Strongly Interacting Fermi Gases: Rotation and Optical Lattices

Hui Zhai

in collaboration with Tin-Lun Ho

Department of Physics, Ohio-State University

Poster on " Correlated States in Degenerate Atomic Gases "
KITP, UC Santa Barbara
April 2007

Ways to achieve strong interactions in quantum gases

Feshbach resonance:

scattering length diverges,
Fermi energy is the only energy scale
===> non-perturbative

Rotation:

each Landau level is highly degenerate,
===> within each Landau level interaction is the only energy scale

Optical Lattices:

wave function localized at each site,
===> tunneling suppressed and interaction enhanced

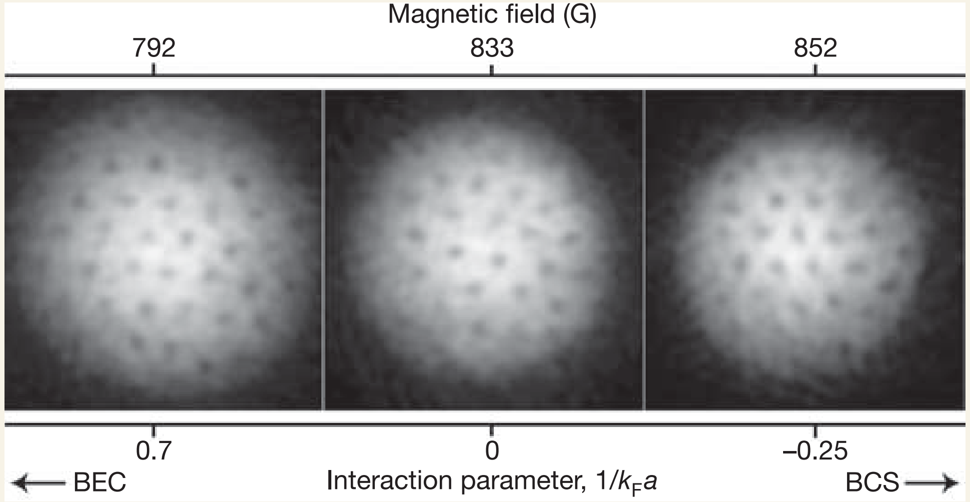
More Intriguing Physics:

resonance + rotation
resonance + optical lattices

Experimental Motivation

Rotating Fermion Superfluid:

Stable vortex lattices created

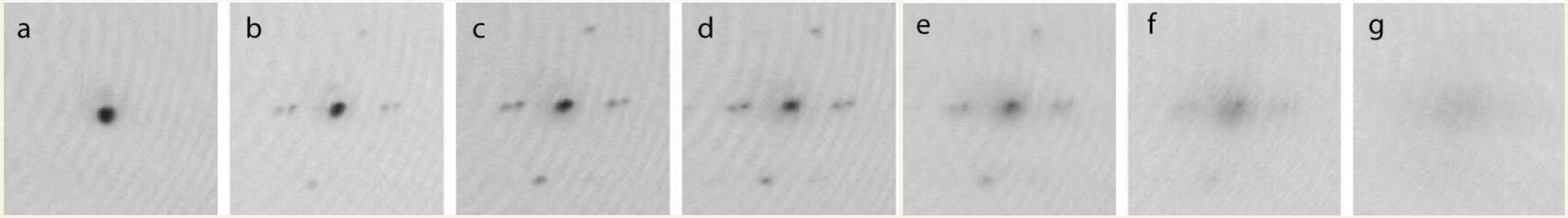


M. W. Zwierlein, J. R. Abo-Shaeer, A. Schirotzek, C. H. Schunck, W. Ketterle: **Nature**, *435, 1047 (2005)*

Fermion Superfluid in Optical Lattices:

Loss of superfluidity as lattice increases

J. K. Chin, D. E. Miller, Y. Liu, C. Stan, W. Setiawan, C. Sanner, K. Xu and W. Ketterle, **Nature**, *443, 961 (2006)*

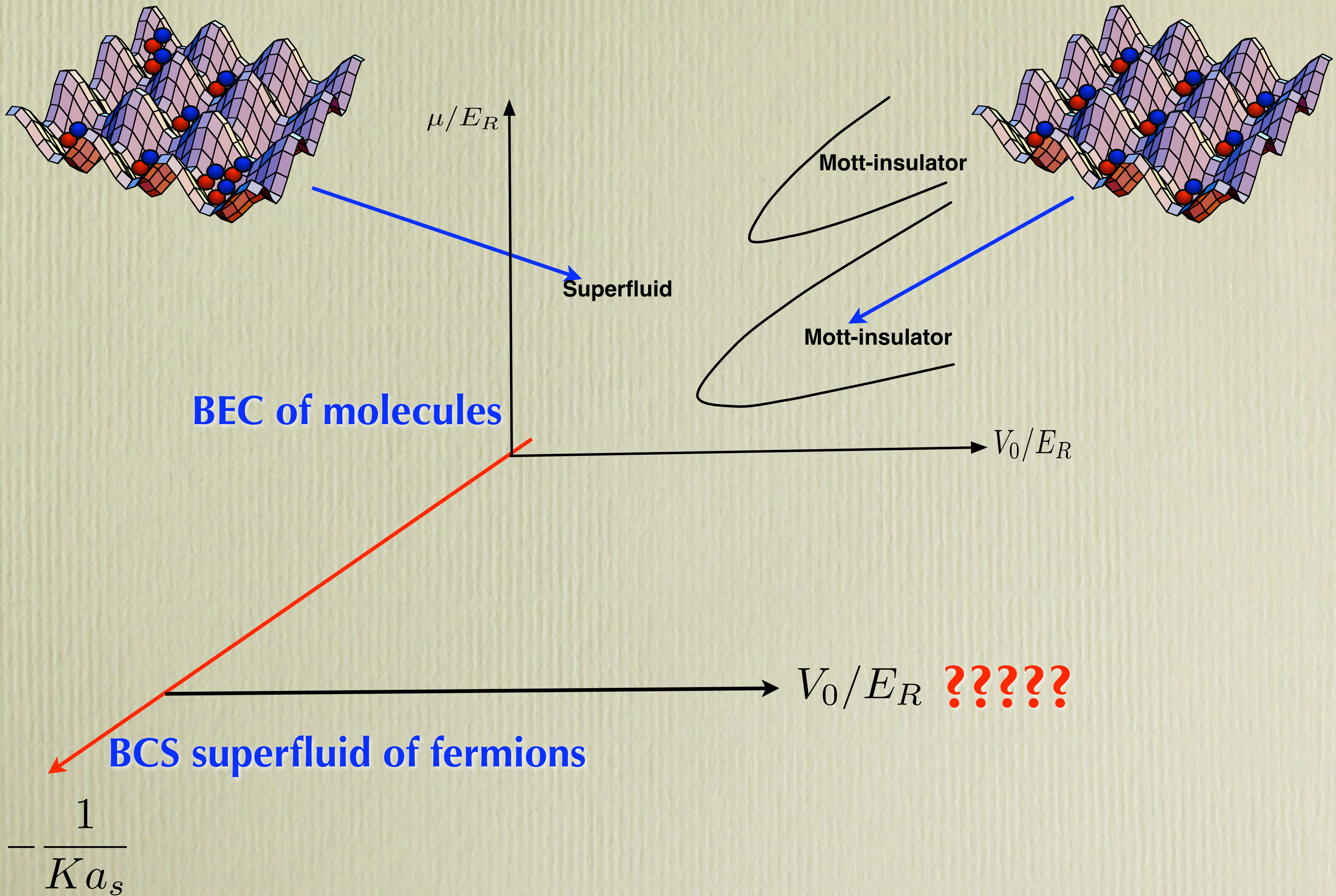


$V_0 = 0$ $V_0 = 2.5$ $V_0 = 4$ $V_0 = 5$ $V_0 = 6$ $V_0 = 7$ $V_0 = 9$

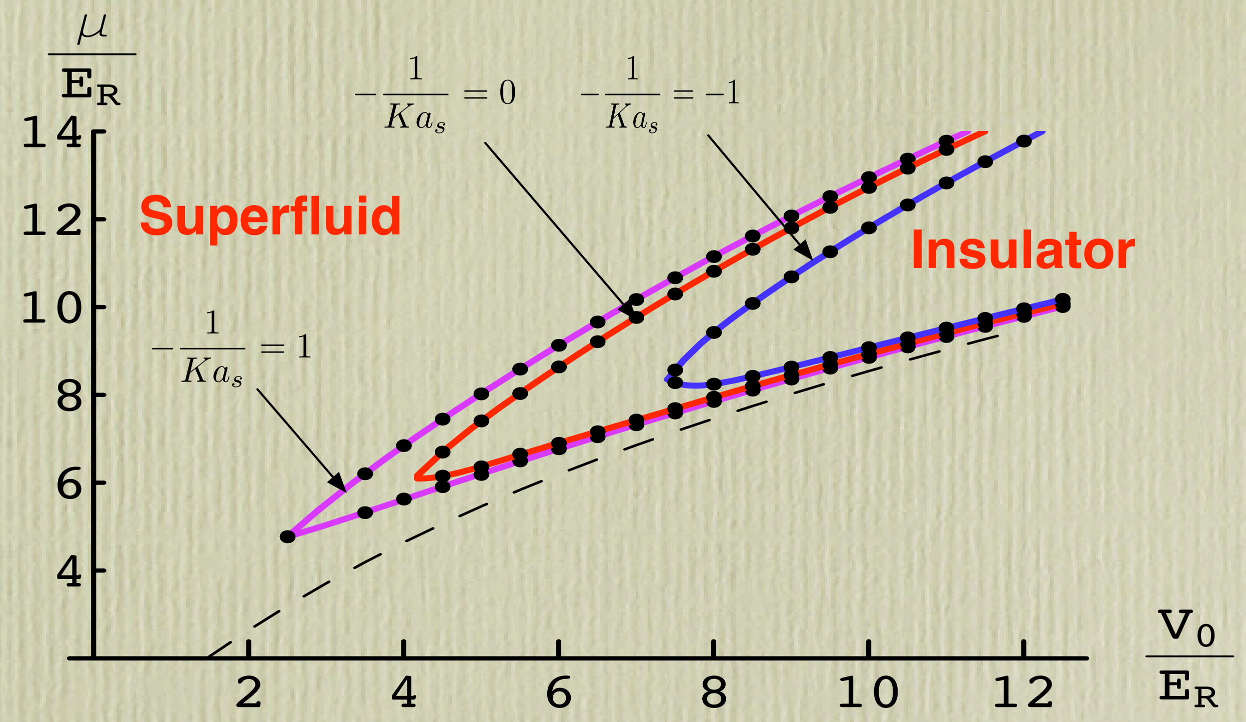
Part I:
Superfluid-Insulator Transition of Strongly Interacting Fermi Gases

