

Quantum Magnetism in Optical Lattices: coherent spin nematics of cold atoms

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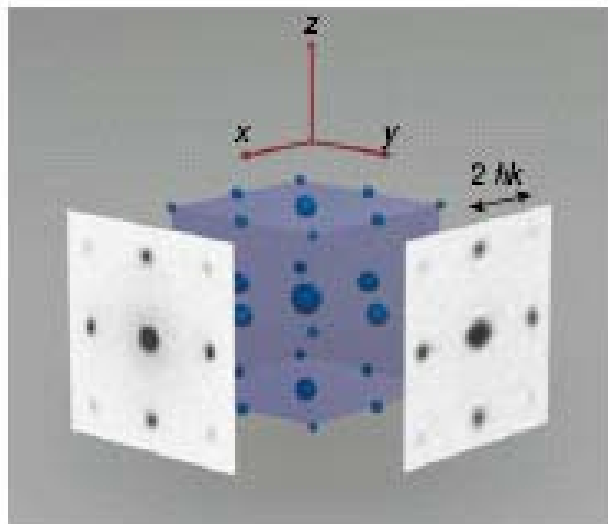
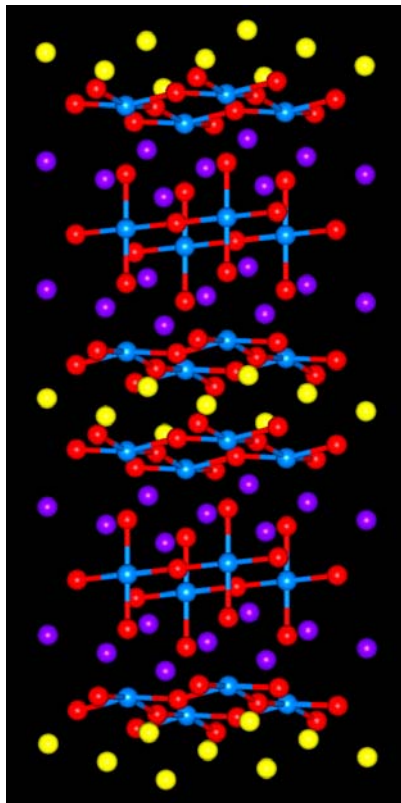
Collaborators:

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Wu Ming Liu, Andrew Ji (CAS, Beijing)

\$: Office of the Dean of the faculty of Science, UBC
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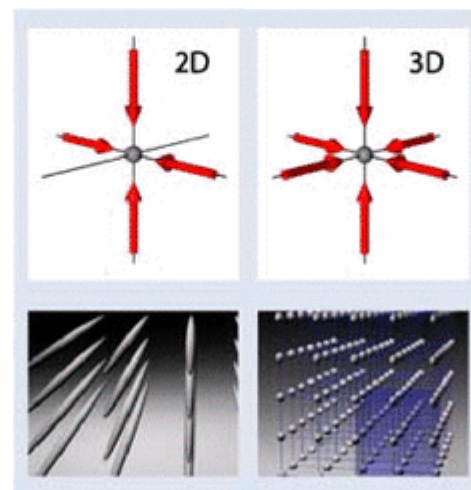
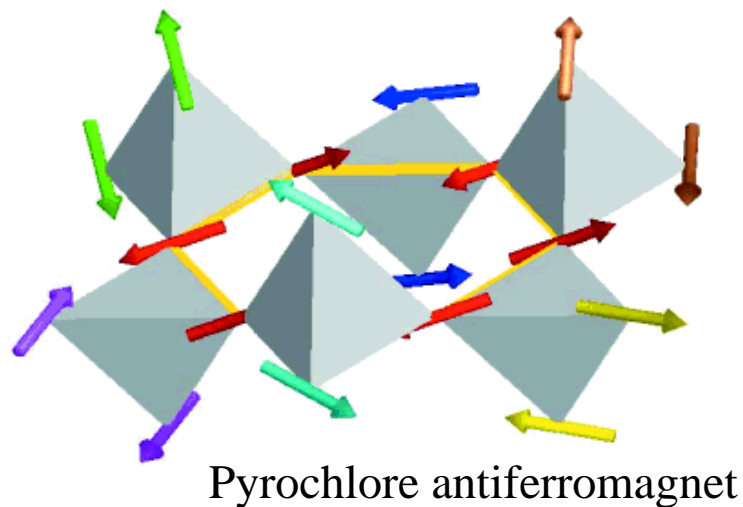
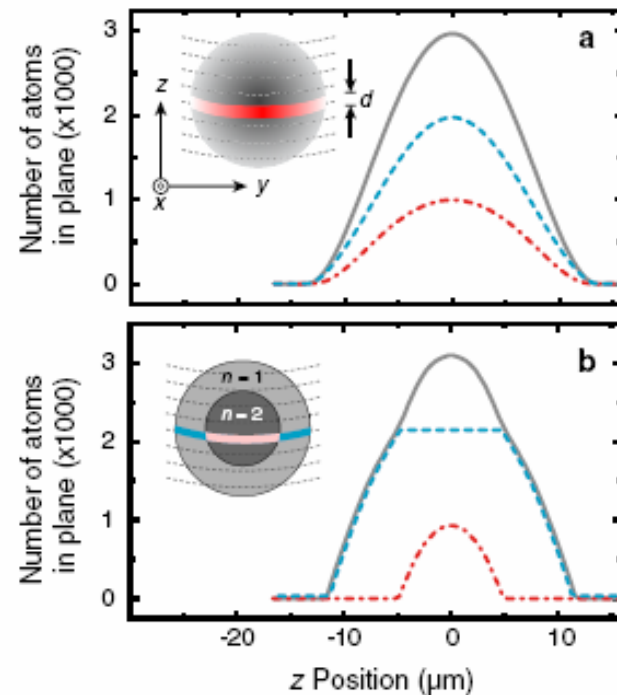
Cuprates

Quantum Magnetism and Optical lattices



M. Greiner et al, 02

Shell structure, Folling et al (Bloch's group), 2006.

