Deconfined Quantum Criticality

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Collaborators

- O. Motrunich (Caltech)
  - Phys. Rev. B **70**, 075104 (2004)

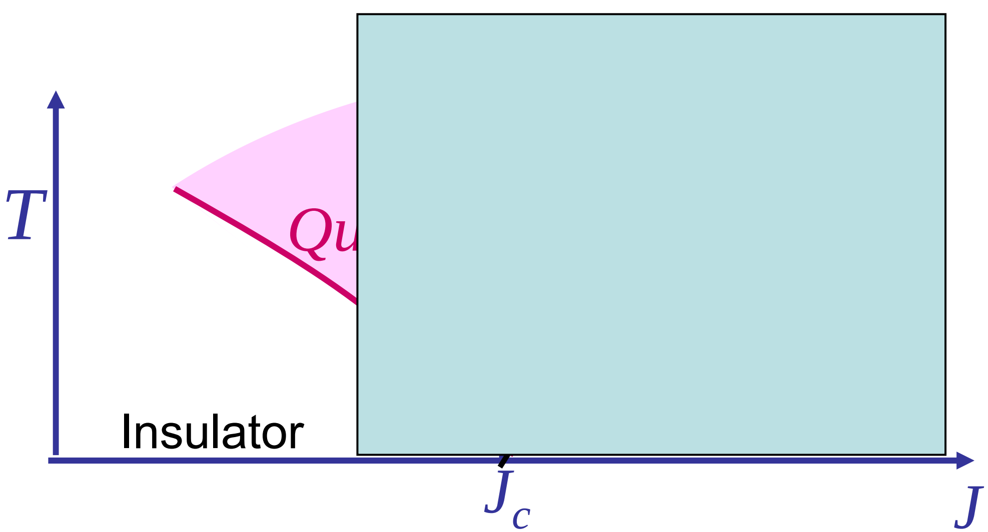
- T. Senthil (MIT)
- L. Balents (UCSB)
- S. Sachdev (Harvard)
- M. P. A. Fisher (KITP)
  - Science 303,1490 (2004).

- Cenke Xu (Berkeley)
  - In preparation

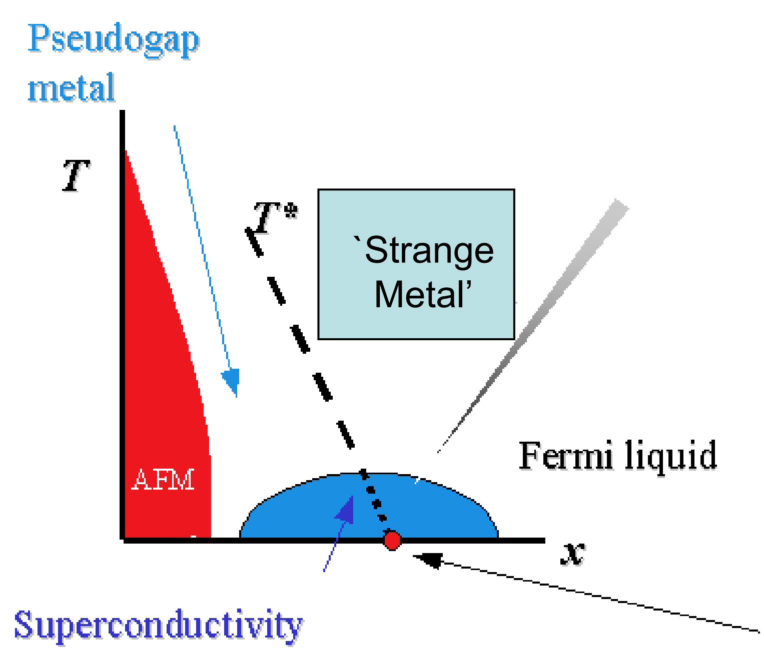
Quantum Phase Transitions

- Phase transition at zero temperature driven by *quantum* fluctuations.
  - Can be first order (sudden) or continuous -> Quantum Criticality.

- Theory:
  - Universality
  - Model of unusual finite temperature behavior.

