Next-to-Leading Order QCD Tools: Status and Prospects

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Introduction

Topics that I will cover:

- $\mathcal{O}(\alpha_s)$ corrections to tree-level processes
  - graphs involving *one* virtual loop
  - no resummation of logarithms
  - no power corrections
  - no matching with parton showers
- When discussing NLO programs, they will not be event generators
  - predictions are parton level only, with no showering, hadronization or detector effects
  - for processes involving jets, one jet will contain at most two partons

Why NLO?

The benefits of higher order calculations are well known

- Less sensitivity to unphysical input scales
  - first predictive normalization of observables at NLO
  - more accurate estimates of backgrounds for new physics searches and (hopefully) interpretation
  - confidence that cross-sections are under control for precision measurements
- More physics
  - jet merging
  - initial state radiation
  - more parton fluxes
- It represents the first step for a plethora of other techniques
  - matching with resummed calculations
  - NLO parton showers

So ....

If all this is true then, given that we have invested heavily (both financially and intellectually) in new upgrades and colliders like Run II of the Tevatron and the LHC:

- What's the current state-of-the-art?
  - NLO tools currently available
- Why are we lacking NLO predictions for many interesting (and crucial) processes?
  - traditional methods
  - difficulties and hurdles
- What's being done about it?
  - promising new directions

An experimenter's wishlist

- Hadron collider cross-sections one would like to know at NLO

Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

NLOJET++

Author(s): Z. Nagy

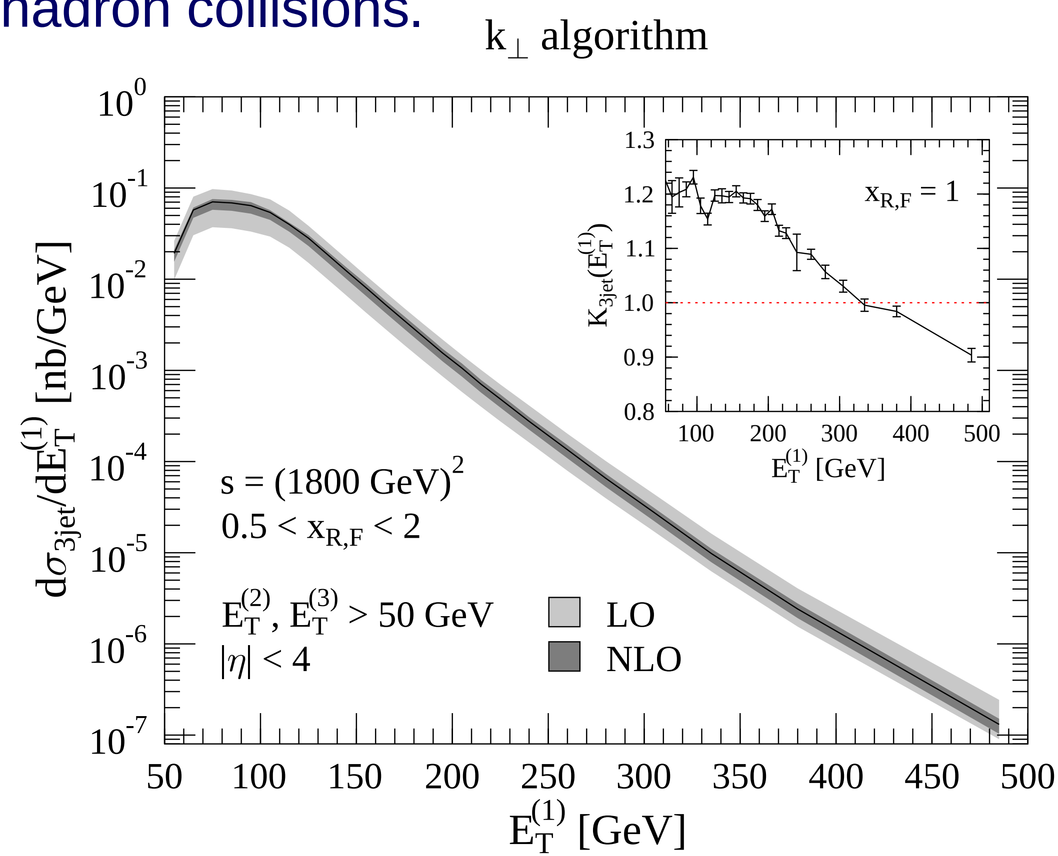
`http://www.ippp.dur.ac.uk/˜nagyz/nlo++.html`

Multi-purpose C++ library for calculating jet cross-sections in e^+e^- annihilation, DIS and hadron-hadron collisions.