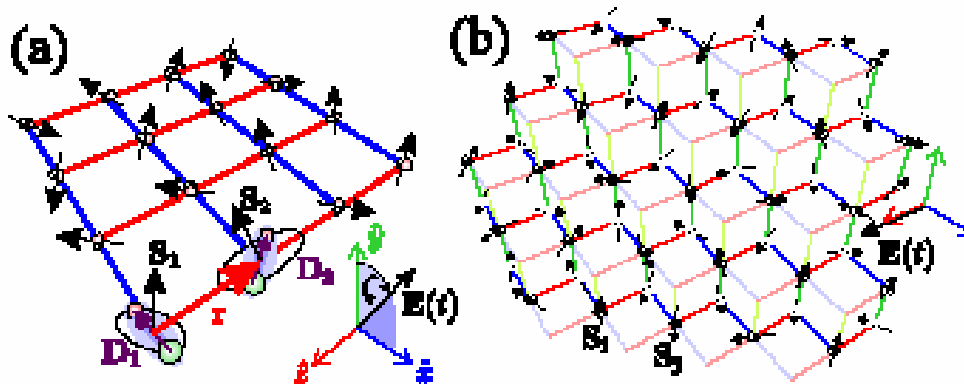


Quantum magnetism in optical lattices (a personal list)

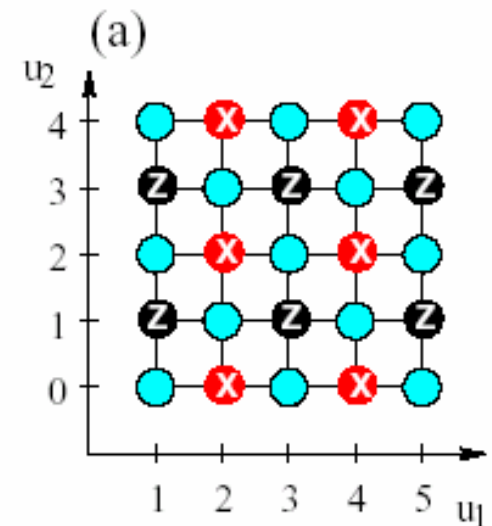
3) Unique opportunities of spin correlated cold atoms in optical lattices (or traps)

- Novel condensates and Mott states of atoms with high hyperfine spins;
- 1D spin correlations, general 1D physics;
- Various quantum phase transitions; exchange driven or field-driven transitions;
- Coherent dynamics and the role of quantum fluctuations.....

Micheli et al (Zoller's group), 05 on cold molecules



Raussendorf et al, 05; Kitaev code by measurements of resource states



Hyperfine spins of cold atoms

${}^6\text{Li}, \dots$ $F=1/2$ (Hulet at Rice, Ketterle at MIT, Salomon at ENS, Grimmer at Innsbruck...)

${}^{23}\text{Na}, {}^{87}\text{Rb}, F=1$ (MIT, Cornell-Wieman at JILA, Stamper-Kurn at Berkeley, Mainz ...)

${}^6\text{Li}, \dots$ $F=3/2$ manifold

${}^{23}\text{Na}, {}^{87}\text{Rb}, F=2$ manifold (Ketterle at MIT, Wieman at JILA, Bloch at Mainz...)

${}^{25}\text{Na}, {}^{85}\text{Rb}, F=2$ (Wieman et al,...)

${}^{40}\text{K}, F=9/2$ (Jin at JILA...)

${}^{52}\text{Cr}, F=3$ (Pfau at Stuttgart...)

Spinor gases: Stenger et al. (Ketterle's group), 98 on magnetic domains; Schmaljohann et al. (Sengstock's group, 04) and Chang et al (Chapman's group, 05) on spin mixing dynamics; Widera et al., 2005-2006 (Bloch's group) on two-body dynamics....

Theory on Spinor condensates, Ho, 98; Ciobanu, Yip, Ho, 00; Ohmi, Nachida, 98; Law, Pu and Bigelow, 00, Ueda and Koashi, 02....

Two-body S-wave scattering lengths in different channels (in atomic units; a.u.=0.529Å)

	a_0	a_2	a_4	Spins of 2-body g.s.	
^{23}Na (F=1)	50.0 (1.6)	55.0 (1.7)	N/A	F=0	*
^{87}Rb (F=1)	105.8 (0.6)	105.0 (0.6)	N/A	F=2	**
^{23}Na (F=2)	34.9 (1.0)	45.8 (1.1)	64.5 (1.3)	F=0	**
^{85}Rb (F=2)	-740 (60)	-570 (50)	-390 (20)	F=0	**
^{87}Rb (F=2)	88.8 (1)	94.8 (1)	103.6 (1)	F=0	**

* Crubellier et al., 1999.

** Data from Roberts (Wieman's group), 1998; Klause et al (Chris Greene's group), 2001.

*** Data from Courteille et al, 98; Wynar (Heinzen's group), 2000; Kempen et al (Verhaar's group), 2002 are not shown here.

What kind of spin correlated ultra cold matter can be formed in optical lattices?

Our recent results on correlated hyperfine spin-2 atoms:
 tensor rep. (Song, Semenov and FZ, 06-07) vs column vector rep.

$$|\alpha\beta\rangle = n_\alpha(\theta, \varphi) n_\beta(\theta, \varphi) - \frac{1}{3} \delta_{\alpha\beta} \Leftrightarrow |2, m_F\rangle = Y_{2, m_F}(\theta, \phi)$$

$$\Psi_{\alpha\beta}^+ \Leftrightarrow \Psi_{m_F}^+$$

Ex:

$$|zz\rangle = n_z n_z - \frac{1}{3} \Leftrightarrow |2, 0\rangle;$$

$$|xx\rangle - |yy\rangle = n_x n_x - n_y n_y \Leftrightarrow |2, -2\rangle + |2, 2\rangle$$

$$\begin{pmatrix} \chi_{xx} & & \\ & \chi_{yy} & \\ & & \chi_{zz} \end{pmatrix} \Leftrightarrow \begin{pmatrix} \chi_{xx} - \chi_{yy} \\ 0 \\ 2\chi_{zz} - \sqrt{2}(\chi_{xx} - \chi_{yy}) \\ 0 \\ \chi_{xx} - \chi_{yy} \end{pmatrix}$$

- 1) Represent an arbitrary state using a traceless symmetric tensor; the matrix elements give the amplitude in a state alpha-beta defined above;
- 2) Convenient for the construction of rotational invariant or invariant operators;
- 3) Plot spin wavefunctions in terms of spherical coordinates and study manifold symmetries.