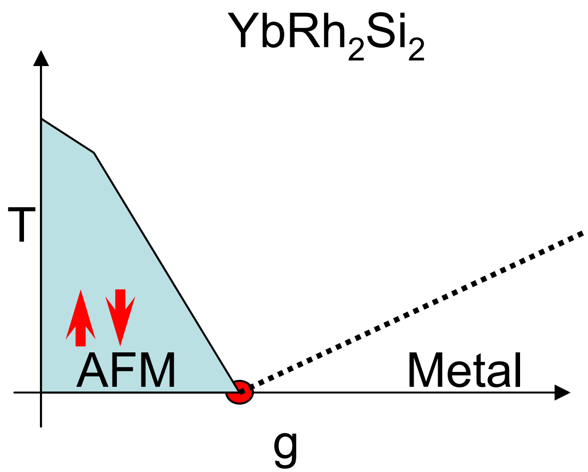


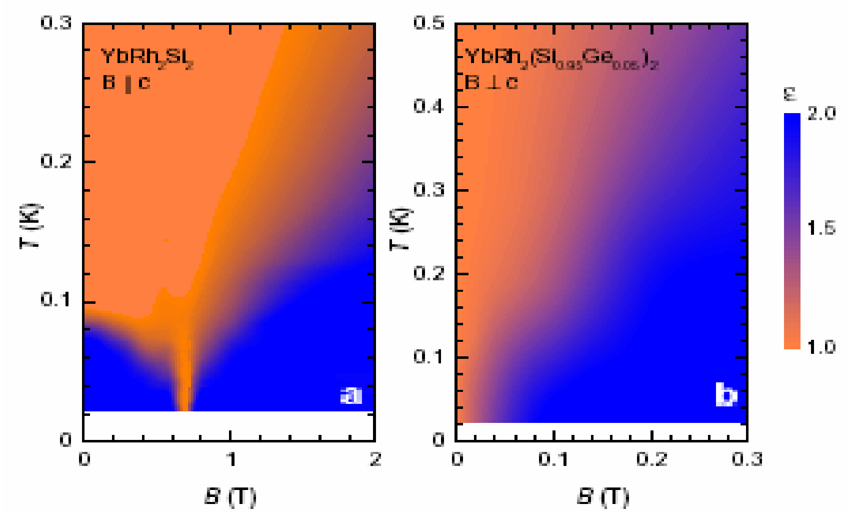
Cuprate High Tc materials



Quantum Phase Transitions

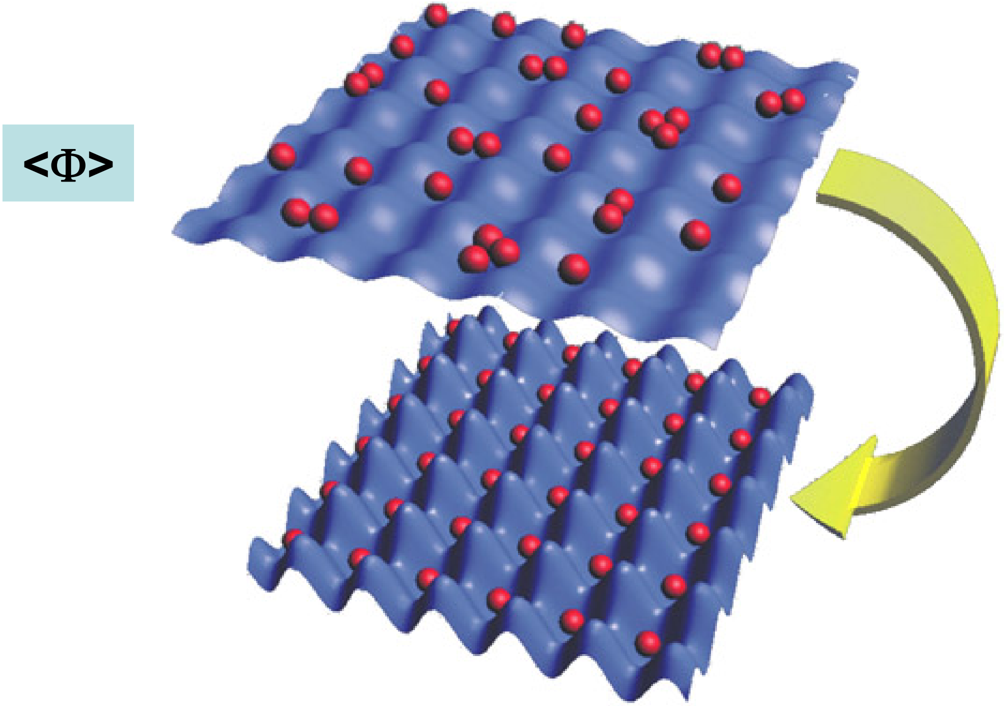
- Experiments:
  - Several experimental realizations of quantum criticality – poorly understood.
  - For example, in ‘heavy fermion’ metals, anomalous properties near anti-ferromagnet to heavy fermi-liquid transition.

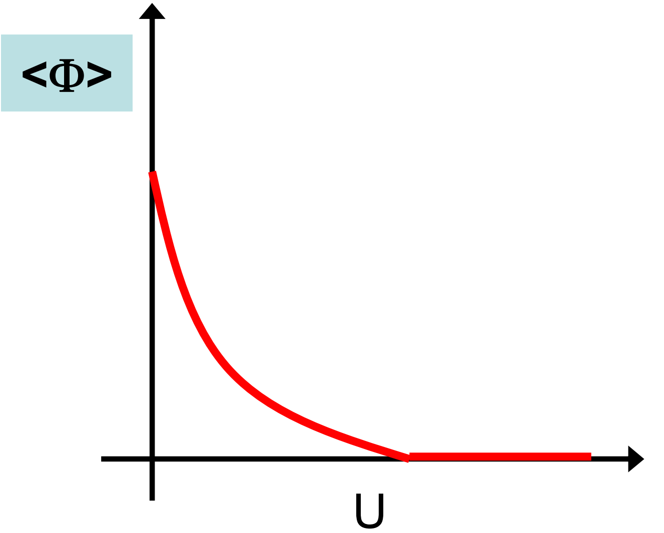




Conventional (Landau) Quantum Criticality

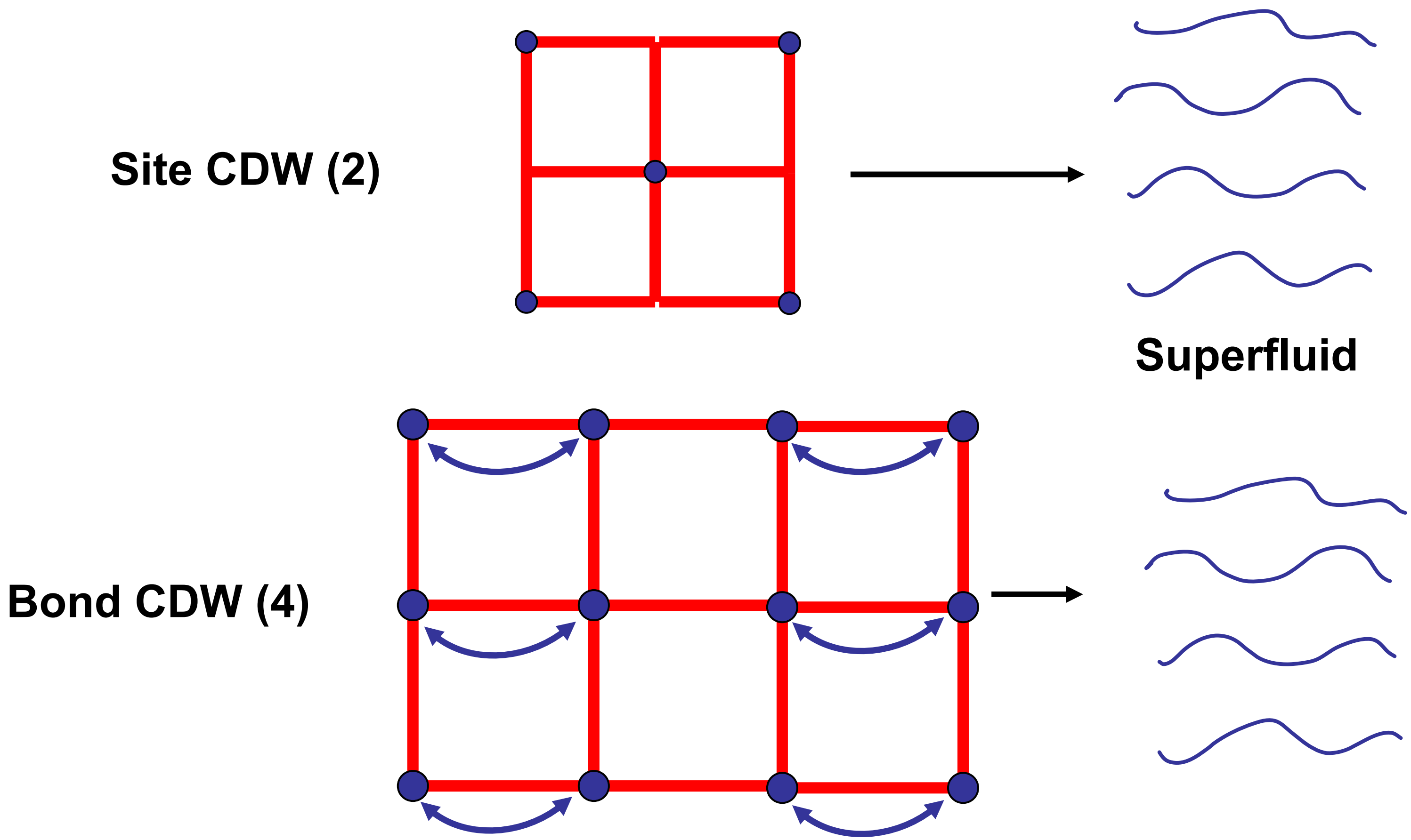
- Symmetry breaking – degenerate ground states related by symmetry.
- Distinguish different ground states by order parameter.
- Critical Point => order parameter fluctuations.





