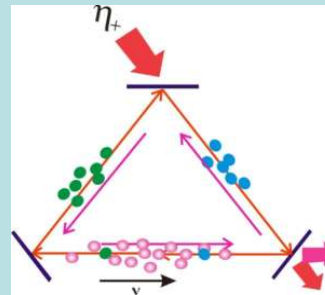




Innsbruck Physics  
Research Center



# Selfordering and sympathetic cooling in optical resonators



Helmut Ritsch  
*Theoretische Physik*  
Universität Innsbruck

*New physics with ultracold molecules*  
KITP, March 12, 2013



Austrian Science Foundation



**Innsbruck Physics  
Research Center**



**Collaborations (theory):**

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Giovanna Morigi (Saarbrücken), Aurelian Dantan (Aarhus)  
Igor Mekhov (Oxford), Maciej Lewenstein (IFCO)*

**PD's + PhD's:**

*(Hashem Zoubi)  
Claudiu Genes  
Wolfgang Niedenzu*

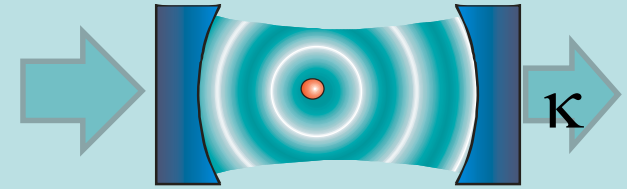
*Tobias Griesser  
Laurin Ostermann  
Kathrin Sandner  
Raimar Sandner  
Matthias Sonnleitner  
Sebastian Krämer*

**Master:**

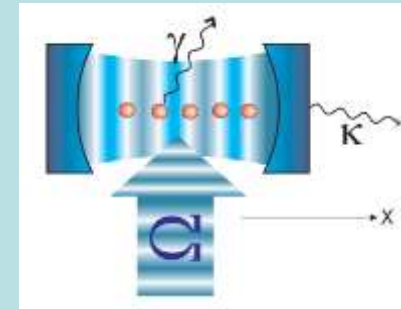
*Stefan Ostermann  
Thomas Maier  
Dominik Winterauer*

# contents

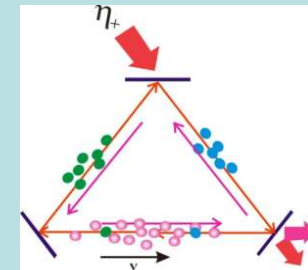
- *Cavity cooling basics and applications*



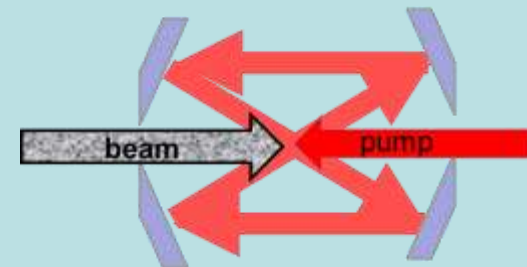
- *Selforganization and superradiant cooling in a cavity generated optical lattice*



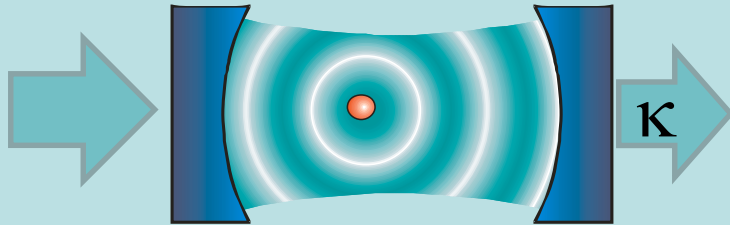
- *Multispecies cavity cooling and sympathetic self organisation*



- *Beam deceleration by cavity enhanced collective backscattering (COMORL)*

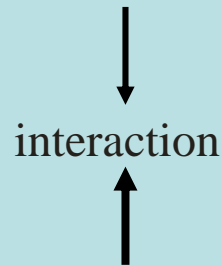


# *Lightforces on polarizable particles in optical resonators*



dispersive regime at  
large laser to particle detuning  
⇒ dipole force dominates

*Light forces* of resonatorfield determine atomic motion



trapping  
friction + diffusion  
correlation and entanglement

*Cavity QED* : atoms change resonator field dynamics

$U(x)$  = optical potential per photon =  
cavity frequency shift per atom

$\gg$

$\gamma(x)$  = photon loss per particle

$$U \sim \kappa \gg \gamma$$

# Classical dynamics in optical resonators

field amplitude:

$$\dot{E} = [-\kappa - \gamma(\mathbf{x}) + i\Delta_c - iU(\mathbf{x})] E - \alpha,$$

momentum:

$$\dot{p} = -|E|^2 \frac{d}{dx} U(\mathbf{x}),$$

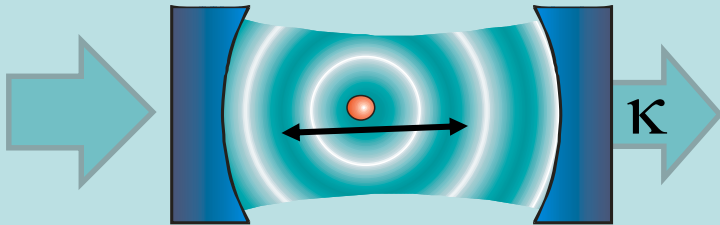
position:

$$\dot{x} = p/m.$$

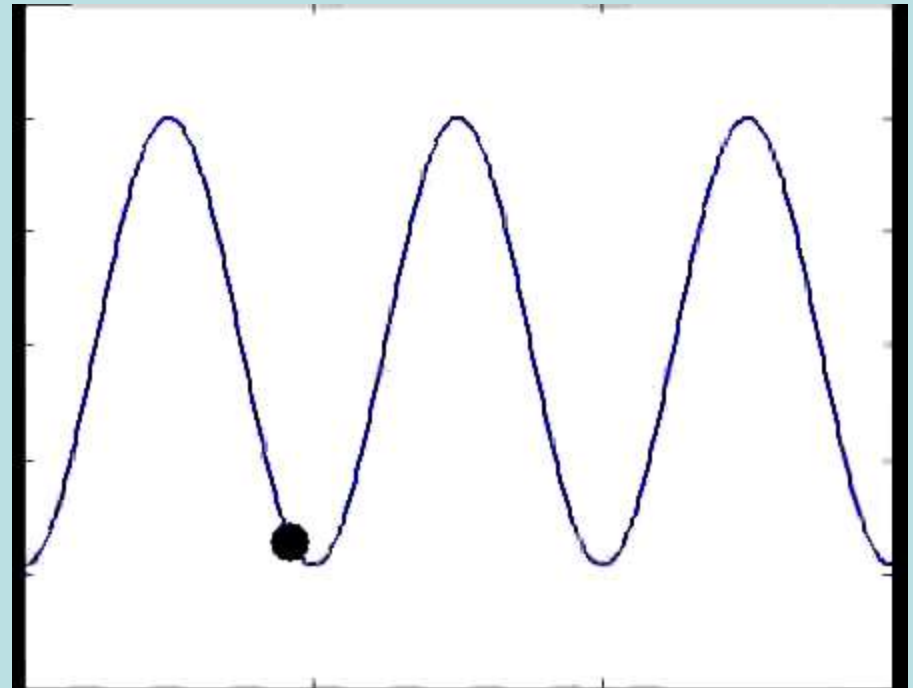
detuning + loss of mode  
depend on atom position

- *red detuning*: atoms drawn to field maxima
- field gets maximal for atom at antinode

particle moving along axis



Lewenstein, PRL 95: ions  
 Horak, PRL 97: atoms  
 Vuletic, Chu, PRL 00: atoms  
 Vitali, PRL 02: mirrors



# Cavity cooling

analytic solution for friction and diffusion for slow point particles

*friction*

$$\overline{F_1} = -k^2 \frac{\eta^2 U_0^2}{4\kappa^4}$$

*diffusion*

$$\overline{D} = k^2 \kappa \frac{\eta^2 U_0^2}{8\kappa^4}$$

*temperature*

$$k_B T = -\frac{\overline{D}}{\overline{F_1}} = \frac{\kappa}{2}$$

$\kappa$  ... cavity linewidth

- **sub-Doppler cooling for  $\kappa < \gamma$  (good cavity)**
- **no spontaneous emission needed**
- **suitable for all polarizable particles (molecules, nanoparticles, beads)**



first dedicated experiment:  
MPQ München, Nature 2004

Ions: PRL 2009

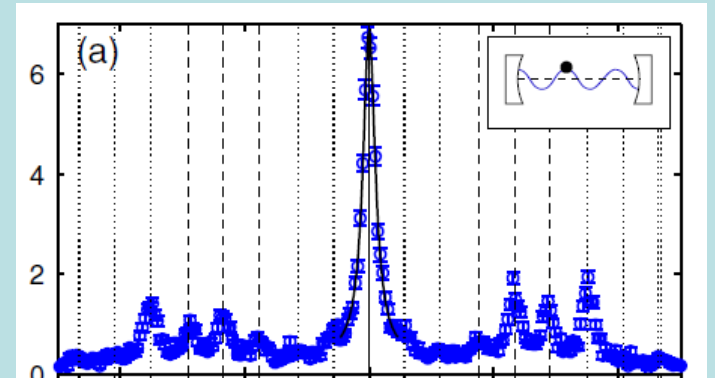
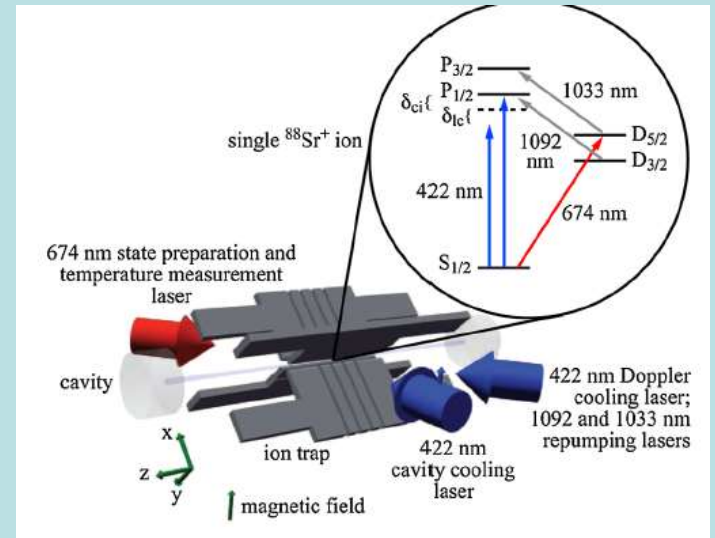
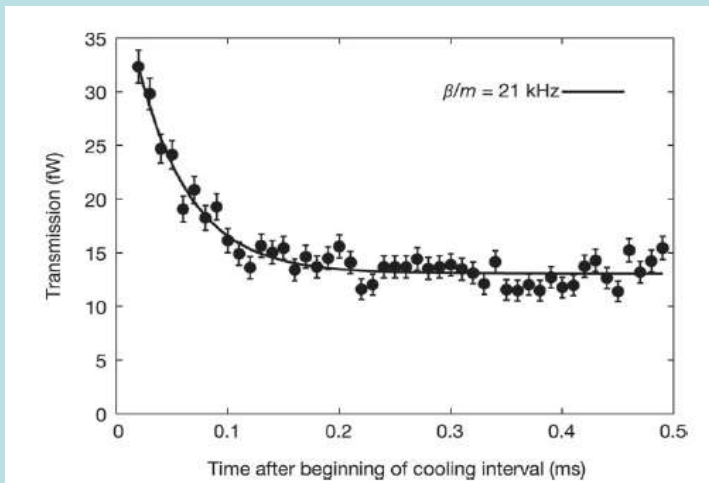
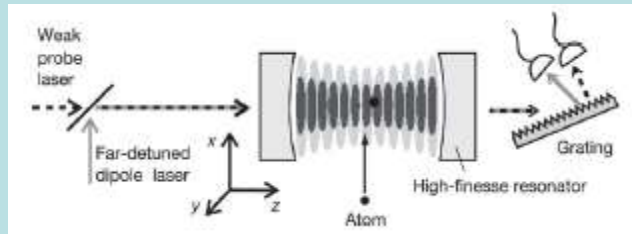
### Cavity Sideband Cooling of a Single Trapped Ion