(Vector Rep. used by Ho et al, 00)

Spin order

$$\text{Cyclic: } \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \omega = \exp(i\frac{2\pi}{3}); \quad \text{nematic: } \begin{pmatrix} \sin(\xi - \frac{\pi}{6}) & 0 & 0 \\ 0 & \sin(\xi - \frac{5\pi}{6}) & 0 \\ 0 & 0 & \sin(\xi - \frac{9\pi}{6}) \end{pmatrix}.$$

Ex nematics:

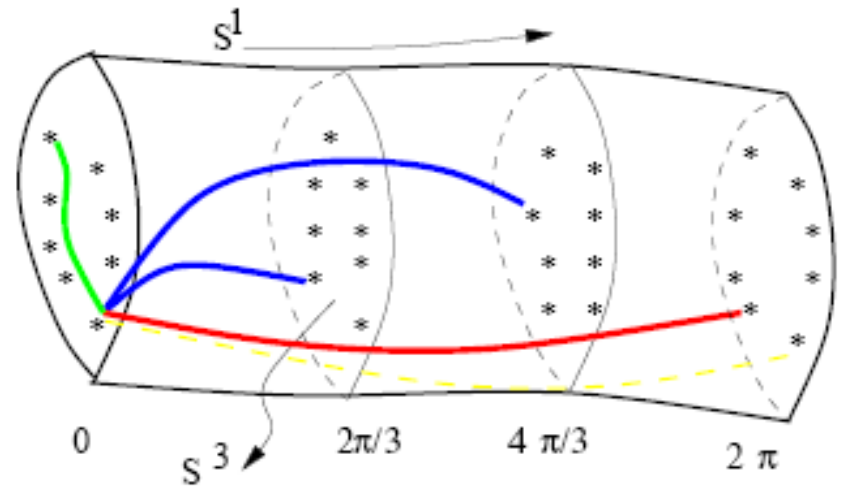
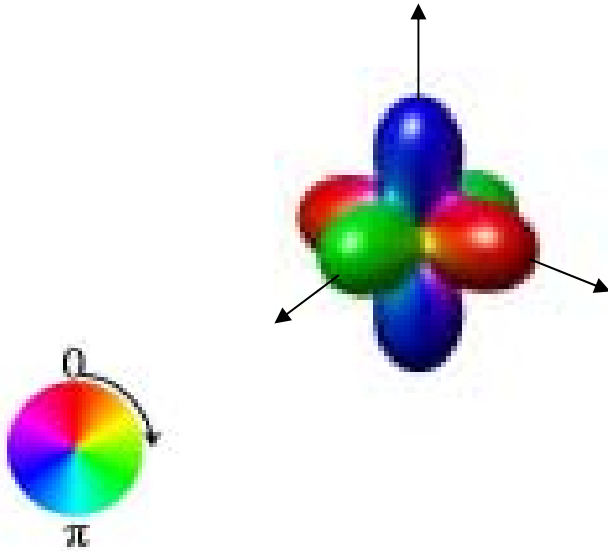
$$\xi = 0, \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \Leftrightarrow |2,0\rangle, \quad \xi = \frac{\pi}{2}, \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} \Leftrightarrow |2,2\rangle + |2,-2\rangle$$

There are three spin ordered phases: Ferromagnetic, cyclic and nematic.

Left up is for cyclic states and Right is for Nematic (up to SO(3) and U(1) rotations).

Mott states recently discussed by Zhou and Semenoff, 06 ; Barnett, Turner and Demler., 06.

1/3-quantum vortices in cyclic condensates



Schematic of the manifold
(Semenoff and Zhou, PRL, 07)

Invariant subgroup:

a) I, and 180° rotation around x,y,z;

b) 120° rotation around (1,1,1), (-1,1,1), (-1, -1, 1) and (1,-1, 1)

accompanied by a phase shift 120° ;

c) 240° degree rotation around (1,1,1), (-1,1,1), (-1,-1,1) and (1,-1, 1)

accompanied by a phase shift 240° .

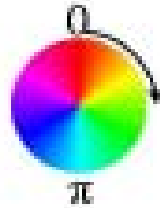
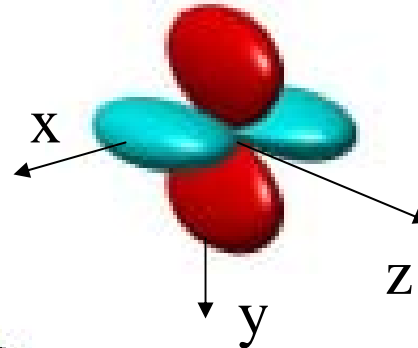
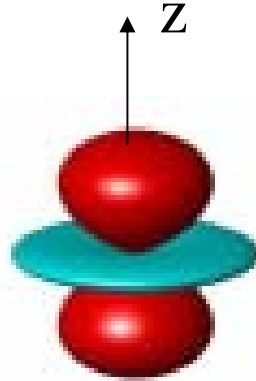
Uniaxial versus biaxial