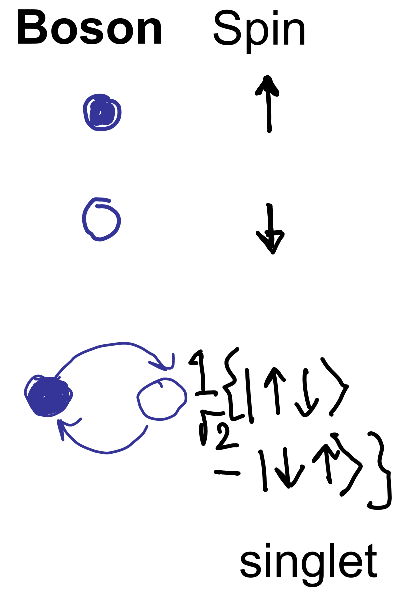
Unconventional Criticality?

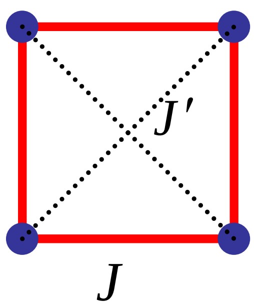
- *Half filled lattice*. What about continuous transitions between Charge Density Wave (CDW) and superfluid?



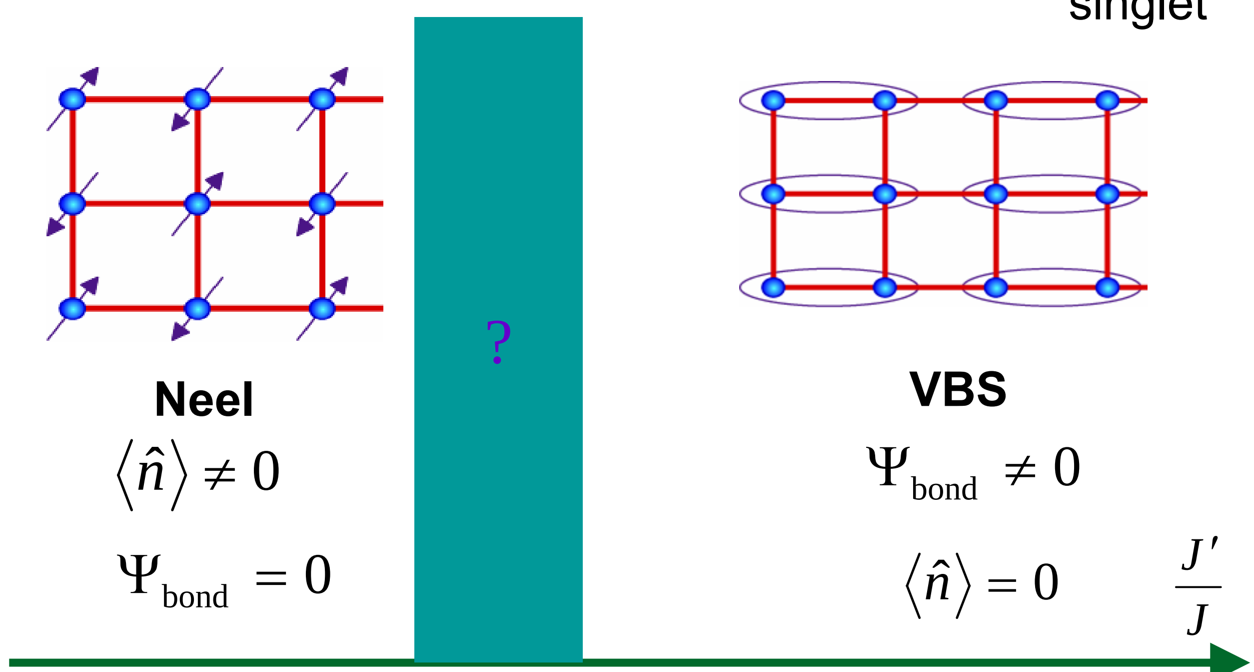
Frustrated Quantum Magnet on the Square Lattice

- Connection to quantum magnetism:
  - Superfluid => Neel order
  - Bond CDW=> Valence bond solid
- 2D Square Lattice S=1/2





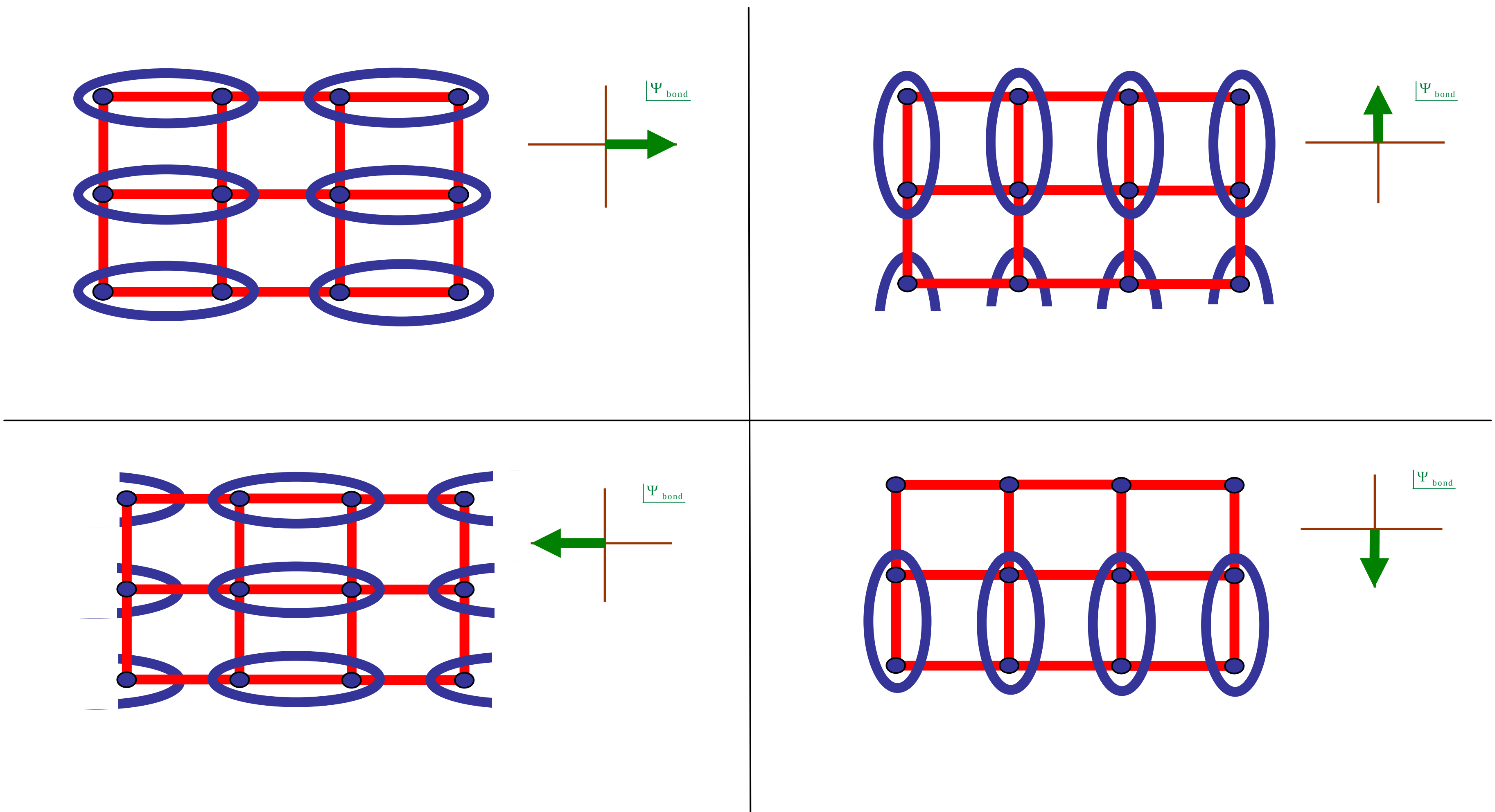
$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



