

The strong-coupling approach to nonequilibrium many-body physics with cold atoms and molecules

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Personnel working on this project

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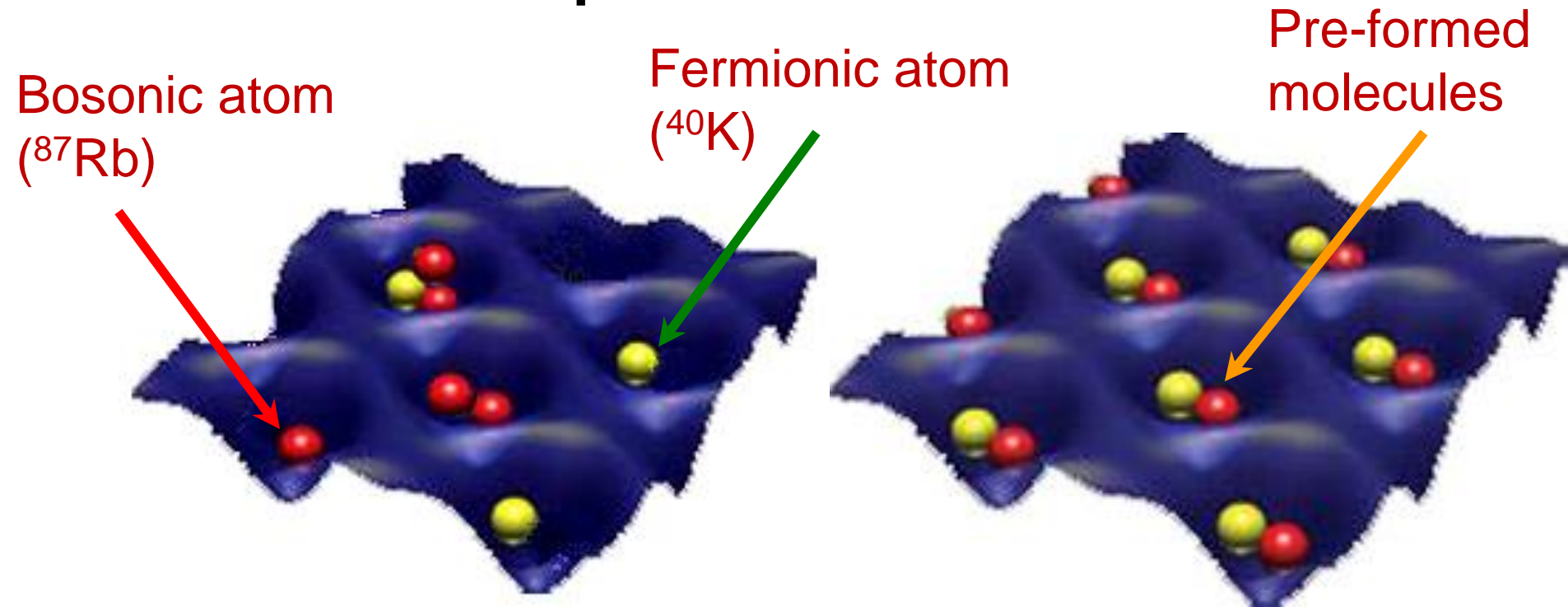
Andreas Dirks (postdoc at GU)

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Karlis Mickelsons (postdoc at GU)

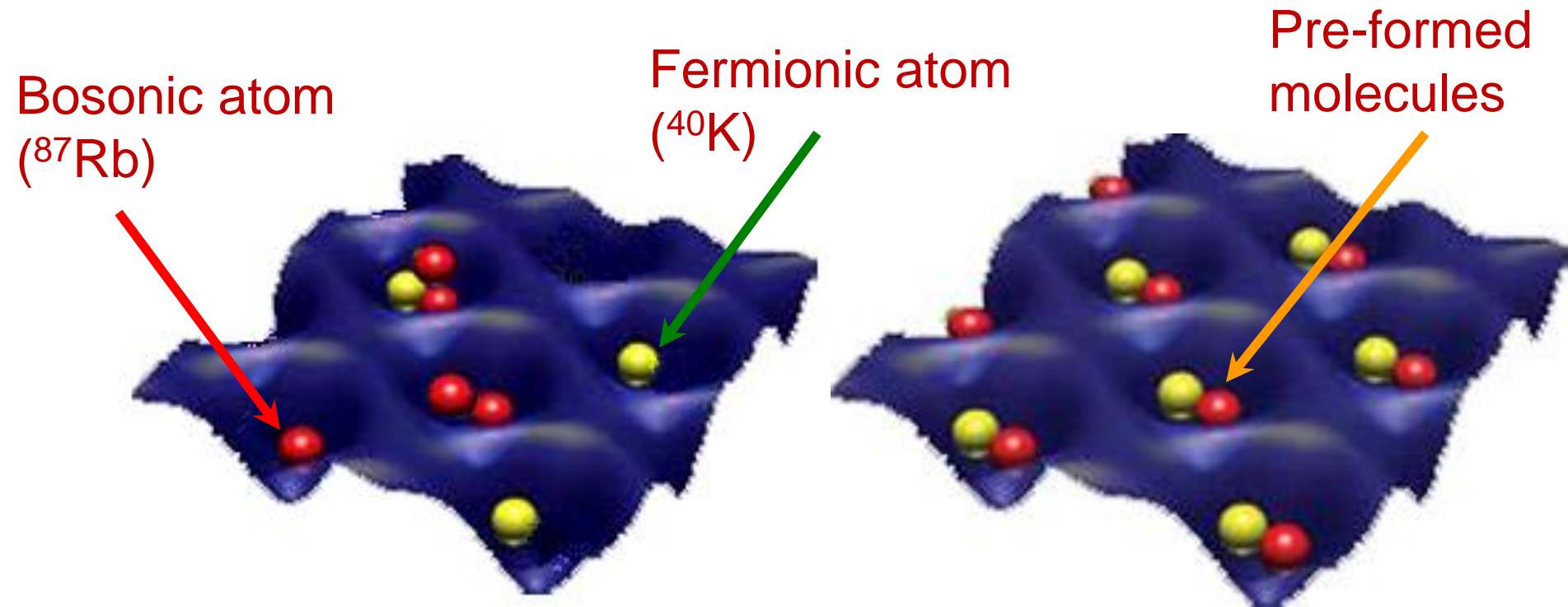


Pre-forming molecules on an optical lattice

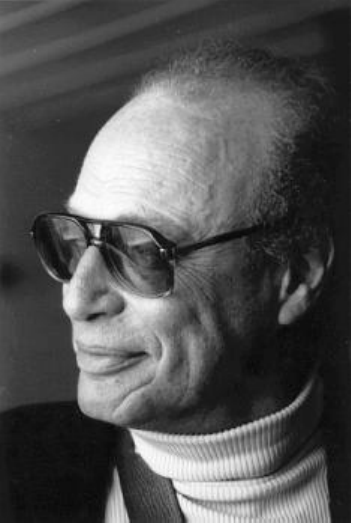


Since the Feshbach molecule formation occurs in a localized region of space, the pre-formed molecule efficiency determines the overall molecule formation efficiency

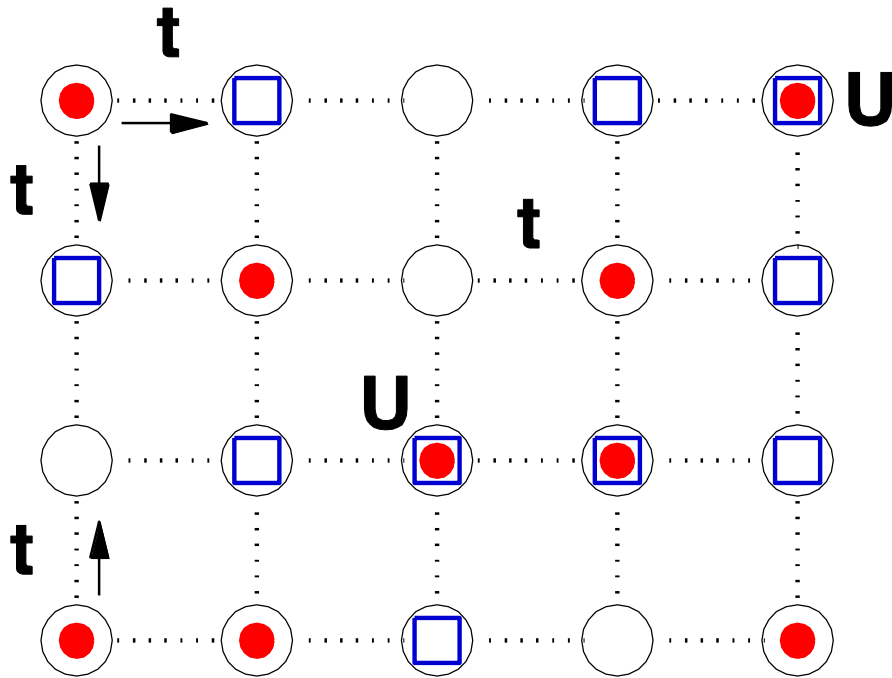
Equilibrium versus Nonequilibrium



Equilibrium calculations determine the long-time average value for the probability of pre-formed molecules **AFTER** thermalization. Most experiments to mix species together will do so using a nonequilibrium process.

