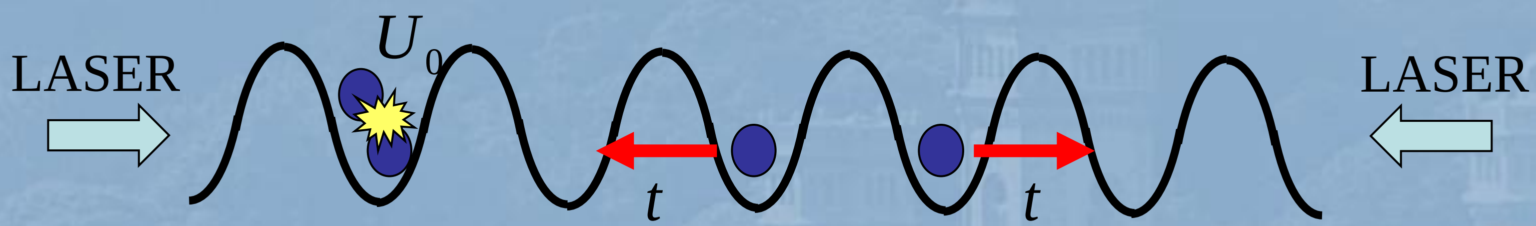
Leibniz
Universität
Hannover

Polar molecules in one-dimensional optical lattices

Luis Santos
Institute of Theoretical Physics and
Center of Excellence QUEST
Leibniz Universität Hannover

Santa Barbara, March 14 , 2013

Ultra-cold gases in optical lattices: lattice models

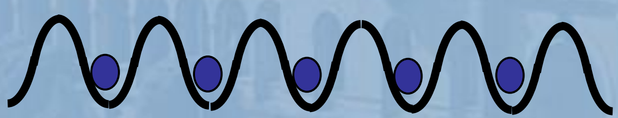


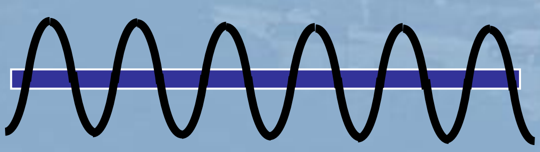
Bose-Hubbard Hamiltonian

[Fisher *et al.*, PRB **40**, 546 (1989) ; Jaksch *et al.*, PRL **81**, 3108 (1998)]

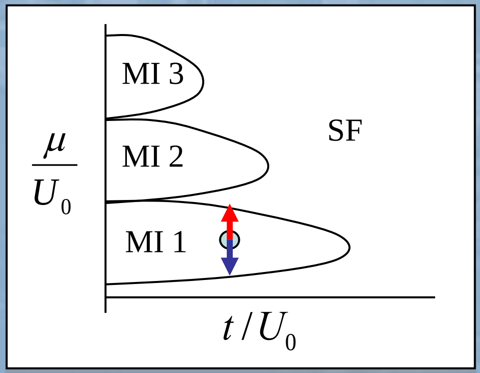
$$H = -t \sum_i \left[b_i^+ b_{i+1} + H.c. \right] + \frac{U_0}{2} \sum_i n_i \left(n_i - 1 \right)$$

Mott insulator (gapped incompressible insulator)



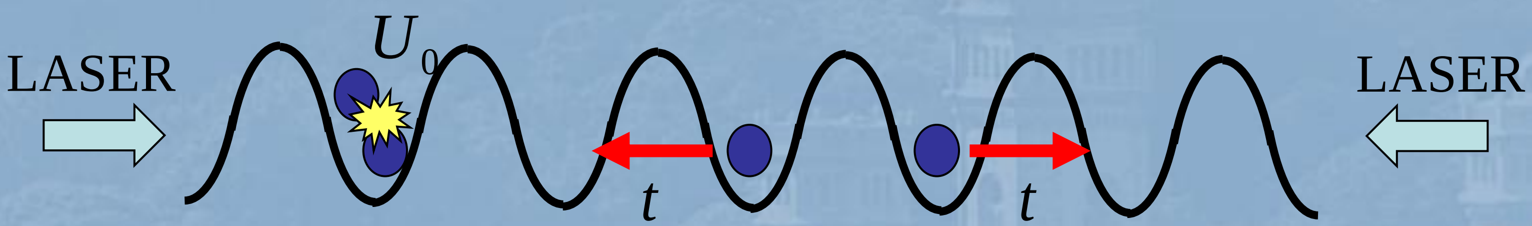


Superfluid



[M. Greiner et al., Nature **415**, 39 (2002)]

Ultra-cold gases in optical lattices: lattice models



Bose-Hubbard Hamiltonian

[Fisher *et al.*, PRB **40**, 546 (1989) ;
Jaksch *et al.*, PRL **81**, 3108 (1998)]

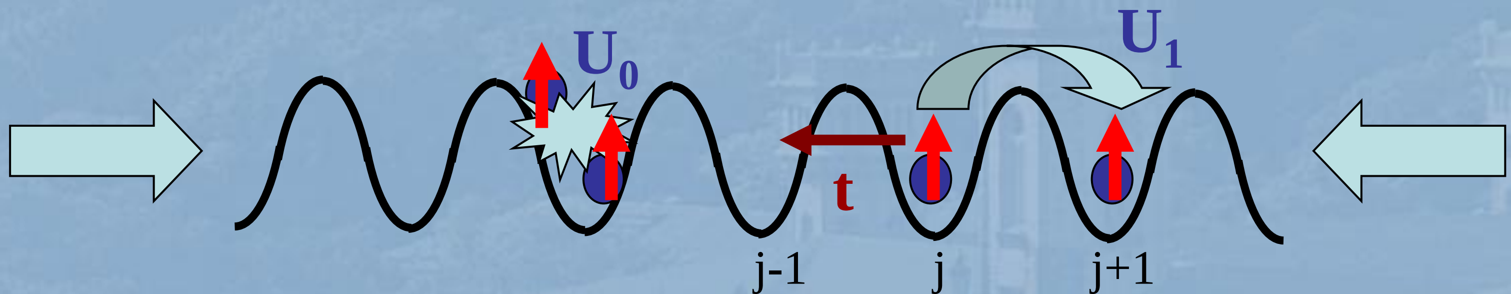
$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c. \right] + \frac{U_0}{2}\sum_i n_i\left(n_i - 1\right)$$

Resemble spin models,
e.g. hard-core bosons (n=0,1)

$$H_{XX} \approx -2t\sum_{<ij>} \left(S_i^x S_j^x + S_i^y S_j^y \right)$$

Dipolar lattice gases: intersite effects

For a dipolar gas inter-site interactions become important



Extended Bose-Hubbard model

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i\left(n_i - 1\right) + U_1\sum_i n_i n_{i+1}$$

Dipolar interactions have been shown already to play an important role in dipolar Chromium BECs in optical lattices (Stuttgart)

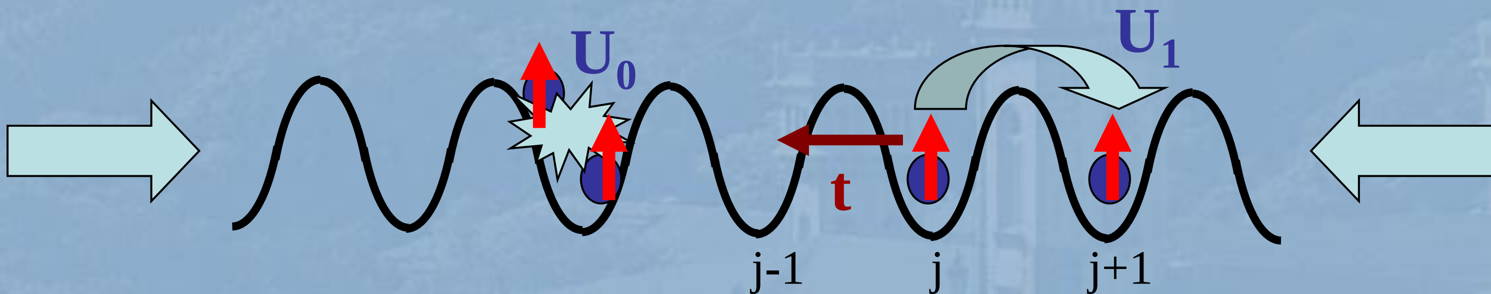
Inter-site destabilization
[Müller et al., PRA **84**, 053601 (2011)]

Time-of-flight-induced collapse of in-lattice stable BECs
[Billy et al., PRA **86**, 051603(R) (2012)]

Also in recent works on spin dynamics of Chromium in lattices (Talk of B. Laburthe)

Dipolar lattice gases: intersite effects

For a dipolar gas inter-site interactions become important



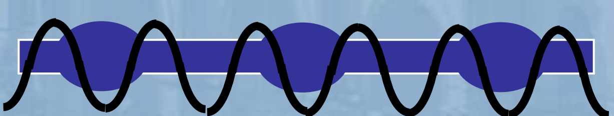
Extended Bose-Hubbard model

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i\left(n_i - 1\right) + U_1\sum_i n_i n_{i+1}$$

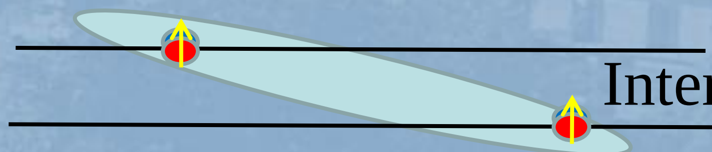
Supersolid, density-waves, self-assembled crystals, metastable states, interlayer superfluids and more…

[Baranov et al, Chemical Reviews **112**, 5012 (2012)]

