Production of Efimov molecules in an optical lattice

Thorsten Köhler

Still at: Clarendon Laboratory, Department of Physics, University of Oxford,
Parks Road, Oxford, OX1 3PU, United Kingdom

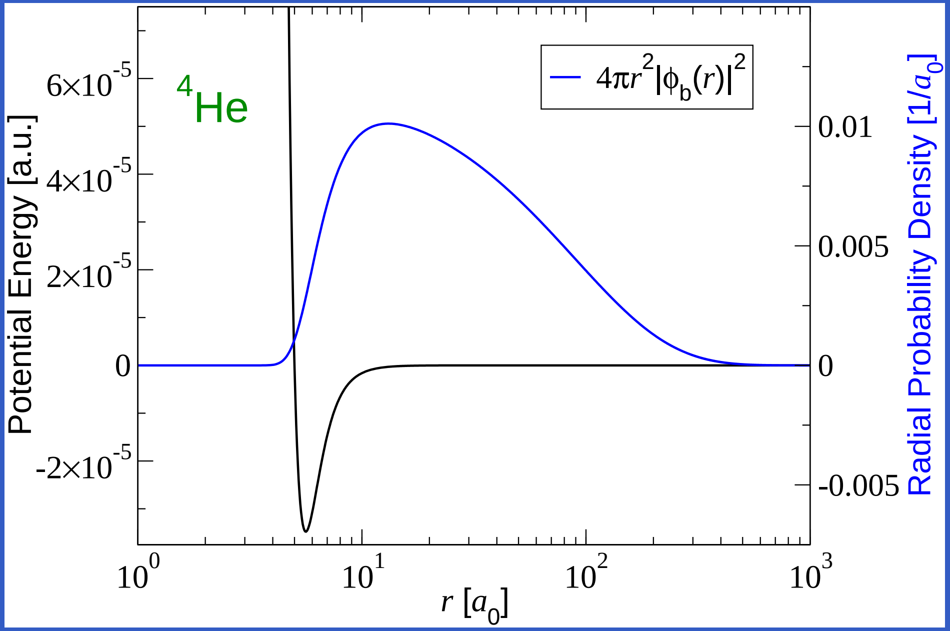
Leaving to: Department of Physics and Astronomy, University College London,
Gower Street, London, WC1E 6BT, United Kingdom

Outline

- Quantum halos
- Feshbach molecules in cold gases
- Efimov states
- Three-body recombination in ^{87}Rb and ^{133}Cs gases
- Excited $^{133}Cs_3$ Efimov states
- Production of excited $^{85}Rb_3$ Efimov states
- Conclusions

Quantum Halos

The average separation between constituent particles exceeds the distance scale set by the classical motion in the potential well!



Classic examples: ^{4}He$_2$ and deuteron

Review: A.S. Jensen, K. Riisager, D.V. Fedorov, and E. Garrido, RMP **76**, 215 (2004)

Quantum Halos

Universal properties of weakly bound diatomic molecules

- Hamiltonian:

$$H_{2B} = -\frac{\hbar^2}{m}\nabla^2 + V(r)$$

- van der Waals potential:

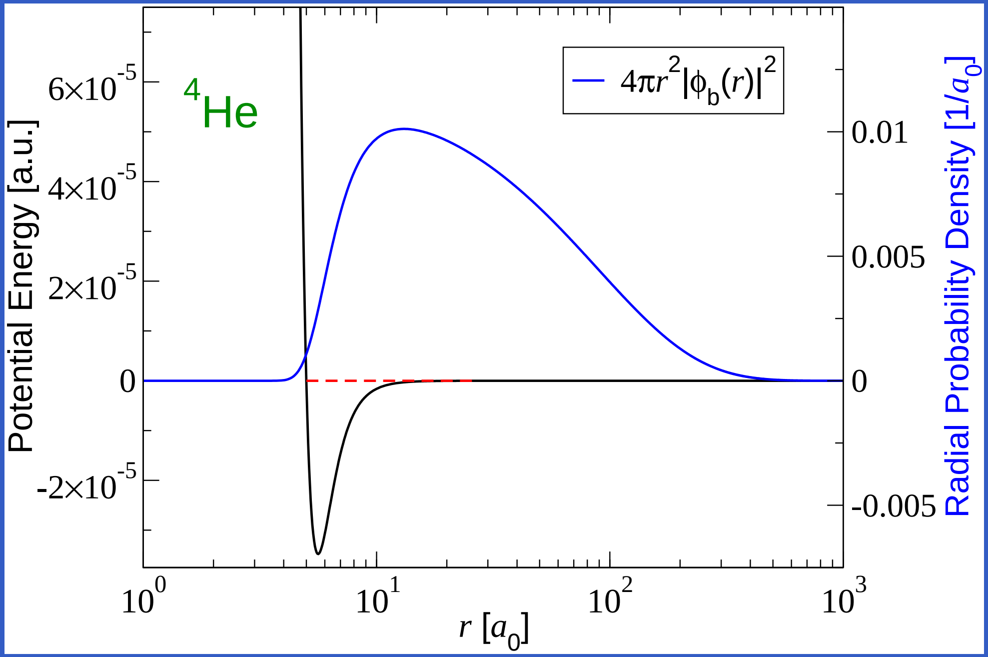
$$V(r) \underset{r\to\infty}{\sim} -C_6/r^6$$

- Bound-state energy:

$$E_b = -\hbar^2/(ma^2)$$

- Bound-state wave function:

$$\phi_b(r) = \frac{1}{\sqrt{2\pi a}}\frac{e^{-r/a}}{r}$$



Quantum Halos

Universality: Physical properties are determined by a single length scale, the s-wave scattering length, a!

- Hamiltonian:

$$H_{2\mathrm{B}} = -\frac{\hbar^2}{m}\nabla^2 + V(r)$$

- van der Waals potential:

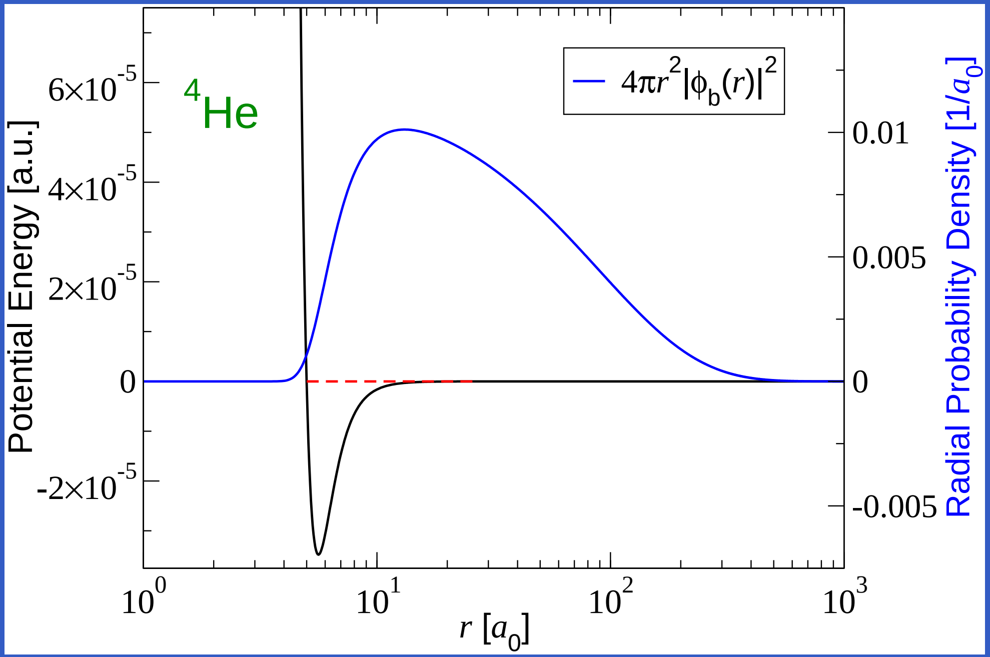
$$V(r) \underset{r\to\infty}{\sim} -C_6/r^6$$

- Bound-state energy:

$$E_{\mathrm{b}} = -\hbar^2/(ma^2)$$

- Bound-state wave function:

$$\phi_{\mathrm{b}}(r) = \frac{1}{\sqrt{2\pi a}}\frac{e^{-r/a}}{r}$$



Quantum Halos

Characteristic length scales

- Bond length:

$$\langle r \rangle = a/2$$

- Classical turning point:

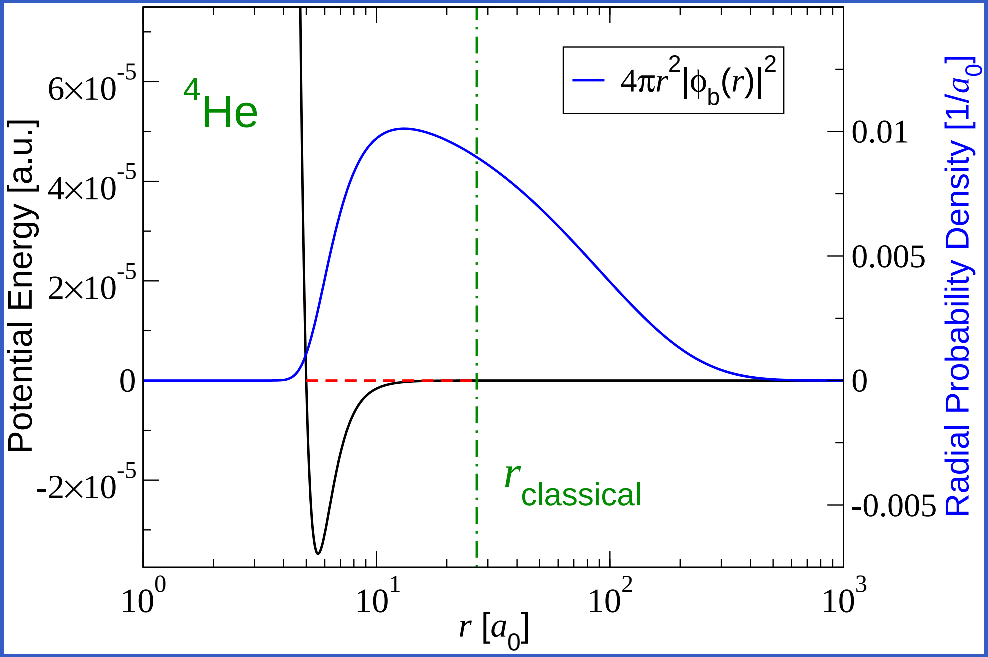
$$r_{\text{classical}} = [a(2l_{\text{vdW}})^2]^{1/3}$$

- van der Waals length:

$$l_{\text{vdW}} = \frac{1}{2}(mC_6/\hbar^2)^{1/4}$$

- Characteristic property:

$$a \gg r_{\text{classical}}$$



Quantum Halos

Properties of the helium dimer

- Bond length:

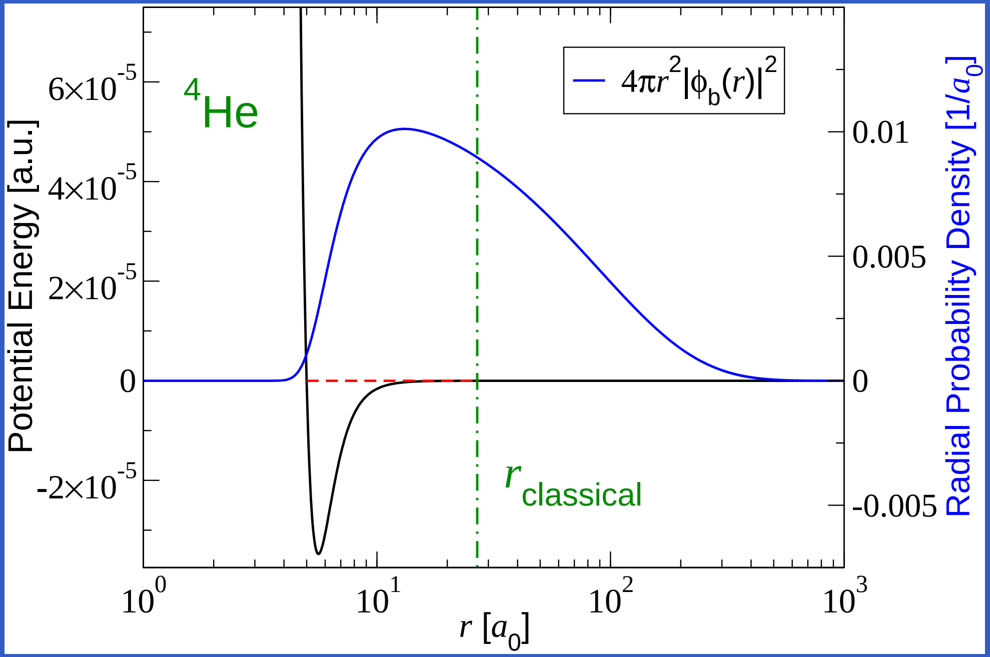
$$\langle r \rangle = 5.2 \pm 0.4\,\text{nm}$$

- Classical turning point:

$$r_{\text{classical}} = 1.4\,\text{nm}$$

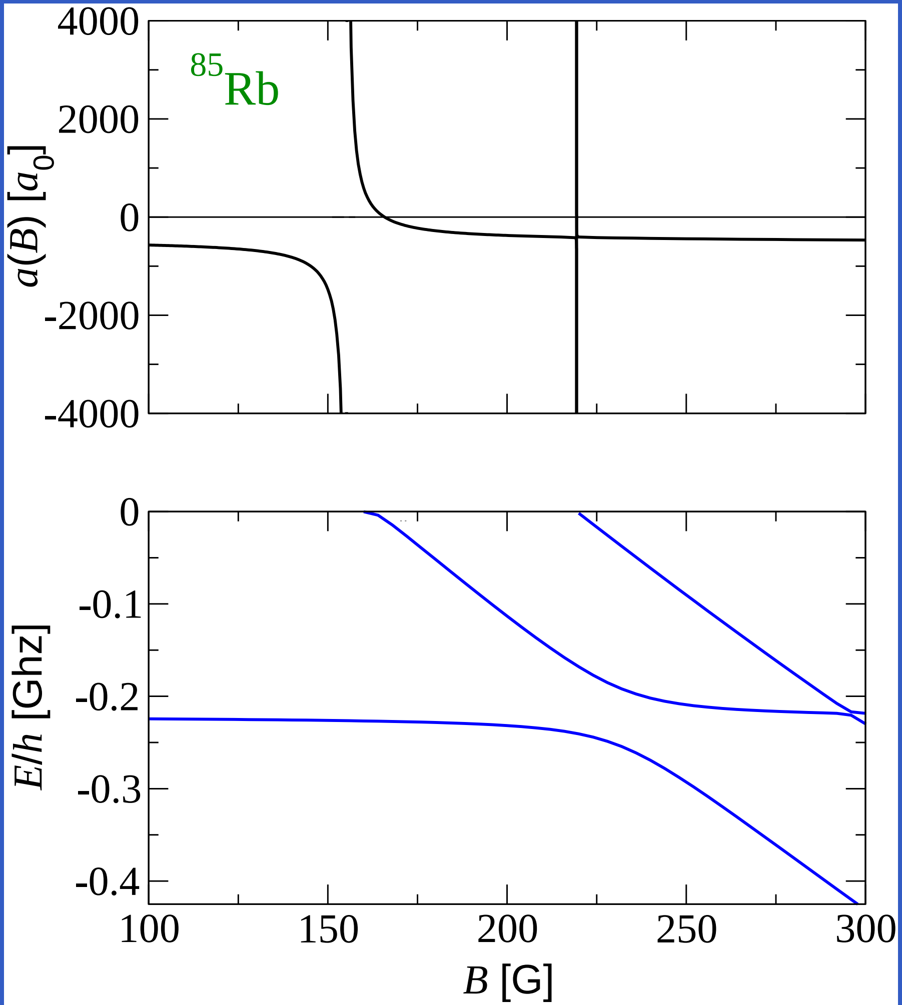
- Characteristic property:

$$a = 10.4\,\text{nm} \gg r_{\text{classical}}$$



R.E. Grisenti, W. Schöllkopf, J.P. Toennies, G.C. Hegerfeldt, TK, and M. Stoll, PRL **85**, 2284 (2000)

Feshbach molecules in cold gases



Scattering length goes to $\pm\infty$.

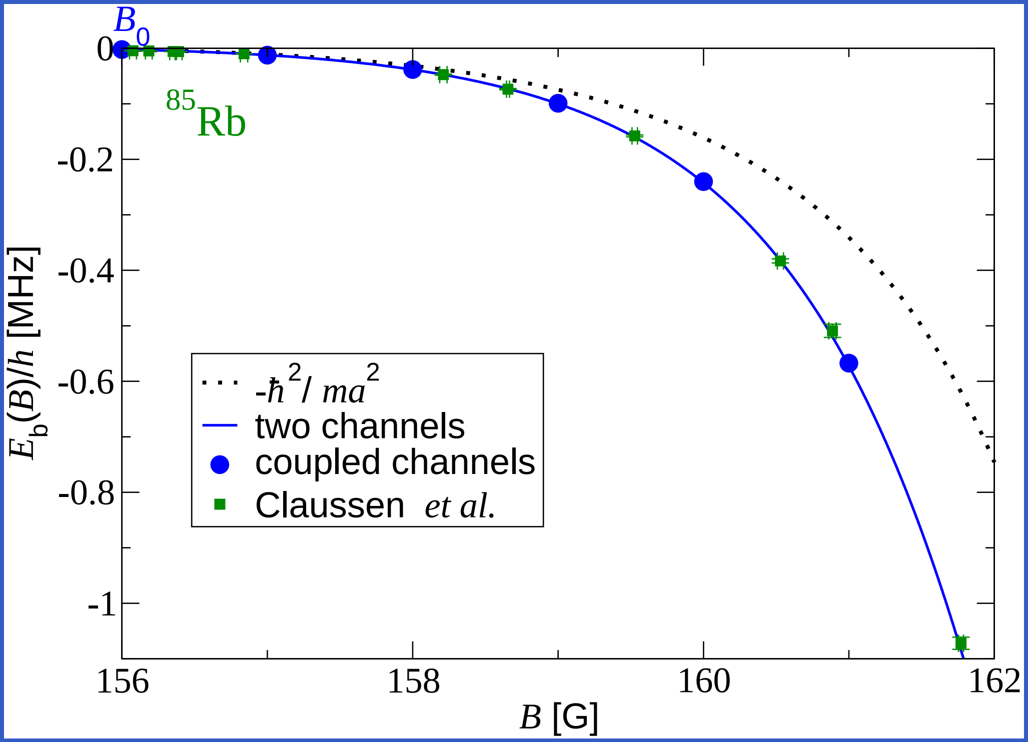
$\Updownarrow$

A new vibrational bound state emerges
at the collision threshold.