VBS Order Parameter

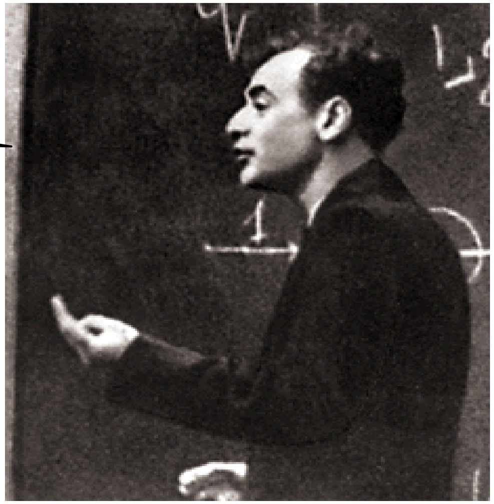
- Broken lattice symmetries. Associate a Complex Number

Ψ_{bond}

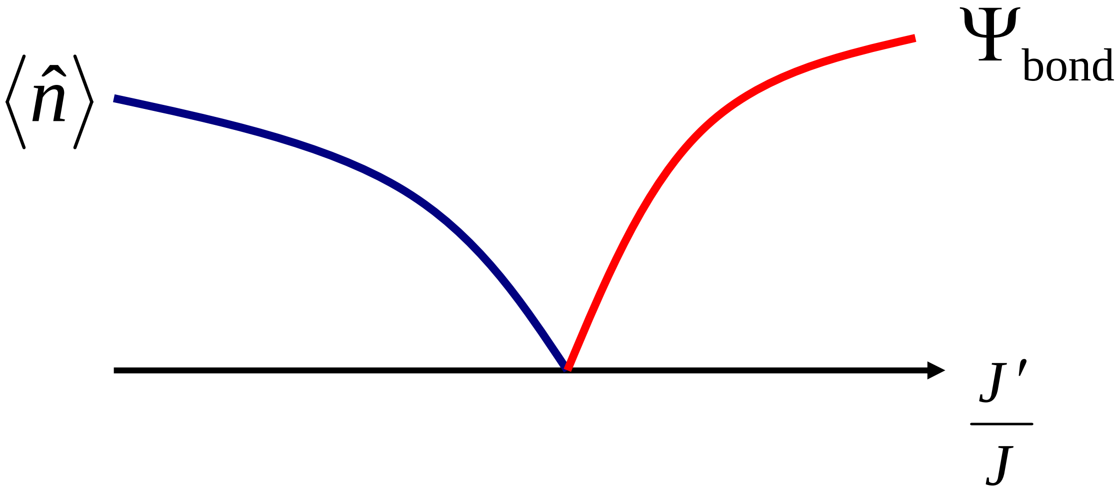


Constraints From Landau's Theory

No direct transition that is continuous



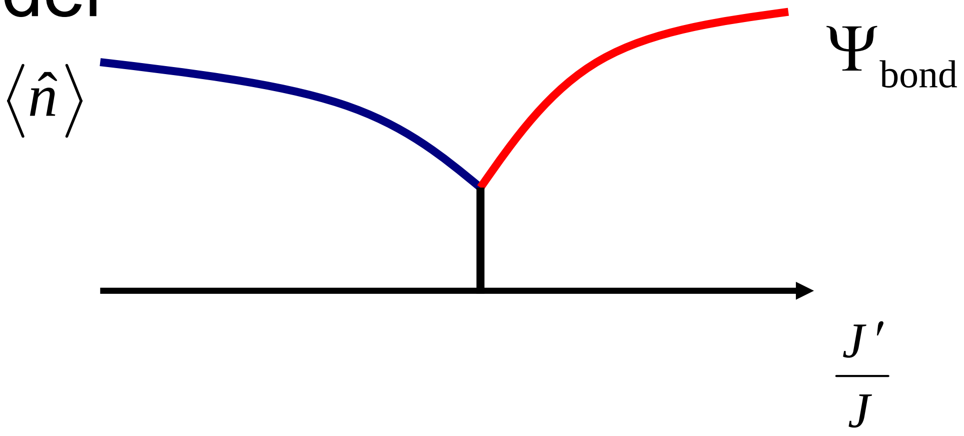
Two unrelated orders – Neel and VBS



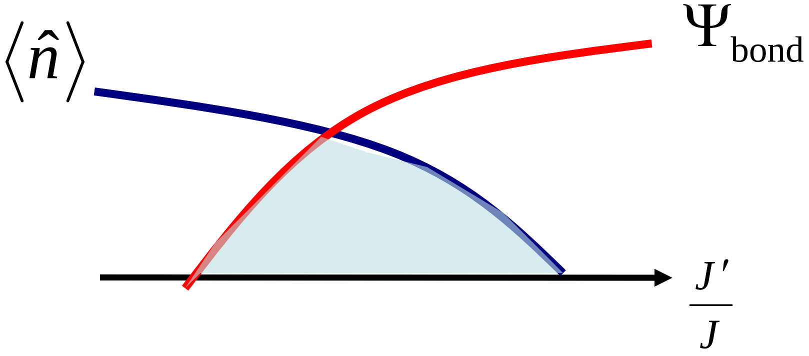
Needs special fine tuning

Generic Possibilities

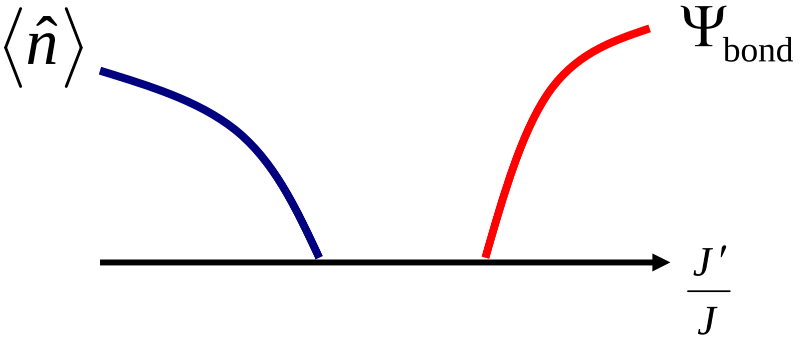
- First Order



- Coexistence

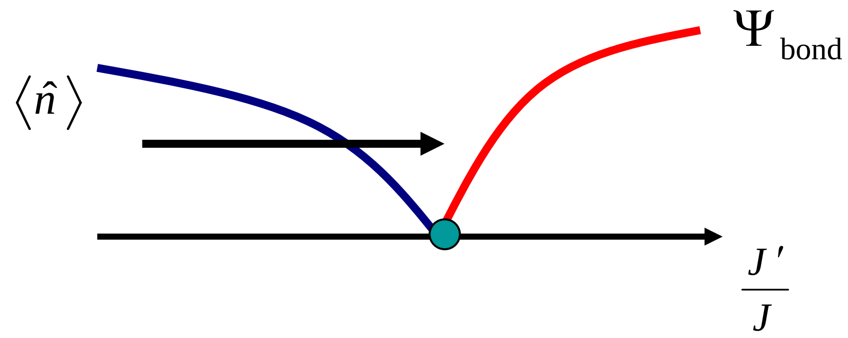


- Or:



Not Always True for Quantum Phase Transitions

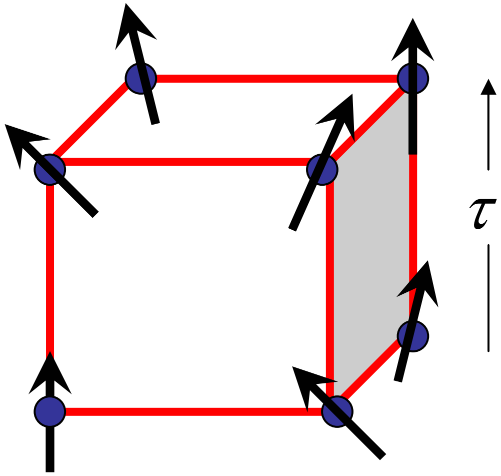
- Continuous transition directly between Neel and VBS is a generic possibility
  - **no** fine tuning required.
  - `Landau forbidden' transition
  - But Phases Conventional



- Critical point **not** described by fluctuations of the order parameters.
  - Theory of the critical point:
    - emergent `photons'
    - fractionalized excitations
    - Topological 'order'.

Describing Quantum Magnets

- Focus on the local Neel vector $\hat{n}$ (*F. D. M. Haldane 1988*).
- Semiclassical limit – spacetime configurations that dominate the partition function.



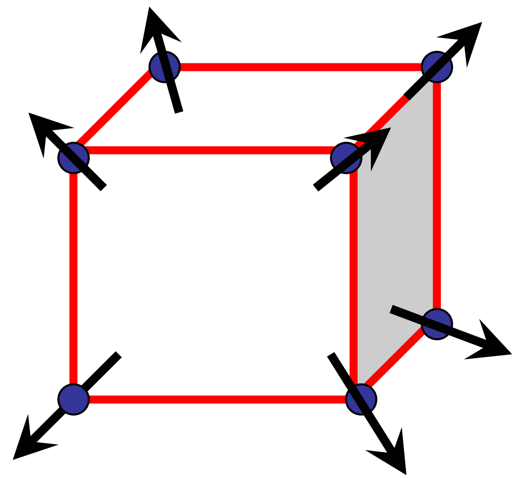
Neel Field in
spacetime

•Neel ordered phase - $\hat{n}$ ordered.
