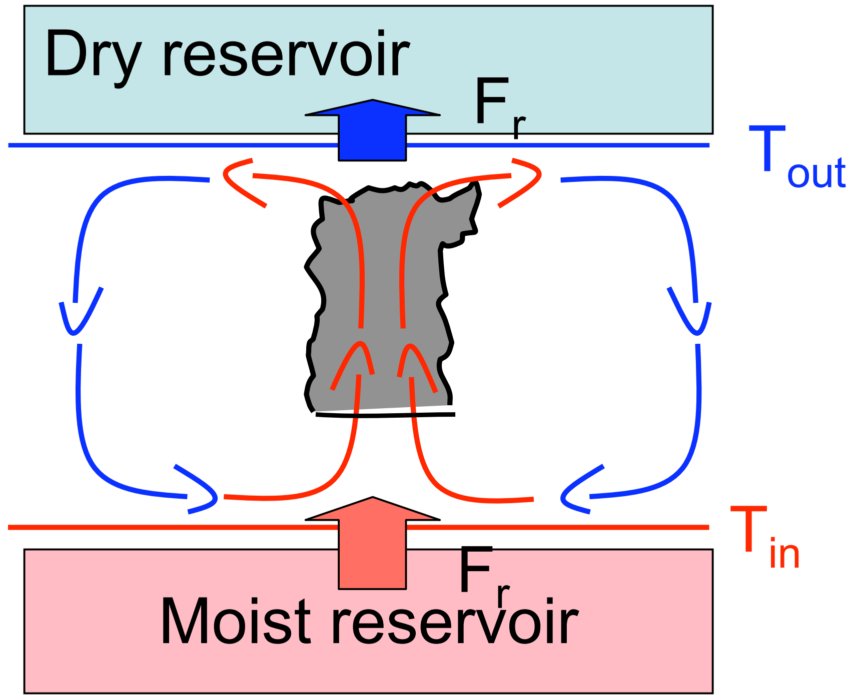
$$\text{Work} = \eta Q_{in} = \left[\frac{T_{in} - T_{out}}{T_{in}} + \frac{1}{1+B}\frac{R_v T_{out}}{L}\ln\frac{H_{in}}{H_{out}}\right] Q_{in}$$





Why saturation affects the efficiency of a humidifier?

- Let us modify the cycle by explicitly including a moist and dry reservoir.
- Assume that these reservoirs are large, i.e. that their (intensive) thermodynamic properties such as temperature, relative humidity, etc… are unaffected by the cycle.
- We force a constant upward flux of water vapor F_r through the layer



Energy budget

- Total energy change:

$$\text{moist reservoir}: \Delta U_m = -LF_r$$
$$\text{cycle}: \Delta U_c = 0$$
$$\text{dry reservoir}: \Delta U_d = +LF_r$$
$$\text{Total change}: \Delta \mathrm{U} = \Delta U_m + \Delta U_c + \Delta U_d = 0$$

- First law of thermodynamics:

$$\Delta U = Q - W \Rightarrow Q = W$$

Entropy budget

- Entropy change:

$$\text{moist reservoir}: \quad \Delta S_m = -\frac{L}{T_{in}} F_r$$

$$\text{cycle}: \quad \Delta S_c = 0$$

$$\text{dry reservoir}: \quad \Delta S_d = \left(\frac{L}{T_{out}} - R_v \ln H_{dry}\right) F_r$$

$$\text{Total change}: \quad \Delta S = \Delta S_m + \Delta S_c + \Delta S_d = \left(\frac{L}{T_{out}} - \frac{L}{T_{in}} - R_v \ln H_{dry}\right) F_r$$

- 2[nd] law of thermodynamics:

$$\Delta S = \frac{Q}{T_{out}} + \Delta S_{irr}$$

$$\rightarrow W = T_{out}(\Delta S - \Delta S_{irr})$$

Loss due to irreversibilities

Thermodynamic forcing depends only on the dry and moist reservoirs

Irreversible entropy production

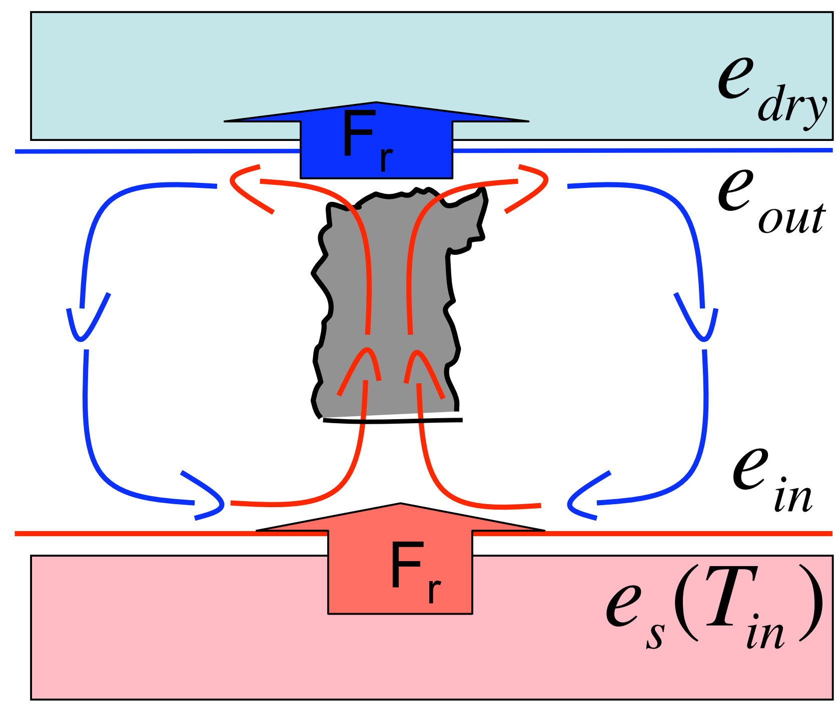
- 2 sources: surface evaporation and diffusion at the top:

$$\Delta S_{irr,sfc} = -R_v F_r \ln H_{in} = R_v F_r \ln \frac{e_s(T_{in})}{e_{in}}$$

$$\Delta S_{irr,top} = R_v F_r \ln \frac{e_{dry}}{e_{out}}$$

$$\rightarrow \Delta S_{irr} = R_v F_r \left[\ln \frac{e_s(T_{in})}{e_{dry}} - \ln \frac{e_{in}}{e_{out}} \right]$$

State of the system affects the irreversible entropy production



- Role of saturation:

$$\ln\frac{e_{in}}{e_{out}} \approx \ln\frac{p_{in}}{p_{out}} \sim \frac{\Delta Z}{8km} \quad \text{(unsaturated case)}$$

$$\ln\frac{e_{in}}{e_{out}} \approx \ln\frac{e_s(T_{in})}{e_s(T_{out})} \sim \frac{\Delta Z}{2.5km} \quad \text{(saturated case)}$$

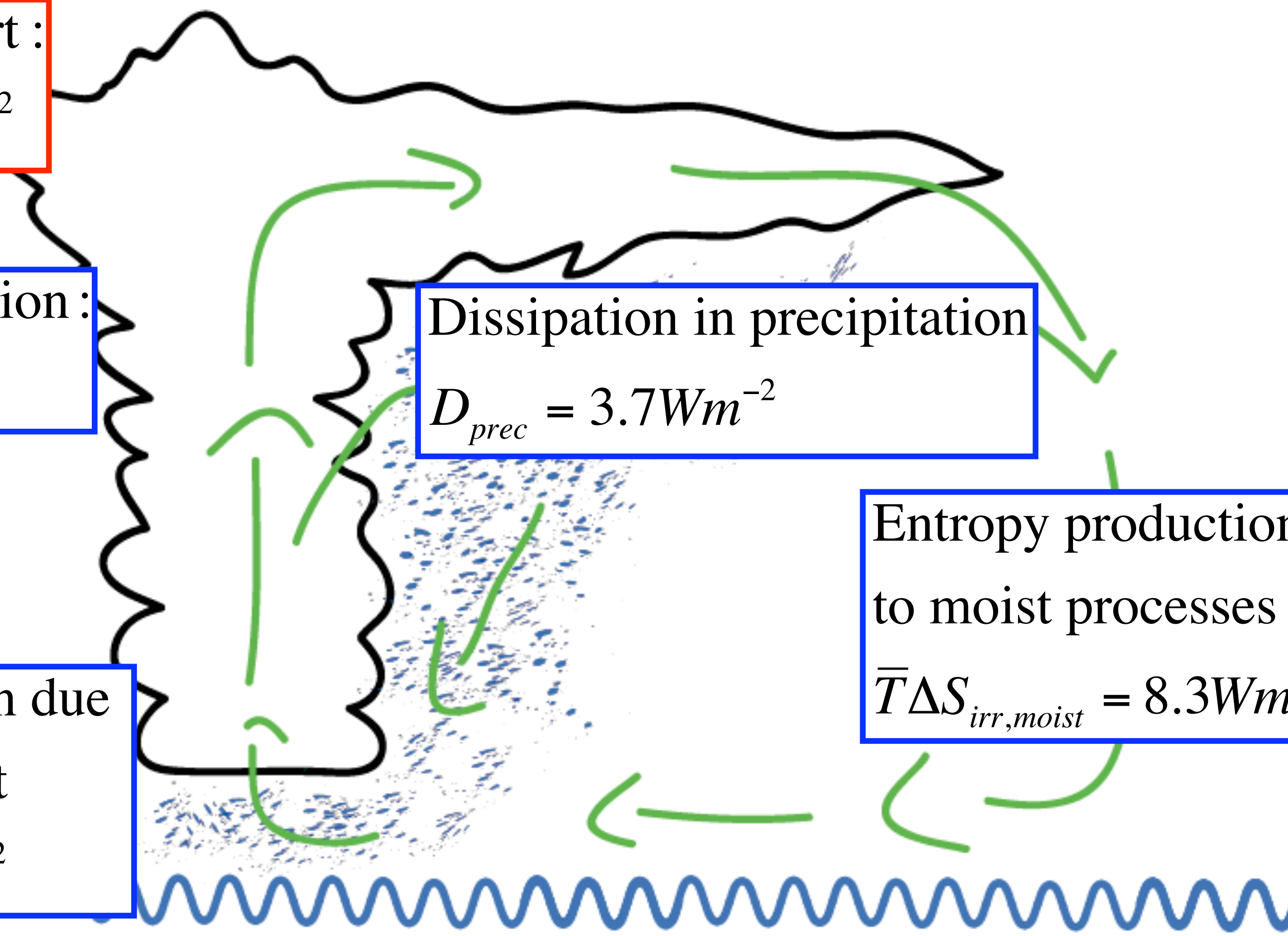
- The partial pressure of water vapor drops much more quickly with height when the atmosphere is saturated than when it is not.
- Hence, the irreversible entropy production will be reduced significantly in a saturated atmosphere.
- This allows the atmosphere to produce more mechanical work

