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Crystals of Light, Disorder, Atomic Mixtures

Ultracold atoms in disordered potentials

✓ *Why disorder?*

- Disorder is a key ingredient of the microscopic (and macroscopic) world
- Fundamental element for the physics of **conduction**
- **Superfluid-insulator transition** in condensed-matter systems

✓ *Why cold atoms?*

- Ultracold atoms are a versatile tool to study disorder-related phenomena
- **Precise control** on the kind and amount of disorder in the system
- Quantum simulation

✓ *Localization effects*

- **Bose glasses**, spin glasses (strongly interacting systems)
- **Anderson localization** (weakly interacting systems)

Different ways to produce disorder

Several proposals for the production of a disordered potential:

✓ *Optical potentials*

B. Damski et al., PRL **91**, 080403 (2003).

R. Roth & K. Burnett, PRA **68**, 023604 (2003).

- Speckle fields
- Multi-chromatic lattices

✓ *Collisionally-induced disorder*

U. Gavish & Y. Castin, PRL **95**, 020401 (2005).

P. Massignan & Y. Castin, cond mat 0604232v2

- Interaction with a different randomly-distributed species

✓ *Magnetic potentials*

H. Gimperlein et al., PRL **95**, 170401 (2005).

- Magnetic field inhomogeneity near an atom chip

✓ *Solid state physics with ultracold atoms*

- Study of quantum transport in periodic potentials (band structure)
- Role of atom-atom interactions (nonlinear systems → solitons, instabilities, ...)

✓ *Strongly correlated systems*

- Superfluid to Mott insulator quantum phase transition
- Systems with low dimensionality (Tonks gases...)

✓ *Atom optics*

- Tools for the implementation of mirrors, beam splitters, diffraction gratings, lenses

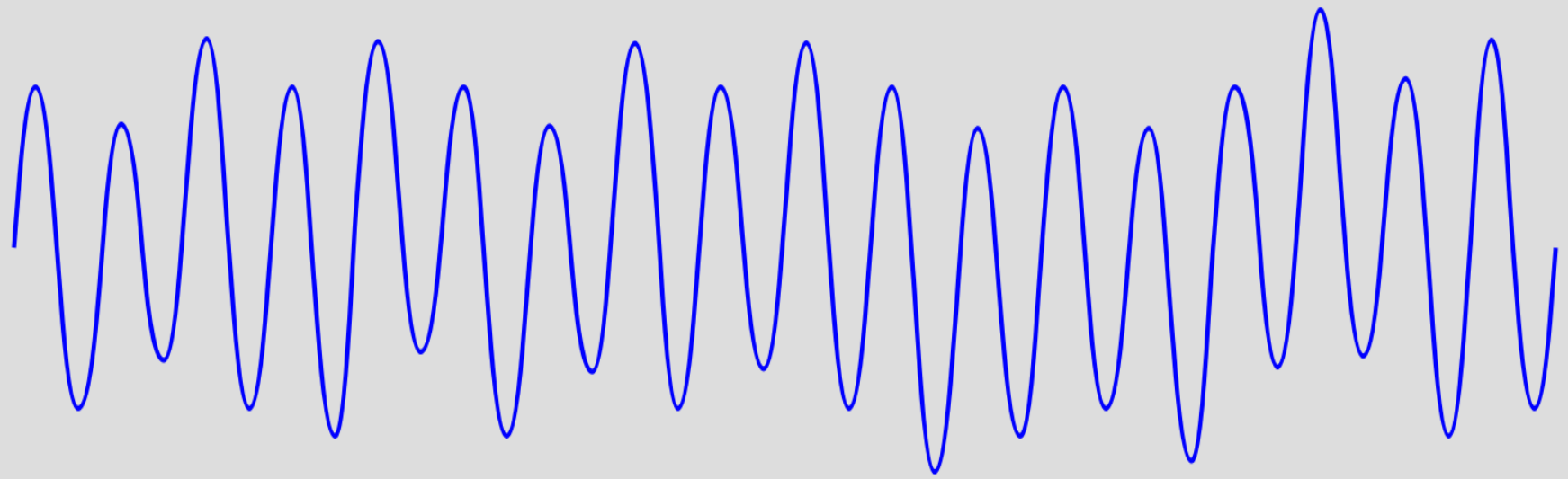
✓ *Quantum computing*

- Quantum registers

Disordered optical lattices

Adding disorder

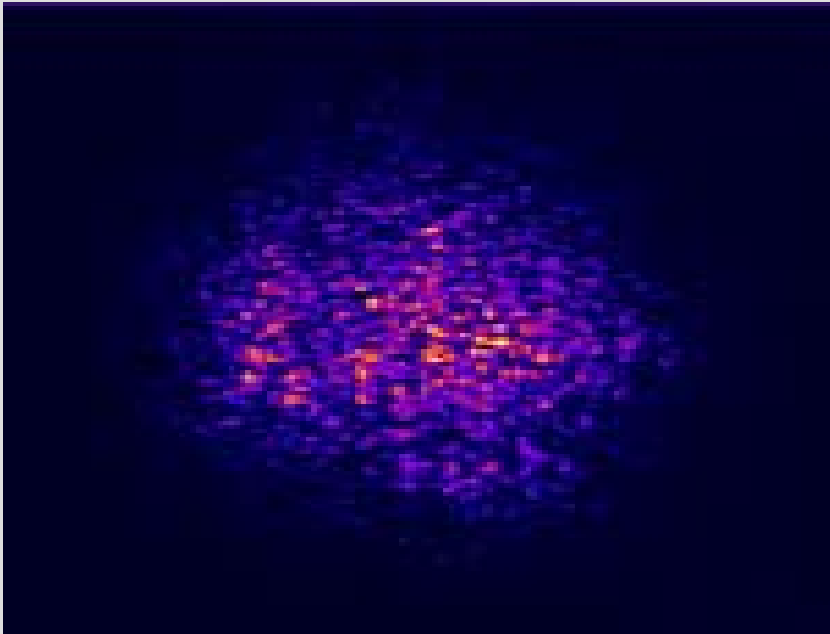
What happens if one adds disorder to the regular lattice structure?



Adding disorder

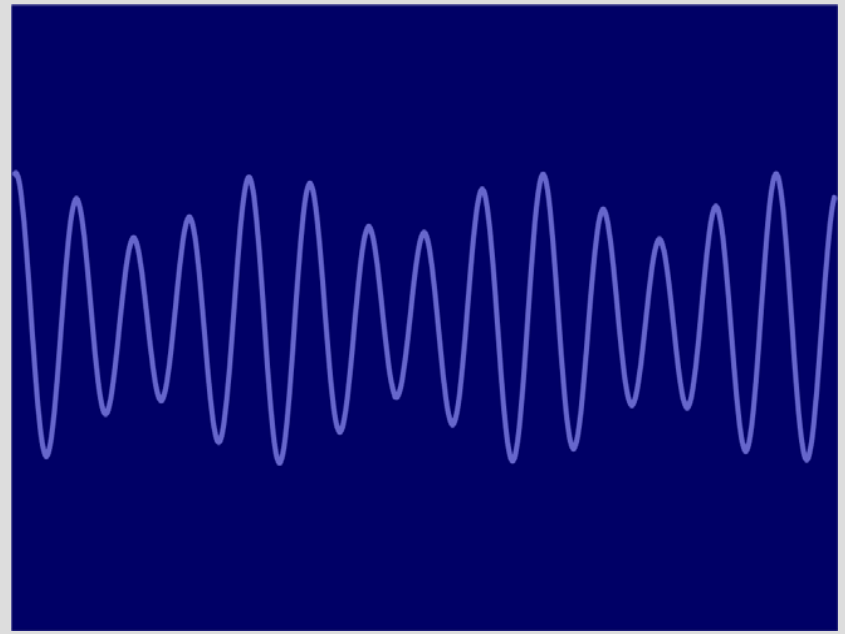
For ultracold atoms in optical lattices one can add optical disorder in two ways:

speckle pattern



- ✓ random potential
- ✗ large length scale (several μm)

bichromatic lattice

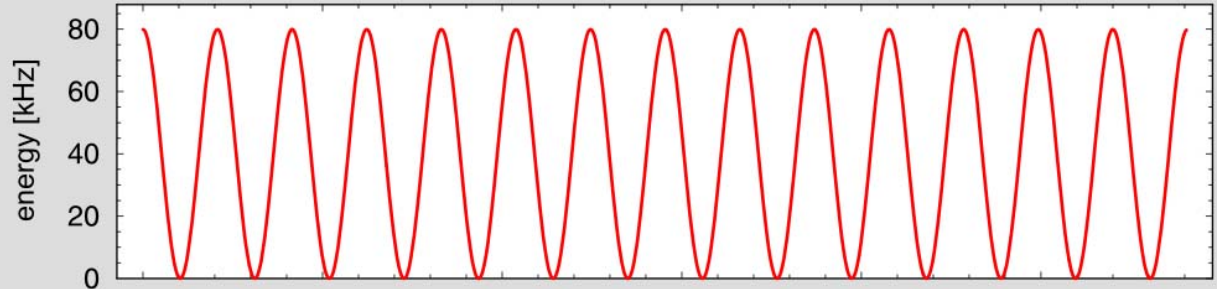


- ✓ quasiperiodic potential
- ✓ smaller length scale (1 μm or less)

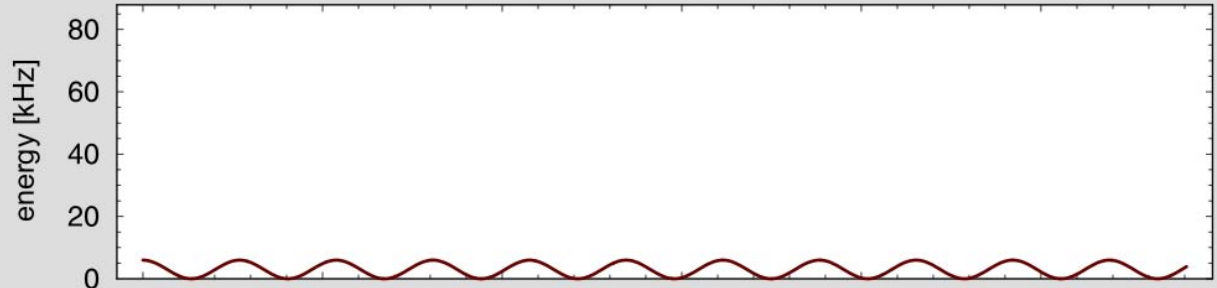
The bichromatic lattice

$$V(x) = s_1 E_{R1} \cos^2(k_1 x) + s_2 E_{R2} \cos^2(k_2 x)$$

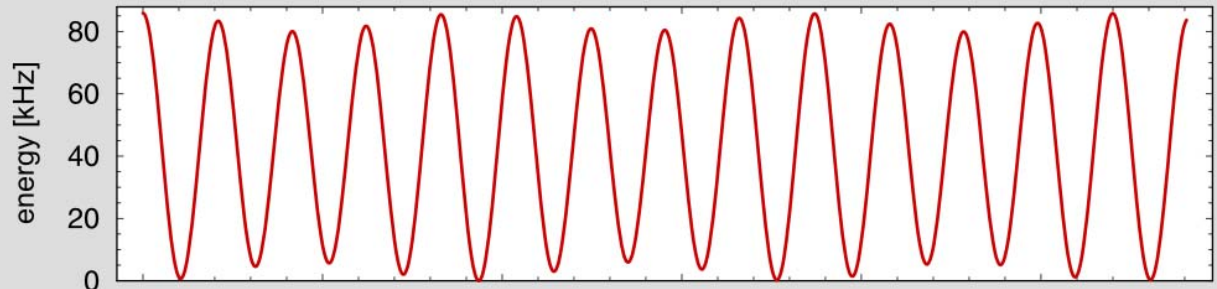
bichromatic lattice



$\lambda = 830$ nm



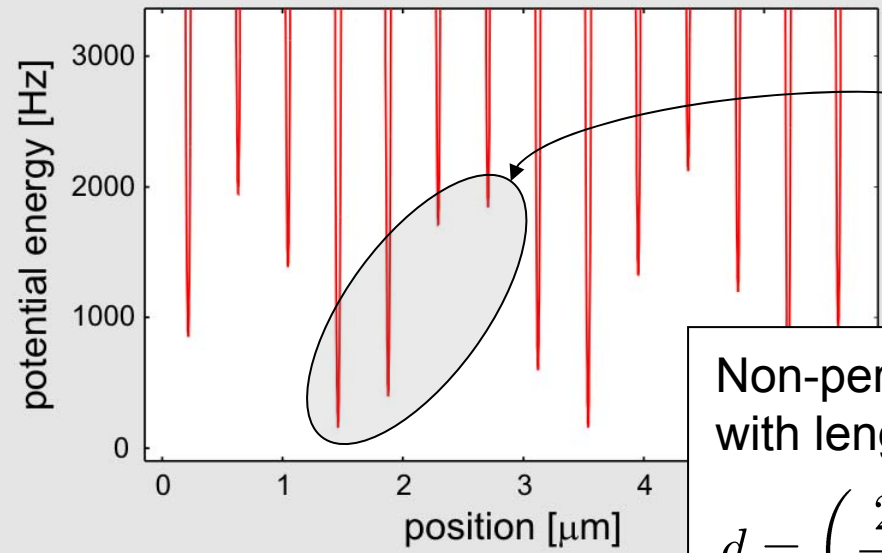
$\lambda = 1076$ nm



$\lambda = 830$ nm
+
 $\lambda = 1076$ nm

0 1 2 3 4 5
position [μm]

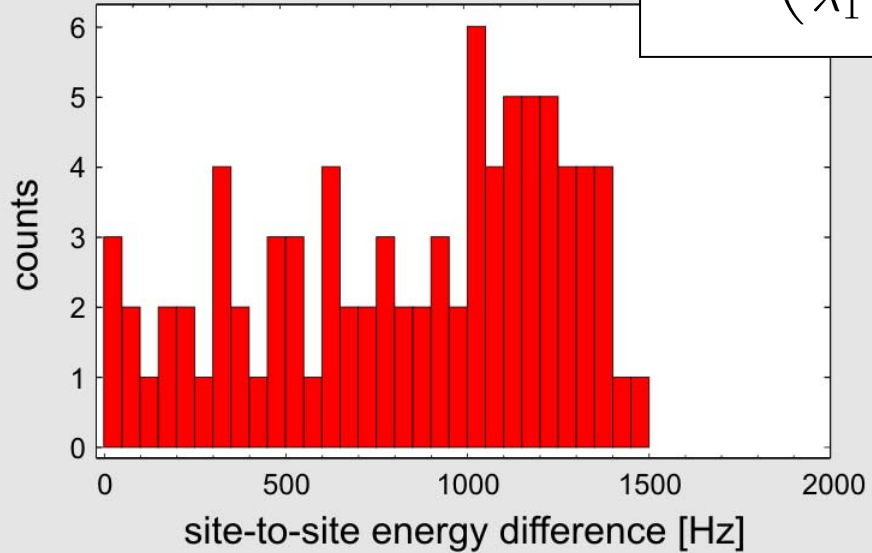
The bichromatic lattice



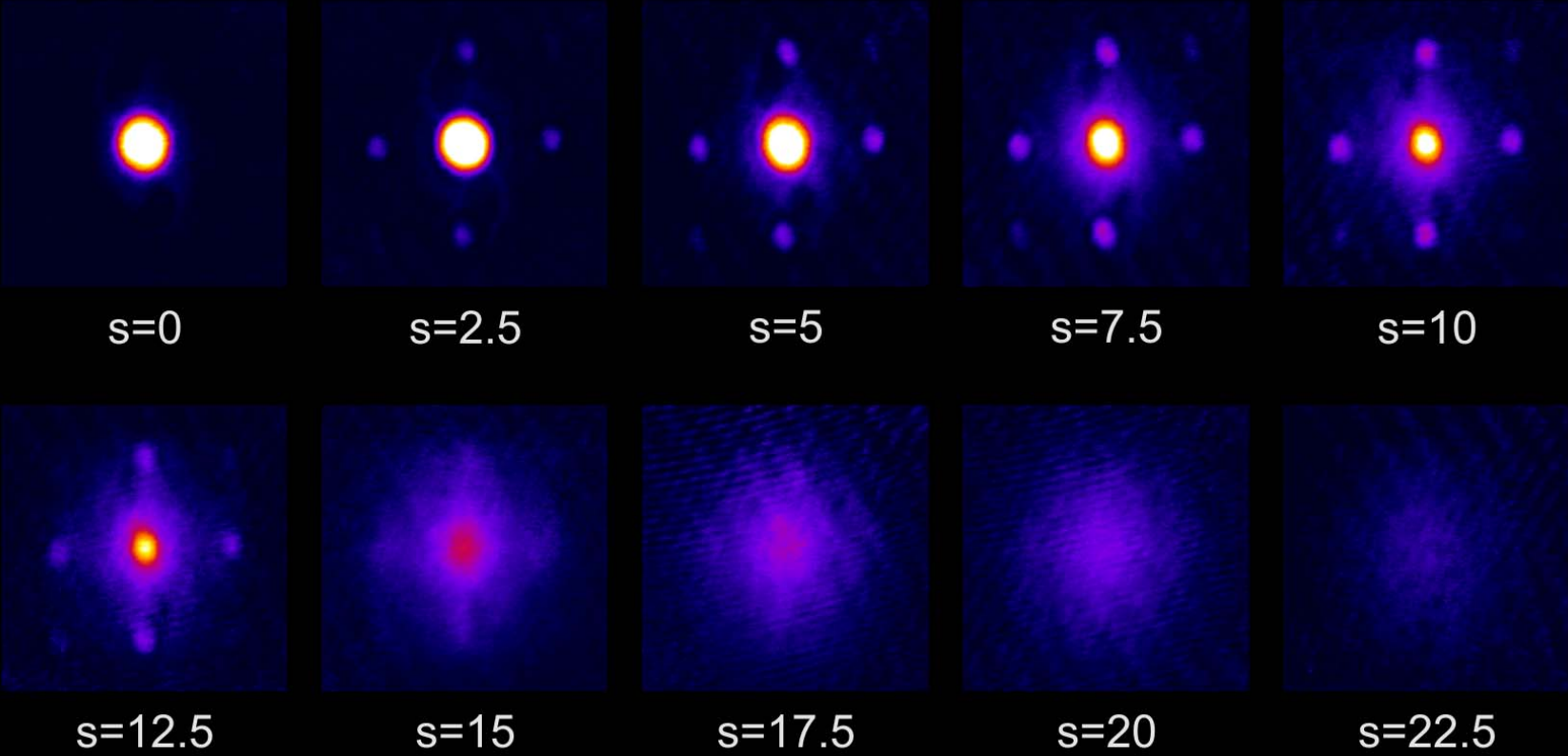
Energy minima of the lattice potential along y direction

Non-periodic modulation of the energy minima with length scale

$$d = \left(\frac{2}{\lambda_1} - \frac{2}{\lambda_2} \right)^{-1} = 1.8 \mu\text{m} = 4.3 \text{ sites}$$