$$\xi = 0,$$

$$3z^2 - 1,$$

$$|2,0\rangle;$$

$$M = \frac{S^2}{Z_2} \times S^1$$



$$\xi = \pi/2,$$

$$x^2 - y^2,$$

$$|2,2\rangle + |2,-2\rangle;$$

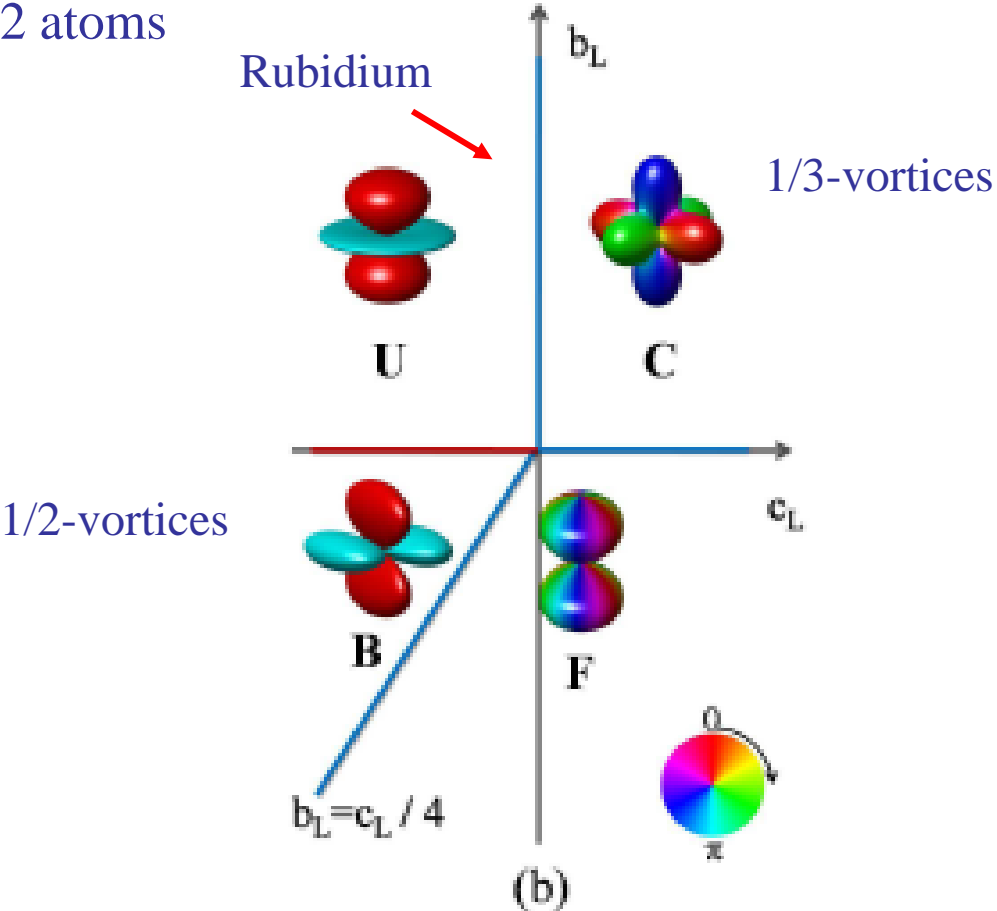
$$M = \frac{SO(3) \times S^1}{Dih4}$$

Invariant under any rotation around the z-axis

invariant under a dihedral-4 group:

- 1) 90, 180, 270 rotations around (0,0,1);
- 2) 180 rotations around (1,0,0), (0,1,0), (1,1,0), (1,-1,0) plus a pi phase shift.

Condensates of F=2 atoms

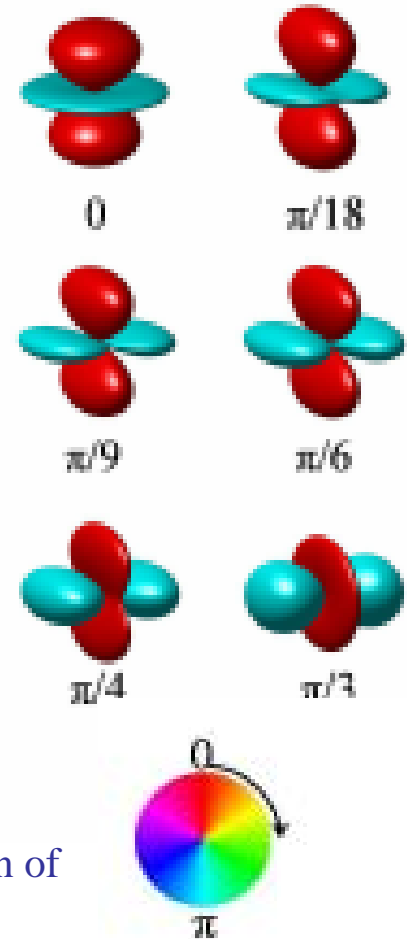
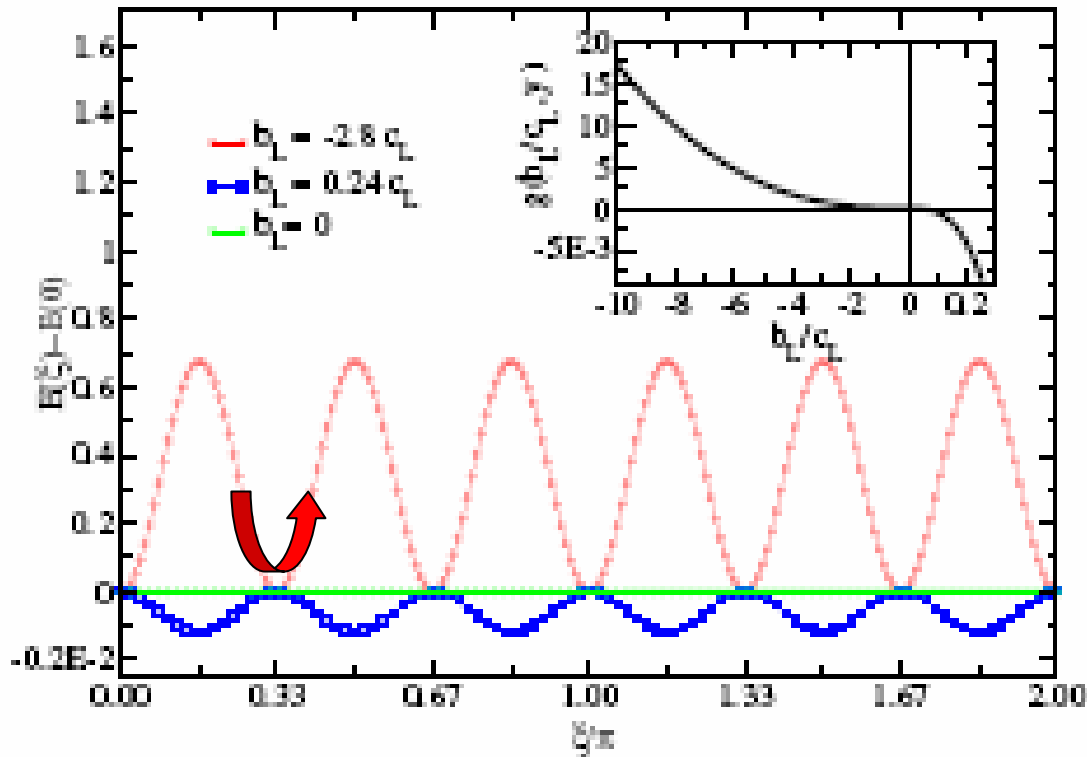