Density wave



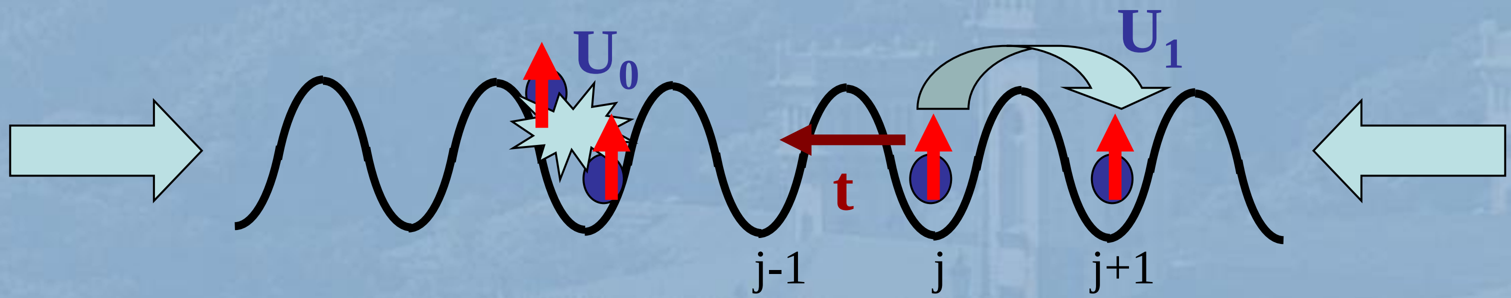
Supersolid



Interlayer SF

Dipolar lattice gases: intersite effects

For a dipolar gas inter-site interactions become important



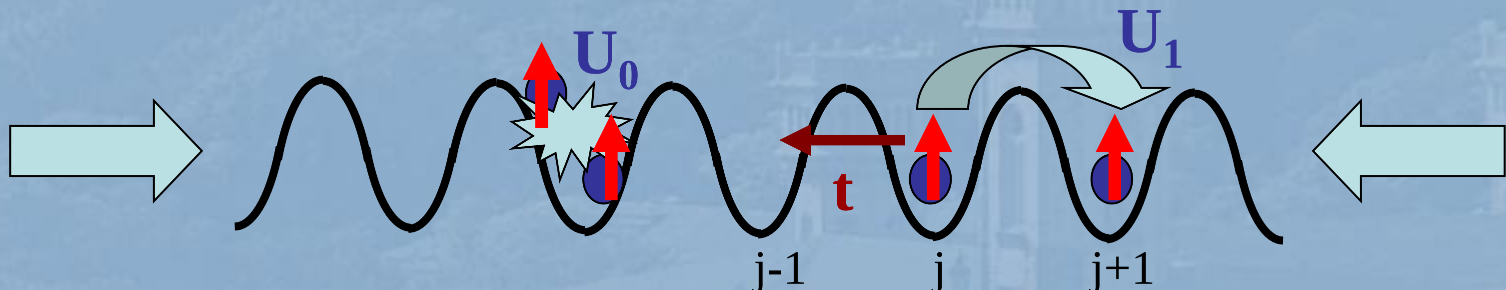
1D polar gases

The intersite interactions lead to a very rich physics for 1D chains and ladders

- Devil's staircase [Burnell et al., PRB 80, 174519 (2009)]
- Haldane insulator [Dalla Torre et al., PRL 97, 260401 (2006)]
- Disorder [Deng et al., arXiv (2012)]
- Simulation of spin-orbital models [Sun et al., PRB **86**, 155159 (2012)]
- More... see. [Chemical Reviews **112**, 5012 (2012]

Dipolar lattice gases: intersite effects

For a dipolar gas inter-site interactions become important



1D polar gases

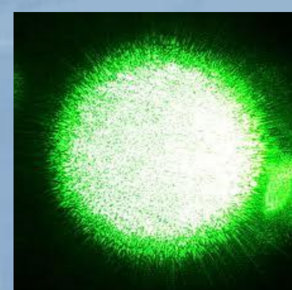
The intersite interactions lead to a very rich physics for 1D chains and ladders

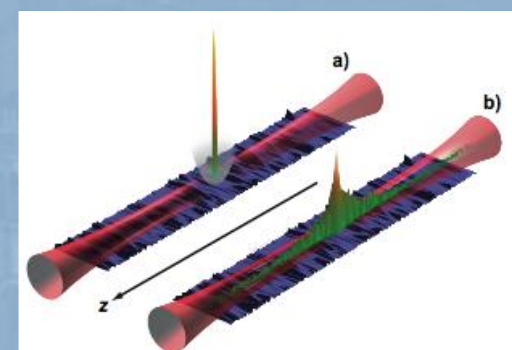
- Devil's staircase [Burnell et al., PRB 80, 174519 (2009)]
- **Haldane insulator** [Dalla Torre et al., PRL 97, 260401 (2006)]
- **Disorder** [Deng et al., arXiv (2012)]
- Simulation of spin-orbital models [Sun et al., PRB **86**, 155159 (2012)]
- More... see. [Chemical Reviews **112**, 5012 (2012)]

Disorder in optical lattices

Speckle

[Billy et al., Nature **453**, 891 (2008)]

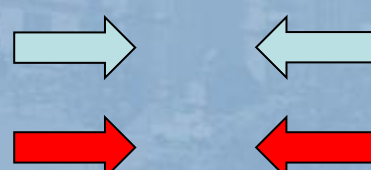




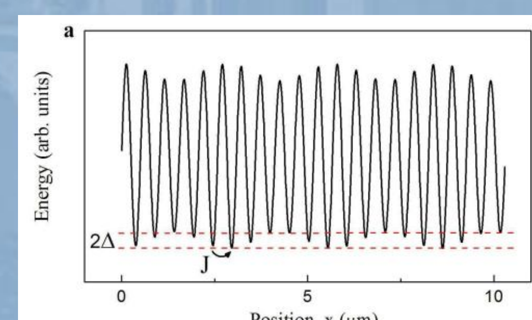
Bichromatic lattices

[Roati et al., Nature **453**, 895 (2008)]

ω



ω_2



Quasi-periodic

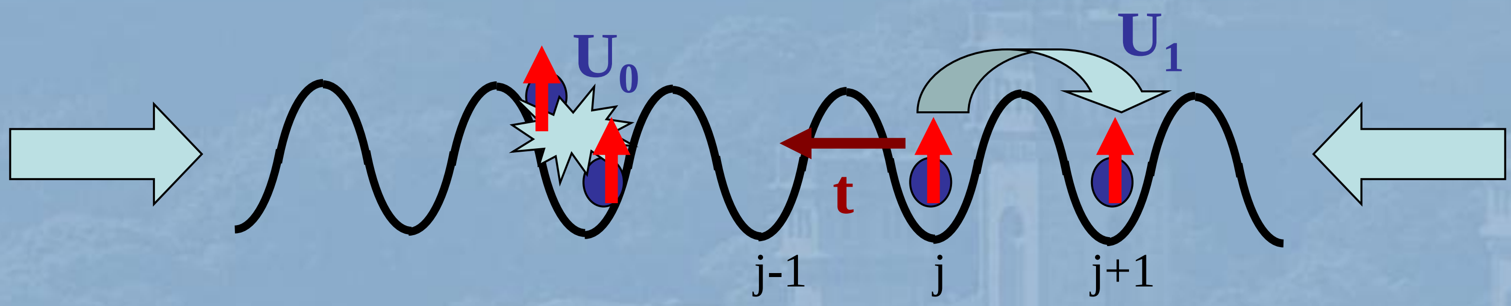
This talk

1D polar gases at unit filling

1D polar gases with uniform bound disorder

1D polar gases in a quasi-periodic lattice

1D polar bosons in optical lattices



Extended Bose-Hubbard model

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i\left(n_i - 1\right) + U_1 \sum_i n_i n_{i+1}$$

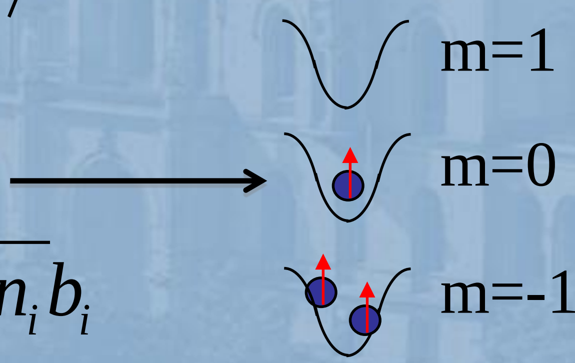
Average filling $\langle n \rangle = 1$

Holstein-Primakoff transformation maps occupation into spin-1

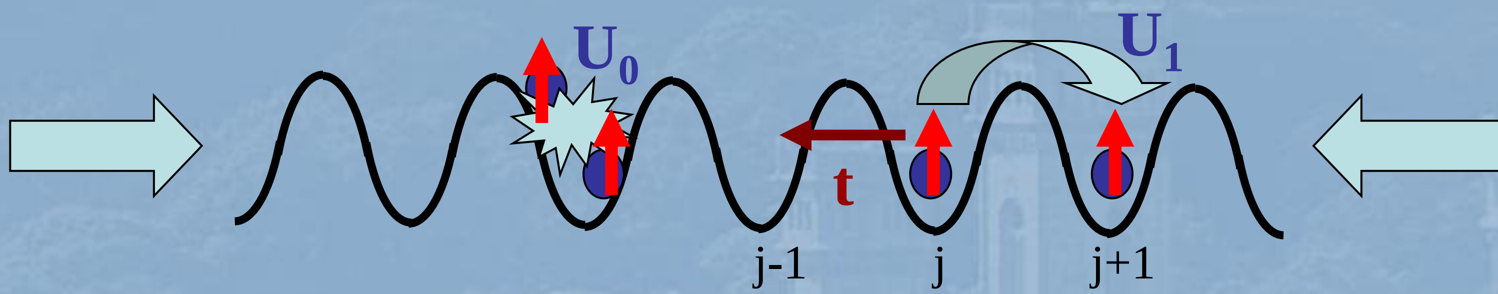
$$S_i^z = 1 - n_i$$

$$S_i^+ = \sqrt{2 - n_i}\, b_i$$

$$S_i^- = b_i^+ \sqrt{2 - n_i}$$



1D polar bosons in optical lattices



Extended Bose-Hubbard model

$$H=-t\sum_i\left[b_i^+ b_{i+1}+H.c.\right]+\frac{U_0}{2}\sum_i n_i\left(n_i-1\right)+U_1\sum_i n_i n_{i+1}$$