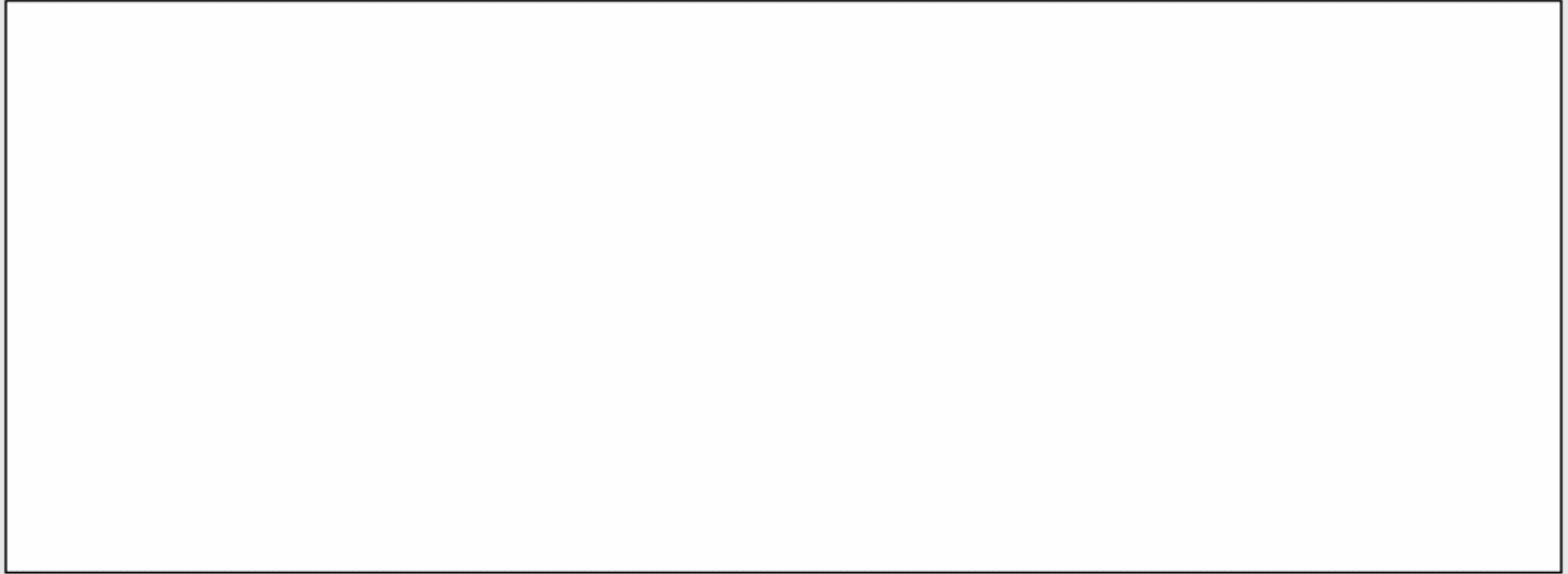


momentum distribution of the atomic sample after expansion
test of phase coherence



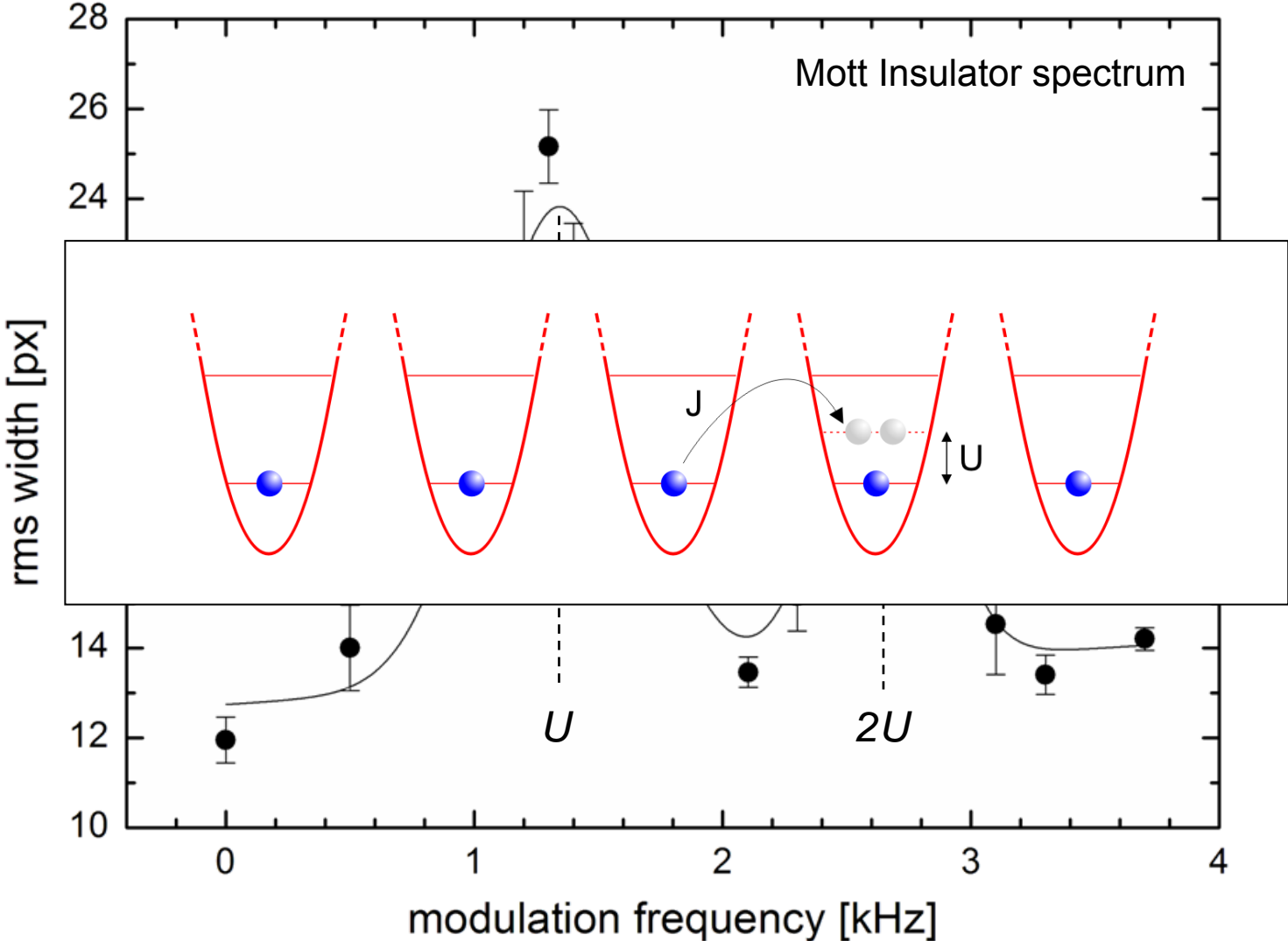
increasing the lattice height \longrightarrow U/J increases

Measuring the excitation spectrum



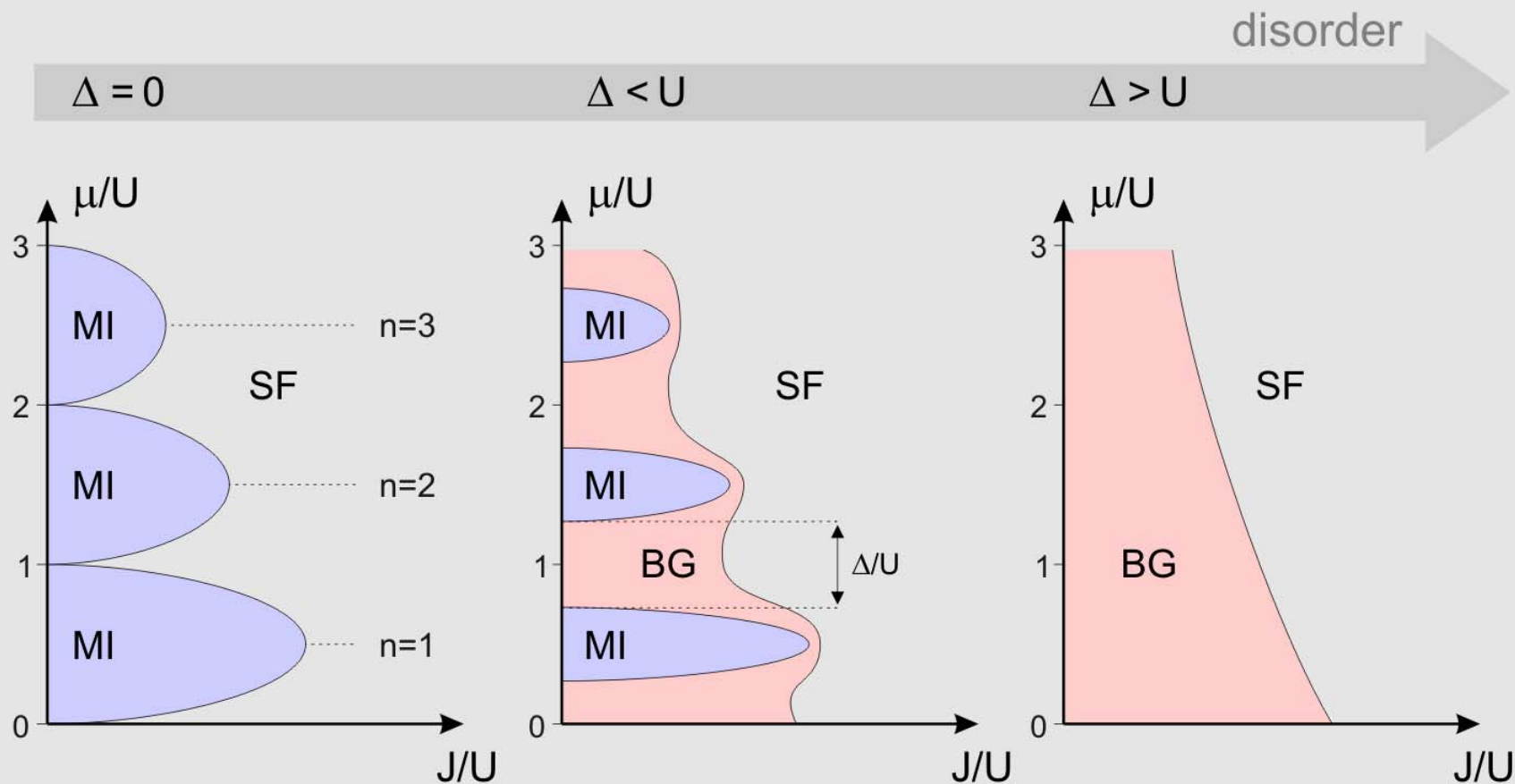
amplitude modulation of the lattice potential
resonant production of excitations (particle-hole pairs)

see also T. Stöferle et al., *PRL* **92**, 130403 (2004)



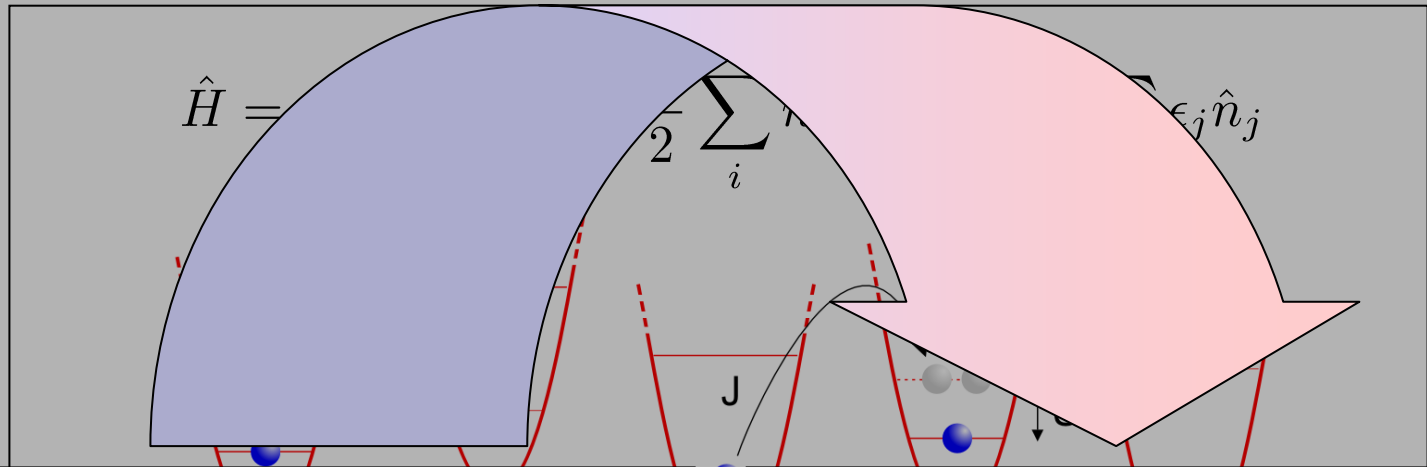
Phase diagrams

Qualitative phase-diagram for an interacting bosons in a disordered lattice:



see M. P. A. Fisher et al., *PRB* **40** 546 (1989).

Bose-Hubbard model with bounded disorder in the external potential $\epsilon_j \in [-\Delta/2, \Delta/2]$



MOTT INSULATOR $\Delta \ll U$

- ✓ No long-range phase coherence
- ✓ Gap in the excitation spectrum
- ✓ Not compressible

BOSE-GLASS $\Delta > U$

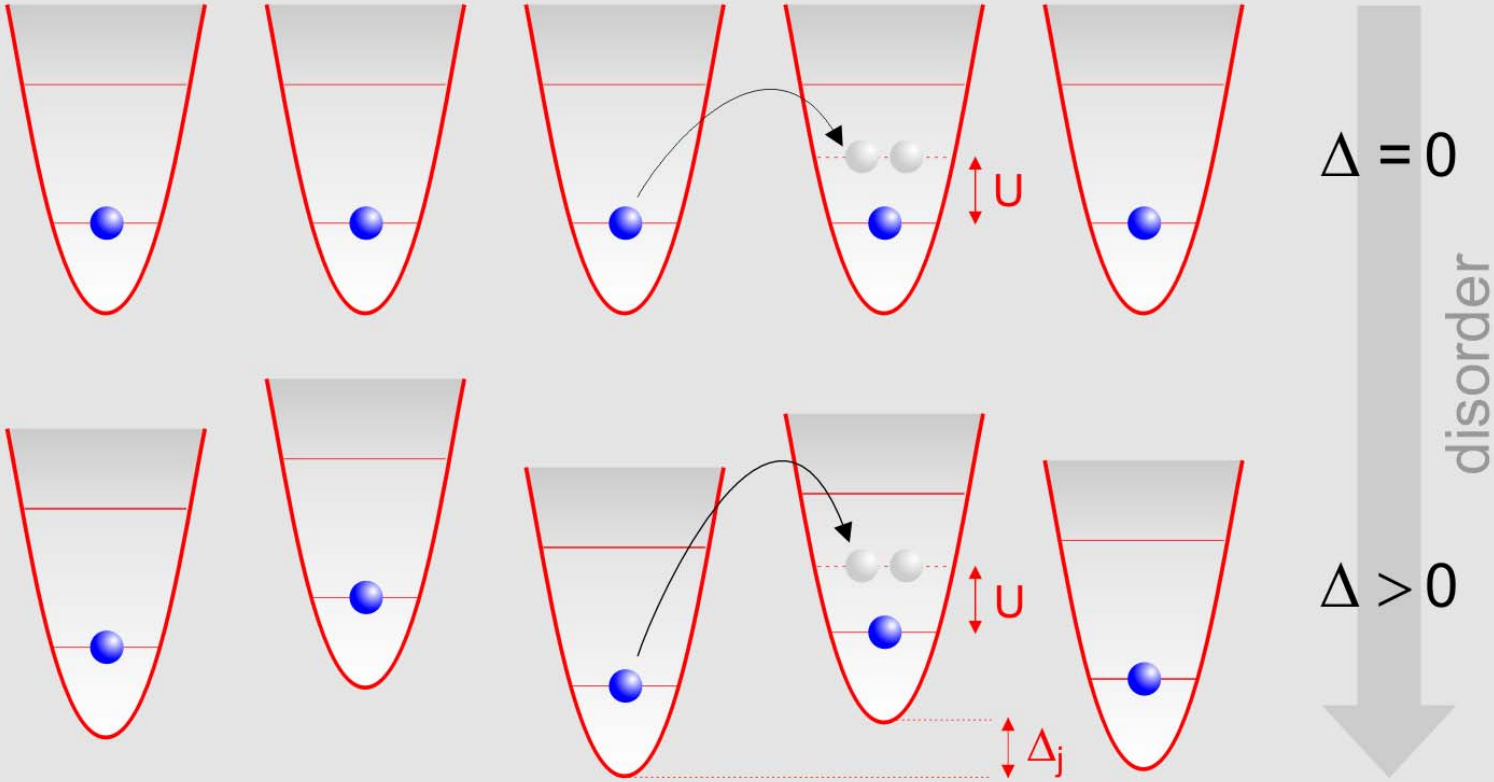
- ✓ No long-range phase coherence
- ✓ **Gapless excitation spectrum**
- ✓ **Finite compressibility**

hopping energy
J

interaction energy
U

disorder
 Δ

Starting from a Mott Insulator and adding disorder, the energy required for the hopping of a boson from a site to a neighboring one becomes a function of position



Excitation spectra

Mott Insulator resonance:

$$f(\nu) = Ae^{-\frac{(\nu-\nu_0)^2}{2\sigma^2}}$$

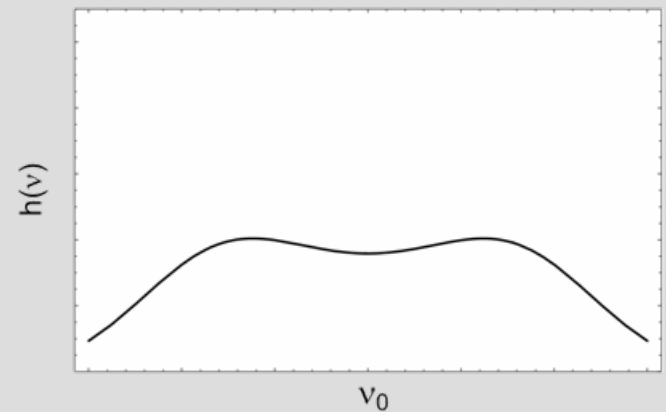
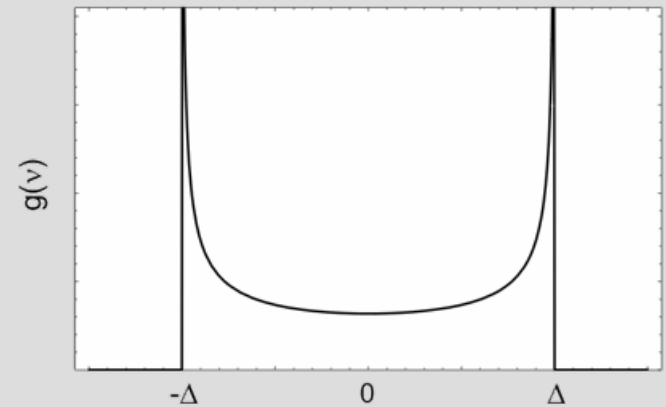
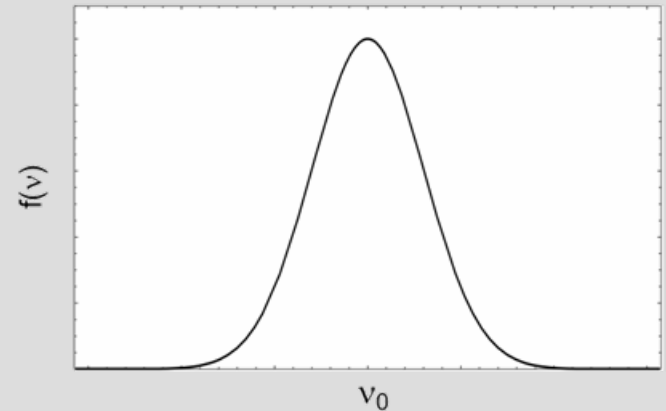
Distribution of energy shifts:

$$g(\nu) = \frac{1}{\pi\Delta} \frac{1}{\sqrt{1 - \left(\frac{\nu}{\Delta}\right)^2}} \theta(\Delta + \nu)\theta(\Delta - \nu)$$