$e^+e^- \longrightarrow \quad \leq 4$ jets

$ep \longrightarrow \quad (\leq 3+1)$ jets

$p\bar{p} \longrightarrow \quad \leq 3$ jets



hep-ph/0110315

AYLEN/EMILIA

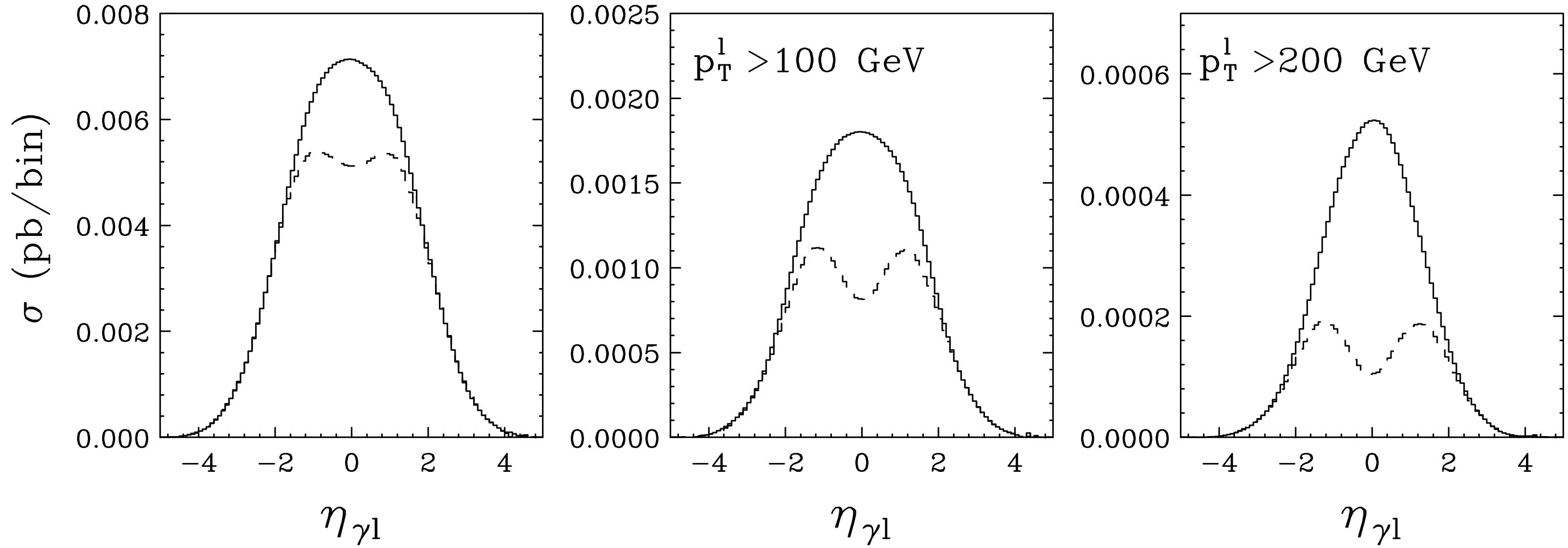
Author(s): L. Dixon, Z. Kunszt, A.Signer, D. de Florian

`http://www.itp.phys.ethz.ch/staff/dflorian/codes.html`

Fortran implementation of gauge boson pair production at hadron colliders, including full spin and decay angle correlations.

$p\bar{p} \longrightarrow VV'$ and $p\bar{p} \longrightarrow V\gamma$ with $V, V' = W, Z$

Anomalous triple gauge boson couplings at the LHC:



hep-ph/0002138

DIPHOX/EPHOX

Author(s): P. Aurenche, T.Binoth, M. Fontannaz, J. Ph. Guillet, G. Heinrich, E. Pilon, M. Werlen

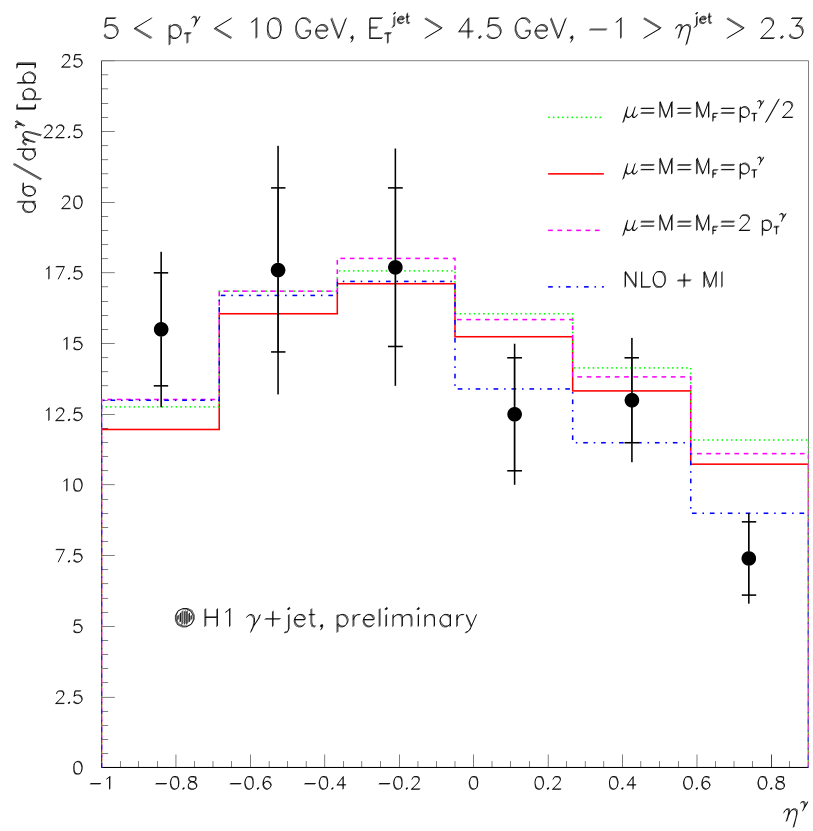
http://wwwlapp.in2p3.fr/lapth/PHOX_FAMILY/main.html

Fortran code to compute processes involving photons, hadrons and jets in DIS and hadron colliders.

