University
of Trento

KITP, March 4, 2013
Fundamental Science and Applications
of Ultra-cold Polar Molecules

CNR-INO

Superstripes and the excitation spectrum of a SO coupled BEC

Yun Li, Giovanni Martone, Lev Pitaevskii and Sandro Stringari

- Recent **experimental realization** of spin-orbit coupled configurations provide new challenging many-body physics in ultra-cold atomic gases

- **Conservation laws are modified** by spin coupling (current is not conserved)

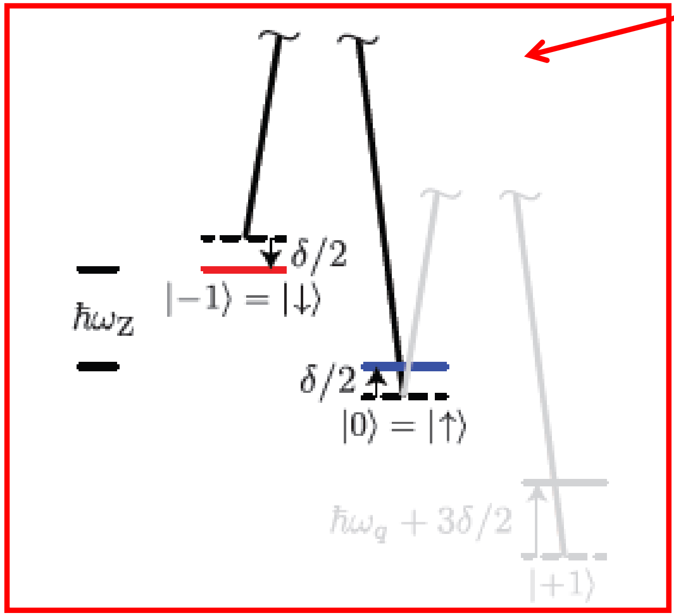
- **New quantum phases** and new phase transitions

- **New dynamic** properties (to be discussed today)

- Focus on **stripe phase** and connections with **supersolidity**

Stripe phase exhibits spontaneous breaking of both gauge (**BEC**) and translational (**crystal**) symmetries

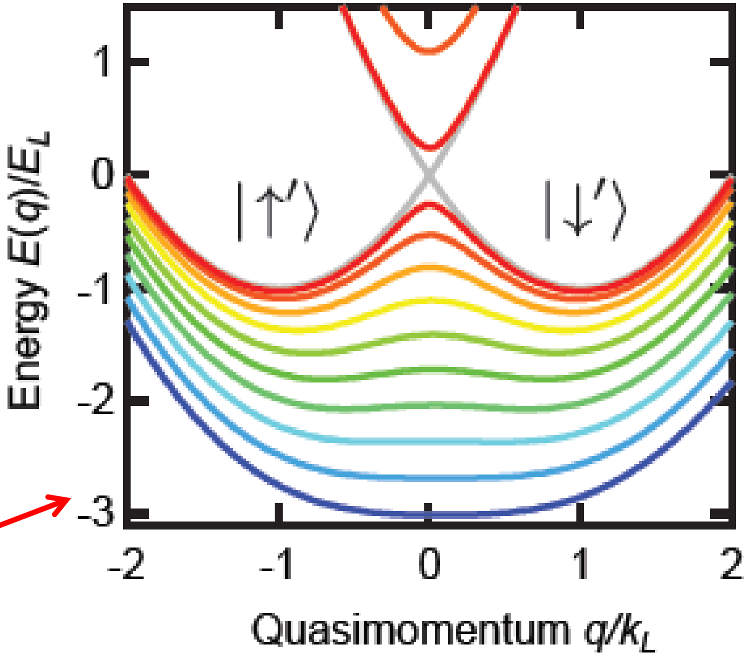
Two polarized laser beams



Local rotation in spin space

$$h_0 = \frac{1}{2}[(p_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z + V_{ext}$$

Raman coupling fixed by laser intensity



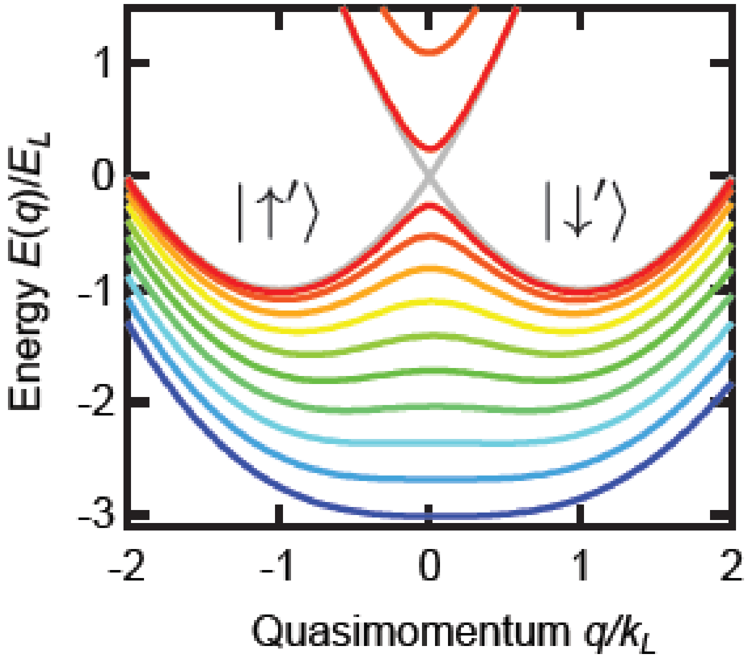
BEC with two minima

If effective magnetic field is zero two minimima are given by

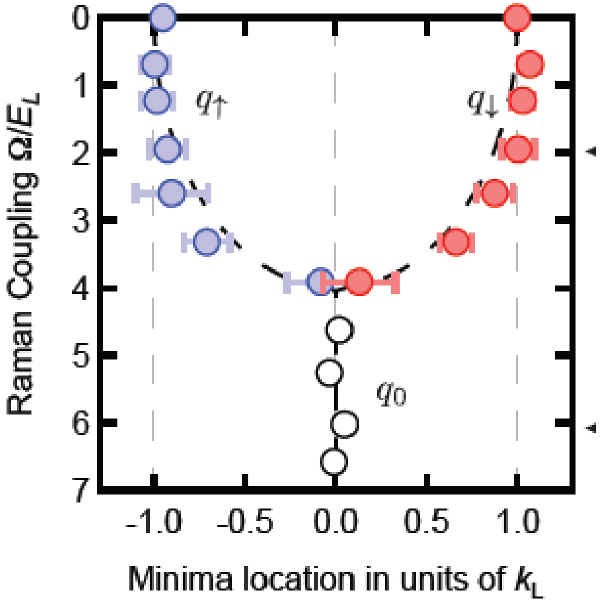
$$\pm k_1 = \pm k_0 \sqrt{1 - \Omega^2 / 4k_0^4}$$