The system resembles to a large extent an AF spin-1 chain with uniaxial single-ion anisotropy

$$H=J\sum_i\left[S_i^x\cdot S_{i+1}^x+S_i^y\cdot S_{i+1}^y+\Delta S_i^z\cdot S_{i+1}^z+D\left(S_i^z\right)^2\right]$$

$$J=2t \qquad D=U_0/4t$$

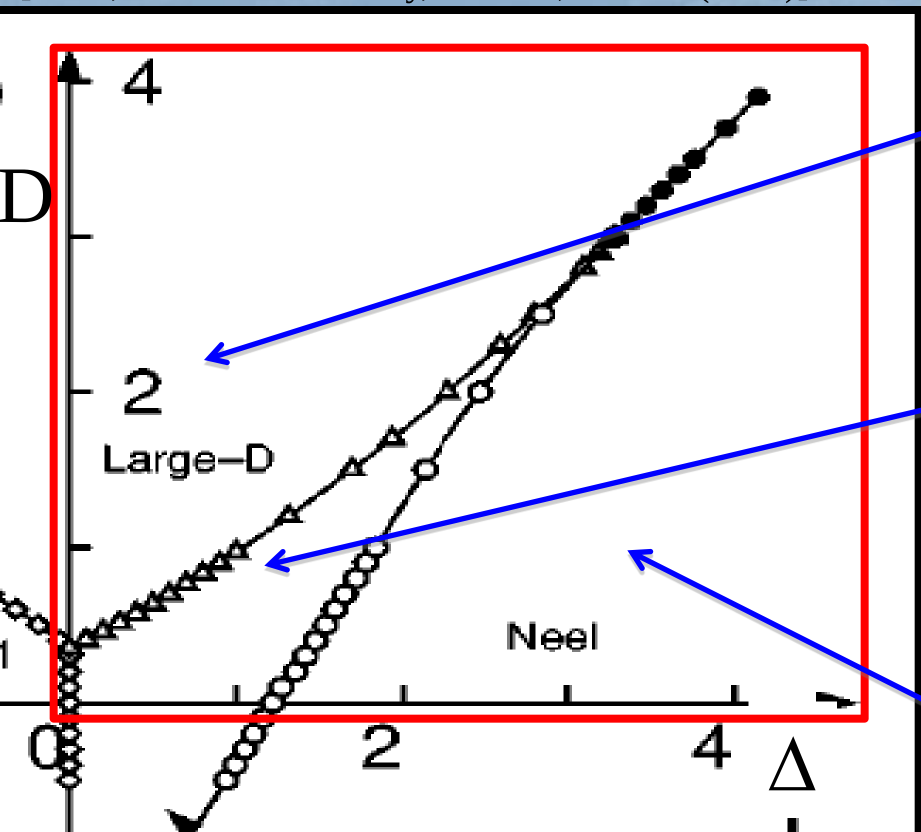
$$\Delta=U_1/2t$$

(imperfect mapping: due to n>2 and extra terms $\sim S_i^-\cdot F\left(S_i^z+S_{i+1}^z\right)\cdot S_{i+1}^+$)

AF spin-1 chains

$$H = J\sum_i \left[S_i^x \cdot S_{i+1}^x + S_i^y \cdot S_{i+1}^y + \Delta S_i^z \cdot S_{i+1}^z + D\left(S_i^z\right)^2 \right]$$

[Chen, Hida and Sanctuary, PRB **67**, 104401 (2003)]



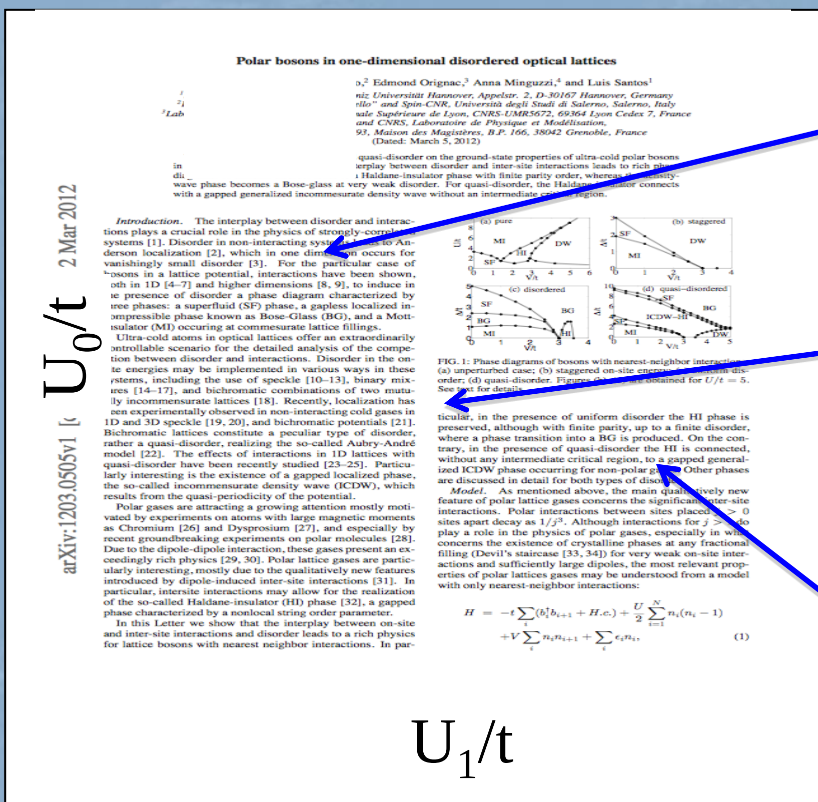
Large-D phase …0+-0…0-+0…0-+0…

Haldane phase …0+0…0-0…0+0…0-0…
(„diluted AF order“)

Néel phase …+-+-+-+-+-+-+-…

1D polar gases in optical lattices: Haldane-insulator phase

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i(n_i - 1) + U_1 \sum_i n_i n_{i+1}$$



Mott-insulator …1021…1201…1200…

Haldane-insulator …101…121…101…121…

[Dalla-Torre, Berg and Altman, PRL **97**, 260401 (2006)]

Density wave …0202020202020202…

String and parity order

$\delta n_i = 1 - n_i$

„fluid of AF ordered defects“

...101...121...101...121...

String order

$$O_S^2 \equiv \lim_{|i-j|\to\infty} \left\langle -\delta n_i \exp\left[i\pi \sum_{l=i+1}^{j-1} \delta n_l \right] \delta n_j \right\rangle \neq 0$$

Parity order

$$O_P^2 \equiv \lim_{|i-j|\to\infty} \left\langle \exp\left[i\pi \sum_{l=i+1}^{j-1} \delta n_l \right] \right\rangle = 0$$

i j

...1101...121...101...121...

...1011...211...011...211...

...0111...112...110...112...

For the Mott insulator $O_S^2 = 0$ $O_P^2 \neq 0$

[Endres et al., Science **334**, 200 (2011)]

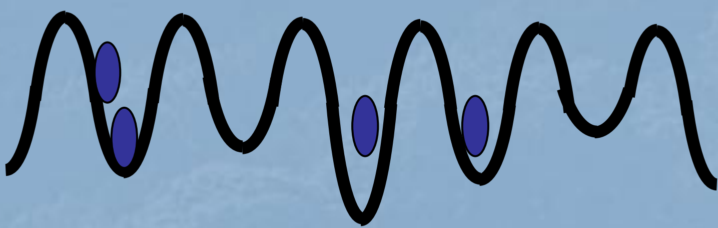
This talk

1D polar gases at unit filling

1D polar gases with uniform bound disorder

1D polar gases in a quasi-periodic lattice

1D gases in disordered optical lattices



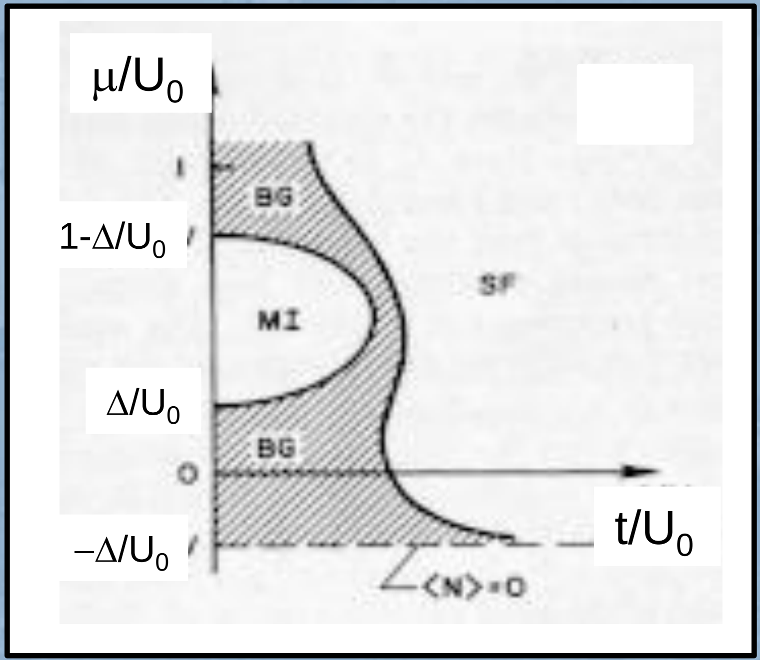
$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i\left(n_i - 1\right) + \sum_i \varepsilon_i n_i$$

Bounded box disorder: $-D < e_i < D$

(similar to speckle)

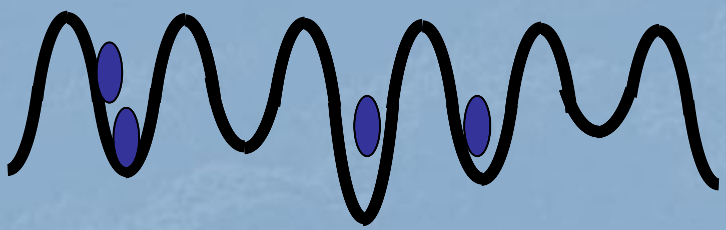
For non-polar bosons: Bose-Glass (gapless, compressible insulator)

[Giamarchi and Schulz, PRB **37**, 325 (1988);
Fisher et al., PRB **40**, 546 (1989);
Rapsch, Schollwöck and Zwerger, EPL **46**, 559 (1999)]



[From PRB **40**, 546 (1989)]