$p\bar{p} \longrightarrow \gamma + \leq 1$ jet

$p\bar{p} \longrightarrow \gamma\gamma$

$\gamma p \longrightarrow \gamma +$ jet



Preliminary H1 data, hep-ph/0312070.

MCFM

Author(s): JC, R. K. Ellis

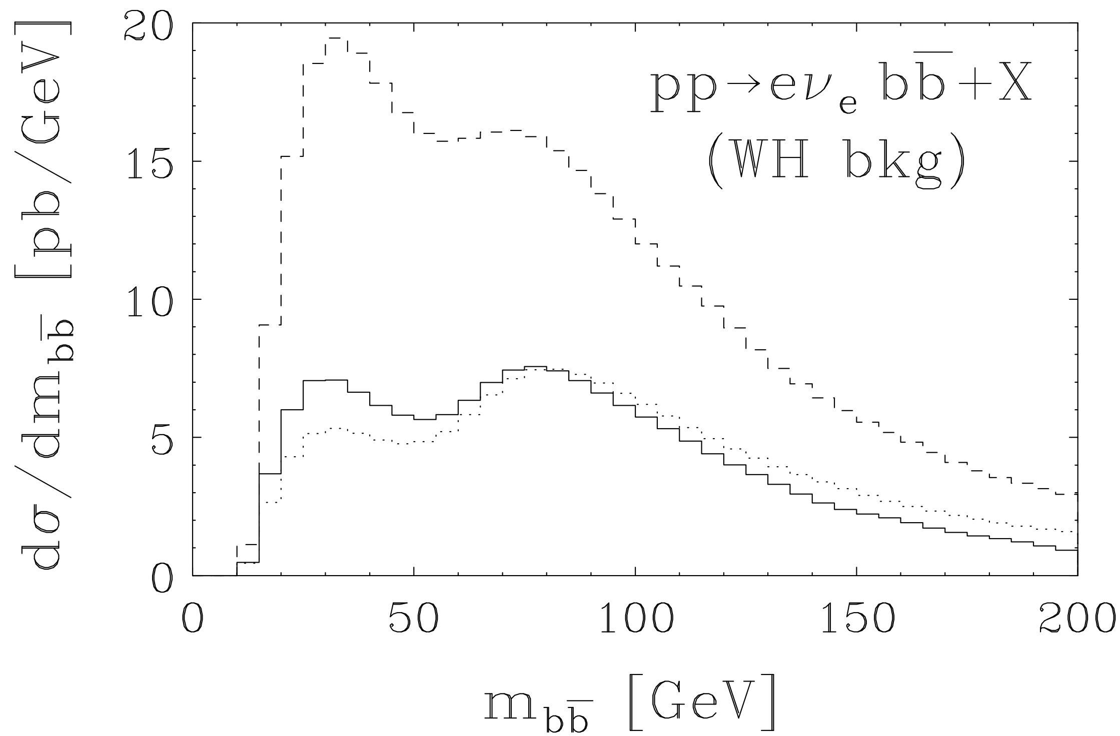
`http://mcfm.fnal.gov`

Fortran package for calculating a number of processes involving vector bosons, Higgs, jets and heavy quarks at hadron colliders.

$p\bar{p} \longrightarrow V + \leq 2$ jets

$p\bar{p} \longrightarrow V + b\bar{b}$

with $V = W, Z$.

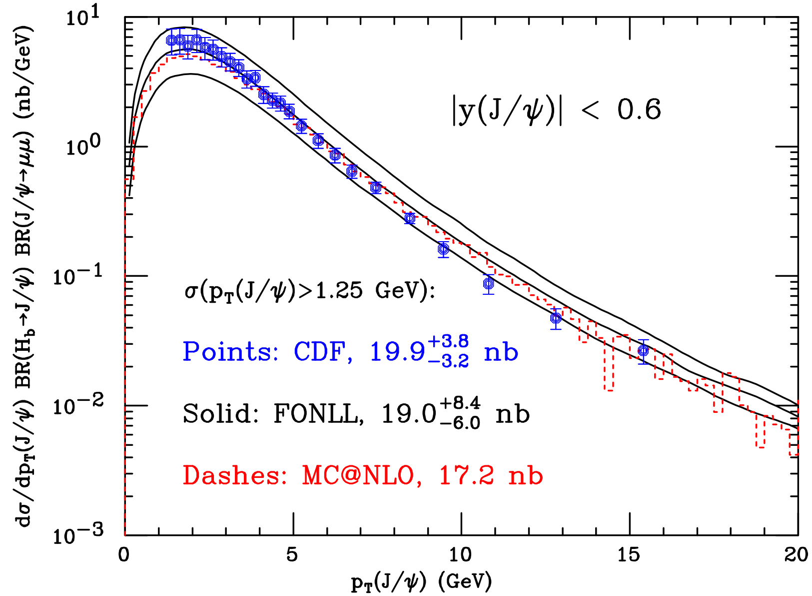


hep-ph/0308195

Heavy quark production

Author(s): M. L. Mangano, P. Nason and G. Ridolfi
http://www.ge.infn.it/˜ridolfi/hvqlibx.tgz
Fortran code for the calculation of heavy quark cross-sections and distributions in a fully differential manner