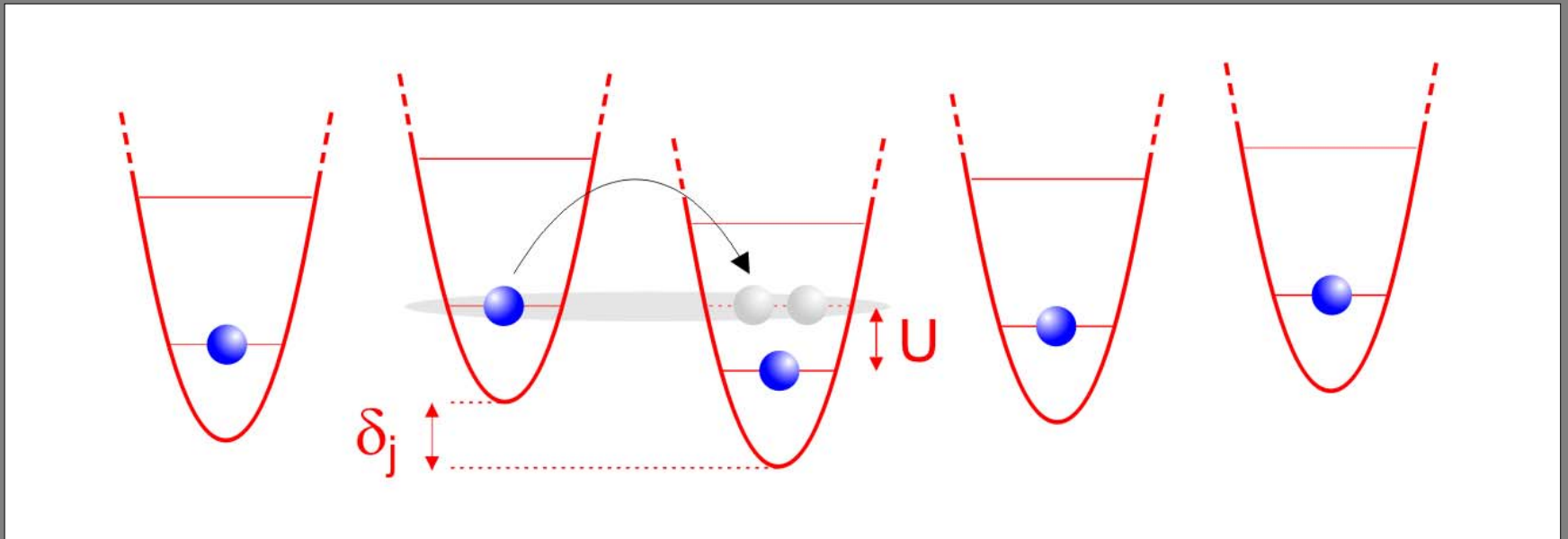
Convolutated spectrum:

$$h(\nu) = \int f(\nu - \bar{\nu})g(\bar{\nu})d\bar{\nu}$$



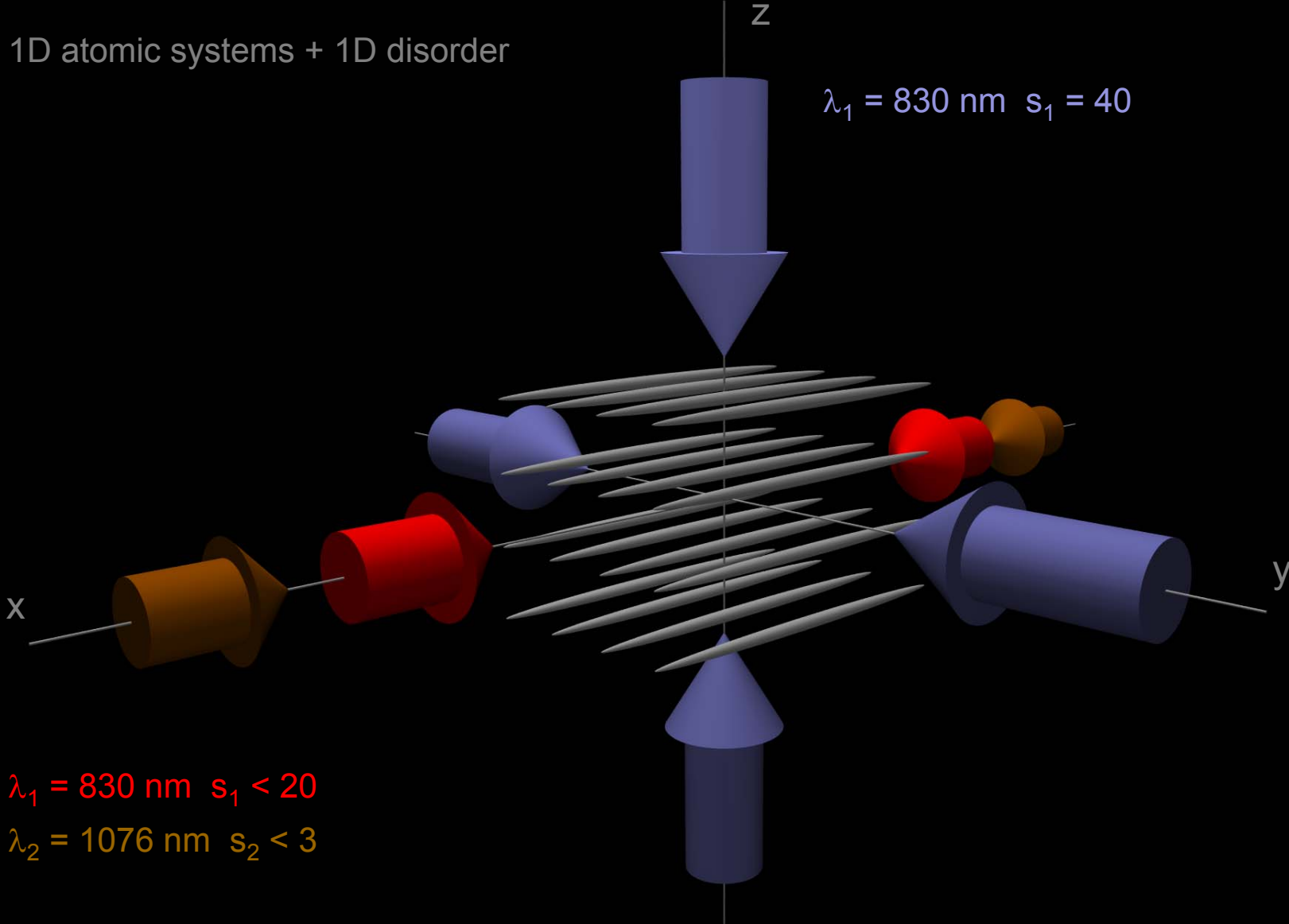
Vanishing gap for strong disorder

For strong disorder $\Delta > U$ the system can be excited at zero energy, the gap vanishes, and different filling configurations in neighboring sites become degenerate.



1D atomic systems + 1D disorder

$\lambda_1 = 830 \text{ nm}$ $s_1 = 40$

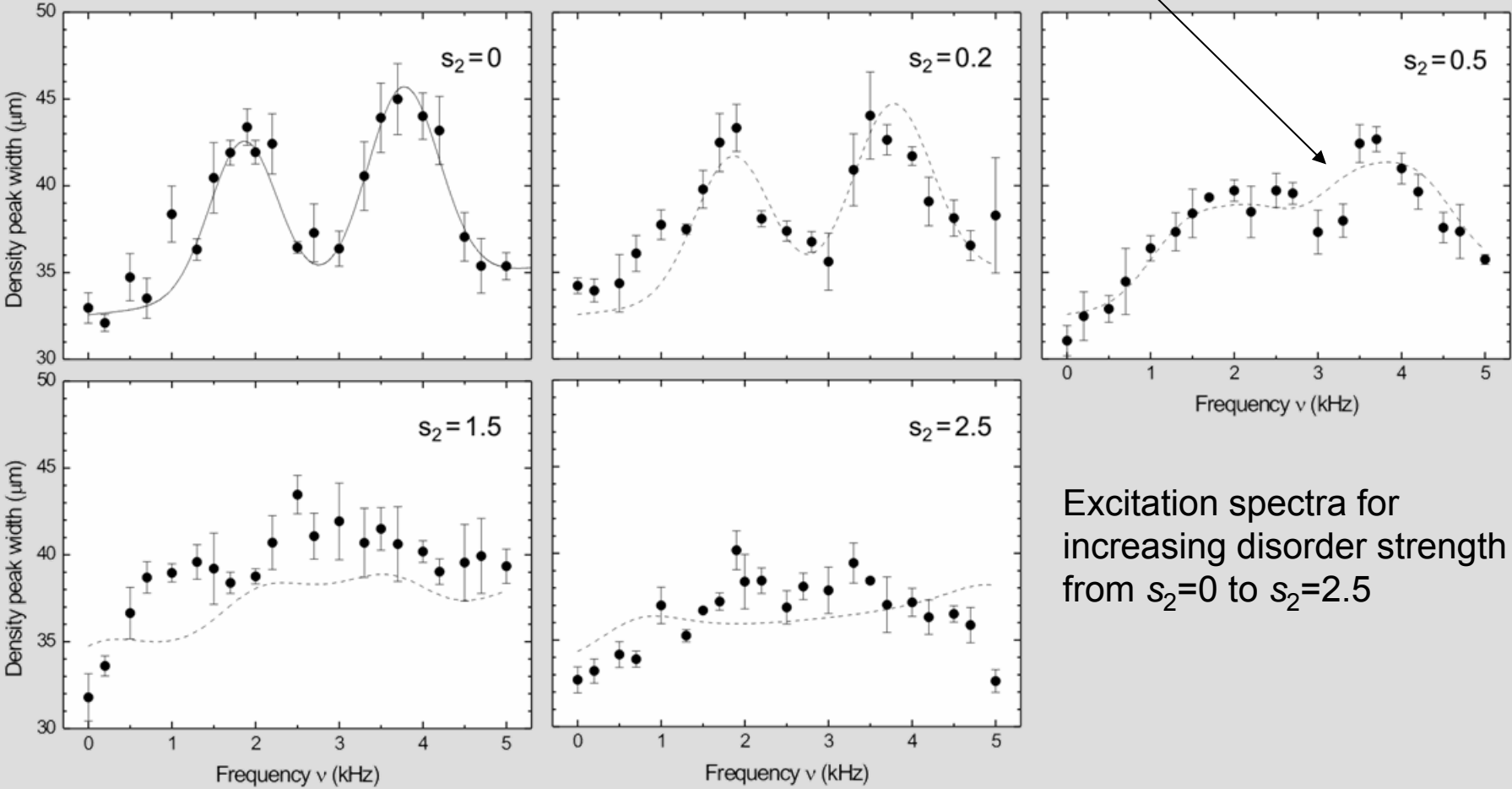


$\lambda_1 = 830 \text{ nm}$ $s_1 < 20$

$\lambda_2 = 1076 \text{ nm}$ $s_2 < 3$

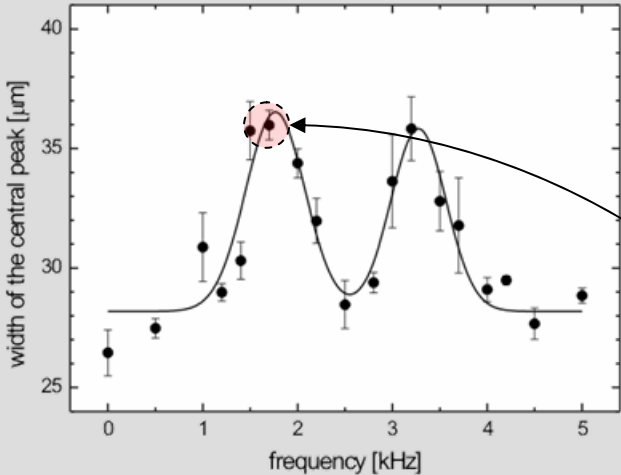
Excitation spectra

Convolution of the MI spectrum at $s_2=0$ with the distribution of site-to-site energy shifts

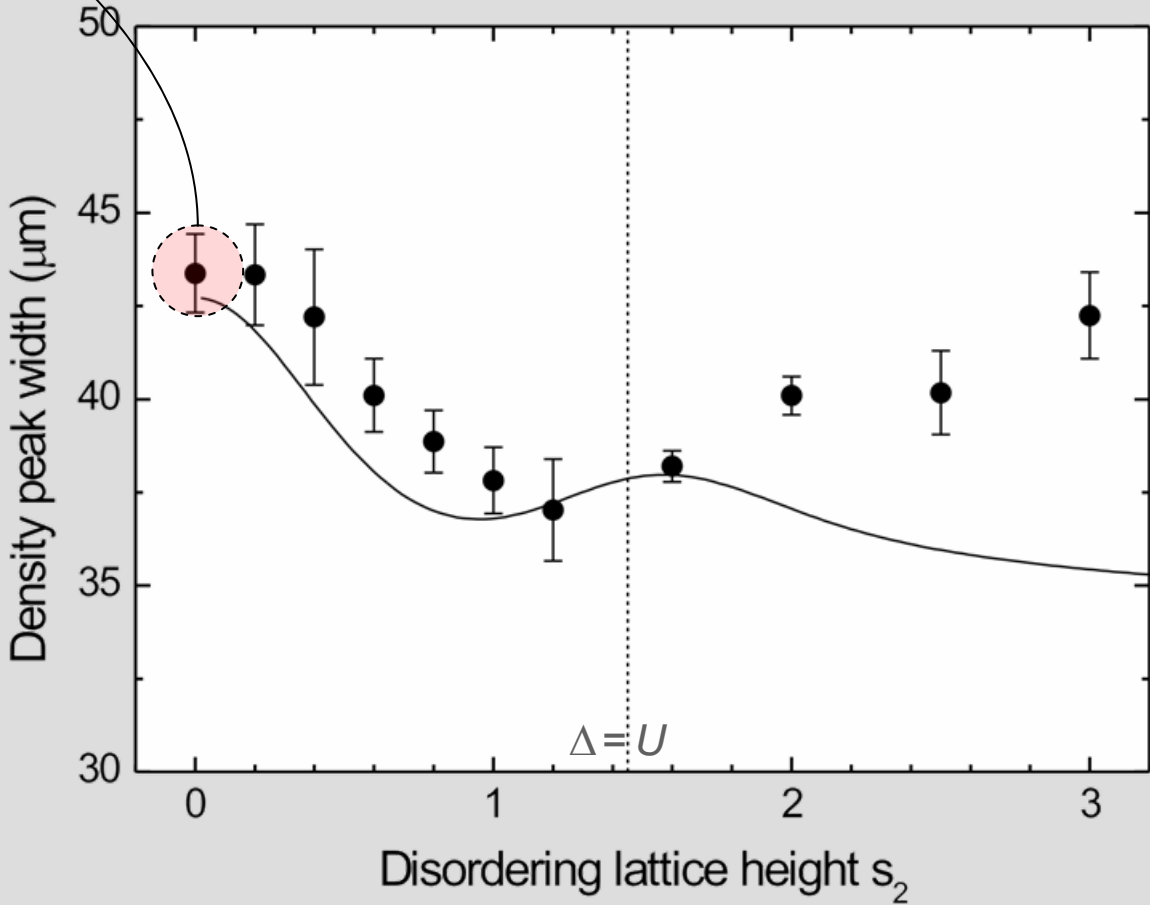


Excitation spectra for increasing disorder strength from $s_2=0$ to $s_2=2.5$

MI spectral broadening

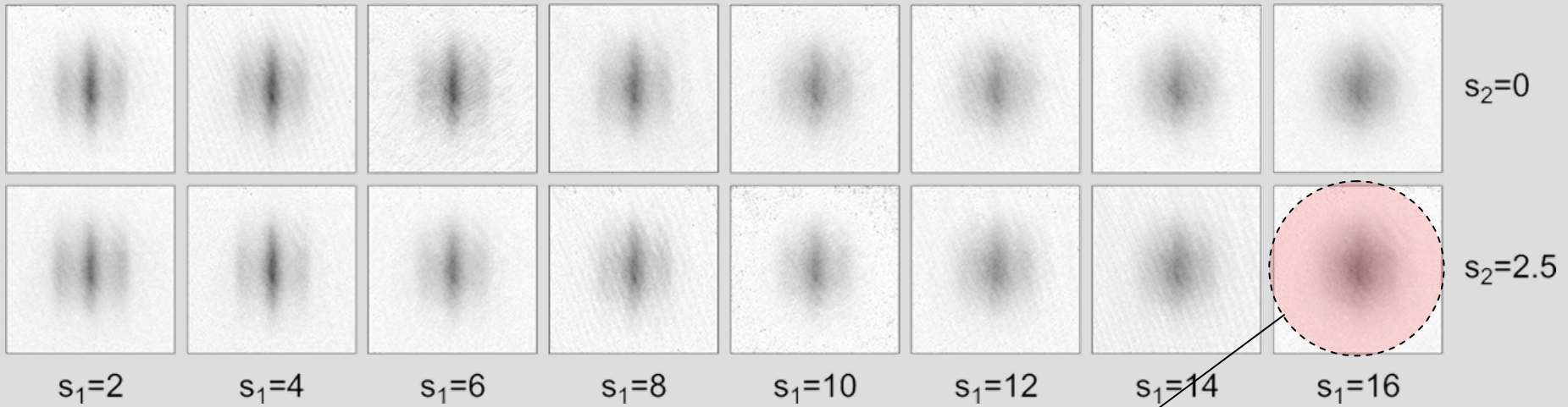


Excitation maximum at U as a function of disorder strength:

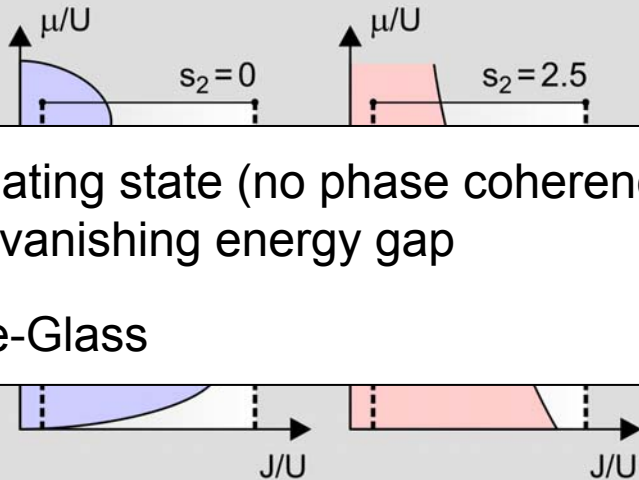


Good agreement with the MI broadening for weak disorder $\Delta < U$

No agreement for strong disorder $\Delta > U$ when the gap goes to zero

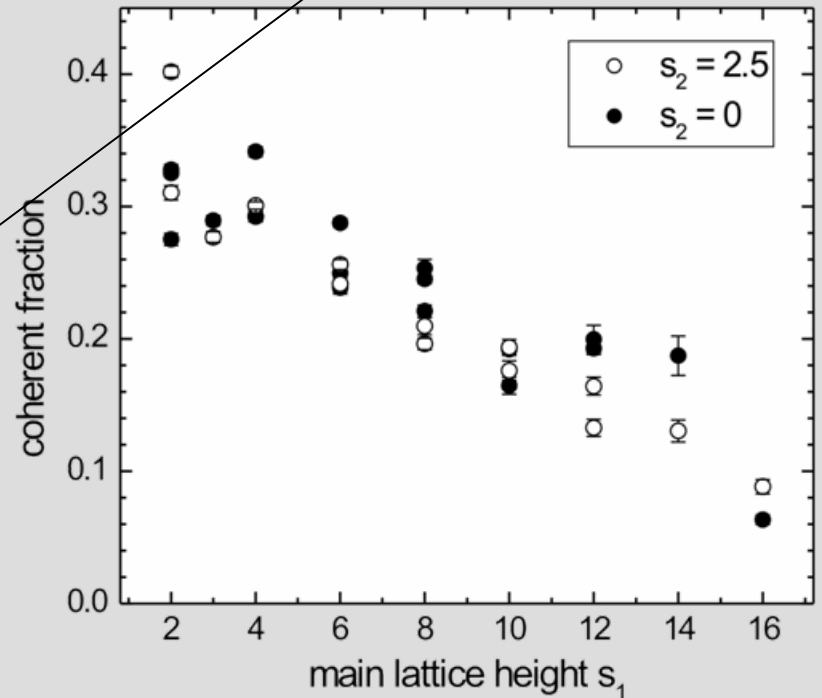


Measurement of the coherent fraction
(visibility of fringes in the TOF images):



Insulating state (no phase coherence)
with vanishing energy gap

Bose-Glass





ultracold bosons in a disordered optical lattice:

- Strongly interacting bosons in a disordered optical lattice :
Bose glass
- **Weakly interacting bosons in a disordered optical lattice: Anderson-like localization**

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON

Bell Telephone Laboratories, Murray Hill, New Jersey

(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

I. INTRODUCTION

A NUMBER of physical phenomena seem to involve quantum-mechanical motion, without any particular thermal activation, among sites at which the mobile entities (spins or electrons, for example) may be localized. The clearest case is that of spin diffusion^{1,2}; another might be the so-called impurity band conduction at low concentrations of impurities. In such situations we suspect that transport occurs not by motion of free carriers (or spin waves), scattered as they move through a medium, but in some sense by quantum-mechanical jumps of the mobile entities from site to site. A second common feature of these phenomena is randomness: random spacings of impurities, random interactions with the "atmosphere" of other impurities, random arrangements of electronic or nuclear spins, etc.