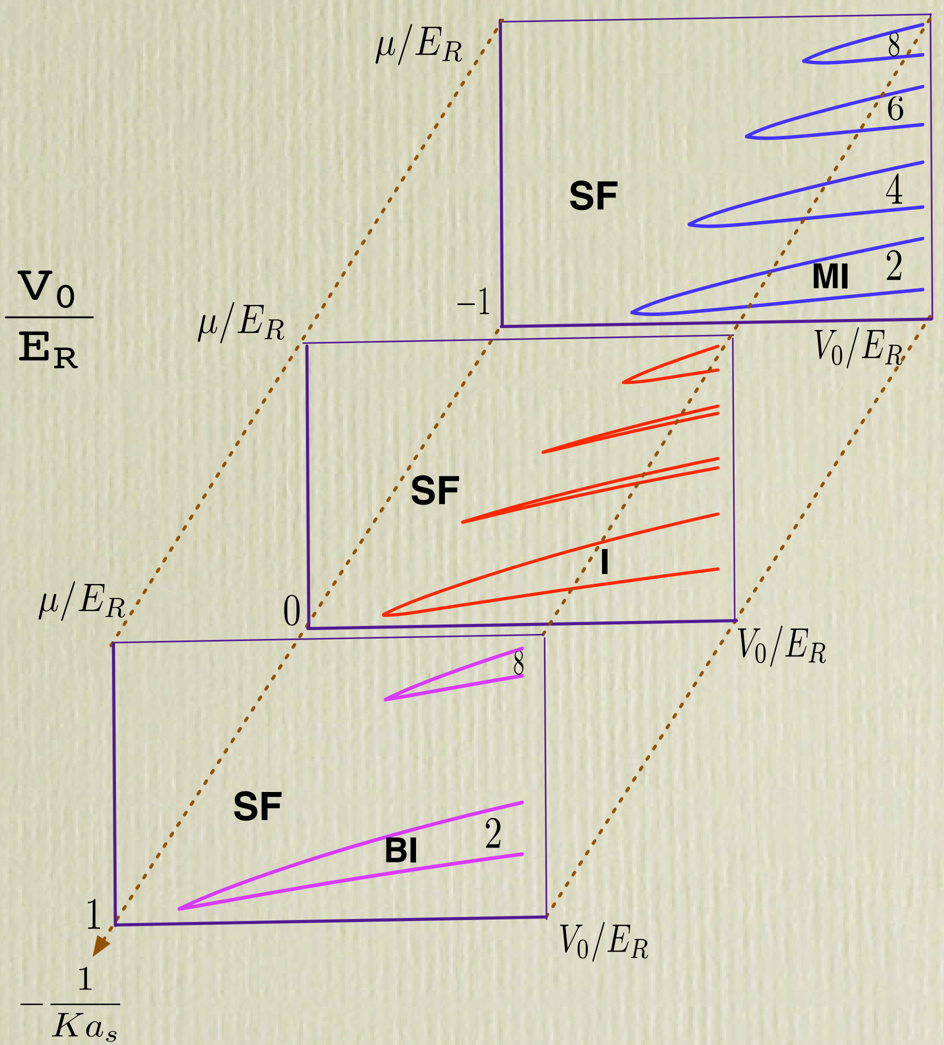
Hui Zhai and Tin-Lun Ho, arXiv: 0704.2957

Question: How SF-Insulator transition depends on a_s



Answer from mean-field theory

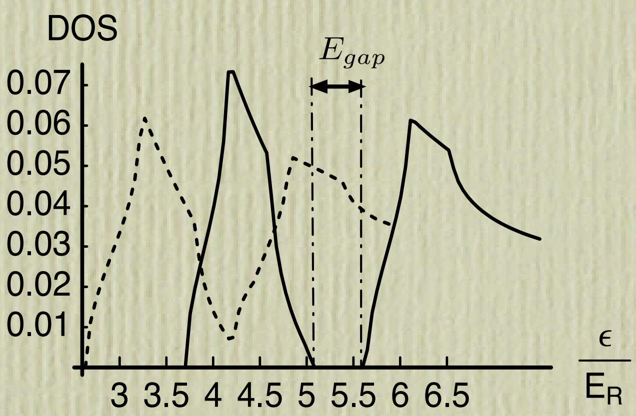




Model

Single particle physics:
band structure

$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 + V_0(\sin^2(Kx) + \sin^2(Ky) + \sin^2(Kz))$$

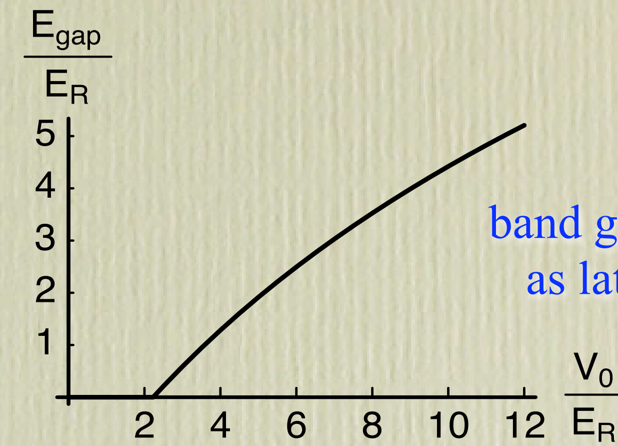


$$V_0 = 2E_R$$

dash line, no band gap

$$V_0 = 3E_R$$

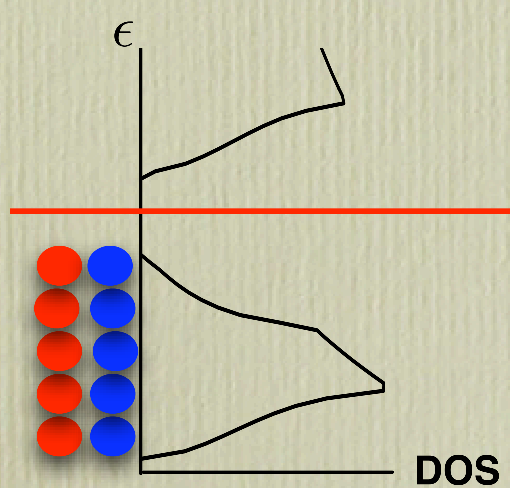
solid line, band gap opens



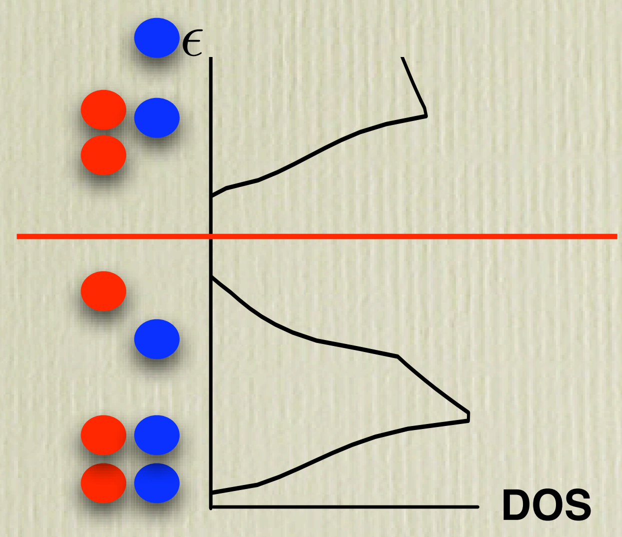
band gap increases
as lattice depth

Many-body physics:
Cooper instability

competition between Energy Gain from Forming Pairs
and Energy Cost to Overcome Band Gaps