$$b_L = -\frac{1}{7}(a_2 - a_4), \quad c_L = \frac{1}{5}(a_0 - a_4) - \frac{2}{7}(a_2 - a_4)$$

Spin wavefunctions are plotted in spherical coordinates.

U: uniaxial nematic; B: Biaxial nematics; C: cyclic; F:ferromagnetic;
 Song, GWS and FZ, 07; Turner, Barnett, Demler and Vishwanath,07.

Quantum-Fluctuation Induced Nematic Order

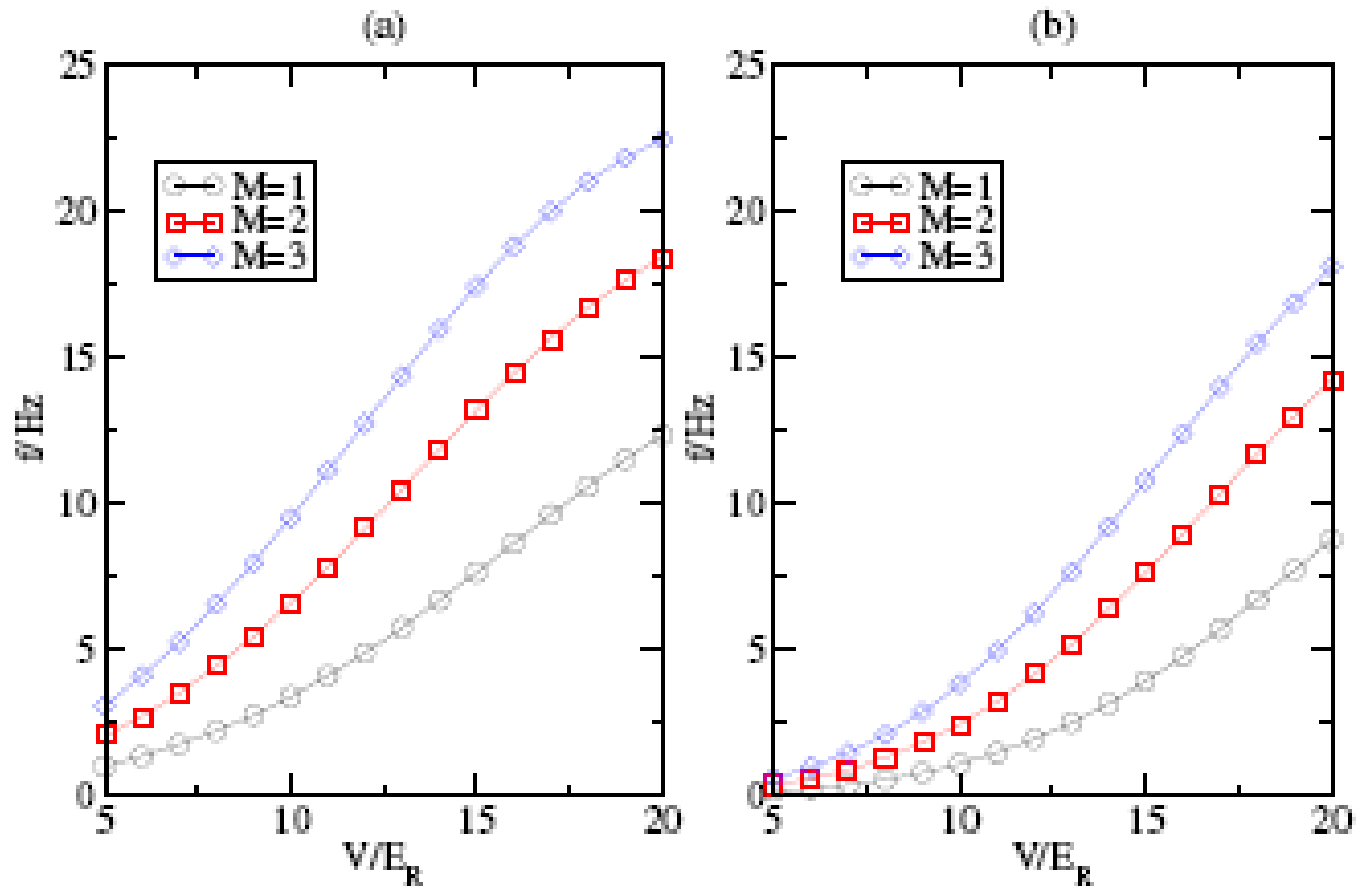


Energy as a function of a parameter which specifies a spin nematic solution of a highly degenerate family.

