

*What do we need to put braneworlds
to the observational test?*

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The new cosmology confronts observation

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Why study brane cosmology?

There seems to be a deep connection between the unification of gravity with other fundamental interactions and the presence of extra spatial dimensions.

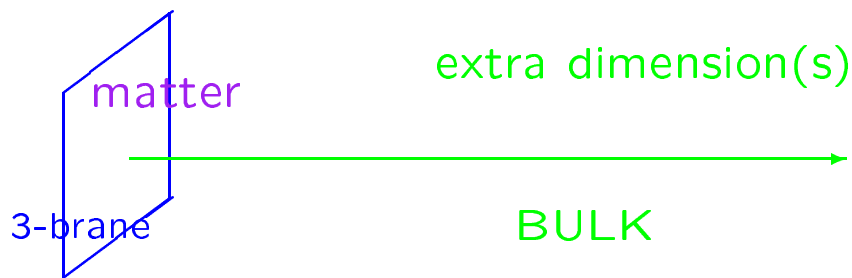
1920's : Th. Kaluza and O. Klein) unify electrodynamics with gravitation by introducing a fifth spacetime (fourth spatial) dimension.

1980's: string theory as a candidate to unify all known fundamental interactions

consistency at the quantum level imposes to choose a number of (spacetime or internal) dimensions larger than four: 10 (resp. 11) in the case of the weakly (resp. strongly) coupled superstring.

In the context of higher-dimensional theories, one may thus envisage two extreme possibilities:

- a standard Kaluza-Klein picture: our own 4-dimensional universe is obtained through the compactification of the higher-dimensional spacetime.
- a **brane-universe picture**: our 4-dimensional world is a 3-brane (3 for the number of spatial dimensions)

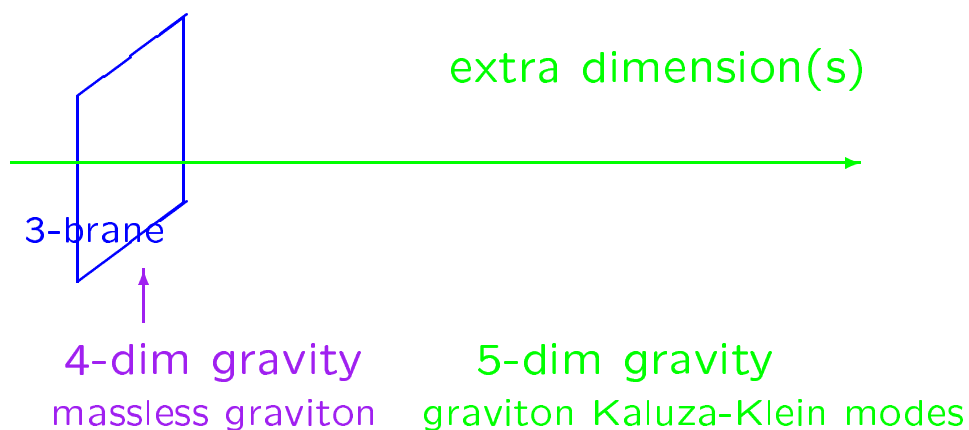


or intermediate situations.

Because of the attractiveness of the brane scenario, many have been tempted to push it beyond the strict string framework where it was defined, and to generalize it to topological defects on which matter is localized.

Two situations are considered:

- the dimensions orthogonal to the brane are compact in order to recover 4-dimensional gravity in the effective low energy theory.
- 4-dimensional gravity is itself localized on the brane
Rubakov, Shaposhnikov; Randall, Sundrum;..