Band Insulator



Fermion Pairs Superfluid

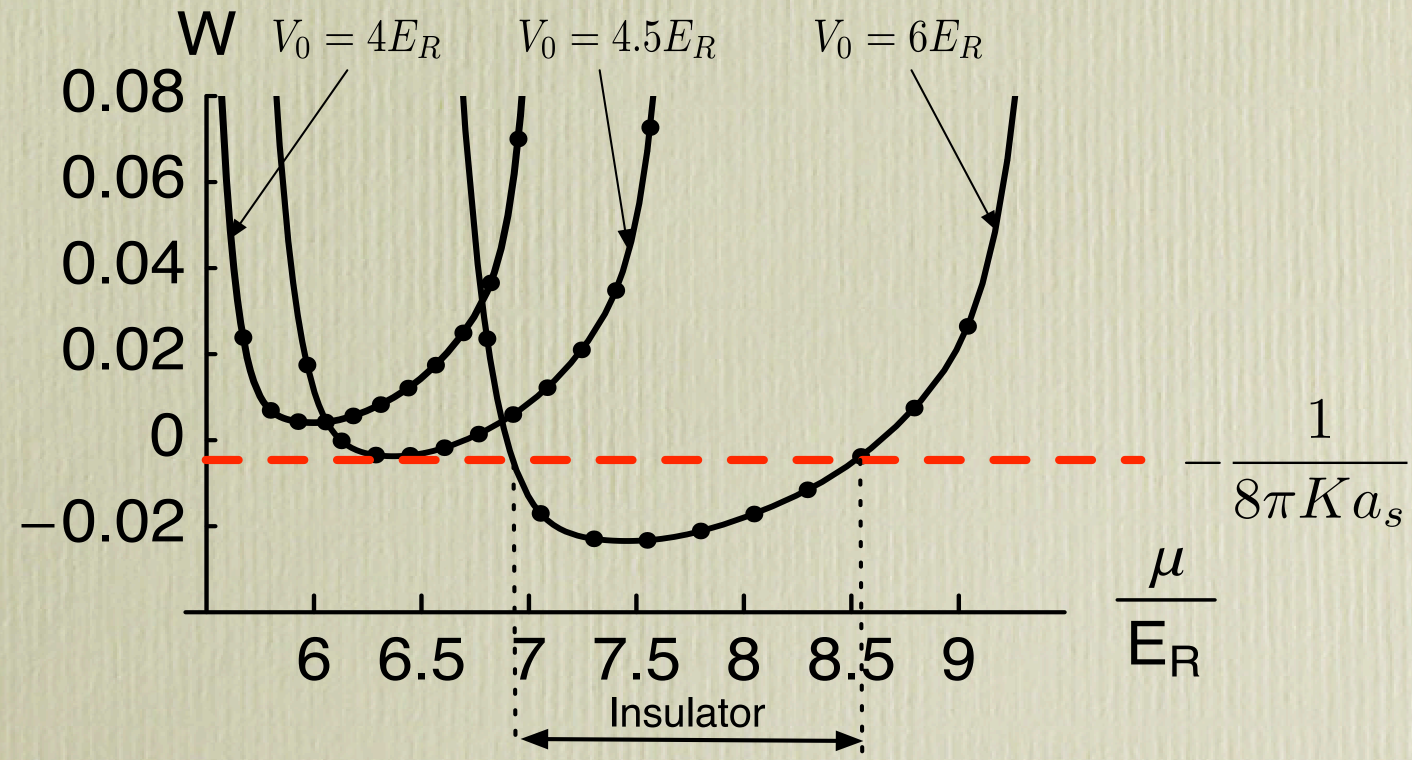
Outline of the calculation: Pairing susceptibility

$$\alpha_{\mathbf{G}} = W_{\mathbf{G}} + \frac{m}{4\pi\hbar^2 a_s}$$

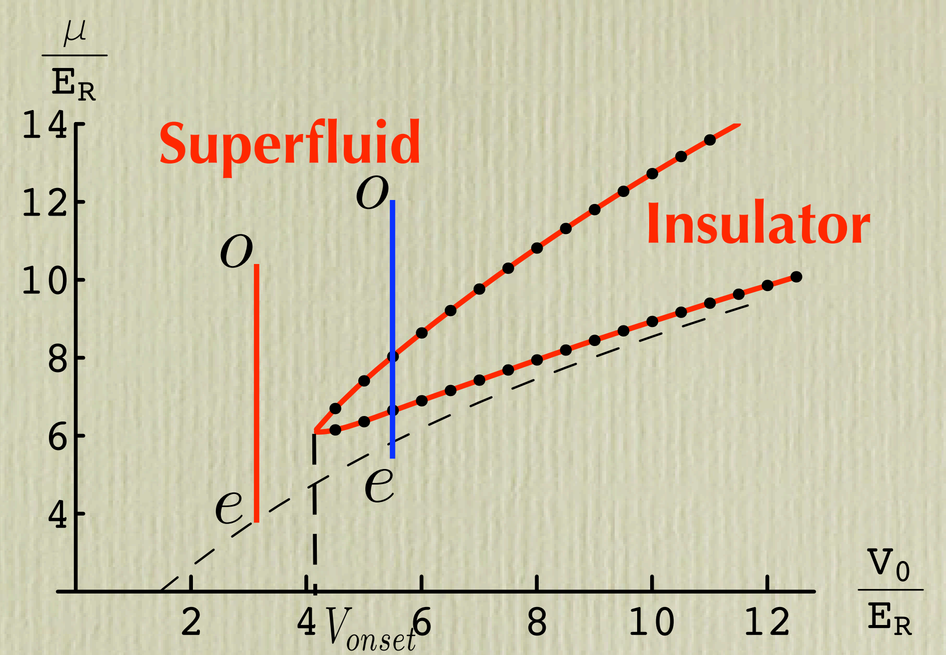
Condition for the onset of superfluidity