Theory: E. Tiesinga, B.J. Verhaar, and H.T.C. Stoof, PRA **47**, 4114 (1993)
Experiment: S. Inouyé *et al.*, Nature **392**, 151 (1998)
Review: TK, K. Góral, and P.S. Julienne, RMP **78**, 1311 (2006)

Feshbach molecules in cold gases

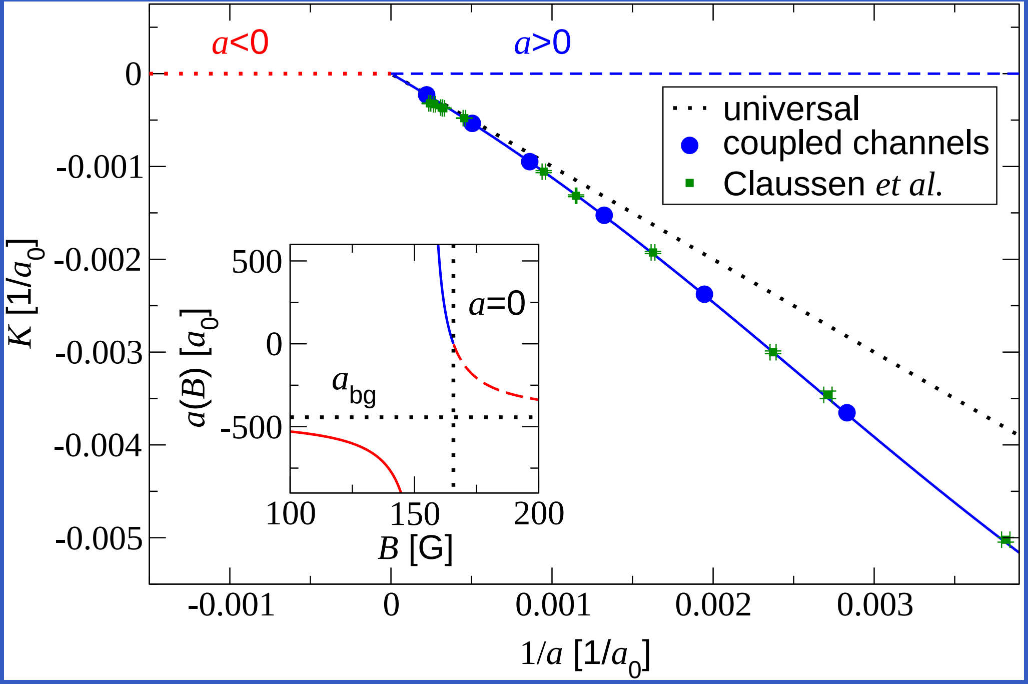
Near-resonant bound-state energies become universal!



Experiment: N.R. Claussen, S.J.J.M.F. Kokkelmans, S.T. Thompson, E.A. Donley, E. Hodby, and C.E. Wieman, PRA **67**, 060701(R) (2003)
Review: TK, K. Góral, and P.S. Julienne, RMP **78**, 1311 (2006)

Feshbach molecules in cold gases

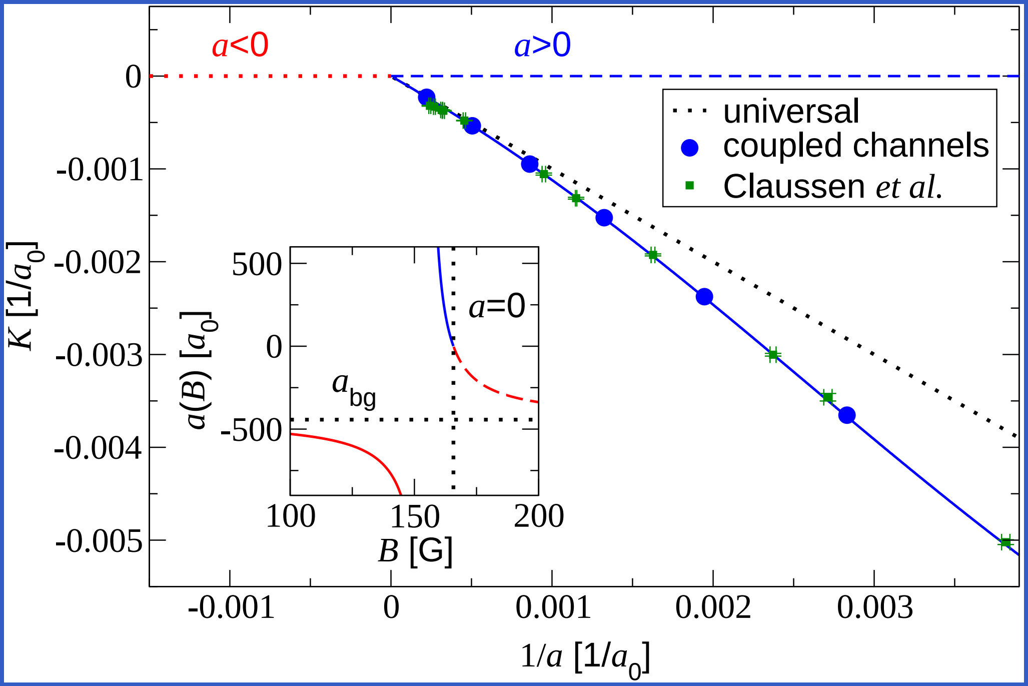
Efimov plot: $K = \sqrt{m|E_\mathrm{b}|/\hbar^2}$ **vs.** $1/a$

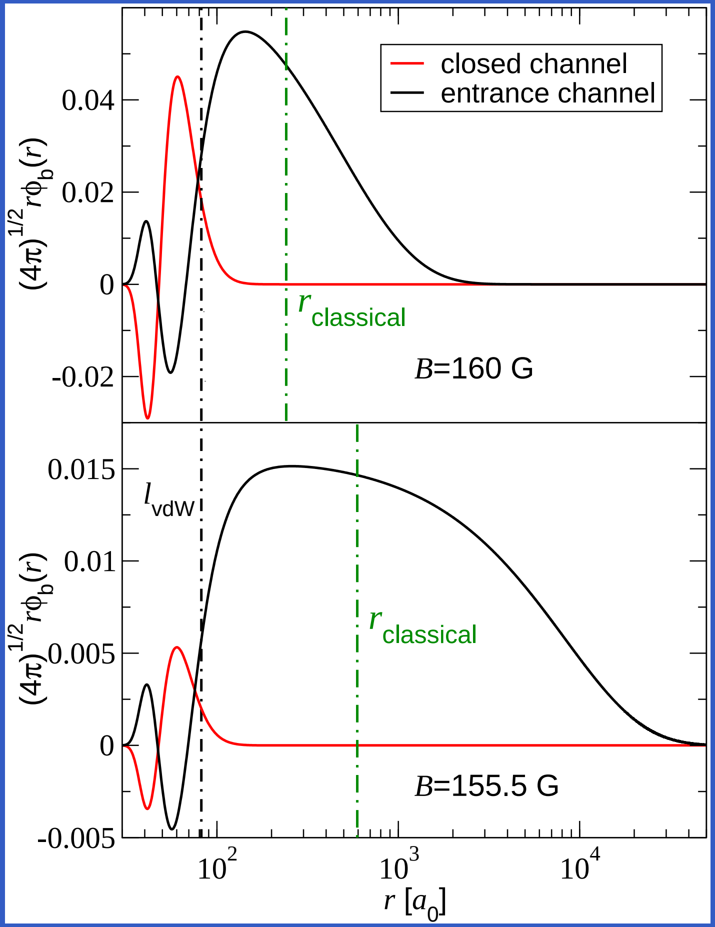


V. Efimov, Phys. Lett. **33B**, 563 (1970)

Feshbach molecules in cold gases

Near-resonant Feshbach molecules are halo states!





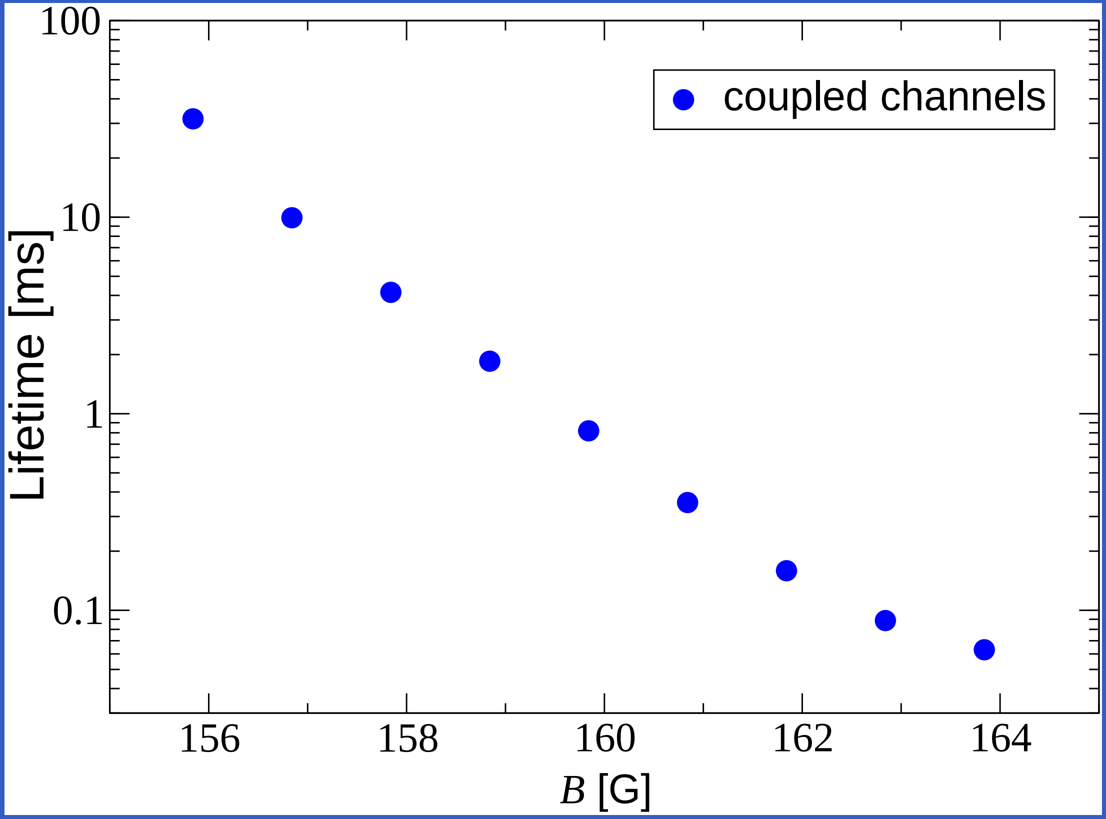
TK, T. Gasenzer, and K. Burnett, PRA **67**, 013601 (2003)
TK, T. Gasenzer, P.S. Julienne, and K. Burnett, PRL **91**, 230401 (2003)

Feshbach molecules in cold gases

^{85}**Rb** $(F = 2, m_F = -2)$ **is an excited state**

$\Leftrightarrow$

^{85}**Rb**$_2$ **Feshbach molecules decay due to spin relaxation!**



Feshbach molecules in cold gases

The lifetime depends on the size of the molecule!

- Average volume of a halo state:

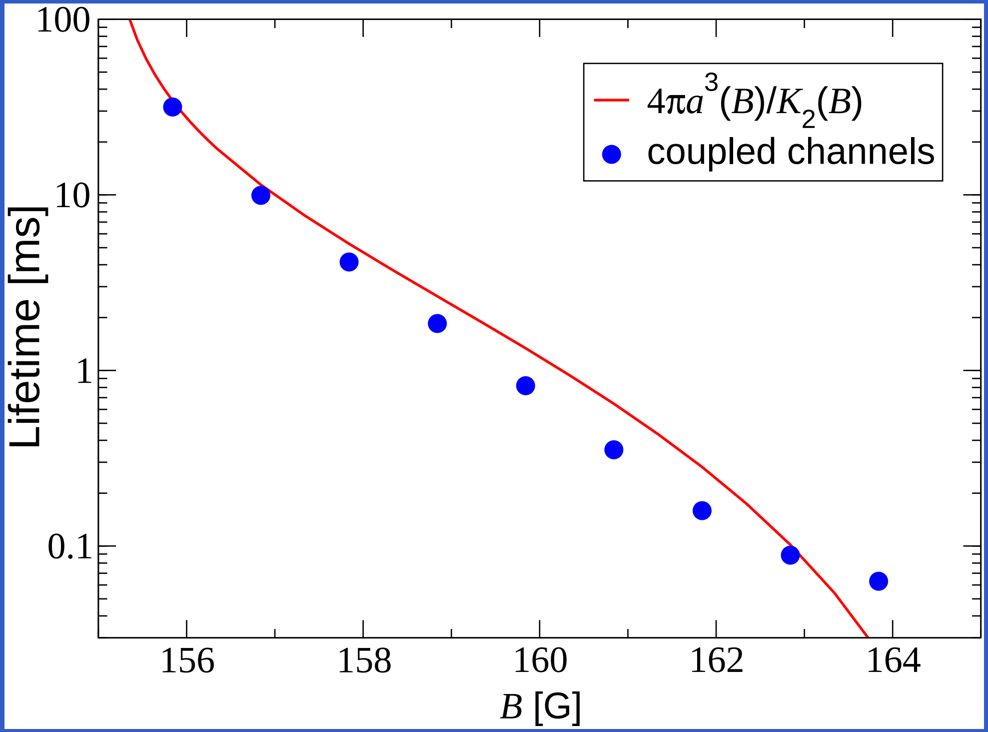
$$\mathcal{V} = 4\pi\langle r^3\rangle/3 = \pi a^3$$

- Rate equation for spin relaxation:

$$\dot{N}(t) = -K_2\langle n(t)\rangle N(t)$$

- Lifetime of the halo state:

$$\tau = \mathcal{V}/[K_2(B)/4]$$
$$= 4\pi a^3(B)/K_2(B)$$



Feshbach molecules in cold gases

Experiment and theory agree!

- Average volume of a halo state:

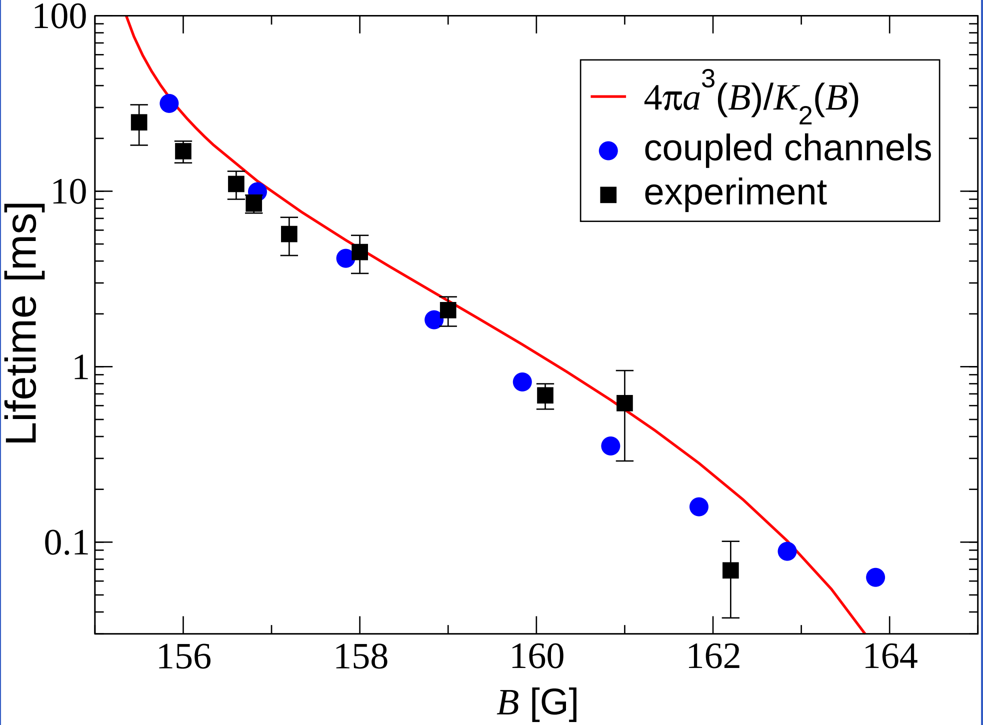
$$\mathcal{V} = 4\pi \langle r^3 \rangle / 3 = \pi a^3$$