1D gases in disordered optical lattices

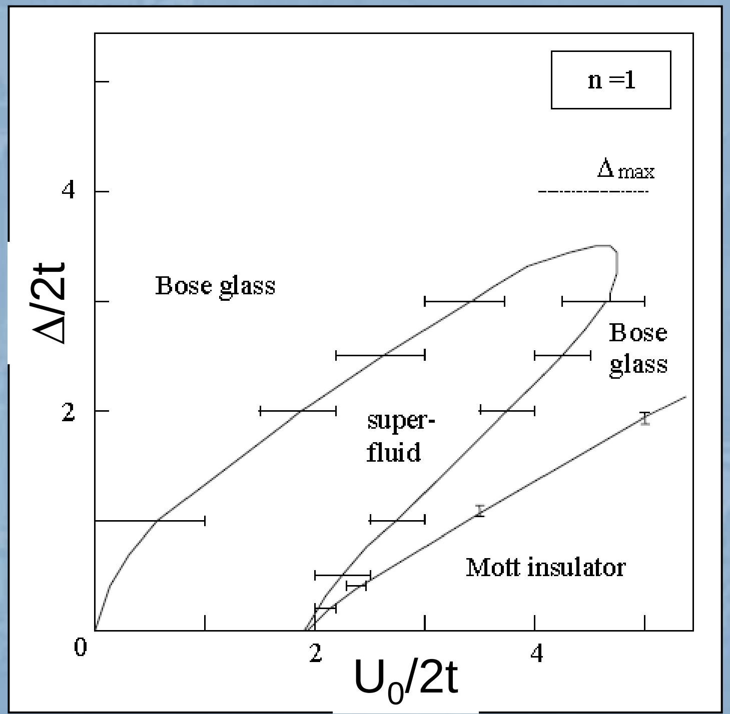


$$H = -t\sum_i \left[b_i^{+} b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i\left(n_i - 1\right) + \sum_i \varepsilon_i n_i$$

Bounded box disorder: $-D < \varepsilon_i < D$

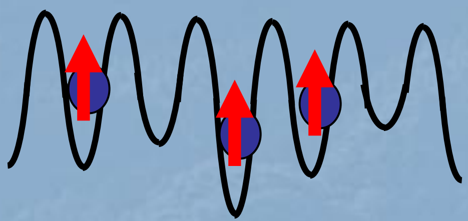
For non-polar bosons: Bose-Glass (gapless, compressible insulator)

[Giamarchi and Schulz, PRB **37**, 325 (1988);
Fisher et al., PRB **40**, 546 (1989);
Rapsch, Schollwöck and Zwerger, EPL **46**, 559 (1999)]



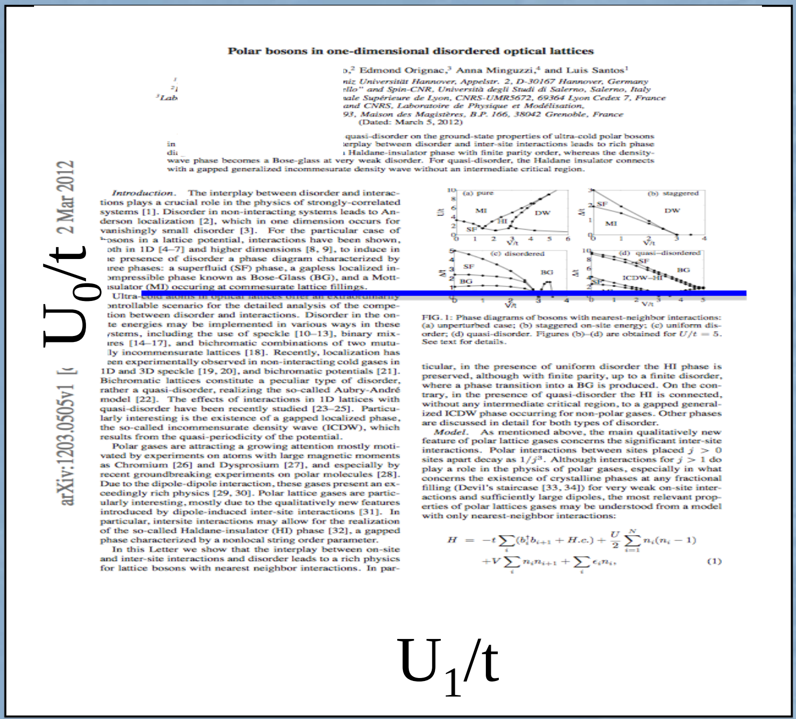
[From EPL **46**, 559 (1989)]

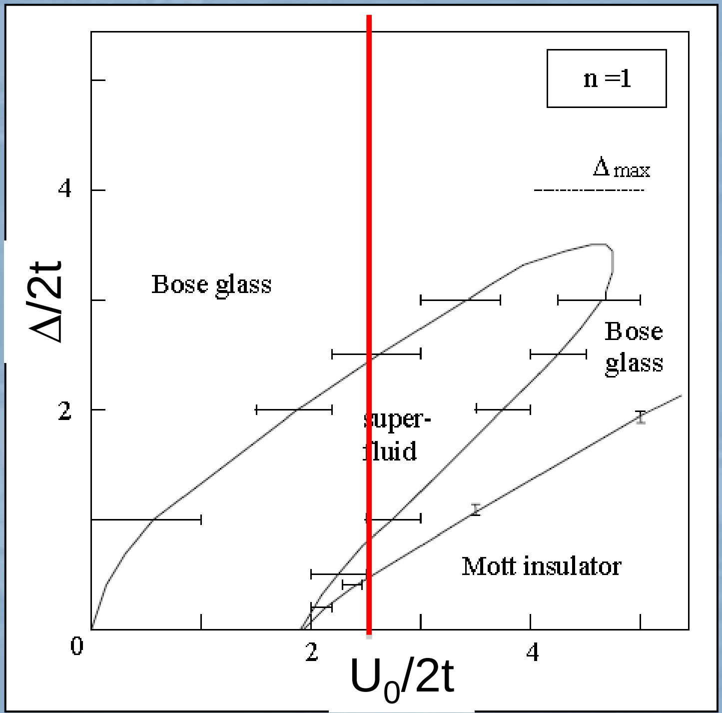
1D polar gases in disordered optical lattices



$$H=-t\sum_i\left[b_i^{+}b_{i+1}+H.c.\right]+\frac{U_0}{2}\sum_i n_i(n_i-1)+U_1\sum_i n_i n_{i+1}+\sum_i \varepsilon_i n_i$$

Bounded box disorder: $-D<\varepsilon_i<D$





[From EPL **46**, 559 (1989)]

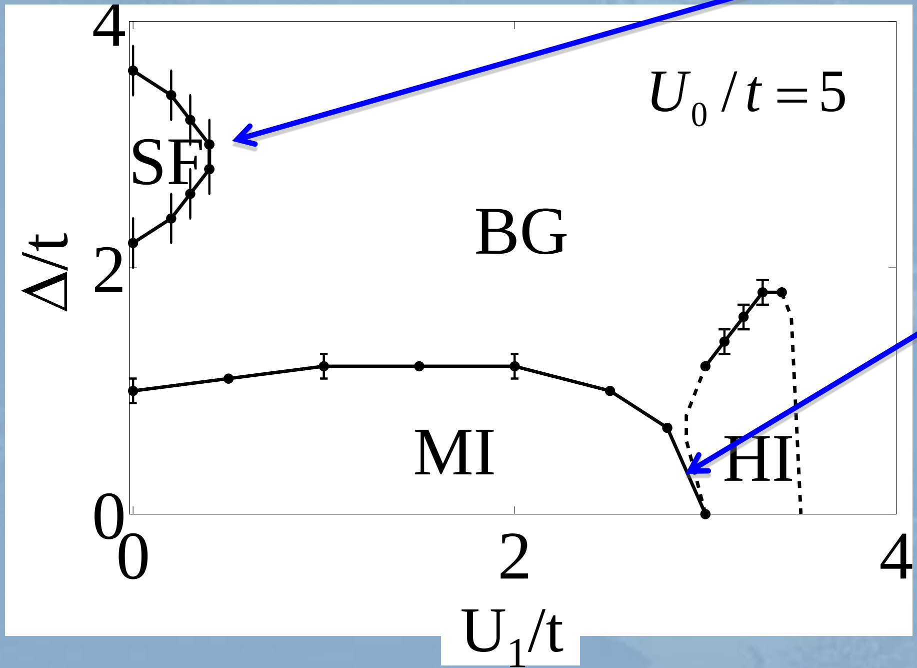
1D polar gases in disordered optical lattices

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i(n_i - 1) + U_1\sum_i n_i n_{i+1} + \sum_i \varepsilon_i n_i$$

Bounded box disorder: $-D < \varepsilon_i < D$

SF vanishes for growing NN interactions (SF-BG at K=3/2)

[Giamarchi and Schulz, EPL **3**, 1287 (1987)]



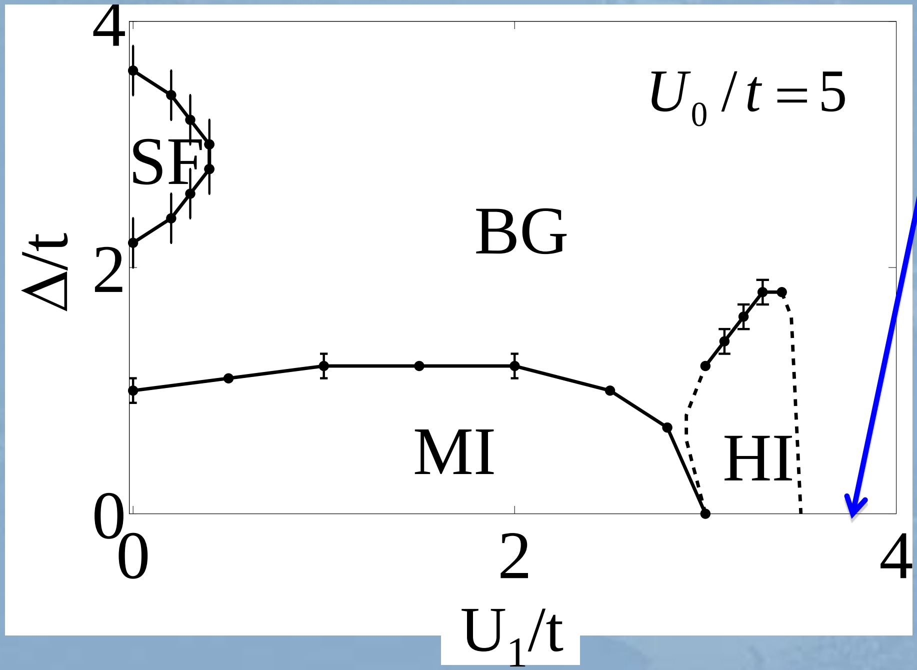
BG goes all the way to zero at the MI-HI for K<3/2

[Deng et al., arXiv:1302.0528]

1D polar gases in disordered optical lattices

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i(n_i - 1) + U_1 \sum_i n_i n_{i+1} + \sum_i \varepsilon_i n_i$$

Bounded box disorder: $-D < \varepsilon_i < D$



DW disappears even for very small disorder.

