Spin-orbit coupling effects in solid state and cold atomic systems

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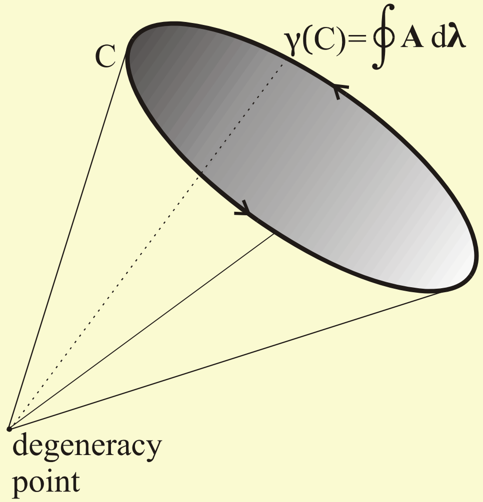
Outline

- Introduction/Motivation for studying spin transport
- Edge (Tamm) states and spin accumulation in clean systems
- Disordered spin-orbit coupled systems
  - Bulk spin diffusion equation
  - Unusual boundary conditions
- Effective spin-orbit coupling in atomic systems
  - An atomic system in spatially varying laser fields
  - Strongly non-equilibrium spin dynamics in a trapped system

Berry phase effects in multiband systems



- Standard Berry's phase picture: Adiabatic evolution of a system described by Hamiltonian $\widehat{\mathcal{H}}(\boldsymbol{\lambda})$ in the $\boldsymbol{\lambda}$-parameter space.

$$|\psi\rangle \longrightarrow e^{i\gamma(C)}|\psi\rangle$$

Berry's phase: $\gamma(C) = \oint_C \mathbf{A} d\boldsymbol{\lambda} = \int_S \mathbf{B}(\boldsymbol{\lambda}) d\mathbf{S}$

Degeneracies in the spectrum are sources of the "magnetic field."

- Example: Electron Bloch states in a crystal, $|u_n(\mathbf{p})\rangle$. Band-crossings are degeneracy points, which produce a non-trivial "gauge field."

$$\mathbf{A}_n(\mathbf{p}) = i\left\langle u_n(\mathbf{p}) \left| \frac{\partial}{\partial \mathbf{p}} u_n(\mathbf{p}) \right. \right\rangle$$

Anomalous velocity

- Classical Eqs. of motion for an electron in a multiband system [Karplus & Luttinger (50's), Sundaram & Niu (99), Haldane (04)]

$$\dot{\mathbf{r}} = \frac{\partial \mathcal{E}_n(\mathbf{p})}{\partial \mathbf{p}} + \mathbf{B}^{(n)}_{\text{dual}} \times \dot{\mathbf{p}}$$

$$\dot{\mathbf{p}} = -e\mathbf{E} + e\mathbf{B} \times \dot{\mathbf{r}}$$

Interesting generalizations: (i) Non-Abelian fields in systems with degenerate bands [Murakami *et al.* (2003)]); (ii) Dual "electric field" in interacting electron systems [Shindou & Balents, cond-mat/0603089].

Important observation: All is needed to produce an anomalous velocity is a degeneracy in the band structure. Periodic potential is *not* necessary.

Degeneracies in spin-orbit coupled systems

- Isotropic Luttinger model (3D p-doped semiconductors)

$$\hat{\mathcal{H}} = \frac{1}{2m}\left[(1+2\gamma)\,p^2 - 2\gamma\left(\mathbf{p}\cdot\hat{\mathbf{J}}\right)^2\right]$$

- Spin-orbit coupling in a 2DEG

$$\hat{\mathcal{H}} = \frac{p^2}{2m} + \mathbf{h}_{\mathbf{p}}\cdot\hat{\boldsymbol{\sigma}}$$

  - Rashba interaction

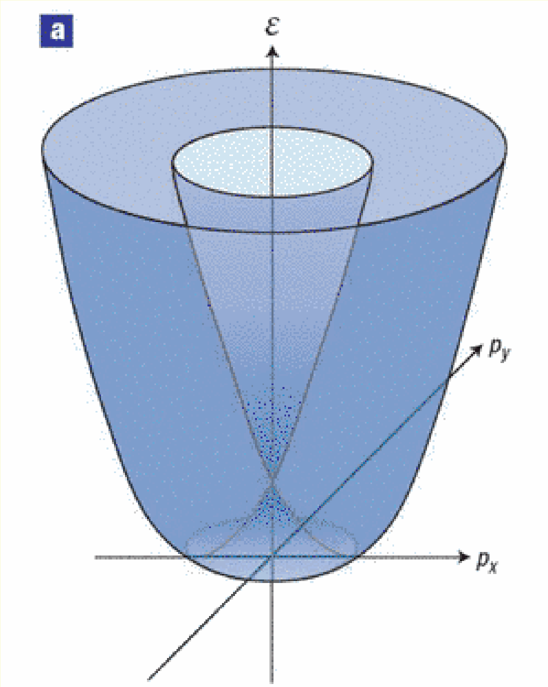
$$\mathbf{h}_{\mathbf{p}} = \alpha\mathbf{p}\times\hat{\mathbf{z}}$$

  - Linear Dresselhaus interaction

$$\mathbf{h}_{\mathbf{p}} = -\beta_1(p_x,\,-p_y)$$

  - Cubic Dresselhaus term

$$\mathbf{h}_{\mathbf{p}} = \beta_3\left(p_x p_y^2,\,-p_y p_x^2\right)$$



Possible relation between the Berry's phase and spin transport

- Model (Murakami *et al.*, 2003): Spin-orbit coupled system, *e.g.*, described by Luttinger model, in an external electric field, $\mathbf{E}$.

- Anomalous velocity:

$$\mathbf{v}_{\text{an}}^{(\lambda)} = \mathbf{B}_{\text{dual}}^{(\lambda)} \times \dot{\mathbf{p}} \propto \frac{\mathbf{p}}{p^3} \times \mathbf{E}, \quad \lambda \text{ is the band index (H- and L-holes)}$$

- Dual field: $\mathbf{B}_{\text{dual}} = \nabla_{\mathbf{p}} \times \mathbf{A}(\mathbf{p})$ and $\mathbf{A}(\mathbf{p}) = i\text{diag}\left[\hat{U}^{\dagger}(\mathbf{p})\partial_{\mathbf{p}}\hat{U}\right]$

  $\hat{U}(\mathbf{p})$ is the matrix, which diagonalizes the Luttinger Hamiltonian.