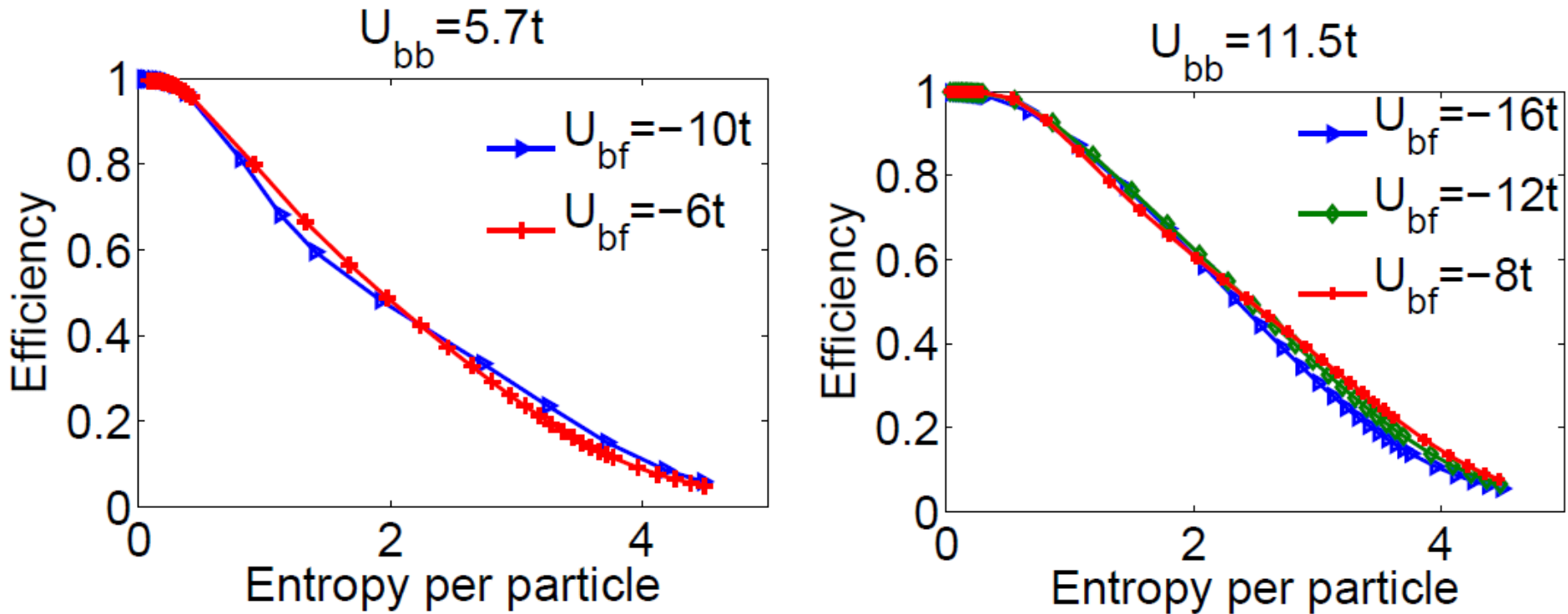


Falicov-Kimball Model

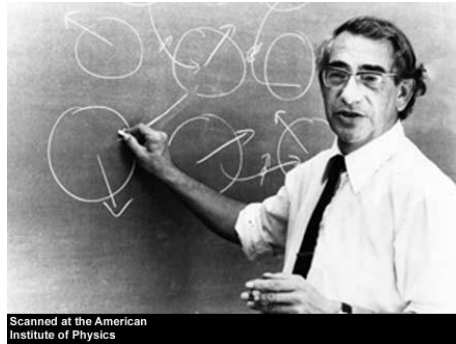


- Two kinds of particles: (i) **light fermions** and (ii) **heavy fermions or bosons**.
- When both particles are on **the same site** they interact with a correlation energy U .
- Many-body physics enters from an **annealed average over all localized electron configurations**.

Efficiency versus entropy (KRb case)



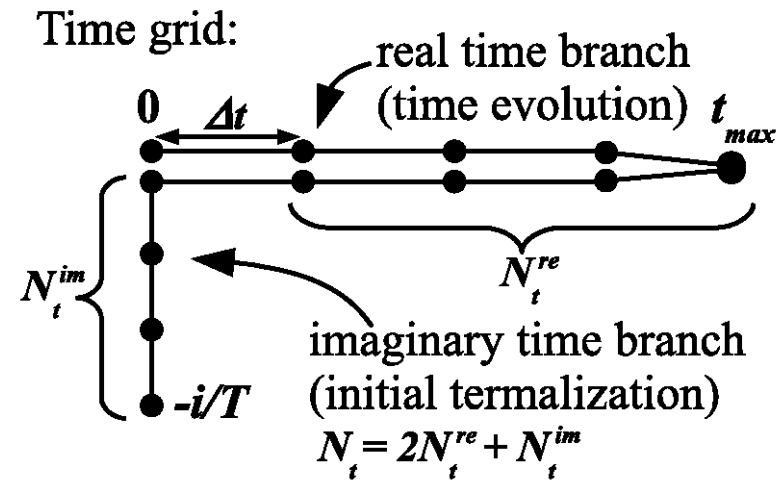
These results are for a 300x300 lattice and show universality. Note how the entropy per particle is larger for high T 's, because the particles are not artificially contained in a too small box.



Strong-coupling approach to nonequilibrium in the Hubbard model

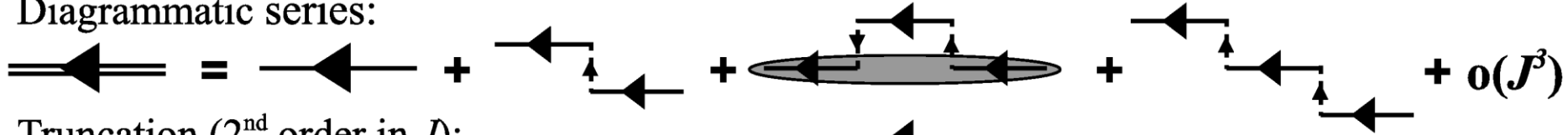
Strong-coupling approach

- Start from the atomic problem (zero hopping) which can be solved **exactly** for any $\mathbf{U}(t)$ in nonequilibrium.
- Turn on a **hopping term $\mathbf{J}(t)$** as a **perturbation**. We need to work on the Kadanoff-Baym-Keldysh contour in time to properly handle the time-dependence.
- Perturbation theory must be developed with care since there is **no Wick's theorem**. We need a **self-consistent PT** for the self-energy to generate damping effects.