(Imry-Ma argument)

[Imry and Ma, PRL **35**, 1399 (1975)]

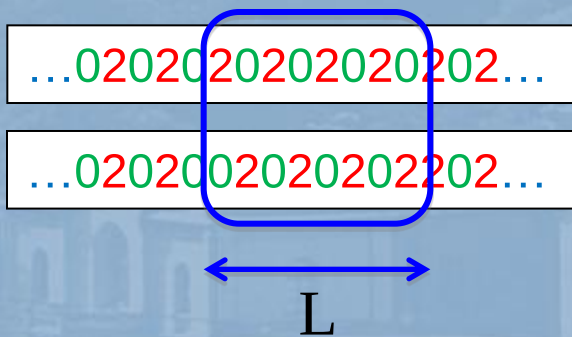
Imry-Ma argument

[Imry and Ma, PRL **35**, 1399 (1975)]

…0202020202020202…

$$\mathcal{O}_{DW} = \lim_{|i-j|\to\infty}\langle(-1)^{i-j}\delta n_i \delta n_j\rangle$$

Gap for flipping a spin



Energy of the domain
coming from the noise: <Δε> ~ Δ $L^{1/2}$

Energy of the domain walls ~ V

Formation of domains of size $L \sim (V/\Delta)^{1/2}$

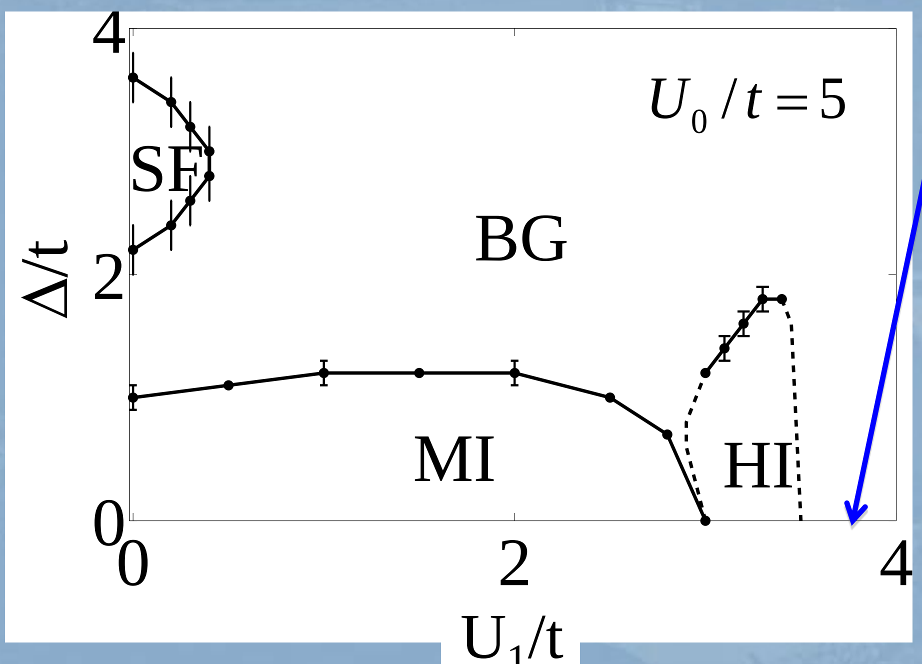
Domain formation destroys the order

The gap is destroyed because now the excitations are simply moving walls, not creating them

1D polar gases in disordered optical lattices

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i(n_i - 1) + U_1 \sum_i n_i n_{i+1} + \sum_i \varepsilon_i n_i$$

Bounded box disorder: $-D < \varepsilon_i < D$



DW disappears even for very small disorder.

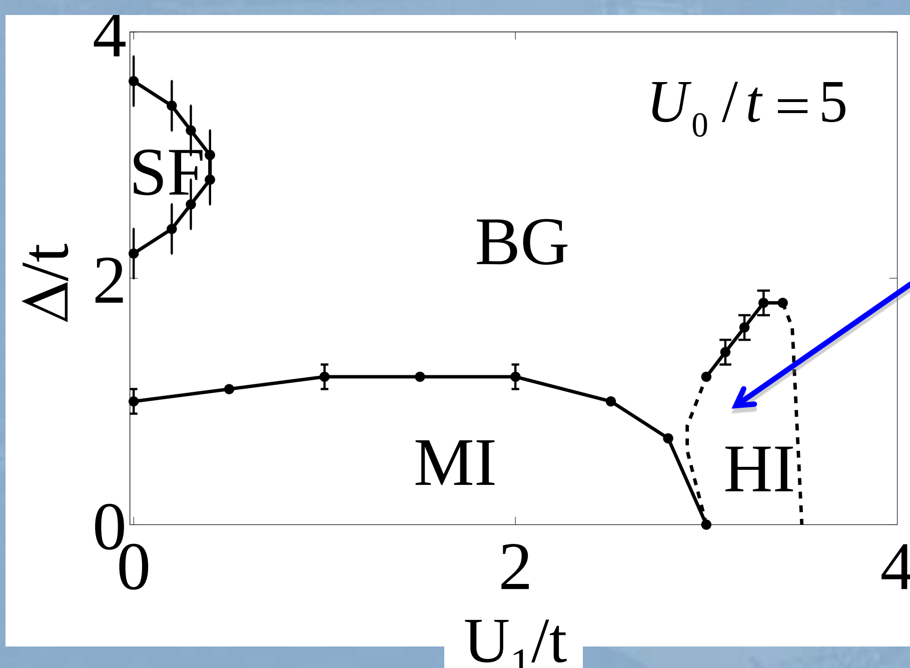
(Imry-Ma argument)

[Imry and Ma, PRL **35**, 1399 (1975)]

1D polar gases in disordered optical lattices

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i\left(n_i - 1\right) + U_1\sum_i n_i n_{i+1} + \sum_i \varepsilon_i n_i$$

Bounded box disorder: $-D < e_i < D$



The HI phase survives, but its character changes

$$O_S^2 \neq 0$$

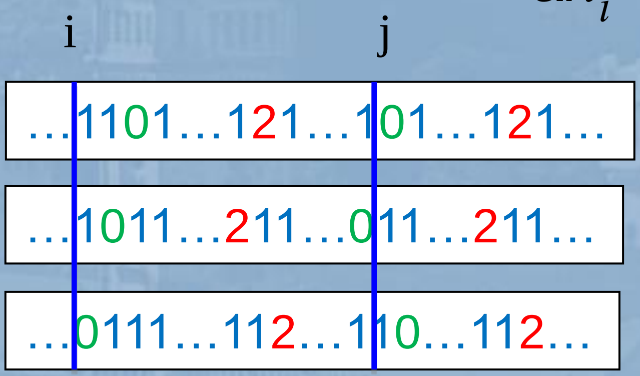
$$O_P^2 \neq 0$$

„Glassy“ Haldane insulator at finite disorder

$$\delta n_i = 1 - n_i$$

Parity order was zero in the HI due to the „fluid“ character of the defects

$$O_P^2 \equiv \lim_{|i-j|\to\infty} \left\langle \exp\left[i\pi \sum_{l=i+1}^{j-1} \delta n_l \right] \right\rangle = 0$$



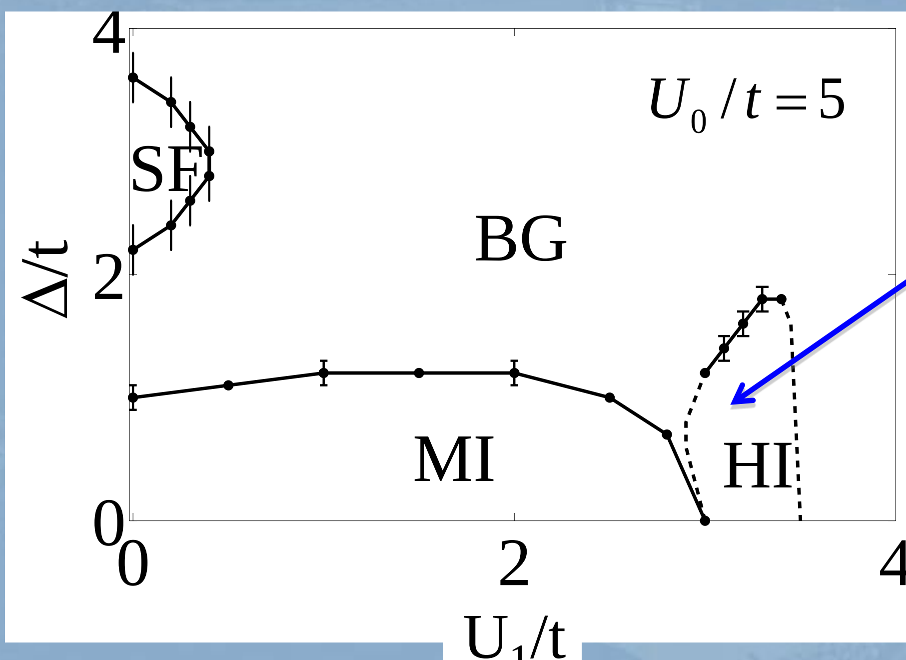
Disorder leads to the localization of defects reducing their mobility (while still keeping their AF ordering)

Parity becomes non-zero due to the glassy defects

1D polar gases in disordered optical lattices

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c. \right] + \frac{U_0}{2}\sum_i n_i (n_i - 1) + U_1 \sum_i n_i n_{i+1} + \sum_i \varepsilon_i n_i$$