Two minima disappear for

$$\Omega \geq 2k_0^2$$



First experimental
Implementation with
BEC's
Spielman, Nature 2011)



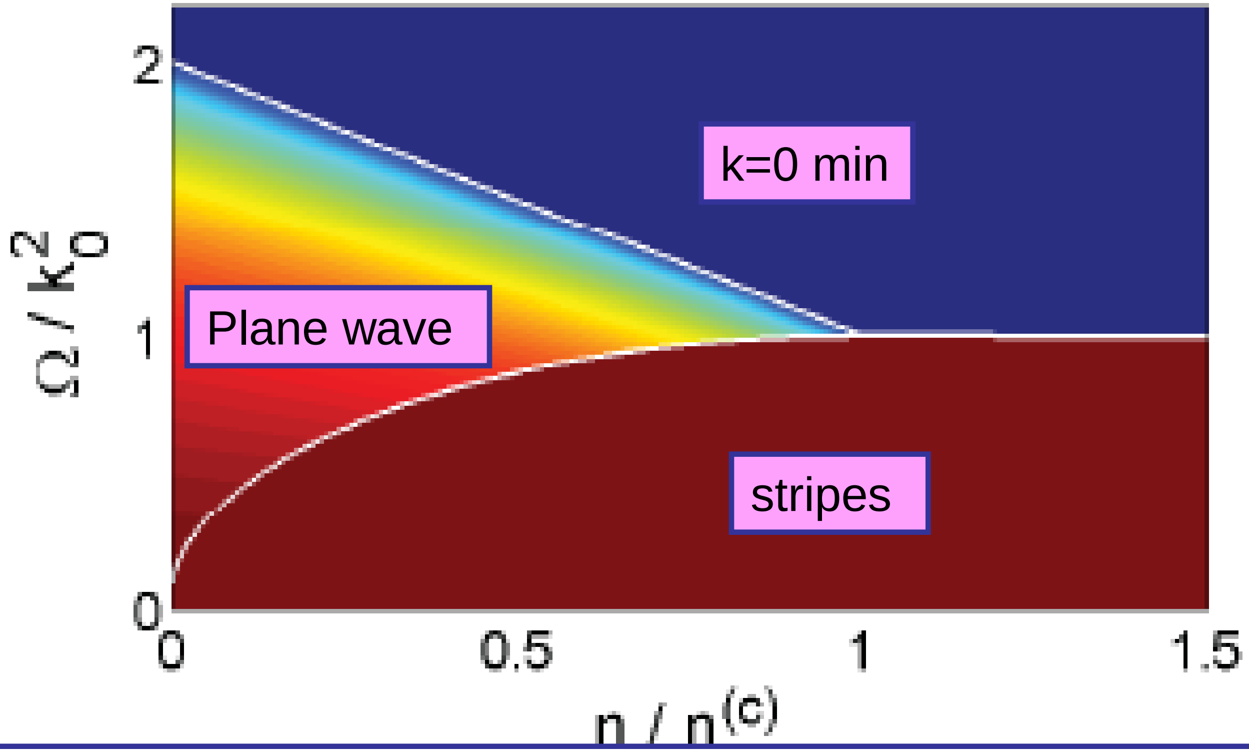
The new quantum phases

Theory:

Ho and Zhang (PRL 2011)

many theoretical papers (……)

Recent Trento paper (Li, Pitaevskii, S.S, PRL 2012)



Order parameter in the new phases

I) **Stripe phase**

$$\boxed{\Psi = \sqrt{\frac{N}{2V}}\left[\begin{pmatrix}\cos\theta \\ -\sin\theta\end{pmatrix}e^{ik_1x} + \begin{pmatrix}\sin\theta \\ -\cos\theta\end{pmatrix}e^{-ik_1x}\right]}$$

\+ higher harmonics

$$\boxed{\cos\frac{\theta}{2} = \frac{k_1}{k_0}}$$

$$\boxed{n(x) = n(1 + \frac{\Omega}{2k_0^2}\cos 2k_1x)}$$

← density modulations

I) **Plane wave phase**

$$\boxed{\Psi = \sqrt{\frac{N}{2V}}\begin{pmatrix}\sin\theta \\ -\cos\theta\end{pmatrix}e^{-ik_1x}}$$

or

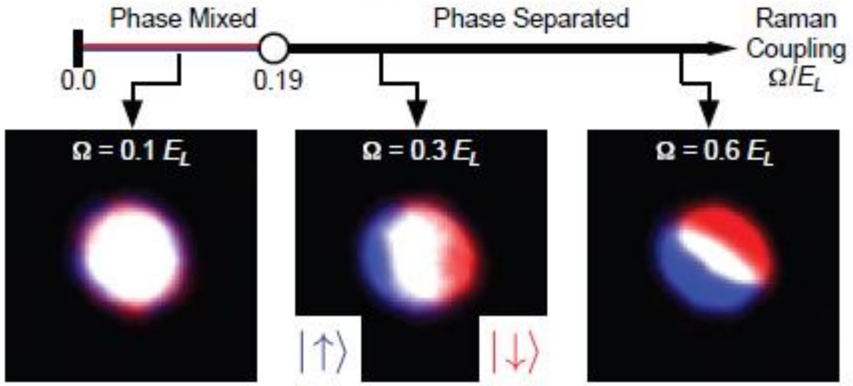
$$\Psi = \sqrt{\frac{N}{2V}}\begin{pmatrix}\cos\theta \\ -\sin\theta\end{pmatrix}e^{ik_1x}$$

$$\boxed{<\sigma_z> = \frac{k_1}{k_0}}$$

II) **Zero momentum phase**

$$\boxed{\Psi = \sqrt{\frac{N}{V}}\begin{pmatrix}1 \\ -1\end{pmatrix}}$$

Transition between mixed (stripe) and spin-separated (plane wave) phases observed at the predicted value of the Raman coupling $\Omega_{cr} = 2k_0^2\sqrt{\frac{2\gamma}{1+2\gamma}}$ with $\gamma = \frac{g_{\uparrow\uparrow}-g_{\uparrow\downarrow}}{g_{\uparrow\uparrow}+g_{\uparrow\downarrow}}$