- Spin current:

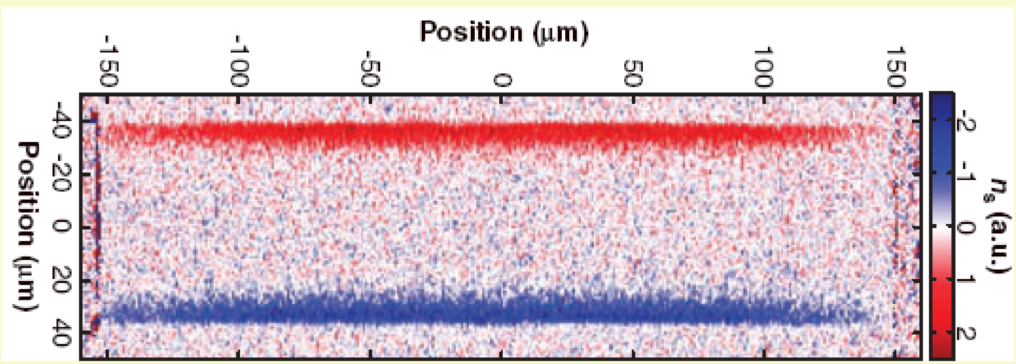
$$\mathbf{J}_{\alpha}^{(s)} = \frac{1}{2}\langle \sigma_\alpha \mathbf{v}_{\text{an}} + \mathbf{v}_{\text{an}}\sigma_\alpha \rangle \propto \sum_{\lambda=\text{bands}} \int_{\mathbf{p}} \langle \lambda|\sigma_\alpha|\lambda\rangle\, \mathbf{v}_{\text{an}}^{(\lambda)}$$

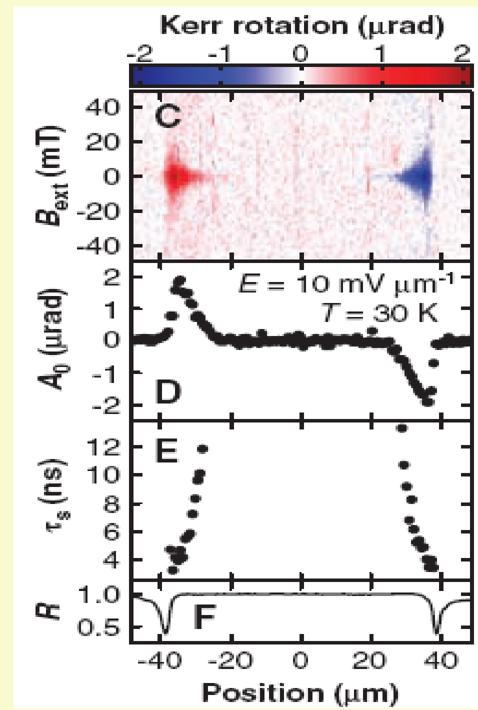
<u>Result:</u> There is a spin-current flowing perpendicular to $\mathbf{E}$.

Conjecture: This spin current leads to spin accumulation at the edges.

Electric-field-induced spin accumulation

Experiment: A thin film of GaAs or InGaAS in the presence of an external electric field.

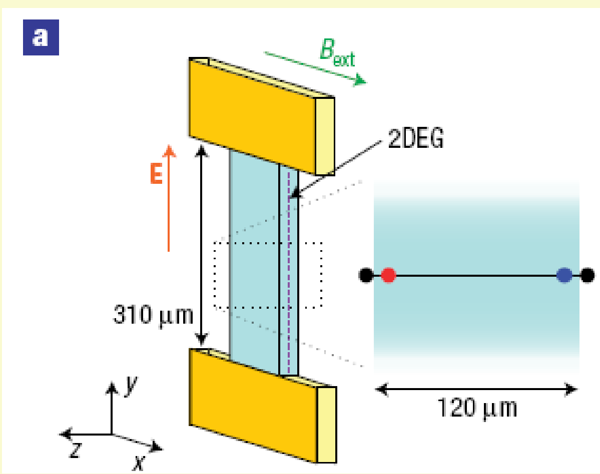


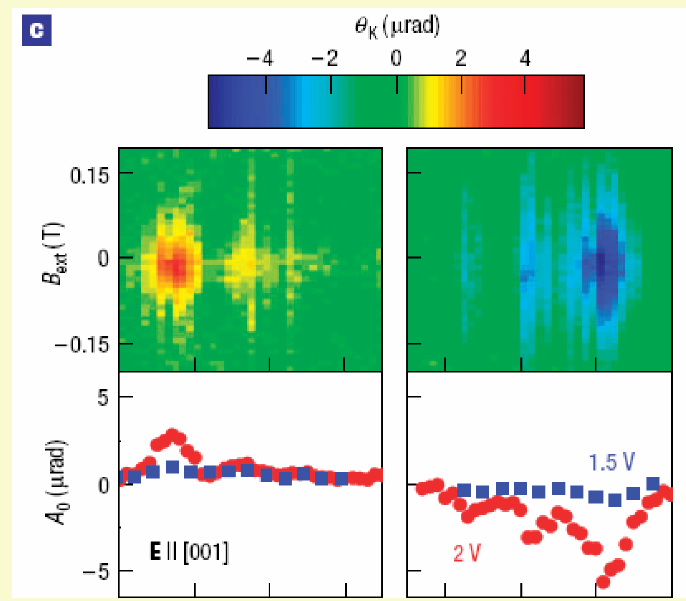


Ref.: Y.K. Kato *et al.*, Science **306**, 1910 (2004)

Electric-field-induced spin accumulation

Experiment: A two-dimensional electron gas in the presence of an external electric field





Ref.: V. Sih *et al.*, Nature Phys. **1**, 31 (2005)

Spin Hall effect: Experiment & Theory

There is a large gap between the experiment, which measures a spin density and the Berry's phase approach, which calculates spin currents. An important question is: What is the connection between them, if any?

Problems of the spin-current-based theory:

1. Spin is not conserved. No continuity equation

$$\frac{\partial S_\alpha}{\partial t} + \nabla \cdot \mathbf{J}_\alpha^{\mathsf{s}} \neq 0$$

2. There is no obvious relation between the spin-Hall current and observables.

3. The concept of Fermi surface Berry's phase can *not* be generalized to disordered systems.

Physics of spin accumulation in a clean

spin-orbit coupled system (Luttinger model)

T. Stanescu & VG [PRB 74, 205331 (2006)]

Refraction and "total internal reflection"

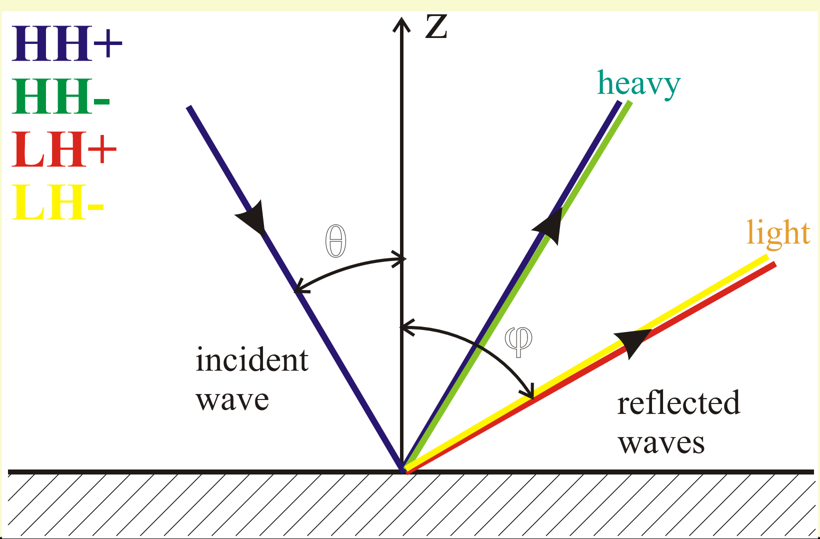
Single-particle QM problem: Luttinger model + hard-wall boundary:

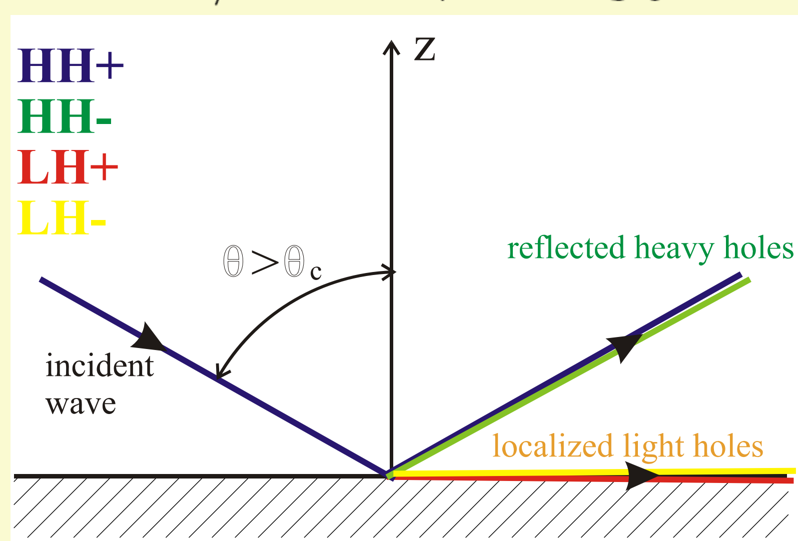
$$\hat{\mathcal{H}}\psi(\mathbf{r}) = \frac{1}{2m}\left[-\left(1+\frac{5}{2}\gamma\right)\nabla^2 + 2\gamma\left(i\hat{\mathbf{J}}\cdot\nabla\right)^2\right]\psi(\mathbf{r}) = E\psi(\mathbf{r}), \quad z \geq 0$$

$\hat{J}$ is a spin-3/2 operator (4×4 matrix). ψ satisfies boundary condition

$$\psi(z=0) = 0$$

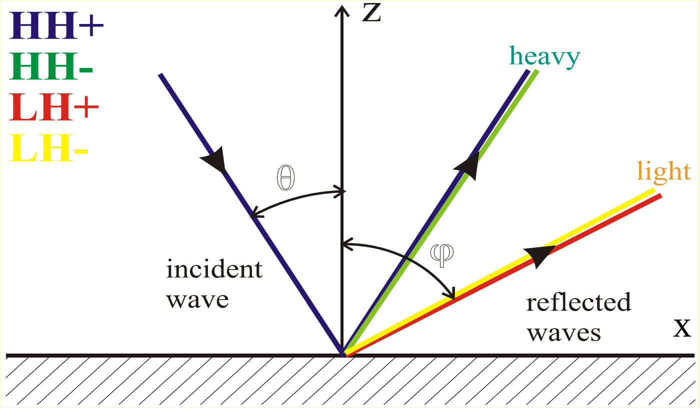
Classification of states in the bulk: Two double degenerate bands, heavy and light holes with chiralities $\pm 3/2$ and $\pm 1/2$ correspondingly.

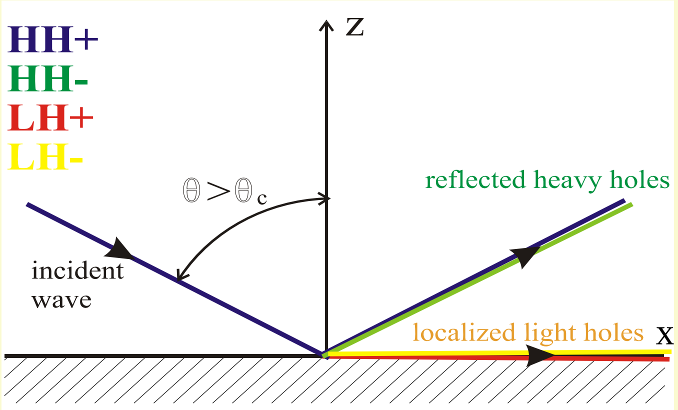




Spin-polarized edge states

$$\psi_{\mathbf{k}}(\mathbf{r}) = \frac{e^{ik_x x + ik_y y}}{\sqrt{C}}\Big[U_{\mathsf{H}}^{+}(\theta)e^{ik_z z} + A_1 U_{\mathsf{H}}^{+}(\pi-\theta)e^{-ik_z z} + A_2 U_{\mathsf{H}}^{-}(\pi-\theta)e^{-ik_z z} + B_1 U_{\mathsf{L}}^{+}(\pi-\phi)e^{-iq_z z} + B_2 U_{\mathsf{L}}^{-}(\pi-\phi)e^{-iq_z z}\Big]$$





- Case I ($\theta < \theta_{\mathsf{c}} = \arcsin\sqrt{\xi},\ \xi = \frac{m_{\mathsf{L}}}{m_{\mathsf{H}}}$)

$$q_z = k\cos\phi,\ \ \phi(\theta) = \arctan\frac{\sin\theta}{\sqrt{\xi - \sin^2\theta}}$$

- Case II ($\theta > \theta_{\mathsf{c}}$)

$$q_z = ik\sqrt{\sin^2\theta - \xi}$$

Important property:

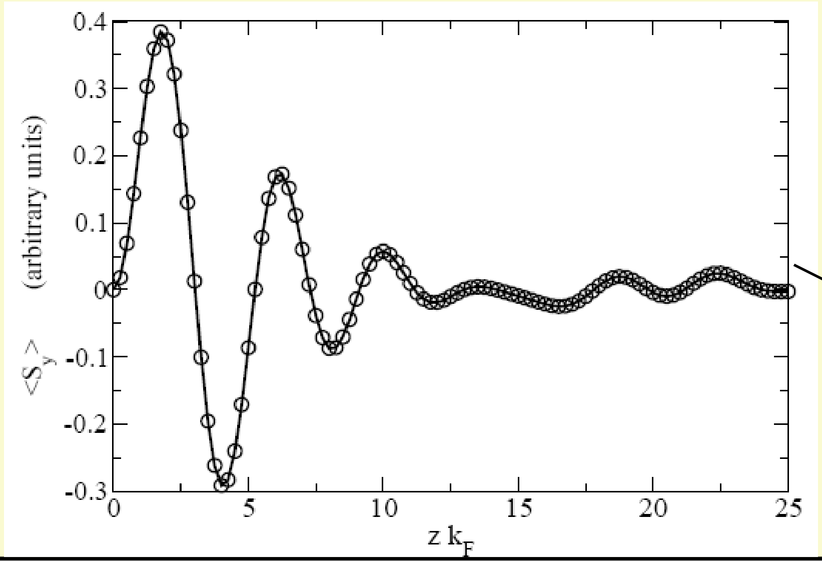
$$\langle \text{localized}+|\,\hat{J}_y\,|\text{localized}+\rangle = \langle \text{localized}-|\,\hat{J}_y\,|\text{localized}-\rangle \neq 0$$

Spin accumulation

It is possible to construct a basis parameterized by the heavy-hole momentum $\mathbf{k}$ and an index λ (certain mixture of chiralities).

$$\langle S_\alpha \rangle_{(\mathbf{r})} = \sum_\lambda \sum_{\mathbf{k}, k_z > 0} \langle S_\alpha \rangle_{(\lambda, \mathbf{k}; \mathbf{r})} F_{\mathbf{k}}(E_F, eV)$$

with $F_{\mathbf{k}}(E_F, eV) = \Theta(E_F + eV/2 - \frac{k^2}{2m_H}) - \Theta(E_F - eV/2 - \frac{k^2}{2m_H})$