Bounded box disorder: $- D < e_i < D$



The HI phase survives, but its character changes

O_S^2 1 0

O_P^2 1 0

This talk

1D polar gases at unit filling

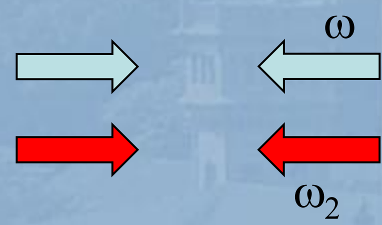
1D polar gases with uniform bound disorder

1D polar gases in a quasi-periodic lattice

1D gases in quasi-periodic optical lattices

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c. \right] + \frac{U_0}{2}\sum_i n_i (n_i - 1) + \sum_i \varepsilon_i n_i$$

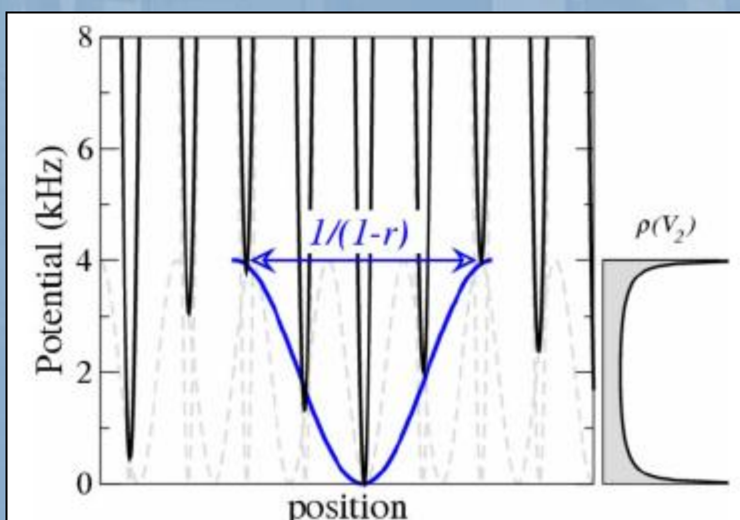
Bichromatic lattices



$$\varepsilon_j = \Delta \cos(2\pi r j + \varphi)$$
$$r = q_2 / q_1$$

Extra phase: incommesurate density wave (ICDW)

[Roscilde PRA **77**, 063605 (2008); Roux et al, PRA **78**, 023628 (2008)]



[From PRA **78**, 023628 (2008)]

Super-wells develop over a characteristic length scale 1/(1-r) („super site“) due to the beating between the two periods of the two lattices.

1D gases in quasi-periodic optical lattices

$$H = -t\sum_i \left[b_i^{+} b_{i+1} + H.c. \right] + \frac{U_0}{2}\sum_i n_i (n_i - 1) + \sum_i \varepsilon_i n_i$$

Bichromatic lattices

ω

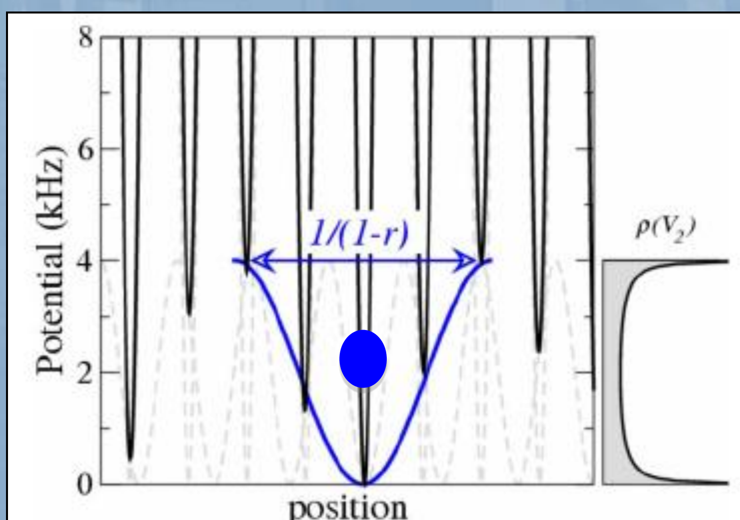
ω_2

$$\varepsilon_j = \Delta\cos(2\pi r j + \varphi)$$

$$r = q_2 / q_1$$

Extra phase: incommesurate density wave (ICDW)

[Roscilde PRA **77**, 063605 (2008); Roux et al, PRA **78**, 023628 (2008)]



[From PRA **78**, 023628 (2008)]

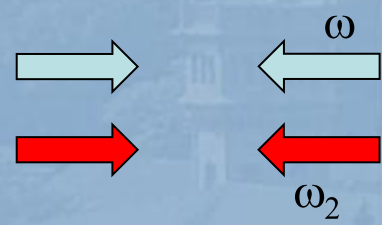
A ICDW phase for <n>=1-r may be interpreted as filling each „super-site“ with one particle.

For n=r it is like having one hole siting at each „super-site“

1D gases in quasi-periodic optical lattices

$$H = -t\sum_i \left[b_i^{+} b_{i+1} + H.c. \right] + \frac{U_0}{2}\sum_i n_i\left(n_i - 1\right) + \sum_i \varepsilon_i n_i$$

Bichromatic lattices

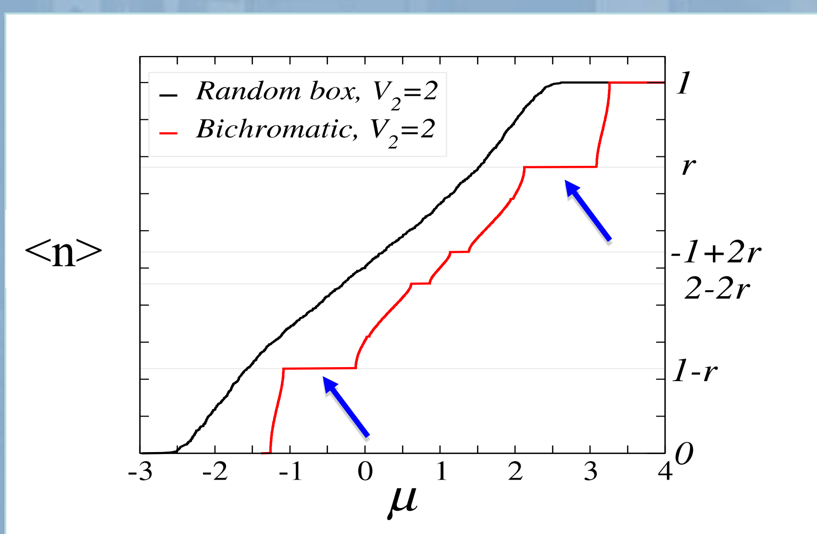


$$\varepsilon_j = \Delta\cos\left(2\pi r j + \varphi\right)$$

$$r = q_2 / q_1$$

Extra phase: incommensurate density wave (ICDW)

[Roscilde PRA **77**, 063605 (2008); Roux et al, PRA **78**, 023628 (2008)]



[From PRA **78**, 023628 (2008)]

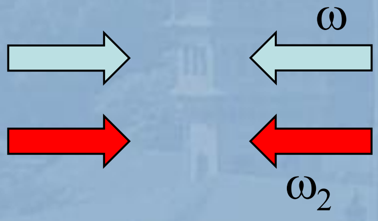
A ICDW phase for <n>=1-r may be interpreted as filling each „super-site" with one particle.

For n=r it is like having one hole siting at each „super-site"

1D gases in quasi-periodic optical lattices

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c. \right] + \frac{U_0}{2}\sum_i n_i (n_i - 1) + \sum_i \varepsilon_i n_i$$

Bichromatic lattices

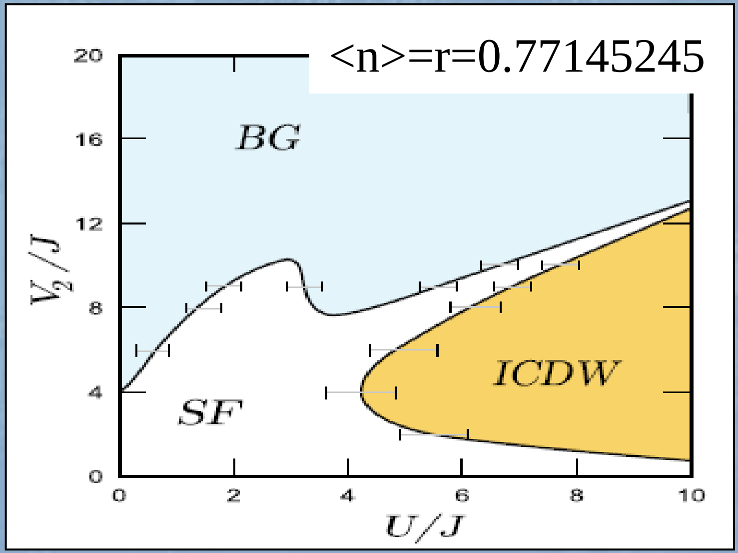


$$\varepsilon_j = \Delta\cos(2\pi r j + \varphi)$$
$$r = q_2 / q_1$$

Extra phase: incommensurate density wave (ICDW)

[Roscilde PRA **77**, 063605 (2008); Roux et al, PRA **78**, 023628 (2008)]

As a result a gapped ICDW appears with a structure factor peaking at the beating periodicity

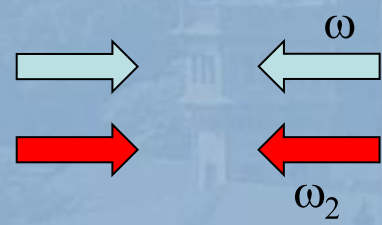


[From PRA **78**, 023628 (2008)]

1D gases in quasi-periodic optical lattices

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c. \right] + \frac{U_0}{2}\sum_i n_i(n_i - 1) + \sum_i \varepsilon_i n_i$$

Bichromatic lattices

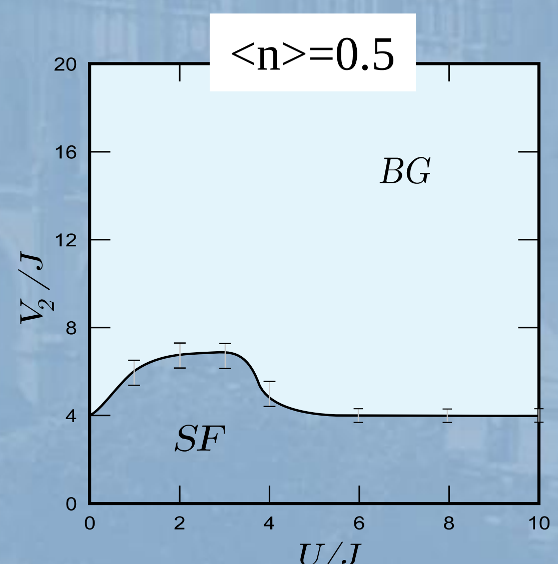


$$\varepsilon_j = \Delta \cos(2\pi r j + \varphi)$$
$$r = q_2 / q_1$$

Extra phase: incommesurate density wave (ICDW)