At least one of $\alpha_{\mathbf{G}} > 0 \Longleftrightarrow W_{\mathbf{G}} > -\frac{m}{4\pi\hbar^2 a_s}$

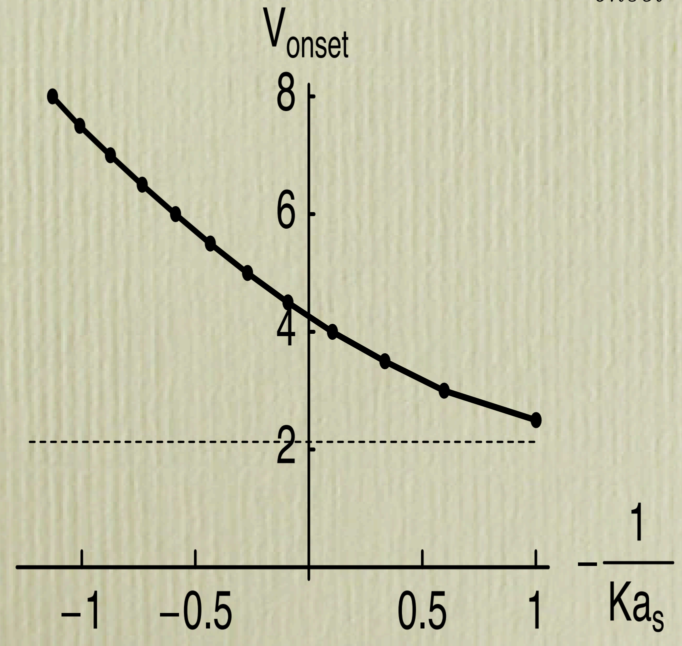
$$W_{\mathbf{G}} = \sum_{\mathbf{k}} \left(\sum_{n,n'} \frac{|Q^{\mathbf{G}}_{n,n',\mathbf{k}}|^2}{\xi_{n\mathbf{k}} + \xi_{n'-\mathbf{k}}} (\Theta(\xi_{n\mathbf{k}}) + \Theta(\xi_{n'-\mathbf{k}})) - \sum_{\mathbf{G}} \frac{1}{\hbar^2(\mathbf{k}+\mathbf{G})^2/m} \right) \qquad Q^{\mathbf{G}=\mathbf{0}}_{nn'k} = \delta_{nn'}$$

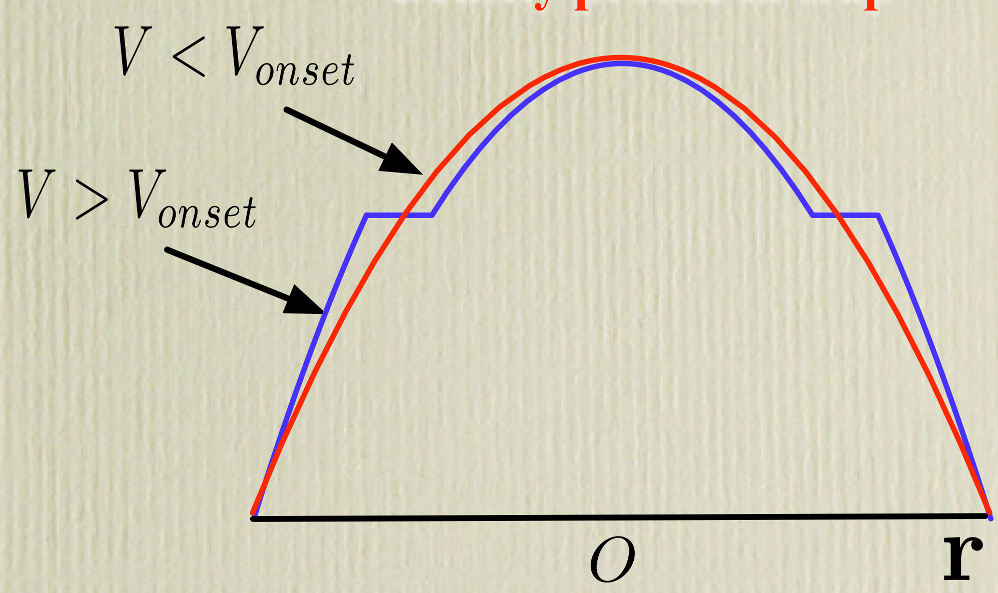


Critical Lattice Depth for the Onset of Insulator



$$\mu(\mathbf{r}) = \mu_0 - V(\mathbf{r})$$

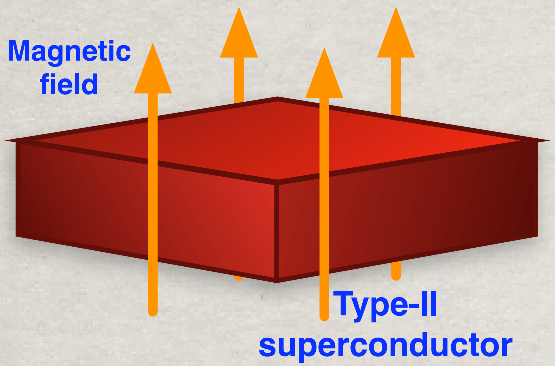


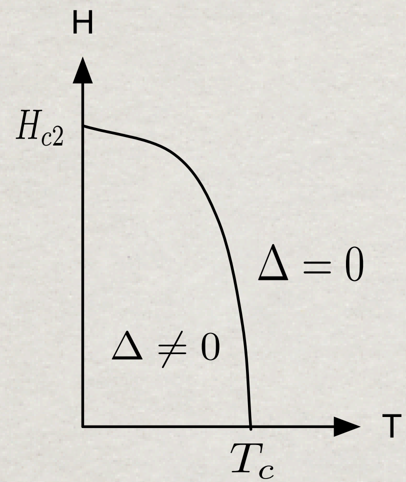


Part II:
Critical Rotational Frequency of Superfluid Fermi Gases across Feshbach Resonance

