Dynamical Schwarzschild models of globular clusters

Glenn van de Ven
Institute for Advanced Study
glenn@ias.edu
Stellar systems

M45
NGC 6397
NGC 5139
NGC 1705
NGC 7742
M51
NGC 4526
NGC 4365
NGC 4526
M32
M87

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Fossil record

• How do stellar systems form and evolve?
• Wealth of structure in morphology and kinematics
• Range in stellar population properties (age, metallicity, ...)
• Link between dynamics and stellar populations?

• Clean fossil record in early-type galaxies and globular clusters
• Galaxies: integral-field spectroscopy
• Globular clusters: discrete stellar kinematics + population properties
Early-type galaxies

- Integral-field spectroscopy
- Schwarzschild’s method
- Triaxial dynamical model of NGC4365
Integral-field spectroscopy

... a spectrum at every position on the plane of the sky
Stellar velocity fields

NGC 4660 [-150/+150 km/s]

NGC 4365 [-58/+58 km/s]

Oblate
Axisymmetric

Triaxial

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Image of numerical orbit
Schwarzschild’s method

- Surface brightness $\rightarrow$ (MGE) gravitational potential
- Grid of $(E,I_2,I_3) \rightarrow$ initial orbit conditions $\rightarrow$ orbit library
- Weighted superposition of orbits that best fits photometry and kinematics $\rightarrow$ dynamical model

Schwarzschild (1979), Richstone & Tremaine (1988), Rix et al. (1997), van der Marel (1998), Cappellari et al. (2002), Gebhardt et al. (2003), ...

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Fitting simultaneously photometry and kinematics in triaxial geometry, including a possible black hole and/or dark matter halo

Intrinsic shape

\[ T = \frac{(1-p^2)}{(1-q^2)} \]

Oblate: \( p=1 \) (a=b>c), \( T=0 \)
Prolate: \( p=q \) (a>b=c), \( T=1 \)

... and viewing direction

\( (\theta, \phi, \psi) = (68^\circ, 73^\circ, 91^\circ) \)
Orbital decomposition

- Long-axis tube orbits
- Short-axis tube orbits

- Prograde short-axis
- Retrograde short-axis

Rotation cancels except center

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Globular clusters: ω Cen

- discrete kinematics: proper motions and line-of-sight velocities
- distance, inclination, M/L
- inner disk and tidal striping

\( \omega \) Cen (NGC 5139)

Proper motions \( \sim 10000 \) (van Leeuwen et al. 2000)

Line-of-sight velocities \( \sim 4000 \) (4 data-sets)

\( \nu \) (km/s) = 4.74 D (kpc) \( \mu \) (mas/yr)

Diameter \( \omega \) Centauri \( \sim 2 \times \) diameter full moon

Loke Kun Tan (StarryScapes)

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Smooth velocity and dispersion fields

- Horizontal: in direction of major $x'$-axis
- Vertical: in direction of minor $y'$-axis
- In direction of l.o.s. $z'$-axis

1 arcmin $\sim$ 1.45 pc

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Corrected velocity field

- **Horizontal**
  - Velocity: 0.16 mas/yr

- **Vertical**
  - Velocity: 0.22 mas/yr

- **L.O.S. z’-axis**
  - Velocity: 6.86 km/s
  - Observation area: x' (arcmin) (-15 to 15)

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Smooth velocity and dispersion fields

1 arcmin \approx 1.45\,\text{pc}

horizontal: in direction of major axis

vertical: in direction of minor axis

in direction of line-of-sight axis

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Schwarzschild model

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Best-fit parameters

\[ D = 4.8 \pm 0.3 \text{ kpc,} \]
\[ i = 50^{\circ} \pm 4^{\circ} \]
\[ \frac{M}{L_{V}} = 2.5 \pm 0.1 \frac{M_{\odot}}{L_{\odot}} \]
\[ L_{V} = 1.0 \pm 0.1 \times 10^{6} L_{\odot} \]
\[ M = 2.5 \pm 0.3 \times 10^{6} M_{\odot} \]
M/L variation with radius?