•Valence Bond Solid Phase? Proliferate *hedgehog* defects.
•Berry phase of hedgehogs leads to VBS (Read and Sachdev)

A hedgehog

Describing Quantum Magnets

$\hat{n}$



$$N_H = +1$$

Unit hedgehog – an instanton

- Neel ordered phase - $\hat{n}$ ordered.
- Valence Bond Solid Phase? Proliferate *hedgehog* defects.
    - Berry phase of hedgehogs leads to VBS (Read and Sachdev)

Hedgehog Free Model

Transition: destroy Neel order + proliferate hedgehogs.

Is it possible to disorder spins without hedgehog defects?

Study a classical 3D hedgehog free model – phase structure?

- Ordered phase.

AND

- A Disordered Phase Exists (Motrunich & A.V.)
- **P*** Phase contains `hidden order'
  - photon like excitations.
  - Spin ½ neutral excitations. (quantum number fractionalization)
- New Transition.(Motrunich & A.V., Kamal and Murthy)

Emergent Photons I

- No Hedgehogs + Spin Disorder => LIGHT

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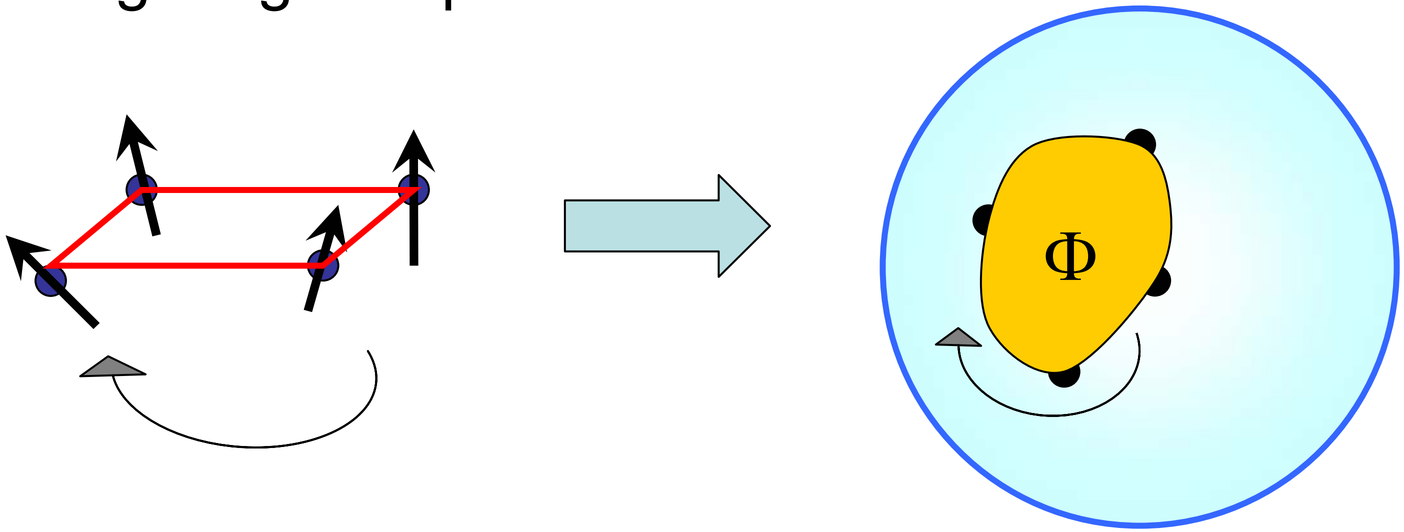


- Associate 'Flux' Φ with each plaquette
- Absence of Hedgehogs = Flux conserved.
  - Obtain Maxwell Electrodynamics
- Prediction: Flux Correlations in disordered phase long ranged. Set by Free Photon. (verified in Motrunich & A.V., 2004)

Fractionalization

Excitations with quantum numbers fundamentally different from electron.