David R. Leibbrandt,\* Jaroslaw Labaziewicz, Vladan Vuletić, and Isaac L. Chuang

### Cavity cooling of a single atom

P. Maunz, T. Puppe, I. Schuster, N. Syassen, P. W. H. Pinkse & G. Rempe

Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1,  
D-85748 Garching, Germany

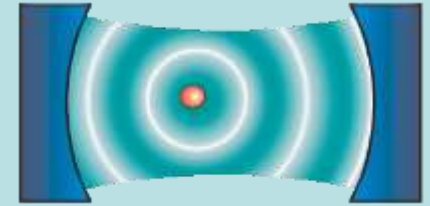


- temperature und cooling rate agree well with theory
- Ions: multiple vibrational modes addressed (Vuletic 2011), Barrett (priv. commun.)
- **new results with atom ensembles (MIT), molecules + nanobeads (Vienna)**





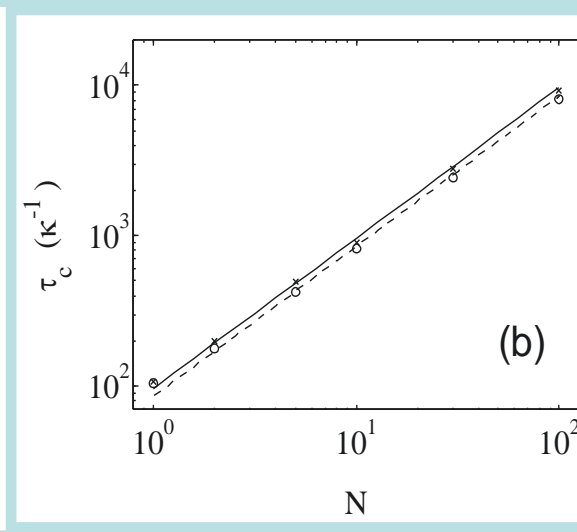
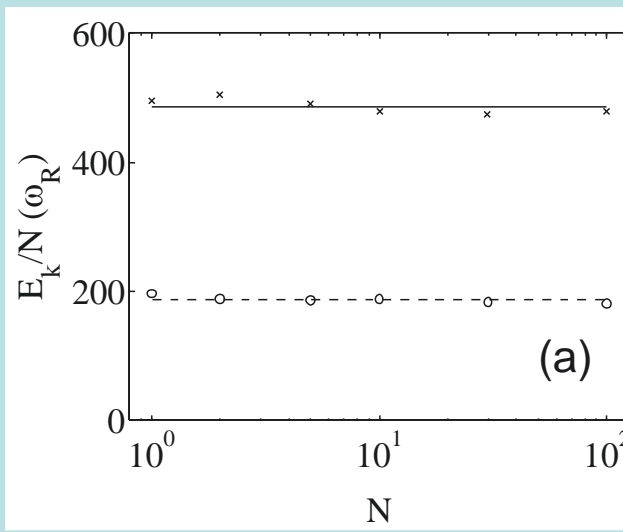
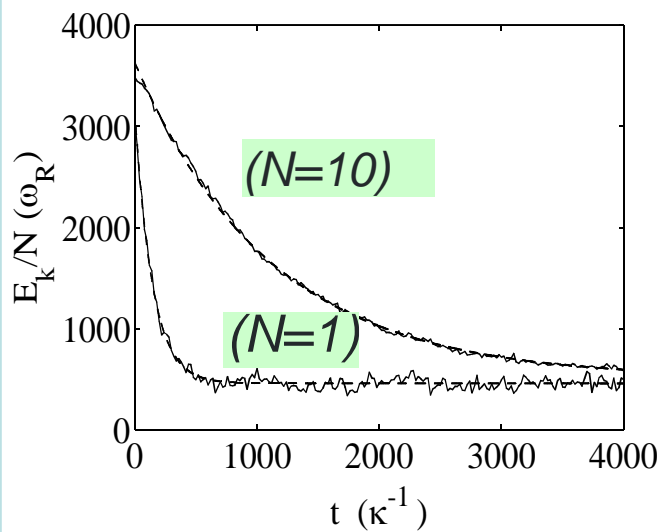
# Scaling of resonator induced cooling



*Is resonator cooling good for larger ensembles ?*

*temperature unchanged*

*cooling time grows*

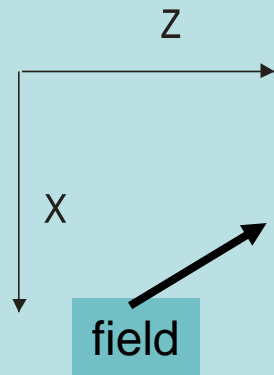
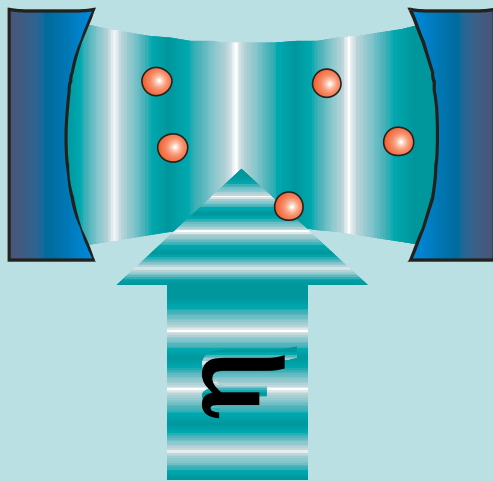


**Slow cooling for large ensembles!**

# Selforganisation of large ensembles through super-radiant light scattering

**New-geometry: transverse pump:**  
direct excitation of atoms from side !

phase of excitation  
light depends on position  $x$



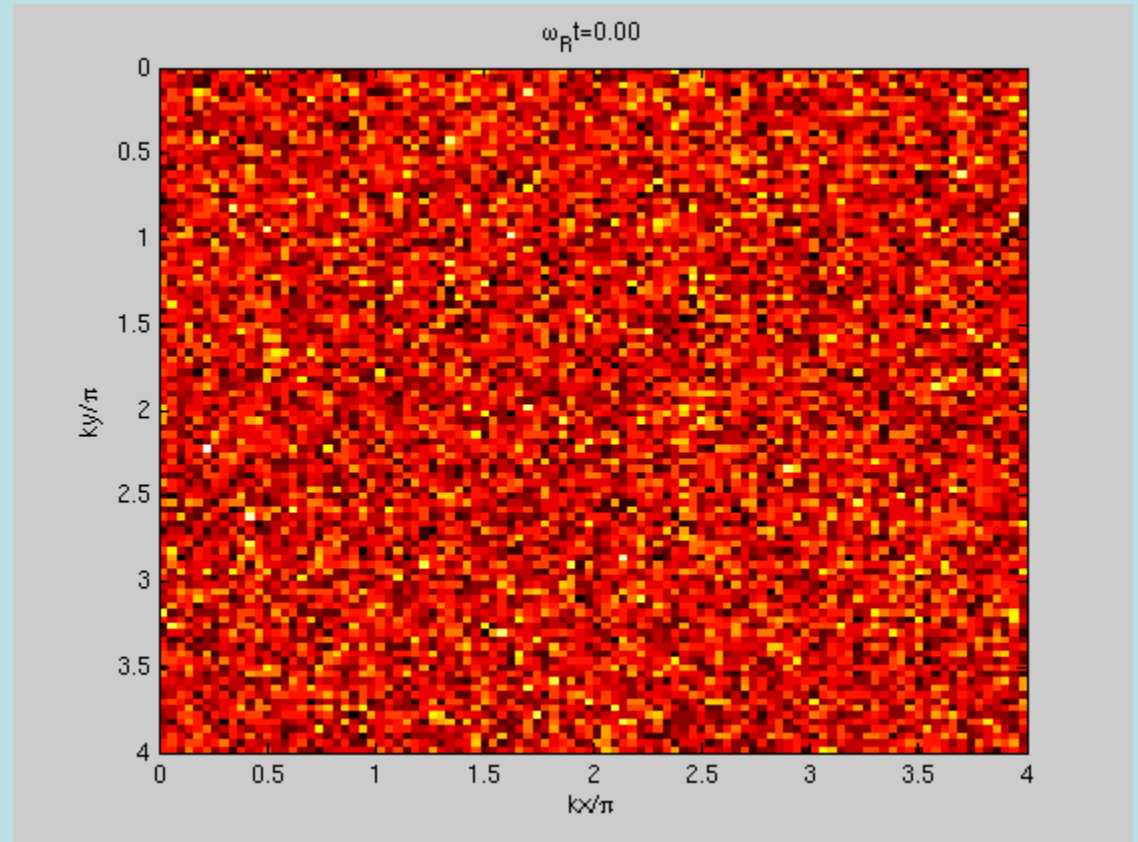
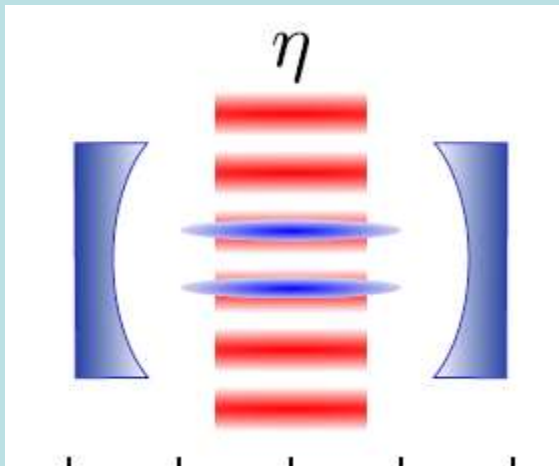
$$\dot{\sigma}_i = (i\Delta_A - \gamma)\sigma_i - g(z_i)a + \eta_x + \xi_A$$

$$\dot{a} = (i\Delta_C - \kappa)a + \underbrace{\sum_{i=1}^N g^*(z_i)\sigma_i}_{\text{collective pumpstrength } R} + \xi_i$$

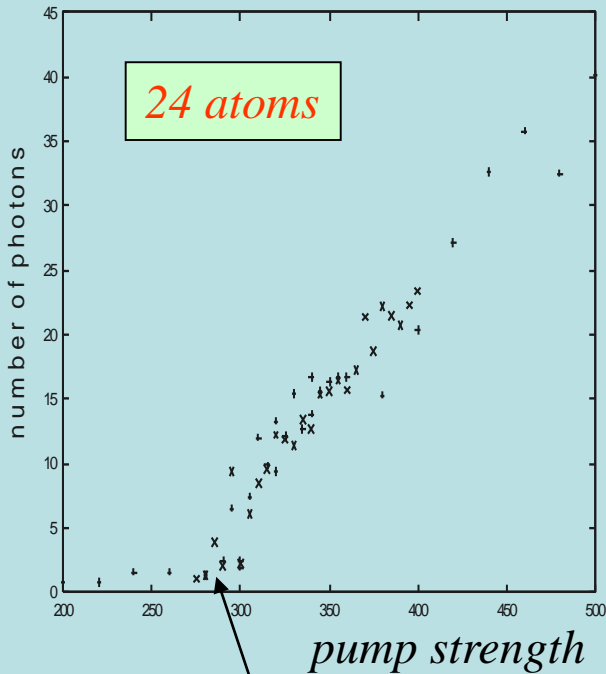
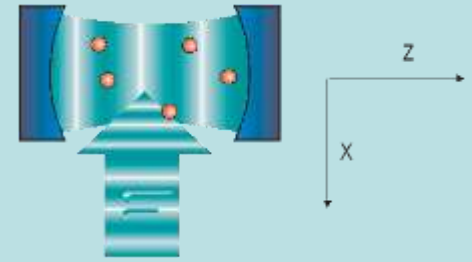
collective pumpstrength **R**