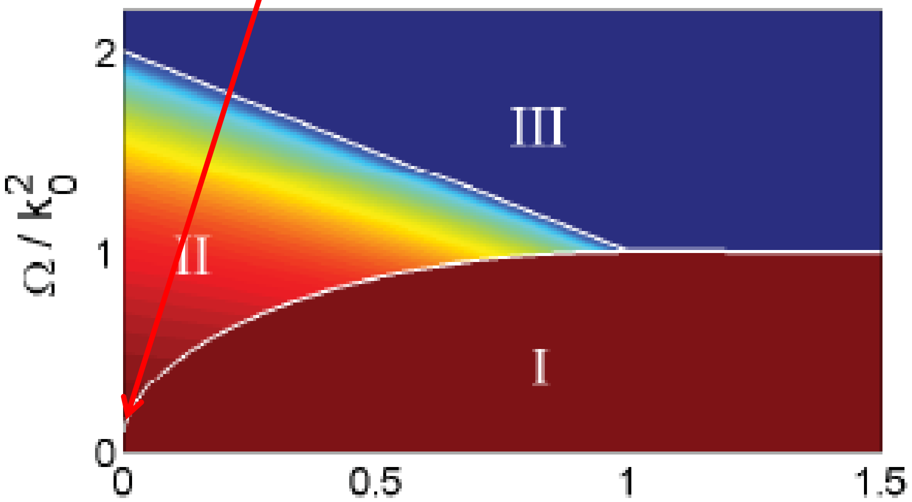


Lin et al., Nature 2011

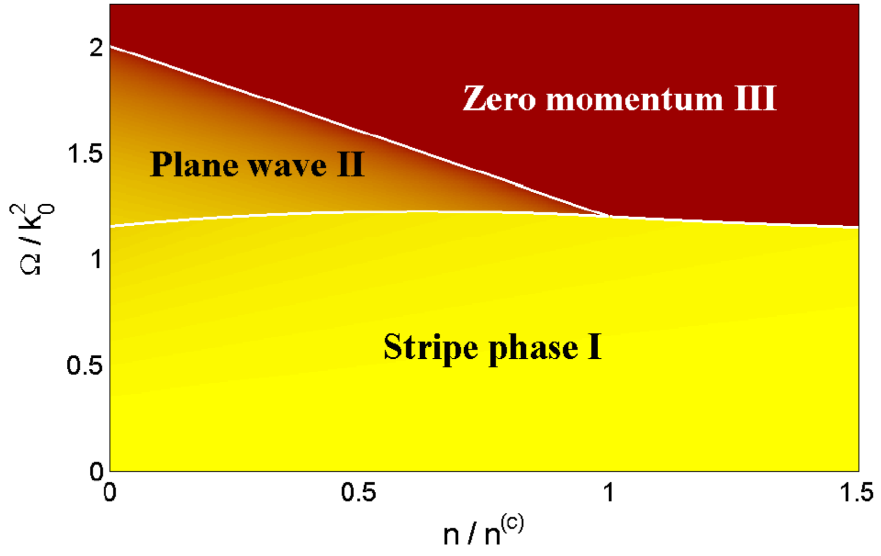
Contrast in density modulations is not visible in **in situ density profile**.

Effects can be more easily revaled by studying dynamic behavior (**excitation spectrum**)

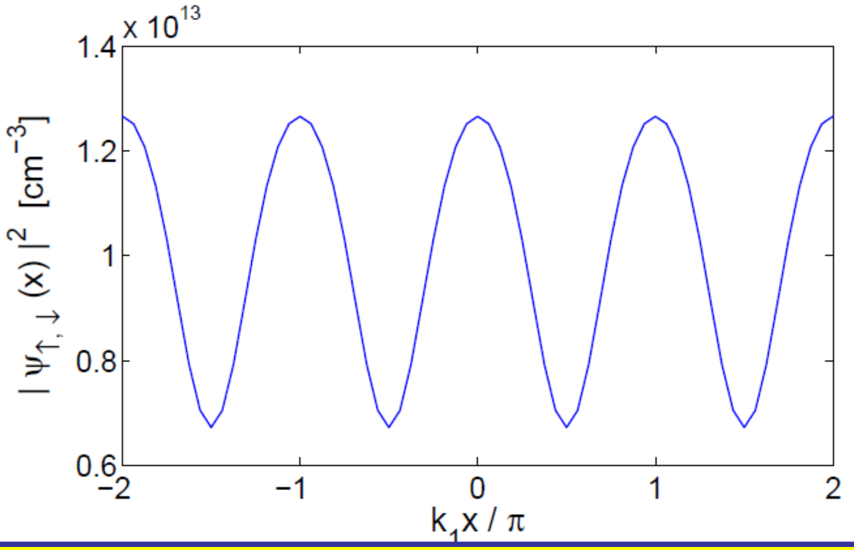
In Rb Ω_{cr} is small. To increase the effects of the contrast (fixed by Ω) choose larger values of $\gamma = \frac{g_{\uparrow\uparrow} - g_{\uparrow\downarrow}}{g_{\uparrow\uparrow} + g_{\uparrow\downarrow}}$