- Rate equation for spin relaxation:

$$\dot{N}(t) = -K_2 \langle n(t) \rangle N(t)$$

- Lifetime of the halo state:

$$\tau = \mathcal{V} / [K_2(B)/4]$$
$$= 4\pi a^3(B)/K_2(B)$$

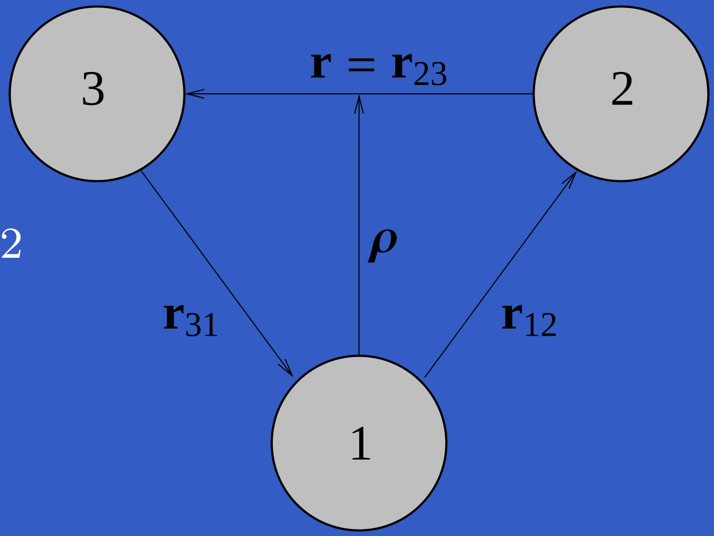


Experiment: S.T. Thompson, E. Hodby, and C.E. Wieman, PRL **94**, 020401 (2005)
Theory: TK, E. Tiesinga, and P.S. Julienne, PRL **94**, 020402 (2005)

Efimov states

Three-body Hamiltonian in Jacobi coordinates

$$
\begin{aligned}
H =& -\frac{\hbar^2}{2(3m)}\nabla^2_{\mathbf{R}} + \frac{1}{2}(3m)\omega^2_{\rm ho}\mathbf{R}^2 - \frac{\hbar^2}{2(\frac{2}{3}m)}\nabla^2_{\boldsymbol{\rho}} \\
&+ \frac{1}{2}\left(\frac{2}{3}m\right)\omega^2_{\rm ho}\boldsymbol{\rho}^2 - \frac{\hbar^2}{2(\frac{m}{2})}\nabla^2_{\mathbf{r}} + \frac{1}{2}\left(\frac{m}{2}\right)\omega^2_{\rm ho}\mathbf{r}^2 \\
&+ V(\mathbf{r}) + V\left(\boldsymbol{\rho} + \frac{1}{2}\mathbf{r}\right) + V\left(\boldsymbol{\rho} - \frac{1}{2}\mathbf{r}\right) \\
=& H_0 + V_1 + V_2 + V_3
\end{aligned}
$$

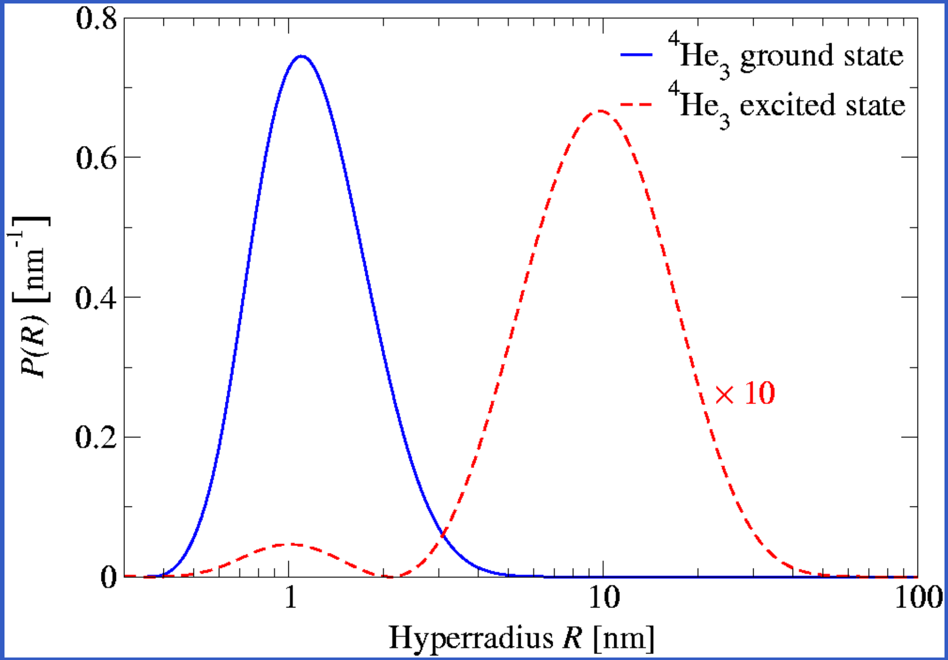


L.D. Faddeev, Soviet Phys. - JETP **12**, 1014 (1961)
E.O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. B **2**, 167 (1967)

Efimov states

The ground and first excited states of the helium trimer molecule

- Scattering length of helium:
$$a = 10.4\,\text{nm}$$
- Effective range:
$$r_{\text{eff}} = 0.7\,\text{nm}$$
- Number of Efimov trimer states as a function of the scattering length:
$$N_{\text{Efimov}} \gtrsim \frac{1}{\pi} \log\left(|a| / r_{\text{eff}}\right) = 0.8$$



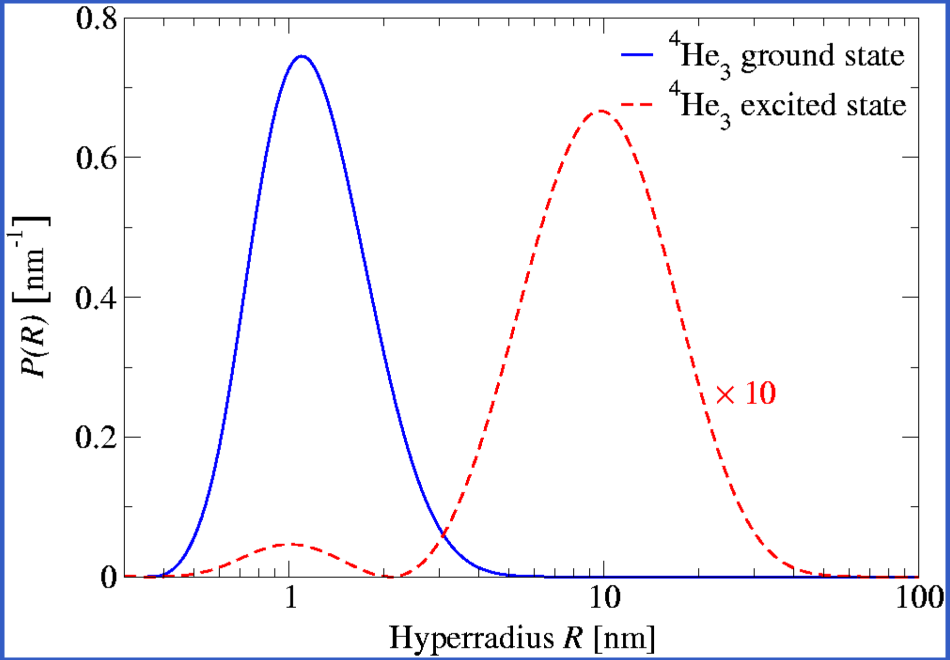
G.C. Hegerfeldt and M. Stoll, PRA **71**, 033606 (2005)

L.H. Thomas, Phys. Rev. **47**, 903 (1935)
V. Efimov, Phys. Lett. **33B**, 563 (1970)
Helium case: T.K. Lim, Sister K. Duffy, and W.C. Damer, PRL **38**, 341 (1977)

Efimov states

The ground and first excited states of the helium trimer molecule

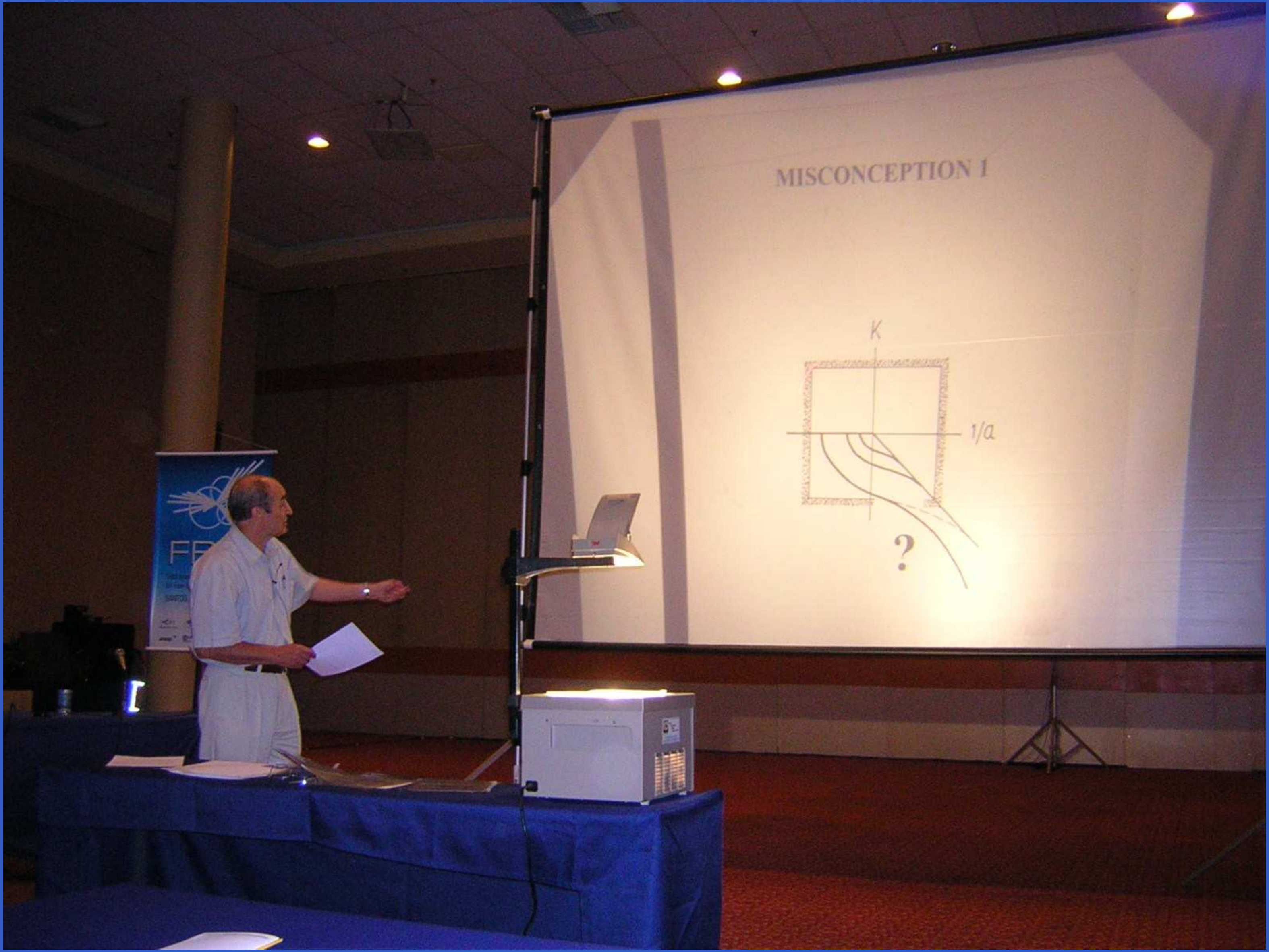
- Scattering length of helium:
  $$a = 10.4\,\text{nm}$$
- Effective range:
  $$r_{\text{eff}} = 0.7\,\text{nm}$$
- Number of Efimov trimer states as a function of the scattering length:
  $$N_{\text{Efimov}} \gtrsim \frac{1}{\pi} \log\left(|a| / r_{\text{eff}}\right) = 0.8$$



G.C. Hegerfeldt and M. Stoll, PRA **71**, 033606 (2005)

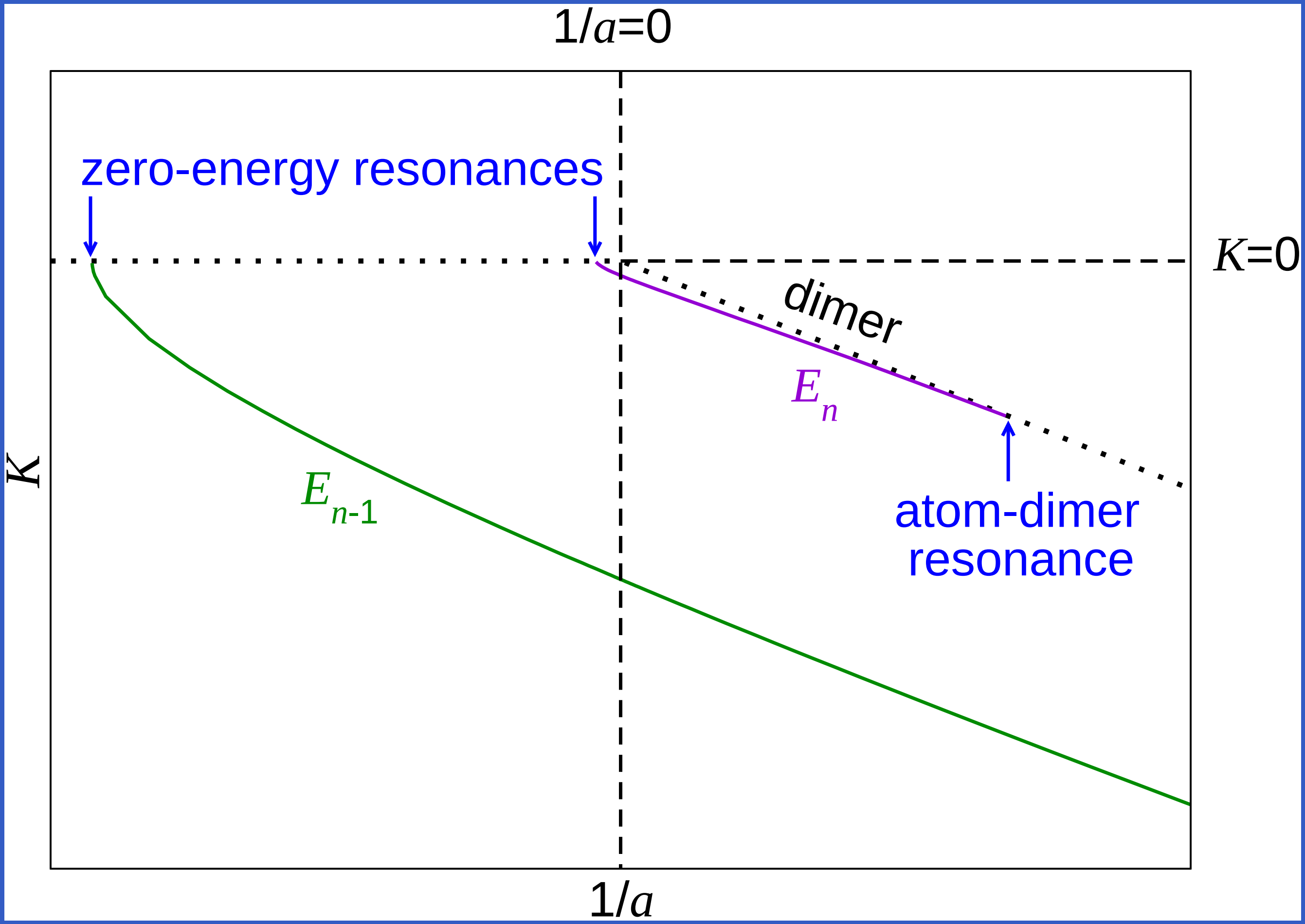
Detection of the ground state: W. Schöllkopf and J.P. Toennies, Science **266**, 1345 (1994)
Measurement of its bond length: R. Brühl, A. Kalinin, O. Kornilov, J.P. Toennies, G.C. Hegerfeldt, and M. Stoll, PRL **95**, 063002 (2005)

Efimov states



Efimov states

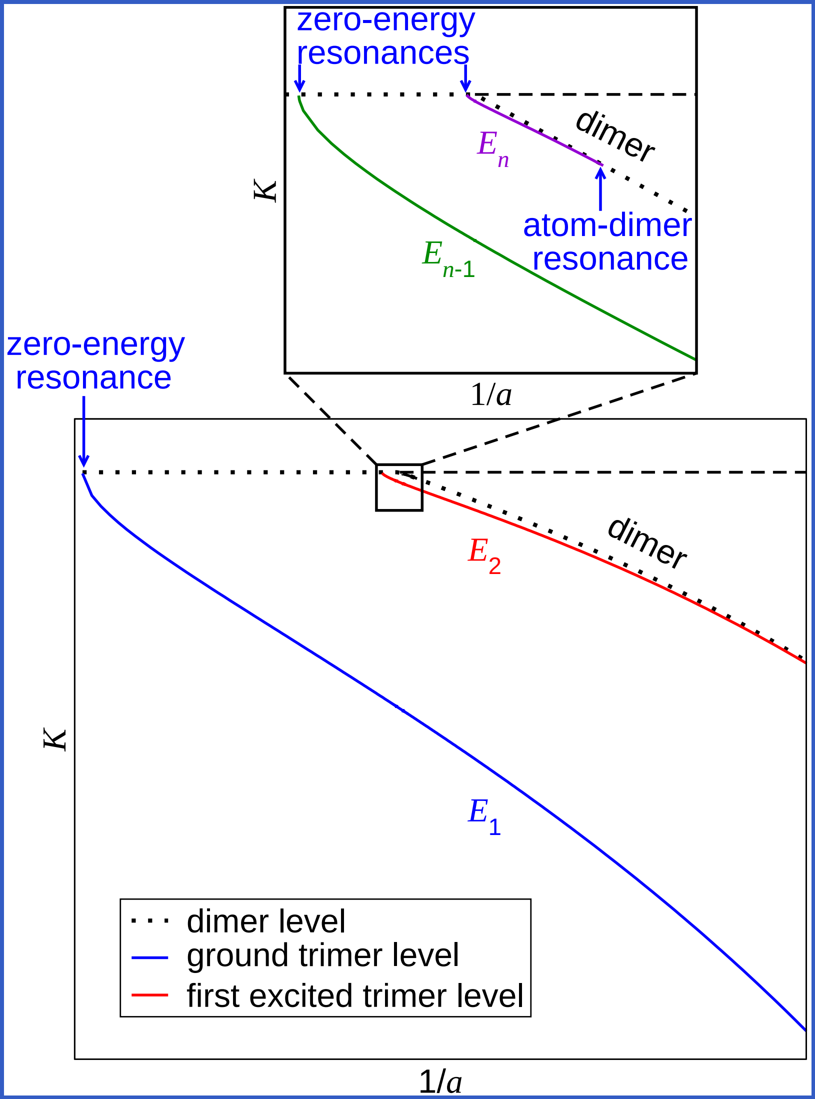
Efimov plot



V. Efimov, Phys. Lett. **33B**, 563 (1970)

Efimov states

Reality is often different!