Our eventual purpose in this work will be to lay the foundation for a quantum-mechanical theory of transport problems of this type. Therefore, we must start with simple theoretical models rather than with the complicated experimental situations on spin diffusion or impurity conduction. In this paper, in fact, we attempt only to construct, for such a system, the simplest model we can think of which still has some expectation of representing a real physical situation

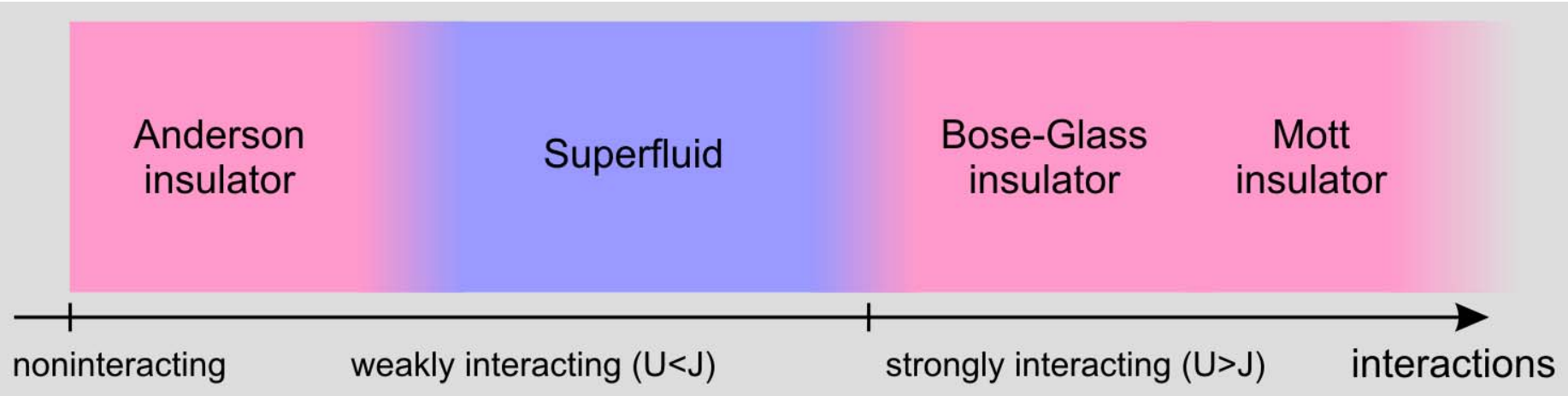
reasonably well, and to prove a theorem about the model. The theorem is that at sufficiently low densities, transport does not take place; the exact wave functions are localized in a small region of space. We also obtain a fairly good estimate of the critical density at which the theorem fails. An additional criterion is that the forces be of sufficiently short range—actually, falling off as $r \rightarrow \infty$ faster than $1/r^2$ —and we derive a rough estimate of the rate of transport in the $V \propto 1/r^2$ case.

Such a theorem is of interest for a number of reasons: first, because it may apply directly to spin diffusion among donor electrons in Si, a situation in which Feher³ has shown experimentally that spin diffusion is negligible; second, and probably more important, as an example of a real physical system with an infinite number of degrees of freedom, having no obvious oversimplification, in which the approach to equilibrium is simply impossible; and third, as the irreducible minimum from which a theory of this kind of transport, if it exists, must start. In particular, it re-emphasizes the caution with which we must treat ideas such as "the thermodynamic system of spin interactions" when there is no obvious contact with a real external heat bath.

The simplified theoretical model we use is meant to represent reasonably well one kind of experimental situation: namely, spin diffusion under conditions of

¹ N. Bloembergen, *Physica* **15**, 386 (1949).² A. M. Portis, *Phys. Rev.* **104**, 584 (1956).³ G. Feher (private communication).

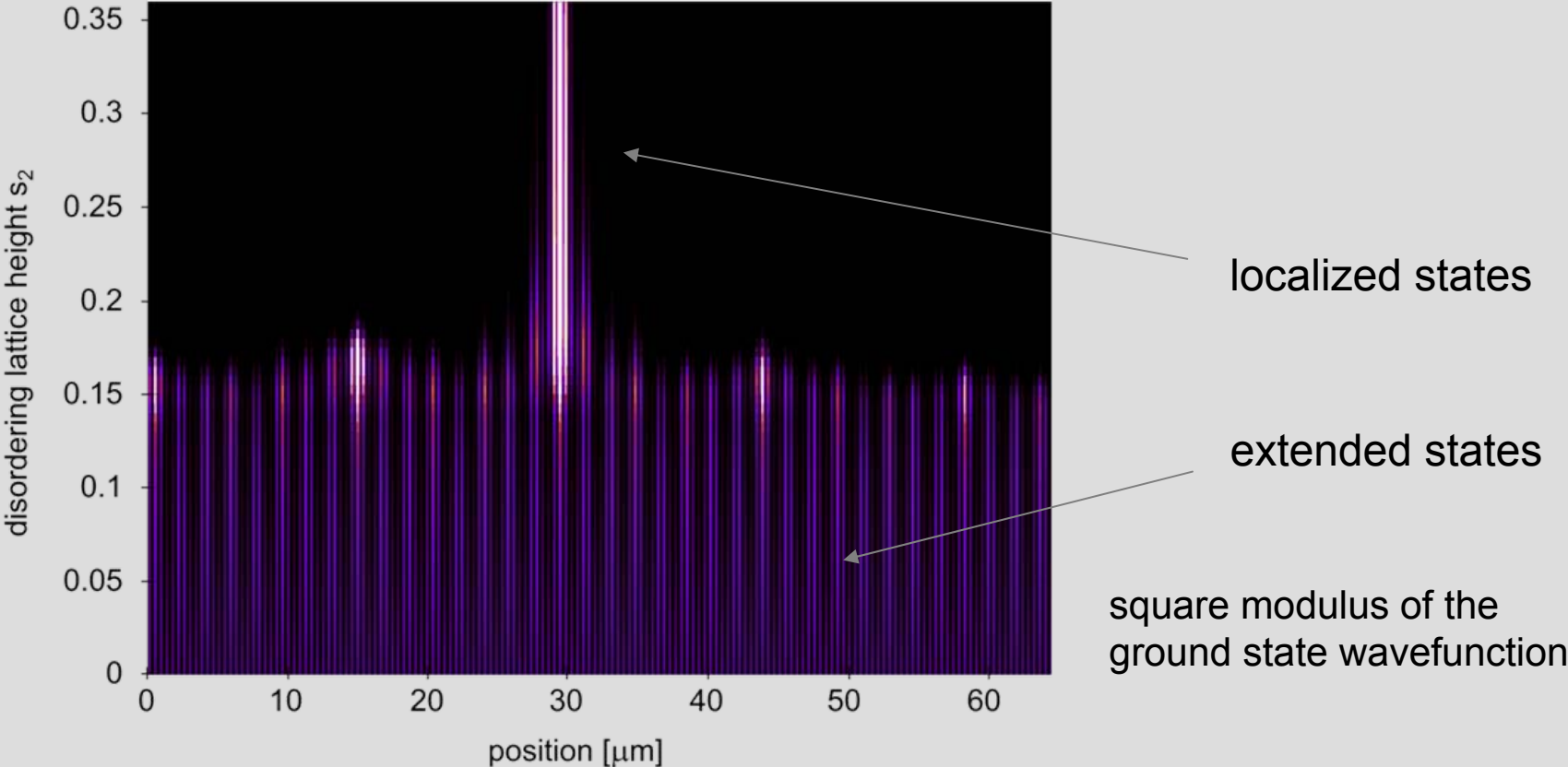
Disordered systems: Role of interactions



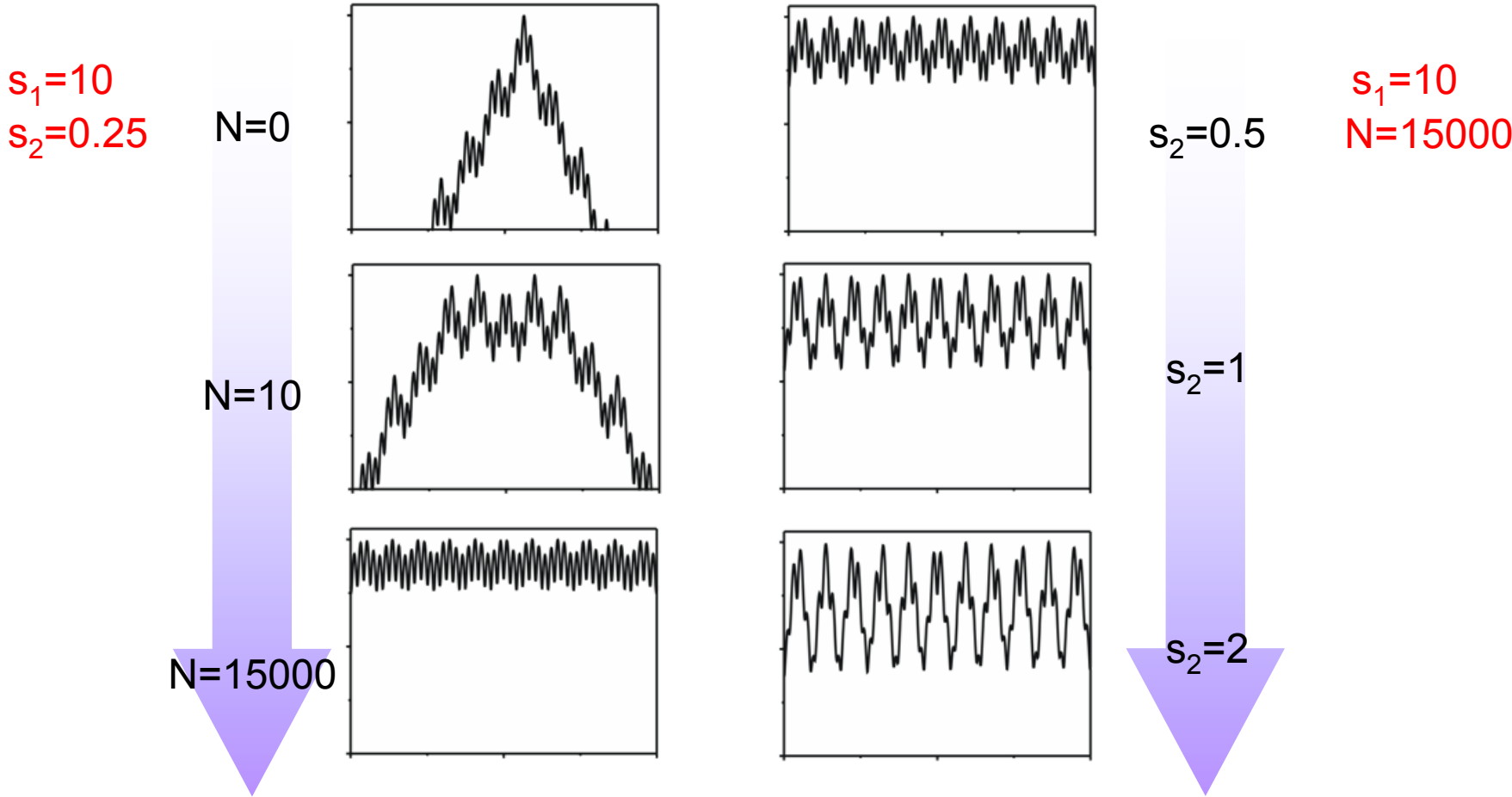
Effects of interaction in the Anderson localization

$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + s_1 E_{R1} \cos^2(k_1 x) + s_2 E_{R2} \cos^2(k_2 y) \right] \psi(x) = E \psi(x)$$

$\lambda_1 = 830 \text{ nm}$ $s_1 = 10$ / $\lambda_2 = 1076 \text{ nm}$

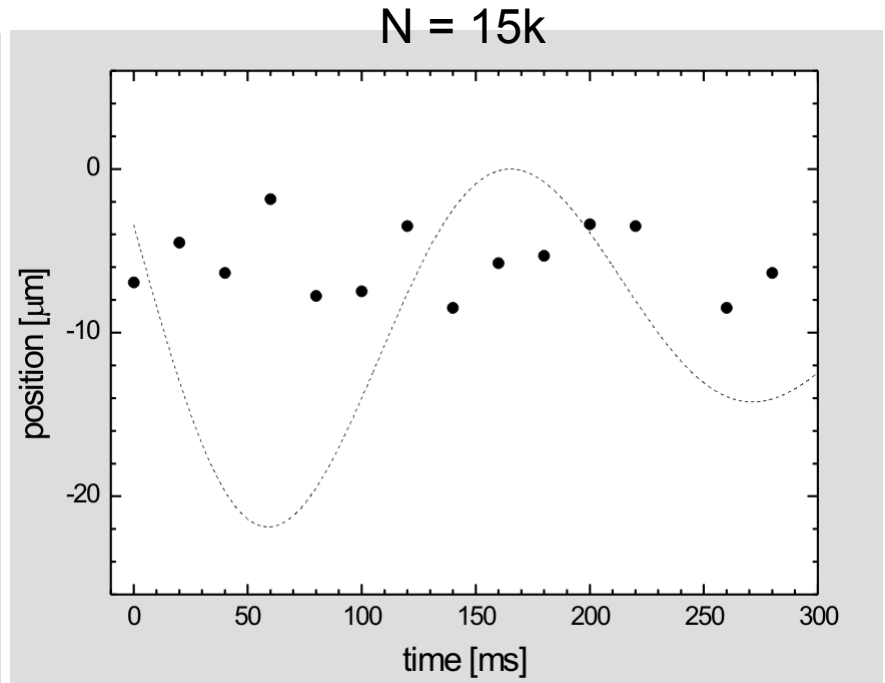
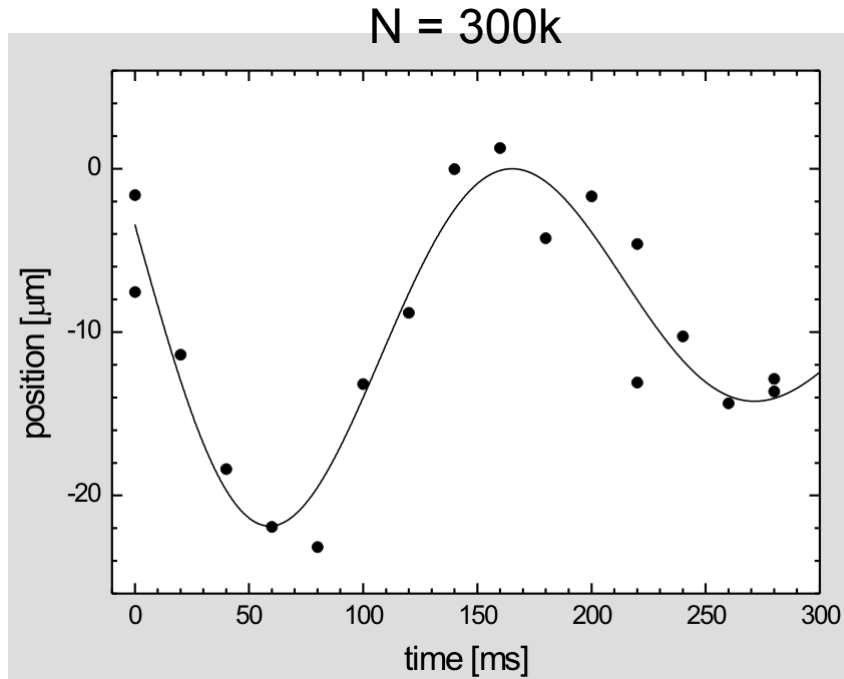


Localization in a quasi-periodic lattice + harmonic trap **adding interactions**



Increasing interactions the “size” of the ground state progressively increases going from a localized state to an extended state...**characterization through transport properties at low atom number**

Decreasing the number of atoms the “localization” effect increases



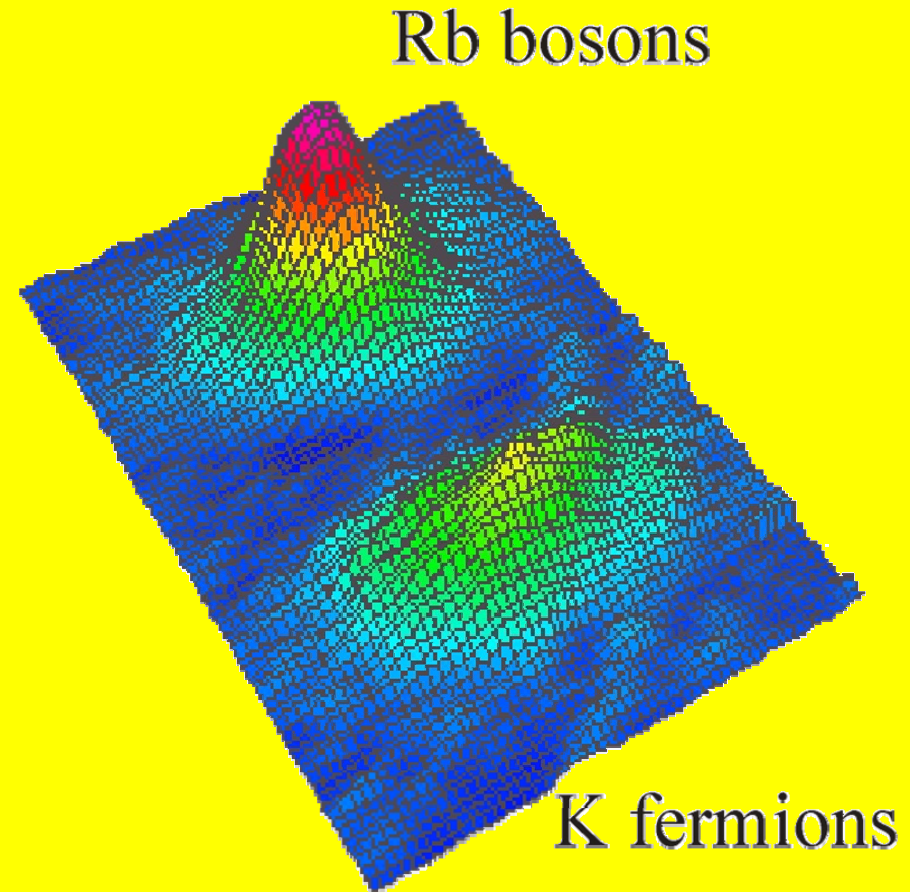
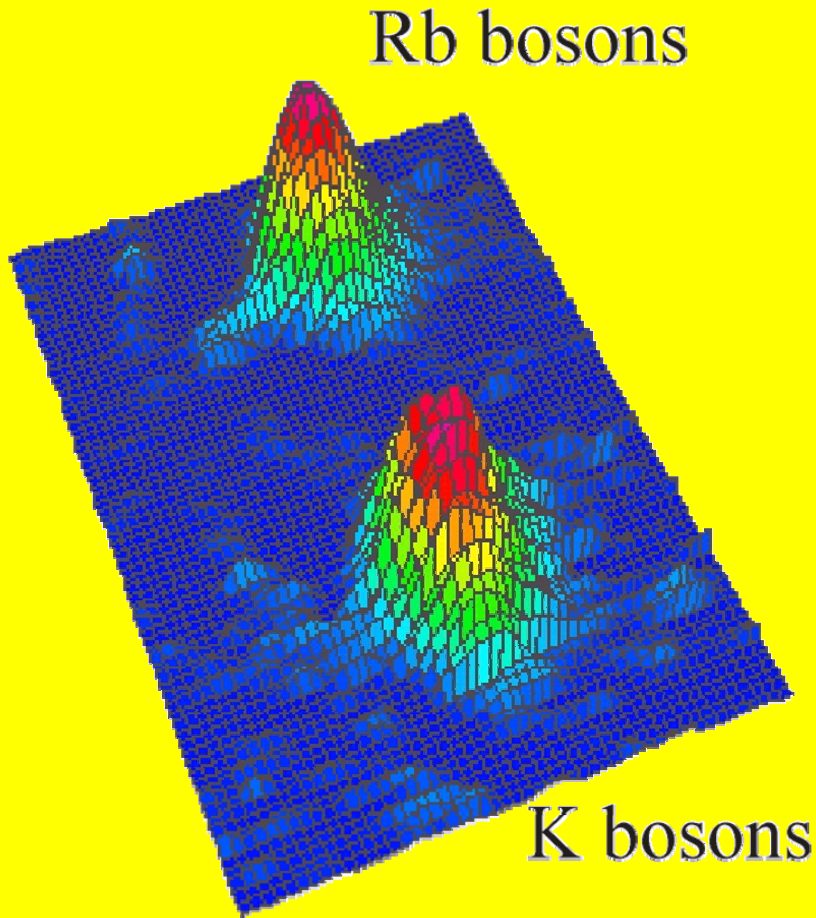
But disorder alone is not the only effect that can lead to localization...