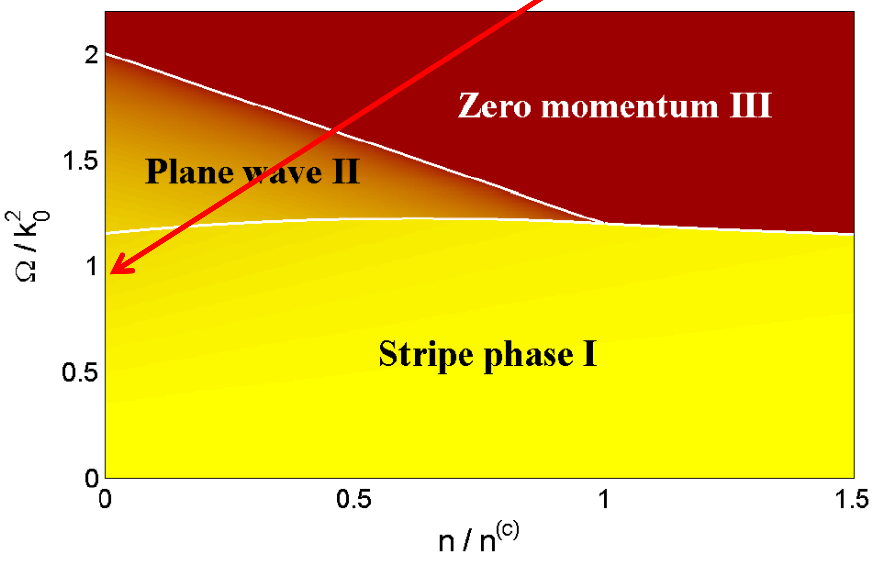
Rb parameters in Spielman exp



Optimized choice with larger γ



Density modulations at $\Omega = k_0^2$



Bogoliubov excitation spectrum

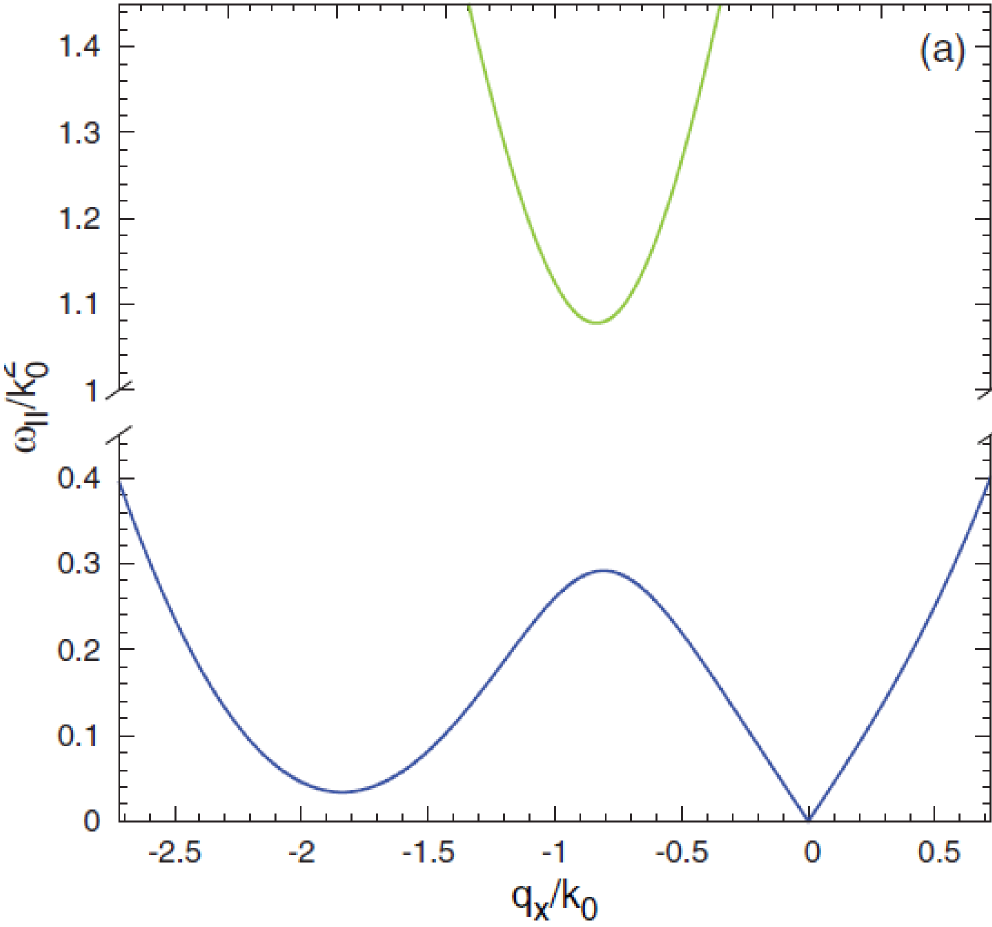
(Martone et al. PRA, 2012)

Plane wave phase

Despite spinor nature the occurrence of Raman coupling gives rise to a **single gapless** branch

Occupation of sp state with finite momentum gives rise to **rotonic structure.**

Anisotropy + parity breaking in excitation spectrum

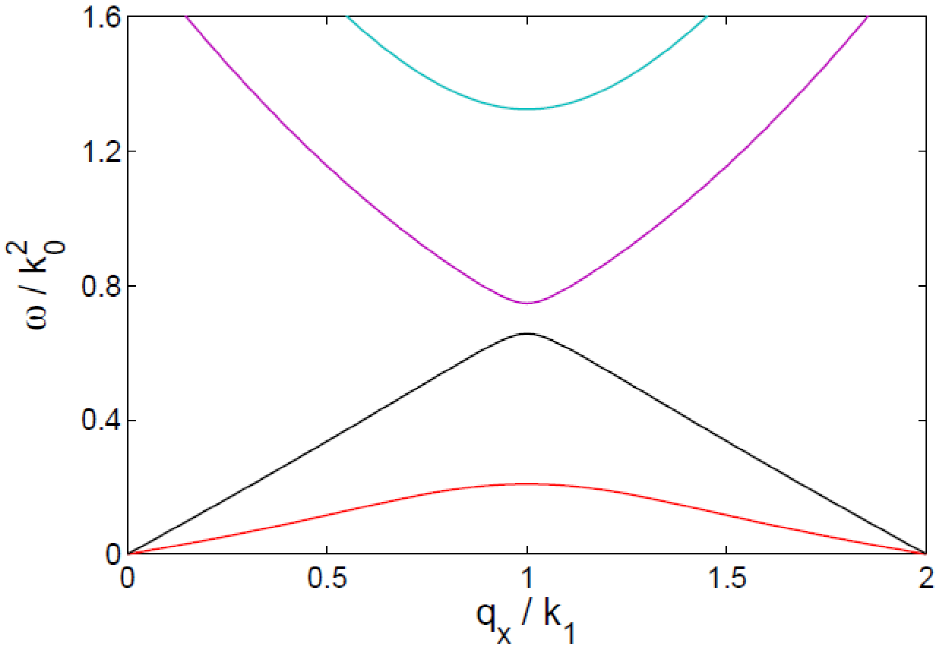


Raman coupling close to transition to stripe phase

Stripe phase

(Li et al, in preparation)

Emergence of two gapless bands



Nature of two gapless branches:

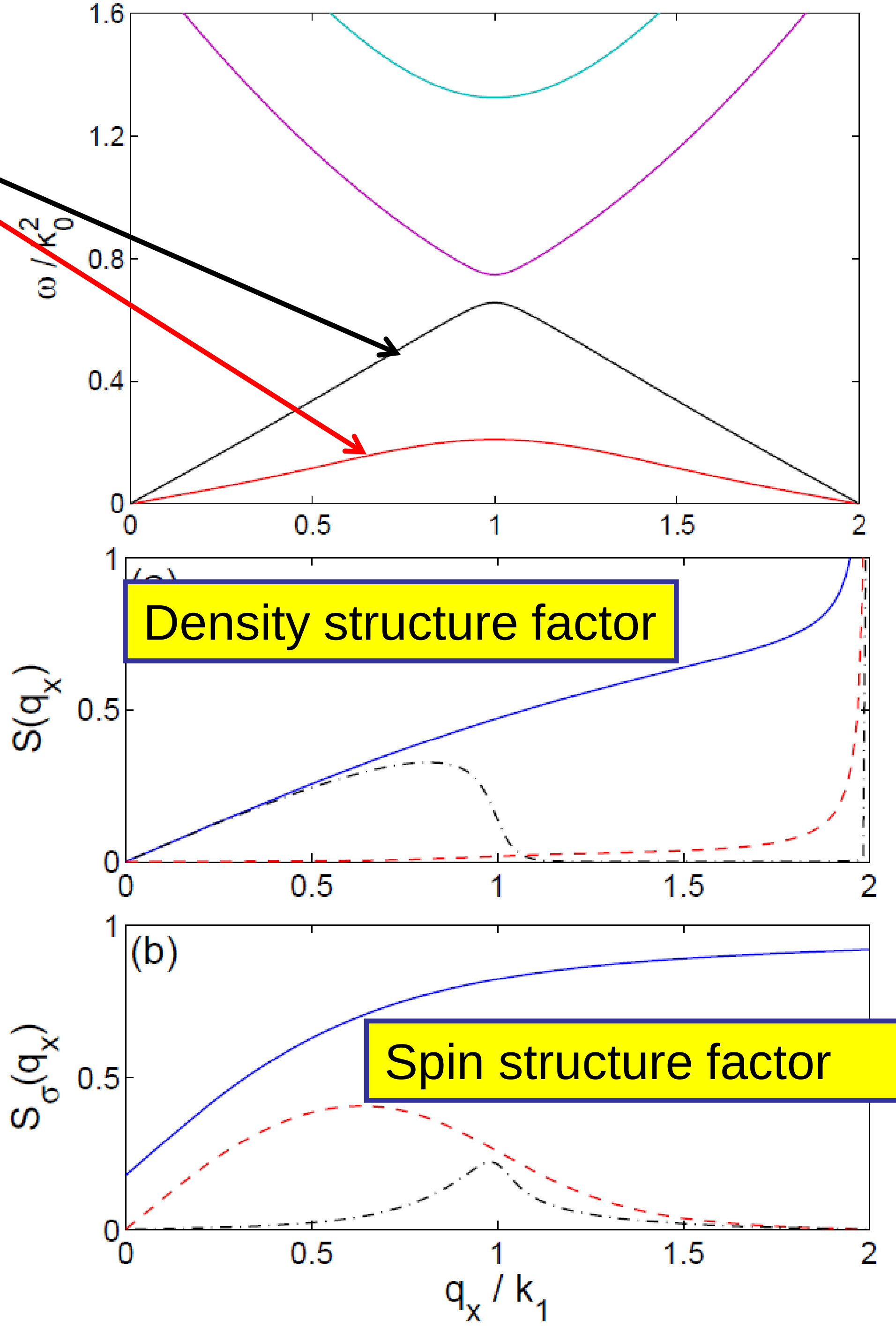
Upper branch is a **density** wave. Exhausts static structure factor

$$S(q)=\int d\omega S(q,\omega)$$

at small q

Lower branch is a **spin** wave at small q. Responsible for **divergent** behavior of S(q) at the Brillouin vector

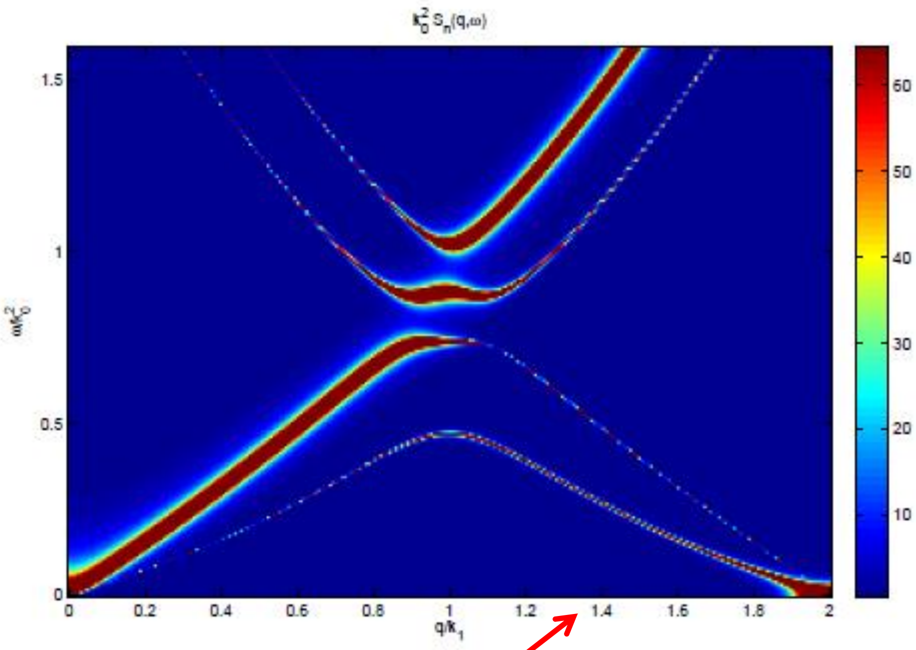
Dynamic structure factor measurable with 2-photon Bragg spectroscopy



Density structure factor

Spin structure factor

Dynamic structure factor



In the interval $1 < q_x / k_1 < 1.8$ lowest bands are not easily excited by density operator

Questions under investigation

- Effect of stripes on quantum depletion of condensate
- Behavior of supercurrents

Relevant papers in BECs with dipolar and finite range soft core potentials.

VOLUME 90, NUMBER 25 PHYSICAL REVIEW LETTERS week ending 27 JUNE 2003

Roton-Maxon Spectrum and Stability of Trapped Dipolar Bose-Einstein Condensates

L. Santos,[1] G.V. Shlyapnikov,[1,2,3] and M. Lewenstein[1]

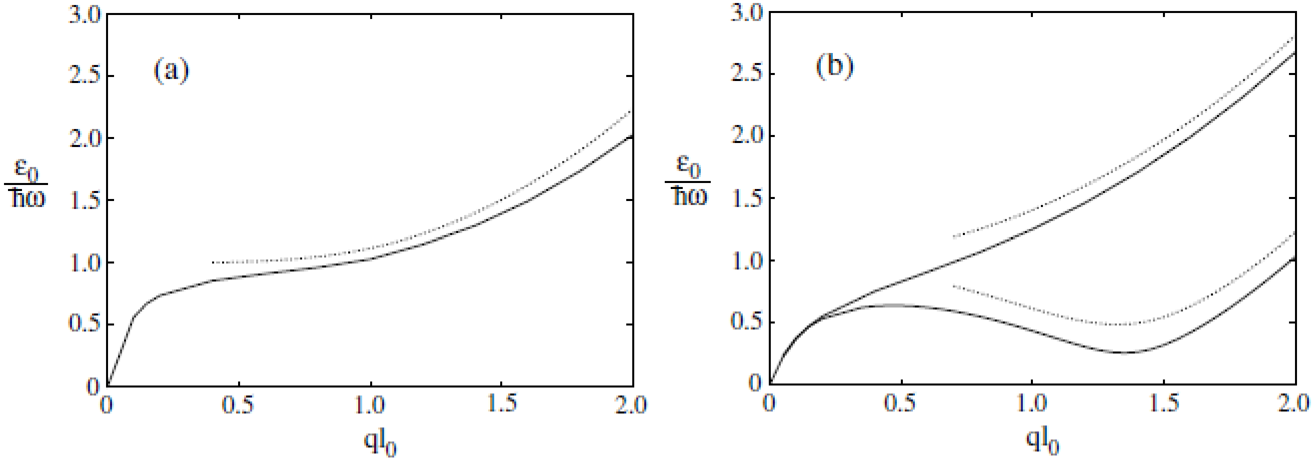
[1]*Institut für Theoretische Physik, Universität Hannover, D-30167 Hannover, Germany*

[2]*FOM Institute for Atomic and Molecular Physics, Kruislaan 407, 1098 SJ Amsterdam, The Netherlands*

[3]*Russian Research Center Kurchatov Institute, Kurchatov Square, 123182 Moscow, Russia*

(Received 24 January 2003; published 27 June 2003)

We find that pancake dipolar condensates can exhibit a roton-maxon character of the excitation spectrum, so far observed only in superfluid helium. We also obtain a condition for the dynamical stability of these condensates. The spectrum and the border of instability are tunable by varying the particle density and/or the confining potential. This opens wide possibilities for manipulating the superfluid properties of dipolar condensates.