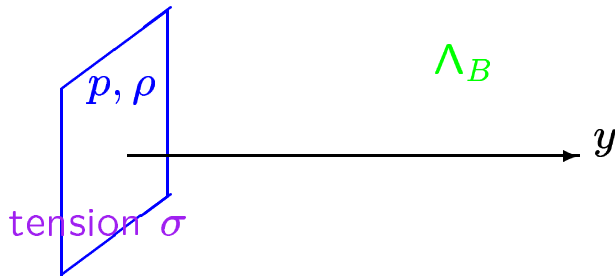


Since brane universes are new set ups, they may provide new twists to old problems. Indeed, whereas most of fundamental physics seems to be well understood, a certain number of problems tend to indicate that we are missing part of the picture :

- **large hierarchies of scales** : why is the scale of quantum gravity $M_{Pl} \sim 10^{19}$ GeV so much larger than the scales of the other fundamental interactions, e.g. the electroweak scale $M_W \sim 10^2$ GeV?
- **baryogenesis** : what is the mechanism that generates the baryon-antibaryon asymmetry observed in the Universe?
- **dark matter**: what is the nature of most of the matter in the Universe?
- **cosmological constant** : what is the cosmological status of vacuum energy? what is the origin of the extinction of most sources of vacuum energy? what is the source of the presently (?) observed vacuum or dark energy?

Life on the brane

Take the simplest 5-dimensional model (single extra dimension)



5-dimensional Einstein equation + Israel junction conditions on the brane

→ generalized Friedmann equation on the brane:

$$H^2 = \frac{1}{6M_5^3} \Lambda_B + \frac{1}{36M_5^6} \sigma^2 + \frac{1}{18M_5^6} \sigma \rho + \frac{1}{36M_5^6} \rho^2 + \frac{C}{a_0^4} - \frac{k}{a_0^2}$$

P.B., Deffayet, Ellwanger, Langlois; Csaki, Graesser, Kolda, Terning; Cline, Grojean, Servant ;...

with $8\pi G_5 \equiv M_5^{-3}$, a_0 cosmic scale factor on the brane,

to be compared with the standard Friedmann equation

$$H^2 = \frac{\lambda}{3} + \frac{1}{3M_P^2} \rho - \frac{k}{a^2}$$

- 4-d cosmological constant λ receives contributions both from the bulk vacuum energy Λ_B and from the string tension σ^2

- Comparing the linear terms in ρ allows a cosmological determination of the 4-dimensional Planck scale M_P

$$M_P^2 = 6 \frac{M_5^6}{\sigma} .$$

- new contribution in ρ^2 . This shows that the cosmology of a 4-dimensional brane universe is generically different from the cosmology of a 4-dimensional universe

↔ notion of extrinsic curvature i.e. the way the brane is curved within the higher-dimensional spacetime: the presence of localized energy induces extrinsic curvature.

importance of the non-conventional ρ^2 term increases as one goes back in time → possible role in the early universe (inflation models)

- the dark radiation term C/a_0^4 represents the effect of the gas of bulk gravitons on the cosmological evolution of the brane. **The brane does not represent a closed system.**

In general relativity, Birkhoff's theorem: a spherically symmetric gravitational field in empty space must be static, with a metric given by the Schwarzschild solution:

$$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \left(1 - \frac{2M}{r} \right)^{-1} dr^2 + r^2 d\Omega^2 \quad .$$

Generalization of Birkhoff's theorem valid for this set up: the general solution for the bulk metric is static with metric given by the the AdS-Schwarzschild solution:

$$ds^2 = -h(y) dt^2 + \frac{1}{h(y)} dy^2 + y^2 d\Sigma_k^2 \quad ,$$

$$h(y) = k - \frac{\mathcal{C}}{y^2} + \frac{y^2}{\ell^2} \quad ,$$

where

$$\ell \equiv \sqrt{\frac{6M_5^3}{|\Lambda_B|}}$$

is called the AdS_5 curvature radius. Thus \mathcal{C} may be interpreted as the mass of the 5-dimensional black hole that is formed in the bulk because of graviton radiation from the brane.

In this set up, time dependence i.e. cosmological evolution only comes from the motion of the brane in the bulk.

When should one recover the 4-dimensional picture?

When the physics on the brane is 4-dimensional, i.e.