How to avoid interactions?

Fermions?

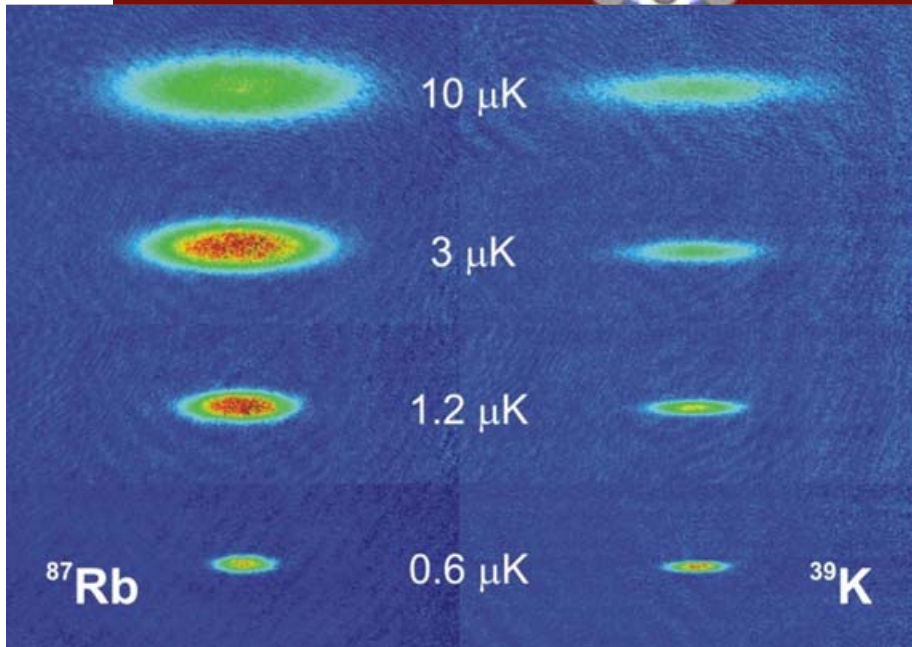
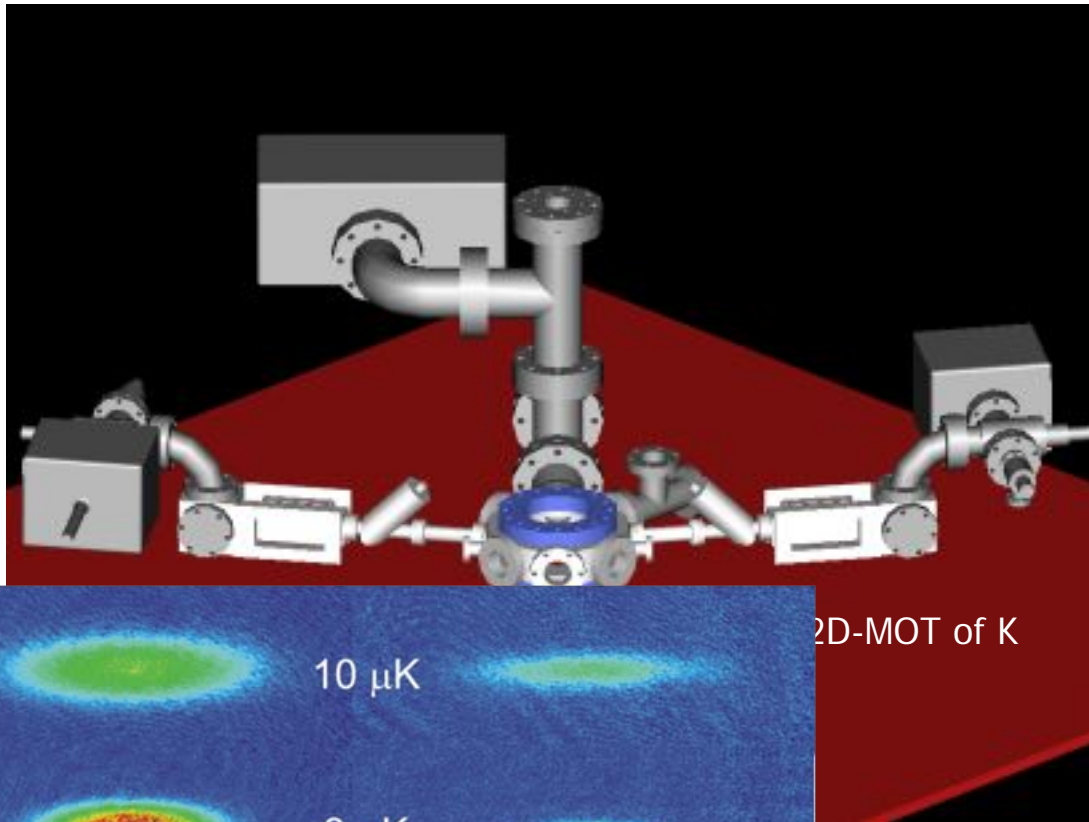
*Tuning the interactions with
Feshbach resonances?*

Sympathetic cooling of Potassium 41 (boson) and 40 (fermion)



G. Modugno, G. Ferrari, G. Roati, R. Brecha, A. Simoni, M. I., *Science* 294, 1320 (2001)

G. Roati, F. Riboli, G. Modugno, M. I. *Phys. Rev. Lett.* 89, 150403 (2002).



In spite of a low interspecies

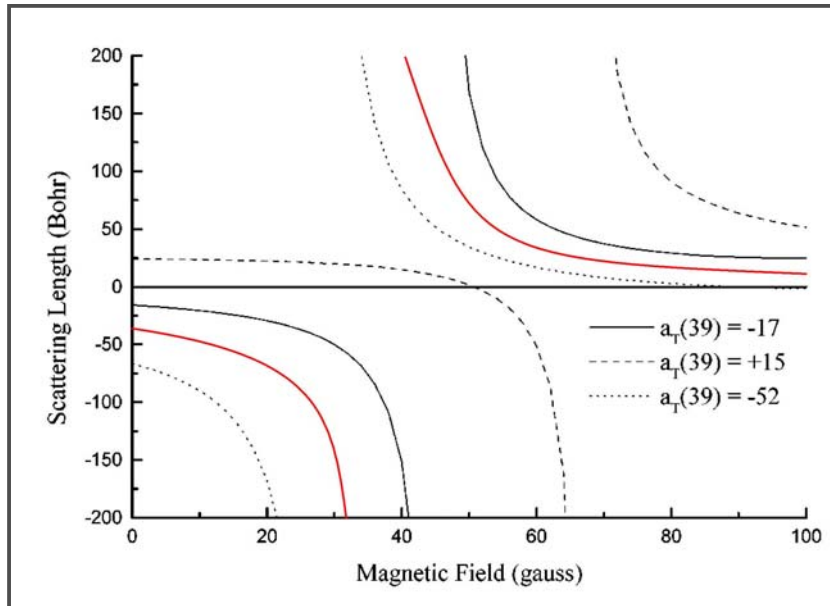
$$a_{39-87} = + 28 a_0$$

but $a_{39-39} = - 33 a_0$

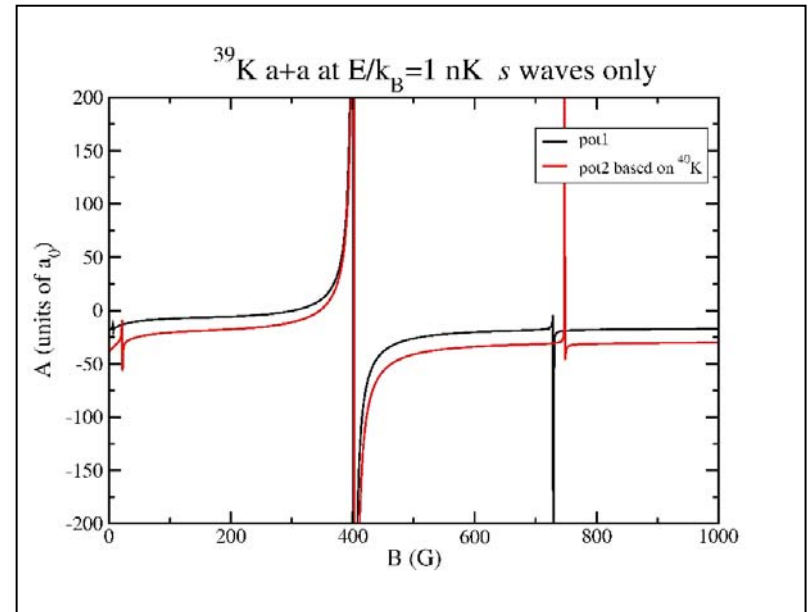
L.De Sarlo, P.Maioli, G.Barontini, J.Catani, F.Minardi, M.I.
Phys.Rev. A75, 022715 (2007)

K39-K39 Feshbach resonances

Broad Feshbach resonances are predicted for different hyperfine states at moderate magnetic fields



Resonance for atoms in $|F=1, m=-1\rangle$
J. Bohn et al., Phys. Rev. A 58, 3660 (1999)



Resonance for atoms in $|F=1, m=1\rangle$
A. Simoni, E. Tiesinga (private communication)

The quantum gas team at LENS : (FERMI) – BOSE MIXTURE

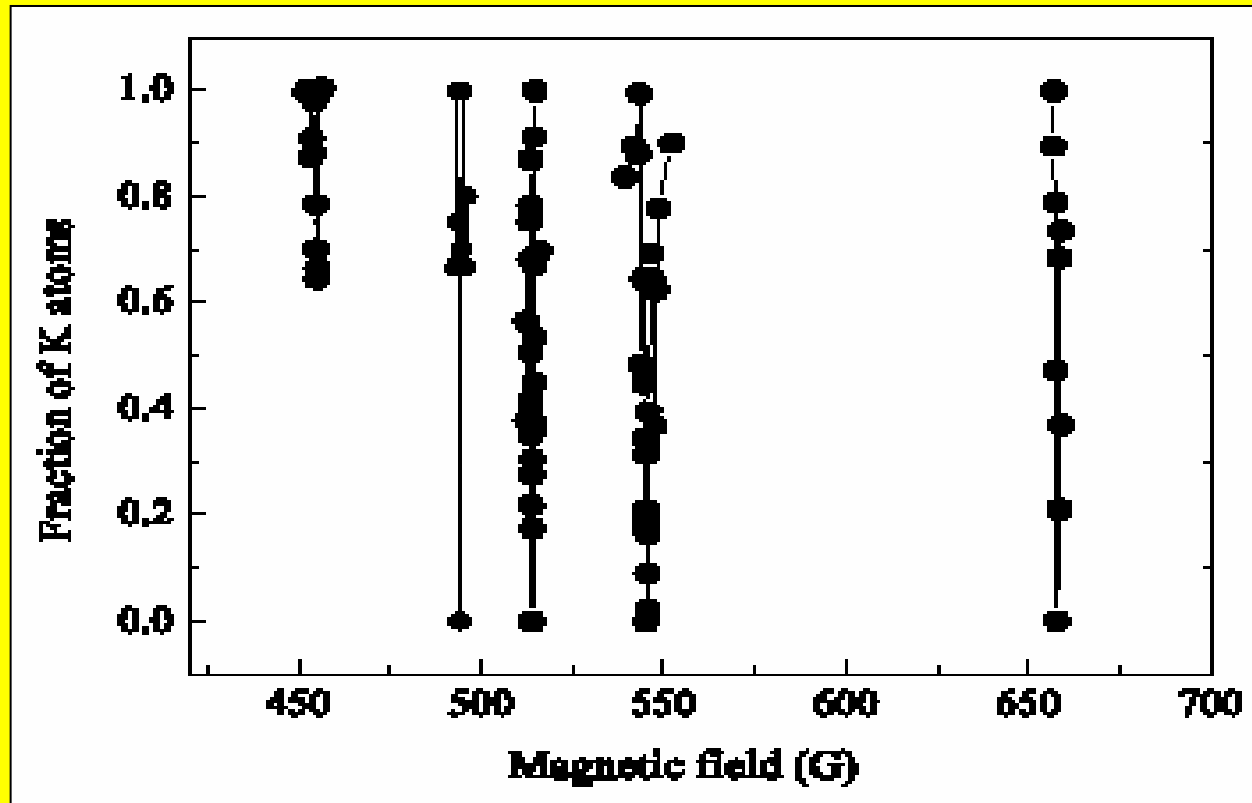


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M.I.

Marco Fattori
(Francesca Ferlaino)
(H. Ott , E. de Mirandes)

from ...Feshbach resonances in ^{87}Rb - ^{40}K



Ferlaino et al, Phys. Rev. A **73**, 040702(R) (2006)

K-Rb scattering lengths and Fano-Feshbach resonances

Interspecies scattering lengths for:
for K-Rb mixture

TABLE II. Calculated singlet and triplet s -wave scattering lengths for collisions between K and Rb isotopes. Sensitivity parameters $\beta_{s,t}$ to the number of bound states (Eq. 1 in our paper) are also shown, with power of ten displayed in parenthesis.

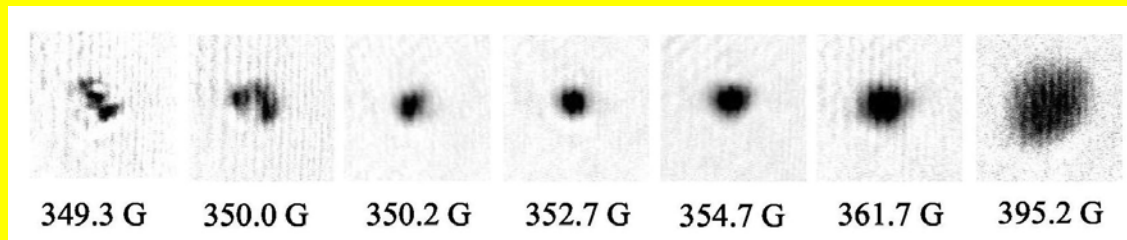
K-Rb	$\bar{a}_s (a_0)$	β_s	$\bar{a}_t (a_0)$	β_t
39-85	26.5 ± 0.9	$-2.8(-2)$	63.0 ± 0.5	$-1.3(-2)$
39-87	824_{-70}^{+90}	$-1.5(-2)$	35.9 ± 0.7	$-1.6(-2)$
40-85	64.5 ± 0.6	$-3.9(-3)$	-28.4 ± 1.6	$-1.8(-2)$
40-87	-111 ± 5		-215 ± 10	
41-85	106.0 ± 0.8	$3.6(-3)$	348 ± 10	$6.5(-3)$
41-87	14.0 ± 1.1	$2.6(-2)$	163.7 ± 1.6	$7.7(-3)$

Resonances for atoms in ground state,

TABLE III. Predicted zero-field s -wave scattering lengths for the absolute ground state of K-Rb isotopes. Feshbach resonance positions and widths are also provided for three selected isotopic pairs. The quoted uncertainties do not include the uncertainty on the number of bound states.

K-Rb	$a (a_0)$	$B_h (G)$	$\Delta_h (G)$
39-85	56.6 ± 0.4		
39-87	27.9 ± 0.9	248.8 ± 1.6	0.26
		320.1 ± 1.6	7.9
		531.9 ± 1.2	2.7
		616.2 ± 1.5	0.10
40-85	-21.3 ± 1.6		
40-87	-185 ± 7		
41-85	283 ± 6	132.5 ± 0.6	0.19
		141.2 ± 1.1	$2 \cdot 10^{-4}$
		147 ± 2	0.025
		184.6 ± 1.0	2.9
		191.4 ± 1.0	0.81
		660 ± 3	3.4
		687 ± 2	16
41-87	1667_{-406}^{+790}	17 ± 5	45
		67 ± 3	8.9
		516 ± 7	82
		688 ± 8	0.059

^{39}K Bose-Einstein condensate with tunable interactions



...also the last stable alkali atom is degenerate

G.Roati, M.Zaccanti, J.Catani, M.Modugno, A.Simoni, M.Inguscio, G.Modugno
arXiv:cond-mat/070314v1

^{87}Rb ^{39}K

Magneto-optical trap:

$$N_{\text{Rb}} = 10^9 \quad T_{\text{Rb}} \sim 100 \mu\text{K}$$

$$N_{\text{K}} = 10^7 \quad T_{\text{K}} \sim 300 \mu\text{K}$$

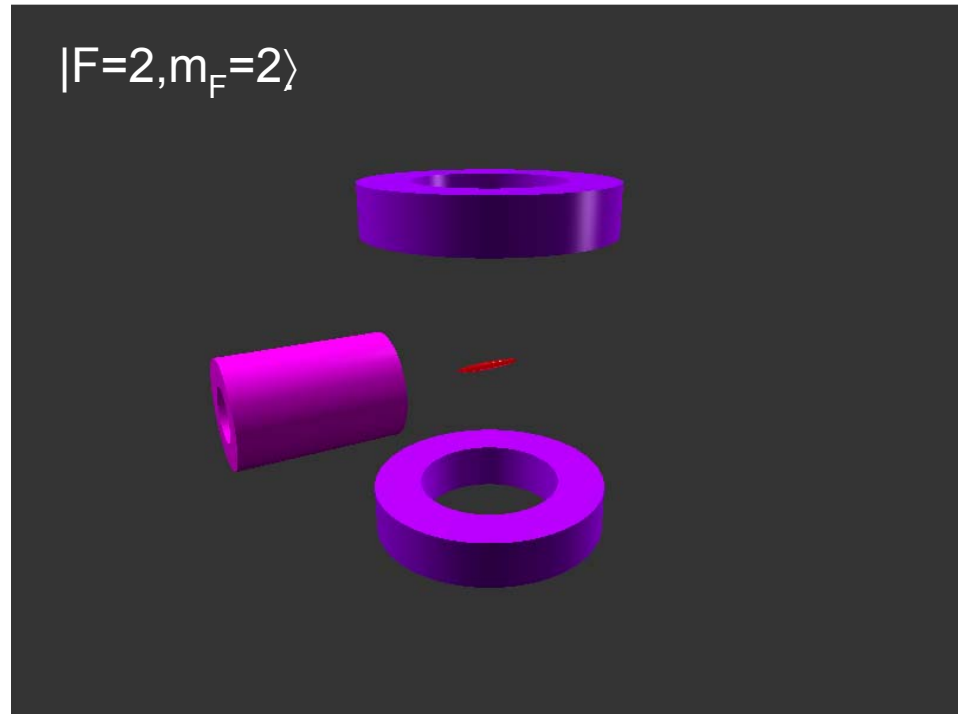
Magnetic trap:

- μ -w evaporation of Rb:
- Sympathetic cooling of K ($a_{\text{KRb}} = 28 a_0$)

$$N_{\text{Rb}} = 10^6 \quad N_{\text{K}} = 2 \cdot 10^5$$
$$T_{\text{K}} \sim 800 \text{ nK}$$

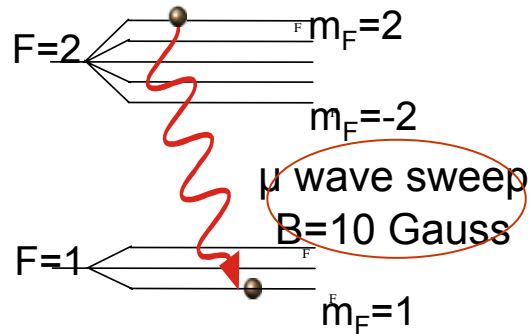
$a_{\text{K}} < 0 \Rightarrow$ condensation would be accompanied by collapse