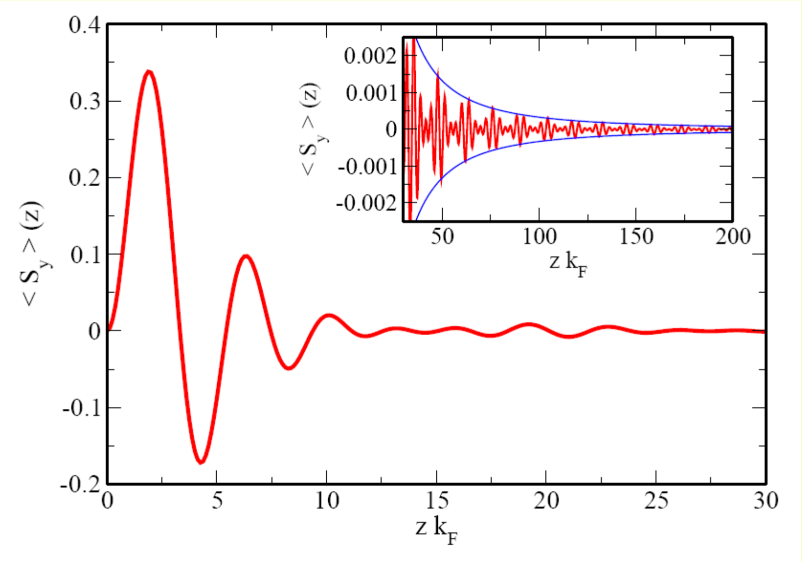
Spin Hall effect is electric-field-induced Friedel oscillations with property

$$S_{\text{tot}} = \int dz \langle S_y \rangle_{(\mathbf{r})} = \kappa E \neq 0$$

Spin density decays as a power law.

(Stanescu & VG, cond-mat/0606670)

Friedel "beatings"



Asymptotically exact result ($zk_F \gg 1$):

$$\langle S_y \rangle(z) = \frac{\Delta k}{8\pi^2}\,\frac{3}{2z^2}\,\left\{\sin\left(2k_F z\right) + \xi \sin\left(2k_F\sqrt{\xi}z\right) - \frac{8\sqrt{\xi}}{(1+\sqrt{\xi})^2}\sin\left[k_F(1+\sqrt{\xi})z\right]\right\}$$

$\xi = m_L/m_H$ is the ration between the light and heavy hole masses.

Disordered spin-coupled systems

Spin diffusion

Spin relaxation and diffusion approximation

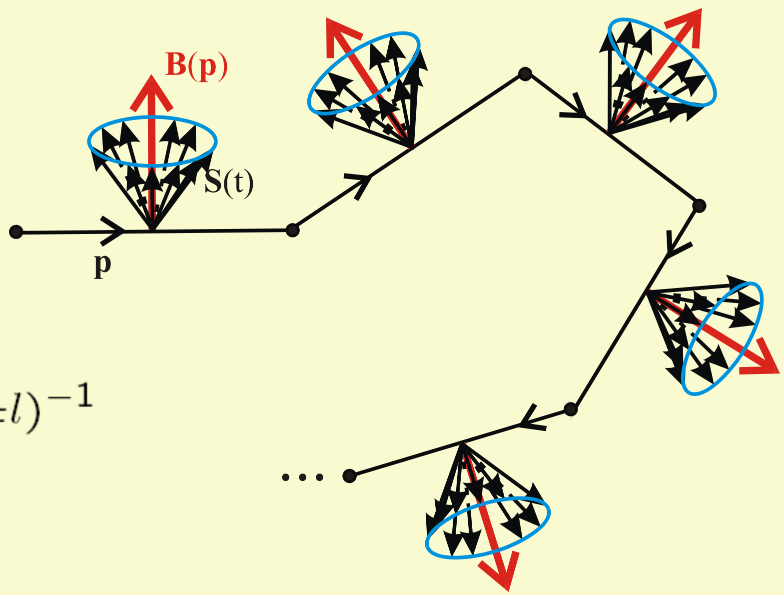
Hydrodynamic approximation holds if the distances over which the densities vary are much larger than the relevant equilibration lengths.

Diffusion equation for spin density

$$\dot{\rho_\alpha} = \mathcal{D}\Delta\rho_\alpha - \frac{1}{\tau_s}\rho_\alpha + \ldots, \quad \tau_s^{-1} \text{ is a spin relaxation rate}$$

"Required" hierarchy of length-scales $p_{\mathsf{F}}^{-1} \ll l \ll L_s = \sqrt{\mathcal{D}\tau_s}$

<u>Spin relaxation mechanism:</u>
Momentum-dependent spin dynamics; Diffusion randomizes momentum and leads to spin relaxation.



Diffusion Eq. is usually Ok if $\epsilon_{\mathsf{SO}}/E_{\mathsf{F}} \ll (p_{\mathsf{F}} l)^{-1}$

- Rashba & Dresselhaus models: YES
- Luttinger model: NO

Gradient expansion

Kernel of the diffusion equation

$$\Pi_{ab}(\omega;\mathbf{r},\mathbf{r}') = \frac{1}{4\pi\nu\tau}\mathrm{Tr}\left\{\hat{\sigma}_a\hat{G}^R(\varepsilon_1;\mathbf{r}-\mathbf{r}')\hat{\sigma}_b\hat{G}^A(\varepsilon_2;\mathbf{r}'-\mathbf{r})\right\}, \;\; \omega=\varepsilon_1-\varepsilon_2$$

Spin-charge diffusion equation (integral form):

$$\rho_a(\mathbf{r}) = \sum_b \int \Pi_{ab}(\mathbf{r},\mathbf{r}')\rho_b(\mathbf{r}')d^d\mathbf{r}'$$

ρ_0 is the charge density and $\rho_{x,y,z}$ are spin densities.

Green's functions are local, $\hat{G}(\mathbf{r}) \propto e^{-r/2l}$. Kernel is almost a δ-function:

$$\lim_{l\to 0}\Pi_{ab}(\omega;\mathbf{r},\mathbf{r}') = \left[1 + i\omega\tau + \mathcal{D}\tau\boldsymbol{\nabla}^2\right]\delta_{ab}\delta(\mathbf{r}-\mathbf{r}') + \sum_n \left(\frac{\epsilon_{\mathrm{SO}}}{E_{\mathrm{F}}}\right)^n \hat{D}_{ab}^{(n)}$$

Usual diffusion — spin-dependent corrections

Bulk spin diffusion equation

General form:

$$\left(\frac{\partial}{\partial t} - D_\alpha \boldsymbol{\nabla}^2\right)\rho_\alpha = P_{\alpha\beta\gamma}\nabla_\gamma \rho_\beta - \frac{1}{\tau_{\alpha\beta}}\rho_\beta + \mathbf{C}_\alpha \boldsymbol{\nabla}\rho_0$$

spin precession $\propto \epsilon_{so}^1$ — spin relaxation $\propto \epsilon_{so}^2$ — spin-charge coupling $\propto \epsilon_{so}^3$ (non-universal)

Uniform polarization far from the boundaries:

$$\rho_\beta^\infty = \tau_{\beta\alpha}\mathbf{C}_\alpha \mathbf{j} = \text{const}$$

Steady state equation for $\bar{\rho}_\beta(\mathbf{r}) = \rho_\beta(\mathbf{r}) - \rho_\beta^\infty$:

$$\left[\mathcal{D}_\alpha \delta_{\alpha\beta}\boldsymbol{\nabla}^2 + P_{\alpha\beta\gamma}\nabla_\gamma - \frac{1}{\tau_{\alpha\beta}}\right]\bar{\rho}_\beta(\mathbf{r}) = 0$$

Replacement $\boldsymbol{\nabla} \to i\mathbf{q}$ defines an eigenvalue problem for $\mathbf{q}$ (cubic equation).