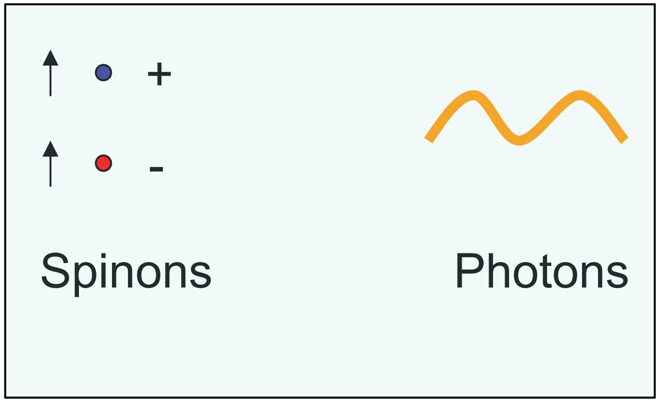
Eg. Spin 1/2, neutral particle.

- In 2D always accompanied by deconfined gauge fields
  - several models of spins that show this physics. (eg. A. Kitaev)

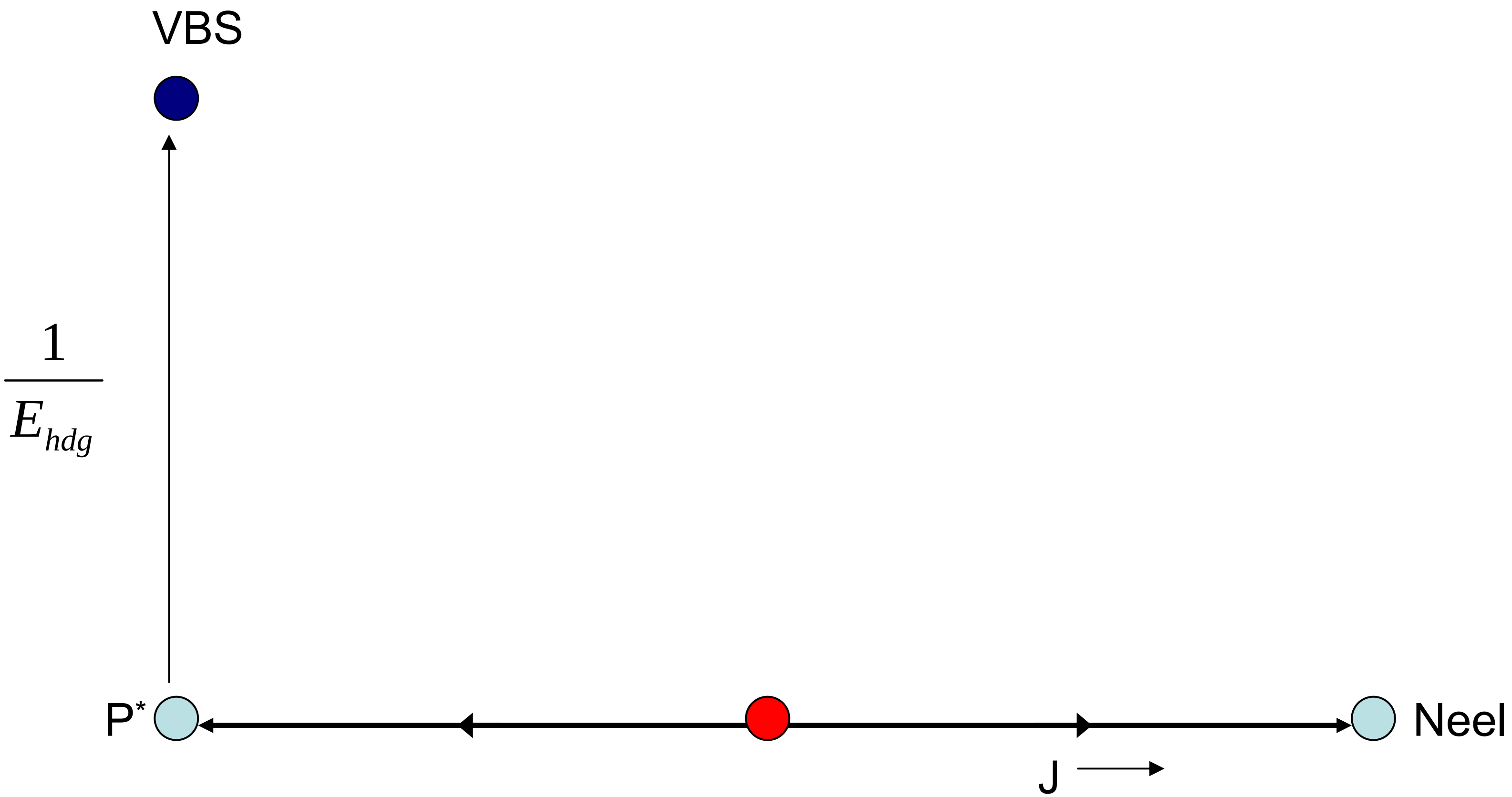
- Here, weakly interacting spin ½ excitations. Carry gauge charge.