PRL **104,** 094501 (2010) PHYSICAL REVIEW LETTERS week ending 5 MARCH 2010

Critical Superfluid Velocity in a Trapped Dipolar Gas

Ryan M. Wilson,* Shai Ronen,† and John L. Bohn
JILA and Department of Physics, University of Colorado, Boulder, Colorado 80309-0440, USA
(Received 29 December 2009; revised manuscript received 28 January 2010; published 1 March 2010)

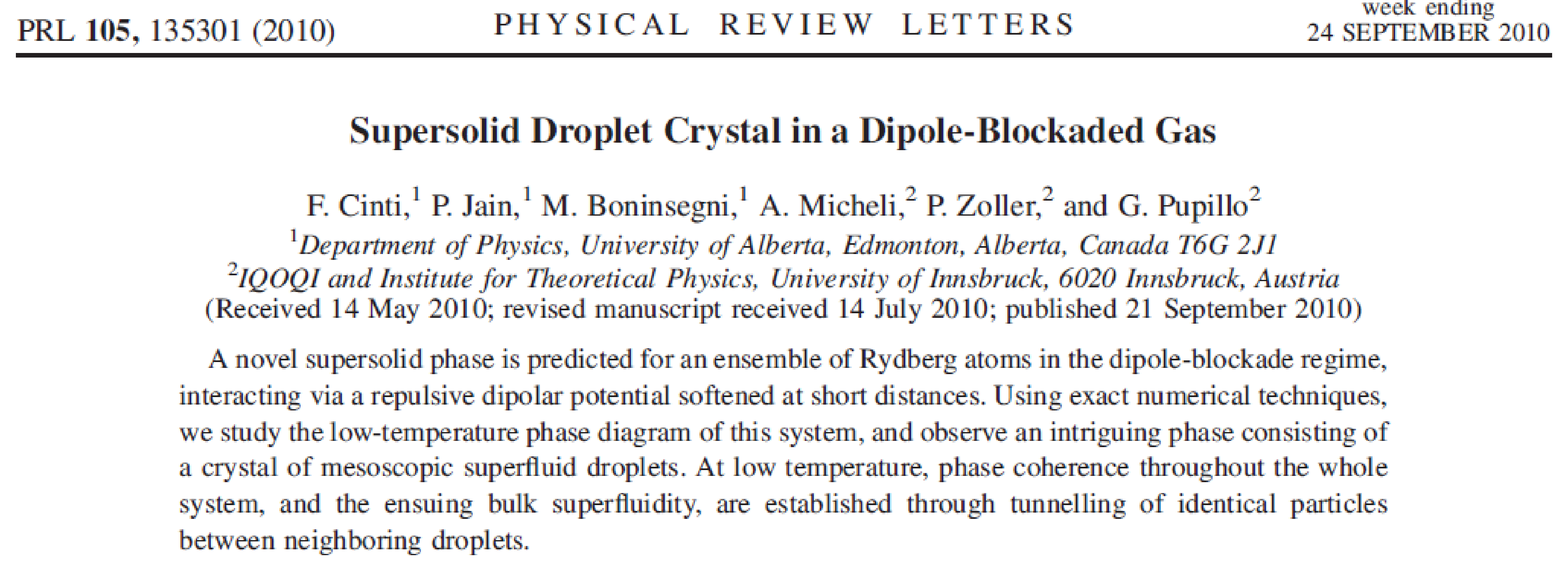
We investigate the superfluid properties of a dipolar Bose-Einstein condensate (BEC) in a fully three-dimensional trap. Specifically, we estimate a superfluid critical velocity for this system by applying the Landau criterion to its discrete quasiparticle spectrum. We test this critical velocity by direct numerical simulation of condensate depletion as a blue-detuned laser moves through the condensate. In both cases, the presence of the roton in the spectrum serves to lower the critical velocity beyond a critical particle number. Since the shape of the dispersion, and hence the roton minimum, is tunable as a function of particle number, we thereby propose an experiment that can simultaneously measure the Landau critical velocity of a dipolar BEC and demonstrate the presence of the roton in this system.

PRL **106,** 065301 (2011) PHYSICAL REVIEW LETTERS week ending 11 FEBRUARY 2011

Anisotropic Superfluidity in a Dipolar Bose Gas

Christopher Ticknor,[1] Ryan M. Wilson,[2] and John L. Bohn[2]
[1]*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*
[2]*JILA, NIST and Department of Physics, University of Colorado, Boulder, Colorado 80309, USA*
(Received 12 November 2010; revised manuscript received 13 December 2010; published 10 February 2011)

We study the superfluid character of a dipolar Bose-Einstein condensate (DBEC) in a quasi-two dimensional geometry. We consider the dipole polarization to have some nonzero projection into the plane of the condensate so that the effective interaction is anisotropic in this plane, yielding an anisotropic dispersion relation. By performing direct numerical simulations of a probe moving through the DBEC, we observe the sudden onset of drag or creation of vortex-antivortex pairs at critical velocities that depend strongly on the direction of the probe's motion. This anisotropy emerges because of the anisotropic manifestation of a rotonlike mode in the system.