**Field in cavity generated only by atoms**  
**R = 0** for random atomic distribution  
**R ~ Ng** for regular lattice distribution

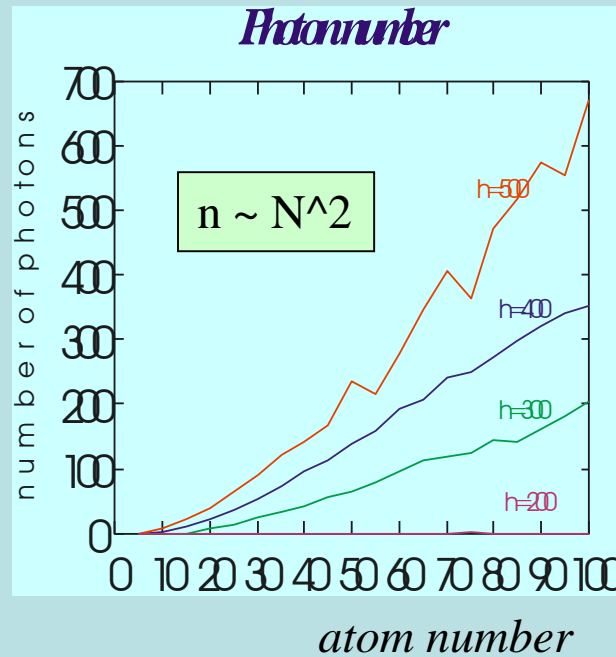
*Numerical simulations of coupled dynamics including atomic motion  
( start with random distribution at Doppler temperature )*



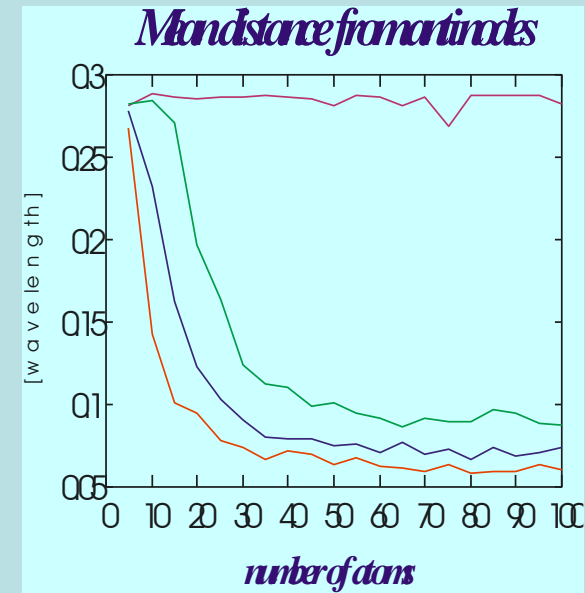
*N*-atoms:  
 Numerical simulations of coupled atom-field dynamics  
 ( start with random distribution at Doppler temperature )



clear threshold !



Superradiance !  
 quadratic dependence on  
 atom number !



Selforganization!



# Atom-field dynamics for very large particle number : => Vlasov equation for particle distribution

Continuous density approximation for cold cloud: single particle distribution function

$$f_s(x, p, t) := \frac{1}{N_s} \left\langle \sum_{j_s=1}^{N_s} \delta(x - x_{j_s}(t)) \delta(p - p_{j_s}(t)) \right\rangle$$

$$\Phi_s(x, \alpha) = \hbar U_{0,s} |\alpha|^2 \sin^2(kx) + \hbar \eta_s (\alpha + \alpha^*) \sin(kx)$$

Vlasov + field equation

$$\frac{\partial f_s}{\partial t} + \frac{p}{m_s} \frac{\partial f_s}{\partial x} - \frac{\partial \Phi_s(x, \langle \alpha \rangle)}{\partial x} \frac{\partial f_s}{\partial p} = 0$$

$$\dot{\alpha} = (i\Delta_c - \kappa) \alpha - i \sum_s \int \left( \alpha U_{0,s} \sin^2(kx) + \eta_s \sin(kx) \right) f_s dx dp$$

stability threshold of  
homogeneous distribution:

$$\frac{N\eta^2}{k_B T} \text{vp} \int_{-\infty}^{\infty} \frac{g'(\xi)}{-2\xi} d\xi < \frac{\delta^2 + \kappa^2}{\hbar|\delta|}$$

Niendenzu W., T. Grieser, H. Ritsch  
(2011), EPL, 43001

threshold at thermal equilibrium:

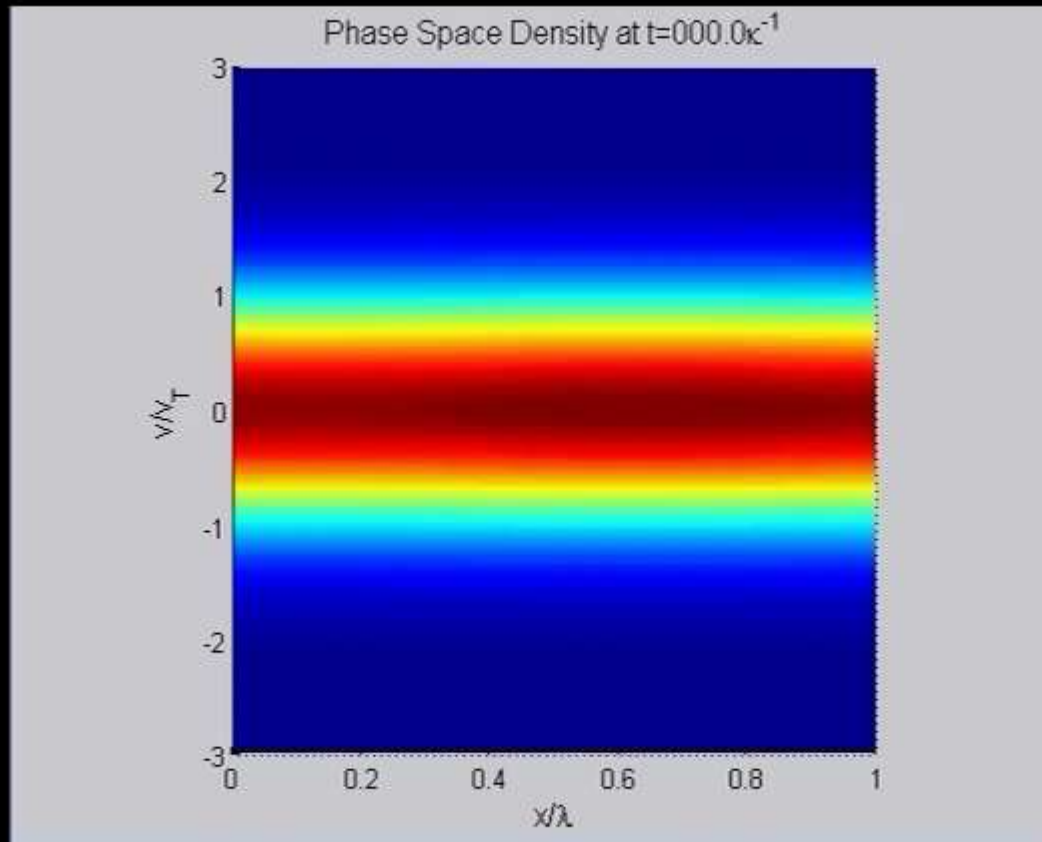
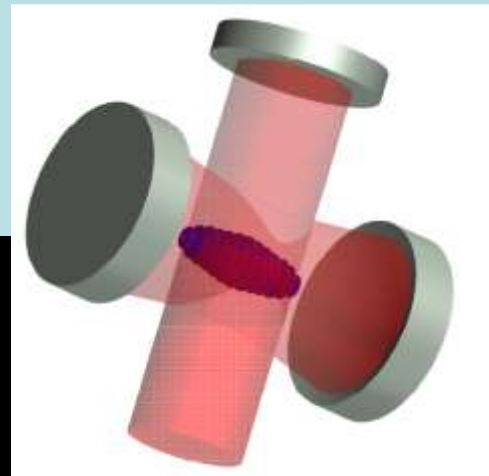
$$U_0 N V_{\text{opt}} > \kappa^2$$

frequency  
shift of cavity

pump laser  
opt. potential

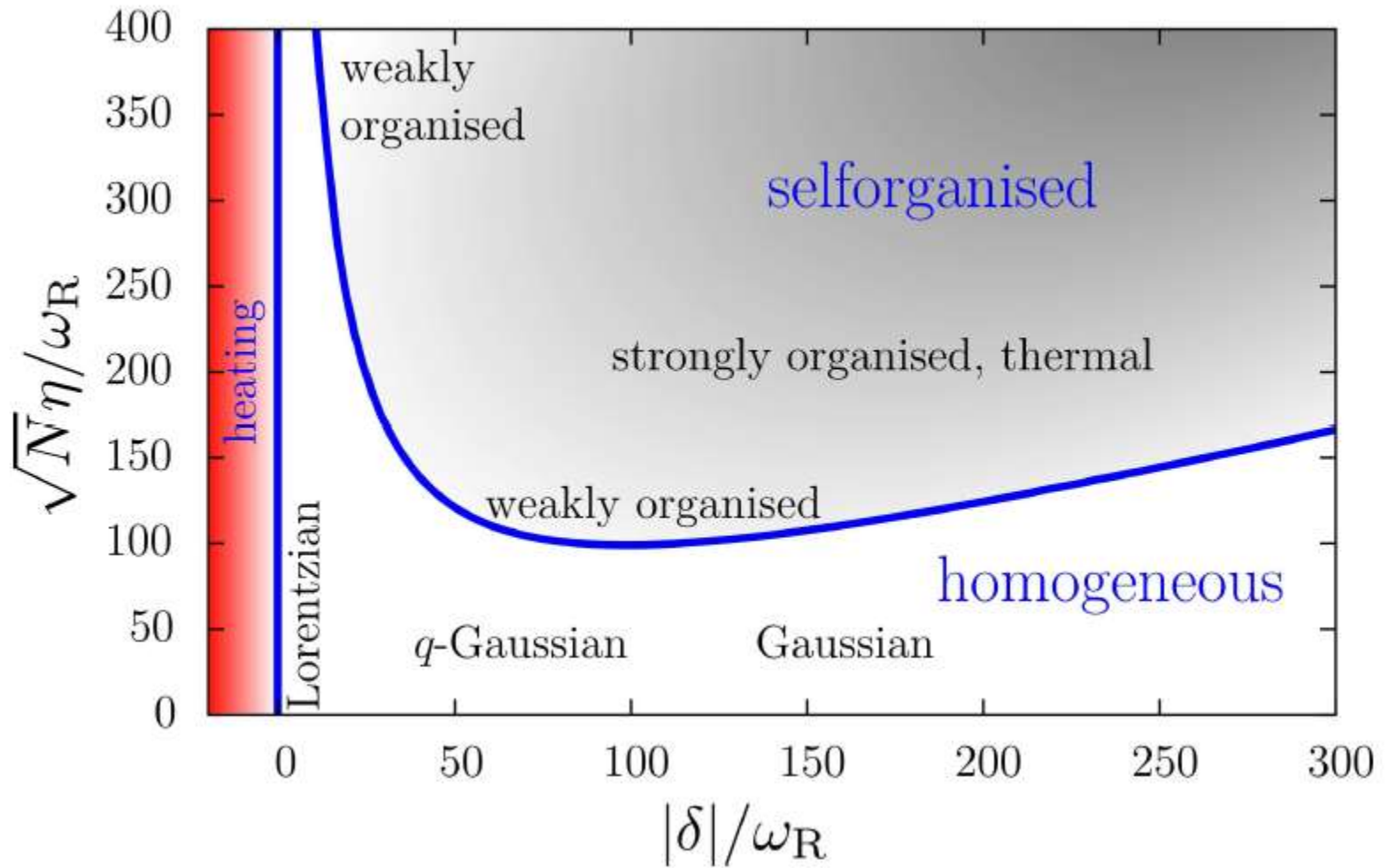
cavity  
damping

Numerical simulation of Vlasov equation  
(single optical period):