$$z = (z_1, z_2)$$ Spin ½ fields

$$\hat{n} = z^{+}\sigma z$$ Neel vector

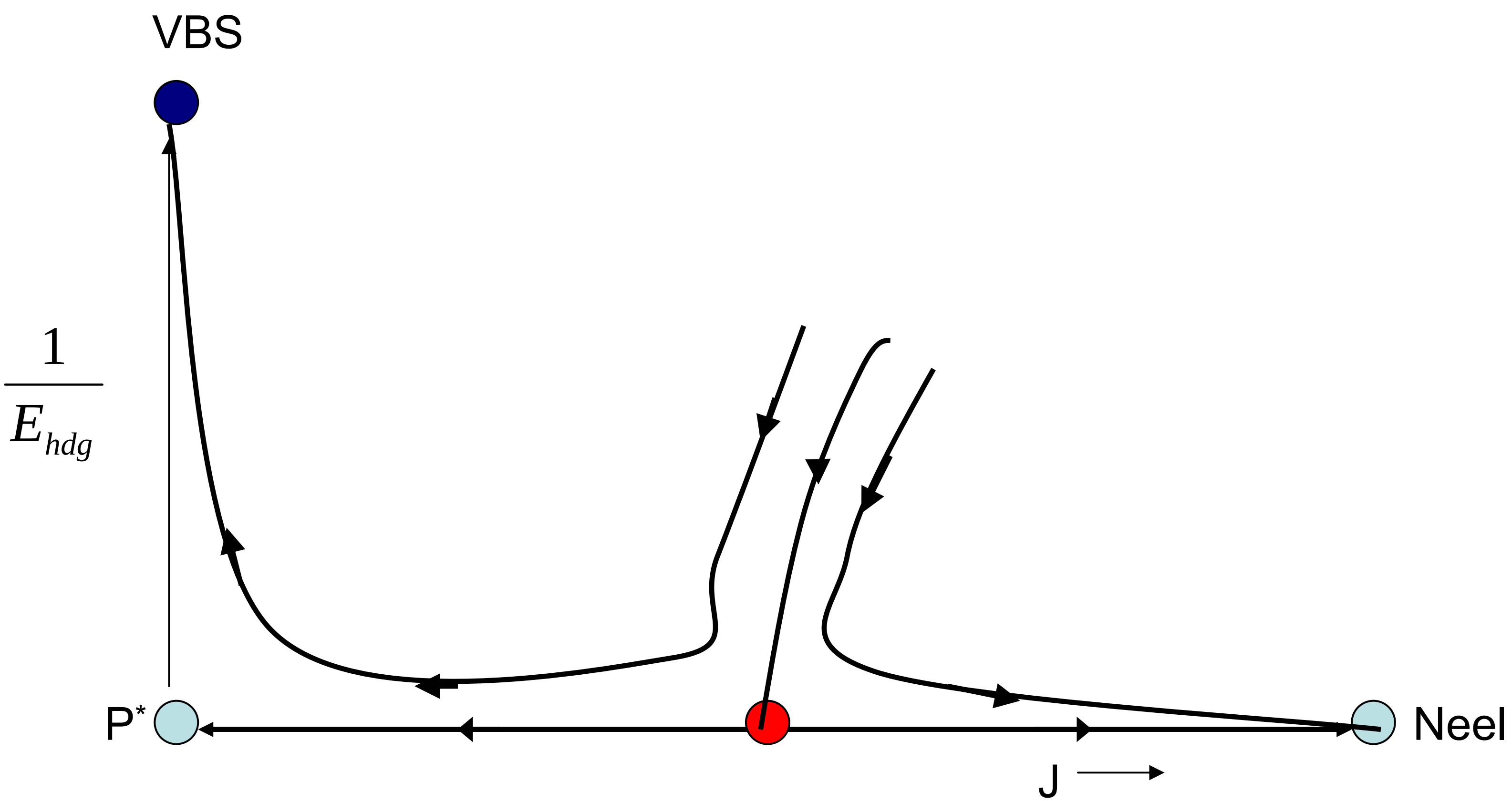


Reintroducing Hedgehogs



Hedgehogs relevant in the disodered phase – leads to VBS, BUT…

Reintroducing Hedgehogs



(Quadrupled) Hedgehogs Irrelevant at Transition. T. Senthil et al.

Properties of the Transition

- Absence of Hedgehogs at the transition.
  - No four-fold anisotropy in VBS distributions.
  - Conserved flux from topology.
- `Deconfined Spinons'

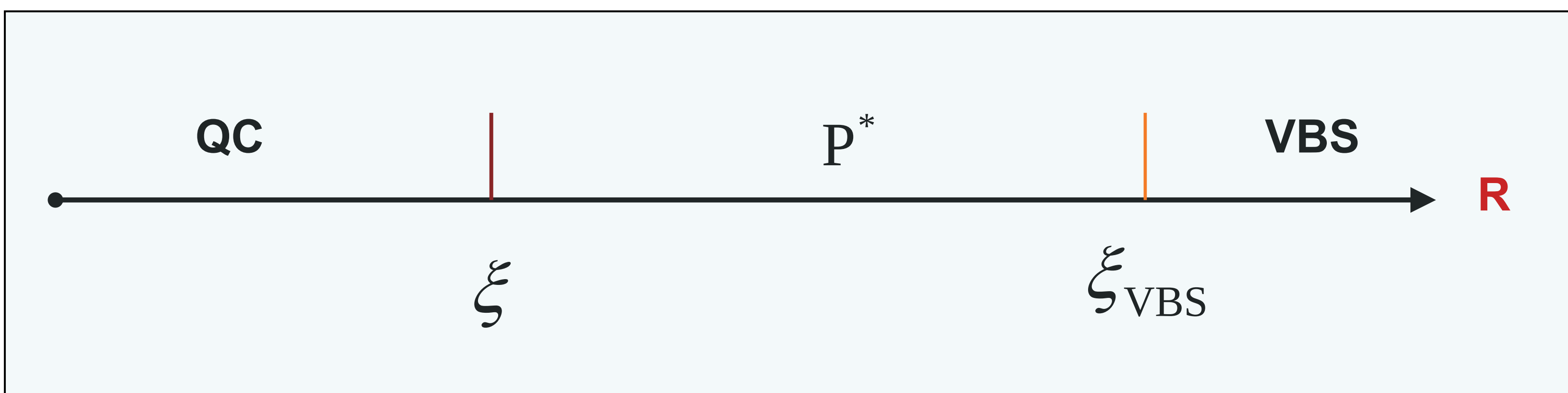
Scaling Dim of Neel field with $[\hat{n}] = \mathrm{L}^{-\frac{1+\eta}{2}}$ η=0.4-0.6 (large!

Compare η(Heisenberg)=0.03)

Because $\hat{n} = z^{+}\vec{\sigma}z$

For free spinons => η=1.0

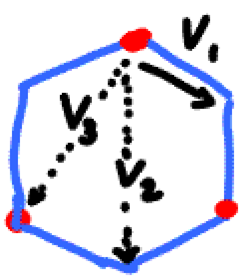
- Two diverging Scales



Microscopic Models of Deconfined Criticality

- Requirements
  - ❑ Model H(x), with $x = 0$ (Phase 1) and $x = \infty$ (Phase 2).
  - ❑ Deconfined fixed point theory -> phases 1 and 2 {minus fluctuations}.
  - ❑ No sign problem.
  - ❑ Preferably, new critical theory {beyond $NCCP_1$}.

- Honeycomb-lattice with charging energy,
  - SF to Insulator transition (C. Xu, A.V. in prep.).