The proof is in the pudding…

$$
\begin{aligned}
W &= T_{out}(\Delta S - \Delta S_{irr}) \\
&= F_r\left[L\frac{T_{in} - T_{out}}{T_{in}} - R_v T_{out} \ln H_{dry} - R_v T_{out}\left(\ln\frac{e_s(T_{in})}{e_{dry}} - \ln\frac{e_{in}}{e_{out}}\right)\right] \\
&= LF_r\left[\frac{T_{in} - T_{out}}{T_{in}} - \frac{R_v T_{out}}{L_v}\ln\frac{H_{in}}{H_{out}}\right] \\
&= LF_r\eta_H
\end{aligned}
$$

- The irreversible thermodynamics approach yields the same expression as the direct calculation (thanks Clausius)…

Entropy budget of moist convection (Pauluis and Held 2002)



Net entropy export: $\overline{T}\Delta S_{exp} = 13.6 Wm^{-2}$

Frictional dissipation: $D_{turb} = 1.0 Wm^{-2}$

Dissipation in precipitation $D_{prec} = 3.7 Wm^{-2}$

Entropy production due to moist processes $\overline{T}\Delta S_{irr,moist} = 8.3 Wm^{-2}$

Entropy production due to diffusion of heat $\overline{T}\Delta S_{irr,dh} = 0.6 Wm^{-2}$

$$\overline{T}\Delta S_{irr} = D + \overline{T}\Delta S_{irr,moist} + \overline{T}\Delta S_{irr,dh}$$

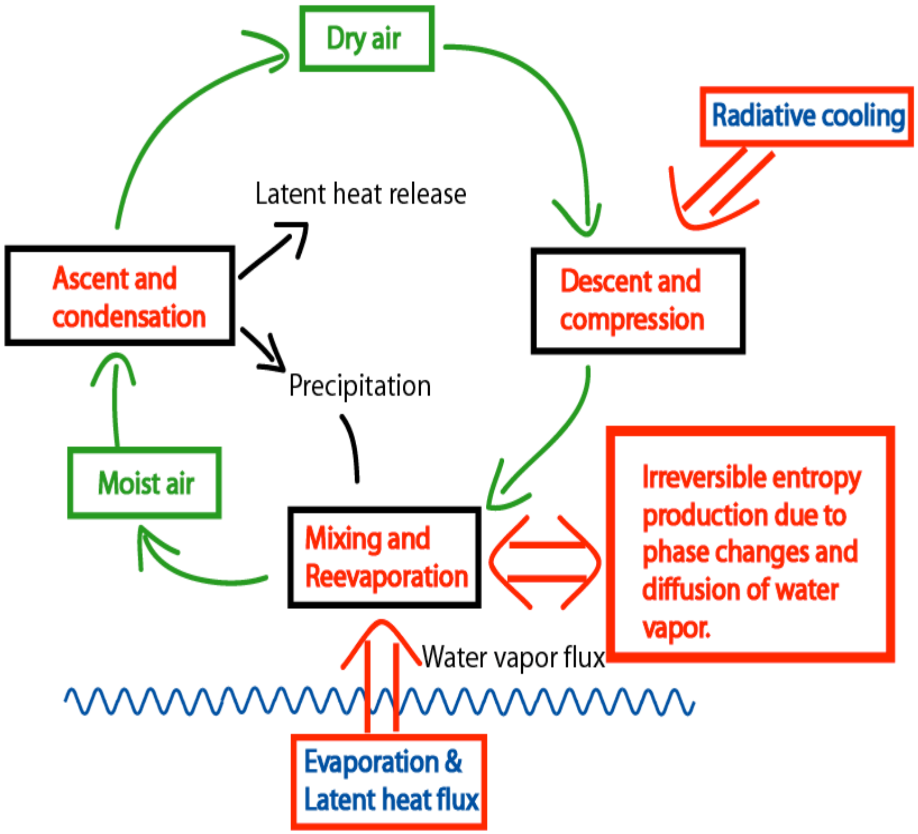
$$13.6 = 4.7 + 8.3 + 0.6 \quad (Wm^{-2})$$