Second-order expansion

Diagrammatic series:







Truncation (2nd order in J):



Resummation:

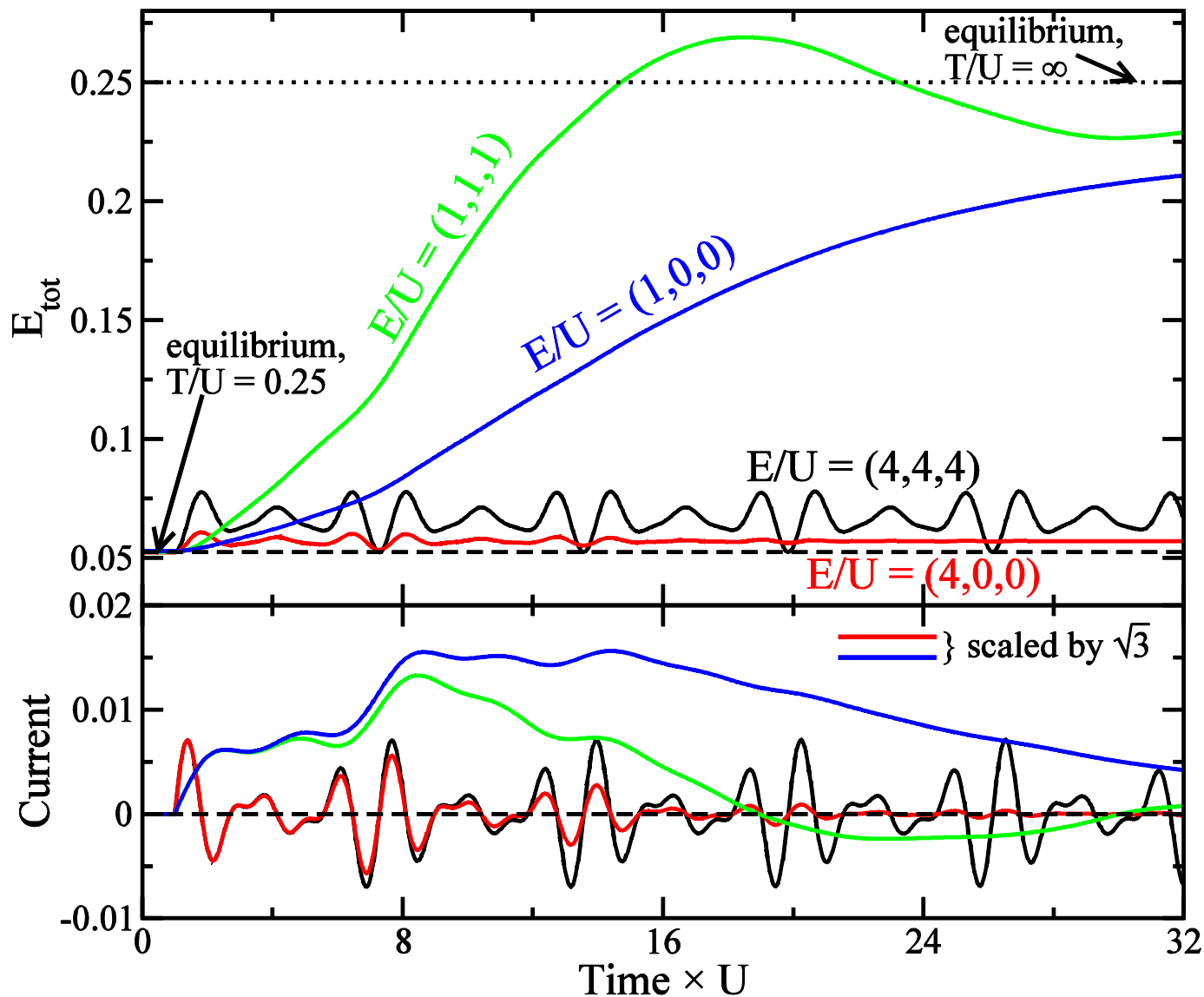


Scales like N^4

-  the bare single-particle Green's function
-  the dressed single-particle Green's function
-  hopping term
-  second-order cumulant

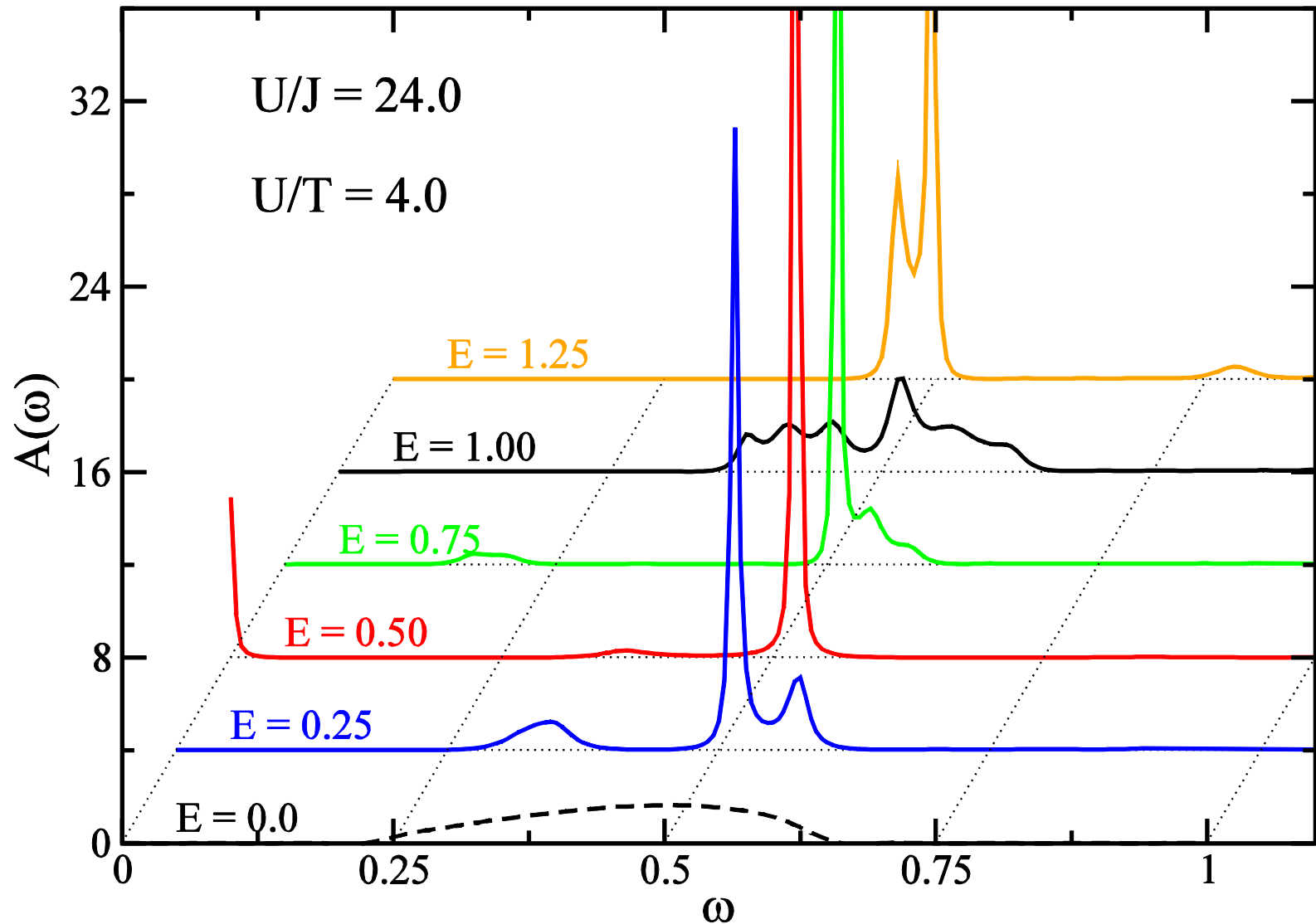
Bloch oscillations for a Fermi Hubbard model on a periodic lattice with a uniform constant electric field