- if the extra dimension is compact and its radius is stabilized

e.g. Hořava-Witten theory

Lukas, Ovrut, Waldram

- if the extra dimension is noncompact but the 4-dim. graviton is localized

Rubakov, Shaposhnikov; Akama; Gogberashvili; Randall, Sundrum

e.g. in Randall-Sundrum (RS-II) case, warped geometry in the extra dimension i.e. the cosmic scale factor has a y dependence:

$$a(y) = e^{-|y|/\ell}, \quad \ell^2 = \frac{6M_5^3}{|\Lambda_B|} \text{ AdS}_5 \text{ curvature radius}$$

$$\text{Minkowski } (M_4) \text{ constraint: } \frac{1}{3}\lambda = \frac{1}{6M_5^3}\Lambda_B + \frac{1}{36M_5^6}\sigma^2 = 0$$

$$\text{Planck scale : } M_P^2 = M_5^3 \int_{-\infty}^{+\infty} e^{-2|y|/\ell} dy = M_5^3 \ell$$

Agrees with the cosmological evaluation:

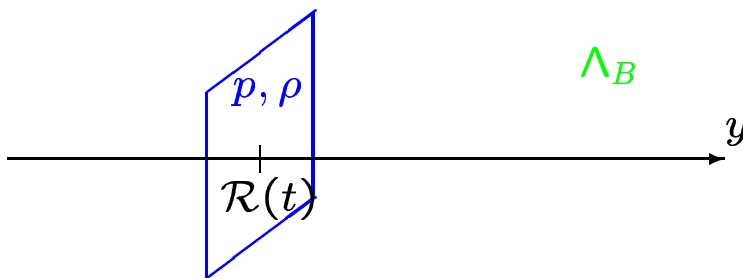
$$M_P^2 = \frac{6M_5^6}{\sigma} = M_5^3 \left(\frac{6M_5^3}{|\Lambda_B|} \right)^{1/2} = M_5^3 \ell$$

The radion field: an example of a modulus field.

In an AdS_5 background, choose static coordinates :

the cosmological evolution on the brane is purely due to its motion in the bulk

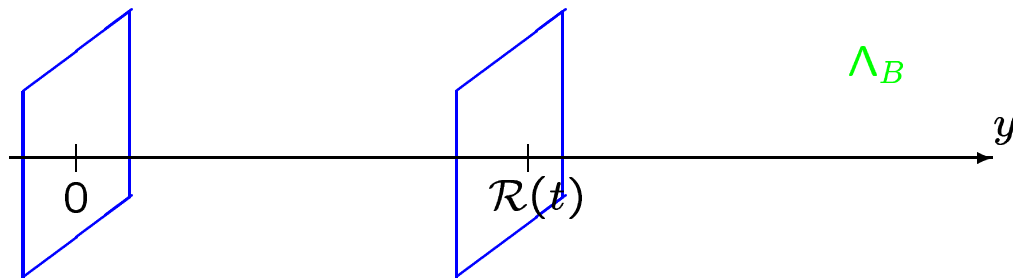
Kraus



$\mathcal{R}(t)$ is the cosmic scale factor on the brane:

$$\begin{aligned} ds^2|_{\text{ind}} &= - \left[h(\mathcal{R}) - \frac{1}{h(\mathcal{R})} \left(\frac{d\mathcal{R}}{dt} \right)^2 \right] dt^2 + \mathcal{R}^2 d\Sigma_k^2 \\ &\equiv -d\tau^2 + \mathcal{R}^2(\tau) d\Sigma_k^2 \end{aligned}$$

In the set up with two branes, the interbrane distance is described by a scalar field or **radion**



The radion field is coupled to the 4-d curvature: it is a **Brans-Dicke field** and the brane 4d-theory is a **scalar-tensor theory of gravity**.

In order to obtain general relativity in the late stages of the evolution of the Universe, one needs to **stabilize the radion field**.

e.g. 2 Minkowski branes \leftrightarrow the potential energy $V(\mathcal{R})$ vanishes (**flat direction of the scalar potential**)

\rightarrow need to stabilize \mathcal{R} .

