Simple Two-Component Models of Stellar Systems

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ABSTRACT

The results of N-body simulations for multi-component systems (e.g. Giersz and Heggie 1996) show that, although the masses segregate as expected during core collapse, **after core collapse** there is self-similar evolution with **very little further evidence of mass segregation** even though the system has **not reached equipartition.** A *single mass* model has a flux of energy at the half-mass radius, which is provided by binary heating in the core. This energy can only be transported to the half-mass radius from the core, if the system is not in thermal equilibrium. Similarly, we suspect that these principles hold in a two-component system (i.e. a stellar system consisting of two populations; the light stars with mass m_3 and the heavy stars with mass m_2, where $m_2 > m_3$). The half-mass radius in a two-component system is in the region dominated by the light stars, while the core is dominated by the heavy stars. In other words, there can only be a flux of energy at the half-mass radius if the heavy stars and light stars are not in equipartition. To investigate this problem and to gain insight into two-component systems , we create a **simple two-component model which only considers gravitational interactions and binary heating.** We also compare our model with our own two-component N-body simulations (performed using a GPU for simulations with larger values of N). We found that our simple model exhibits behaviour similar to our N-body simulations; the post-collapse evolution is self-similar and there is no further mass segregation even though the system is not in equipartition. **We conclude that the processes considered in our simple model are enough to prevent mass segregation to develop further in stellar systems which have undergone core collapse.**

Constructing the Model

We construct the analytical model by first creating two simpler models; a single mass model with a core radius and a two-component model without a separate core. We then embed the single mass model into the two- component model. The two-component model considers the gravitational interactions (highlighted in red) in terms of the half-mass radius of the light stars and the heavy stars, r_2 and r_3, respectively. The single mass model (highlighted in yellow) considers gravitational interactions in the core radius and the effects of binary heating on the core radius, r_1, and r_2.

The simple model can be written as,

$$\dot{r}_3 = \frac{f\varphi}{C_1 t_{eq}},$$

$$\dot{r}_2 = g\frac{f\varphi}{C_1 t_{eq}} + C_2\frac{r_2\dot{T}_b}{T_2},$$

$$\dot{r}_1 = \frac{r_1}{t_{rc}}\left(-0.01 + C_2\frac{\dot{T}_b t_{rh}}{T_h}\right) + \frac{r_1}{r_2}\dot{r}_2,$$

where, C_1 and C_2 are constants, $\dot{T}_b$ is the binary heating term, t_{rc} is the core relaxation time, t_{rh} is the half-mass relaxation time and t_{eq} is the equipartition time. The functions f, g and φ are described by,

$$f(r_3, r_2) = [r_3^4 r_2 M_3^2 M_2 (M_3 + M_2)]^{-1},$$

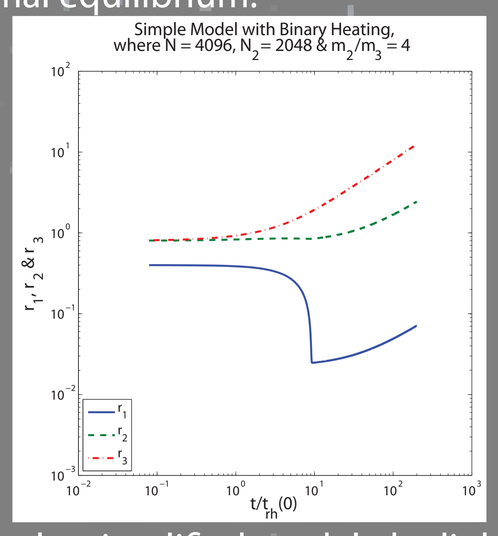
$$g(r_3, r_2) = \frac{M_3 r_2^2 (3M_2 r_2^2 - M_3 r_3^2 - M_2 r_3^2)}{M_2 r_3 (M_2 r_3^3 + 2M_3 r_2^3)},$$

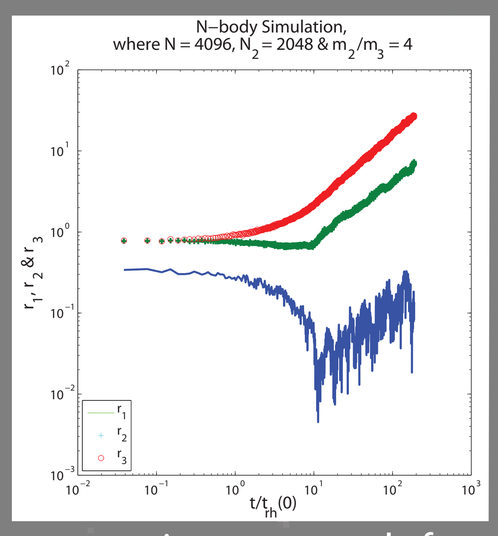
$$\varphi\left(\frac{r_2}{r_3}\right) = \frac{M_2}{M_3}\frac{m_2}{m_3} + \frac{m_2}{m_3}\left(\frac{r_2}{r_3}\right)^3 - \left(1 + \frac{M_2}{M_3}\right)\frac{r_2}{r_3}.$$

where M_2 and M_3 are the total mass of the heavy stars and the total mass of the light stars, respectively. These expressions arise from estimating the velocity dispersion of the two components from their masses and radii, using the virial theorem. Also, we consider only three-body interactions.

Comparing with N-body Simulations

We compare the analytical model (left) with two-component N-body simulation (right). As there are fundamental differences between these two models, we do not expect the simple model to emulate the N-body simulation precisely. However, we do expect the simple model to contain similar key features to the simulation. The plots below show r_1, r_2 and r_3 with respect to time, where time is expressed as a multiple of the initial half-mass relaxation time. Both of these models exhibit mass segregation prior to core collapse and display little further mass segregation after core collapse, even though these systems are not in thermal equilibrium.





In the simplified model, the light stars receive energy only from the heavy stars (whose energy is sustained by binary heating). Transfer of energy from the heavy stars to the light stars requires that they are not in equipartition. We suggest that this also explains why equipartition is not reached after core collapse in N-body systems.

Conclusions

We've constructed a two-component model which only considers gravitational interactions and binary heating. We also find that **binary heating is enough to prevent mass segregation after core collapse in a system that is not in equipartition.**