Precipitating convection as an atmospheric dehumidifier

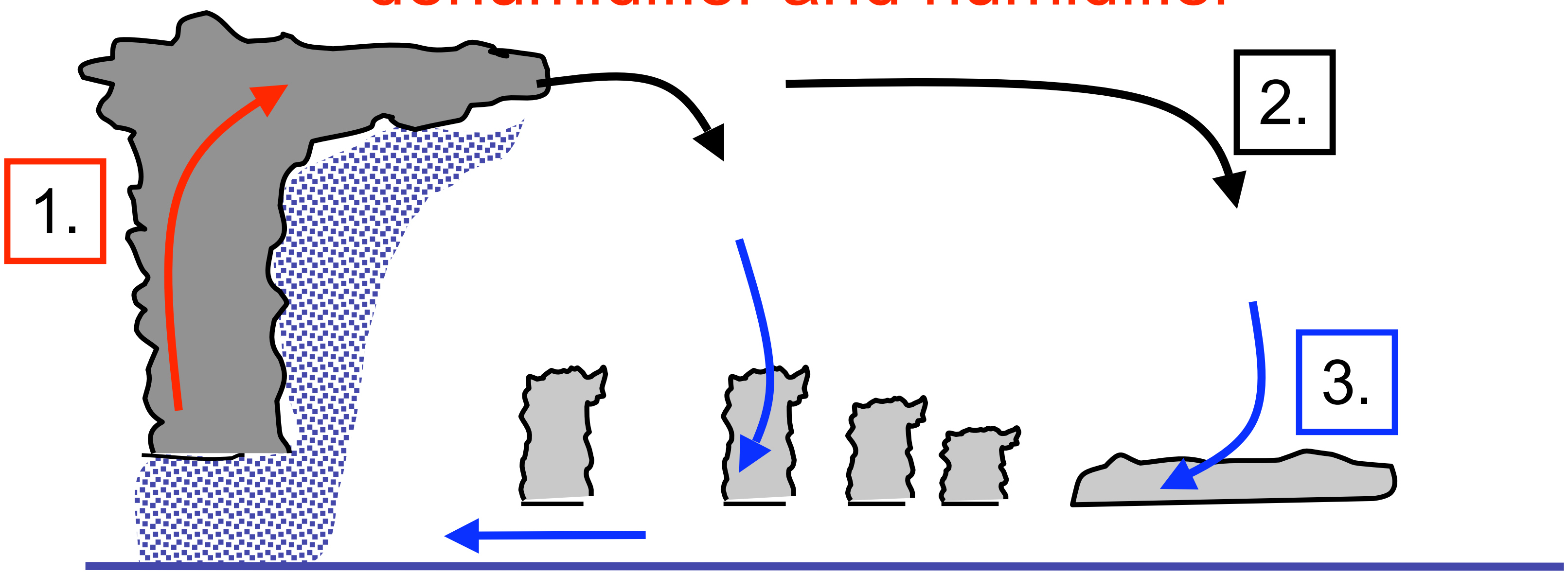
- precipitating convection acts as an atmospheric dehumidifier that continuously removes water vapor from the atmosphere through condensation and precipitation.

- The dehumidification is closely linked to the upward transport latent heat.

- Re-moistening of dry air is associated with irreversible phase changes and diffusion of water vapor. These can thus be viewed as the irreversible counter-part to the dehumidification.



The global circulation as a combination of dehumidifier and humidifier



1. Deep convection in the equatorial regions dehumidifies the atmosphere.

2. The global circulation exports dry air to the subtropics.

3. Shallow convection taps into the thermodynamic desequilibrium between the ocean surface and the dryer troposphere

Conclusion

- Shallow non-precipitating convection can be viewed as a humidifier that transports water vapor from a moist source to a dryer sink.
- Work done by atmospheric heat engines depends on:
  - Energy transport
  - Temperature difference
  - Relative humidity
  - Bowen ratio.
- Global circulation can be viewed as a combination of dehumidification by deep convection and re-humidification by shallow convection.