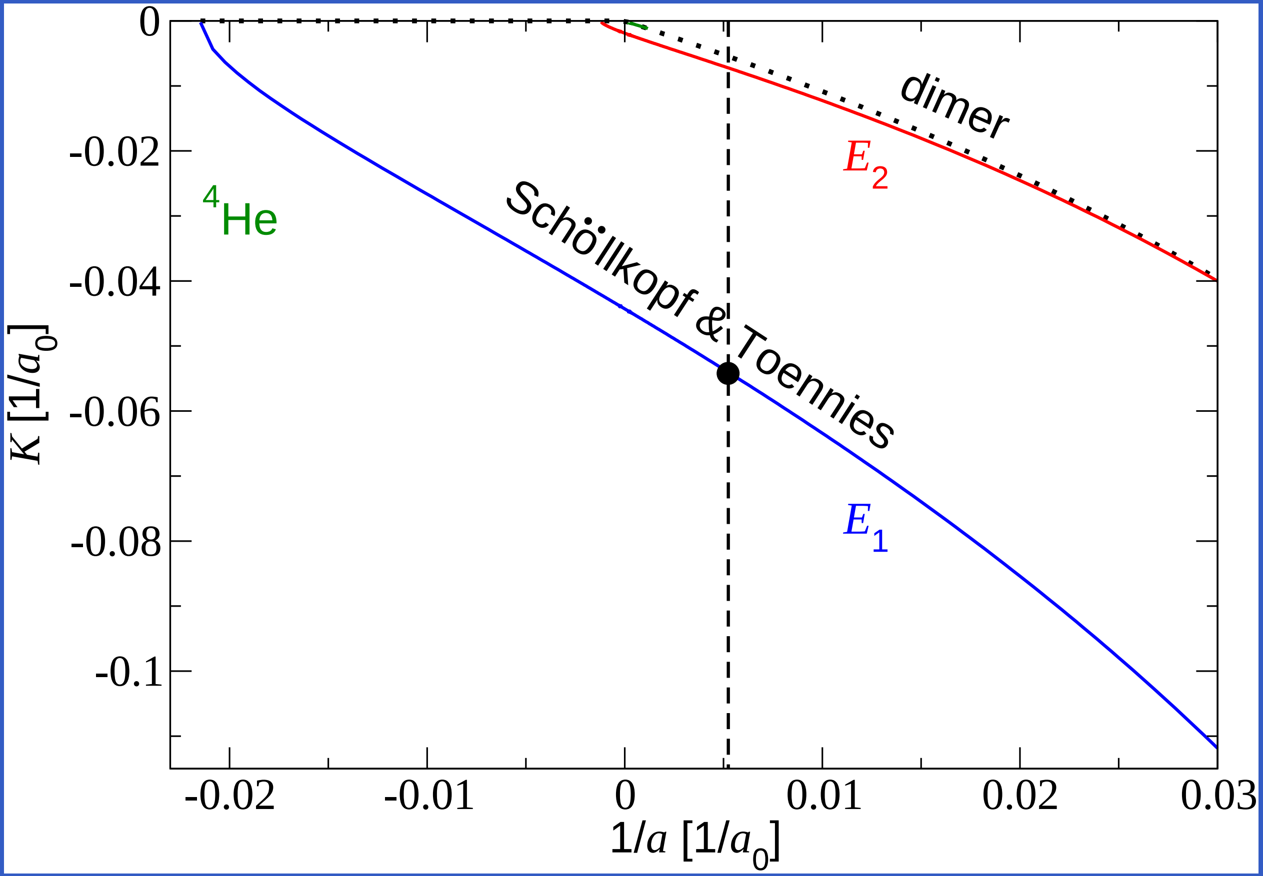


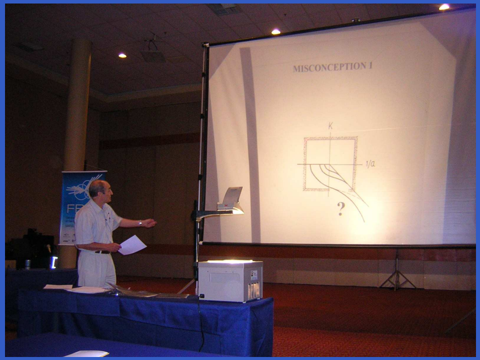
Only ground levels have been experimentally observed which inevitably connect to non-universal physics.

L.W. Bruch and K. Sawada, PRL **30**, 25 (1973)
V. Efimov, Nucl. Phys. A **362**, 45 (1981)

Efimov states

Efimov spectrum of ^{4}He

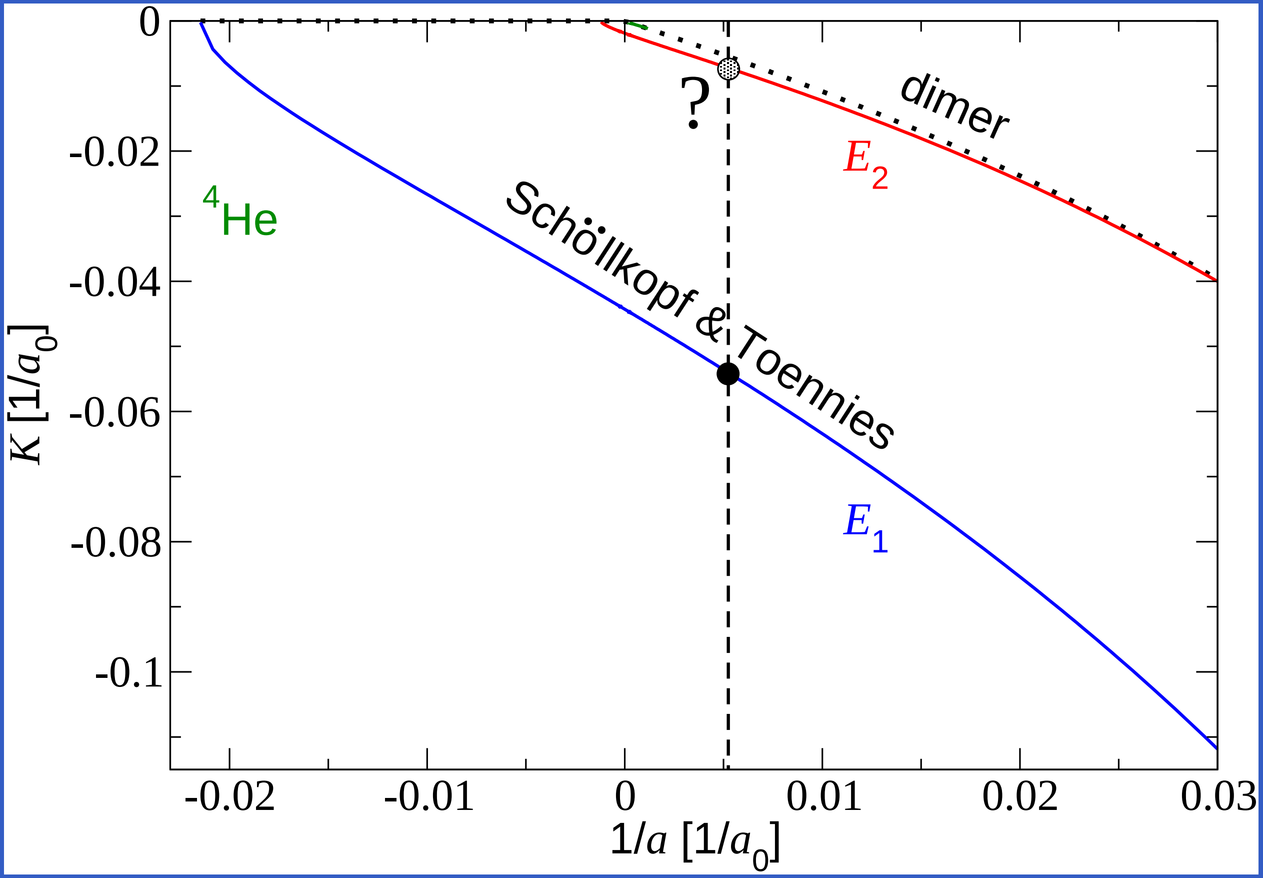


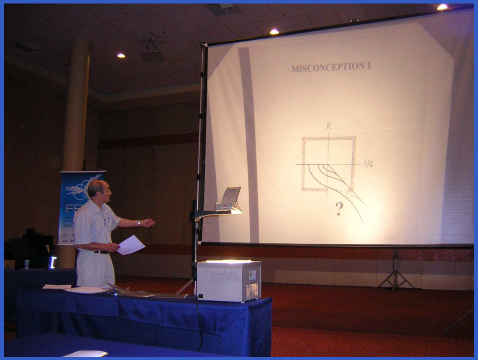


W. Schöllkopf and J.P. Toennies, Science **266**, 1345 (1994)

Efimov states

The excited $^4He_3$ level has not been found!

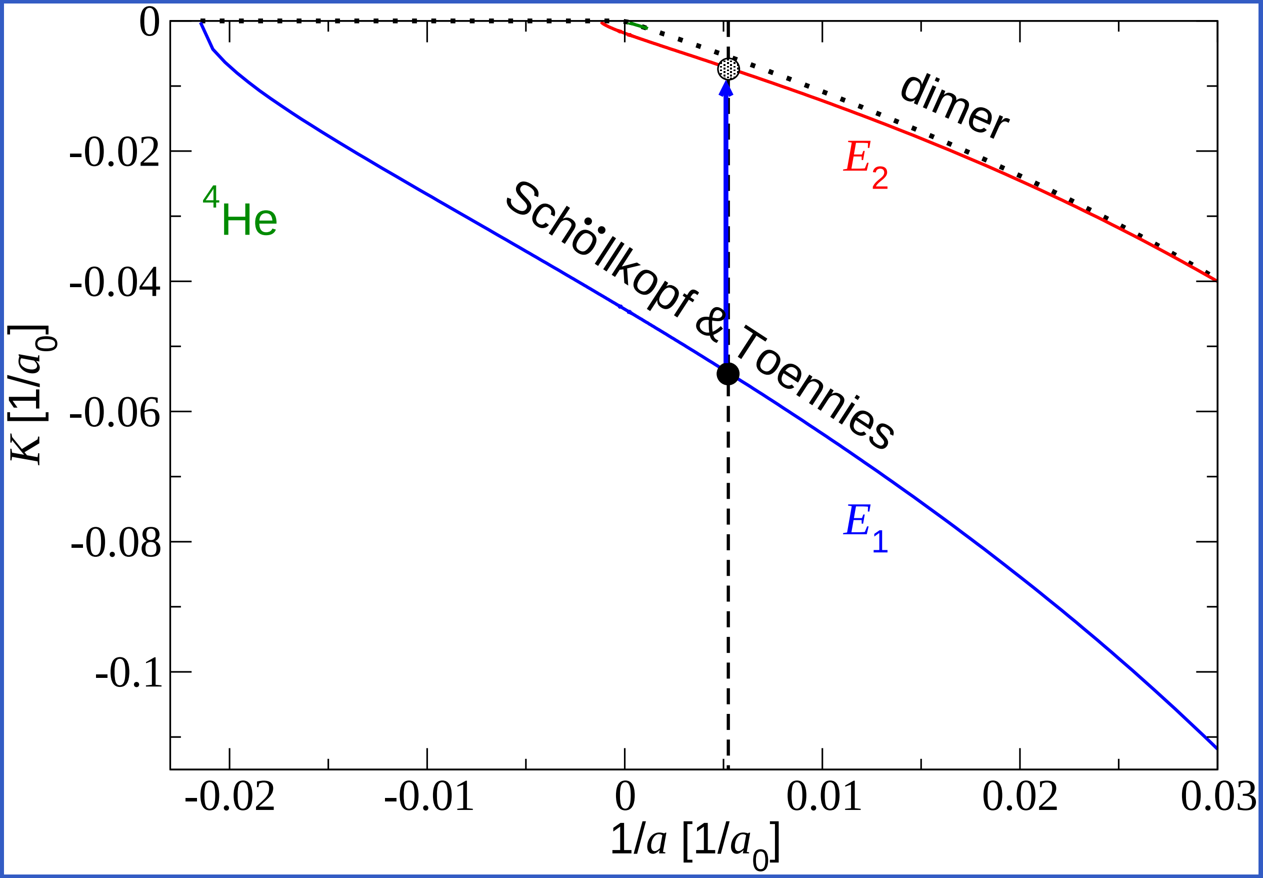


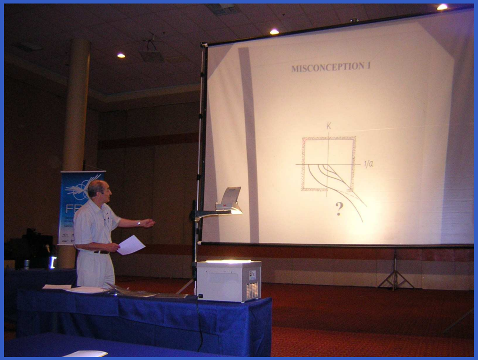


R. Brühl, A. Kalinin, O. Kornilov, J.P. Toennies, G.C. Hegerfeldt, and M. Stoll, PRL **95**, 063002 (2005)

Efimov states

The $^4He_3$ system is universal!

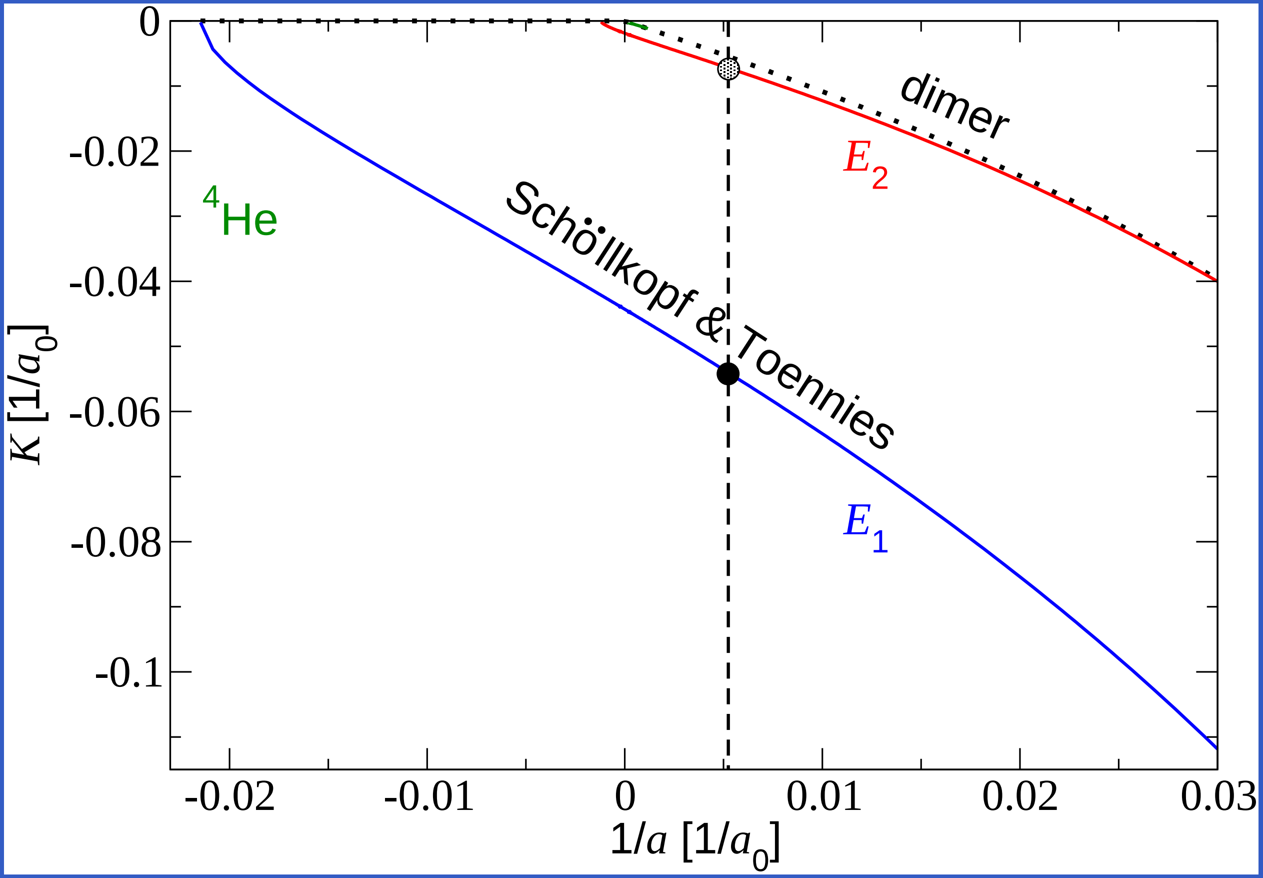


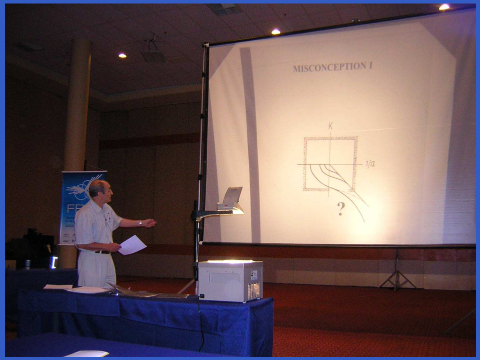


Interpretation of predictions on $^4He_3$: E. Braaten and H.-W. Hammer, PRA **67**, 042706 (2003)
See also triton case: M.L.E. Oliphant, P. Harteck, and E. Rutherford, Proc. Roy. Soc. A **144**, 692 (1934)

Efimov states

The ground level is never universal throughout!