Bloch oscillations

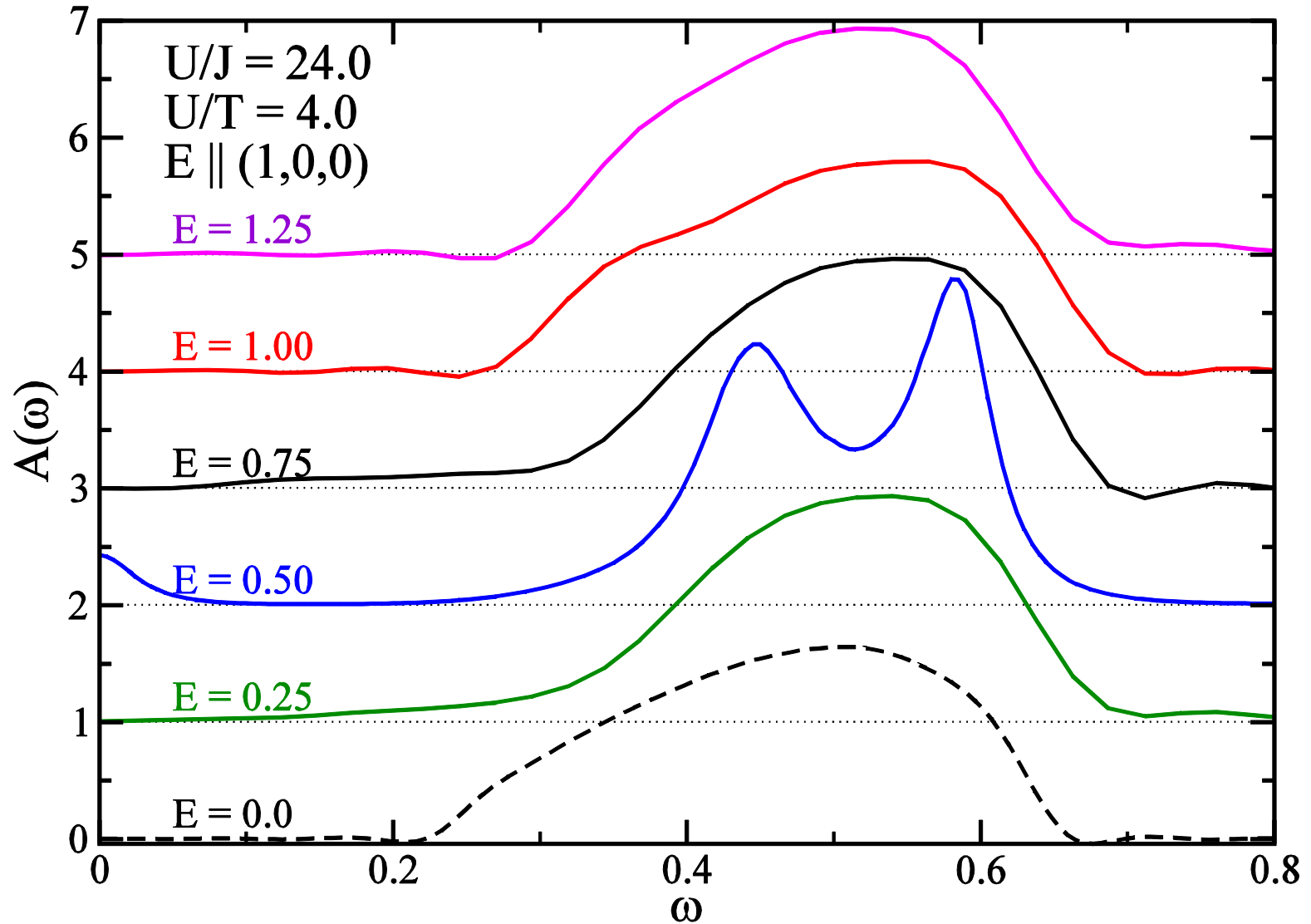


Rapid thermalization when $E=U$. Much slower thermalization when E is different from U due to a metastable transient state.

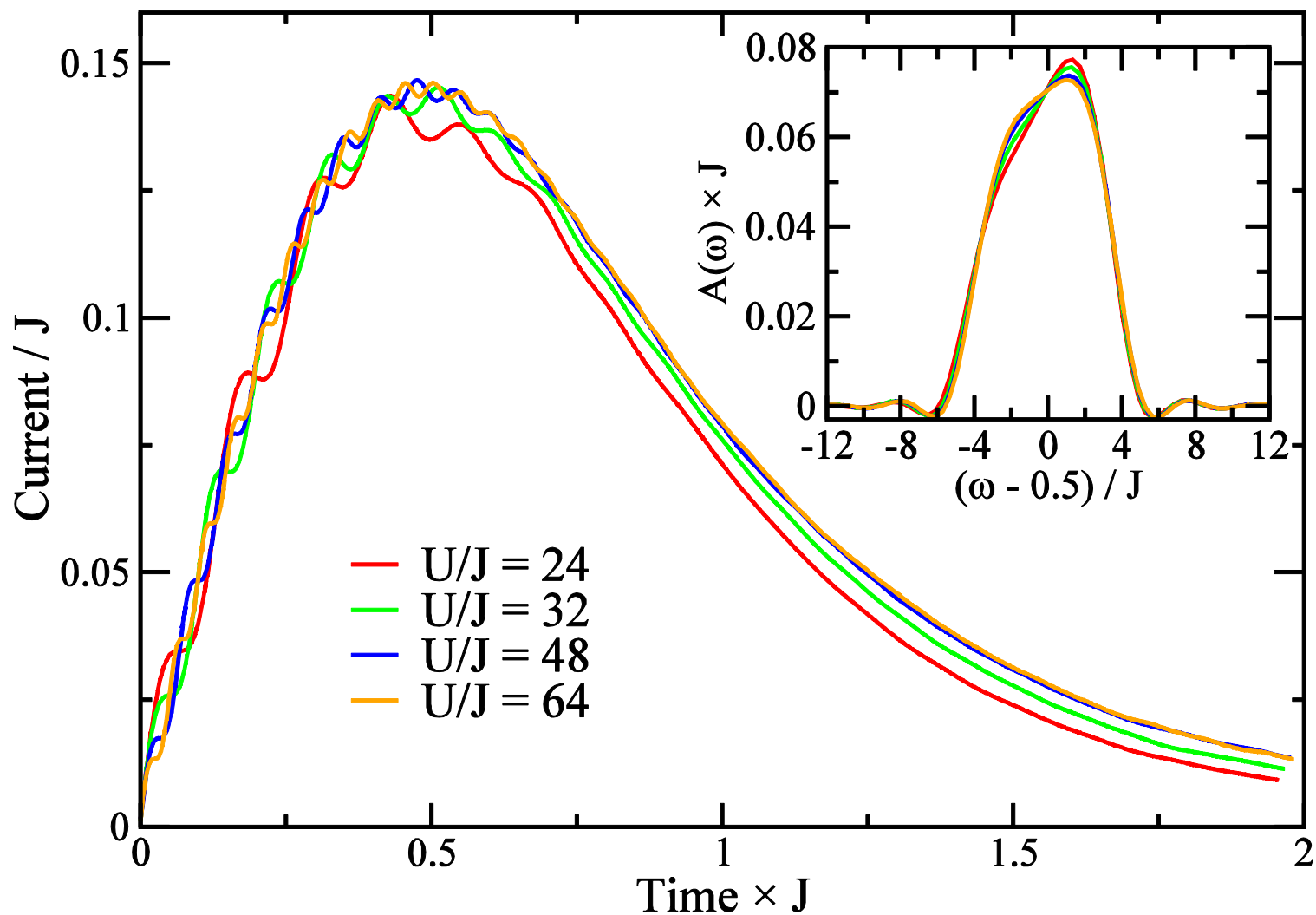
“steady state” density of states (E diagonal)



“steady state” density of states (E axial)

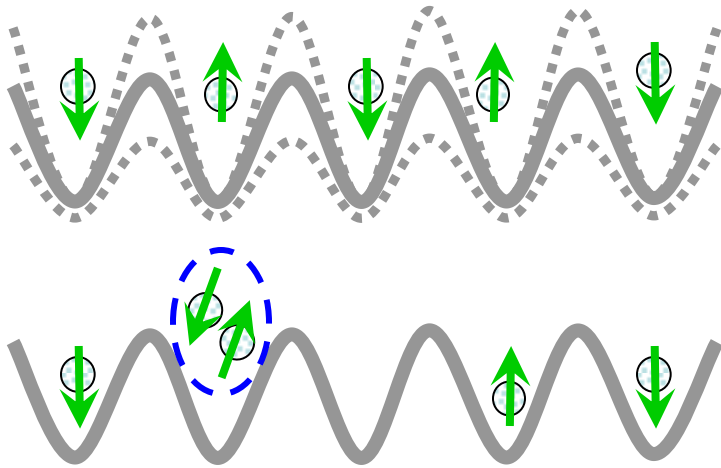


Quasi-universal scaling on resonance



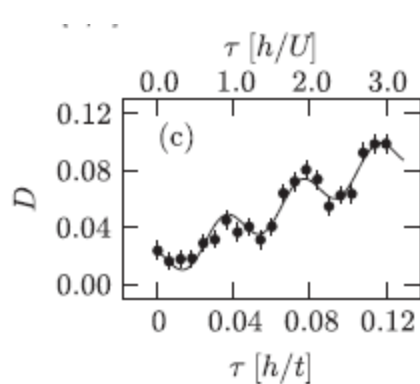
Lattice modulation spectroscopy

Modulation spectroscopy experiment



Modulate lattice potential V_0

Measure number of doubly occupied sites that are formed



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PHYSICAL REVIEW LETTERS

week ending
8 APRIL 2011

Probing Nearest-Neighbor Correlations of Ultracold Fermions in an Optical Lattice