Homonuclear Feshbach resonance \Rightarrow tuning a_{K} to positive value

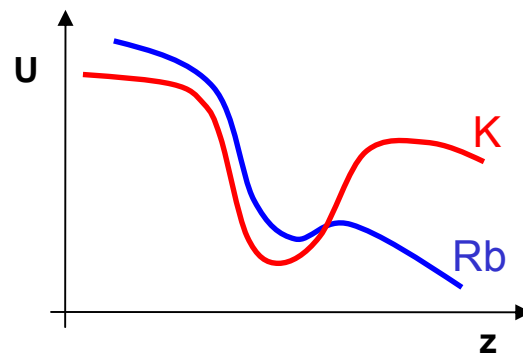
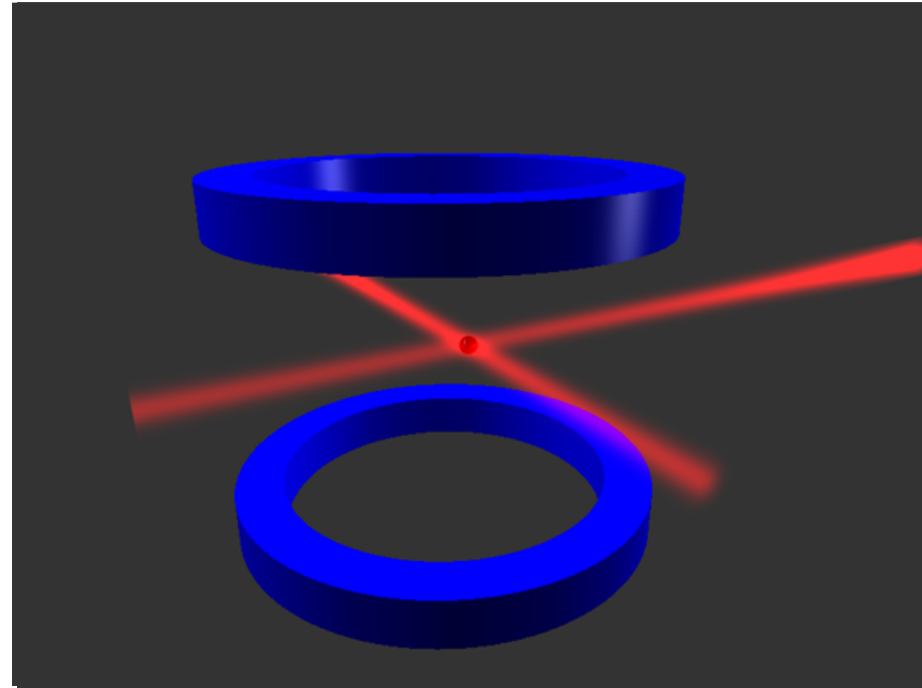


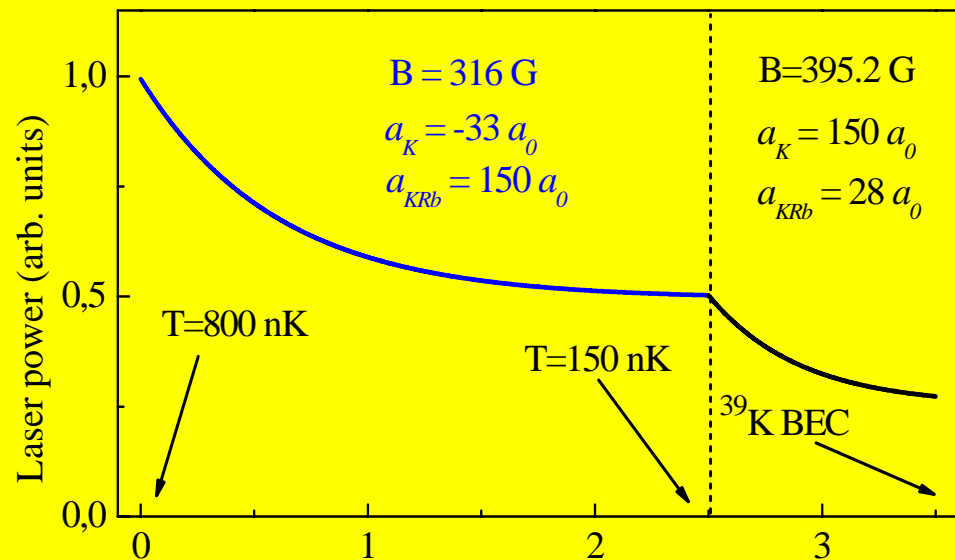
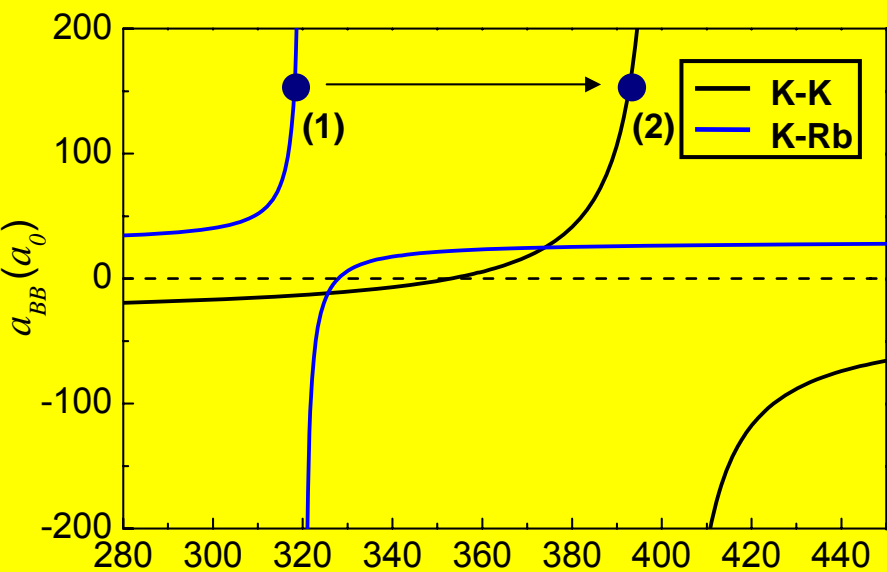
Optical trap:

- Transfers to right Zeeman sublevels $|1,1\rangle$

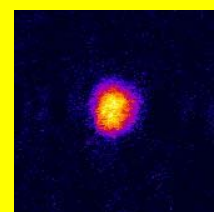
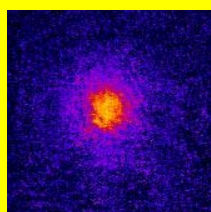
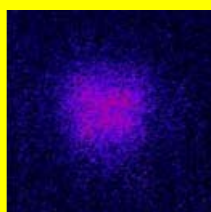


- Evaporation in presence of a homogeneous magnetic field
- We choose the trap parameters in order to evaporate mainly Rb atoms





B (G)



Time (s)

$B_0 = 317.9$ G

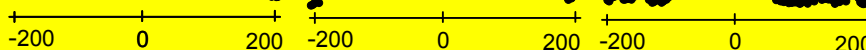
$B_0 = 402.4$ G

Density distribution

3 s

3.2 s

3.5 s

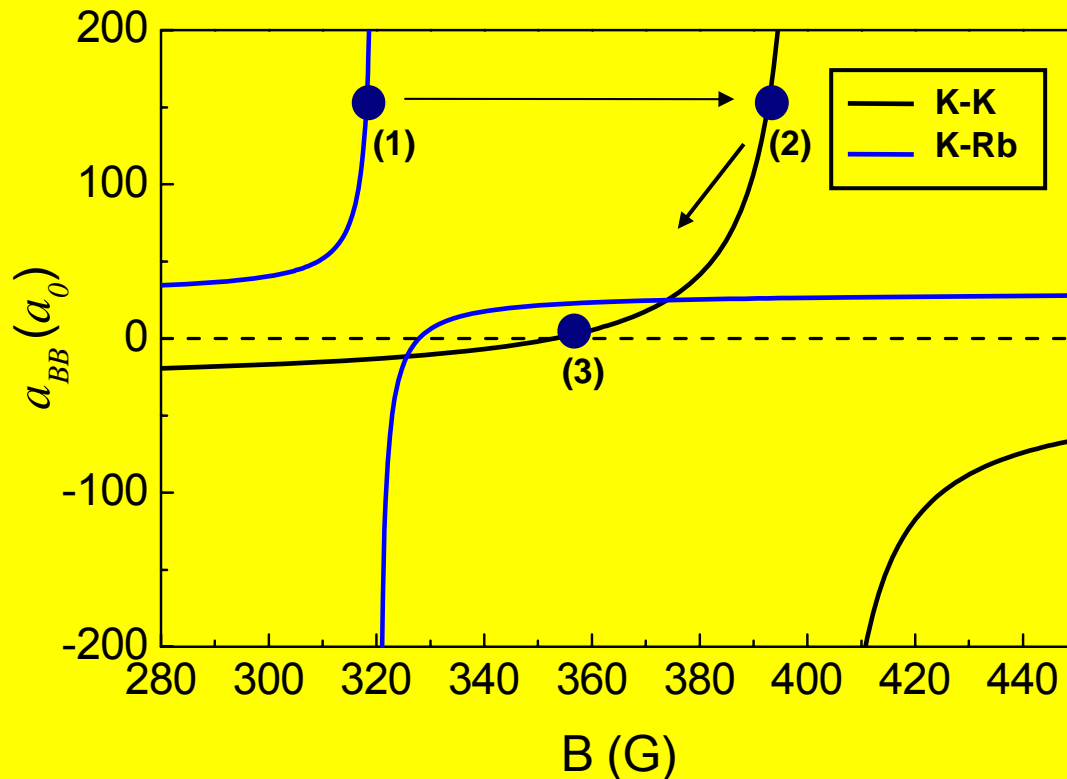


Horizontal position (μm)

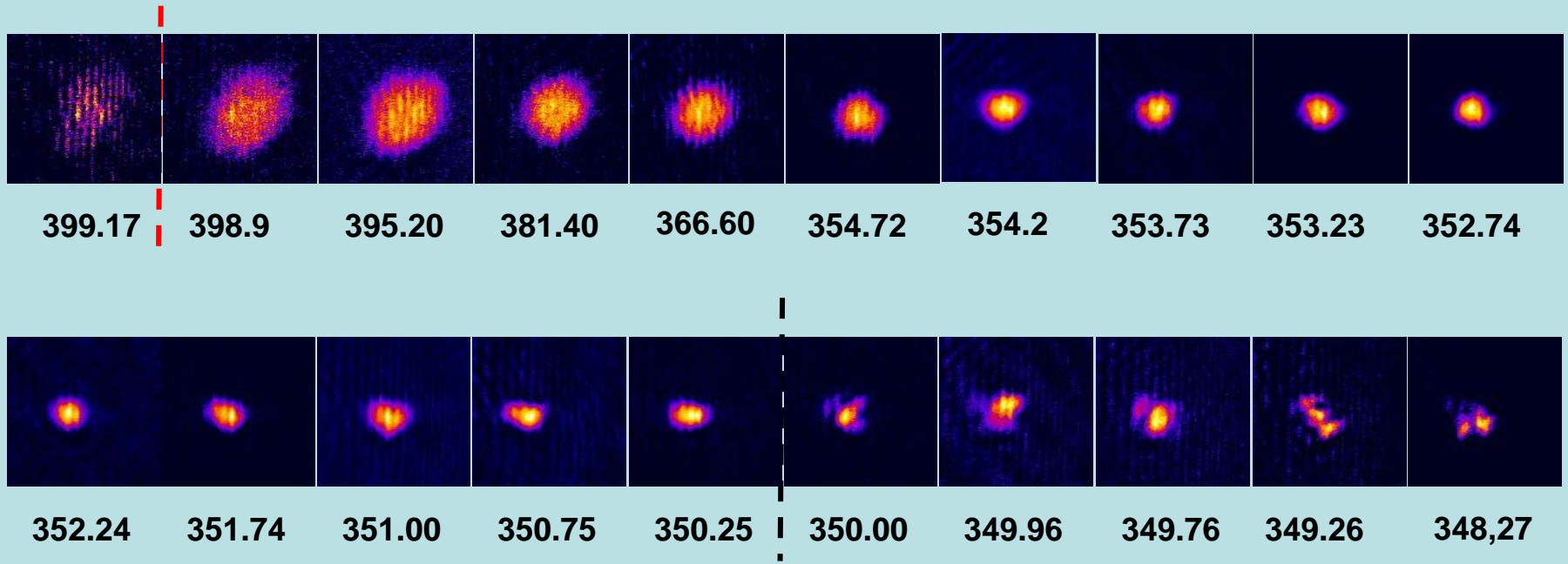
$N_K = 4 \cdot 10^4$
 $T_c \sim 100$ nK

Tuning the interactions

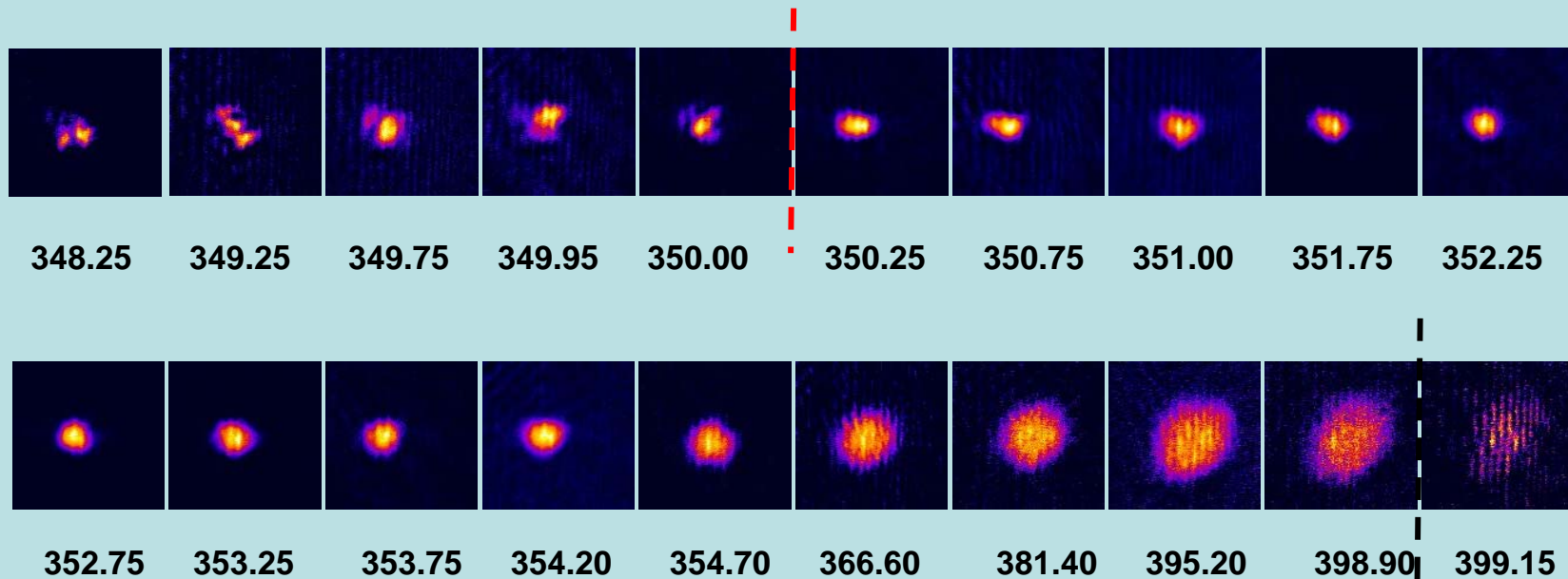
We have explored the 52 G-wide magnetic-field region below the homonuclear resonance in which the condensate is stable



Probing the tunability of the ^{39}K BEC: results

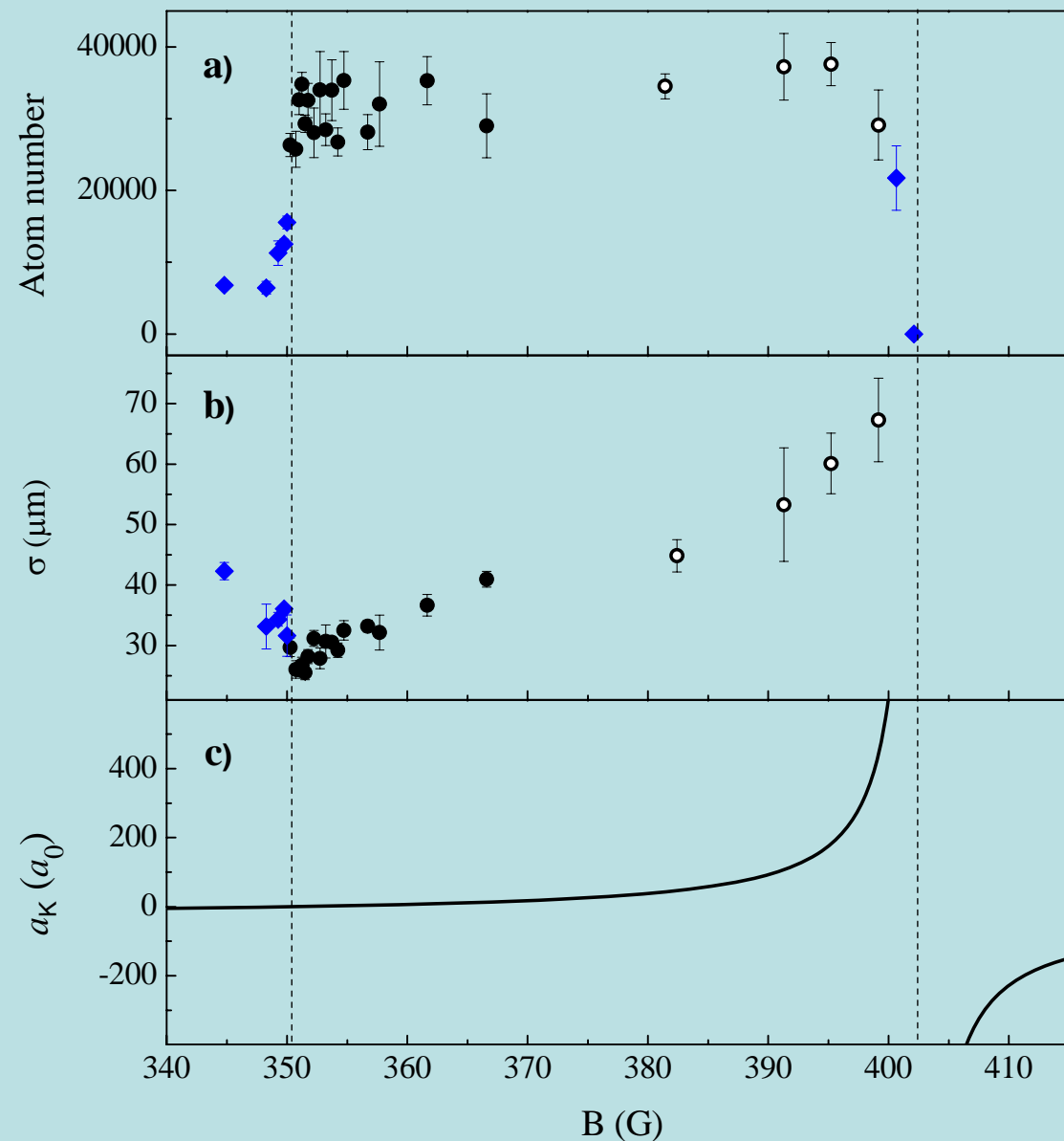


Tuning the interactions



- Between 350 G and 350.25 G we observe a sudden drop of the atom number that can be attributed to the collapse of the BEC for too large negative a (expected for $a(B) \leq -0.4 \div -0.8 a_0$)
- On the other side, moving towards the FR center, BEC is destroyed by enhanced three-body recombination ($K_3 \propto a^4$)

Tuning the interactions



➤ The width of the condensate decreases by almost a factor three from FR centre to zero-crossing

➤ This is due to the variation of the interaction strength in the condensate:

• $\sigma \propto (E_{int})^{1/2}$

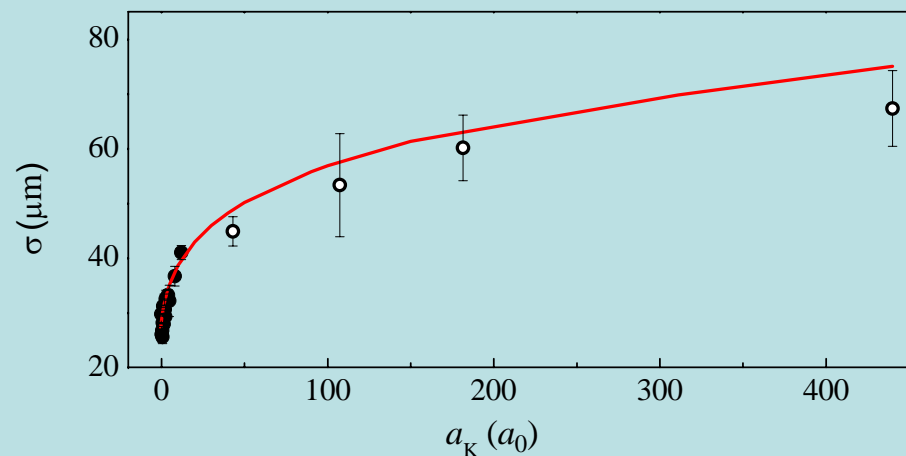
• $E_{int} \propto a^{2/5}$ in the TF limit for large positive a_K

• to reach exactly the ground-state energy of the trapping potential for $a = 0$)

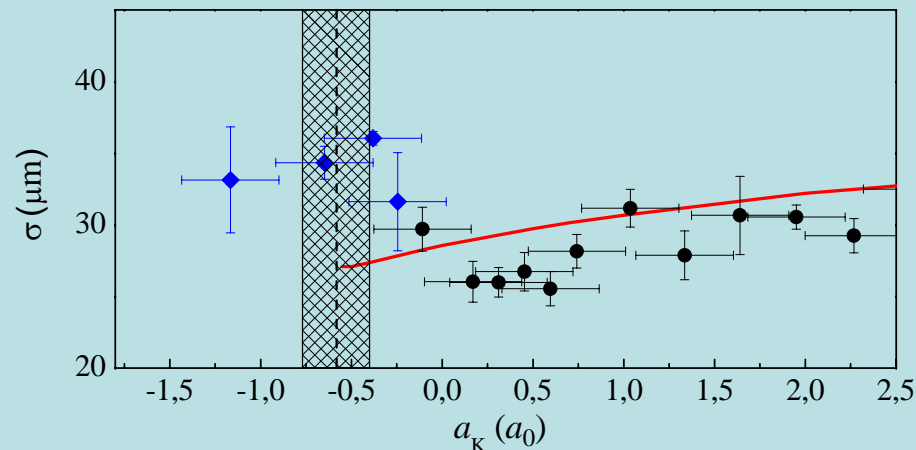
Tuning the interactions

The experimental data is compared to a numerical calculation based on the GP theory, using the $a(B)$ determined by our quantum collisional model

- The good agreement between theory and experiment indicates that precise tuning of a around zero is possible



- The hatched region indicates the critical scattering length for collapse ($a_c = -0.57 a_0$) for the nominal atom number in the experiment



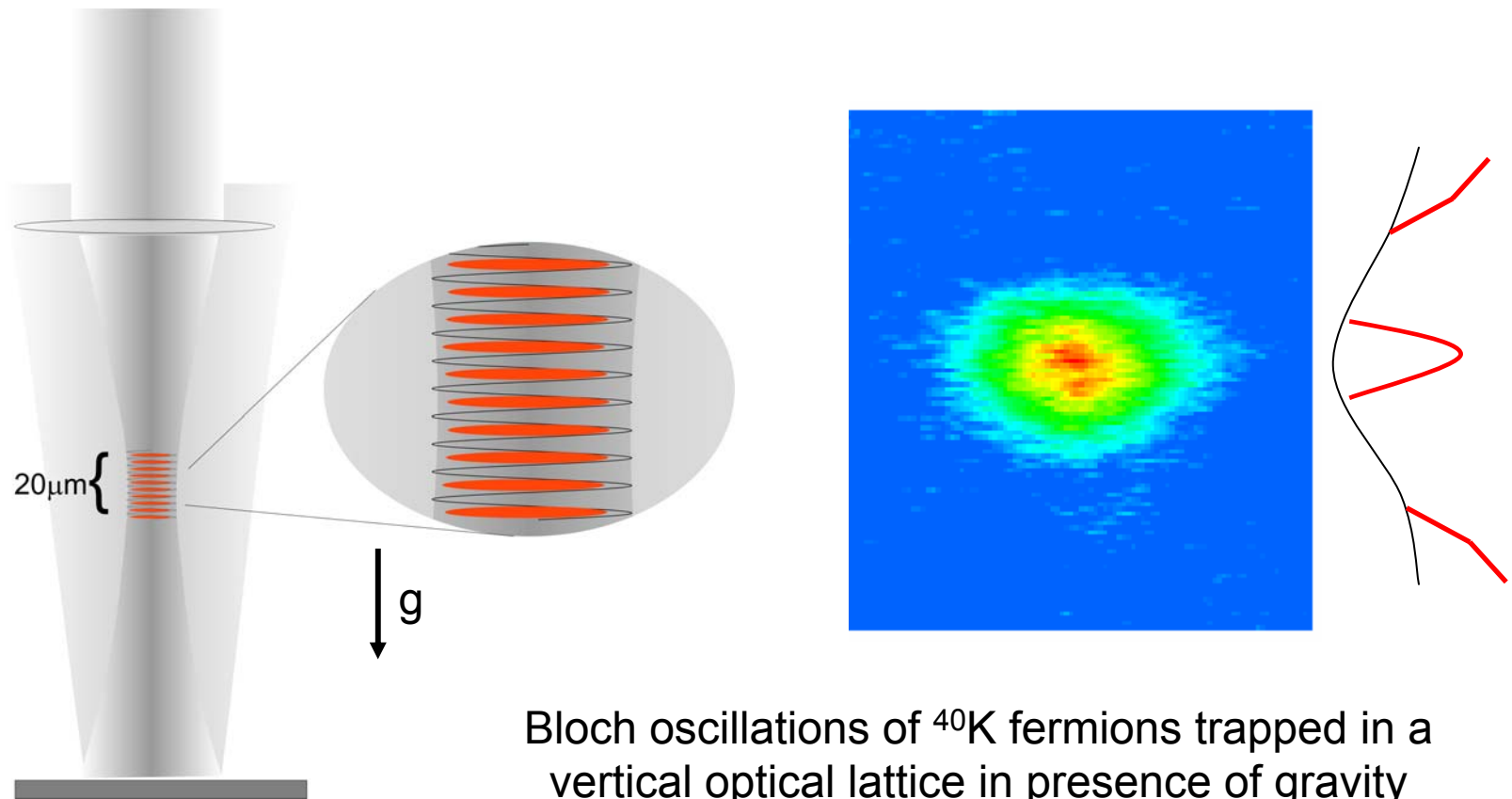
^{39}K BEC

is promising for future experiments with weakly interacting Bose gases, Anderson localization, Bloch oscillations and precision atomic interferometry...

attractive condensates in optical lattices ...

... in addition to the possible tuning of U independently of J

Also Cs, ^7Li



Bloch oscillations of ^{40}K fermions trapped in a vertical optical lattice in presence of gravity

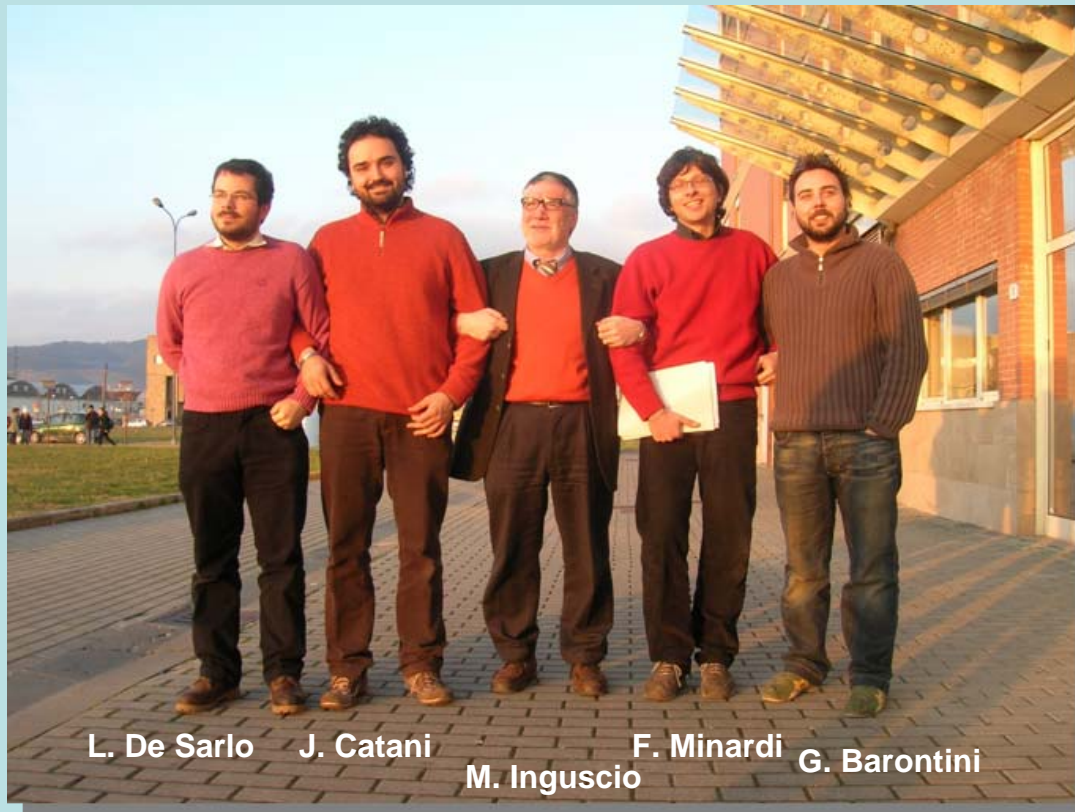
Back to ⁴¹K

Double Species

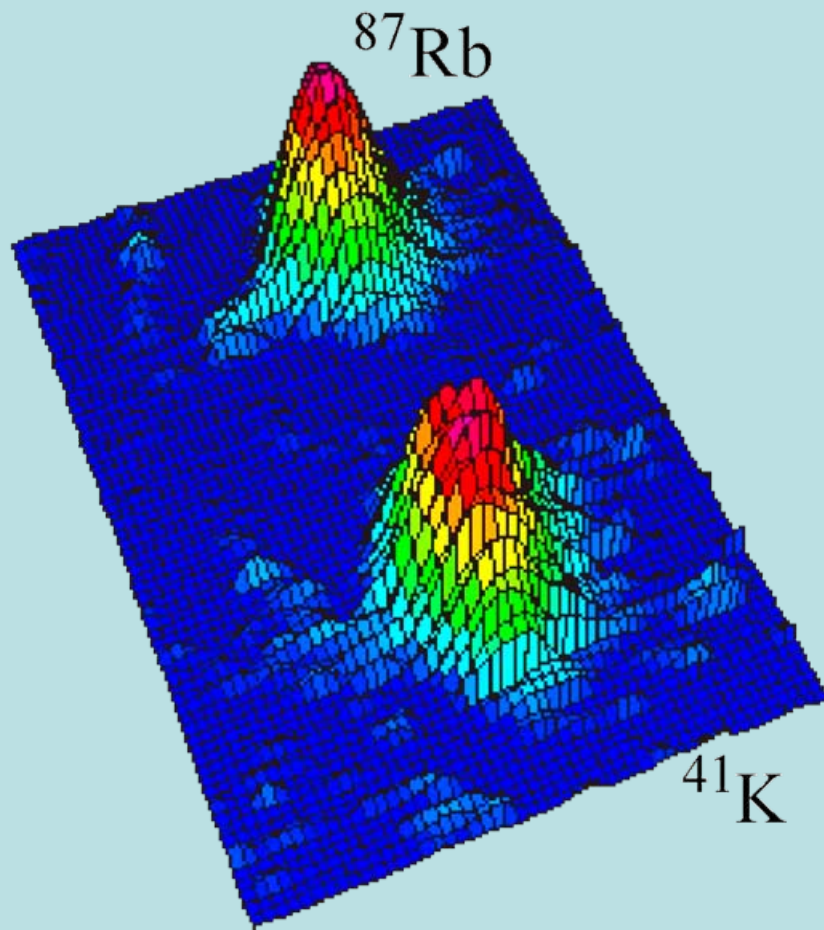
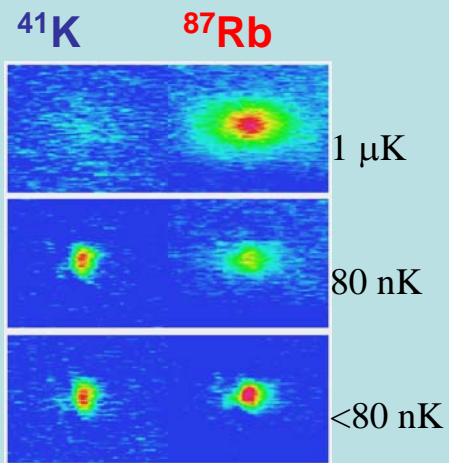
BEC

in

Optical Lattices



A two-species BEC



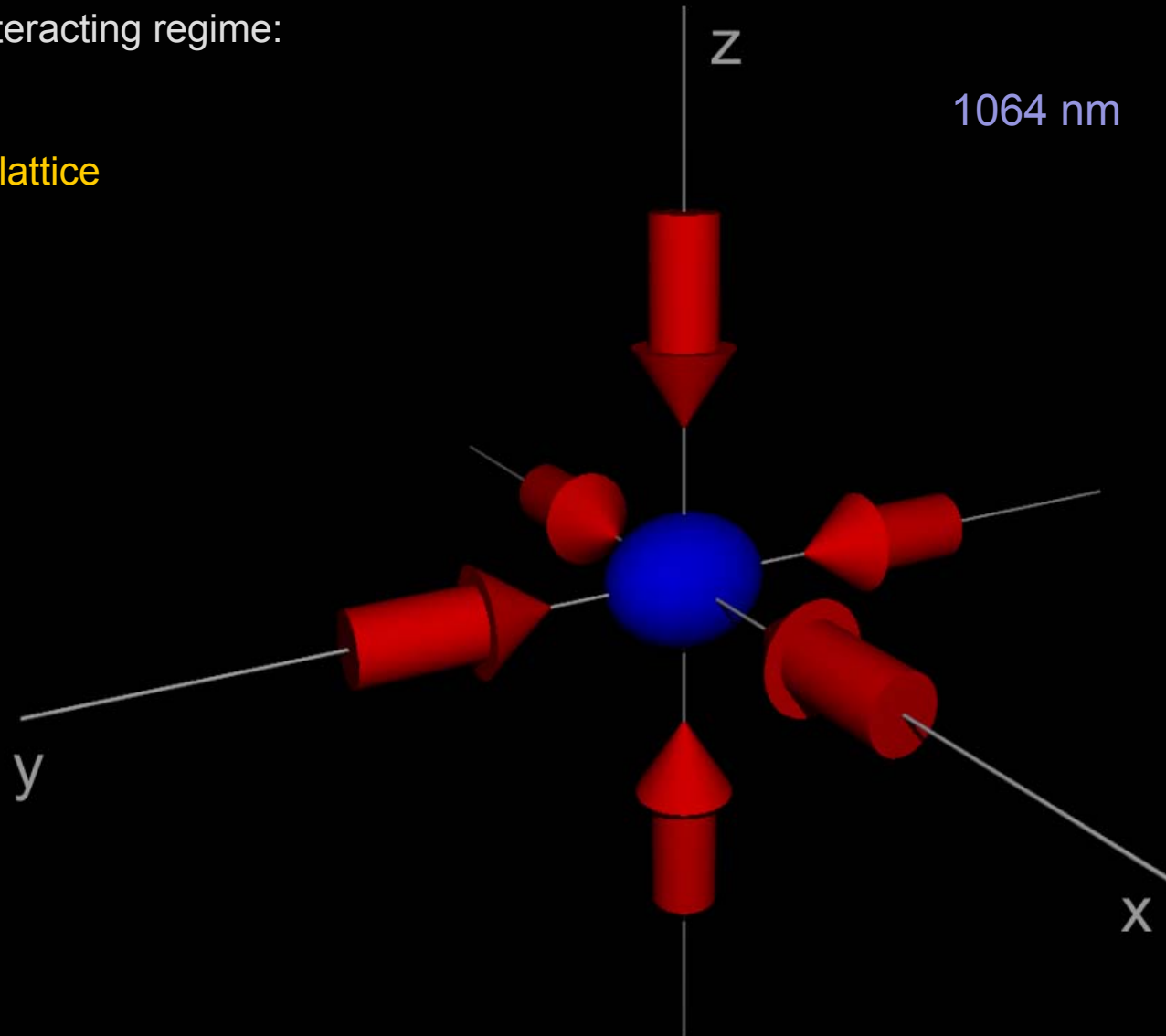
$$a_{\text{KRb}} = +163a_0 \quad a_{\text{K}} = +60a_0 \quad a_{\text{Rb}} = +90a_0$$

ADDING THE 3D OPTICAL LATTICE

strongly interacting regime:



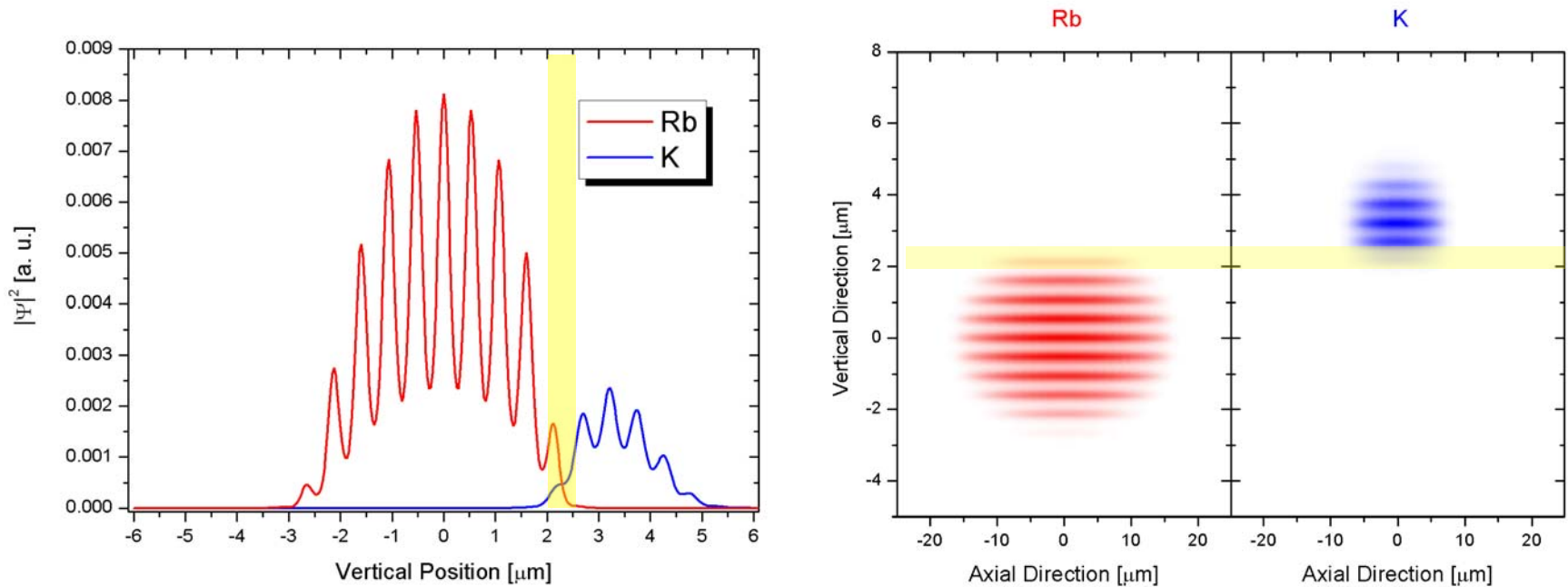
3D optical lattice



41K 87Rb

TIGHT TRAP (70 A, $\omega_R=202$ Hz)