- Based on the more inclusive calculations of Dawson et al, Beenakker et al.
- Does not include multiple gluon radiation, $\log(p_T/m_b)$ (FONLL)
  Cacciari et al., hep-ph/9803400
- These are the same matrix elements that are incorporated into MC@NLO
  Frixione et al., hep-ph/0305252



hep-ph/0312132

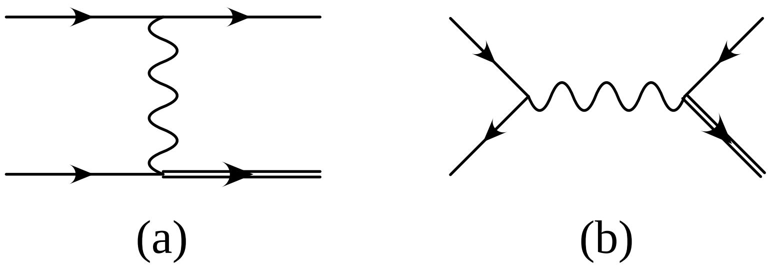
Single top production

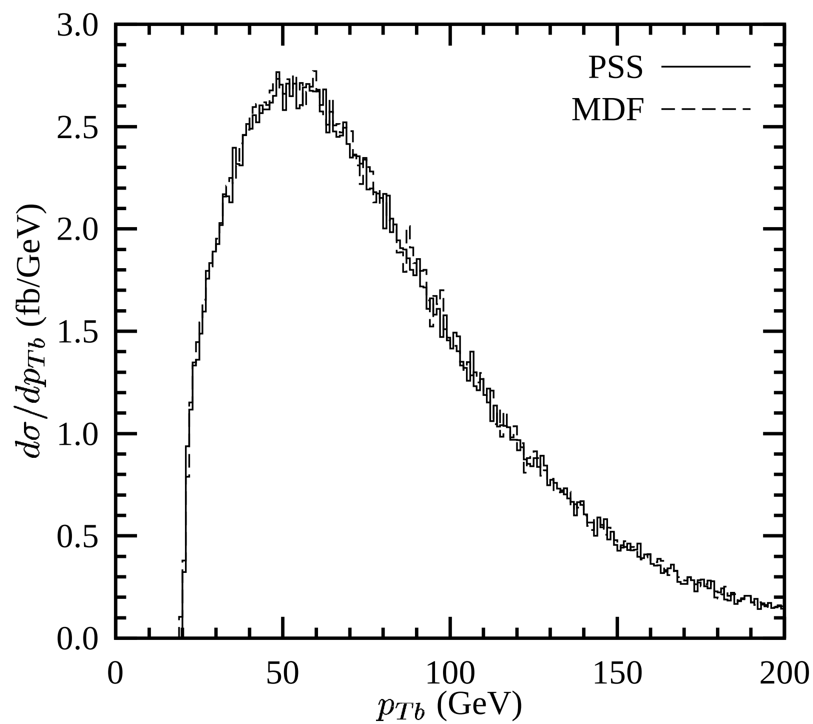
Author(s): B. W. Harris, E. Laenen, L. Phaf, Z. Sullivan, S. Weinzierl
(No public code released)

Fully differential calculation of single top production in hadron-hadron collisions, via both channels:

(a) $u + b \longrightarrow t + d$

(b) $u + \bar{d} \longrightarrow t + \bar{b}$





hep-ph/0207055

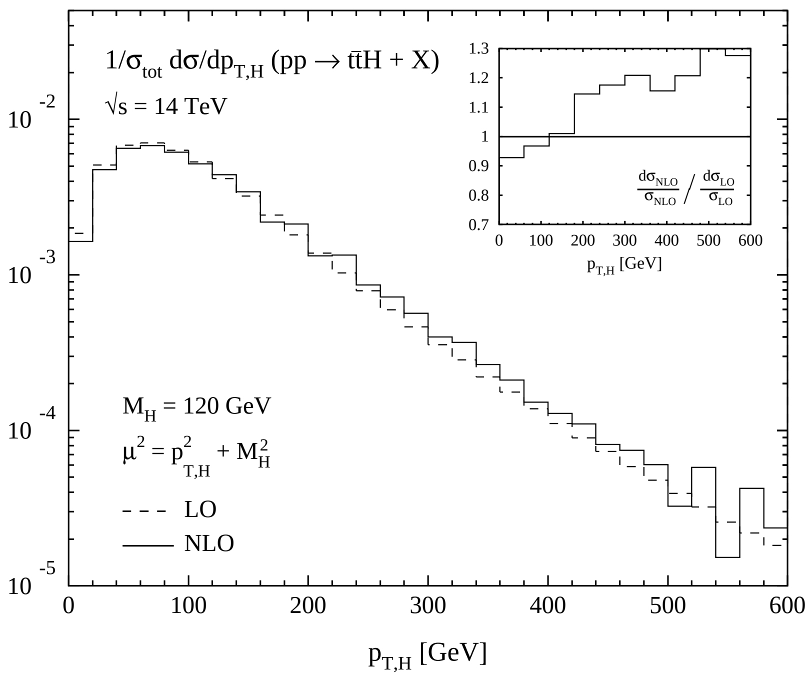
$Higgs + Q\bar{Q}$

Author(s): S. Dawson, C. B. Jackson, L. H. Orr, L. Reina, D. Wackeroth; W. Beenakker, S. Dittmaier, M. Kramer, B.Plumper, M. Spira, P. Zerwas