Problem in a half-space: Eigenvalues and amplitudes

Consider a problem in a half-space with a boundary at $x = 0$.

Eigenvalues: $q = \pm k_1 + \frac{i}{\lambda_1}, \frac{i}{\lambda_2}$

General form of the solution

$$\bar{\rho}_\alpha(x) = A_\alpha \cos(k_1 x + \phi_\alpha)\, e^{-x/\lambda_1} + B_\alpha e^{-x/\lambda_2}$$

(2 amplitudes) $\times$ (3 spin components) $+$ (3 phase-shifts) $= 9$

Diffusion equation applied for two eigenvectors provides six relations. Three more relations are needed...

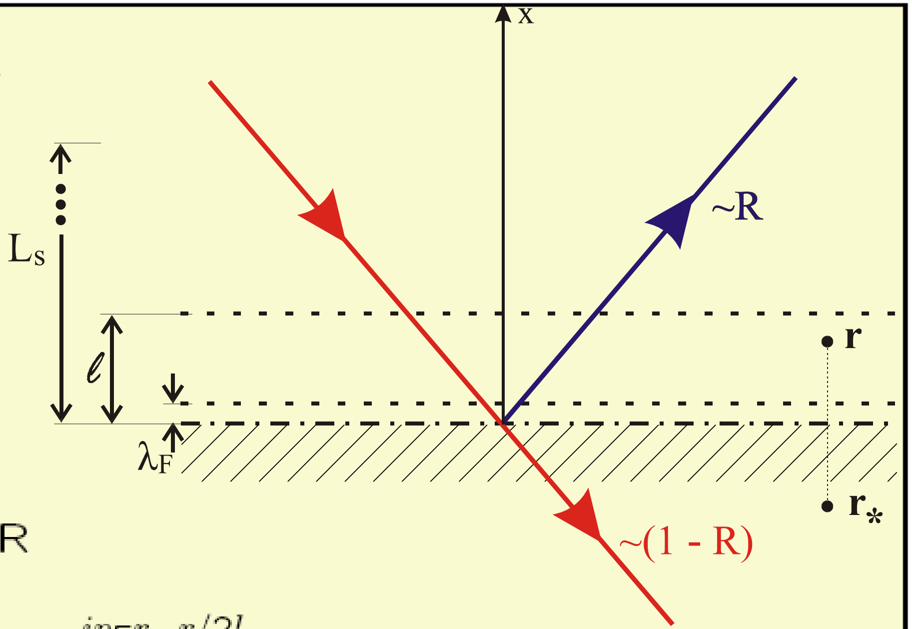
Boundary conditions are the "missing" equations

$$\hat{L}_{\alpha\beta}\, \rho_\beta(x)\Big|_{x=0} = 0$$

Derivation of boundary conditions

BC's can be derived by expanding the integral equation near the edge [VG, Burkov, & Das Sarma PRB **74**, 115331 (2006)]

$$\rho_\alpha(0) = \lim_{\lambda_F \ll \mathbf{r}\cdot\hat{\mathbf{n}} \ll l} \int_{x'>0} \Pi_{\alpha\beta}(\mathbf{r},\mathbf{r}')\rho_\beta(\mathbf{r}')d^d\mathbf{r}'$$



Example: Boundary with a "reflectivity" R

$$\tilde{G}(\mathbf{r},\mathbf{r}') = G_0(\mathbf{r}-\mathbf{r}') - RG_0(\mathbf{r}_*-\mathbf{r}'),\ \ G_0(r) \propto e^{ip_F r - r/2l}$$

$$\tilde{\Pi}(\mathbf{r},\mathbf{r}') \sim GG + R^2 G_* G_* - \cancel{RGG_* - RG_*G}$$

$$\rho_\alpha(0) = \frac{1+R^2}{2}\rho_\alpha(0) + cl\partial_{\mathbf{n}}\rho_\alpha(0) + \ldots$$

Two types of boundary conditions for spin

Expansion of the "integral" diffusion equation near the interface:

$$\rho_\alpha(0) = \frac{1+R^2}{2}\rho_\alpha(0) + cl\partial_{\mathbf{n}}\rho_\alpha(0) + \dots$$

- If $(1-R^2) \sim 1$ (e.g., a transmitting boundary with a ballistic contact or a medium with a long mean-free path)

$$\rho_a(0) = 0, \quad \text{Dirichlet BCs}$$

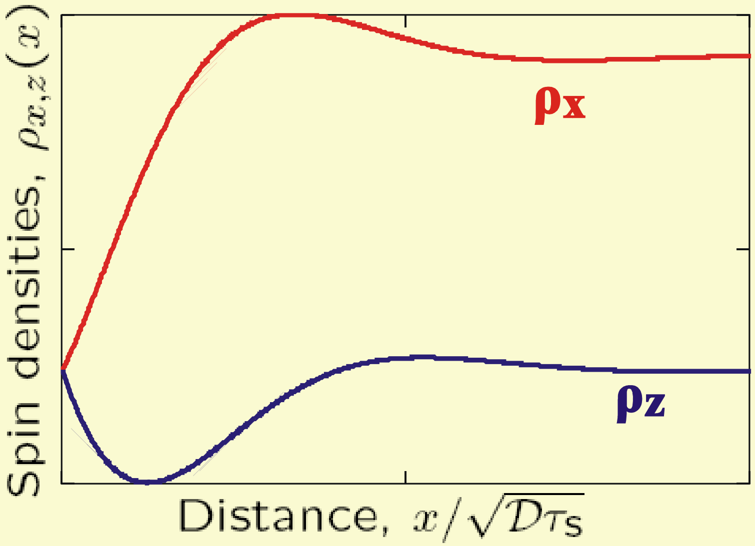
- If $R = 1$ (e.g., hard wall), SO corrections become important:

$$l\mathbf{n}\boldsymbol{\nabla}\rho_a(\mathbf{0}) = B_{ab}\rho_b(\mathbf{0}) + \mathbf{C}_a\boldsymbol{\nabla}\rho_0(\mathbf{0}), \quad \text{von Neumann BCs}$$