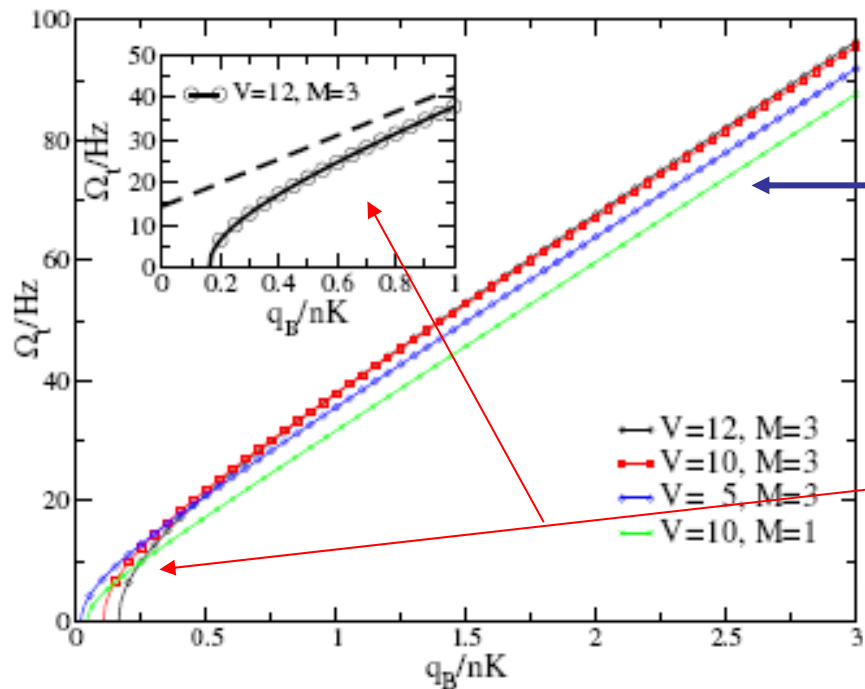
Related phenomena in other fields:

- Shender (82) –Henley(89)’s order from disorder physics in AF,
- Halperin-Lubensky-Ma’s fluctuation-induced 1st order transitions, 74;
- Coleman & Weinberg symmetry breaking via radiative corrections, 72

Mapping potential via Quantum-fluctuation controlled spin dynamics (QFCSD) ?>>>>>>



Oscillation (around uniaxial nematics) frequency versus optical potential depth V



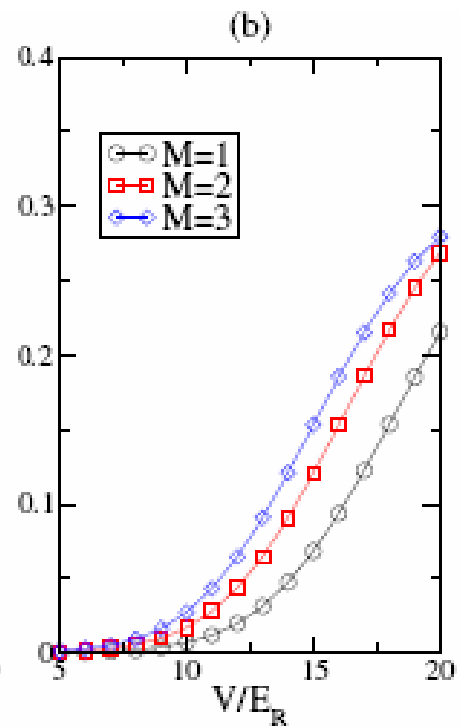
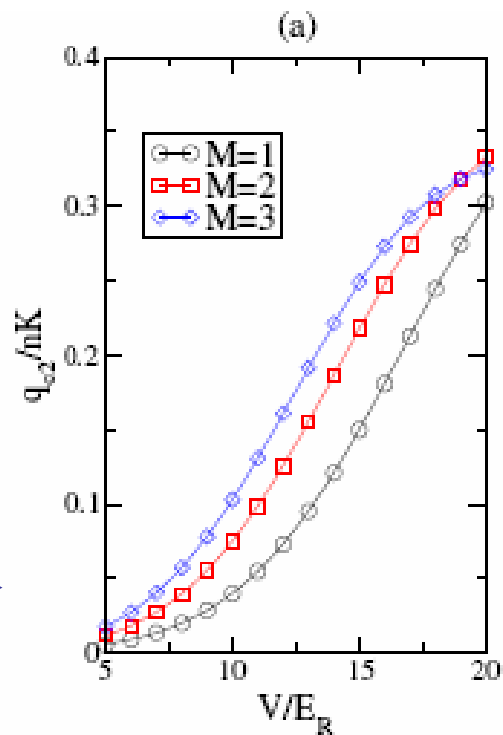
MF dynamics