[Roscilde PRA **77**, 063605 (2008); Roux et al, PRA **78**, 023628 (2008)]

Away from these fillings the ICDW disappears

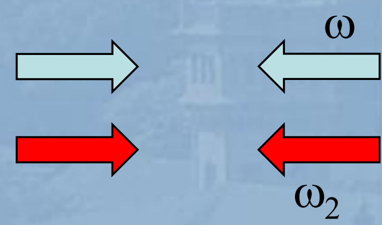


[From PRA **78**, 023628 (2008)]

1D gases in quasi-periodic optical lattices

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c. \right] + \frac{U_0}{2}\sum_i n_i (n_i - 1) + \sum_i \epsilon_i n_i$$

Bichromatic lattices

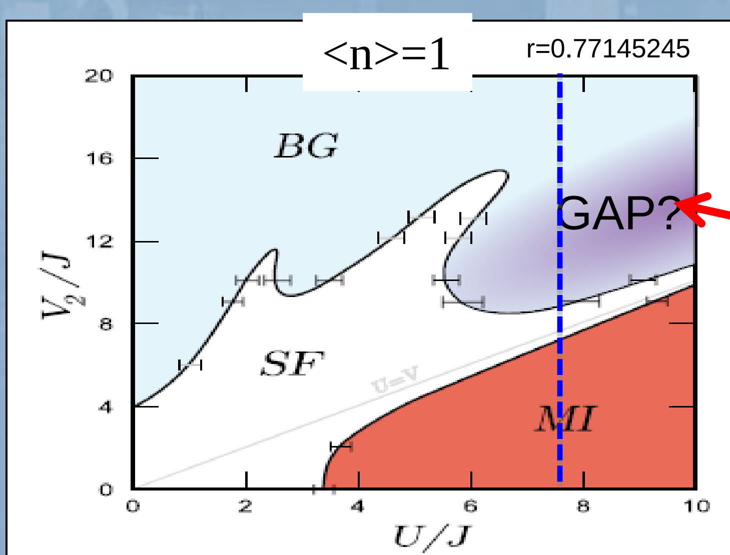


$$\epsilon_j = \Delta \cos(2\pi r j + \varphi)$$

$$r = q_2 / q_1$$

Extra phase: incommesurate density wave (ICDW)

[Roscilde PRA **77**, 063605 (2008); Roux et al, PRA **78**, 023628 (2008)]



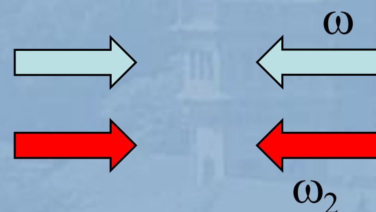
[From PRA **78**, 023628 (2008)]

For a filling <n>=1 a possible gapped phase may occur as well (generalized ICDW)

1D polar gases in disordered optical lattices

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i(n_i - 1) + U_1\sum_i n_i n_{i+1} + \sum_i \varepsilon_i n_i$$

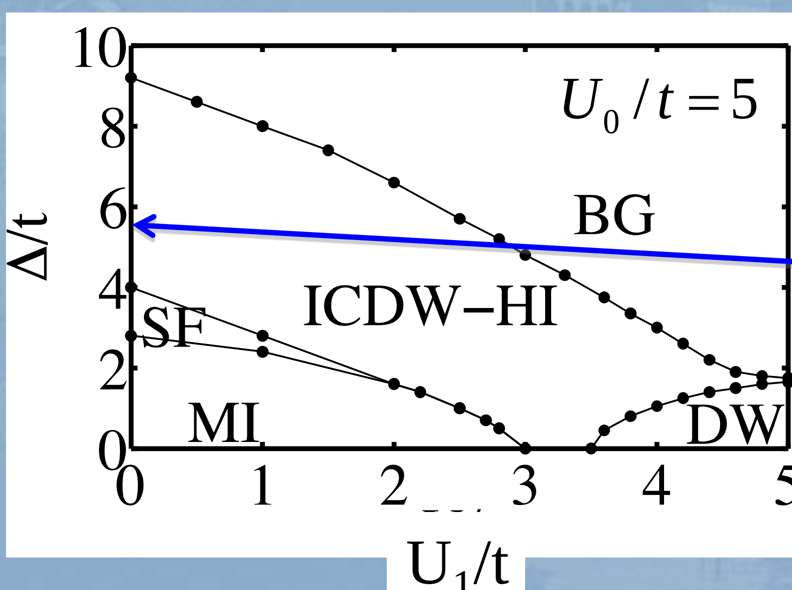
Bichromatic lattices

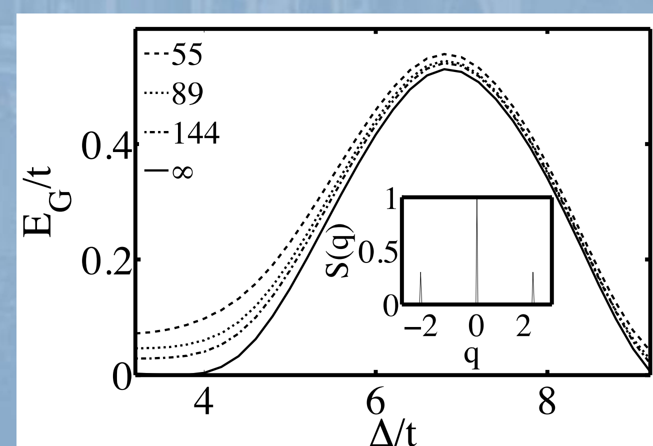


$e_j = D\cos(2prj + j)$

$r = q_2 / q_1 = (\sqrt{5} - 1)/2$

A generalized ICDW phase appears for <n>=1 for a sufficiently large 2nd lattice

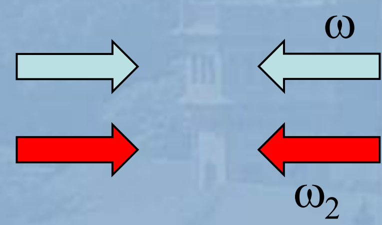




1D polar gases in disordered optical lattices

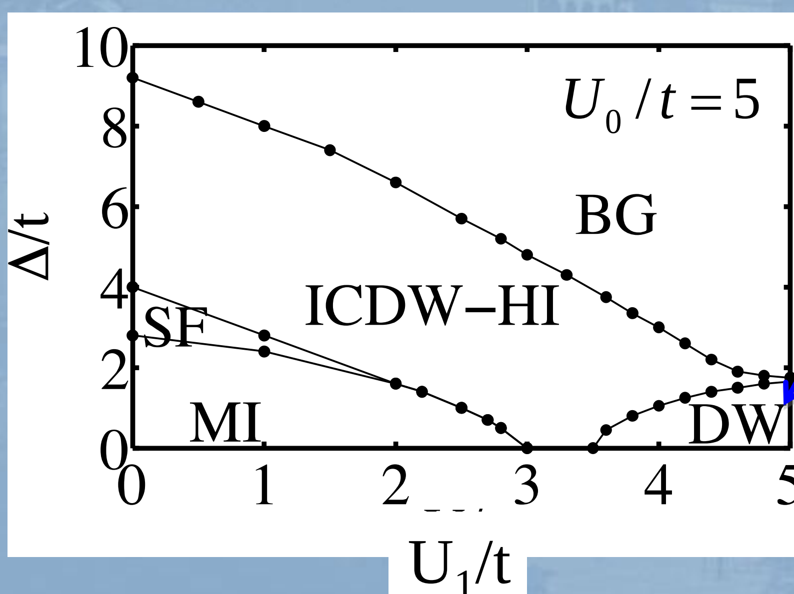
$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i(n_i - 1) + U_1\sum_i n_i n_{i+1} + \sum_i \varepsilon_i n_i$$

Bichromatic lattices



$$\varepsilon_j = \Delta\cos(2\pi r j + \phi)$$

$$r = q_2 / q_1 = \left(\sqrt{5} - 1\right)/2$$

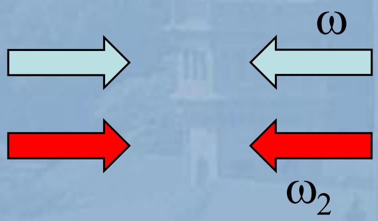


The DW phase survives for a small disorder (no Imry-Ma argument here)

1D polar gases in disordered optical lattices

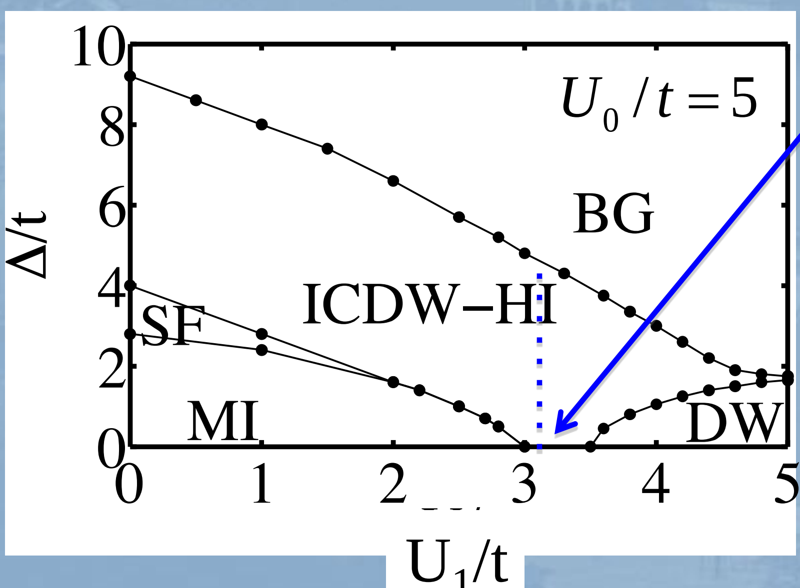
$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i(n_i - 1) + U_1\sum_i n_i n_{i+1} + \sum_i \varepsilon_i n_i$$