Results in the Rashba model

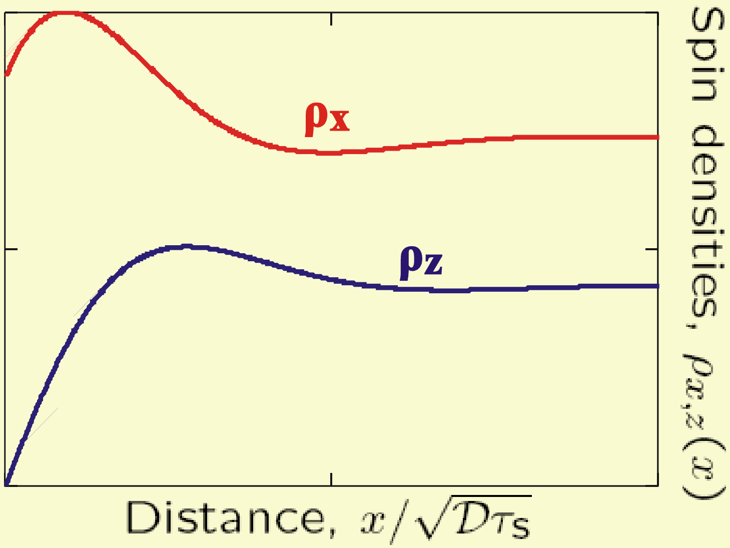
- Partially transparent boundary

$\rho_{x,z}(0) = 0$



- Impenetrable boundary

$\rho_x'(0) = -\rho_z(0)$ and $\rho_z'(0) = \rho_x(0) + \eta\rho_x^\infty$,
η is a non-universal constant
(spectrum and boundary-dependent)
For hard wall and quadratic spectrum, $\eta = 0$.



$$\rho_\alpha(x) = A_\alpha \cos(kx + \phi_\alpha)e^{-\frac{x}{\lambda}}, \quad k + \frac{i}{\lambda} = \sqrt{\frac{1 + i\sqrt{7}}{2\mathcal{D}\tau_s}}$$

Published in *Nature*, **437** pp. 1330-3 (27 Oct. 2005)

Observation of spin Coulomb drag in a two-dimensional electron gas

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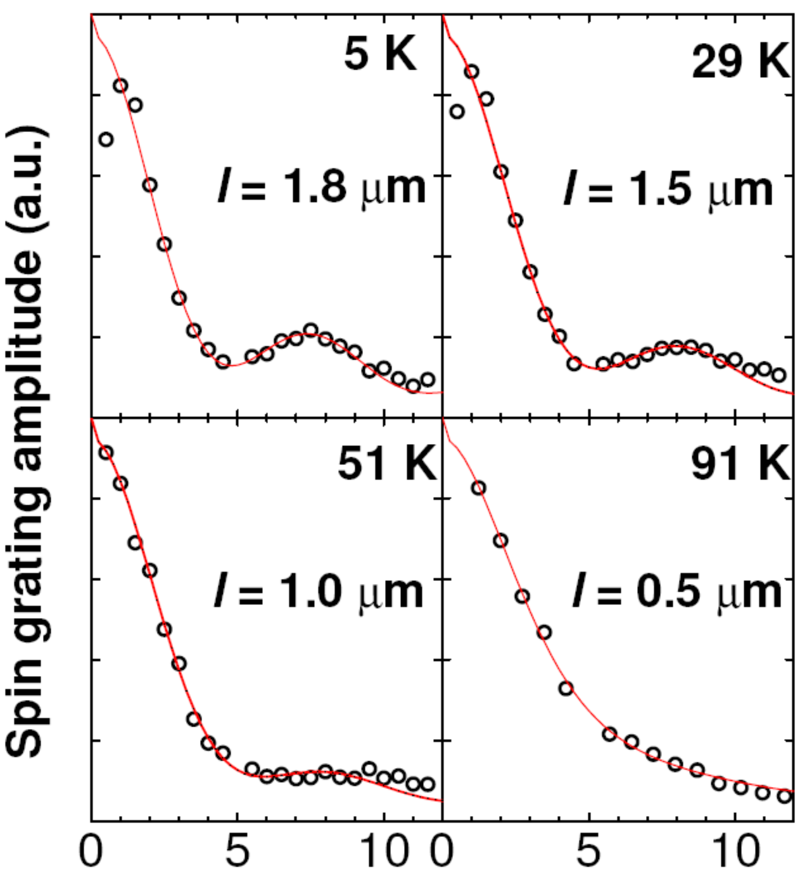


FIG. 2: Time-dependence of the spin-grating's amplitude. The lines are fits of the data to $S(q, \omega)$. The values of l determined from these fits are indicated in each panel. Due to laser heating, the temperature T_e of the electron gas is higher than the lattice temperatures indicated.

Relaxation of injected spin density

Diffusion Eq. in Fourier space:

$$\rho_a(\omega, \mathbf{q}) = \Pi_{ab}(\omega, \mathbf{q})\rho_b(\omega, \mathbf{q})$$

Green's function of the spin-charge-coupled diffusion Eq.

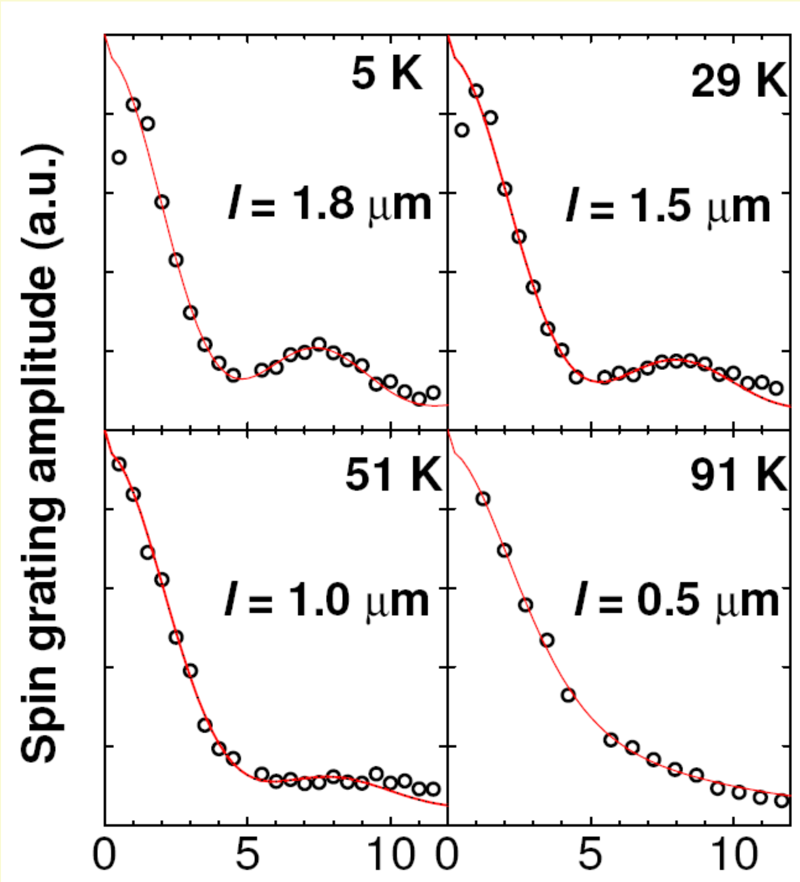
$$\hat{D}(\omega, \mathbf{q}) = \frac{1}{1 - \hat{\Pi}(\omega, \mathbf{q})}$$

The real space-time dependence of the diffuson describes a diffusive relaxation of the corresponding component of the spin density. E.g., if at $t = 0$, we had $\rho_z(\mathbf{r}) = \delta(\mathbf{r})$, $D_{zz}(t, \mathbf{r})$ gives the subsequent dynamics.

Without spin-orbit coupling, we would have the usual diffuson

$$D(t, \mathbf{r}) = \frac{1}{(2\pi Dt)^{d/2}} \exp\left[-\frac{\mathbf{r}^2}{4Dt}\right]$$

Spin relaxation in spin grating experiments



Spin-grating experiments. probe spin relaxation at a finite $\mathbf{k}$-vector:

$$\mathrm{D}_{ij}(t,\mathbf{k}) = \sum_{l=0}^{3} A_l(\mathbf{k}) e^{-i\omega_l(\mathbf{k})\; t}$$

If all $i\omega_l(\mathbf{k})$ are real - no oscillations.