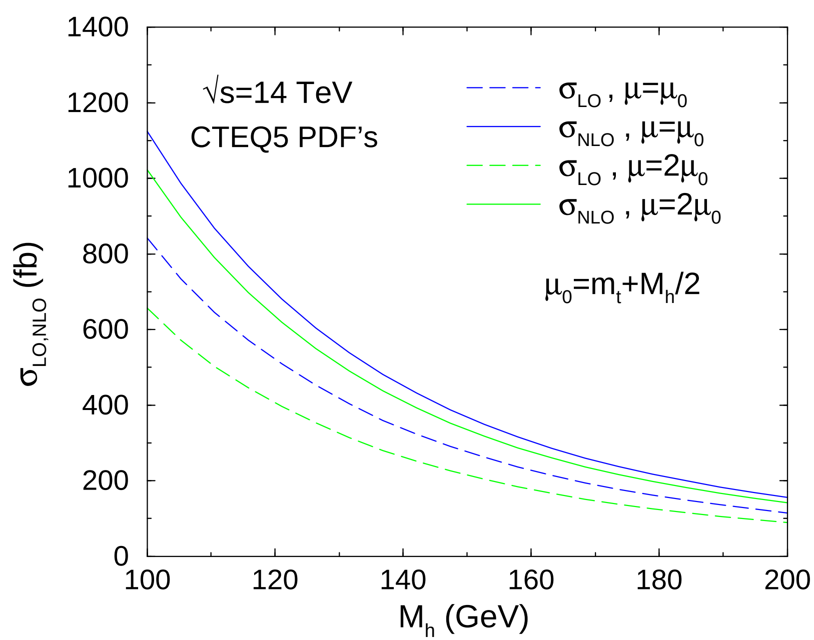
(No public code released)

Associated production of a Higgs and a pair of heavy quarks,

$$p\bar{p} \longrightarrow Q\bar{Q}H, \quad \text{with } Q = t, b.$$



hep-ph/0211352



hep-ph/0311216

Theoretical status

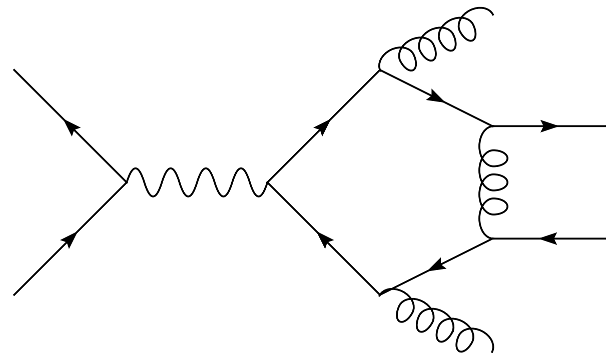
- Much smaller jet multiplicities, some categories untouched

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 2j$	$WW + \leq 0j$	$WWW + \leq 3j$	$t\bar{t} + \leq 0j$
$W + b\bar{b} + \leq 0j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 0j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 2j$	$ZZ + \leq 0j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 0j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 0j$
$Z + c\bar{c} + \leq 0j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 0j$
$\gamma + \leq 1j$	$\gamma\gamma + \leq 1j$		$b\bar{b} + \leq 0j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 0j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 0j$		
	$Z\gamma + \leq 0j$		

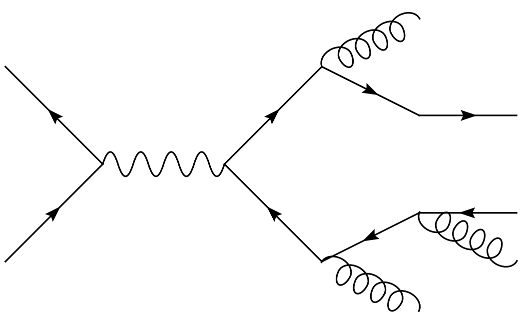
NLO basics

VIRTUAL

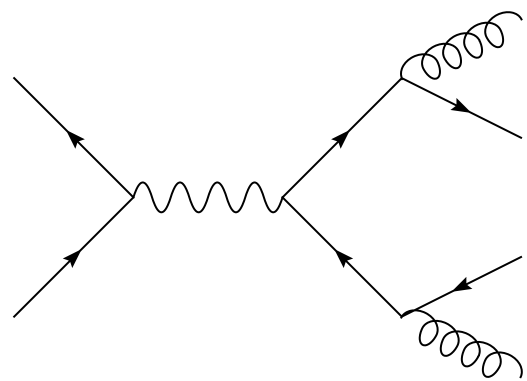
$$\int d^{4-2\epsilon}\ell \;\; 2\mathcal{M}^*_{loop}\,\mathcal{M}_{tree}$$
$$= \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right)\,|\mathcal{M}_{tree}|^2$$



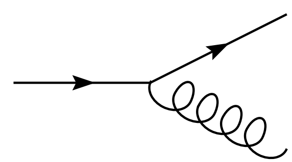
REAL



$$\longrightarrow$$



$$\otimes$$



$$|\mathcal{M}_{tree+1}|^2$$

$$|\mathcal{M}_{tree}|^2$$

$$\int (Split)\, dPS$$
$$= -\left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right)$$

Slow progress

- Why has progress been so slow?