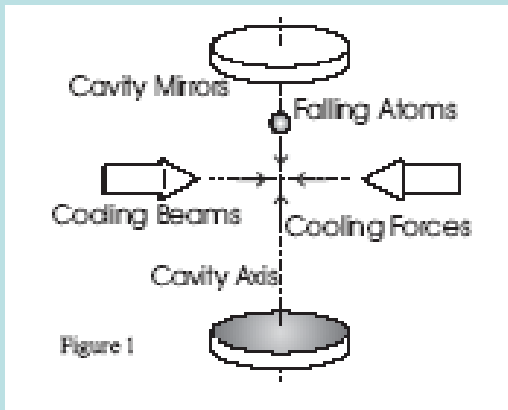




„phase diagram“

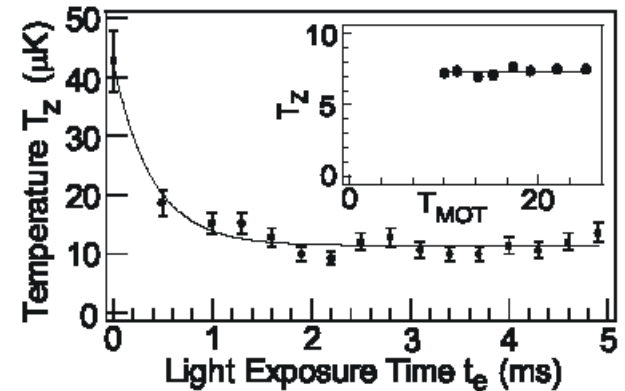


## Experiment with atoms:



Vladan Vuletic,  
Stanford University (=>MIT)

$10^6$  Caesium atoms  
in resonator with  
transverse coherent  
pump field

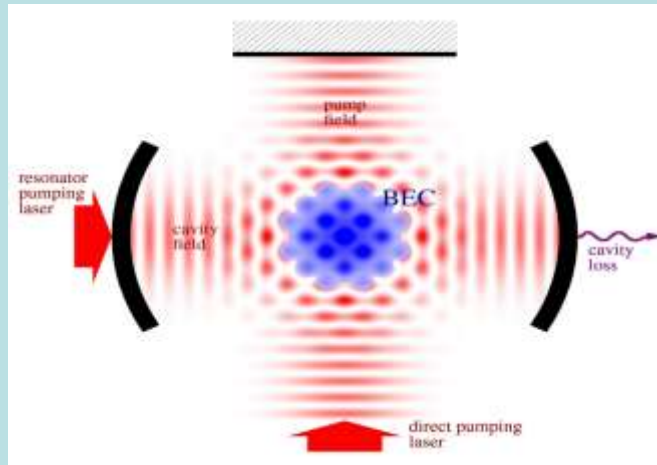


Phase stability of coherent emission  
with Pi-jumps (bistable pattern)

\*  $>10^6$  Atoms trapped and cooled to  $\sim\text{mK}$  with simultaneous coherent light emission

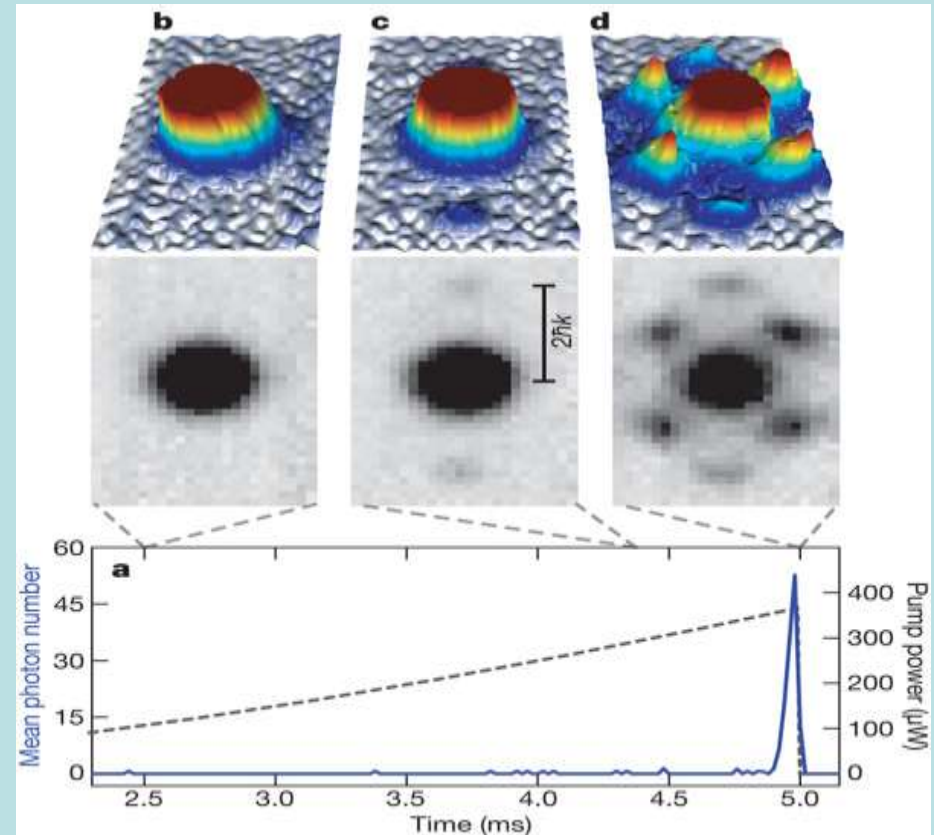
- *Experiment works better than predictions and even close to cavity resonance*
- *2-nd experiment with accelerations of  $>10^6$  g at very low saturation*
- *recent experiments: Renzoni (London), M. Baden + K. Arnold (Singapore)*
- *more theory: multimode selfordering : P. Goldbart + B. Lev, Sadchdev, ...*

# BEC: Observation of the phase transition to new phase with coherence + ordering present

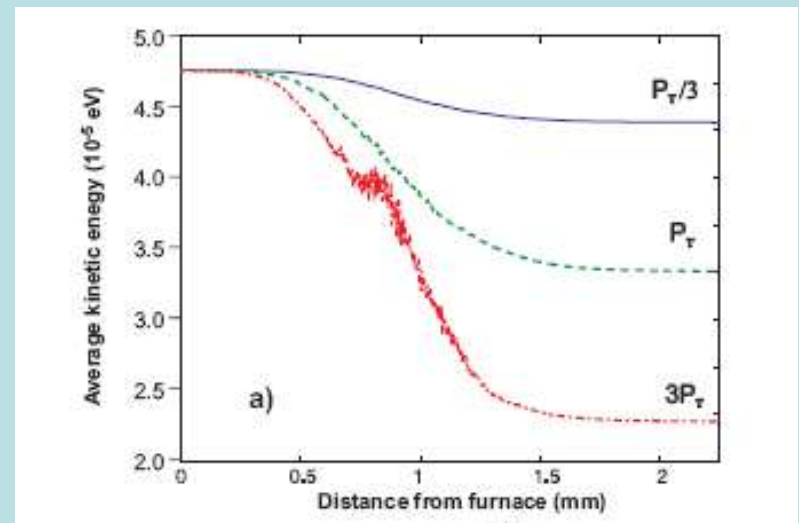
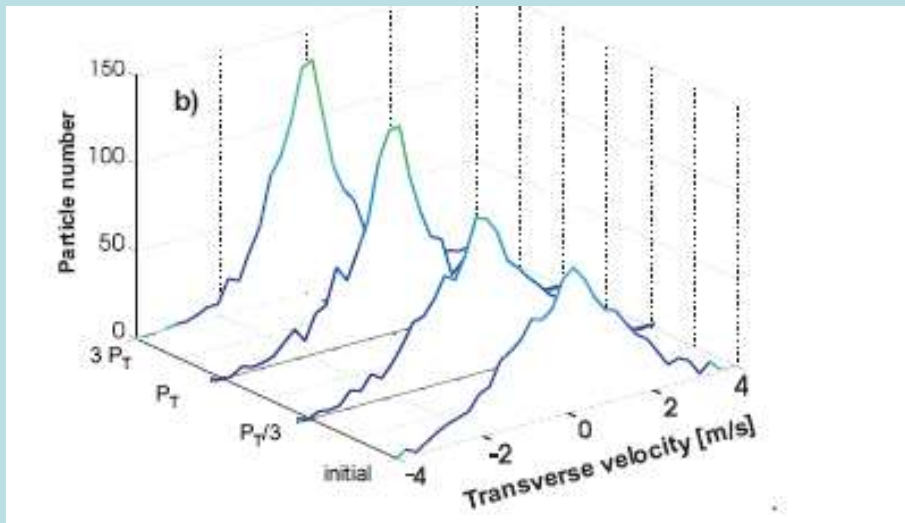
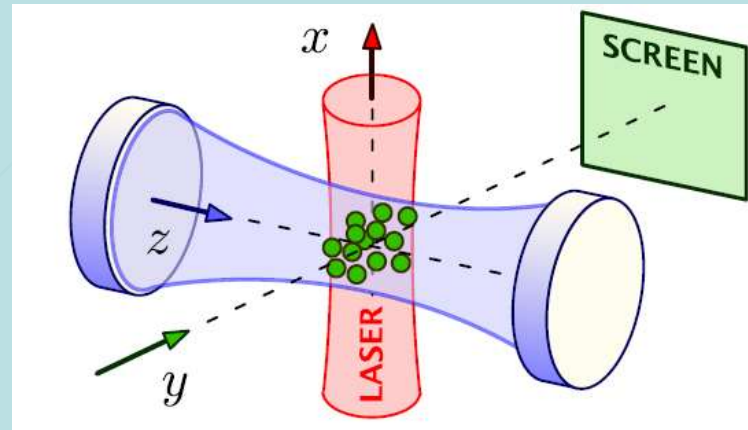


$$\Psi(x) = \frac{1}{\sqrt{L}}c_0 + \sqrt{\frac{2}{L}}c_1 \cos kx$$

$$H = -\delta_C a^\dagger a + \omega_R \hat{S}_z + iy(a^\dagger - a)\hat{S}_x/\sqrt{N} + ua^\dagger a \left(\frac{1}{2} + \hat{S}_z/N\right)$$



## 2D – transverse selforganization for collimation of a fast beam

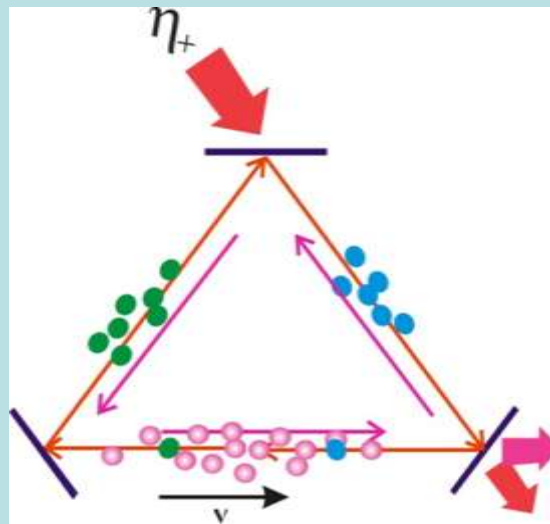


parameters for perfluorinated C60,  
M. Arndt, Vienna

T. Salzburger, NJP 11, 55025 (2009)

# Part III

## *Sympathetic selforganization and multispecies cooling*



# No (?) experiments with molecules yet !?

## Why ?

B. Lev, et. al.  
*Phys. Rev. A* **77** 023402

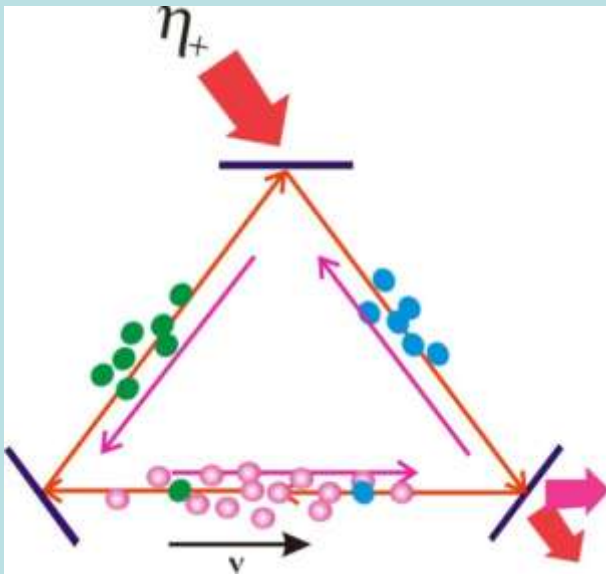
- Need suitable initial conditions to reach threshold !
- sufficient high starting phase space density:  $U * N \sim \kappa$   
or
- sufficient intracavity power :  $V_{opt} \sim \kappa$