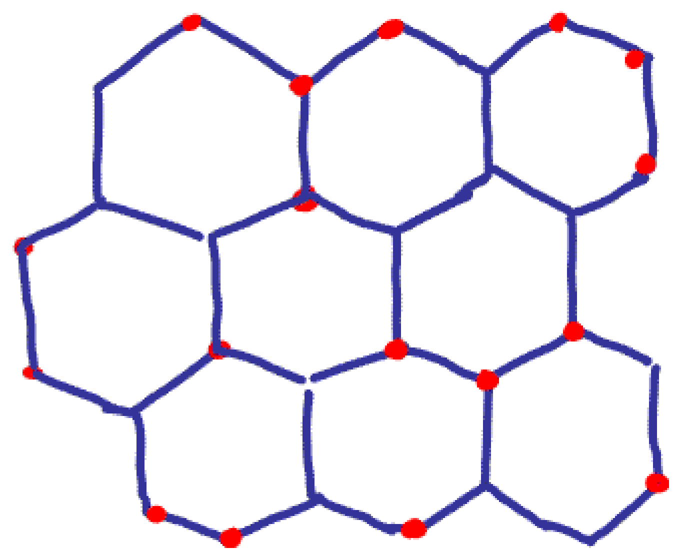
$$H_0 = U \sum \left[\sum_{\hexagon} (n_i - 1/2)\right]^2 - t \sum_{\langle ij \rangle} b_i^\dagger b_j + h.c.$$

$$\frac{1}{2} V_1 = V_2 = V_3$$

Honeycomb Lattice Model with Charging Term I

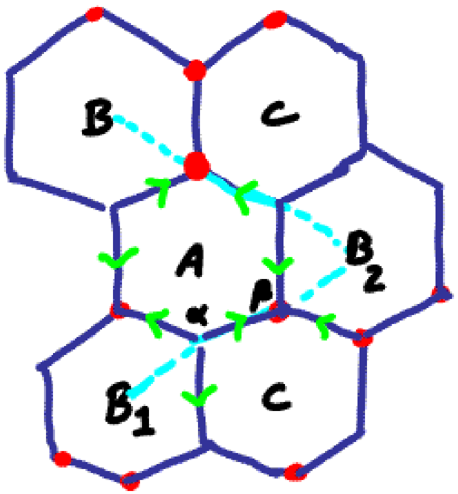
- Insulator
  - U large - many degenerate ground states (hard core bosons)

$$H_0 = U \sum \left[\sum_{\hexagon} (n_i - 1/2) \right]^2$$



Honeycomb Lattice Model with Charging Term II

- Insulator
  - Quantum Fluctuations lift degeneracy – is a uniform (liquid) phase possible? [No]
  - Field theory: Degenerate manifold -> complex height field ψ. Dual to compact U(1)xU(1) gauge field.
  - Instantons lead to confinement (ψ smooth) - always symmetry broken solid states.



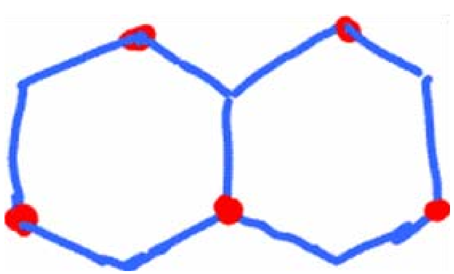
$$\Psi_{B_2} = \Psi_{B_1} + (n_\alpha - \tfrac{1}{2}) + (n_\beta - \tfrac{1}{2})$$

etc.

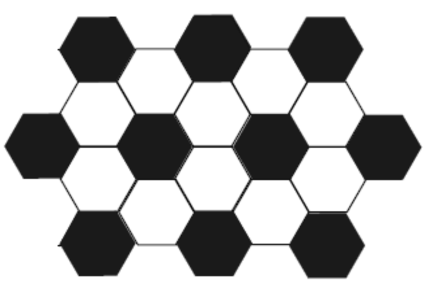
or $\Psi_A + \Psi_B + \Psi_C = \pm(n - \tfrac{1}{2})$ Δ, ∇

slow fields

$$\Psi_R = \mathrm{Re}\left[\psi\, e^{i\frac{4\pi}{3}R_x}\right]$$



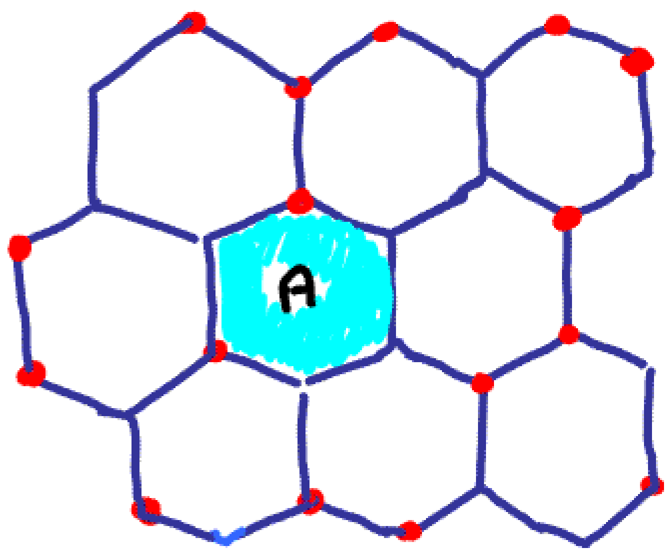
OR



Honeycomb Lattice Model with Charging Term III

- Insulator

  Charge Excitations: 1/3 charge *defect* hexagons, 3 flavors (A,B,C).



Field	Gauge Charge Q
Φ_A	$\frac{2}{\sqrt{3}}(0,1)$
Φ_B	$\frac{2}{\sqrt{3}}(\frac{\sqrt{3}}{2},+\frac{1}{2})$
Φ_C	$\frac{2}{\sqrt{3}}(\frac{\sqrt{3}}{2},-\frac{1}{2})$

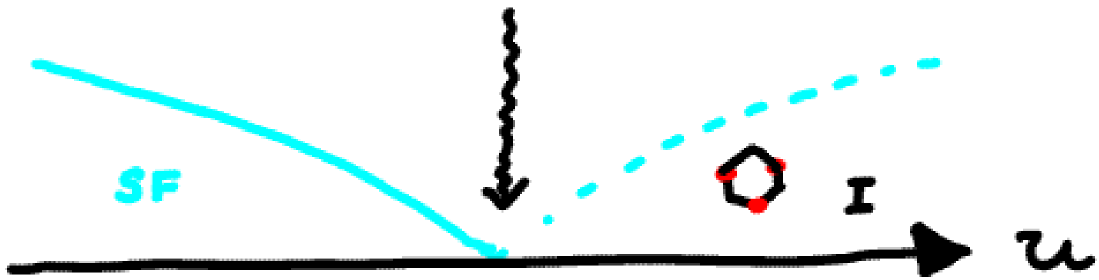
Generalized Dirac Condition (M: instanton charge): M.Q=2πn

Honeycomb Lattice Model with Charging Term V

- Insulator to superfluid transition

  Condensation of 1/3 charges.

$$\mathcal{L}_c = \sum_{\alpha=1}^{3} \left| (\nabla_\mu - i\, \vec{q}_\alpha \cdot \vec{a}_\mu)\, \Phi_\alpha \right|^2 + V_c(|\Phi_\alpha|^2) + \vec{F}_{\mu\nu} \cdot \vec{F}_{\mu\nu}$$



Effect of instantons at criticality and fluctuations?

Open Questions

- Deconfined criticality in D=3?