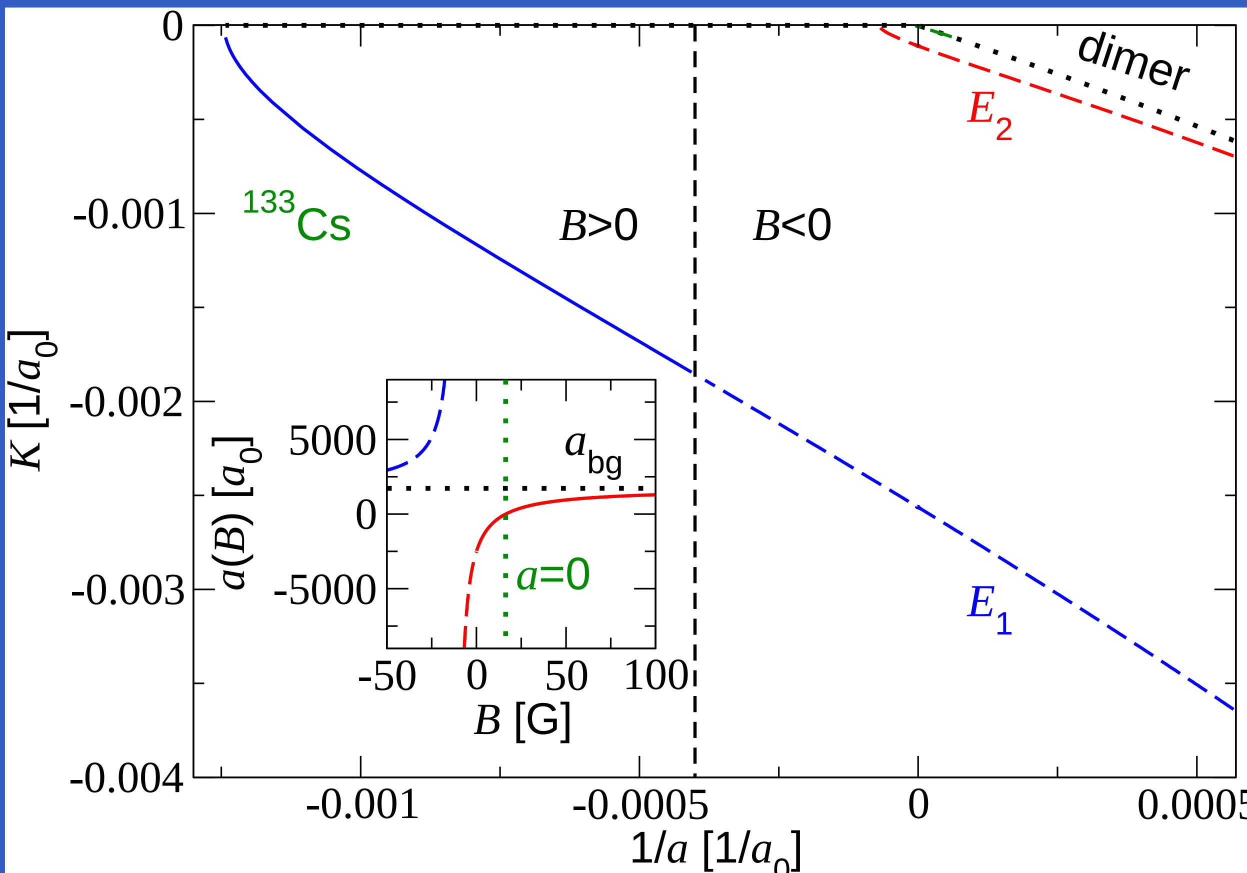


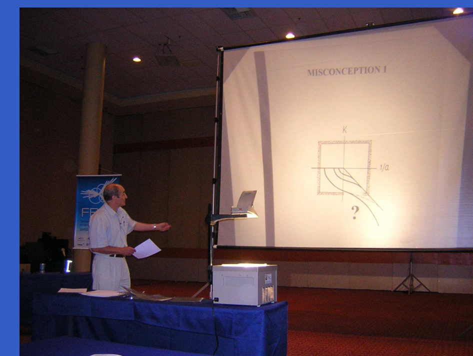


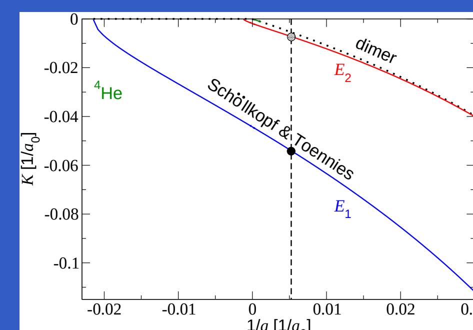
L.W. Bruch and K. Sawada, PRL **30**, 25 (1973)
V. Efimov, Nucl. Phys. A **362**, 45 (1981)

Efimov states

Efimov spectrum of ^{133}Cs at low magnetic fields

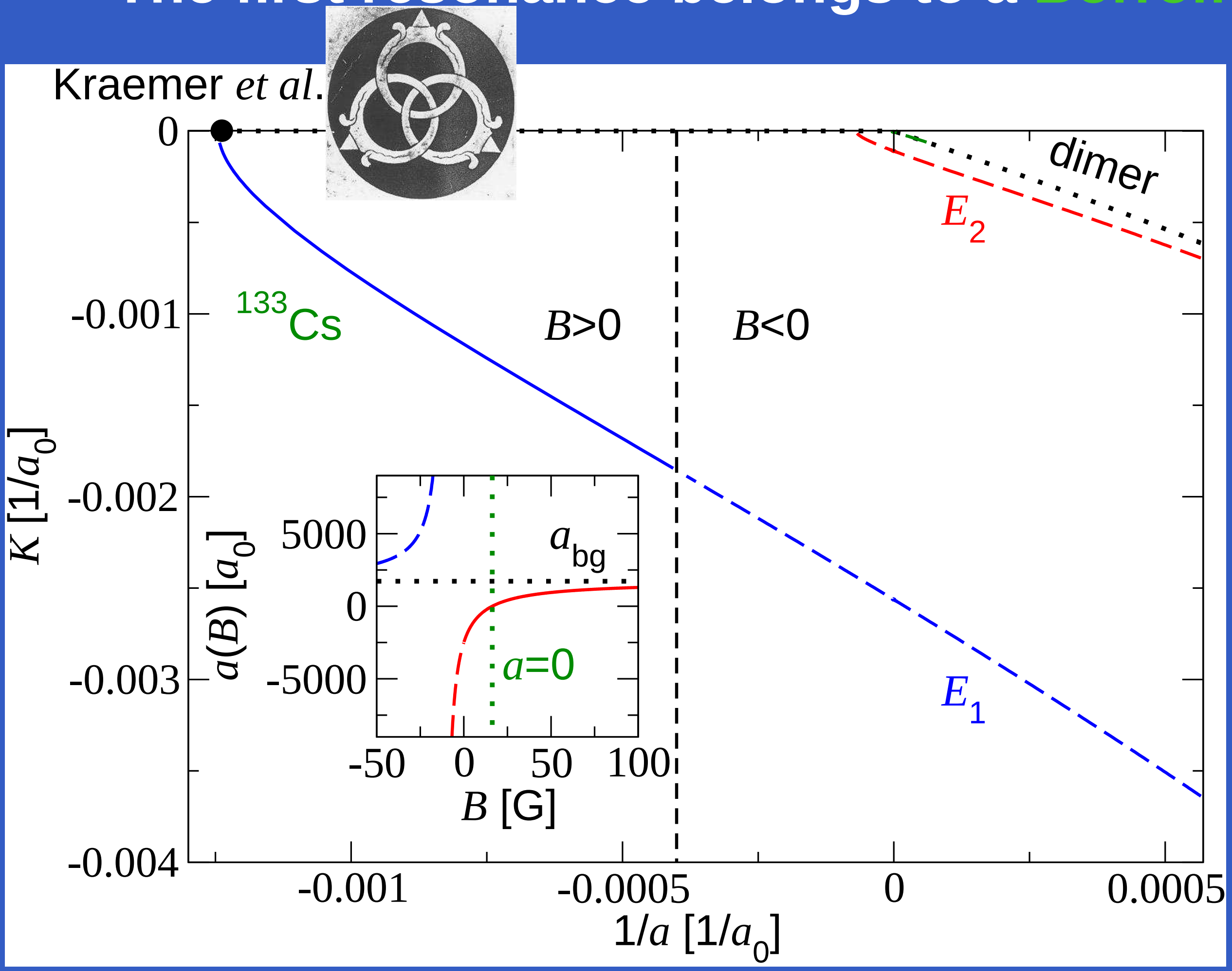


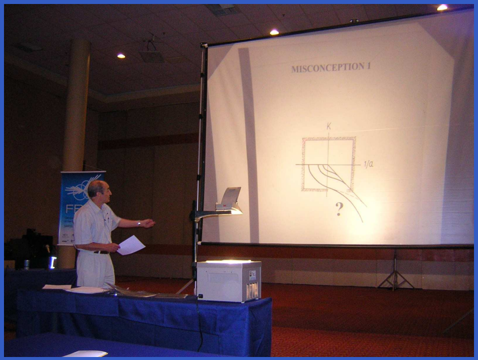


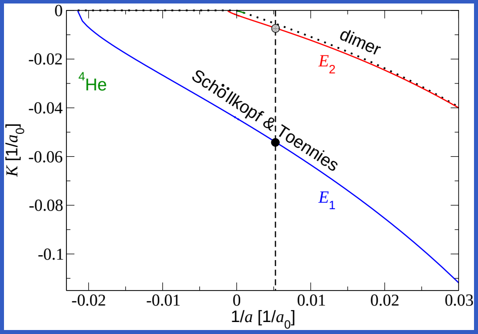


Efimov states

The first resonance belongs to a Borromean ground level!



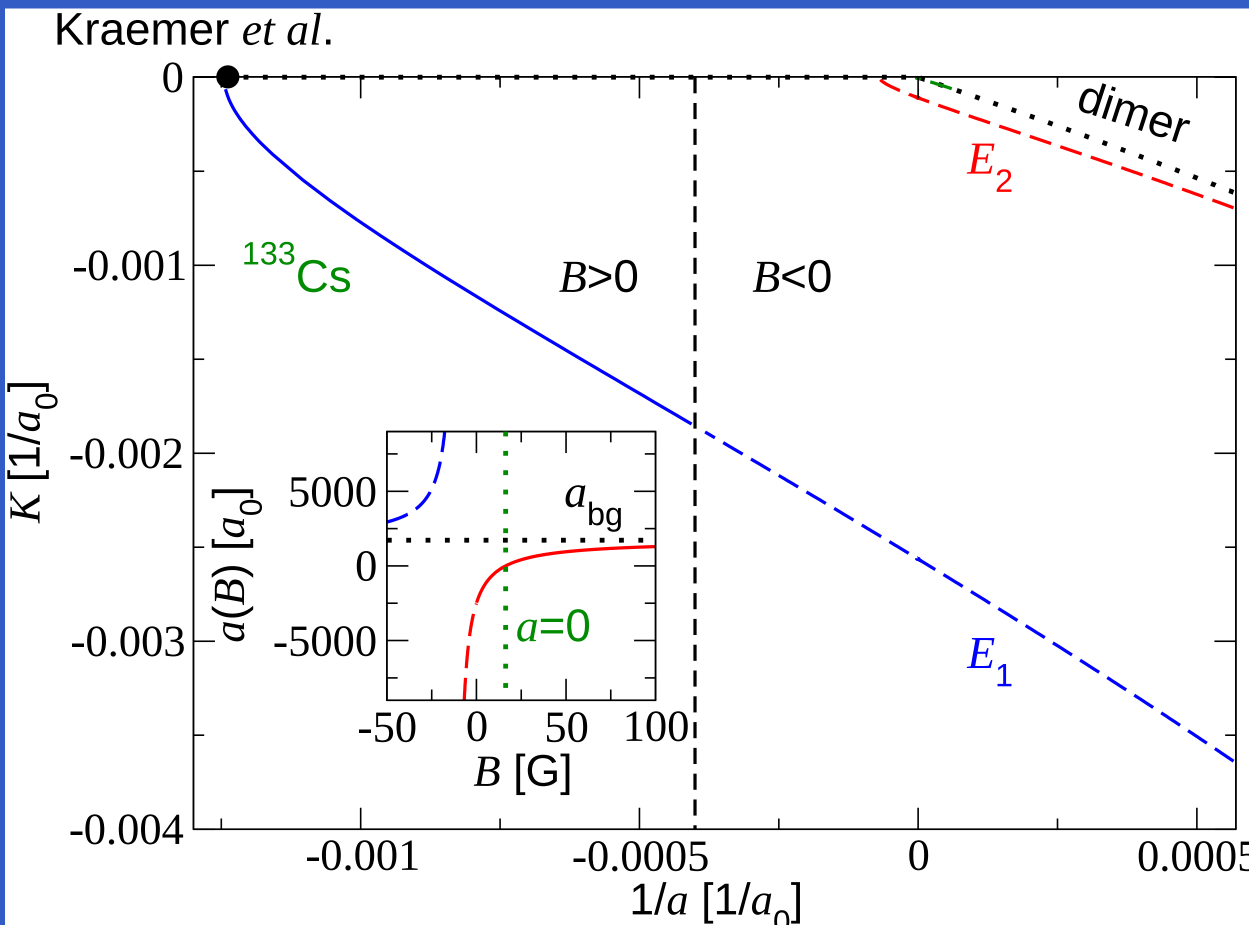


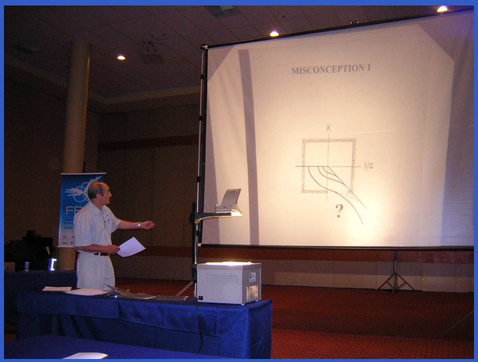


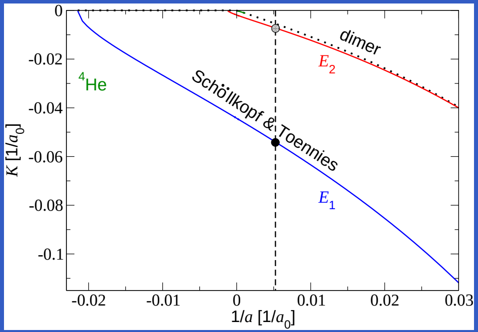
T. Kraemer, M. Mark, P. Waldburger, J.G. Danzl, C. Chin, B. Engeser, A.D. Lange, K. Pilch, A. Jaakkola, H.-C. Nägerl, and R. Grimm, Nature **440**, 315 (2006)

Efimov states

Efimov-state story to date







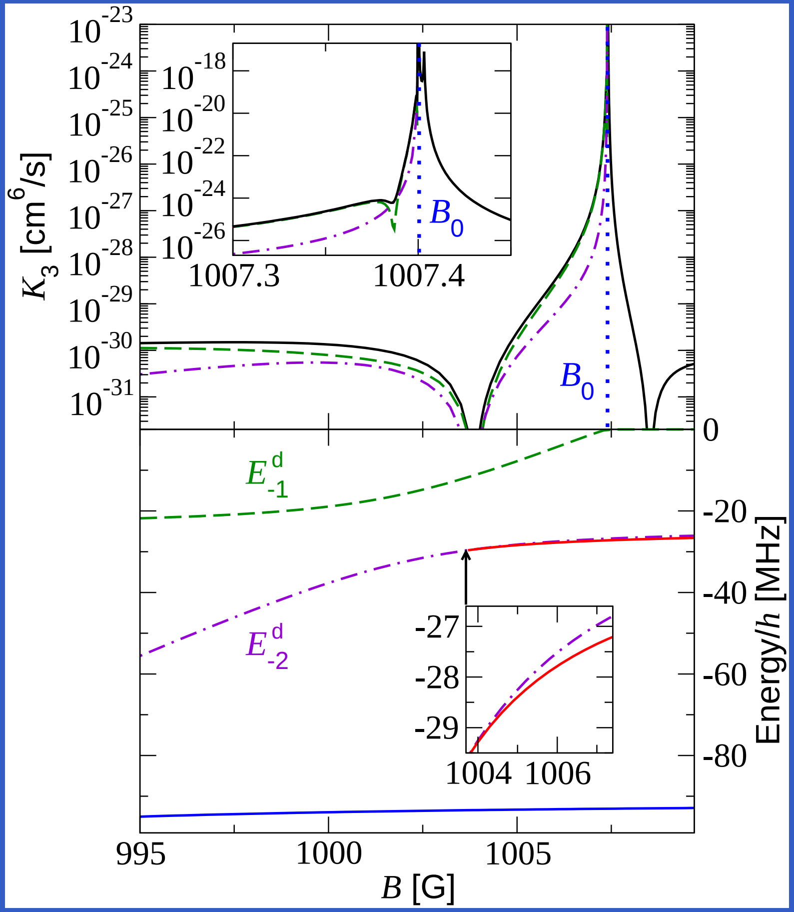
RMP colloquium: TK, R. Grimm, P.S. Julienne, H.-C. Nägerl, W. Schöllkopf, and J.P. Toennies