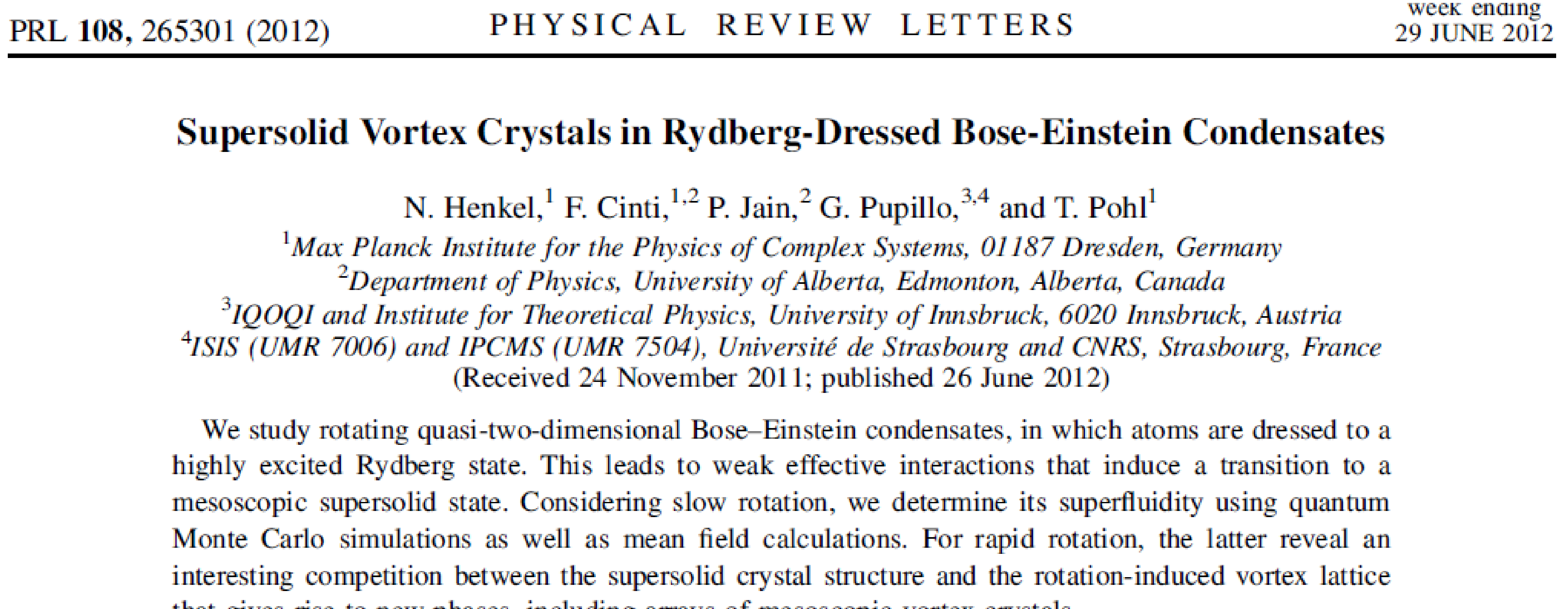


PRL **105,** 135301 (2010) PHYSICAL REVIEW LETTERS week ending 24 SEPTEMBER 2010

Supersolid Droplet Crystal in a Dipole-Blockaded Gas

F. Cinti,[1] P. Jain,[1] M. Boninsegni,[1] A. Micheli,[2] P. Zoller,[2] and G. Pupillo[2]
[1]*Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2J1*
[2]*IQOQI and Institute for Theoretical Physics, University of Innsbruck, 6020 Innsbruck, Austria*
(Received 14 May 2010; revised manuscript received 14 July 2010; published 21 September 2010)

A novel supersolid phase is predicted for an ensemble of Rydberg atoms in the dipole-blockade regime, interacting via a repulsive dipolar potential softened at short distances. Using exact numerical techniques, we study the low-temperature phase diagram of this system, and observe an intriguing phase consisting of a crystal of mesoscopic superfluid droplets. At low temperature, phase coherence throughout the whole system, and the ensuing bulk superfluidity, are established through tunnelling of identical particles between neighboring droplets.



PRL **108,** 265301 (2012) PHYSICAL REVIEW LETTERS week ending 29 JUNE 2012

Supersolid Vortex Crystals in Rydberg-Dressed Bose-Einstein Condensates

N. Henkel,[1] F. Cinti,[1,2] P. Jain,[2] G. Pupillo,[3,4] and T. Pohl[1]
[1]*Max Planck Institute for the Physics of Complex Systems, 01187 Dresden, Germany*
[2]*Department of Physics, University of Alberta, Edmonton, Alberta, Canada*
[3]*IQOQI and Institute for Theoretical Physics, University of Innsbruck, 6020 Innsbruck, Austria*
[4]*ISIS (UMR 7006) and IPCMS (UMR 7504), Université de Strasbourg and CNRS, Strasbourg, France*
(Received 24 November 2011; published 26 June 2012)

We study rotating quasi-two-dimensional Bose–Einstein condensates, in which atoms are dressed to a highly excited Rydberg state. This leads to weak effective interactions that induce a transition to a mesoscopic supersolid state. Considering slow rotation, we determine its superfluidity using quantum Monte Carlo simulations as well as mean field calculations. For rapid rotation, the latter reveal an interesting competition between the supersolid crystal structure and the rotation-induced vortex lattice

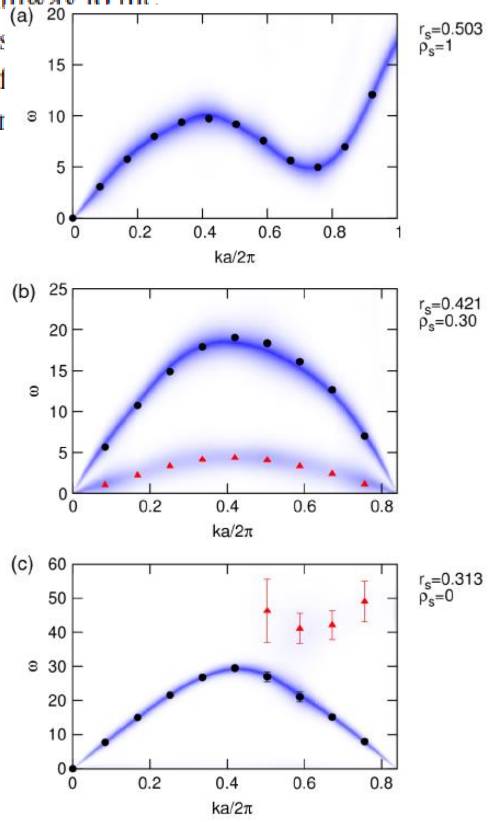
PRL **108,** 175301 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

Excitation Spectrum of a Supersolid

S. Saccani,[1] S. Moroni,[1] and M. Boninsegni[2]
[1]*SISSA Scuola Internazionale Superiore di Studi Avanzati and DEMOCRITOS National Simulation Center, Istituto Officina dei Materiali del CNR Via Bonomea 265, I-34136, Trieste, Italy*
[2]*Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2G7*

(Received 16 September 2011; revised manuscript received 17 February 2012; published 24 April 2012)