Results for the Rashba case:

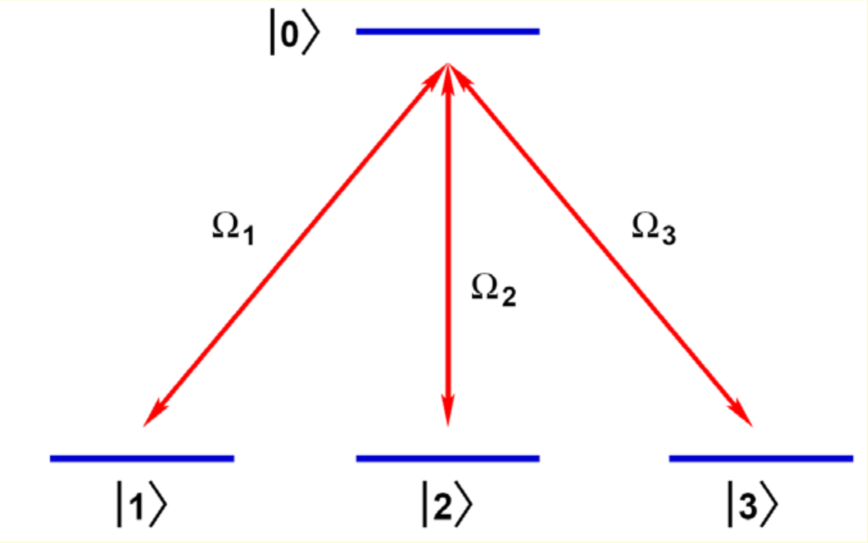
$$\begin{cases} i\omega_0(\mathbf{k}) = \frac{1}{2} + k^2 - \frac{1}{2}\sqrt{1 - 32\pi^2 g^2 \alpha^2 k^2} \\ i\omega_1(\mathbf{k}) = \frac{1}{2} + k^2 + \frac{1}{2}\sqrt{1 - 32\pi^2 g^2 \alpha^2 k^2} \\ i\omega_2(\mathbf{k}) = \frac{3}{2} + k^2 - \frac{1}{2}\sqrt{1 + 16k^2} \\ i\omega_3(\mathbf{k}) = \frac{3}{2} + k^2 + \frac{1}{2}\sqrt{1 + 16k^2} \end{cases}$$

Effective spin-orbit interaction and spin dynamics in atomic systems

T. Stanescu, C. Zhang, and VG

[cond-mat/0703500]

Atomic system in a spatial-varying laser field



Hamiltonian: $\hat{\mathcal{H}} = \hat{\mathcal{H}}_{\text{kin}} + \hat{V}_{\text{trap}} + \hat{\mathcal{H}}_{\text{a-l}}$

Atom-laser interaction:

$$\hat{\mathcal{H}}_{\text{a-l}} = -\left[\Omega_1(\mathbf{r})|0><1| + \Omega_2(\mathbf{r})|0><2| + \Omega_3(\mathbf{r})|0><3|\right] + \text{h. c.}$$

Degenerate dark states

Useful parametrization of the *position-dependent* frequencies

$$\begin{aligned}\Omega_1(\mathbf{r}) &= \Omega \quad \sin\theta \quad \cos\phi \quad e^{iS_1}\\ \Omega_2(\mathbf{r}) &= \Omega \quad \sin\theta \quad \sin\phi \quad e^{iS_2}\\ \Omega_3(\mathbf{r}) &= \Omega \quad \cos\theta \quad e^{iS_3}\end{aligned}$$

One can find a matrix $\hat{U}(\mathbf{r})$, which diagonalizes $\hat{\mathcal{H}}_{\text{a-l}}$.

New "band structure" includes a pair of degenerate dark states

$$\begin{aligned}|u_1\rangle &= \sin\phi \; e^{-iS}|1\rangle - \cos\phi \; e^{-iS}|2\rangle,\\ |u_2\rangle &= \cos\theta \; \cos\phi \; e^{-iS}|1\rangle + \cos\theta \; \sin\phi \; e^{-iS}|2\rangle - \sin\phi|3\rangle, \quad E_1 = E_2\end{aligned}$$

But (!) the kinetic part produces new terms in the Hamiltonian:

$$\delta\hat{\mathcal{H}} = \hat{U}^\dagger(\mathbf{r})\left(-\frac{\nabla^2}{2m}\right)\hat{U}(\mathbf{r})$$

Non-Abelian gauge field vs. pseudo-spin-orbit coupling

E.g., consider the case: $S_1 = S_2 = S(r) = mv_S y$, $S_3 = 0$, and $\phi = mv_\phi x$

- **Picture I:** **Particles in a non-Abelian gauge potential**

$$\hat{H}_{\text{eff}} = \frac{1}{2m}\left(-i\nabla\hat{I} - \hat{\mathbf{A}}\right)^2 + \hat{V}$$

with a vector potential

$$\hat{\mathbf{A}} = \begin{pmatrix} \nabla S & -i\cos\theta\nabla\phi \\ i\cos\theta\nabla\phi & \cos^2\theta\nabla S \end{pmatrix}$$

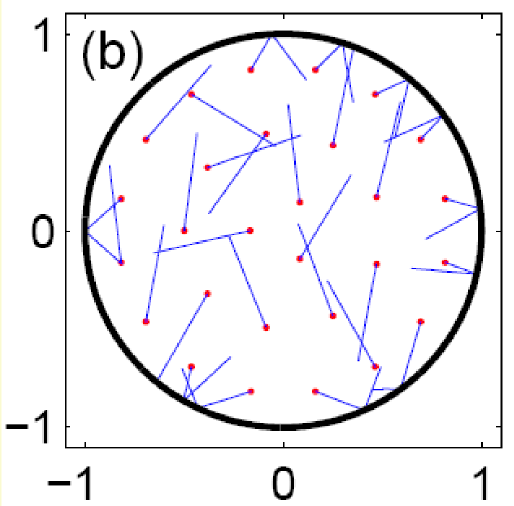
- **Picture II:** **Particles with (pseudo) spin-orbit interaction**

$$\hat{H}_{\text{eff}} = \left(\frac{p^2}{2m} + \frac{m\omega^2}{2}r^2\right)\hat{I} + \delta_0\hat{\sigma}_z + \hat{H}_{SO}$$

Pseudo-spin-orbit coupling is:

$\hat{H}_{SO} = v_0 p_x \hat{\sigma}_y - v_1 p_y \hat{\sigma}_z,$ $v_0 = -v_\phi \cos\theta$ and $v_1 = \sin^2\theta v_s/2$