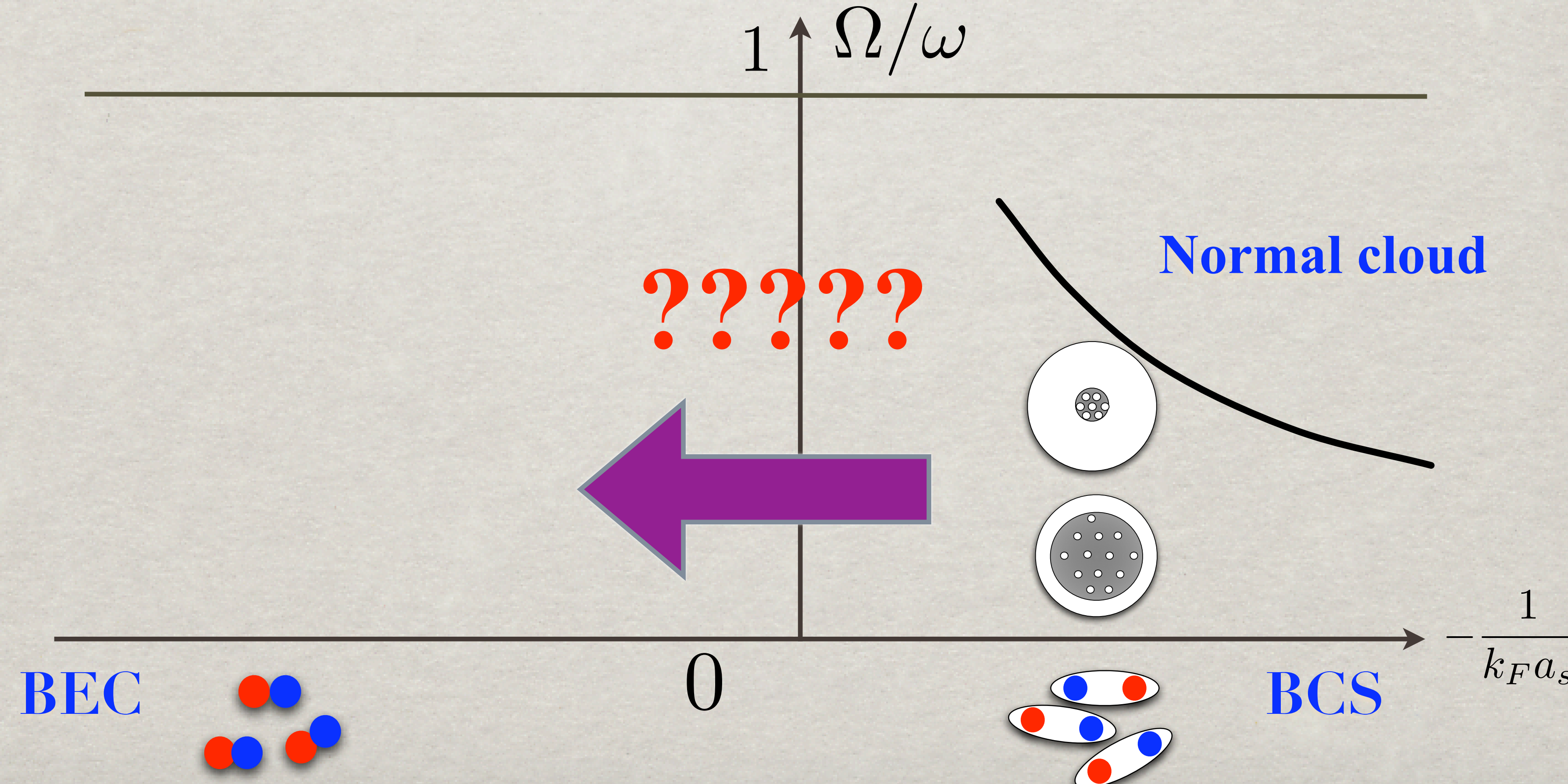
Hui Zhai and Tin-Lun Ho, Physical Review Letters, 97, 180414 (2006)

Question: How critical frequency depends on a_s

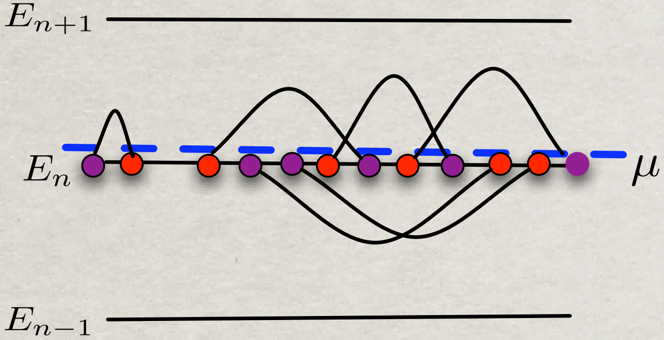




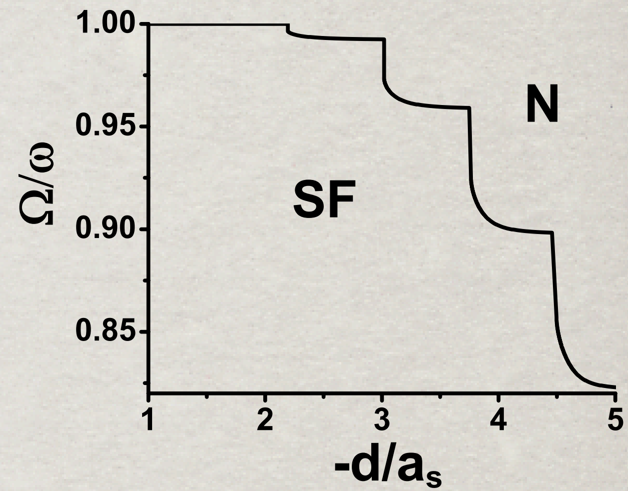
Critical rotational frequency <==> Analogy of Hc in superconductor