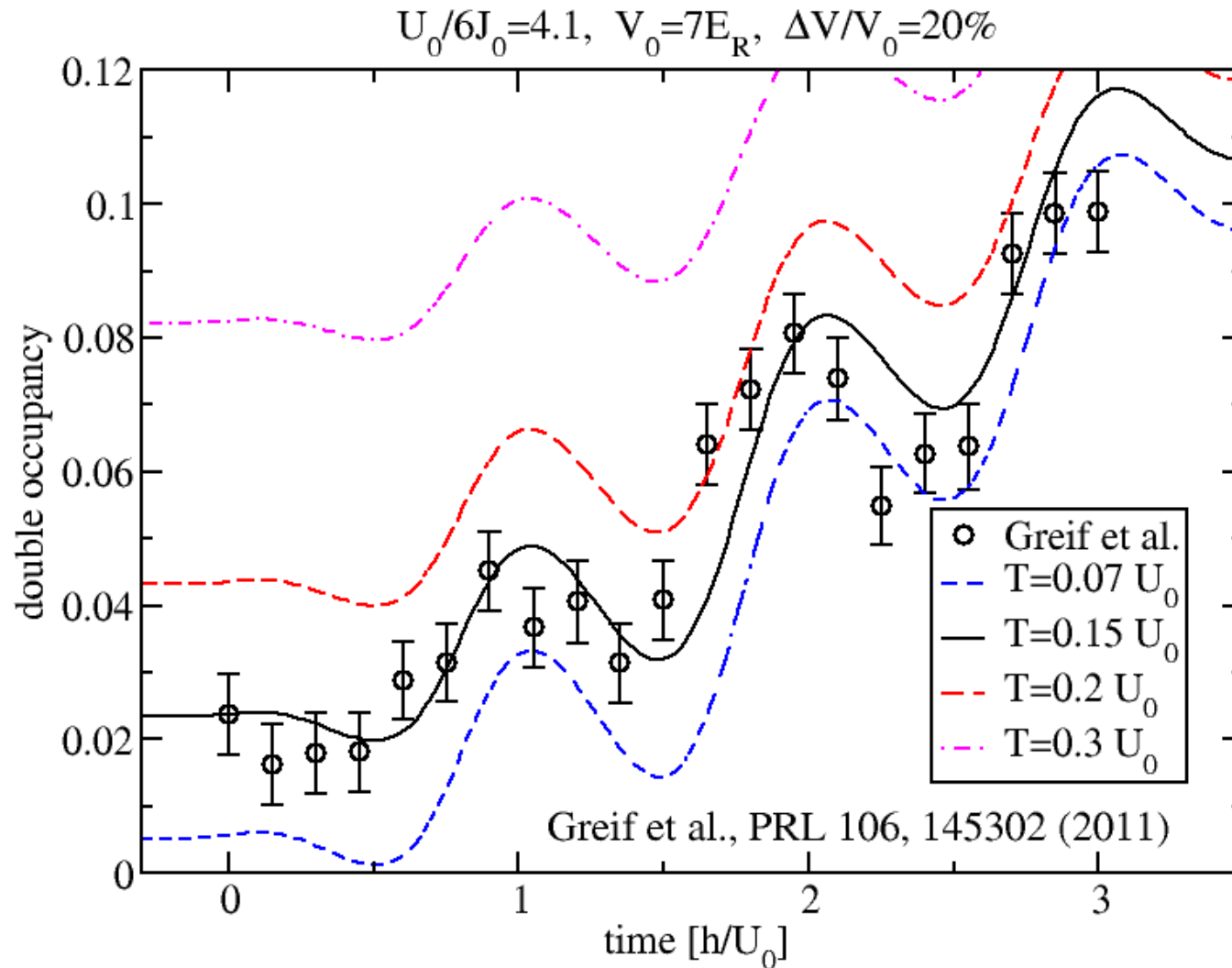
Daniel Greif, Leticia Tarruell,* Thomas Uehlinger, Robert Jördens, and Tilman Esslinger

Institute for Quantum Electronics, ETH Zurich, 8093 Zurich, Switzerland

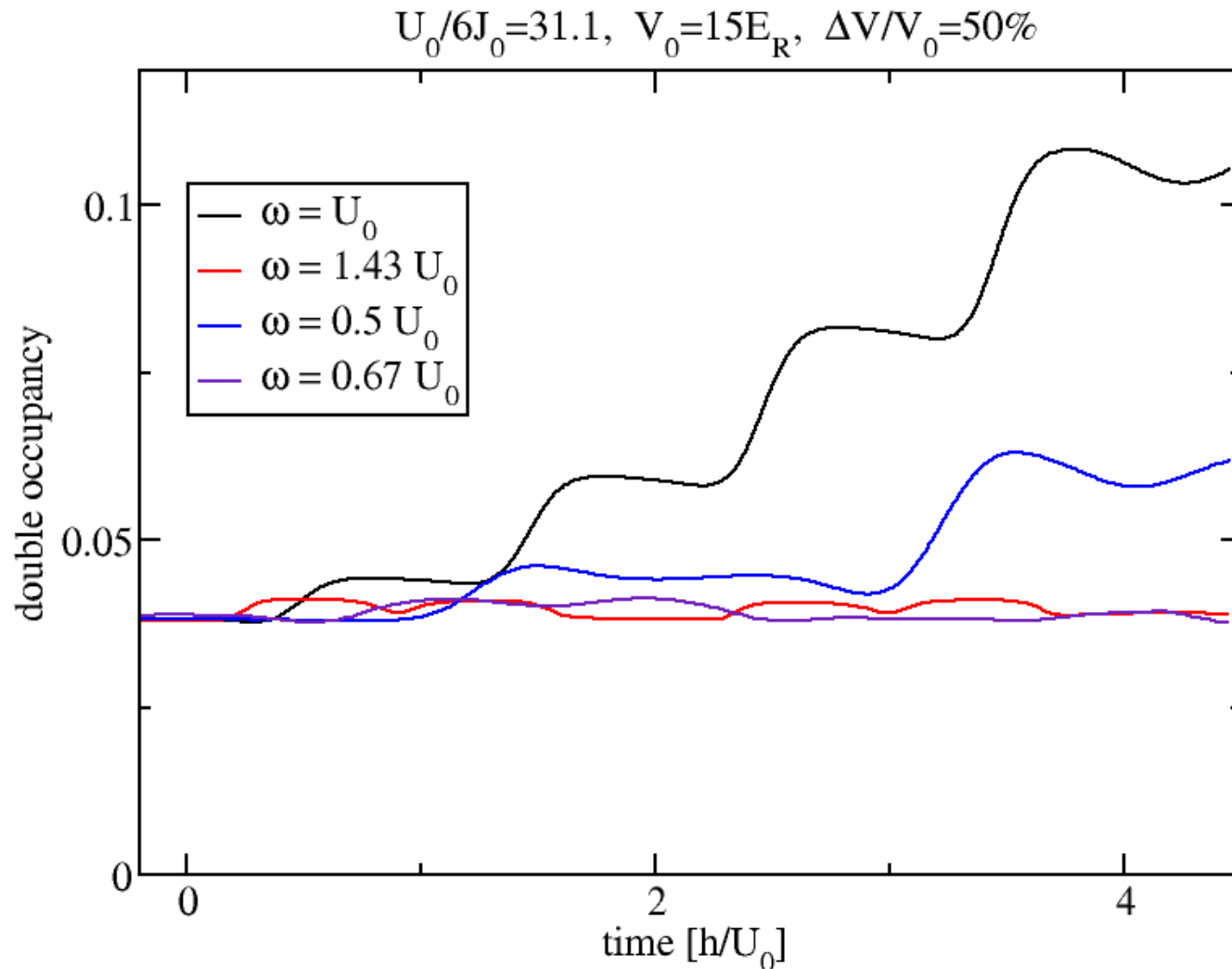
(Received 2 December 2010; revised manuscript received 1 February 2011; published 5 April 2011)

We demonstrate a probe for nearest-neighbor correlations of fermionic quantum gases in optical lattices. It gives access to spin and density configurations of adjacent sites and relies on creating additional doubly occupied sites by perturbative lattice modulation. The measured correlations for different lattice temperatures are in good agreement with an *ab initio* calculation without any fitting parameters. This probe opens new prospects for studying the approach to magnetically ordered phases.