Consistent with constant $M/L_V = 2.5 \pm 0.1 \ M_\odot/L_\odot$

1 arcmin $\sim 1.45$ pc
Phase-space distribution

Main component central non-rotating ‘bulge’

Outwards increasing rotation and flattening

Inner maximum rotating disk (~4% mass)
Tidal interaction and multiple populations

- $\omega$ Centauri mean rotation ($L_z > 0$) and orbit around Milky Way center opposite $\rightarrow$ prograde orbits ($L_z < 0$) tidally removed?

- Impulse approximation: $|\Delta v| \sim \sigma$ around 16 arcmin (tidal radius around 45 arcmin)  
  \[ \text{Dinescu et al. (1999)} \]

- Multiple stellar populations 
  \[ \text{(Freeman & Rodgers 1975, Norris et al. 1997, Pancino et al. 2003, ...)} \]

Metal-rich $[\text{Ca/H}] > -1.2$
- Centrally concentrated
- No apparent rotation
- Nearly round

Metal-poor $[\text{Ca/H}] < -1.2$
- Throughout galaxy
- Rapidly rotating
- Flattened

Non-rotating ‘bulge’? 
Rotating flattened component?

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Globular clusters: M15

- mass-to-light ratio
- IMBH or dark remnants?

M15 (NGC 7078)

- D=10 kpc (1"=0.05 pc)
- SB profile: $r_c=0.05$ pc, central slope=$-0.62\pm0.06$
- 1540 l.o.s. velocities
- 703 HST proper motions
Distance and inclination

![Graph showing distance and inclination](image)

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M/L variation with radius


M/L per Gaussian
mass/luminous density
constant M/L = 1.6 ± 0.2

1 arcmin ~ 3 pc

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IMBH or dark remnants?

- $D = 10.3 \pm 0.05$ kpc
- $i = 60 \pm 15$ degrees

- From $M_{\text{BH}} - \sigma$ relation:
  $\sim 10^3 M_\odot$ ($\sigma \sim 11 \text{ km/s}$)

- $r_{\text{BH}} \sim 0.5'' \sim 0.025$ pc

- Within $r_c = 0.05$ pc
  $M_c = 3.4 \times 10^3 M_\odot$
  $\rho_c = 7.4 \times 10^6 M_\odot/\text{pc}^3$

- $M_{\text{tot}} = 4.4 \times 10^5 M_\odot$
Outlook and summary
Next steps

- HST proper motions: IMBH?
- Correlated and higher order velocity moments, or...
- Fitting directly discrete kinematics with Max. Likelihood methods
- Including color, metallicity and age indicators, etc.:
  link kinematics and stellar properties in single model
- Synergy with particle-based models?
Summary in figures

ω Cen

M15

Summary in figures

ω Cen

M15

R_e=1.23  R_e=1.53  R_e=1.91  R_e=2.37  R_e=2.95  R_e=3.67  R_e=4.57  R_e=5.69  R_e=7.08  R_e=8.81  R_e=11.0  R_e=13.6

ω Cen

M15

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...the end
Extra: $\omega$ Cen
Some properties

- Most massive GC in Milky Way: \( M \approx 2-5 \times 10^6 \, M_\odot \)
- One of most flattened GCs: \( q' \approx 0.9 \)
- Relatively loosely bound: \( \log(r_t/r_c) \approx 1.24 \)
  tidal radius \( r_t \sim 45' \), core radius \( r_c \sim 2.6' \)
- Small heliocentric distance: \( D \approx 5 \, \text{kpc} \)
  RR Lyrae and eclipsing binary
- Complicated composition with multiple stellar populations
  - Self-enrichment (isolated cluster/nucleus dwarf galaxy)?
  - (Subsequent) interaction/merger GCs?

Smooth velocity and dispersion fields

- Horizontal: in direction of major x'-axis
- Vertical: in direction of minor y'-axis
- In direction of l.o.s. z'-axis

1 arcmin ~ 1.45 pc

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Corrected velocity field

0.16 mas/yr

0.22 mas/yr

6.86 km/s

Horizontal

Vertical

I.o.s. z'-axis

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Measuring ‘solid-body’ rotation

Any axisymmetric object:

\[
\langle v_z \rangle(x', y') = -4.74D \tan i \langle \mu_y \rangle(x', y')
\]

Residual solid-body rotation

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Constraint on inclination

- Best-fit solid-body rotation
  \[ sbr = 0.029 \text{ mas/yr/arcmin} \]