$e^+e^- \longrightarrow 3$ jets c. 1980

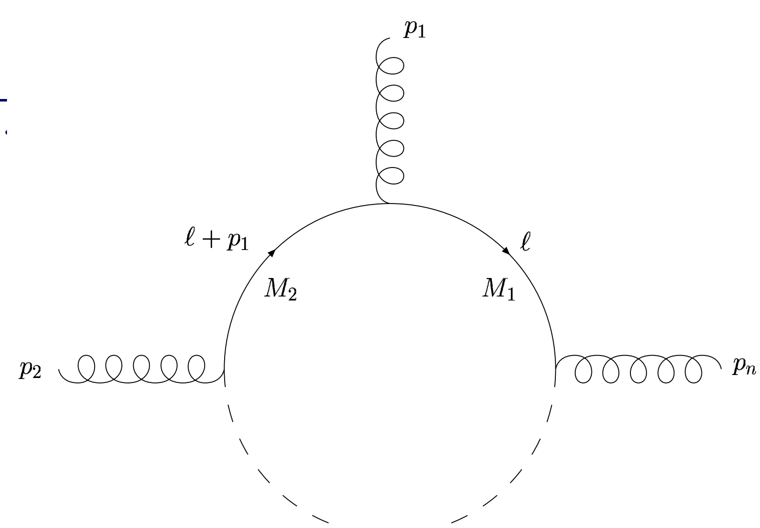
R. K. Ellis et al., 1981

$e^+e^- \longrightarrow 4$ jets c. 2000

Bern et al., Glover et al., 1996-7

- More particles $\rightarrow$ many scales $\rightarrow$ lengthy analytic expressions
- Integrals are complicated and process-specific:

$$\int d^{4-2\epsilon}\ell \, \frac{1}{(\ell^2-M_1^2)((\ell+p_1)^2-M_2^2)\ldots}$$



- different for:

$p_i^2 \neq 0$ W, Z, H

$M_i^2 \neq 0$ t, b, $\ldots$

Complications

- Fermions and non-Abelian couplings lead to more complicated tensor integrals:

$$\int d^{4-2\epsilon}\ell \, \frac{\ell^\mu}{(\ell^2 - M_1^2)((\ell + p_1)^2 - M_2^2)\ldots}$$

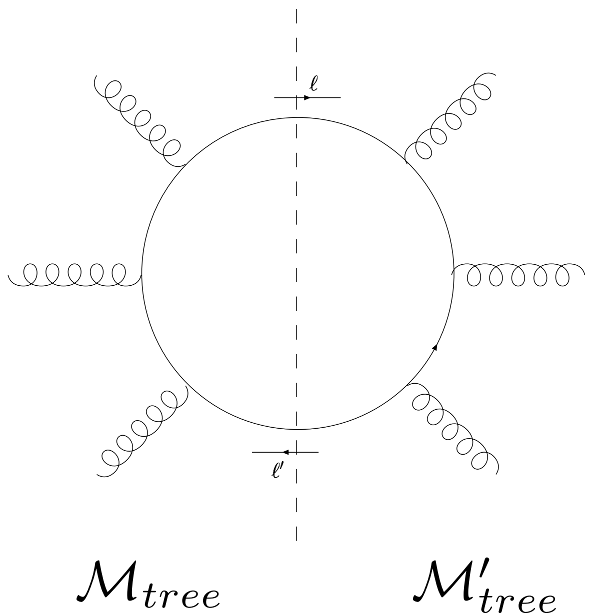
- Passarino-Veltman reduction in terms of scalar integrals:

$$\longrightarrow c_1 p_1^\mu + \ldots c_{n-1} p_{n-1}^\mu$$

  where the c_i are given by the solutions of $(n-1)$ equations

- This gives rise to the $(n-1) \times (n-1)$ Gram determinant, $\Delta = \det(2p_i \cdot p_j)$.
  - large intermediate expressions
  - spurious singularities

Unitarity technique



$$= \int dPS(\ell, \ell')\, \mathcal{M}_{tree} \times \mathcal{M}'_{tree}$$

- Standard tree-level tricks can be used to simplify amplitudes, yielding compact results

e.g. Dixon, hep-ph/9601359

- Rational functions of invariants cannot be obtained easily with this method
- Not easy to generalize and automate, simplification by hand

Hexagons and beyond

- There is little computational experience with N-point intgerals beyond pentagons, $N = 5$: the NLO frontier is at $2 \to 3$ processes
- However, we know that all integrals with $N > 4$ can be written as a sum of known box integrals