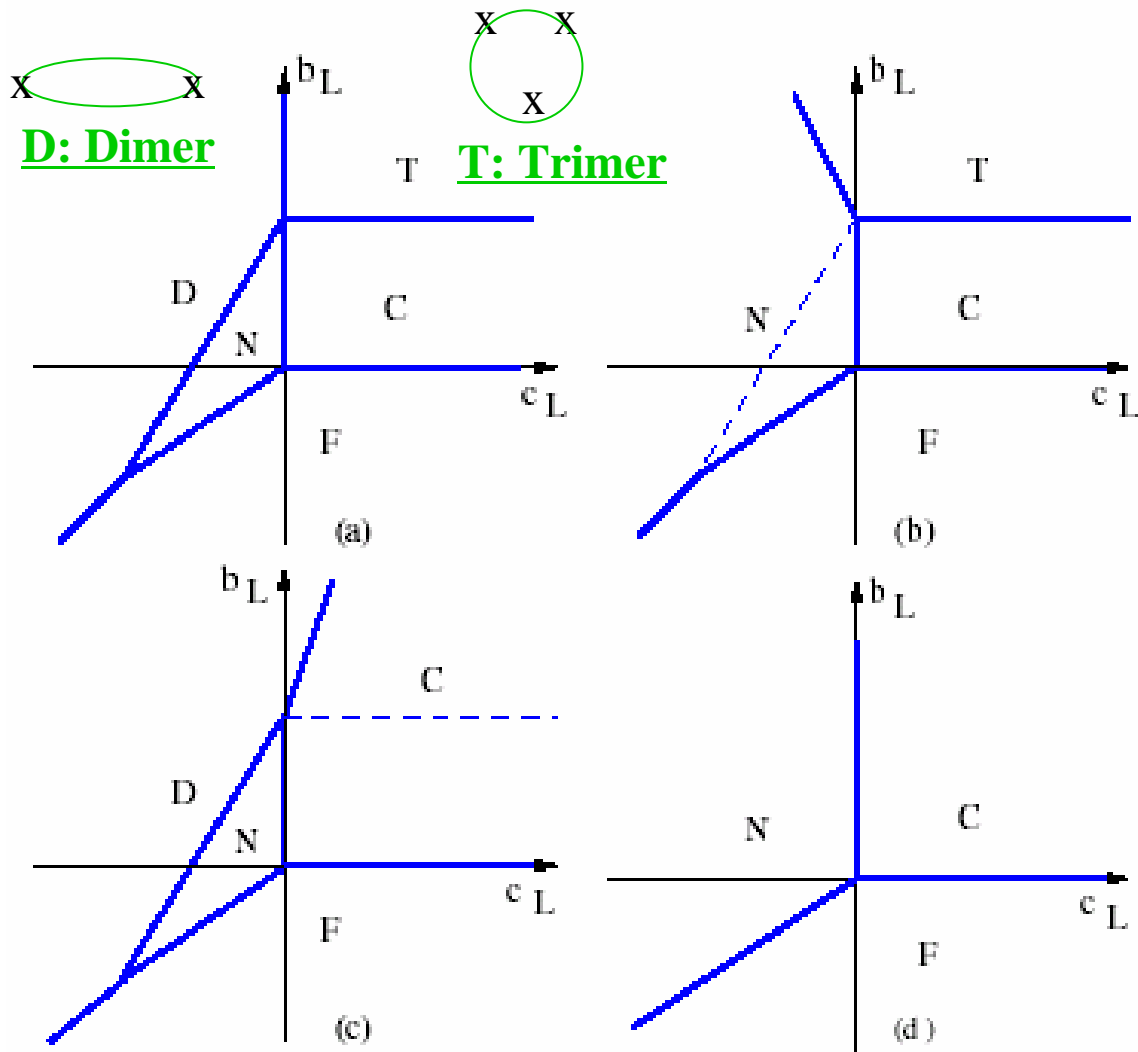
QFCSD

QFCSD within 100ms

Frequency versus quadratic Zeeman
(around biaxial point)

Also threshold versus potential depth V →





Mott states of F=2 atoms:

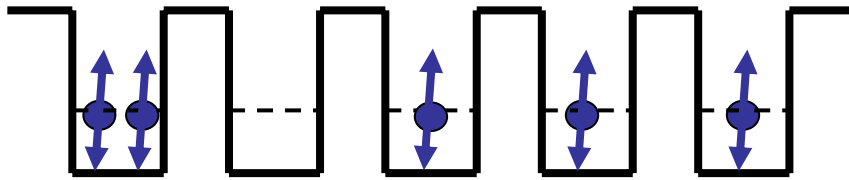
N: Nematic state; C: Cyclic state; F: ferromagnetic

(a) 6 atoms per site; (b) 3 particles per site; (c) 2 particles per site; (d) 1 particle

$$b_L = -\frac{1}{7}(a_2 - a_4), \quad c_L = \frac{1}{5}(a_0 - a_4) - \frac{2}{7}(a_2 - a_4)$$

(Zhou and Semenoff, PRL, 2006)

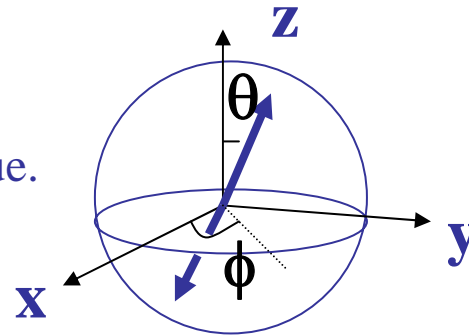
Earlier results on spin-nematic condensates of $F=1$ Na atoms



Spin projection along the director has zero eigenvalue.
Each condensate is specified by the director.

$$U_F(r_1 - r_2) = \delta(r_1 - r_2) g_F,$$

$$g_F = \frac{4\pi\hbar a_F}{M}, \quad \underline{a_2} > a_0, \quad F = 0, 2.$$



Directors live on a two-sphere; however
Inversion of the director leaves the state
invariant up to a pi-phase.

Spin Nematics, Half-vortices (HV) and 1D superfluids, Zhou, 01;

Optical lattices, Demler and Zhou, 02; Immabekov, Lukin, Demler, 03; Snoek, Zhou, 03.

1d Dimerized valence bonds versus activated Ising gauge fields, Zhou and Snoek, 03...