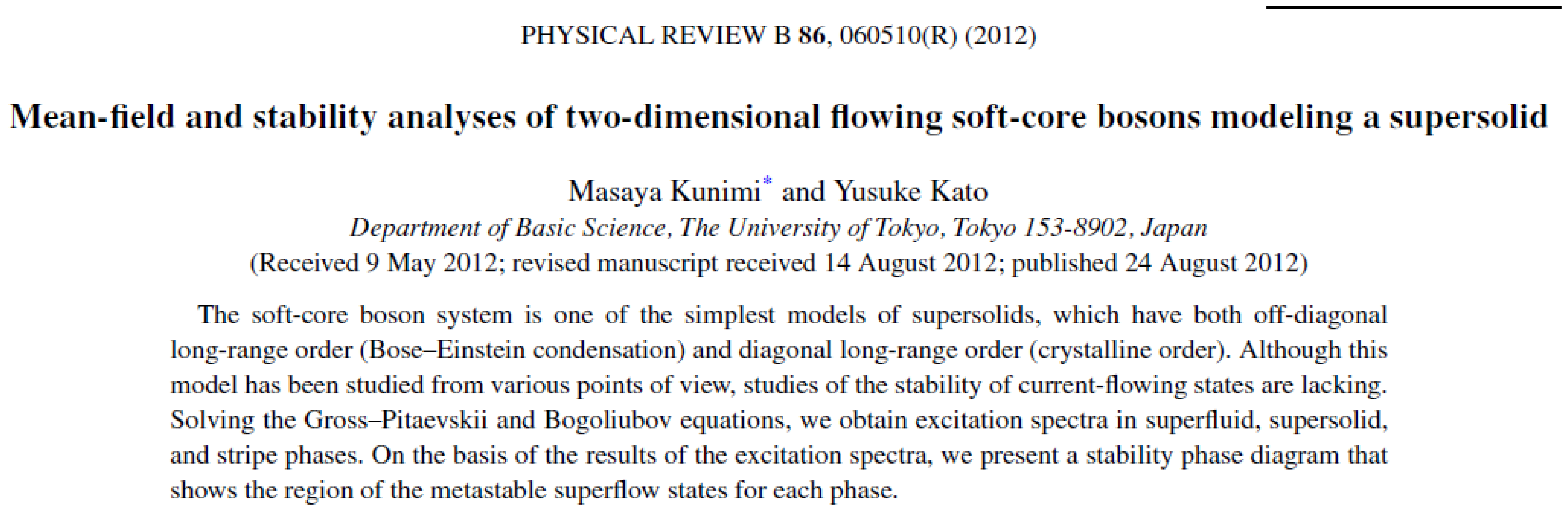
Conclusive experimental evidence of a supersolid phase in any known condensed matter system is presently lacking. On the other hand, a supersolid phase has been recently predicted for a system of spinless bosons in continuous space, interacting via a broad class of soft-core, repulsive potentials. Such an interaction can be engineered in assemblies of ultracold atoms, providing a well-defined pathway to the unambiguous observation of this fascinating phase of matter. In this Letter, we study by firs computer simulations the elementary excitation spectrum of the supersolid, and show that it f distinct modes, namely, a solidlike phonon and a softer collective excitation, related to broken and gauge symmetry, respectively.



Superfluid

Supersolid

Normal Solid



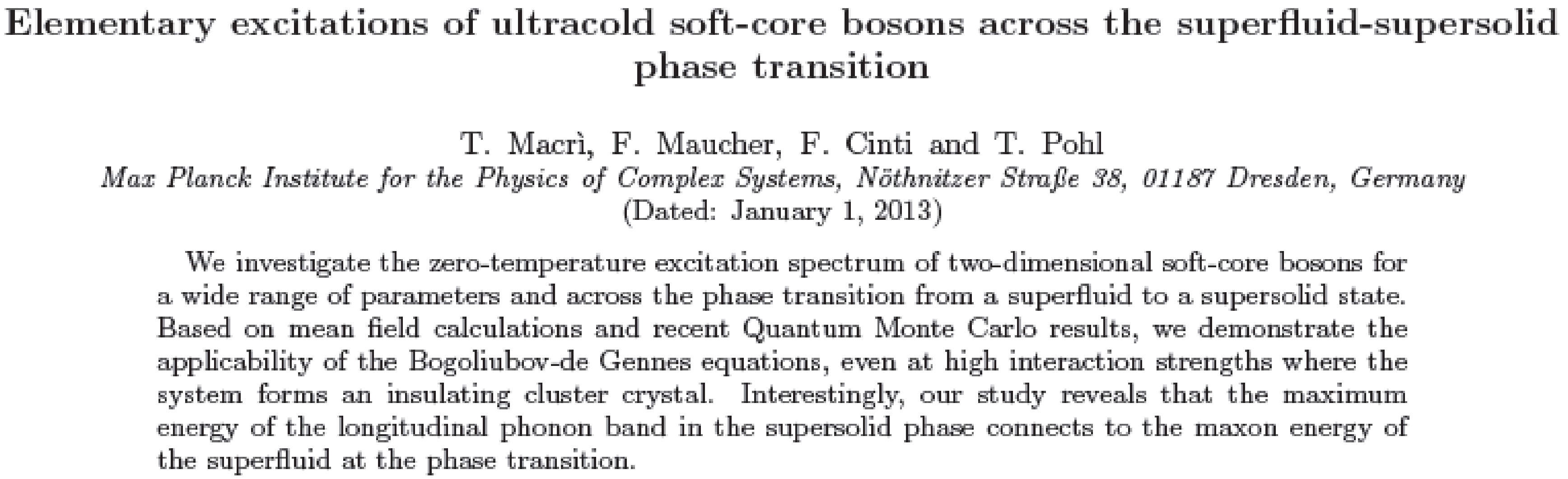
PHYSICAL REVIEW B **86**, 060510(R) (2012)

Mean-field and stability analyses of two-dimensional flowing soft-core bosons modeling a supersolid

Masaya Kunimi[*] and Yusuke Kato
Department of Basic Science, The University of Tokyo, Tokyo 153-8902, Japan

(Received 9 May 2012; revised manuscript received 14 August 2012; published 24 August 2012)

The soft-core boson system is one of the simplest models of supersolids, which have both off-diagonal long-range order (Bose–Einstein condensation) and diagonal long-range order (crystalline order). Although this model has been studied from various points of view, studies of the stability of current-flowing states are lacking. Solving the Gross–Pitaevskii and Bogoliubov equations, we obtain excitation spectra in superfluid, supersolid, and stripe phases. On the basis of the results of the excitation spectra, we present a stability phase diagram that shows the region of the metastable superflow states for each phase.



Elementary excitations of ultracold soft-core bosons across the superfluid-supersolid phase transition

T. Macrì, F. Maucher, F. Cinti and T. Pohl
Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Straße 38, 01187 Dresden, Germany

(Dated: January 1, 2013)

We investigate the zero-temperature excitation spectrum of two-dimensional soft-core bosons for a wide range of parameters and across the phase transition from a superfluid to a supersolid state. Based on mean field calculations and recent Quantum Monte Carlo results, we demonstrate the applicability of the Bogoliubov-de Gennes equations, even at high interaction strengths where the system forms an insulating cluster crystal. Interestingly, our study reveals that the maximum energy of the longitudinal phonon band in the supersolid phase connects to the maxon energy of the superfluid at the phase transition.

Smoking gun for supersolidity ?

- Density modulations
- Double gapless band structure
- Divergent behavior of S(q)