Three-body problem in ^{87}Rb

- Rate equation for three-body recombination:

$$\dot{N}(t) = -K_3(B)\langle n^2(t)\rangle N(t)$$

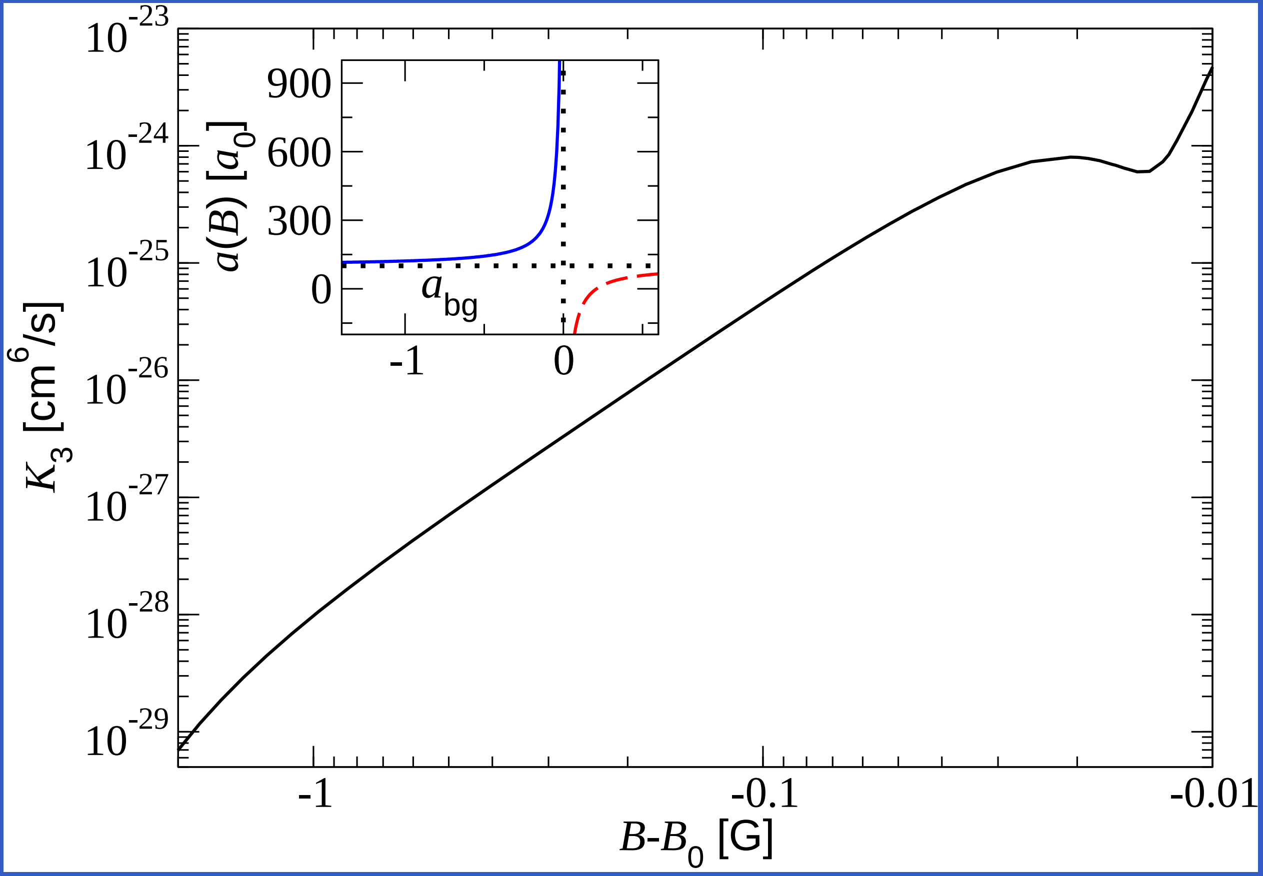
- Interferences in $K_3(B)$ occur due to the distortion of the two-body energy spectrum by the Feshbach resonance.
- The Efimov spectrum is largely inaccessible due to the narrow magnetic-field range.



Theoretical approach: E.O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. B **2**, 167 (1967)
Calculations for ^{87}Rb gases: G. Smirne, R.M. Godun, D. Cassettari, V. Boyér, C.J. Foot, T. Volz, N. Syassen, S. Duerr, G. Rempe, M.D. Lee, K. Góral, and TK, PRA **75**, 020702(R) (2007)

Three-body recombination in ^{87}Rb

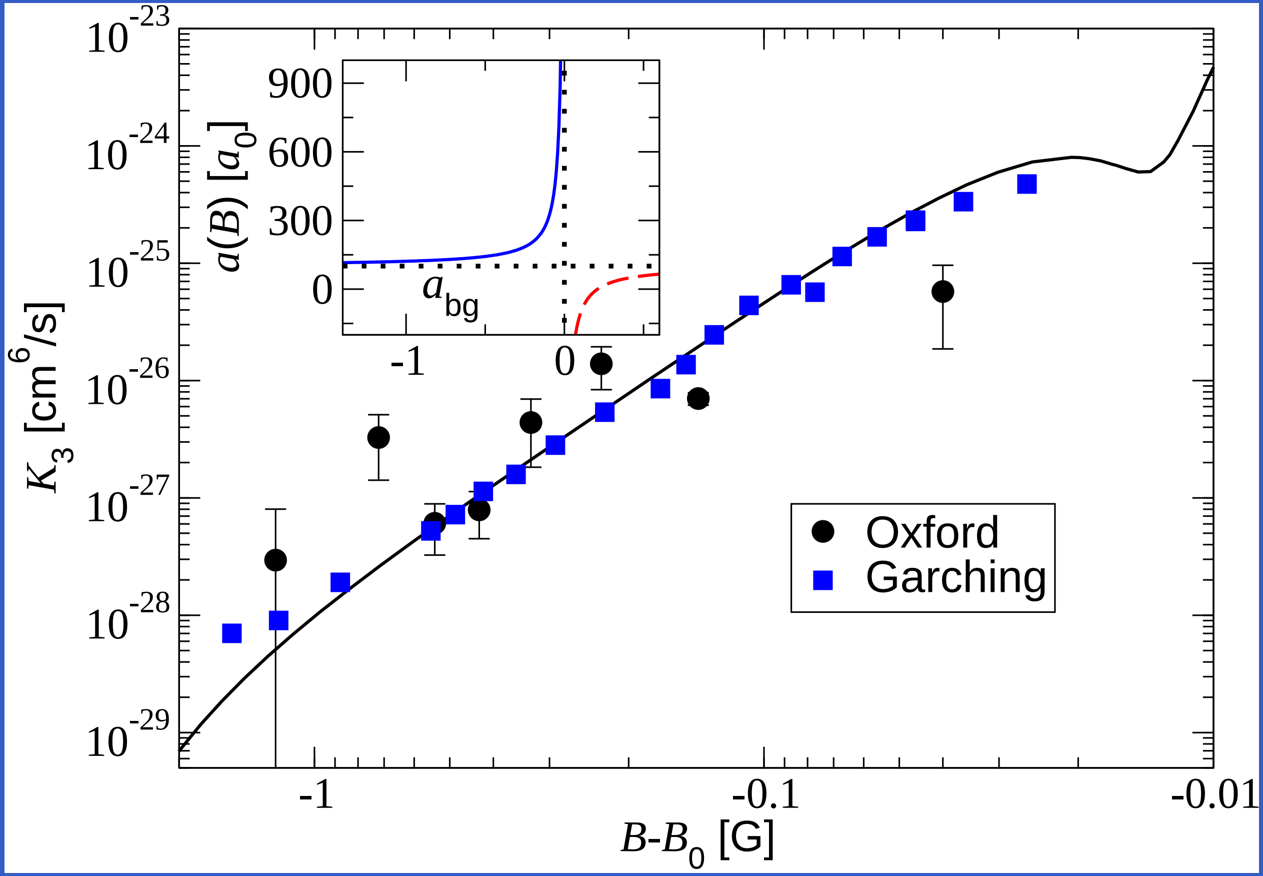
Near-resonant three-body recombination rate constants



G. Smirne, R.M. Godun, D. Cassettari, V. Boyér, C.J. Foot, T. Volz, N. Syassen, S. Duerr, G. Rempe, M.D. Lee, K. Góral, and TK, PRA **75**, 020702(R) (2007)

Three-body recombination in ^{87}Rb

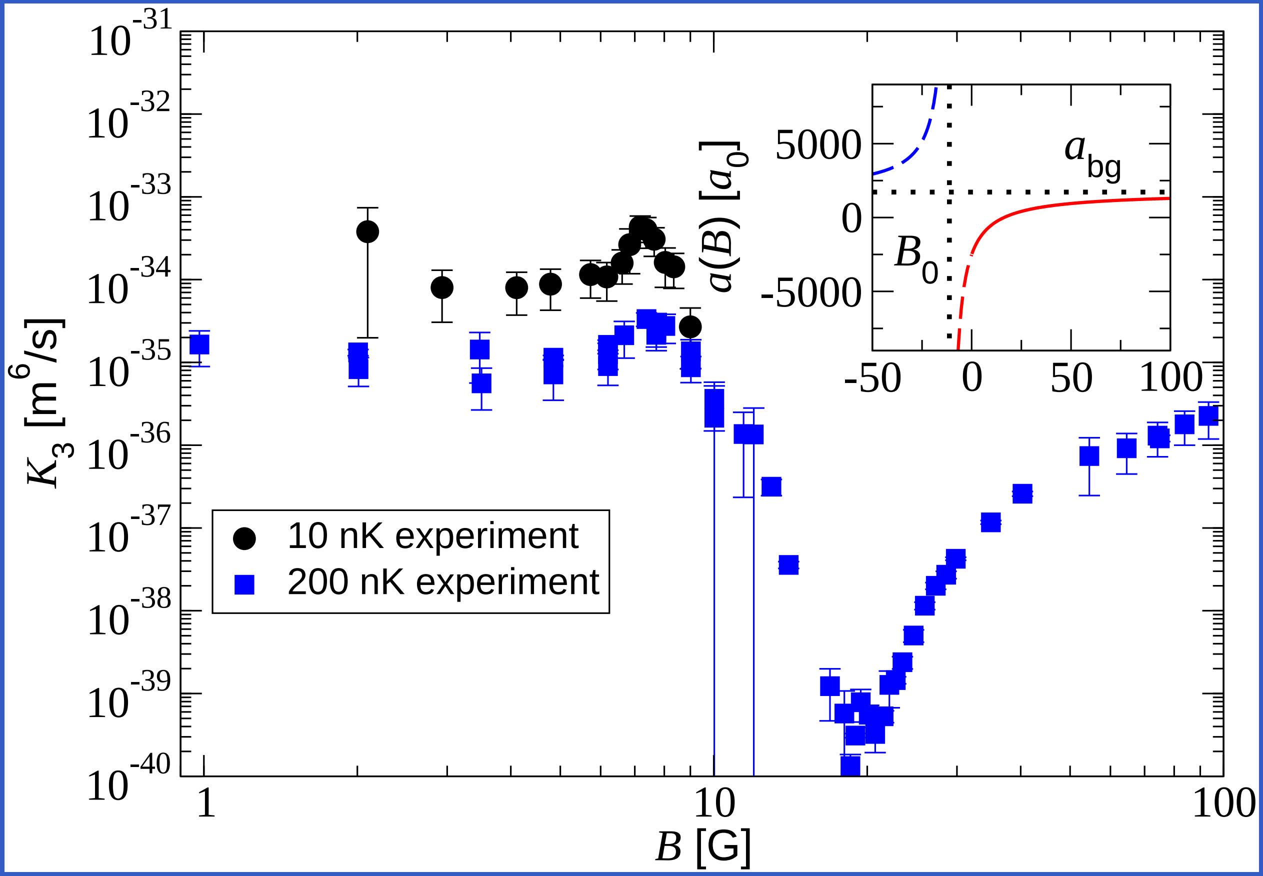
Comparison with experiment



G. Smirne, R.M. Godun, D. Cassettari, V. Boyér, C.J. Foot, T. Volz, N. Syassen, S. Duerr, G. Rempe, M.D. Lee, K. Góral, and TK, PRA **75**, 020702(R) (2007)

Three-body recombination in ^{133}Cs

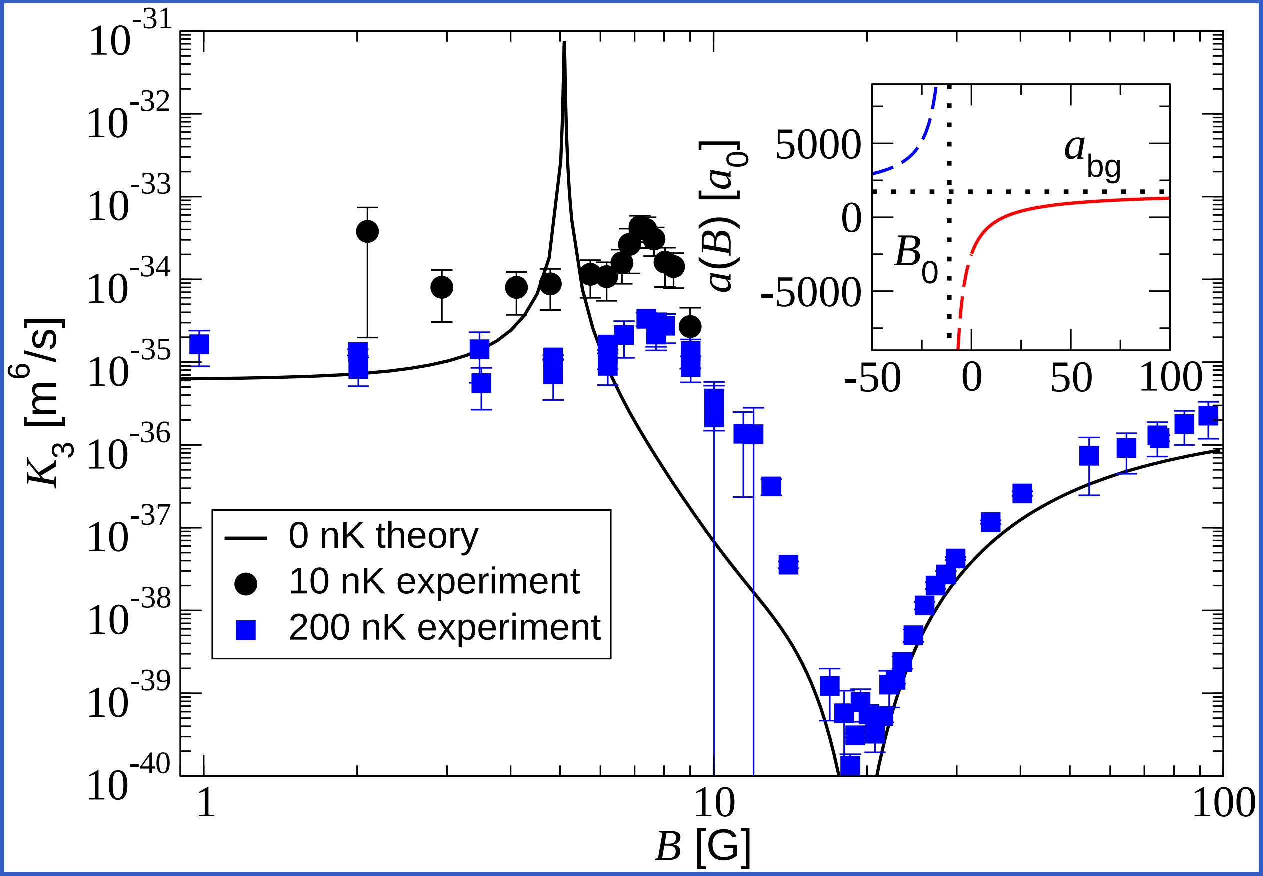
Three-body recombination rate constants in ^{133}Cs



Experiment: T. Kraemer, M. Mark, P. Waldburger, J.G. Danzl, C. Chin, B. Engeser, A.D. Lange, K. Pilch, A. Jaakkola, H.-C. Nägerl, and R. Grimm, Nature **440**, 315 (2006)

Three-body recombination in ^{133}Cs

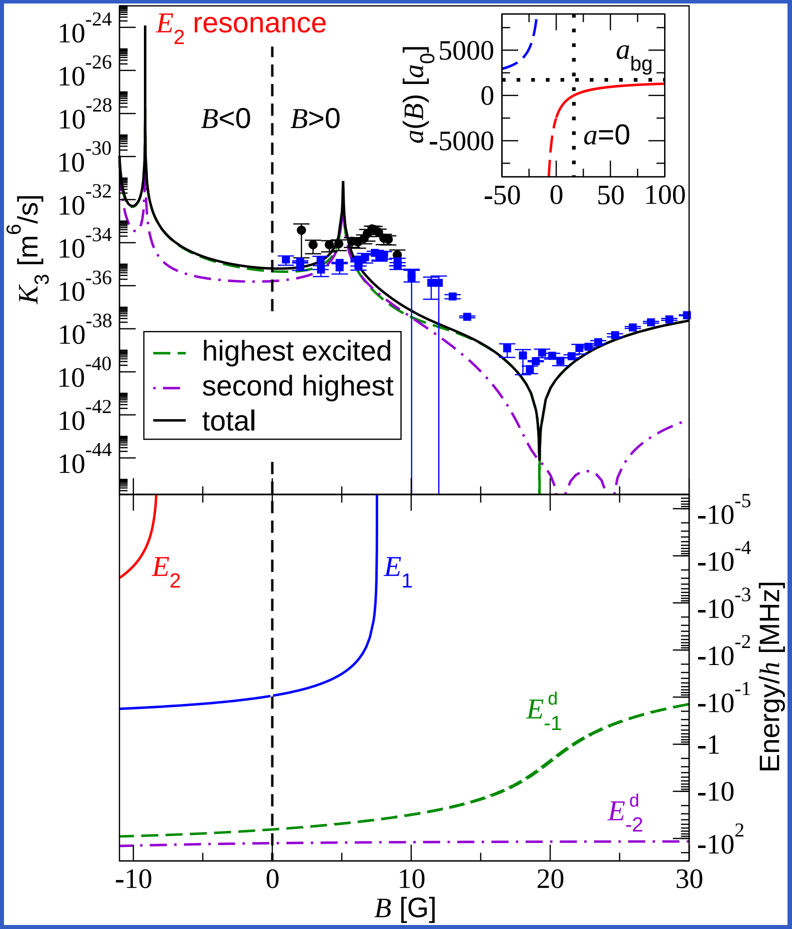
Three-body zero-energy resonance in ^{133}Cs