Possible alternative solution:

## **Sympathetic** selforganization and cooling

i.e: take an atom experiment  
and  
add some molecules !

both components selforganize  
in the same pattern





# Sympathetic multispecies dynamics

Field commonly scattered by all species

$$\dot{\alpha} = (-\kappa + i\Delta_c)\alpha - i \sum_{l=1}^S N_l \int (U_{0,l}\alpha \sin^2(kx) + \eta_l \sin(kx)) f_{K,l}(x, p) dx dp$$

Joint threshold condition

$$\sum_{l=1}^S \frac{N_l \eta_l^2}{k_B T_l} \left( \text{P} \int_{-\infty}^{\infty} \frac{g'_l(u)}{-2u} dx \right) > \frac{\kappa^2 + \delta^2}{\hbar |\delta|}$$



(,thermal')

$$\sum_{l=1}^S \frac{N_l \eta_l^2}{k_B T_l} > \frac{\kappa^2 + \delta^2}{\hbar |\delta|}$$

common threshold  
strictly lower !

$$g(v/v_T) := Lv_T f(v)$$

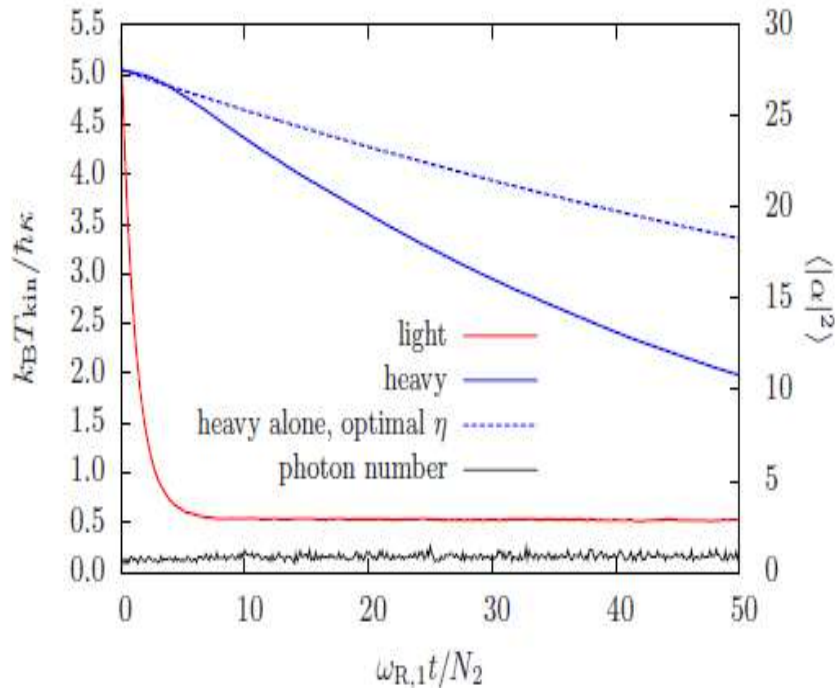
coupled kinetic equations individual ensembles:

$$\begin{aligned} \dot{Q}_{2 \rightarrow 1} &= -\frac{N_1 m_1}{N_2 m_2} \dot{Q}_{1 \rightarrow 2} \simeq N_1 \eta_2^2 \eta_1^2 \frac{4\sqrt{\pi} \hbar \delta^2}{(\kappa^2 + \delta^2)^2} \times \\ &\times \sqrt{\frac{\hbar \omega_{R,1}}{k_B T_1}} \left(1 - \frac{T_2}{T_1}\right) \left(1 + \frac{m_1 T_2}{m_2 T_1}\right)^{-3/2} \end{aligned}$$

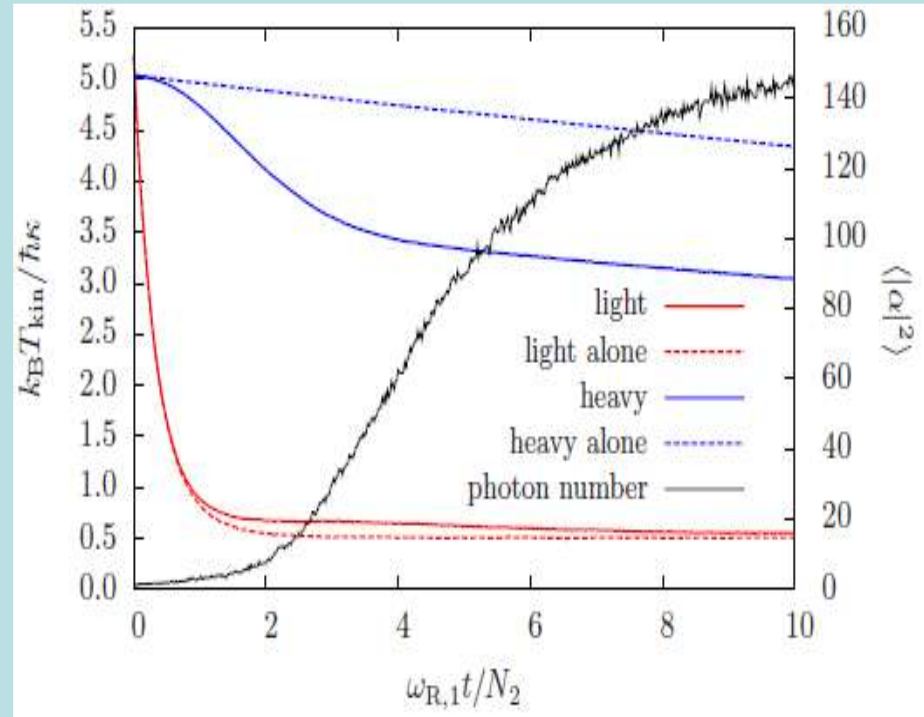
interspecies  
energy flux !!

# Sympathetic two-species cooling at same initial temperature

$$m_2 = 200m_1, \quad N_1 = 200, \quad N_2 = 200.$$



Below threshold:  
enhanced cavity cooling

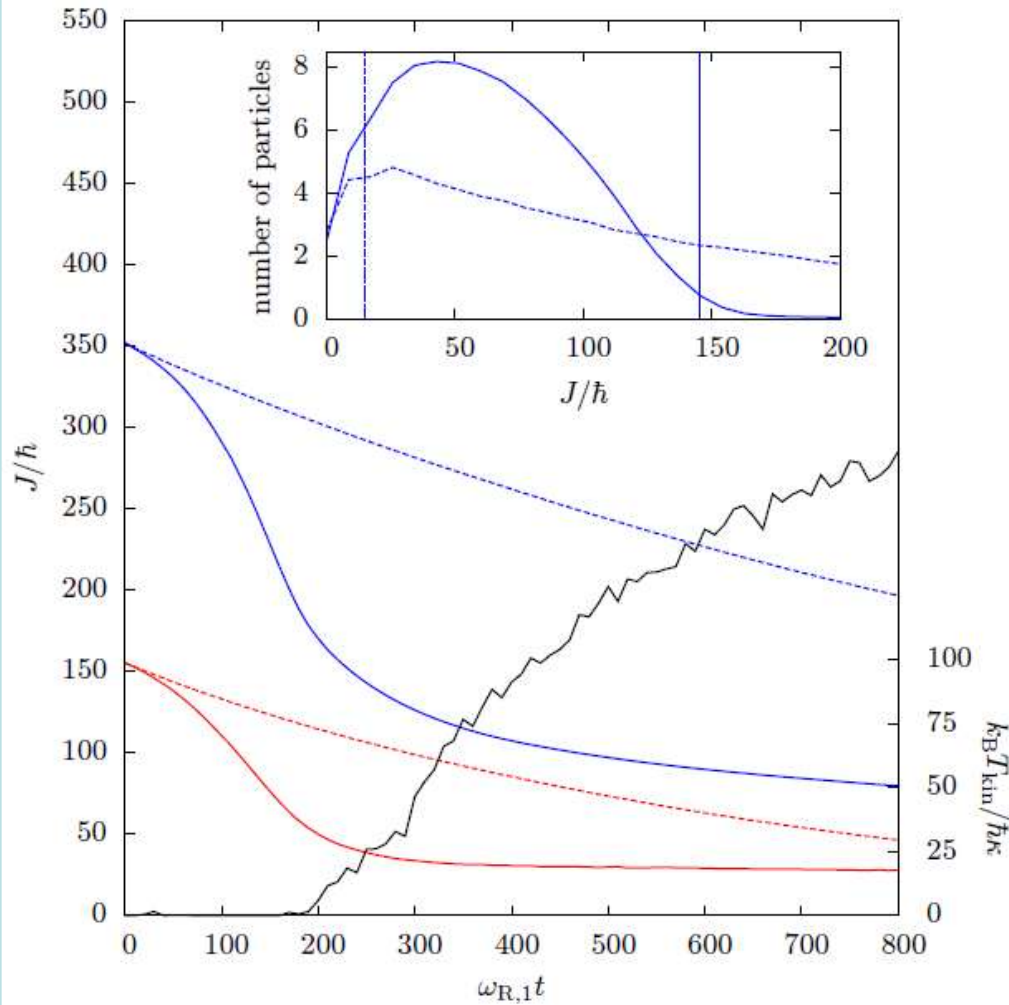


above threshold:  
common self ordering

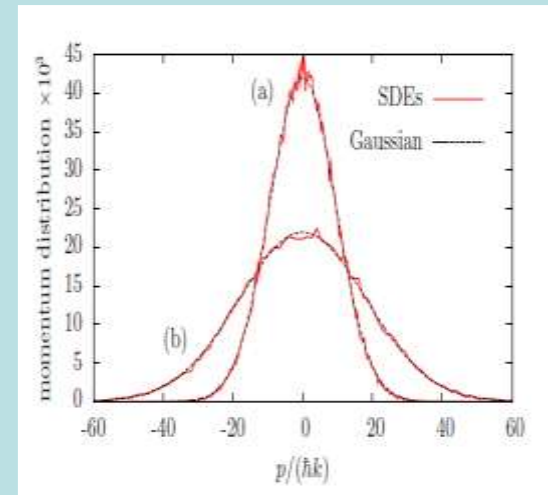
- *Cooling enhanced below and above threshold*
- *Cooling slows in fully organized phase*