Meeting observation

- Perturbations: computing the CMB spectrum
Kodama, Ishibashi, Seto; Maartens; Langlois; van de Bruck, Dorca, Brandenberger, Lukas; Koyama, Soda; Deruelle, Dolezel, Katz; Langlois, Maartens, Sasaki, Wands; Deffayet; Riazuelo, Vernizzi, Steer, Durrer...

To this day no complete evaluation from the higher-dimensional point of view of the C_l distribution for the cosmic microwave background.

problems already arise in the simplest case of the Randall-Sundrum model because the Kaluza-Klein modes do not decouple, even to linear order: this is a reflection of the fact that **the brane is not a closed system**.

In the two-brane case, need to define precisely which **brane fluctuation modes** are allowed (dynamical brane, orbifold fixed point, orientifold...) and, if needed, to properly **stabilize the radion field**.

- possibility of specific violations of Lorentz invariance
*Chung, Freese, Chung, Kolb, Riotto; Csaki, Erlich, Grojean;
Grojean, Quevedo, Tasinato, Zavala*

e.g. models where the metric in the bulk has the form

$$ds^2 = - h(y) dt^2 + y^2 dx^2 + \frac{1}{h(y)} dy^2$$

gravity waves are allowed to travel in the **bulk**

e.m. waves are confined to the **brane at $y = 0$**

⇒ speed of light travelling between two points on the brane may be different from the speed of gravitational waves

leads to violations of Lorentz invariance which might be detectable once gravitational waves are observed.

also poses the **cosmological horizon problem** in a new light

Modification of gravity at large distance

- Precursor model: GRS model

Gregory, Rubakov, Sibiriakov

where a collection of massive graviton modes contribute to form a 4-dimensional **unstable** massless graviton bound-state on the brane.

→ 5-d gravity (r^{-3}) recovered at large distance.

- multi-gravity models

Kogan, Mouslopoulos, Papazoglou, Ross

e.g. bi-gravity: set-up corresponds to two branes, each localizing gravity: for a finite distance between the branes, the corresponding graviton zero modes mix, giving a massless graviton **plus a very light graviton whose presence modifies gravity at large distance.**

- induced gravity models

Dvali, Gabadadze, Porrati

gravity includes 5d gravity plus induced 4d gravity on the brane:

$$\mathcal{S} = \int d^5x \sqrt{-g} M_5^3 \frac{1}{2} R^{(5)} + \int_{\text{brane}} d^4x \sqrt{-h} M_p^2 \frac{1}{2} R^{(4)}$$

For distances r larger than a critical distance r_c given by

$$r_c = \frac{M_{\text{Pl}}^2}{2M_5^3}$$

gravity “leakage” into the extra dimension: gravitational force in r^{-3} .

Note: heated debate about van Dam-Veltman-Zakharov discontinuity

For all these models, the critical distance must be cosmological: CMB [P.B., J. Silk](#), weak lensing [Bernardeau, Uzan](#)

Cosmology of induced gravity model

Deffayet; Deffayet, Dvali, Gabadadze

$$H^2 = \left(\sqrt{\frac{\rho}{3M_{\text{Pl}}^2} + \frac{1}{4r_c^2} + \frac{1}{2r_c}} \right)^2 - \frac{k}{a^2}$$

Hence acceleration at late time, without a need for a cosmological constant!

Conclusions

- Braneworld set-up is not unique. String theory consistency should constrain the number of possibilities.
- Most braneworld backgrounds do not lead to satisfactory late time cosmology.
- If a satisfactory braneworld background is found, then it is in principle possible to compute perturbations: the fact that **the brane is not a closed system** imposes to perform a computation in the higher-dimensional theory (if one wants to keep intact the brany characteristics)
- Braneworld cosmology may lead to unexpected effects: violations of Lorentz invariance, modification of gravity at cosmological distances, late acceleration...
- The braneworld idea has given a new set up to discuss old problems but has not (yet?) provided striking new solutions.