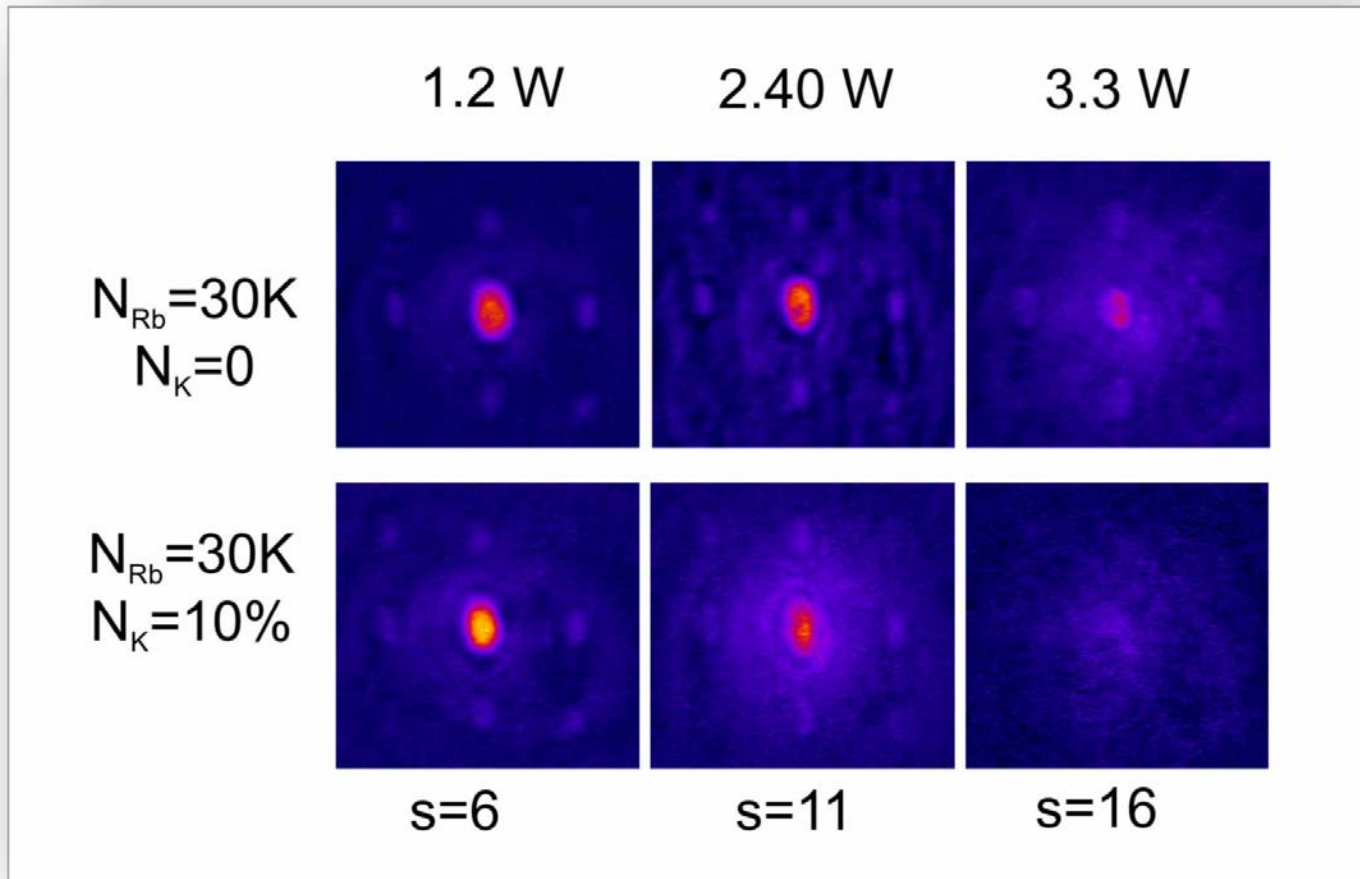
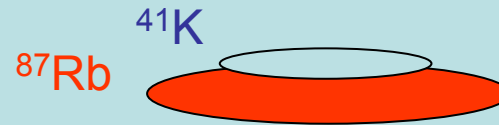
Numerical 3D-GPE simulation for harmonic trap and vertical lattice (s=20)



Numerical 3D-GPE simulation for harmonic trap and vertical lattice s=20

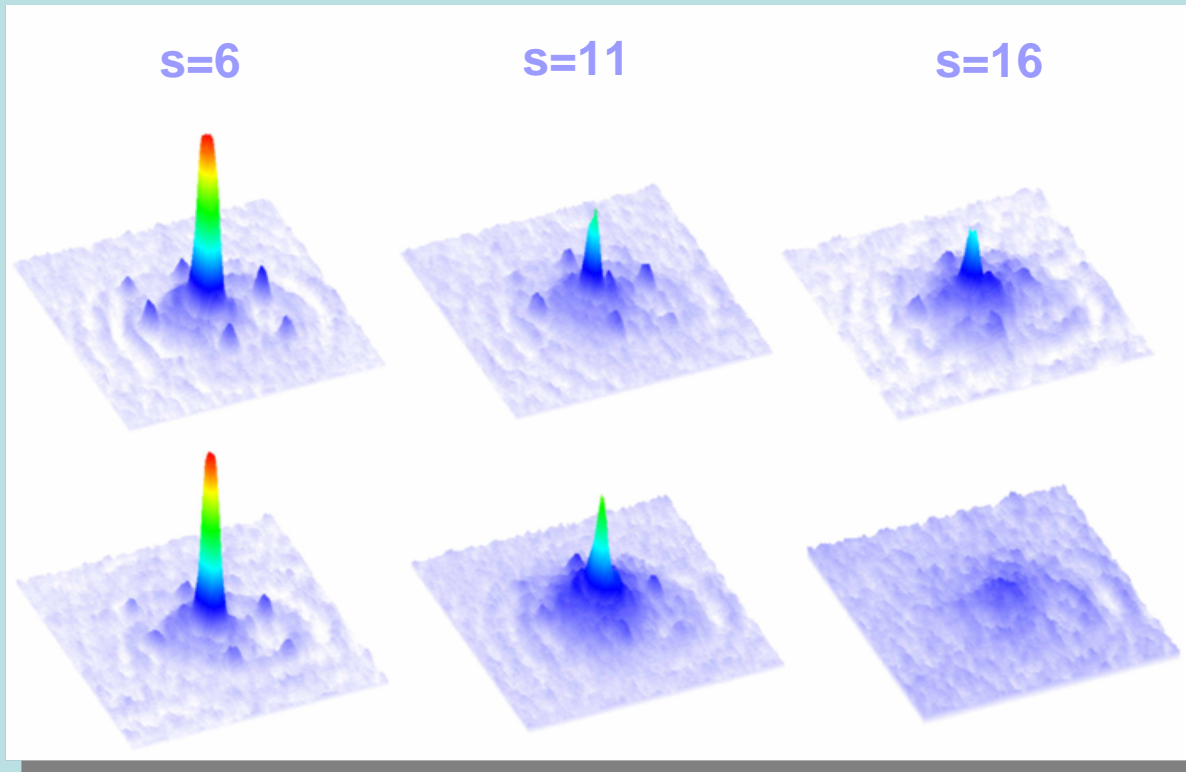
Tight Magnetic Trap (70 A, $\omega_R=202$ Hz)

VERTICAL SAG = 3.2 μm



TIGHT TRAP (70 A, $\omega_R=202$ Hz), partial overlap

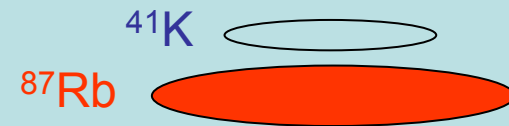
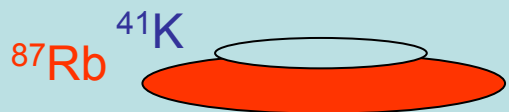
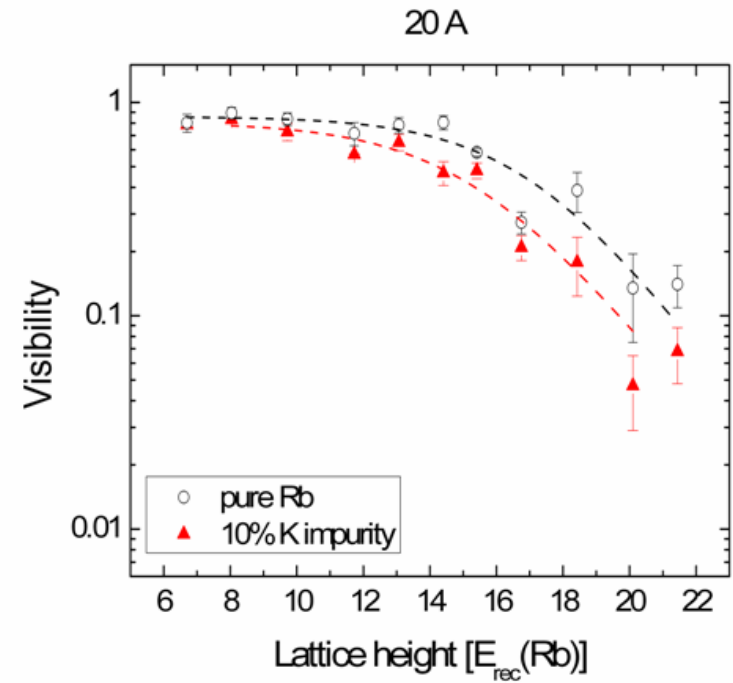
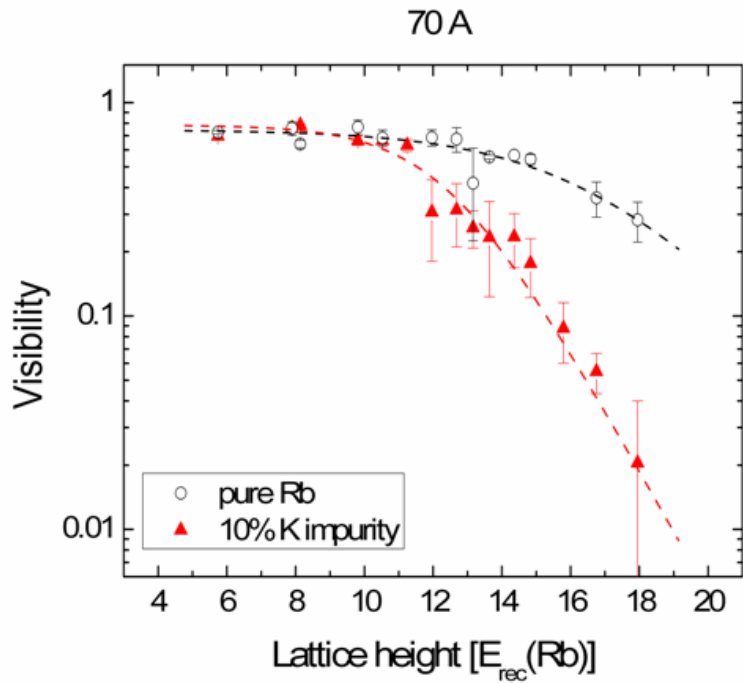
87Rb



87Rb + 41K

F.Minardi, G.Barontini, J.Catani, L.De Sarlo, M.I.

VISIBILITY for DIFFERENT OVERLAP between ^{41}K and ^{87}Rb



PRL **96**, 180402 (2006)

PHYSICAL REVIEW LETTERS

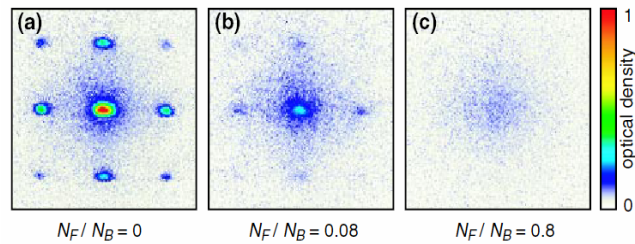
week ending
12 MAY 2006

Bose-Fermi Mixtures in a Three-Dimensional Optical Lattice

Kenneth Günter, Thilo Stöferle, Henning Moritz, Michael Köhl,* and Tilman Esslinger

Institute of Quantum Electronics, ETH Zürich, CH-8093 Zürich, Switzerland

(Received 4 April 2006; published 9 May 2006)



PRL **96**, 180403 (2006)

PHYSICAL REVIEW LETTERS

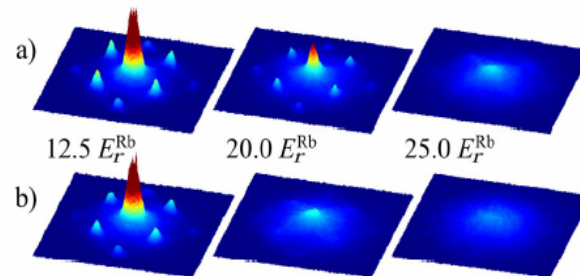
week ending
12 MAY 2006

Localization of Bosonic Atoms by Fermionic Impurities in a Three-Dimensional Optical Lattice

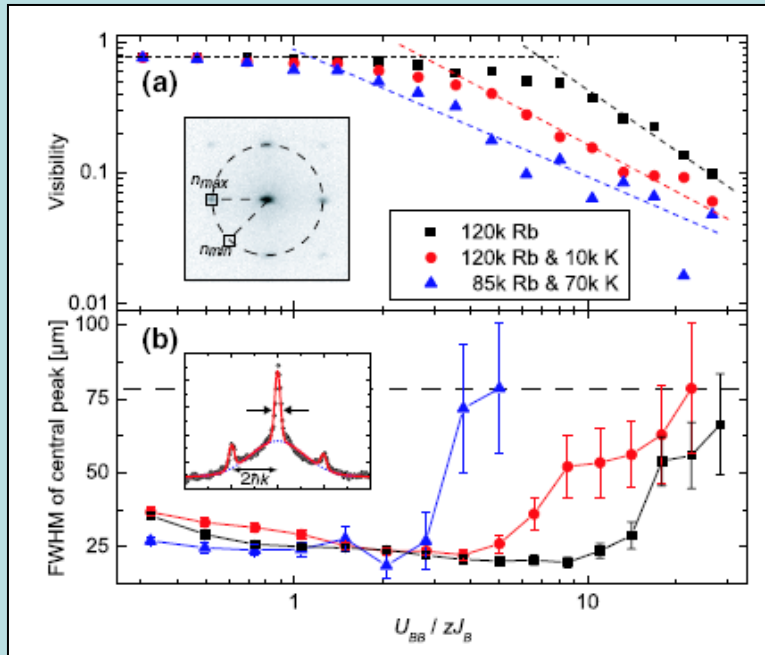
S. Ospelkaus, C. Ospelkaus, O. Wille, M. Succo, P. Ernst, K. Sengstock, and K. Bongs

Institut für Laserphysik, Luruper Chaussee 149, 22761 Hamburg, Germany

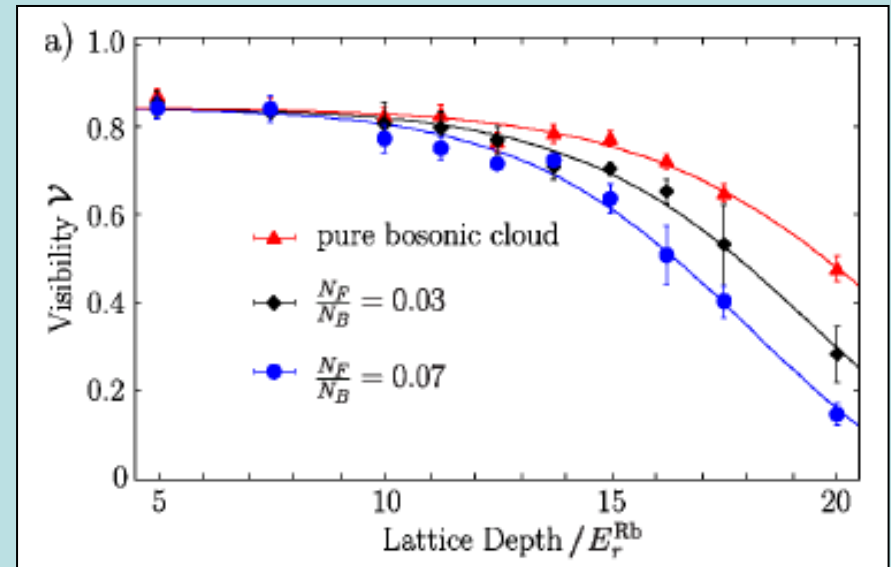
(Received 6 April 2006; published 9 May 2006)



MEASUREMENTS in ^{40}K - ^{87}Rb Bose-Fermi Mixtures with attractive character



K. Guenter *et al.*, PRL **96**, 180402 (2006).



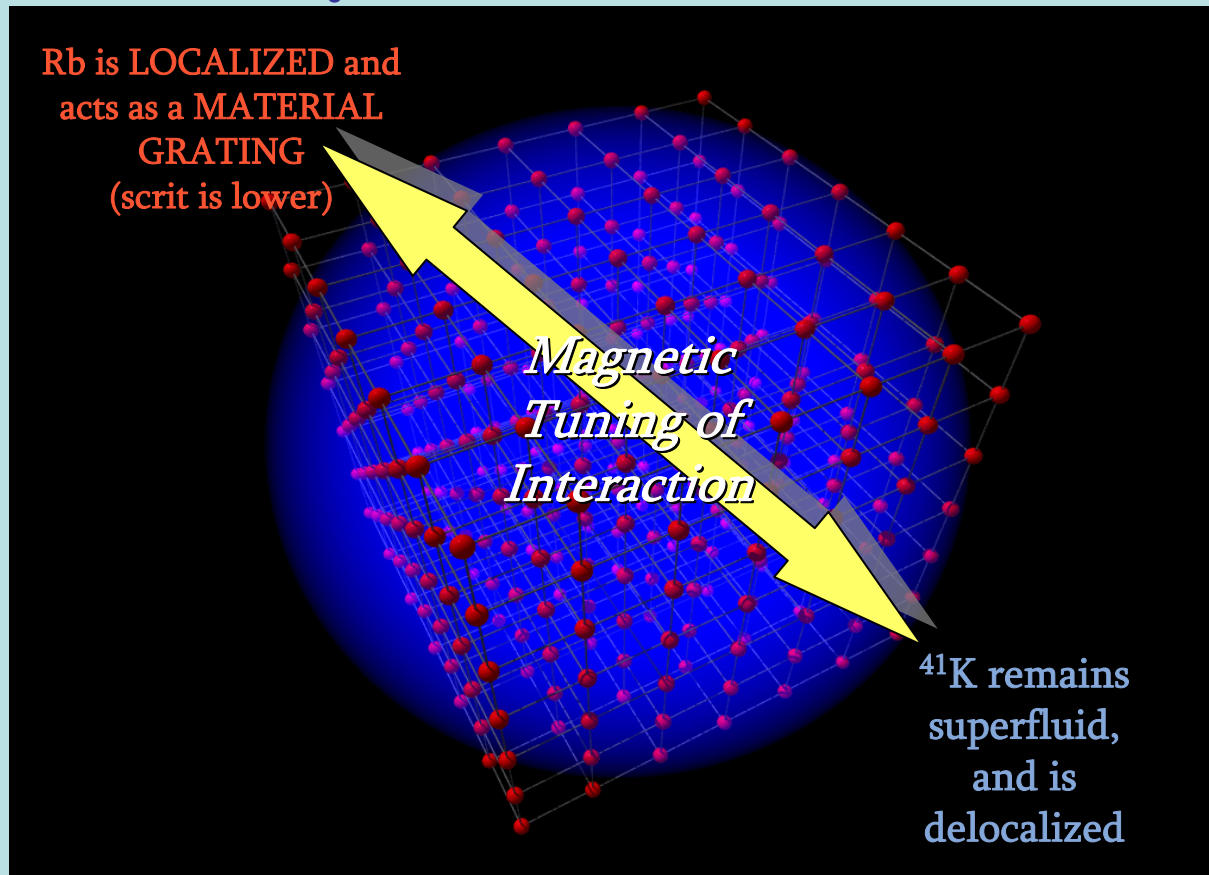
S. Ospelkaus *et al.*, PRL **96**, 180403 (2006).

Interspecies Fano-Feshbach Resonances for ^{41}K - ^{87}Rb Mixture

$|1,1\rangle$ levels, absolute ground state of the system

$B_0 = 67$ Gauss, $\Delta = 9$ Gauss

$B_0 = 516$ Gauss, $\Delta = 82$ Gauss



Perspectives

New quantum phases:

with a single boson per species in every site,

with a single boson per species every two sites (a b a b a ...)

Polarons physics

Dipolar pairs bound by repulsion ...

DISORDER (presence of localized atoms of the other species)

suggestions!

The coldest side of Florence



<http://quantumgases.lens.unifi.it>