Binoth et al., hep-ph/9911342

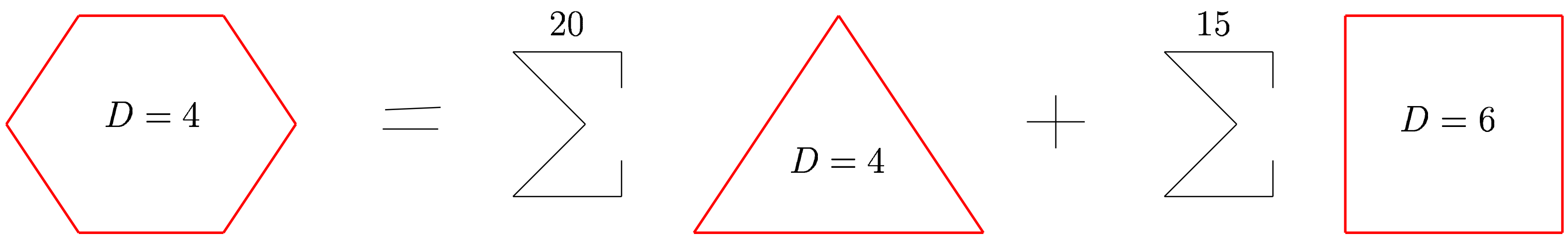
- Analytic result is:

$$N - \text{point finite part} = \sum^{m} \text{dilogarithms} + \text{simpler functions}$$

- For a hexagon integral with masses, $m > 1000$. This may lead to large cancellations in some kinematic regions and thus numerical instabilities
- Perhaps a numerical method could be just as good, or better

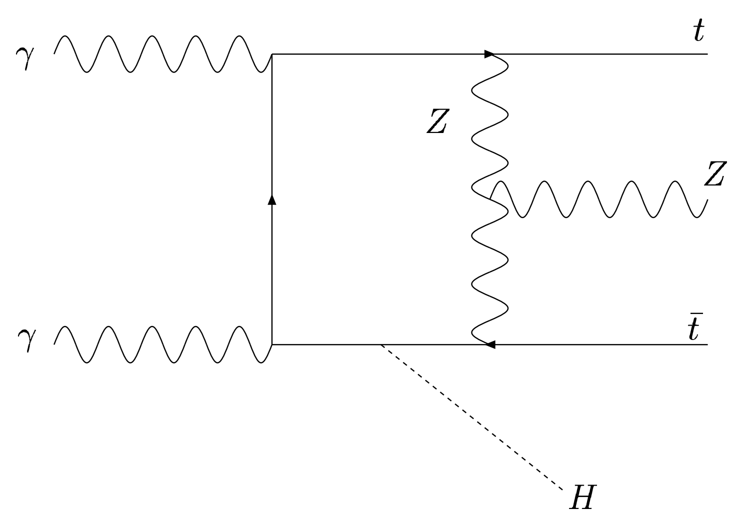
Binoth et al., hep-ph/0210023
Ferroglia et al., hep-ph/0209219

Numerical recipe



Hexagon reduction in terms of triangles and boxes

- A sector decomposition is used to simplify the integrals
- triangles $\longrightarrow$ 1-dim. integral
- boxes $\longrightarrow$ 2-dim. integral
- Integration by a combination of standard techniques and Monte Carlo



IR-divergent loop integrals

- The IR singularities can be isolated from the loop integrals using a simple technique

Dittmaier, hep-ph/0308246

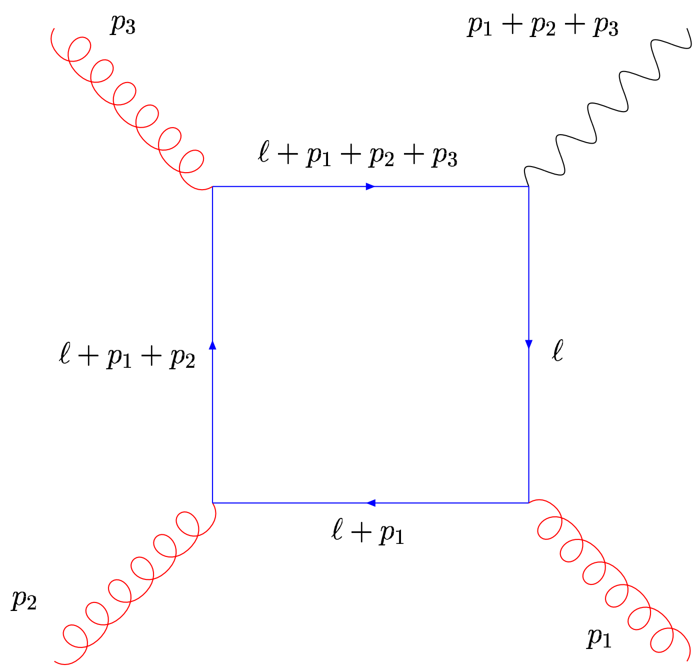
- Singularities occur when:

a massless external particle splits into two massless internal lines — COLLINEAR

two external on-shell particles exchange a massless particle — SOFT

- These result in $\frac{1}{\epsilon}$, $\frac{1}{\epsilon^2}$ poles
- By identifying all the soft and collinear configurations in an integral, one can extract all the IR poles and obtain a finite integral that can be evaluated in 4 dimensions.
- Singular pieces are given in terms of related triangle integrals