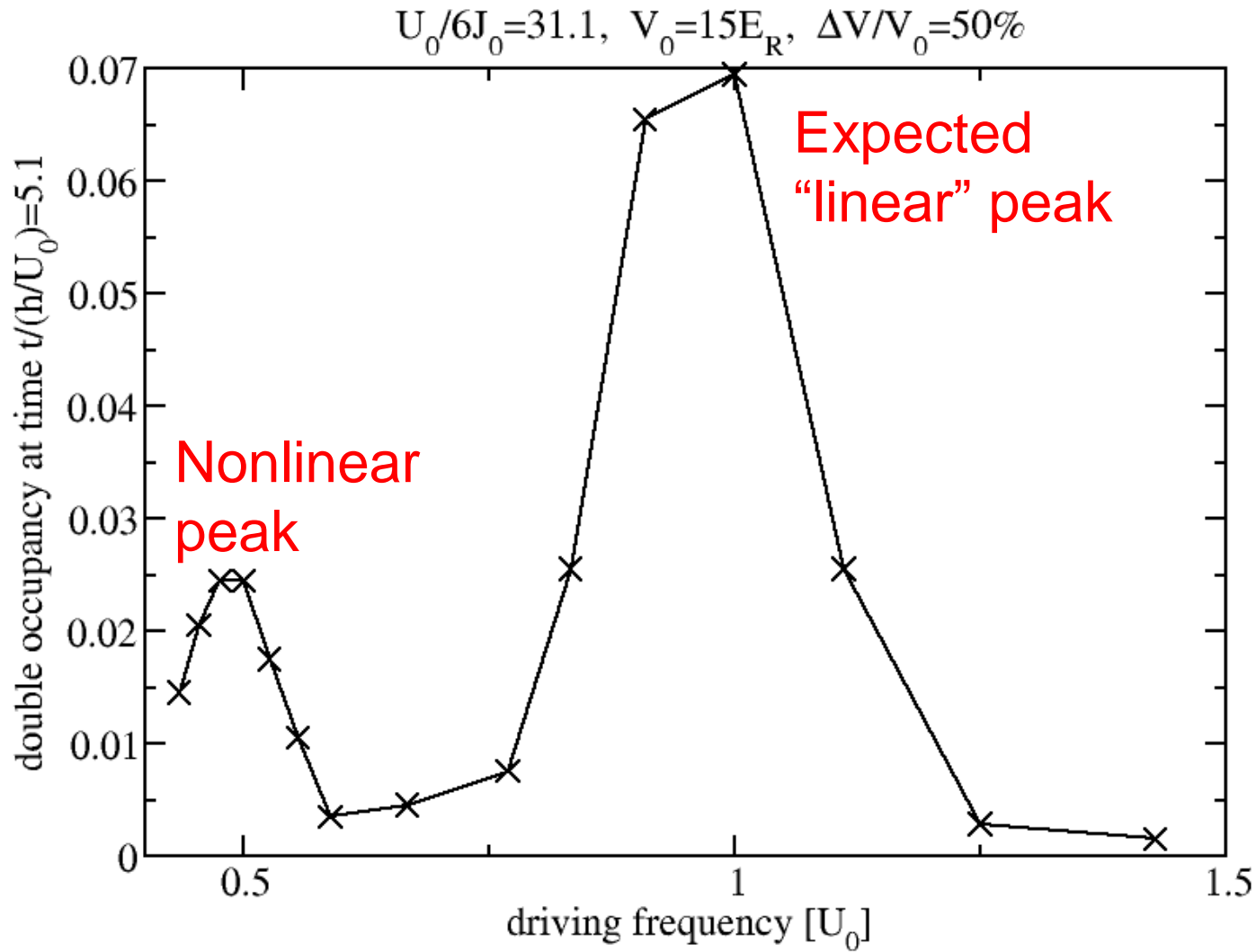
Modulation spectroscopy theory



Resonance when driving frequency= U , $U/2$



High amplitude spectroscopy



Thermalization from the real time perspective

“Causality” for the two-time Green’s function

Regions in time

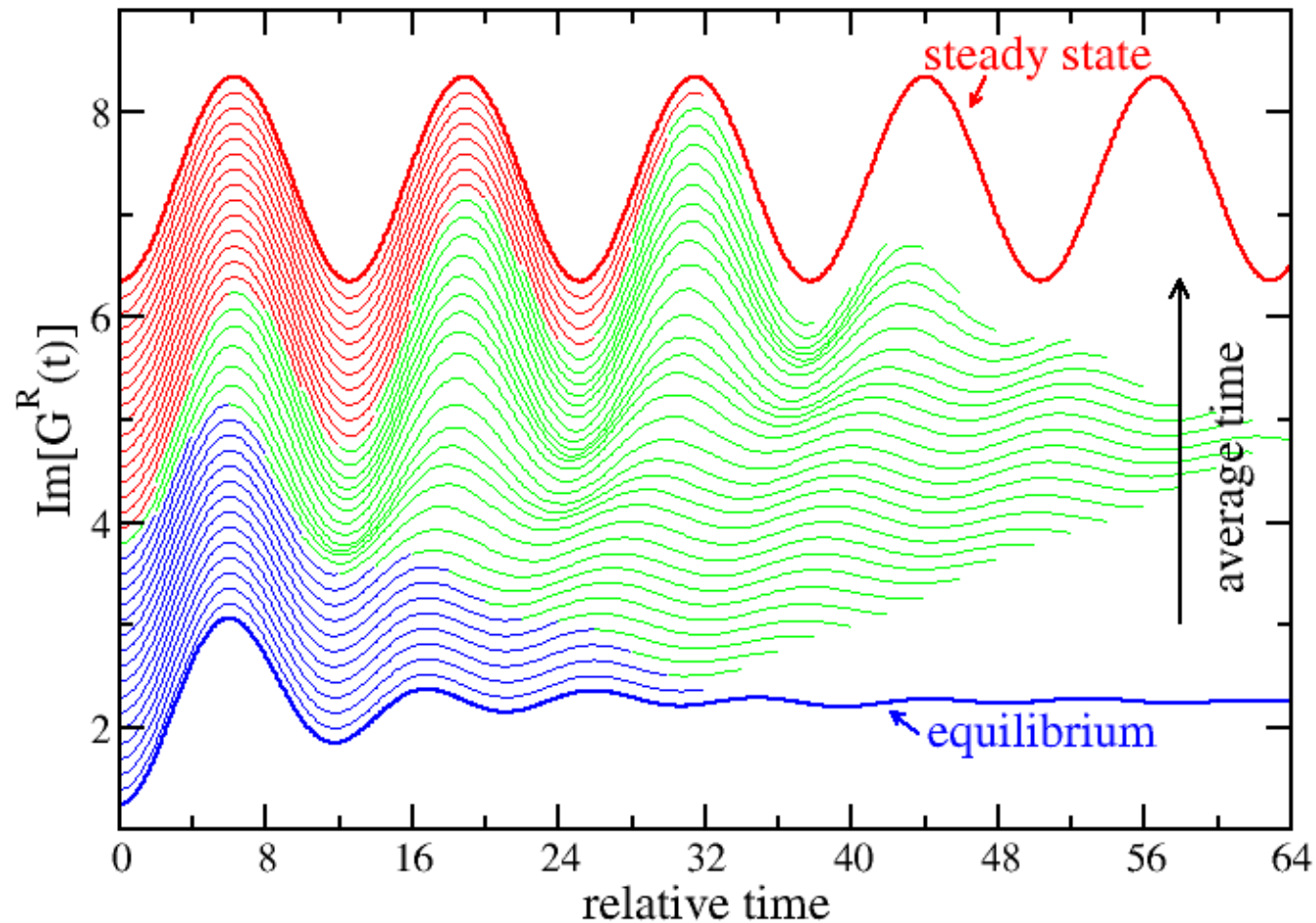
Both times before field turned on gives the equilibrium retarded Green's function

Both times after the field is turned on gives the steady state nonequilibrium retarded Green's function

