



## Introduction

- ▶ The density wave instability is the onset of a periodic modulation of the density, which is caused by repulsive interactions.
- ▶ Fermionic dipoles are aligned by an external field and confined to in equidistant quasi 2D layers at zero temperature.
- ▶ In a conserving Hartree-Fock approximation, we calculate the critical coupling strength for the formation of density waves as a function of the dipole orientation and the distance between layers.

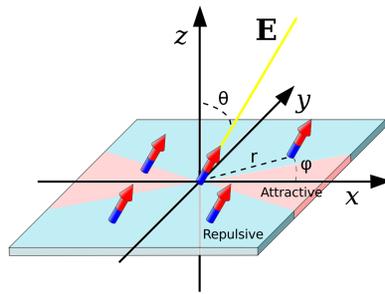
## Dipole-dipole Interaction

The interaction between dipoles with dipole moment  $p$  is **anisotropic** when the external field is tilted.

The intralayer interaction,

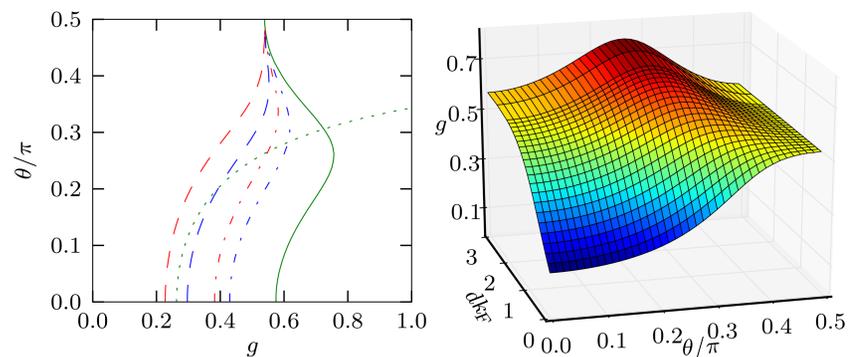
$$V_{2D}(\mathbf{r}) = D^2 \frac{1 - 3 \sin^2 \theta \cos^2 \varphi}{r^3},$$

where  $D = \frac{p^2}{4\pi\epsilon_0}$ , is illustrated in the figure on the right.



Intralayer interaction. Image: [1]

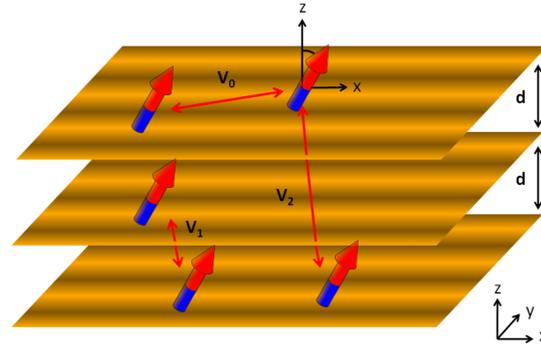
## Results



(a) The phase boundary between the normal (left) and the striped phase (right) in the  $(g, \theta)$  plane. **Singlelayer** HFA (full) and RPA (dotted). **Bi-** and **trilayer**: The dashed lines are for  $dk_F = 0.5$ , while the dash-dotted (---) are for  $dk_F = 1$ .  
 (b) The full  $dk_F$  and  $\theta$  dependency for a bilayer system

## System

- ▶ The dipolar molecules are confined to quasi two dimensional layers by means of a deep one dimensional optical lattice.



- ▶ Quasi 2D single particle states

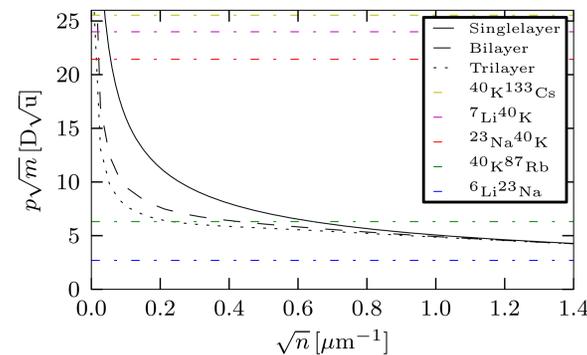
$$\psi_{l\mathbf{k}}(\vec{r}) \propto e^{-(z-ld)^2/(2w^2)} \cdot e^{i\mathbf{r}\cdot\mathbf{k}}$$

- ▶ Hopping suppressed in deep lattice

$$\mathcal{H} = \sum_{\mathbf{k}, l} \left( \frac{k^2}{2m} - \mu \right) \hat{c}_{\mathbf{k}, l}^\dagger \hat{c}_{\mathbf{k}, l} + \frac{1}{2A} \sum_{l, l'} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} V_{l-l'}(\mathbf{q}) \hat{c}_{\mathbf{k}+\mathbf{q}, l}^\dagger \hat{c}_{\mathbf{k}'-\mathbf{q}, l'}^\dagger \hat{c}_{\mathbf{k}', l'} \hat{c}_{\mathbf{k}, l}$$

- ▶ Dimensionless **coupling strength**  $g \equiv \frac{4mD^2k_F}{3\pi\hbar^2}$

## Comparison to Experimental Values



For a typical layer separation  $d = 1064 \text{ nm}/2$  and perpendicular polarization  $\theta = 0$ , the critical value of dipole moment times the square root of the mass,  $p\sqrt{m}$ , as a function of the square root of the density. Vertical lines: Permanent dipole moments for five dipolar fermionic molecules of alkali metals

## Methods

- ▶ A divergence in the static density-density response function at a momentum  $\mathbf{q}$  signifies spontaneous fluctuations in the density with this wave vector.
- ▶ The divergences of response function are found in mean field theory using a **conserving Hartree-Fock approximation** (HFA).

$$\chi_{ij}(q) = \delta_{ij} \left[ \text{bubble}(k, i; k, i) + \text{bubble}(k, i; k', j; k', j) \right] + \text{exchange}(k, i; k', j; k, i; k', j)$$

The Bethe-Salpeter equation for the particle-hole scattering matrix

$$\Gamma_{l,p}(k, k', q) = \Gamma_{l,p}(k, k', q) + \Gamma_{l,p}(k, k'', q) \Gamma_{l',p}(k'', k', q)$$

$$I_{l,p}^{HF}(k, k', q) = V_{l,p}(q) - \delta_{l,p} V_0(k-k')$$

HFA for the irreducible particle-hole interaction. Intralayer: Direct and exchange interactions, interlayer: Only direct interactions (distinguishable)

## Conclusions

- ▶ **Exchange** correlations within each layer **suppress** the density wave instability significantly.
- ▶ Conversely, interactions between dipoles in **different layers** **enhance** the density wave instability. This effect, which is strongest when the dipoles are oriented perpendicular to the planes, also causes the density waves in neighboring layers to be **in-phase** for all orientations of the dipoles.
- ▶ For several fermionic molecules of alkali metals, the density wave instability is expected to lie in the regime of **experimentally realisable densities**, where the multilayer effects are significant.

## References

Published in: Block J K, Zinner N T and Bruun G M 2012 *New J. Phys.* **14** 105006  
 [1] Image adapted from: Bruun G M and Taylor E 2008 *Phys. Rev. Lett.* **101** 245301