Bichromatic lattices

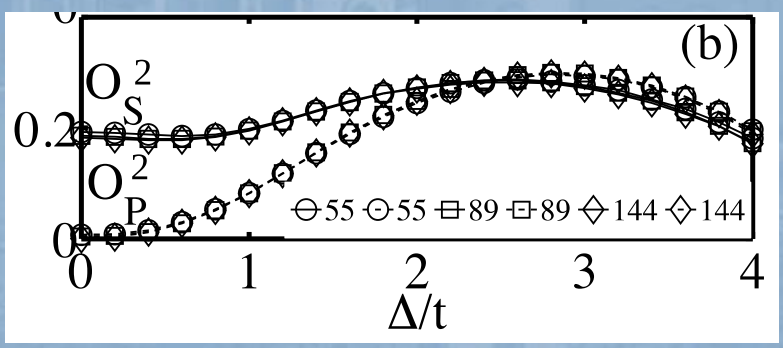


$$\varepsilon_j = \Delta\cos(2\pi r j + \varphi)$$

$$r = q_2 / q_1 = \left(\sqrt{5} - 1\right)/2$$



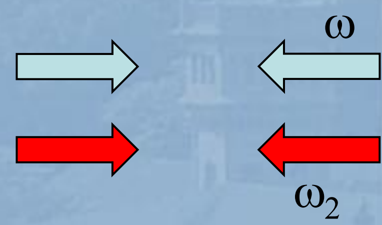
„Glassy“ HI phase due to the pinining of defects



1D polar gases in disordered optical lattices

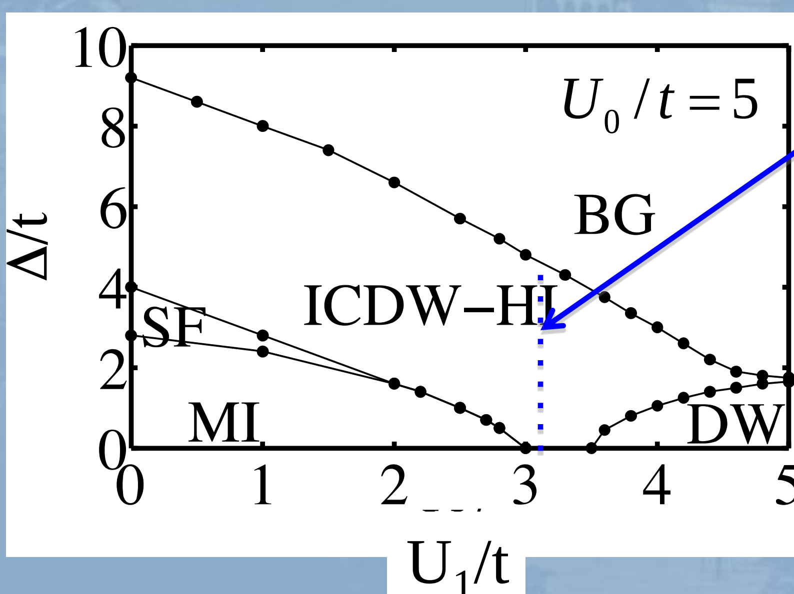
$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i(n_i - 1) + U_1\sum_i n_i n_{i+1} + \sum_i \varepsilon_i n_i$$

Bichromatic lattices

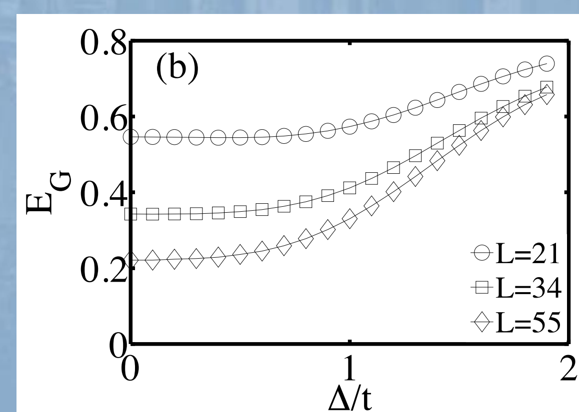


$$\varepsilon_j = \Delta\cos(2\pi r j + \phi)$$

$$r = q_2/q_1 = \left(\sqrt{5} - 1\right)/2$$



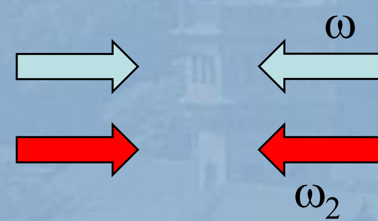
The „glassy“ HI phase connects adiabatically with the ICDW (which has also O_S^2, $O_P^2 > 0$)



1D polar gases in disordered optical lattices

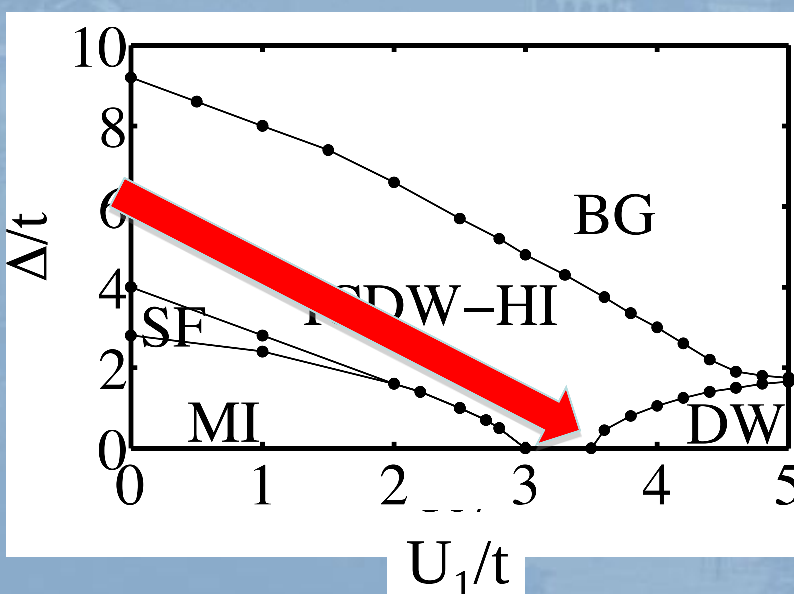
$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i\left(n_i - 1\right) + U_1\sum_i n_i n_{i+1} + \sum_i \varepsilon_i n_i$$

Bichromatic lattices



$$e_j = D\cos\left(2prj + j\right)$$

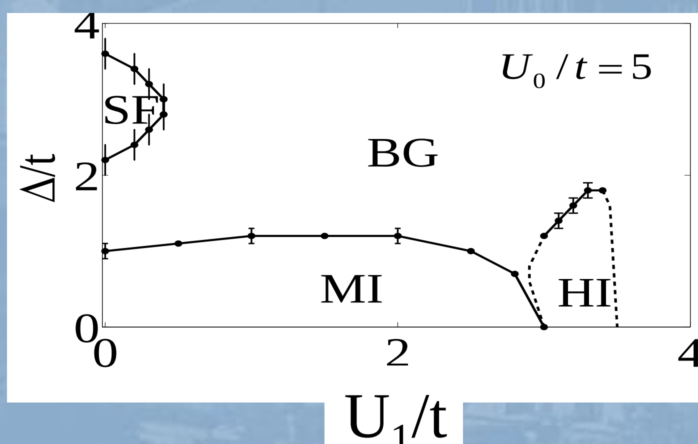
$$r = q_2 / q_1 = \left(\sqrt{5} - 1\right)/2$$

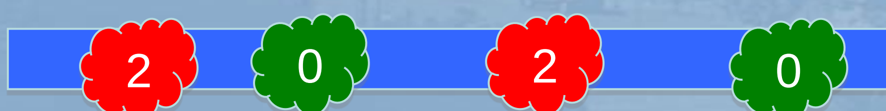


This may open an interesting route (protected by a gap) for the creation of the HI phase

Summary

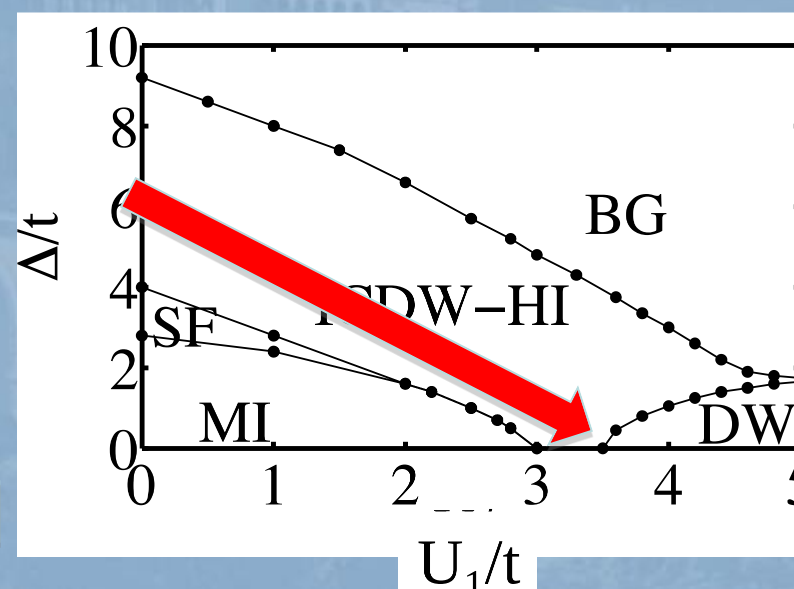
Polar gases behave very differently in uniformly disordered lattices and quasi-periodic lattices





Uniform disorder: glassy HI (dissapearing into a BG) and rapidly vanishing DW

Quasi-disorder: finite DW, glassy-HI and ICDW adiabatically connected



People

X. Deng
(Leibniz Uni.)

E. Orignac
(ENS, Lyon)

R. Citro
(Univ. Salerno)

A. Minguzzi
(Univ. Grenoble)

T. Vekua

G. Sun

+ G. Jackeli (MPI-FKP, Stuttgart)