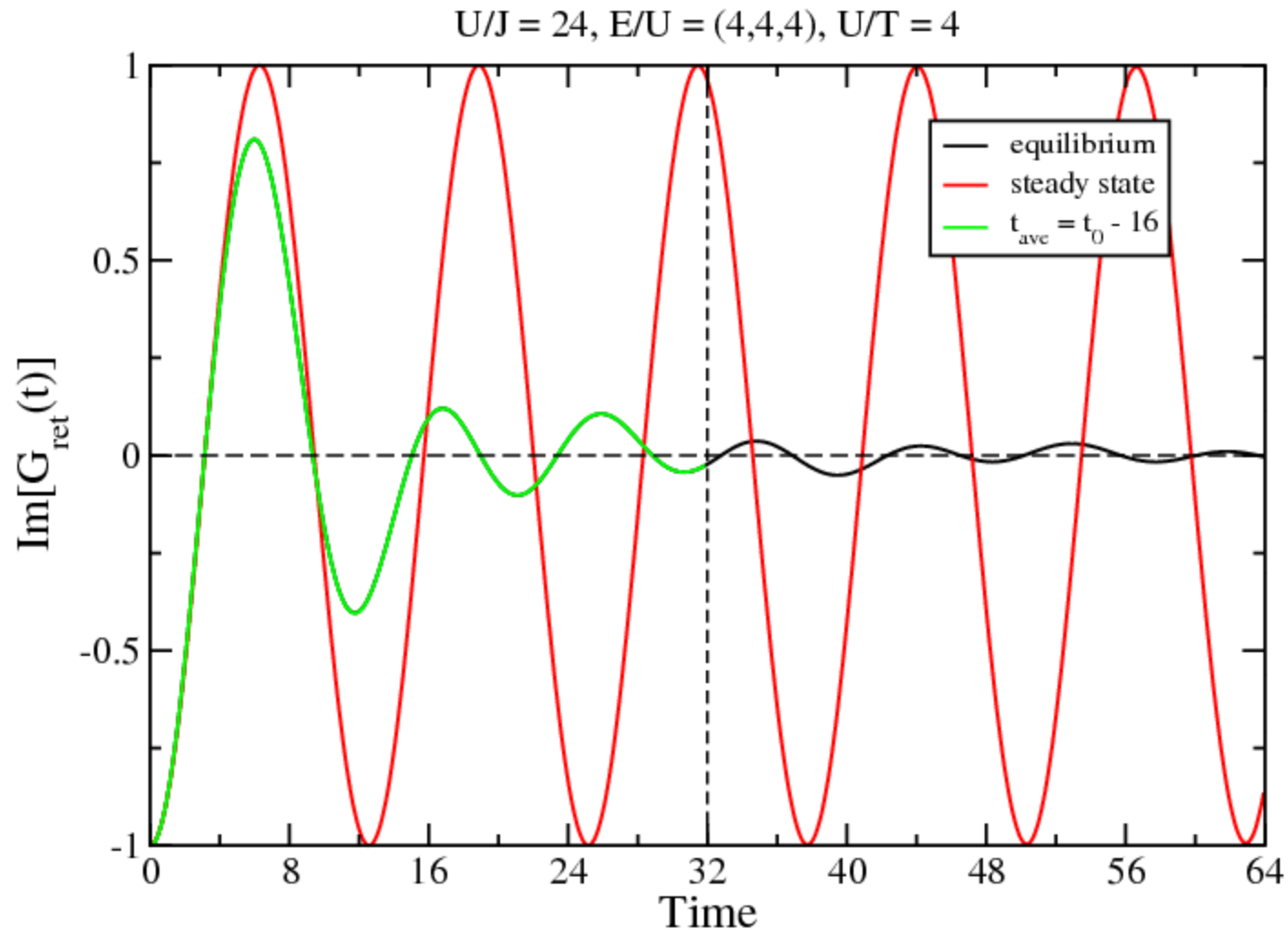
One time before and one time after gives the mixed retarded Green's function which interpolates between the two

Retarded GF (Hubbard model)