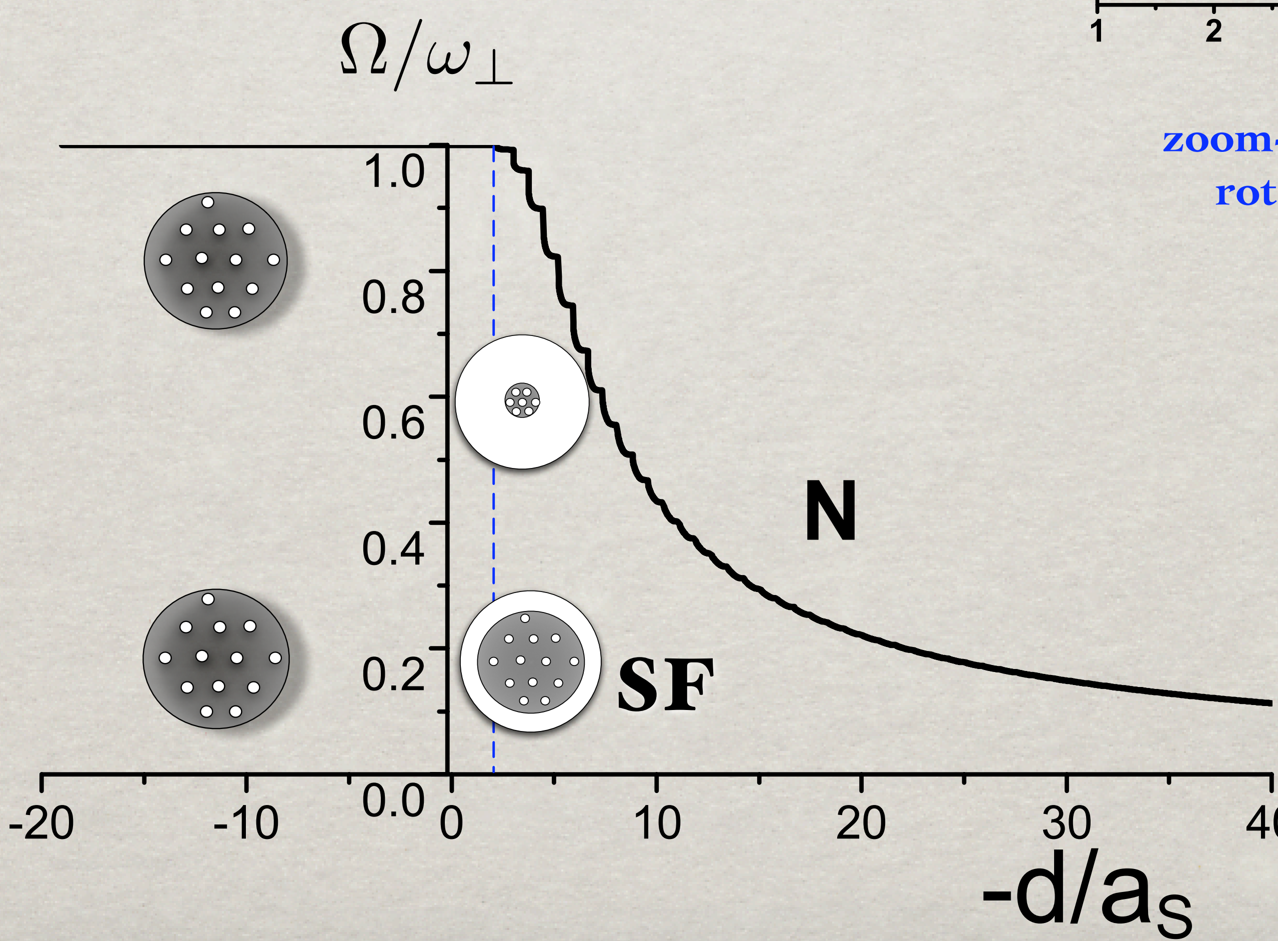
Answer: Seeing integer Landau levels



A sequence of jumps reveal the features of Landau levels



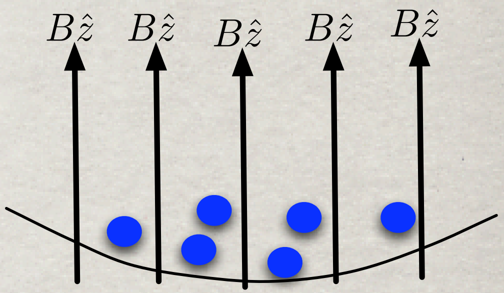
zoom-in plot of high rotational limit



Model

Single particle Hamiltonian

$$H_0 = \frac{\hbar^2}{2m}\nabla^2 - \Omega L_z + \frac{1}{2}m\omega^2 r^2 + \frac{1}{2}m\omega_z^2 z^2$$



$$\Rightarrow \quad H_0 = \frac{1}{2m}(\mathbf{p} - m\Omega\hat{z}\times\mathbf{r})^2 + \frac{1}{2}m(\omega^2 - \Omega^2)r^2 + \frac{1}{2}\omega_z^2 z^2$$