Example



$p_1^2 = p_2^2 = p_3^2 = 0$

$\ell = -p_1 - p_2$
yields soft singularities

$\ell = xp_1$ for any arbitrary x
leads to collinear singularities

$$\frac{1}{(\ell+p_1+p_2)^2(\ell+p_1+p_2+p_3)^2} \longrightarrow \frac{A}{(\ell+p_1+p_2)^2} + \frac{B}{(\ell+p_1+p_2+p_3)^2}$$

- This method has already been applied to pentagon integrals involved in the calculation of $t\bar{t}H$ production at NLO

Numerical approach

- If all singularities can be subtracted, perhaps loop integrals can be done numerically
- This method has many advantages:
  - a general solution for many processes, regardless of internal and external masses
  - extension to large final-state multiplicites limited only by CPU power
  - presence of masses in general should simplify the procedure (less singularities) rather than requiring much more work (cf. analytical approach)
- Problem: loop integrals also contain UV divergences

$$\int d^{4-2\epsilon}\ell \frac{\ell^\mu \ell^\nu}{\ell^2(\ell+p_1)^2(\ell+p_1+p_2)^2}$$

First attempt

- Problem of UV subtraction solved and outlined by Nagy and Soper

  Nagy and Soper, hep-ph/0308127
- At the moment, limited to QCD with $m_Q = 0$
- Schematically,

$$\sum \underbrace{(\mathrm{Graph} - \mathrm{CT})}_{\text{finite}} + \underbrace{\left(\sum \mathrm{CT}\right)}_{\text{simple}}$$

  where CT stands for the sum of UV, soft and collinear counter-terms
- Loop integration can then be performed numerically
- General algorithm laid out, but the details of the numerical integration provide a topic for further study

  see also e.g. Soper, hep-ph/9804454
- No implementation to-date

Real contribution

- Relatively simple - diagrams and phase space can already be generated efficiently by tree level programs
- Methods for dealing with singular regions are well-developed, such as phase-space slicing and dipole subtraction
- However, for high multiplicity final states, the number of singular regions is large, resulting in:
  - Very many dipoles
  - Time-consuming calculation of subtraction terms
- Modifications to the original formalism have been made that limit the subtraction region and thus speed up the code

  Z. Nagy, hep-ph/0307268
- There's room for investigation of this implementation and further ideas

A different approach

- Try to construct infrared finite amplitudes for gauge theories with massless fermions

  Forde and Signer, hep-ph/0311059

- Finite amplitudes would have many benefits:
  - Simple numerical approach
  - Easy matching to a parton shower

S-matrix elements in normal Fock space	$\longleftarrow$	soft and collinear divergences
$\Downarrow$		
order-by-order dressed states including all long-range interactions	$\longleftarrow$	free of infrared divergences

Basic idea

- Basic assumption when constructing amplitudes normally:

$$\underbrace{e^{-\imath tH}}_{\text{full Hamiltonian}} \underbrace{|\Psi(t)\rangle}_{\text{exact state}} \longrightarrow \underbrace{e^{-\imath tH_0}}_{\text{free Hamiltonian}} \underbrace{|\Phi(t)\rangle}_{\text{free state}} \quad \text{as } t \to \pm\infty$$

- This assumption is not true for QCD: massless gauge bosons have long-range interactions that do not vanish sufficiently quickly $\longrightarrow$ IR singularities
- Introduce an asymptotic Hamiltonian that contains the long-range interactions that give rise to soft and collinear splittings:

$$e^{-\imath tH_A} \, |\Omega(t)\rangle$$

- Diagrammatic rules similar to Feynman rules, but time-ordered
- So far, only demonstrated on a test case ($e^+e^- \to 2$ jets): no hadronic initial state, no triple-gluon coupling

Summary

- NLO tools are an invaluable aid to experimental studies now and will continue to be so in the future
- There are many programs currently available for predictions at both existing and proposed colliders
  - author-controlled
    single top, $H + Q\bar{Q}$
  - single class of processes
    $V\gamma$, $Q\bar{Q}$
  - generic programs
    NLOJET++, PHOX-family, MCFM
- Despite recent progress towards NNLO predictions, there's still much left to be done at the one-loop level

Workshop outlook

- Obviously, NLO computations generally involve time-scales longer than the length of this workshop. However, it would be useful to set some experimentally-motivated priorities as a field
- Are there (feasible) calculations that desperately need to be done at NLO?
  - e.g. $p\bar{p} \longrightarrow WQ\bar{Q}$ with the quark mass?
- If so, should such a calculation be undertaken using existing techniques, or is now the time for a new approach?
- How can existing algorithms be improved?
  - technical improvements to current slicing/subtraction procedures, particularly regarding how they cope with higher numbers of singular regions
  - implementation of a numerical approach to loop integrations
  - how to better integrate upcoming (and existing) results with new approaches such as MC@NLO

Long-term outlook

- It seems clear that performing NLO calculations on a case-by-case basis is not the way of the future
- An automated approach, combining algebraic and numerical recipes, appears both promising (in terms of physics output) and feasible
  - Perhaps one day we'll have an ALPGEN@NLO or MadLoop
- However, even if such ambitious projects can be realized, the story does not end there
  - interpretation and grooming of results will still be very process-specific
  - jet-clustering, photon fragmentation, threshold effects, resummation and more will need to be considered