General concept: E. Nielsen and J.H. Macek, PRL **83**, 1566 (1999)
B.D. Esry, C.H. Greene, and J.P. Burke, PRL **83**, 1751 (1999)
Quantitative approach: G. Smirne *et al.*, PRA **75**, 020702(R) (2007)

Excited $^{133}Cs_3$ Efimov states

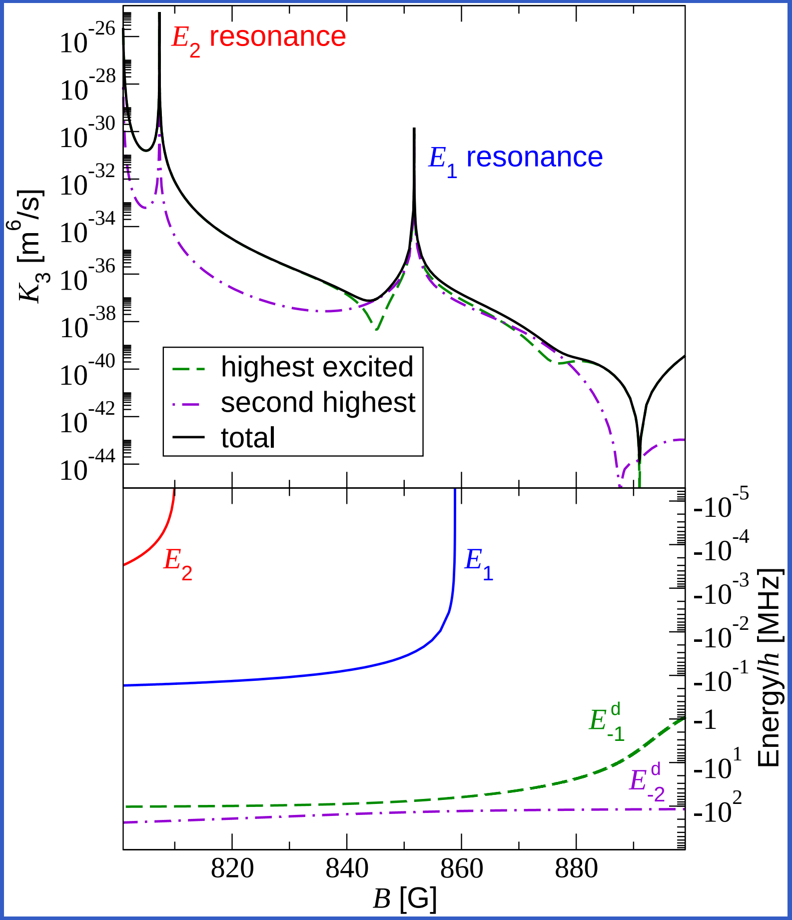
Measurement of excited-state resonances of the mirror-spin-channel might be hampered by spin relaxation!



J.L. Roberts, N.R. Claussen, S.L. Cornish, and C.E. Wieman, PRL **85**, 728 (2000)

Excited $^{133}Cs_3$ Efimov states

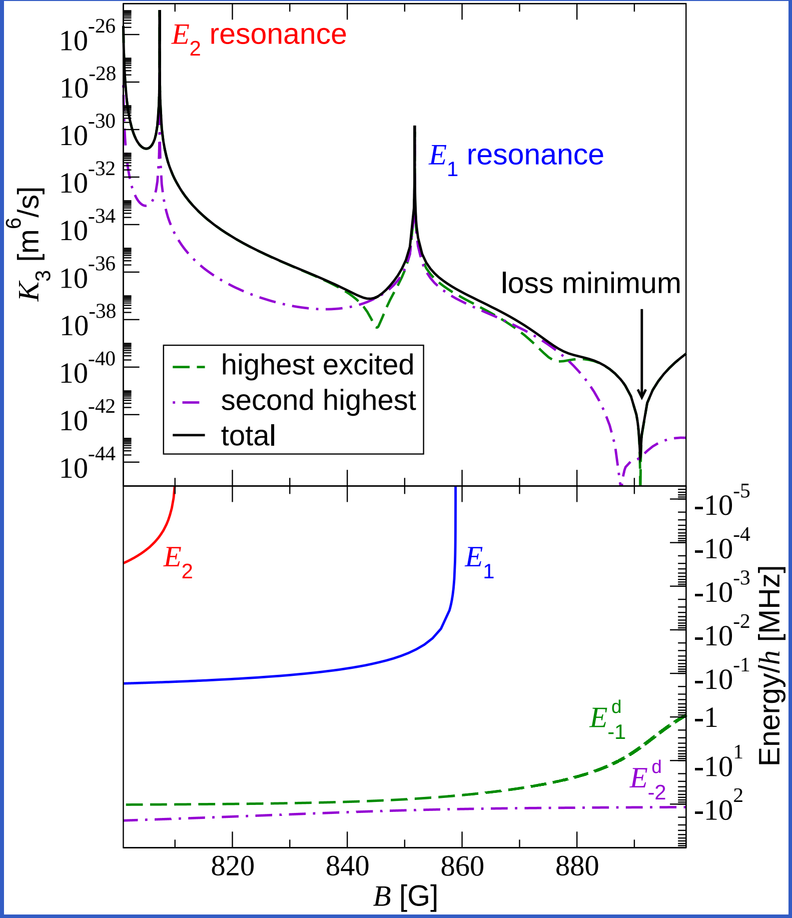
Alternative: Measurement of three-body zero-energy resonances in the vicinity of 800 G



M.D. Lee, TK, and P.S. Julienne, e-print cond-mat/0702178

Excited $^{133}Cs_3$ Efimov states

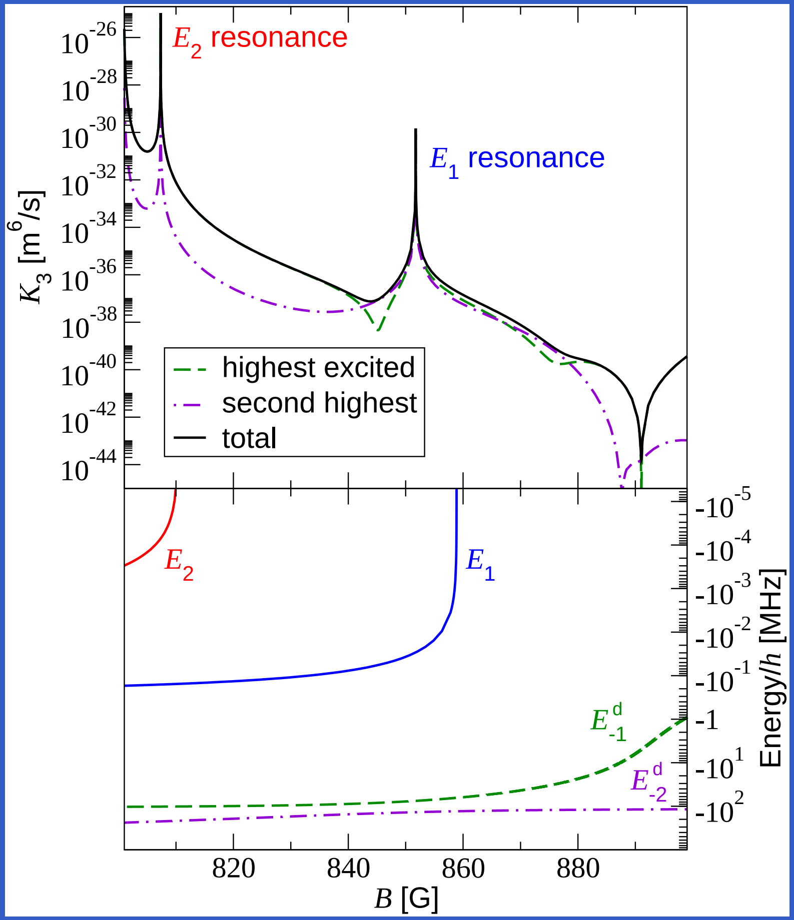
The loss minimum in the vicinity of 891 G might allow Bose-Einstein condensation of cesium!

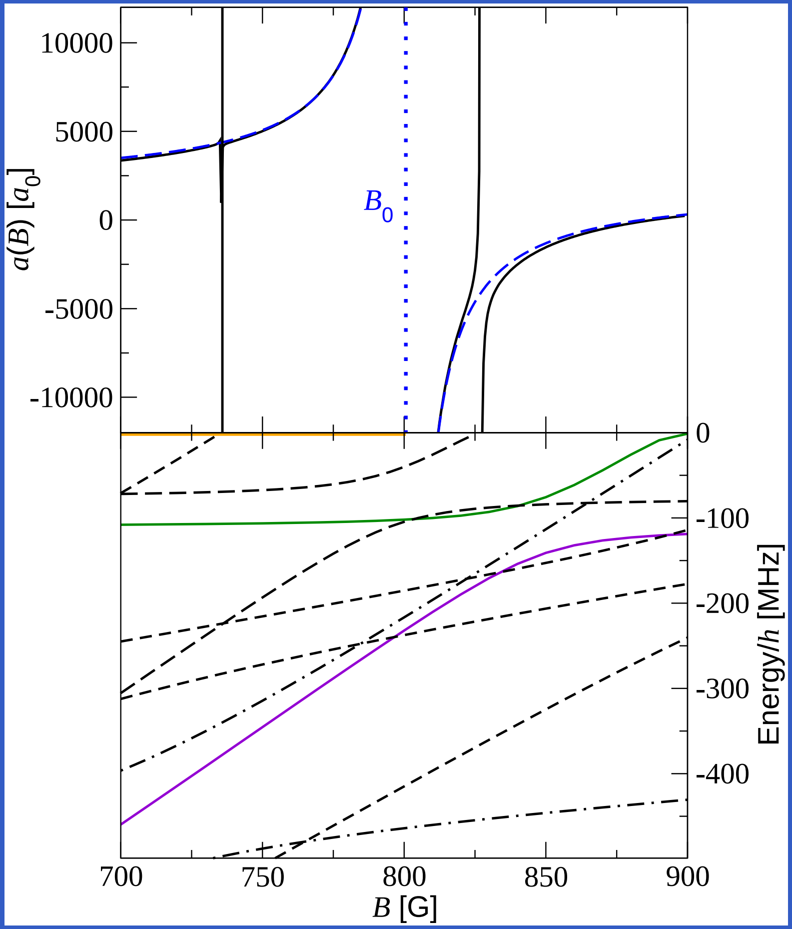


T. Weber, J. Herbig, M. Mark, H.-C. Nägerl, and R. Grimm, Science **299**, 232 (2003)

Excited $^{133}Cs_3$ Efimov states

The theory does not account for the full complexity of cesium interactions!

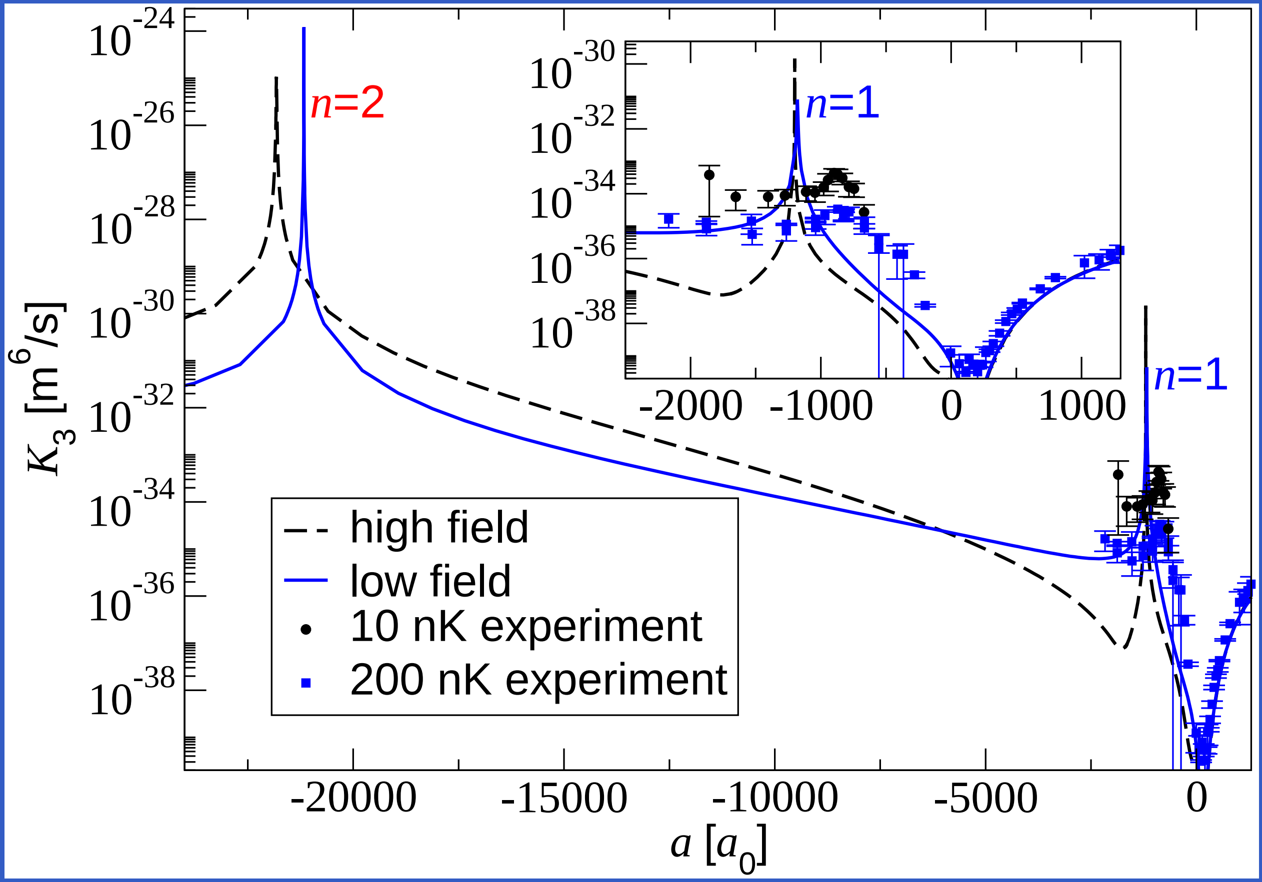




M.D. Lee, TK, and P.S. Julienne, e-print cond-mat/0702178

Excited $^{133}Cs_3$ Efimov states

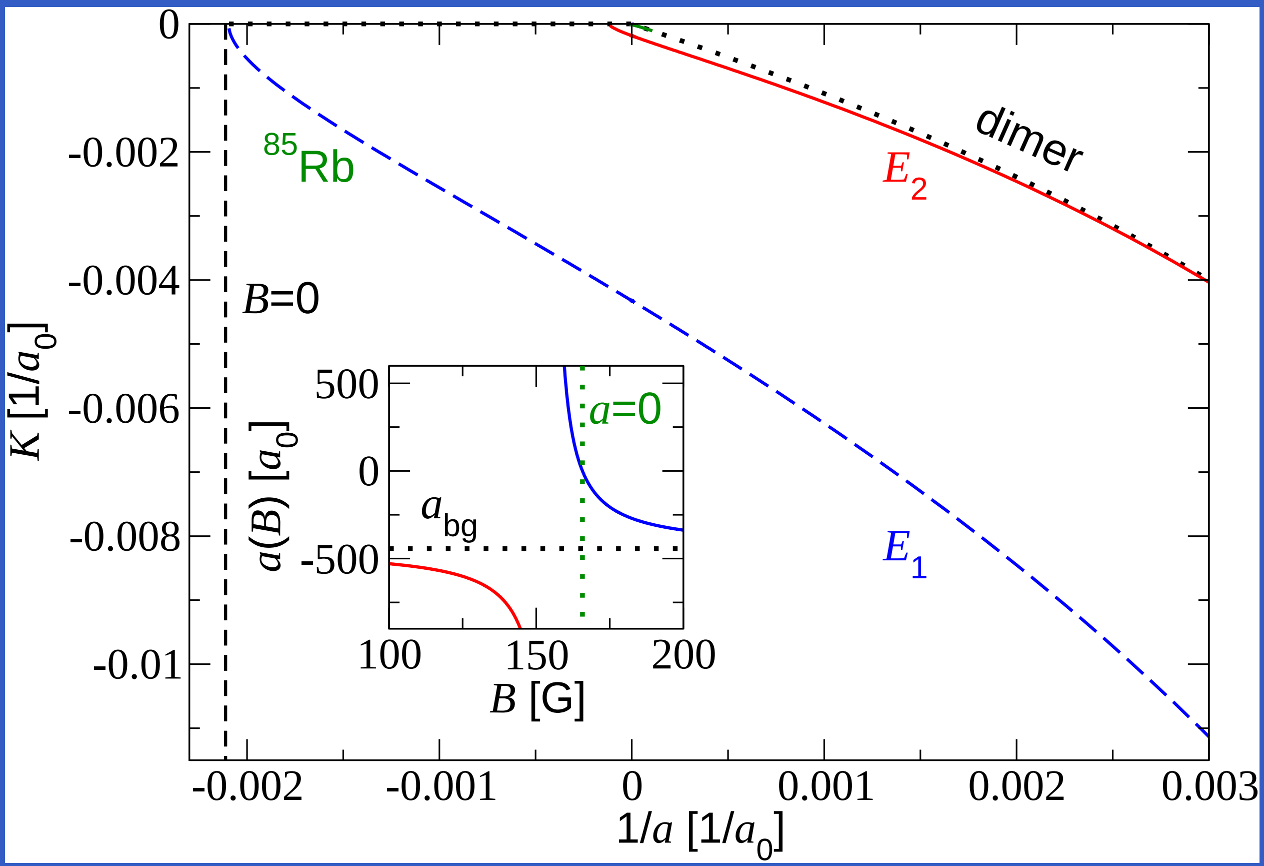
The diatomic target states at negative scattering lengths are not uniform in the magnetic field!