*phase space „density“ evolution for two-species*

$$N_1 = 1500, N_2 = 100, m_2 = 80m_1$$

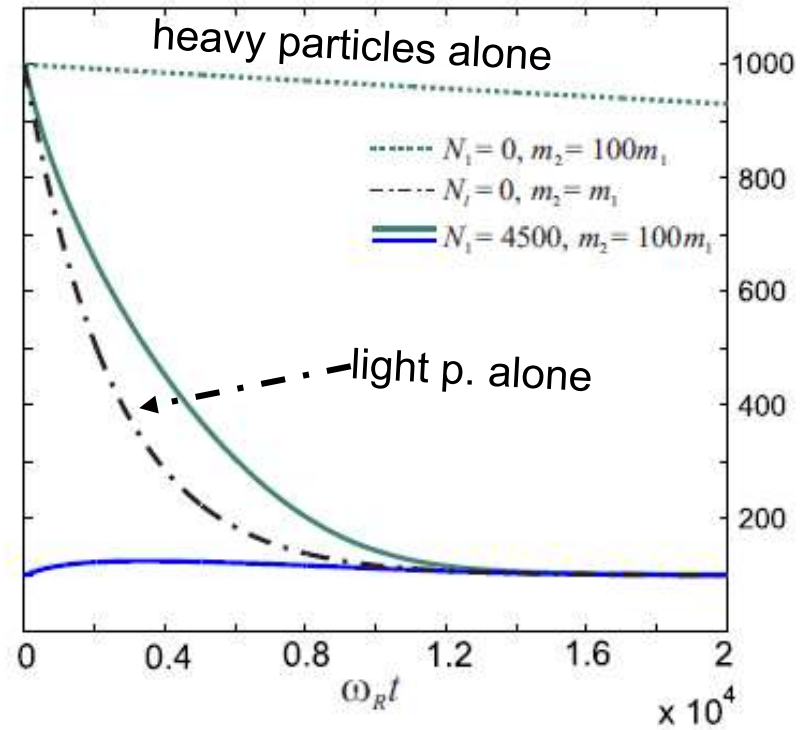
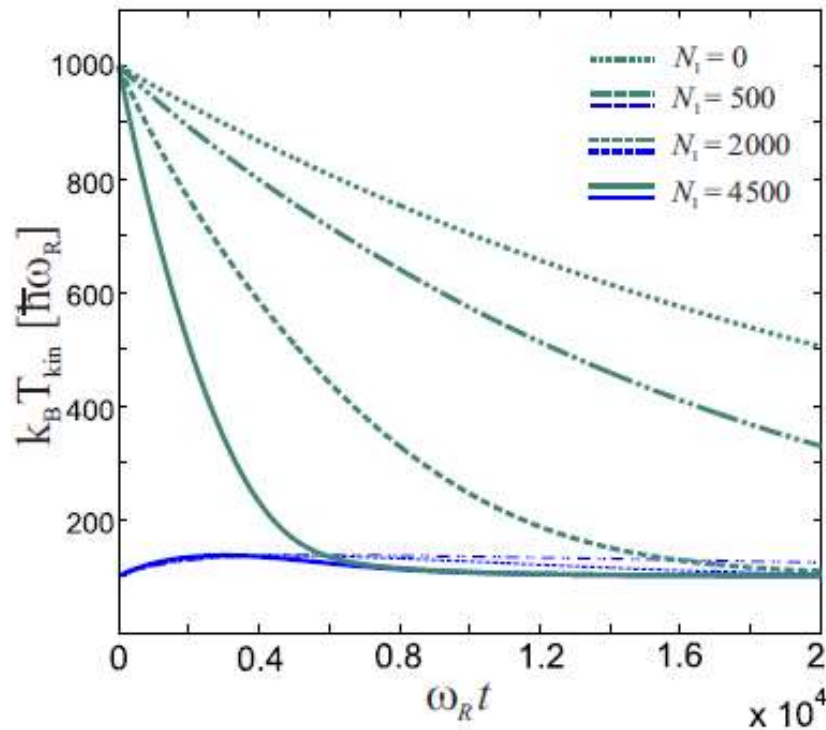


*momentum distributions*



*selfordering increases phase space density !*

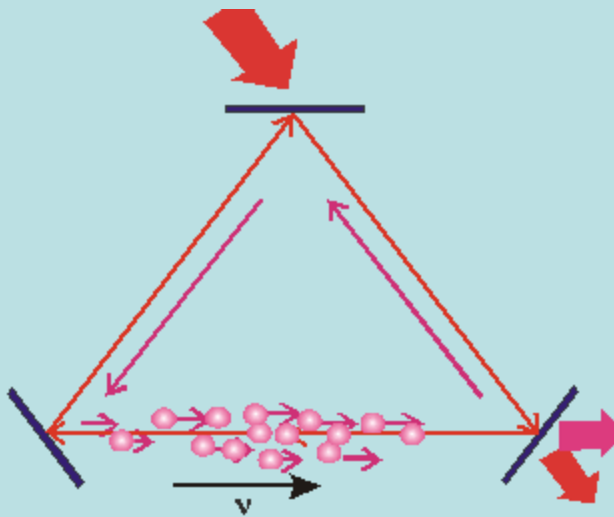
*Energy flow for different initial temperatures  
(cold atoms + hot molecules)*



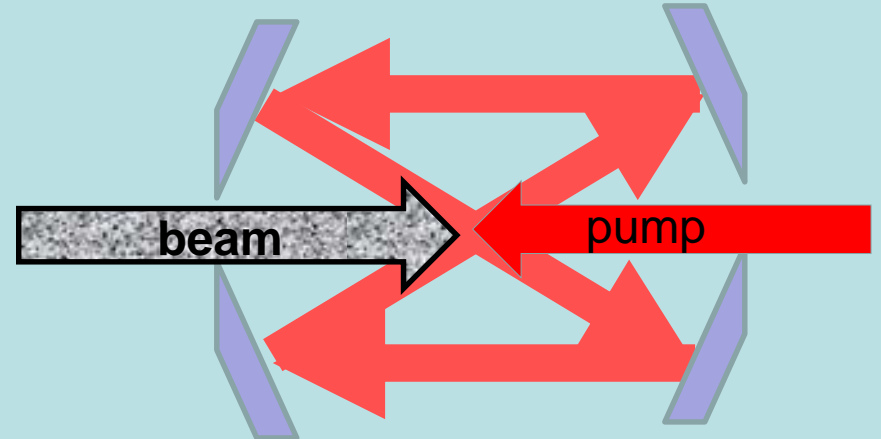
*Fast thermalization and common cooling !*

# Part IV

## *Fast beam deceleration in ring cavity*



theoretical idea



theoretist's idea of  
realistic experiment

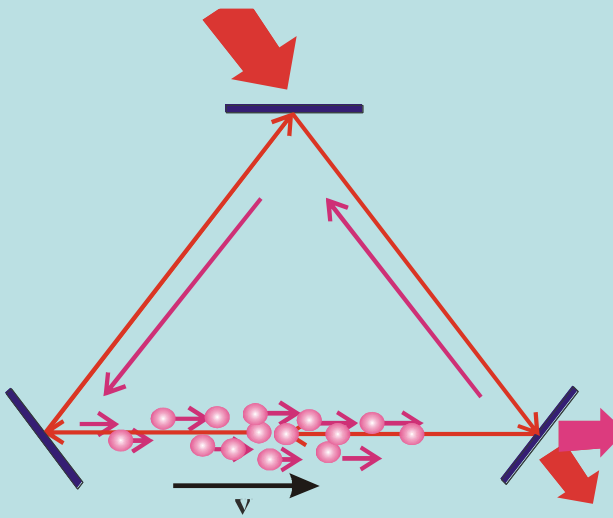
*collective dynamics of a polarisable cold gas in a ring resonator with **single-side** pumping (COMORL - related work on FEL / CARL)*

**particles rest frame**

$$\frac{d}{dt}a_{\pm}(t) = [i\Delta_{\pm} - iNU_0 - \kappa]a_{\pm}(t) - iNU_0 \langle e^{\mp 2ikx} \rangle a_{\mp}(t) + \eta_{\pm}$$

*collective backscattering acts like gain*

$$a_+ a_-^* = |\eta|^2 \frac{(\Delta_- - NU_0 + i\kappa) NU_0 |R_-| e^{-ikx_0}}{|(\Delta_+ - NU_0 + i\kappa)(\Delta_- - NU_0 + i\kappa) - N^2 U_0^2 |R_-|^2|^2}$$



$$R_- = \langle e^{2ikx} \rangle$$

*interference of pump and backscattered field creates periodic potential*

*selfordering threshold to 1/2 periodic grating = moving Bragg lattice*

$$\eta_{thresh} > \sqrt{\left| \frac{k_B T \sqrt{(\Delta_- - NU_0)^2 + \kappa^2} ((\Delta_+ - NU_0)^2 + \kappa^2)}{NU_0^2} \right|}$$

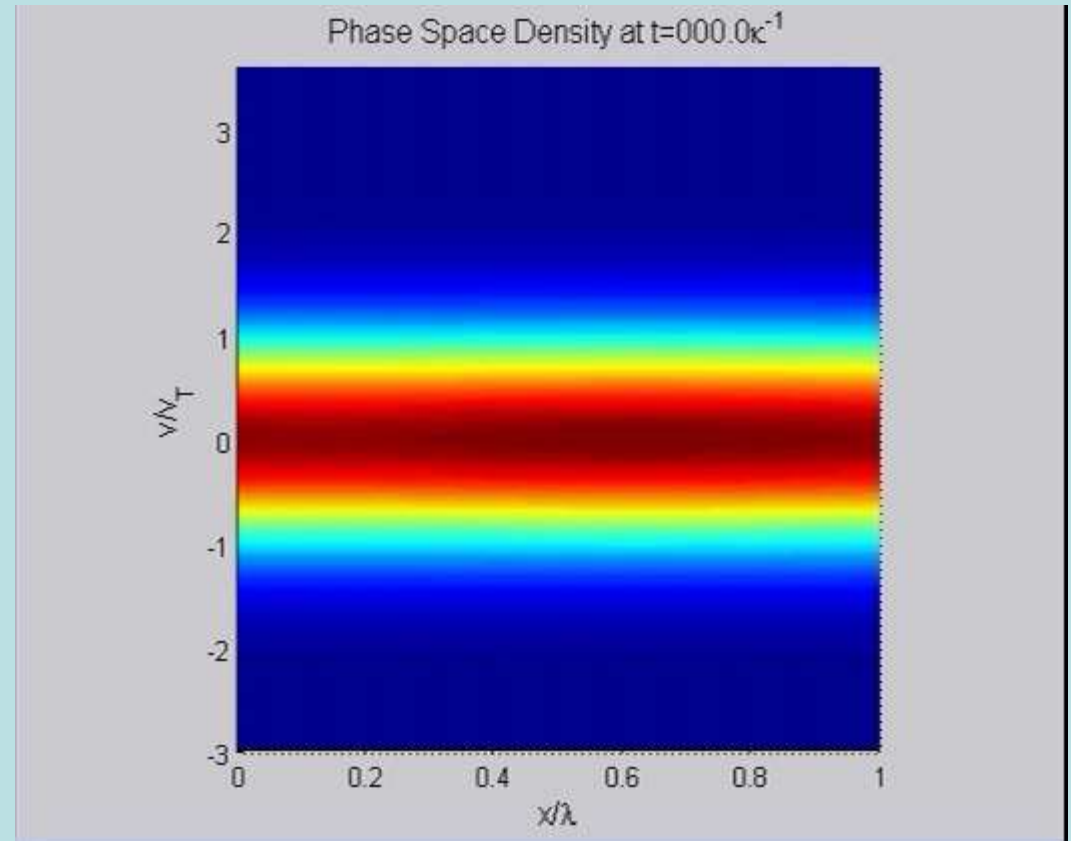
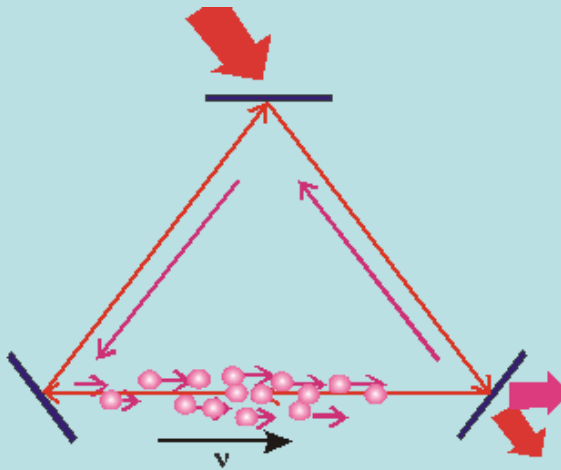
**at resonance:**

$$N_{cav} = \left( \frac{\eta_{thresh}}{\kappa} \right)^2 > \frac{k_B T \kappa}{NU_0^2}$$