- canonical \( D = 5.0 \pm 0.2 \text{ kpc} \)
  inclination \( i = 41 - 57^\circ \)

- flattening:
  observed \( q' = 0.88 \pm 0.01 \)
  intrinsic \( q = 0.78 \pm 0.03 \)
MGE mass model

- 1D: 8 Gaussians → 2D: flattening profile Geyer et al. (1983)
- \( E(B-V) = 0.11, D = 5.0 \pm 0.2 \text{ kpc}: L_V \sim 1.0 \pm 0.1 \times 10^6 L_\odot \)

1 arcmin ~ 1.45 pc
Polar grid of apertures

Reflected to first quadrant, around 80 stars per aperture

- Proper motions: 28 apertures, total 2295 stars
- L.o.s. velocities: 27 apertures, total 2223 stars

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Averaged kinematics

- Fitting to average kinematics \((V,\sigma,...)\) of stars within aperture:
  - Linear method ensures global best-fit
  - Faster than using discrete velocities

- How to extract velocity moments?
  - Gaussian fit to velocity histograms
  - Instrumental dispersion: \(\sigma_{fit}^2 = \sigma^2 + \sigma_{ins}^2\)
  - Maximum likelihood estimation:

\[
L(V,\sigma,...) = \prod_{i=1}^{n} \int_{-\infty}^{\infty} L(v) \frac{1}{\sqrt{2\pi\sigma_i}} e^{-\frac{1}{2} \left( \frac{v_i-v}{\sigma_i} \right)^2} dv
\]
Rotational/pressure supported

- max $V/\sigma$ at $R \sim 8$ arcmin
  $\sim$ at maximum $v_{\text{los}}$
- $V/\sigma > 0.5$ above isotropic oblate rotator in $(V/\sigma, \varepsilon)$
  $\sim$ rotational support
- Outwards (partly) pressure supported

$$\sigma_{\text{RMS}}^2 = \left(\sigma_R^2 + \sigma_\theta^2 + \sigma_\phi^2\right)/3$$

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Anisotropy

- Principal axis velocity ellipsoid $\sigma_+, \sigma_-, \text{and } \sigma_\phi$
- In meridional plane:
  - Almost isotropic near equatorial plane
  - Tangential anisotropic towards symmetry axis
- 3D (including azimuthal)
  - Radial anisotropic center
  - Tangential anisotropic in outer parts
- Not two-integral $F(E,L_z)$

$$\sigma_t^2 = \left(\sigma_\theta^2 + \sigma_\phi^2\right)/2$$

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Intrinsic velocity moments

\[ \sigma_\phi^2 = \langle v_\phi^2 \rangle - \langle v_\phi \rangle^2 \]

\[ \sigma_\theta^2 = \langle v_\theta^2 \rangle \]

\[ \sigma_R^2 = \langle v_R^2 \rangle \]

\[ \sigma_{R\theta}^2 = \langle v_R v_\theta \rangle \]

Axisymmetric:

\[ \langle v_R \rangle = \langle v_\theta \rangle = \langle v_R v_\phi \rangle = \langle v_\theta v_\phi \rangle = 0 \]
Conclusions ω Centauri

- Significant perspective and residual solid-body rotation
- Amount solid-body rotation and D·tan(i) directly from data
- Axisymmetric anisotropic Schwarzschild model:
  \[ D = 4.8 \pm 0.3 \, \text{kpc}, \quad M/L_V = 2.5 \pm 0.1 \, \text{M}_\odot/\text{L}_\odot, \quad M = 2.5 \pm 0.3 \times 10^6 \, \text{M}_\odot \]
- Substructure in distribution function:
  - Main component center non-rotating ‘bulge’
  - Outwards increasing rotation and flattening
  - Inner (~1-3 arcmin) maximum rotating disk ~ 4% mass

... linked with multiple stellar populations?
... tidally stripped dwarf galaxy?
Extra: M15
Dispersion profiles

1 arcmin ~ 3 pc


1540 l.o.s. velocities

Kinematic maps

703 HST proper motions

1 arcmin ~ 3 pc
MGE mass model

1 arcmin ≈ 3 pc

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Schwarzschild model

horiz.

vert.

l.o.s.

1 arcmin ~ 3 pc

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