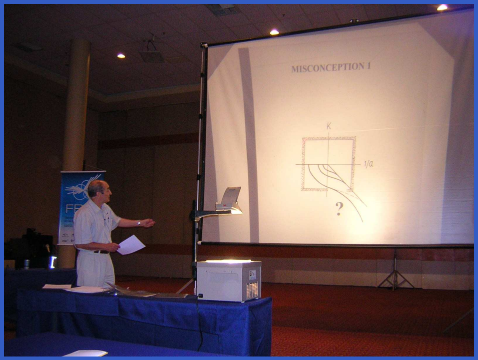


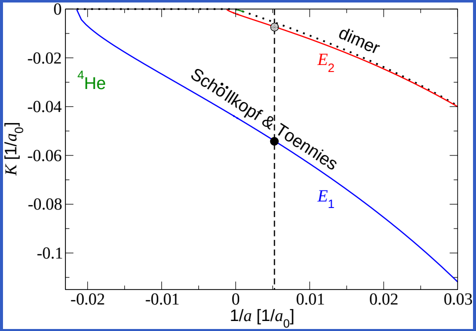
M.D. Lee, TK, and P.S. Julienne, e-print cond-mat/0702178

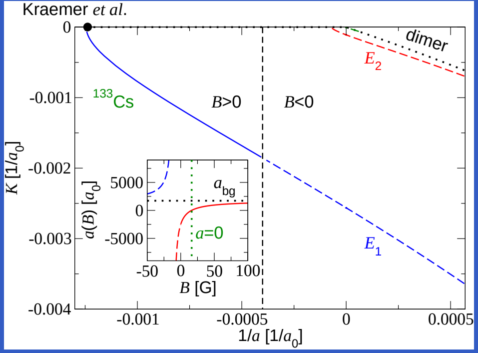
Production of excited $^{85}Rb_3$ Efimov states

Efimov spectrum of ^{85}Rb





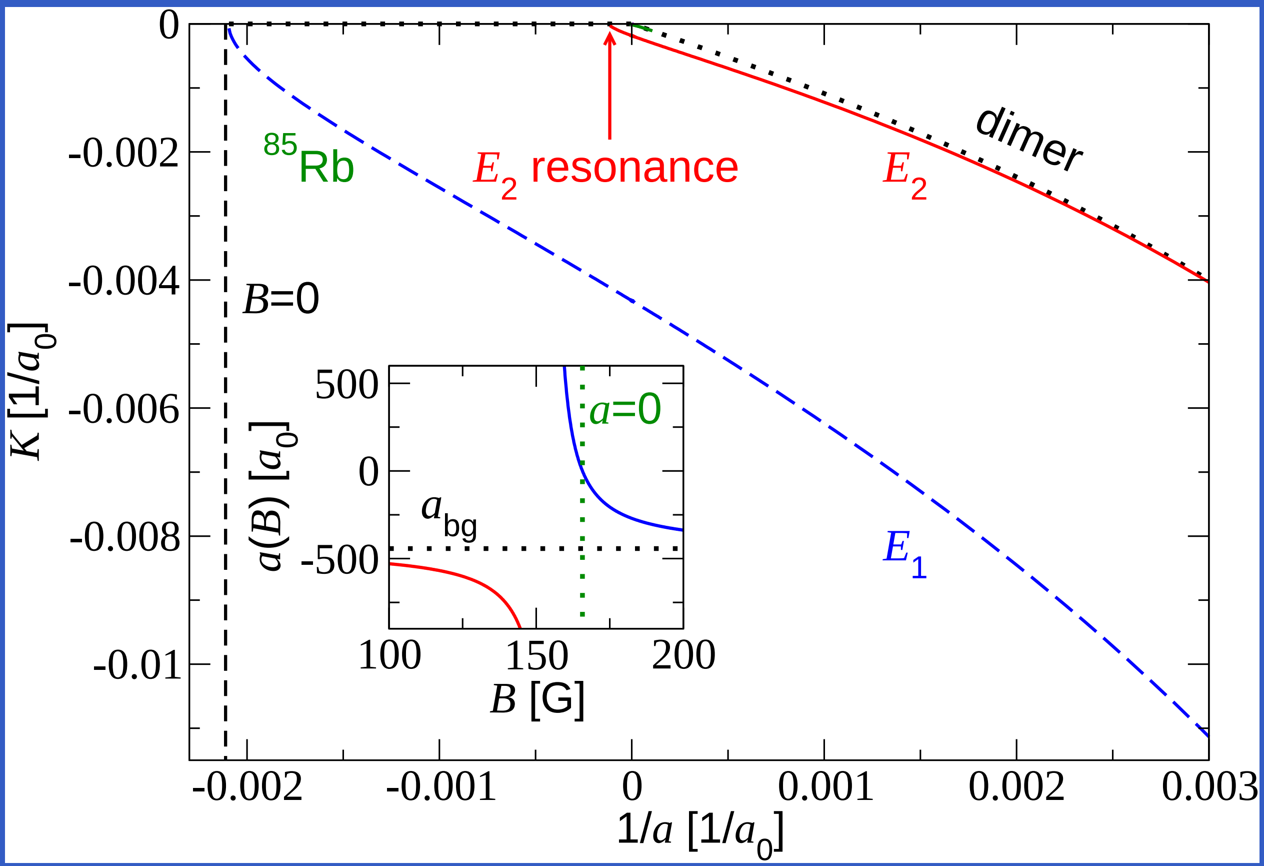


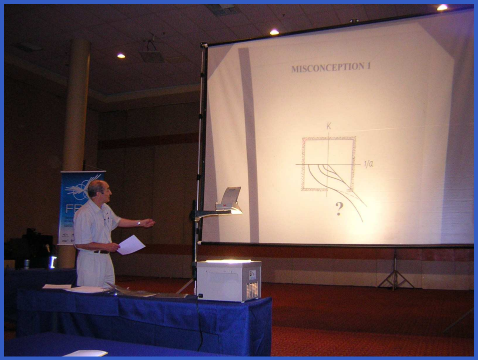


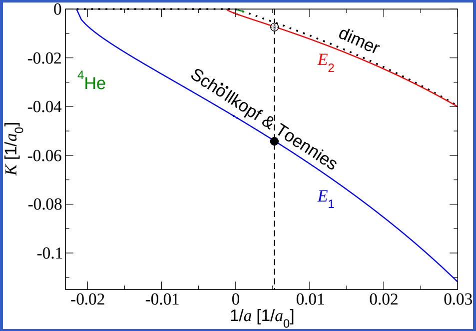
M. Stoll and TK, PRA **72**, 022714 (2005)

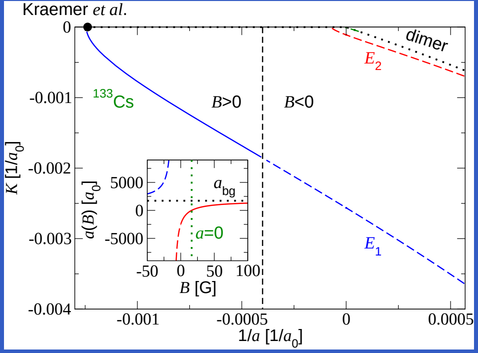
Production of excited $^{85}Rb_3$ Efimov states

^{85}Rb gases allow easy access to the E_2 resonance!





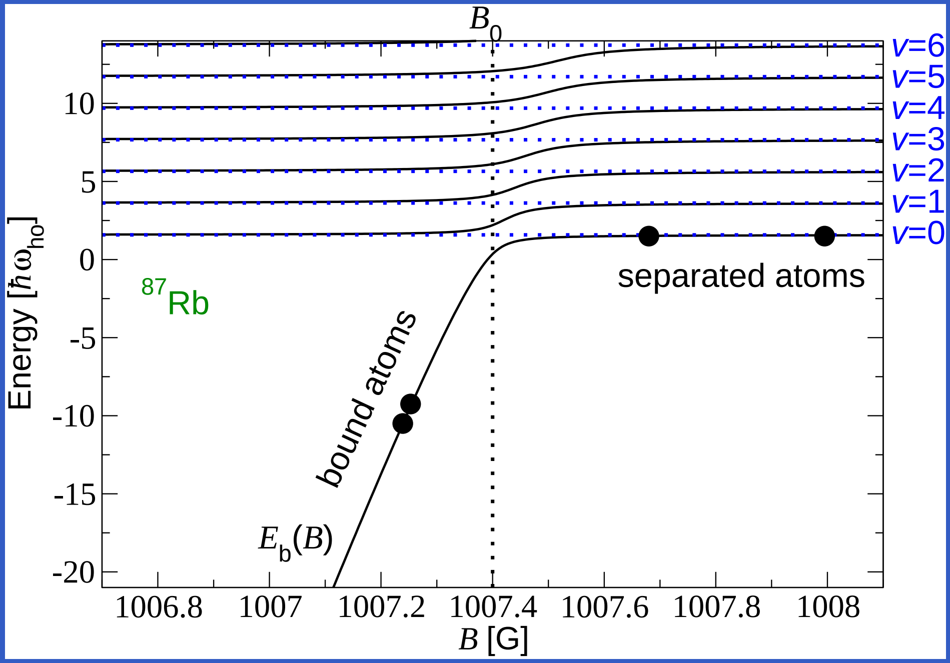




M. Stoll and TK, PRA **72**, 022714 (2005)

Production of excited $^{85}Rb_3$ Efimov states

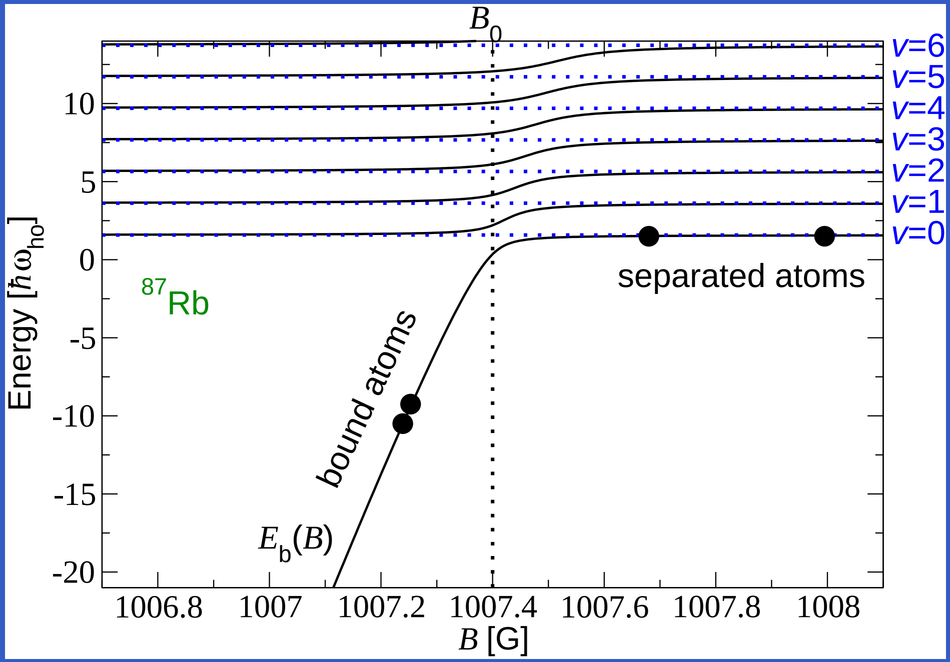
Association of dimer molecules in an optical lattice site

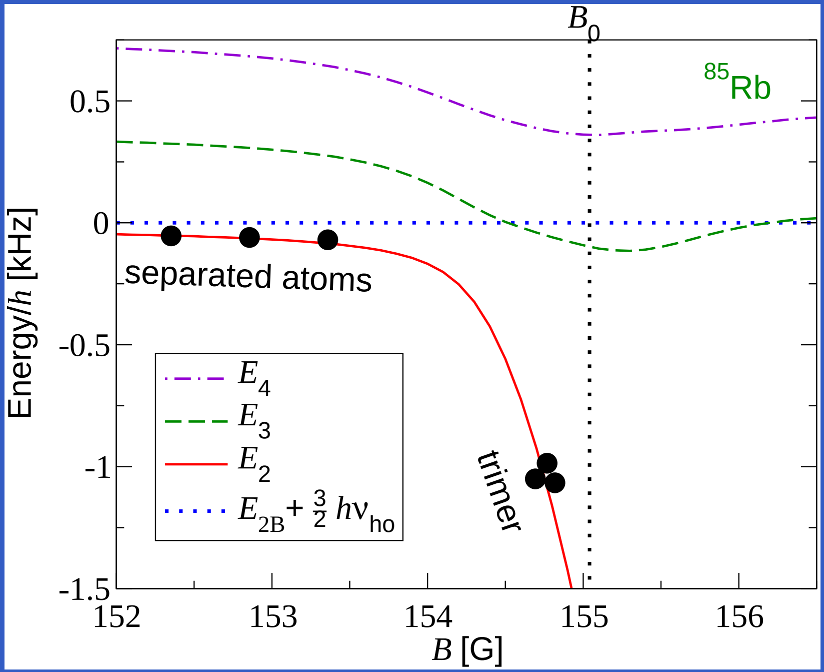


G. Thalhammer, K. Winkler, F. Lang, S. Schmid,
R. Grimm, and J.H. Denschlag, PRL **96**, 050402 (2006)

Production of excited $^{85}Rb_3$ Efimov states

Association of trimer molecules in an optical lattice site





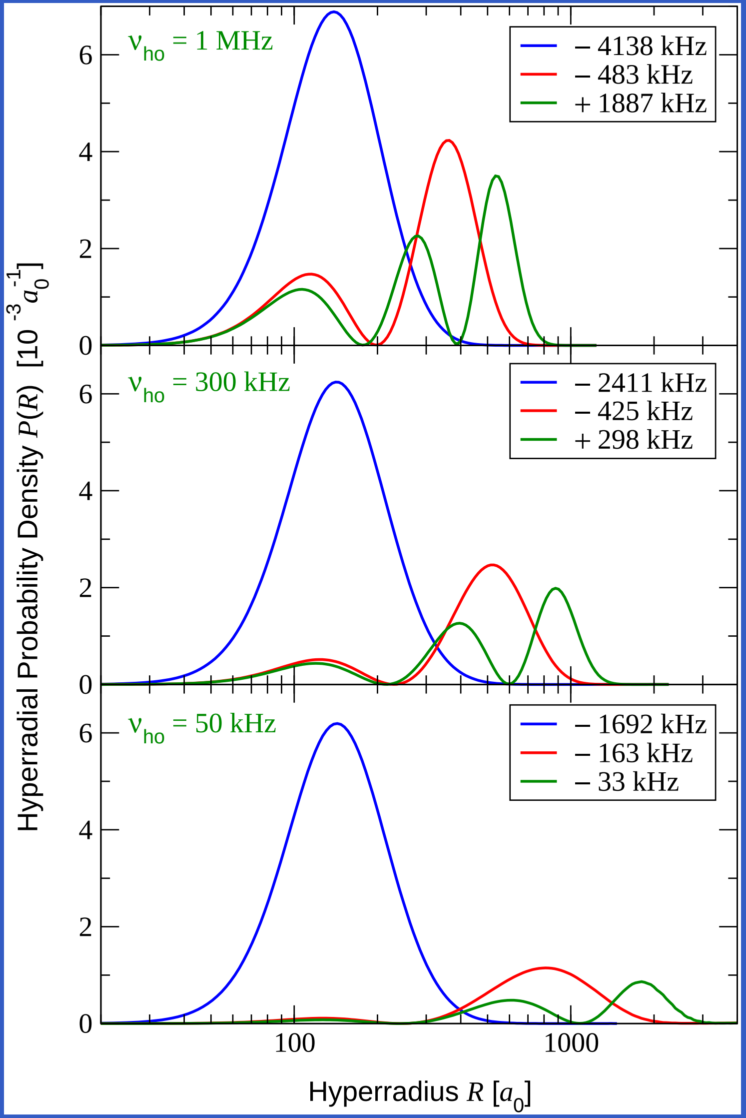
G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J.H. Denschlag, PRL **96**, 050402 (2006)

M. Stoll and TK, PRA **72**, 022714 (2005)
See also: S. Jonsell, H. Heiselberg, and C.J. Pethick, PRL **89**, 250401 (2002)

Production of excited $^{85}Rb_3$ Efimov states

Efimov trimer molecules in an optical lattice site

- The magnetic-field dependent binding energies and spatial extents of E_2-trimer molecules are comparable to those of the associated diatomic Feshbach molecules.
- The lifetimes of Efimov-trimer molecules increase with increasing bond length.



M. Stoll and TK, PRA **72**, 022714 (2005)
See also: S. Jonsell, Europhys. Lett. **76**, 8 (2006)

Conclusions

- Near-resonant diatomic Feshbach molecules are halo states.
- The halo nature of diatomic Feshbach molecules implies Efimov's effect in the associated three-body problem.
- Measurements of near resonant three-body recombination in cold gases of ^{133}Cs to date prove the existence of a Borromean ground state of the Efimov spectrum.
- Excited-state Efimov zero-energy resonances might be detected in cold gases of ^{133}Cs at magnetic fields in the vicinity of 800 G.
- Metastable excited-state Efimov trimers might be produced in cold gases of, e.g., ^{85}Rb using magnetic-field sweep techniques in analogy to the association of diatomic Feshbach molecules.

Thanks to

- DPhil students: Thomas Hanna and Hugo Martay
- Postdocs: Marzena Szymańska, Mark Lee, and Martin Stoll
- Group leader: Keith Burnett
- Coupled-channels calculations: Paul Julienne (NIST)
- Experimental groups at: Oxford, JILA, Innsbruck, and Munich
- Discussions with: Wieland Schöllkopf, Peter Toennies, and Vitaly Efimov

THE ROYAL SOCIETY

EPSRC