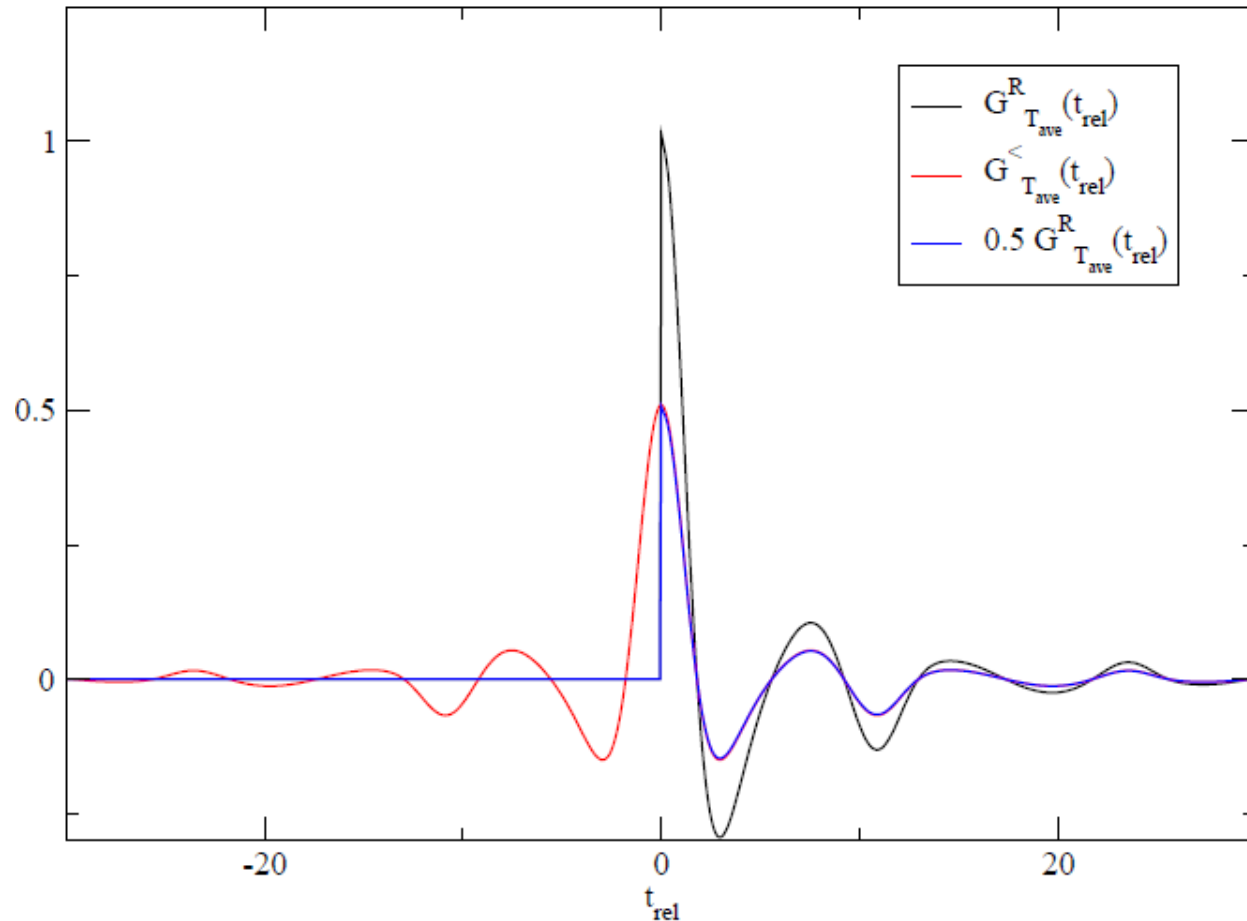
$U/J = 24, U/T = 4, E/U = (4,4,4)$



Movie of evolution



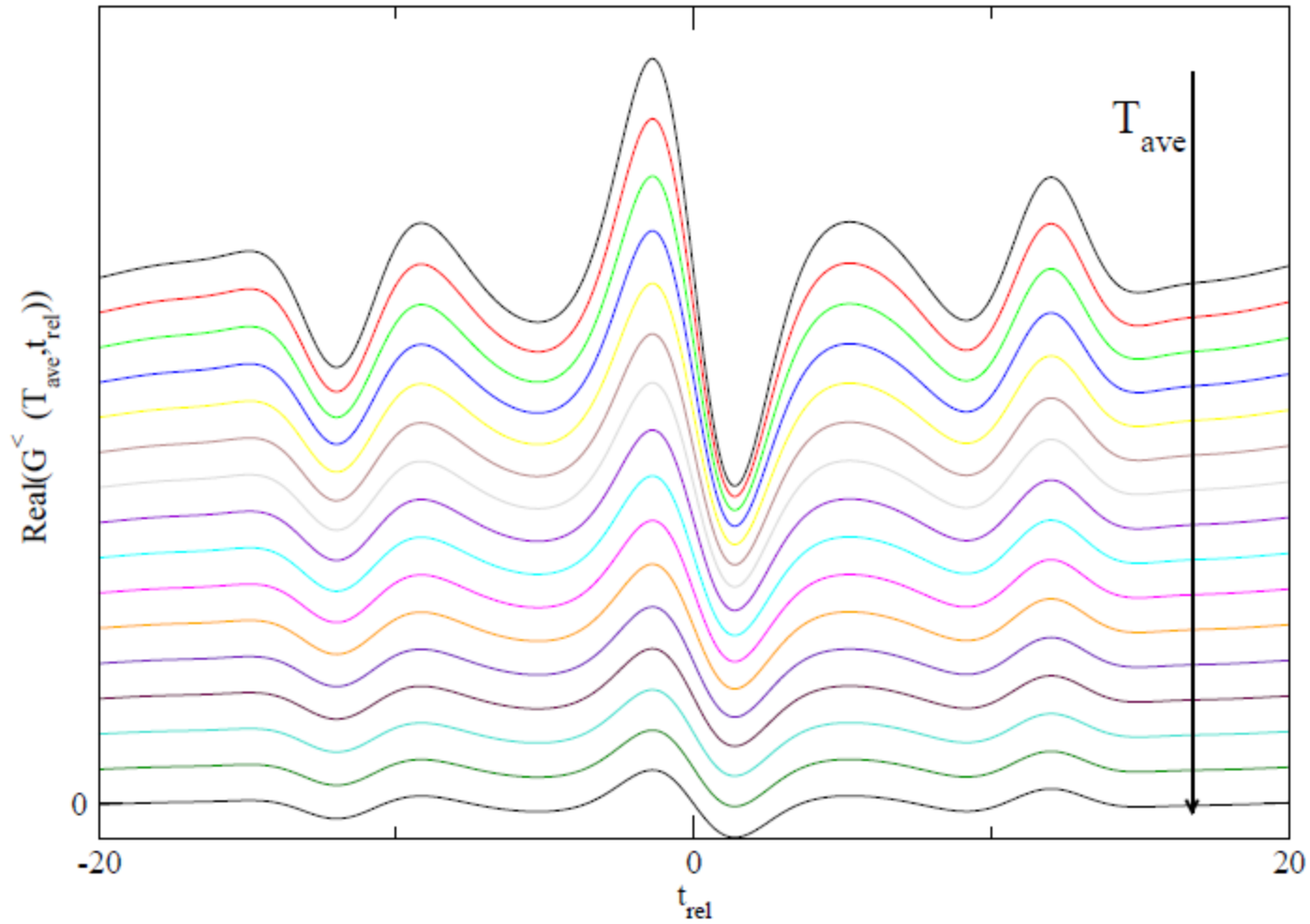
$\text{Im } G^< = G^R/2$ FK model (also Hubbard)



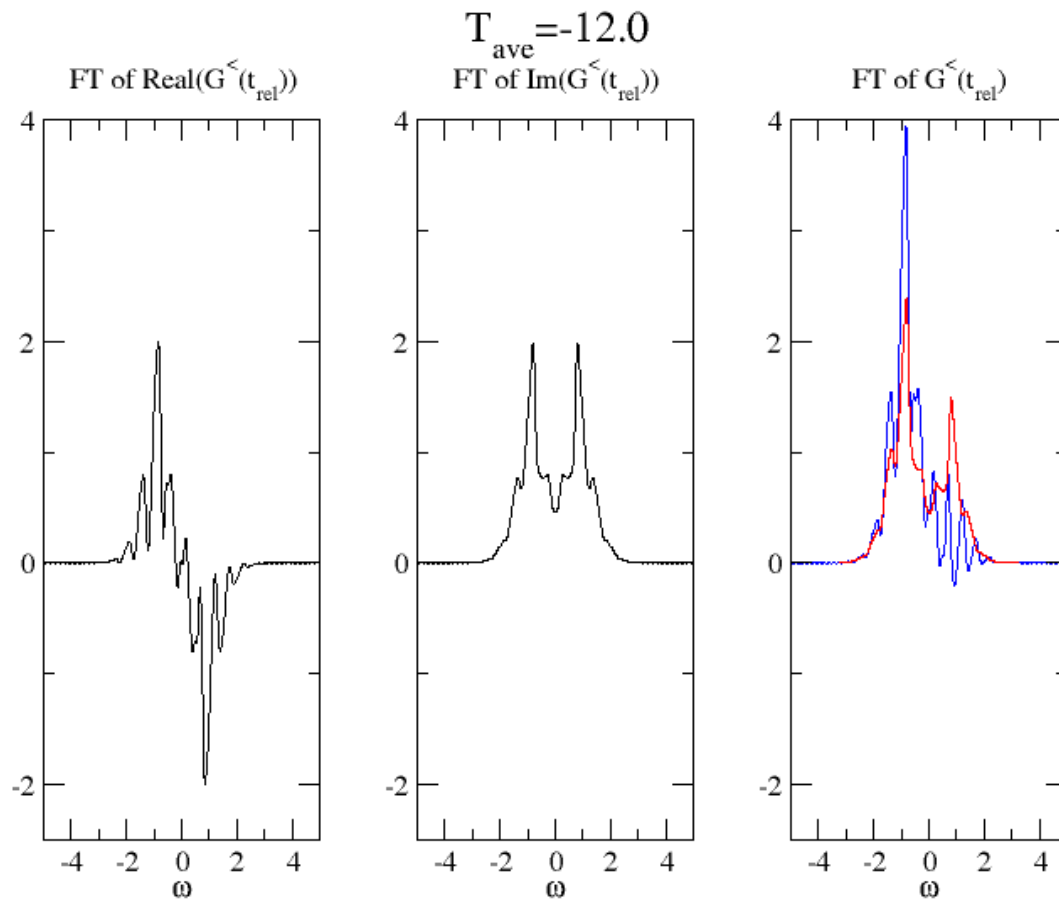
Thermalization comes from $\text{Re}G^<$

In the Hubbard model, there is no simple way that the $\text{Re}G^<$ goes to zero as the steady state is approached. But for the Falicov-Kimball model, it is often just a simple exponential decay.

Decay of $\text{Re } G^< \text{ (FK model)}$



Evolution of local lesser GF (FK)



Nonequilibrium on an optical lattice

Second-order expansion is the same

Diagrammatic series:

$$\text{Diagrammatic series: } \leftarrow\!\!\leftarrow = \leftarrow + \begin{array}{c} \leftarrow \\ \vdots \\ \leftarrow \end{array} + \text{cumulant} + \begin{array}{c} \leftarrow \\ \vdots \\ \leftarrow \\ \vdots \\ \leftarrow \end{array} + o(J^3)$$

Truncation (2nd order in J):

$$\text{Truncation (2}^{\text{nd}} \text{ order in } J): \leftarrow\!\!\leftarrow \approx \leftarrow + \begin{array}{c} \leftarrow \\ \vdots \\ \leftarrow \end{array} + \text{cumulant}$$

Resummation:

$$\text{Resummation: } \leftarrow\!\!\leftarrow \approx \leftarrow + \begin{array}{c} \leftarrow \\ \vdots \\ \leftarrow\!\!\leftarrow \end{array} + \text{cumulant}$$

- the bare single-particle Green's function
- the dressed single-particle Green's function
- hopping term
- second-order cumulant

But have to work in real space and perform self-consistency using properties of neighbors

Optical lattice with a trap

- Problem requires significant extra computation due to the need to find the inverse of a large block sparse matrix multiplying a large number of different vectors.
- Currently, we are employing algorithms to do this based on iterative sparse-matrix techniques like GMRES and on local truncation in (relative) space and time coordinates due to short coherence lengths of excitations.
- Expect we will have results for two-dimensional nonequilibrium systems with modulation spectroscopy soon.

Conclusions

Showed how strong coupling approach should be useful to apply to preformed molecule problem

Developed a nonequilibrium approach that incorporates damping effects

Applied to Bloch oscillations, modulation spectroscopy, and thermalization problems