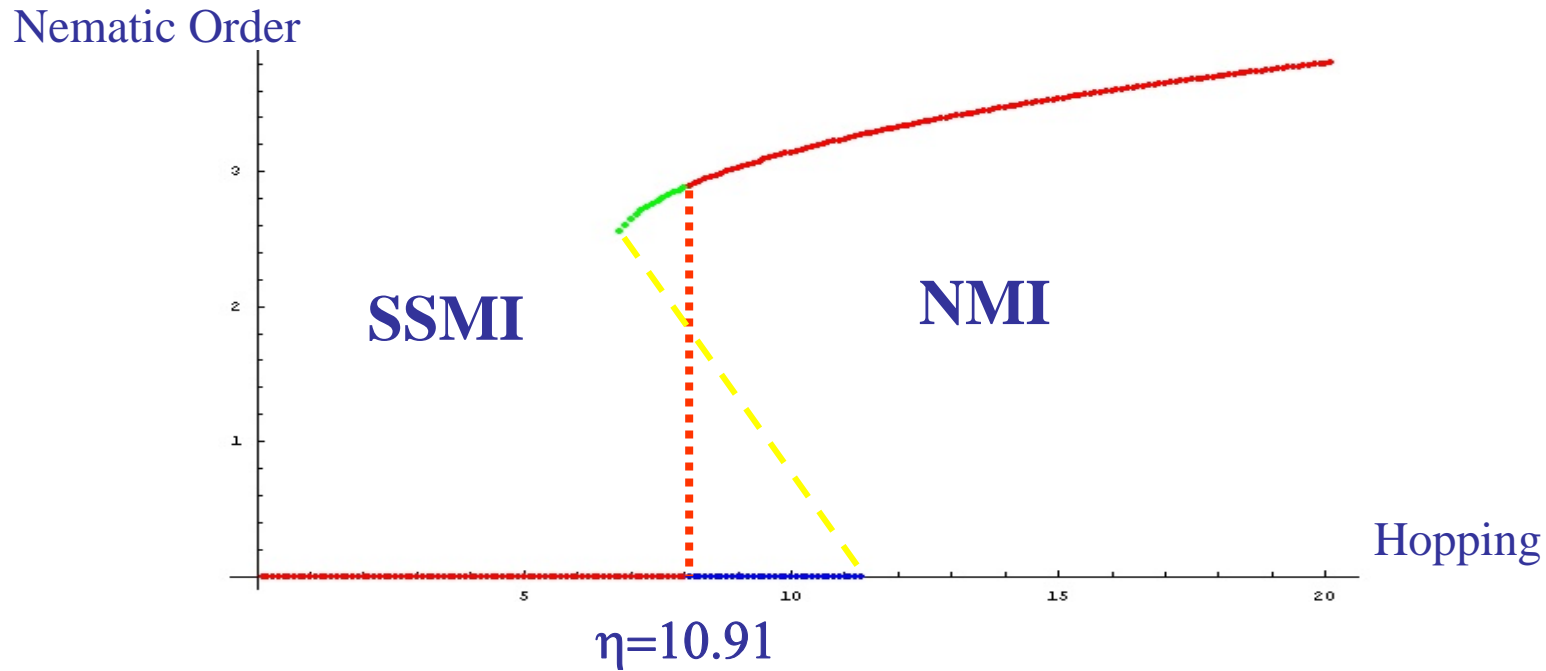
Other works:

Fermion spin-3/2 superfluids, Wu, Hu and Zhang, 03;

KT transitions and half vortices, Mukerjee, Xu and Moore, 06;

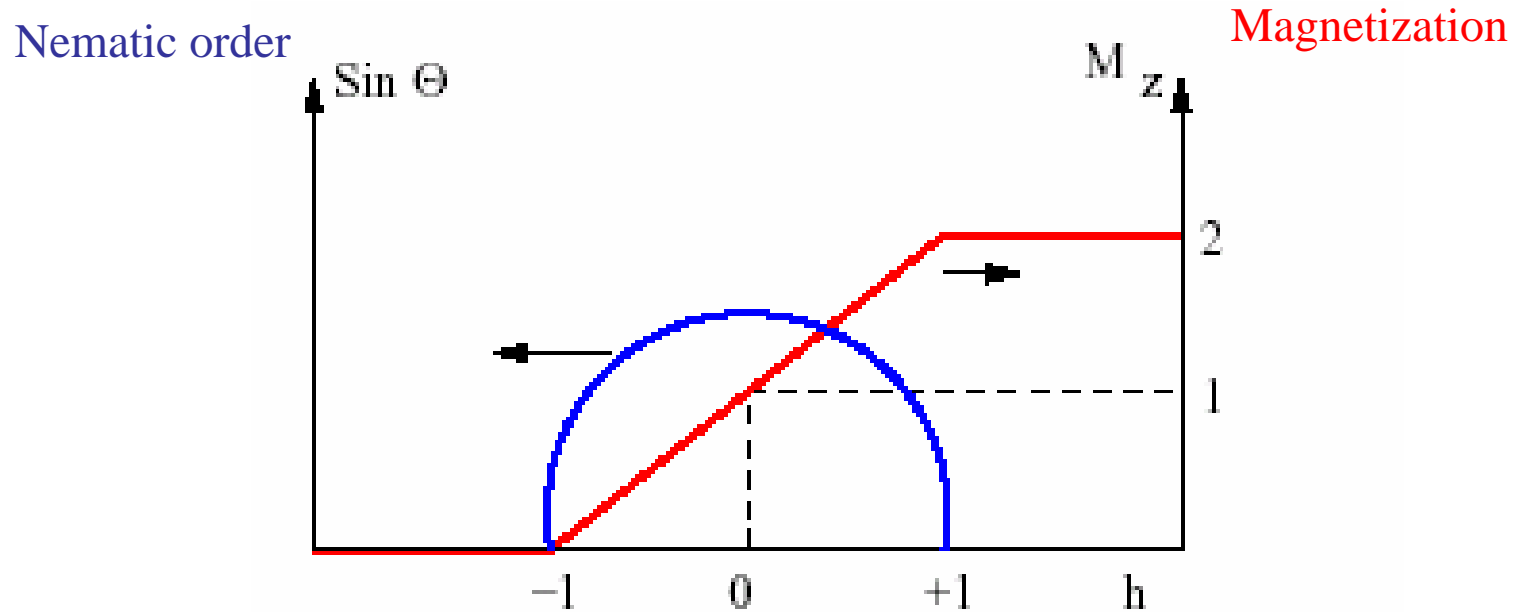
Condensates of chromium atoms, Diener and Ho, 05, Barnett et al (Demler' group), 06.....

Hopping-driven Spin singlet-Nematic Mott state transition



Snoek and Zhou, 03; also Immabekov, Demler and Lukin, 03.

Field-induced Nematic states (2-atoms per site)



Critical exponents and universality in 1D (free fermion) and 3D (free boson),
Wiemer, Snoek, Zhai, Affleck and FZ, 2005-2006.

Conclusions

- Quantum spin nematics have been suggested to be ground states of sodium and rubidium atoms in optical lattices.
- There are fractionalized vortices in spin correlated condensates.
- Quantum fluctuations select nematics with higher symmetries (either uniaxial or biaxial but with dihedral-four symmetries). QF controlled spin dynamics is a novel class of phenomena to be studied in the future.
- Optical lattices is a new platform to study the exciting subject of quantum magnetism. not only will lead to better understanding of how spin correlated cold atoms organize themselves at nk temperatures but also might shed light on traditional CM subjects such as HTC, fractionalized liquids, and topological quantum computation etc.