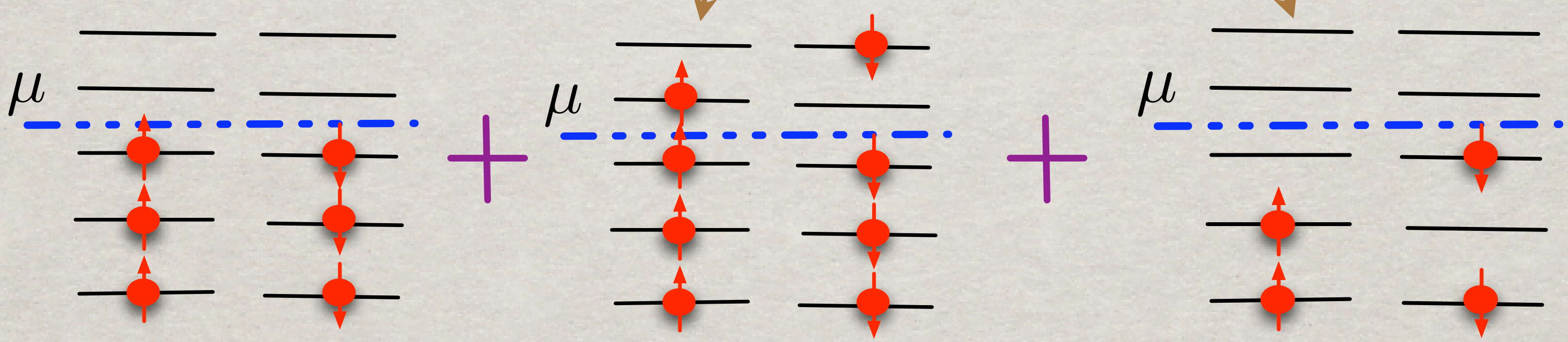
"electrons" in magnetic field: Landau levels

local density approximation

$$\mu \to \mu - \frac{1}{2}m(\omega^2 - \Omega^2)r^2 - \frac{1}{2}\omega_z^2 z^2$$

Many-body Hamiltonian

$$\mathcal{H} = \sum_{\nu\sigma}\xi_\nu c^\dagger_{\nu\sigma}c_{\nu\sigma} + \Delta\sum_{\nu\nu'}\alpha_{\nu\nu'}c^\dagger_{\nu\uparrow}c^\dagger_{\nu'\downarrow} + \Delta^*\sum_{\nu\nu'}\alpha^*_{\nu\nu'}c_{\nu\uparrow}c_{\nu'\downarrow} + \frac{|\Delta|^2}{g}$$



$$F = F_0 + \alpha|\Delta|^2 + \beta|\Delta|^4 + \ldots\ldots$$

$\alpha < 0$ **Superfluid**

$\alpha > 0$ **Normal**

Outline of the calculation: Pairing susceptibility

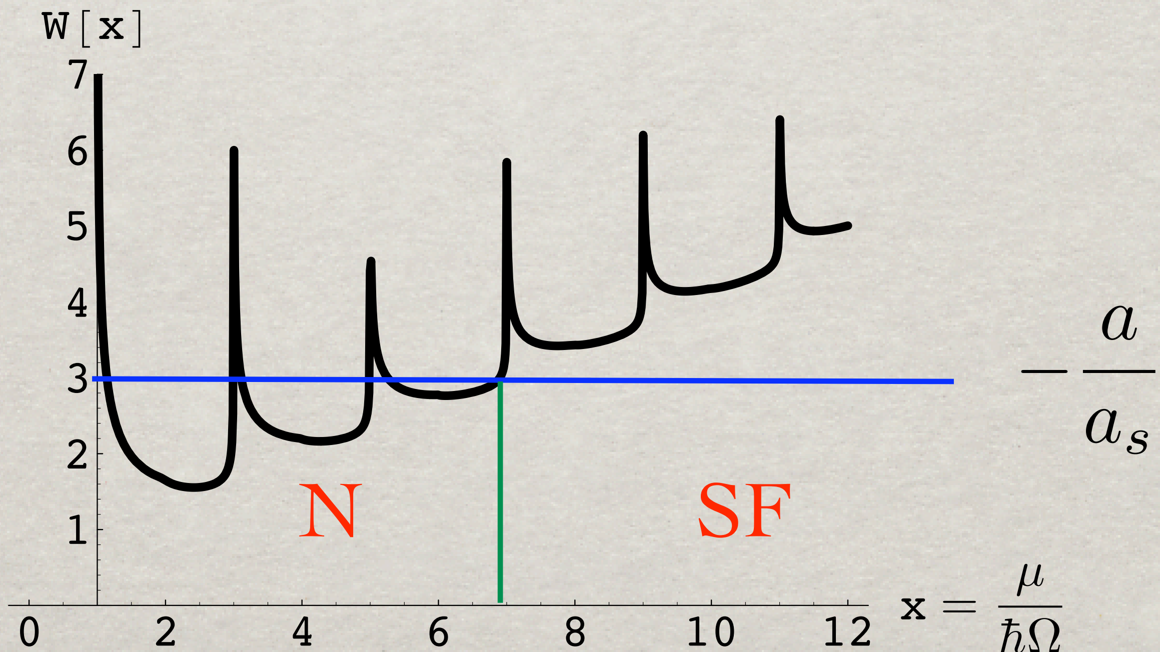
$$\alpha = -W\left(\frac{\mu}{\hbar\Omega}\right) - \frac{a}{a_s}$$

$$a = \sqrt{\frac{\hbar}{2m\Omega}}$$

$$W = \frac{\pi\sqrt{2}a\hbar^2}{2m}\sum_{\nu\nu'}\frac{\tanh(\frac{\beta\xi_\nu}{2}) + \tanh(\frac{\beta\xi_{\nu'}}{2})}{\xi_\nu + \xi_{\nu'}}|\alpha_{\nu\nu'}|^2 - \sum_N \frac{1}{4\sqrt{N+1}}$$

$W > -\frac{a}{a_s}$ **Superfluid**

$W < -\frac{a}{a_s}$ **Normal**



Thank You Very Much for Your Attention