Spin relaxation in a harmonic trap



Let us polarize all spins
in the z-direction, $\mathcal{P}(0) = 1$

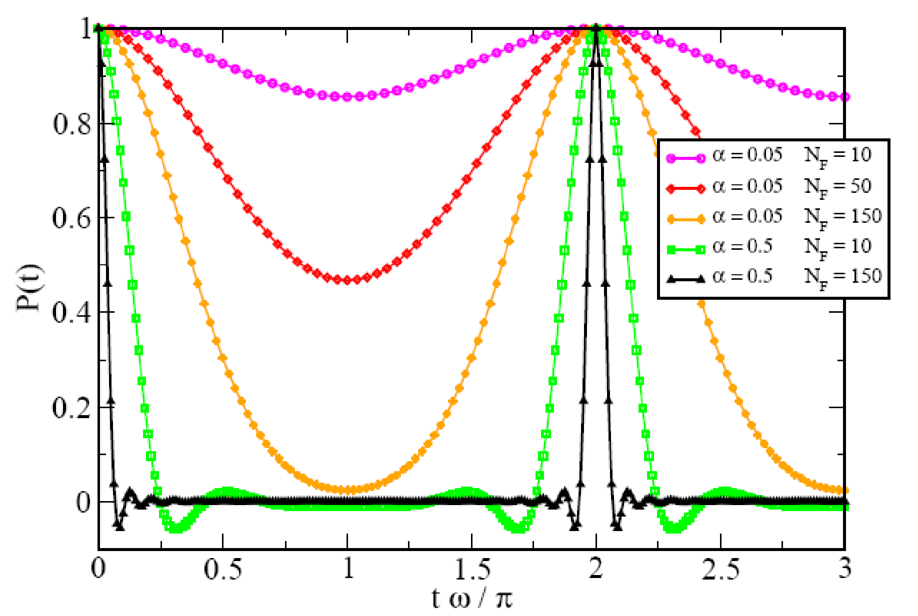
Spin polarization dynamics, $\mathcal{P}(t) = \frac{1}{N}\langle\Phi_0|\widehat{P}_z(t)|\Phi_0\rangle$

Polarization operator, $\widehat{P}_z = \int d^2r\ \widehat{\Psi}^\dagger(\mathbf{r})\widehat{\sigma}_z\widehat{\Psi}(\mathbf{r})$

The Heisenberg representation, $\widehat{P}_z(t) = \widehat{\mathcal{U}}^{-1}(t)\widehat{P}_z\widehat{\mathcal{U}}(t)$

Time-evolution operator, $\widehat{\mathcal{U}}(t) = \exp\left(-i\widehat{\mathcal{H}}t\right)$

Spin relaxation in a symmetric trap

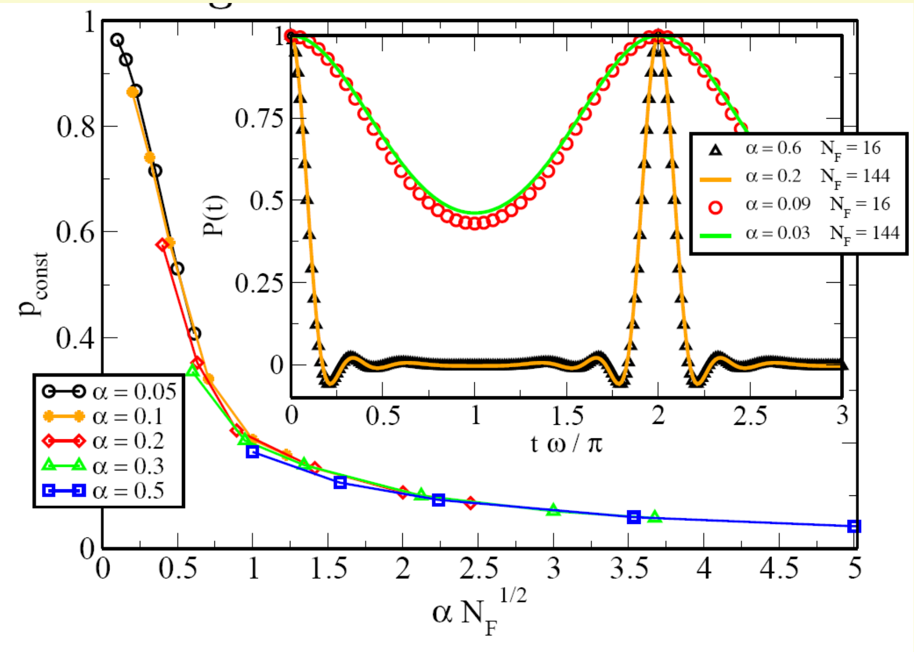


Polarization in the weak-coupling limit

$$\mathcal{P}(t) \approx 1 - \frac{8\alpha^2 N_\mathrm{F}}{3}\left[1 - \cos(\omega t)\right]$$

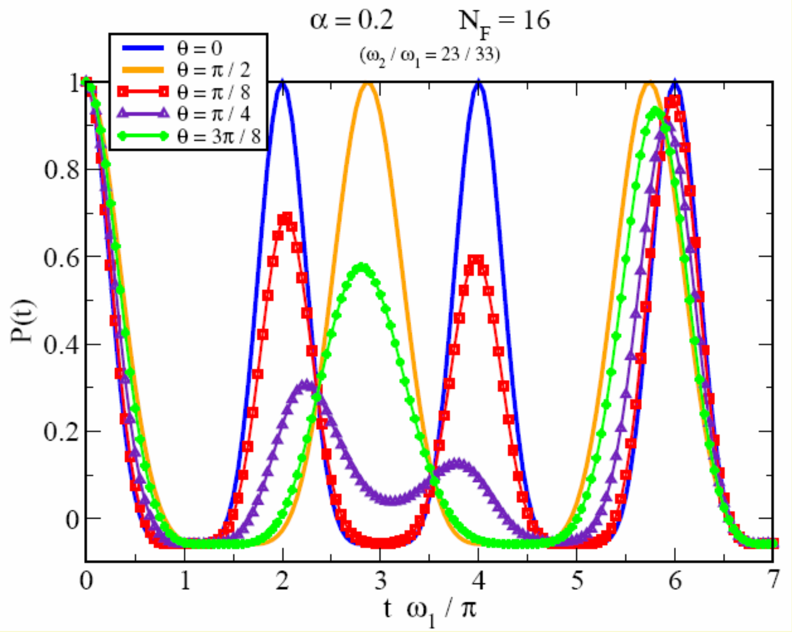
The $2\pi/\omega$ periodicity is due to the equal spacing between the harmonic oscillator levels. When the SO coupling is large, the polarization exhibits fast relaxation followed by periodic echoes.

Remanent “magnetization” of the cloud



The spin polarization dynamics depends both on the strength of SO coupling and the number of particles. In thermodynamic limit the quantum dynamics are delta-function-like peaks followed by periods of zero polarization.

Spin relaxation in an elliptic trap



Relaxation curves in an elliptic trap $\omega_1 > \omega_2$. If the lasers are oriented along the principal axes ($\theta = 0$ or $\theta = \pi/2$), the relaxation curves are identical to those of an isotropic trap with the corresponding ω (blue and orange curves). If $\omega_2/\omega_1 = p/q$, the period of the relaxation curves becomes $q2\pi/\omega_1 = p2\pi/\omega_2$.

Summary

- In clean SO coupled systems, the effect of spin accumulation is due to the localized edge states.
- In disordered systems, spin transport is described in terms of a matrix spin diffusion equation; spin accumulation depends on the structure of the edges and may occur without bulk spin currents.
- Observed temporal oscillations in spin-grating experiments may be due to strong spin-charge coupling, which leads to qualitatively different time dynamics of spin-charge diffusion.
- Spatial-varying laser fields in cold atomic systems, lead to an effective SO coupling. The spin dynamics in a trapped system is similar to spin relaxation in spin-orbit-coupled quantum dots.