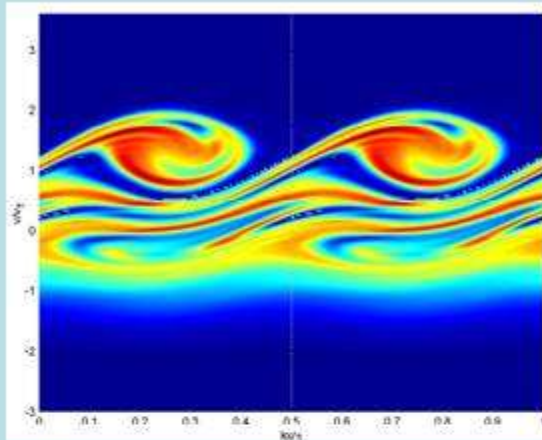
# Collective particle accelerator

**very large particle number  
continuous distribution**

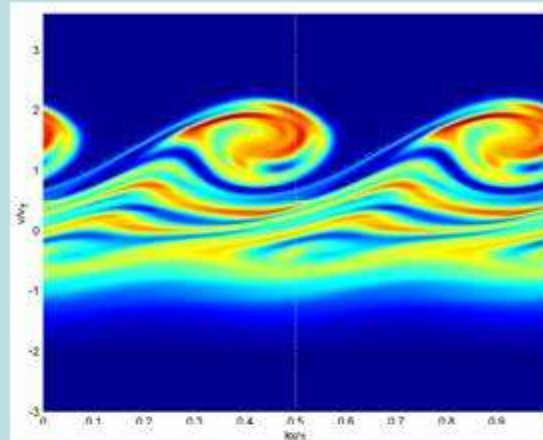


- *backscattered light and gets amplified*  
=> *selfconsistent accelerated/decelerated lattice (FEL / CARL)*

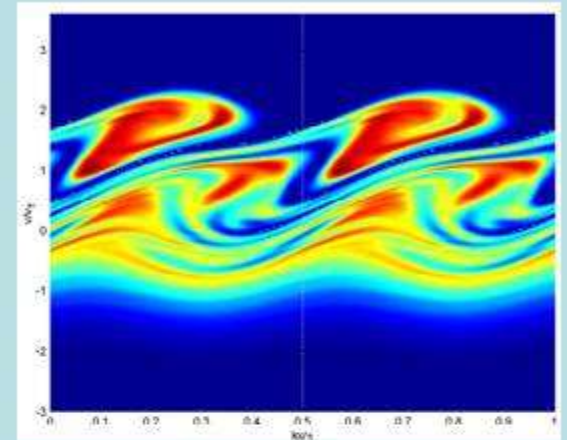
# Example 1: Instability in unidirectional cavity



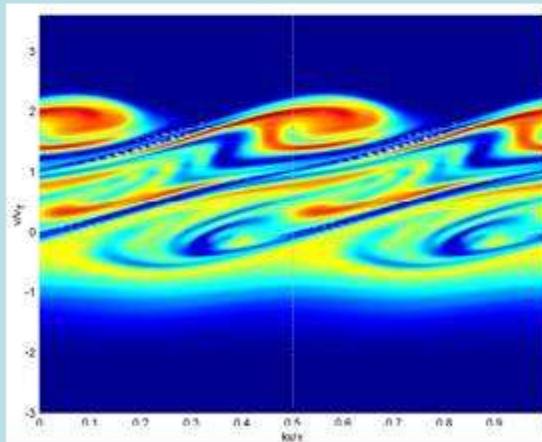
$t = 19\kappa$



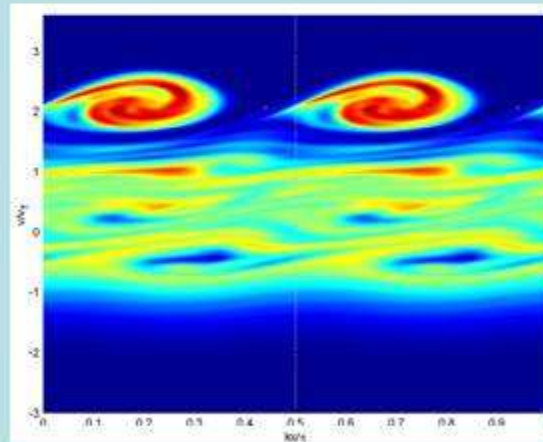
$t = 21\kappa$



$t = 27\kappa$



$t = 30\kappa$

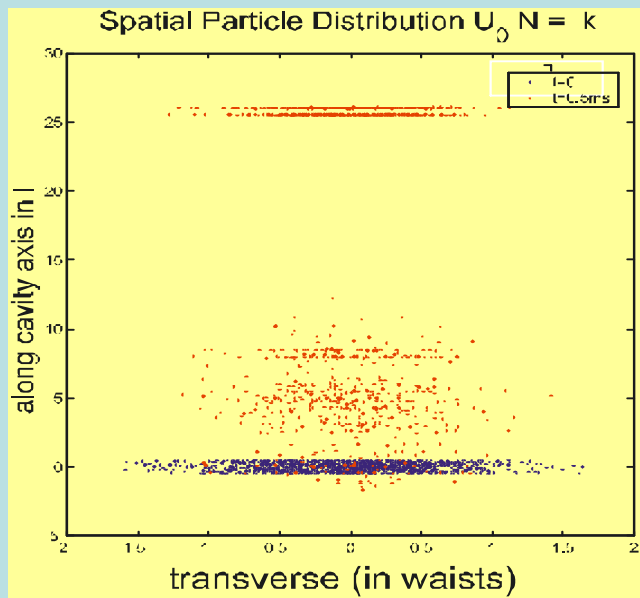


$t = 45\kappa$

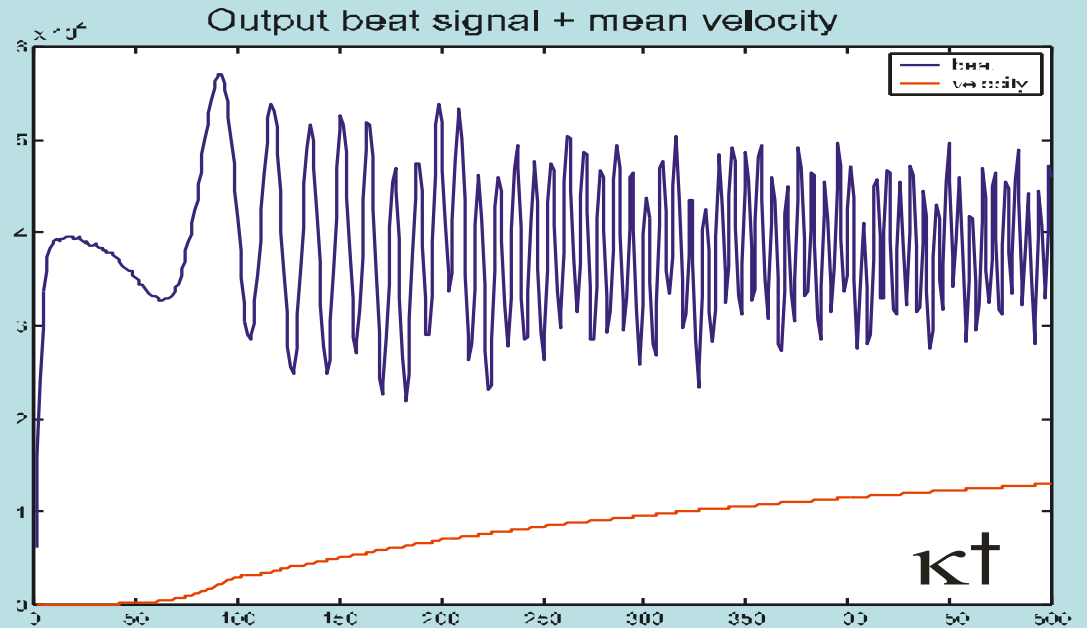
**Space-time order forms !**

*3D-particle simulation of ring cavity selfordering: inverse CARL  
collective atomic recoil laser (analogous to free electron laser)*

particles:  
initial - final



field evolution

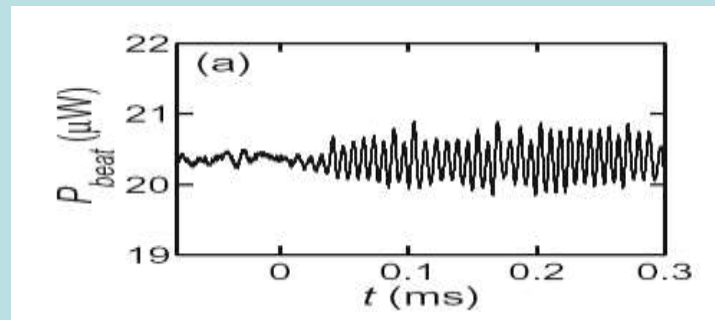
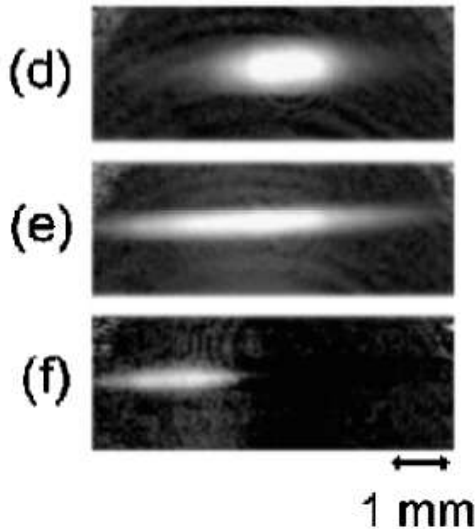
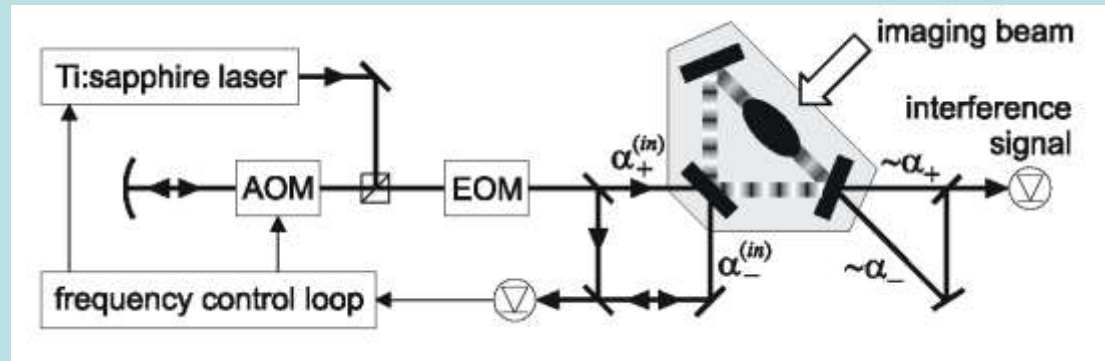


# Experiments with ring resonators:

C. Zimmermann  
Universität Tübingen

cold atoms at MOT temperature  
large detuning: 5 nm

Setup with BEC  
in Tübingen

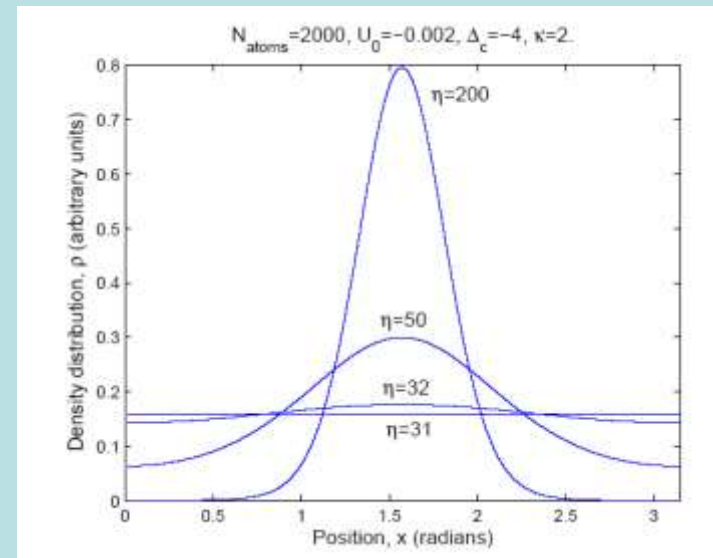
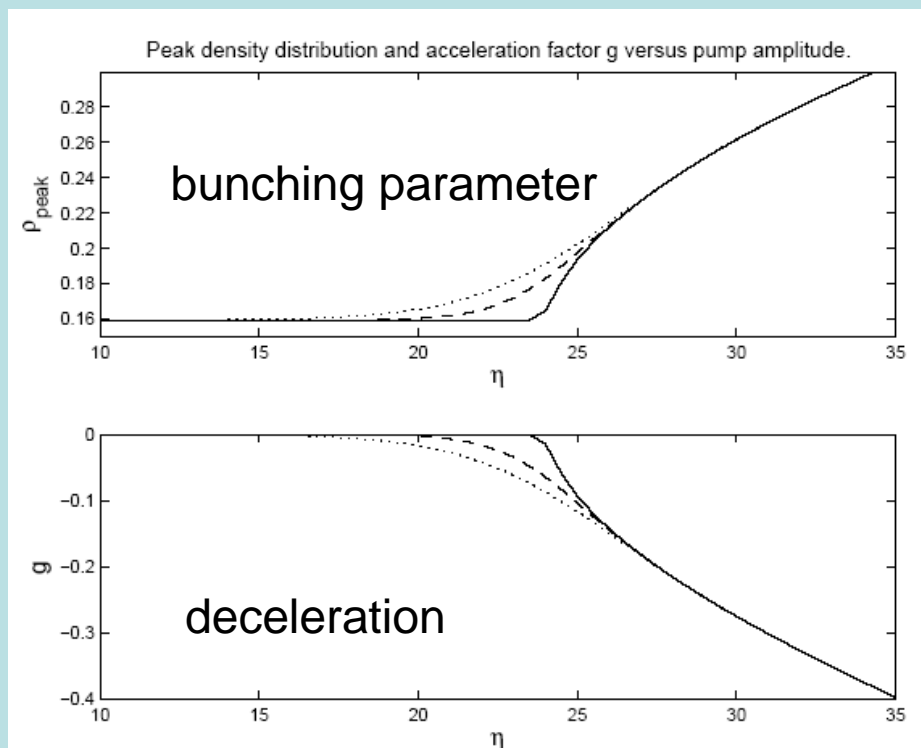
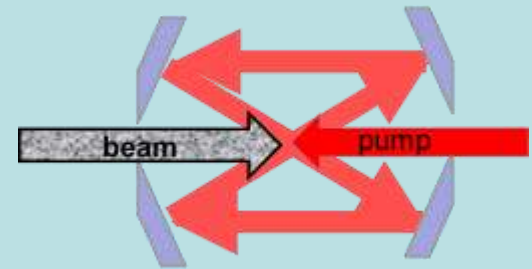


C. Zimmermann:  $>10^6$  atoms collectively accelerated  
A. Hemmerich: collective transverse oscillations of  $10^6$  atoms

*Inverse setup:*  
*effective centre of mass deceleration*  
*~ number of backscattered photons / particle*

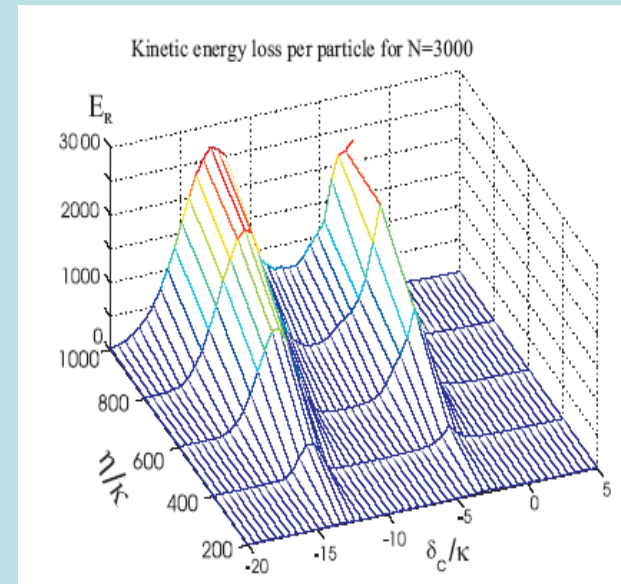
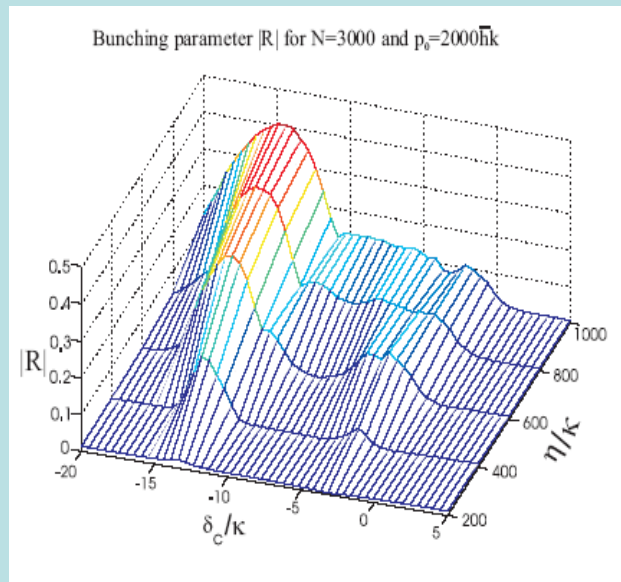
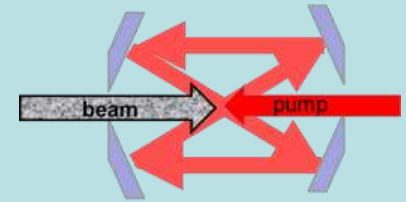
$$mg = -4\kappa |a_-^* a_-| \frac{k}{N}$$

$$g = -|\eta^2| \frac{2k}{m} \frac{N |U_0|^2 |R_-| \kappa}{|(\Delta_+ - NU_0 + i\kappa)(\Delta_- - NU_0 + i\kappa) - N^2 U_0^2 |R_-|^2|^2}$$



Force only limited by power – build in shutoff at low velocities

# Stopping of a fast beam (particle simulation)



for a fast beam two resonance frequencies appear:

- \* pump beam resonance and backscattering beam resonance
- \* backscattering resonance exhibits lower threshold and more efficient stopping



# Summary

- *Light forces are strongly modified in optical resonators:  
e.g: dipole force turns into a dissipative force through cavity decay*
- *self trapping and cooling with suppressed spontaneous emission in driven mode for any sufficiently polarizable particle*
- *selforganisation and collective superradiance into prescribed mode  
(cold gas nearly at rest) allows for fast trapping/cooling of large ensembles*
- *CARL type deceleration with suppression of fluorescence*
- *Sympathetic cooling (deceleration) of many species at same time*



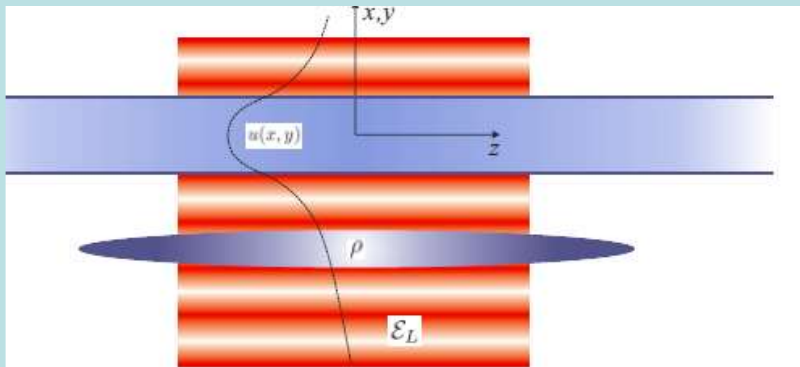
*Thanks for your attention !*

**Innsbruck University – visitors welcome !**





# Crystallization in 1D Systems

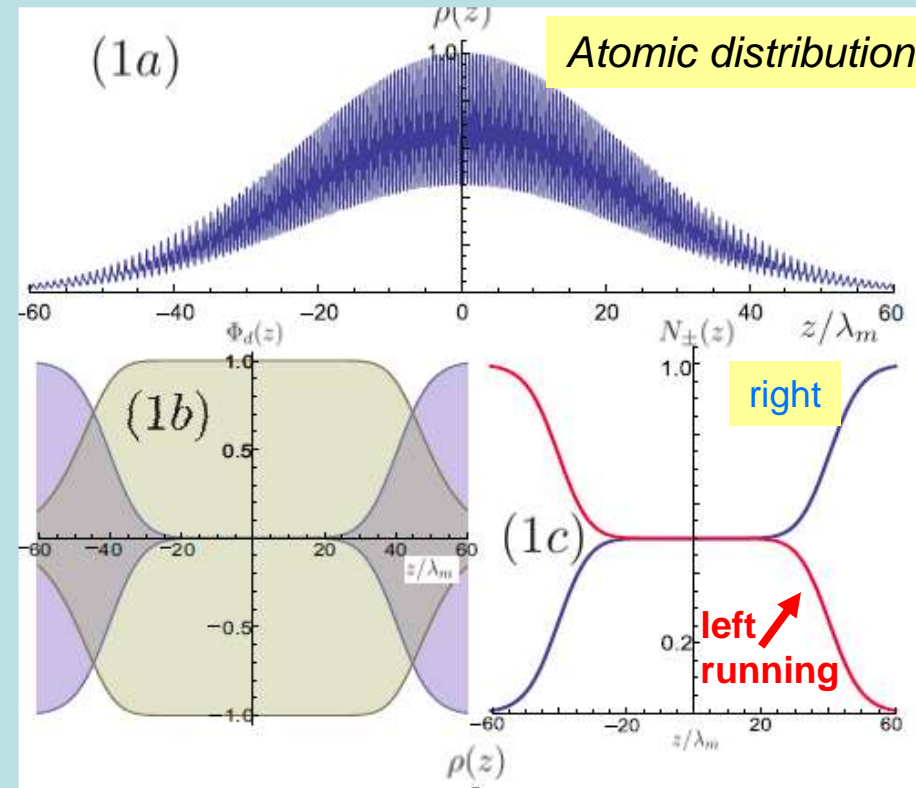


Cigar-shaped atomic gas alongside optical nanofiber.

$$\frac{\partial^2 E}{\partial z^2} + (\beta_m^2 + k_L^2 \tilde{\chi}) E = -k_L^2 \tilde{\chi} E_L, \quad (1)$$

$$\frac{\partial f}{\partial t} + \frac{p_z}{m} \frac{\partial f}{\partial z} - \frac{\partial}{\partial z} (U - \alpha[|E|^2 + 2E_L E_r]) \frac{\partial f}{\partial p_z} = 0.$$

Coupled Maxwell + mean field equation



Atomic distribution

Field distribution

(J. Kimble + coworkers, PRL 2013)