KITP, UCSB, 29 January 2004

Infrared-Finite Amplitudes For Massless Gauge Theories

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overview

Infrared Singularities	• where they come from
	• how to deal with them
	• how to avoid them
Infrared-Finite Amplitudes	• set the stage (previous work)
	• set the target
Good News	• construction of IR-finite amplitudes ! !
Bad News	• computation of IR-finite amplitudes ??

Work done in collaboration with
Darren Forde and Mark Morley-Fletcher

you are about to enter a perturbative area

this talk will not contain any non-perturbative physics. . .

you are about to enter a perturbative area

this talk will not contain any non-perturbative physics. . .

. . . and no string theory, I'm afraid

Collider Workshop; KITP, UCSB – p. 3/24

where they come from

basic assumption (in Schrödinger picture): $|\Psi^{(\pm)}(\alpha, t)\rangle = \Omega^{(\pm)}|\Phi(\alpha, t)\rangle$

- $|\Psi\rangle$: full state, evolving with $H = H_0 + H_I$
- $|\Phi\rangle$: free state, evolving with H_0
- $\Omega^{(\pm)} = \lim_{\tau \to \mp\infty} e^{iH\tau} e^{-iH_0\tau}$: Møller operators

scattering matrix elements: ($\Rightarrow$ interaction picture)

$$\langle\Psi^{(-)}(\beta, t)|\Psi^{(+)}(\alpha, t)\rangle = \langle\Phi(\beta, t)|\Omega^{(-)\dagger}\Omega^{(+)}|\Phi(\alpha, t)\rangle \equiv \langle\Phi(\beta, t)|S|\Phi(\alpha, t)\rangle$$

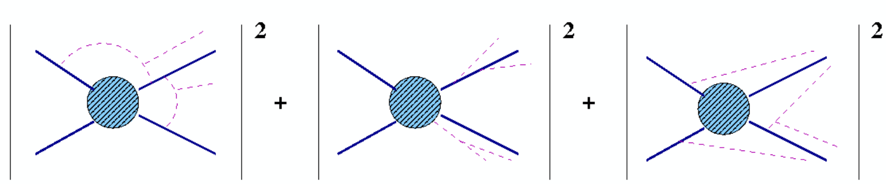
- time-ordered perturbation theory: $S = U(\infty, t_0)\, U^\dagger(-\infty, t_0)$ with $U(t_1, t_2) = \mathrm{T}\ \exp\left(-i\int_{t_2}^{t_1} \mathrm{d}\tau\, H_I(\tau)\right)$
- covariant perturbation theory: Green functions, LSZ reduction

In a gauge theory we have massless gauge bosons $\Longrightarrow$ interactions $\sim 1/r$
The above assumption is not justified $\Longrightarrow$ IR singularities
More precisely: soft singularities $E \sim 0$ and collinear singularities $\angle \sim 0$

how to deal with them

cross-section method

- Use IR-regulator (dimensional regularization)
- Sum different but physically indistinguishable cross sections
- Singularities cancel (for IR-safe observables) between different cross sections, Bloch-Nordsieck, Kinoshita-Lee-Naueberg theorem



- Soft singularities and final state collinear singularities cancel
- Initial state collinear singularities are factorized into pdf's

how to avoid them

modify basic assumption (in Schrödinger picture): $|\Psi^{(\pm)}(\alpha,t)\rangle = \Omega_A^{(\pm)}|\Xi(\alpha,t)\rangle$

- $|\Psi\rangle$: full state, evolving with $H = H_0 + H_I = H_A + H_h = (H_0 + H_\Delta) + H_h$
- $|\Xi\rangle$: asymptotic state, evolving with H_A
- $\Omega_A^{(\pm)} = \lim_{\tau\to\mp\infty} e^{iH\tau}e^{-iH_A\tau}$: asymptotic Møller operators

scattering matrix elements: ($\Rightarrow$ interaction picture)

$$\langle\Psi^{(-)}(\beta,t)|\Psi^{(+)}(\alpha,t)\rangle = \langle\Xi(\beta,t)|\Omega_A^{(-)\dagger}\Omega_A^{(+)}|\Xi(\alpha,t)\rangle \equiv \langle\Xi(\beta,t)|S_A|\Xi(\alpha,t)\rangle$$

if the $|\Xi\rangle$ form a complete set: $H\Omega_A^{(\pm)} = \Omega_A^{(\pm)}H_A$ and therefore $[S_A, H_A] = 0$

- can we find H_A such that the modified basic assumption holds?
  $\Rightarrow$ YES (good news)
- what are the states $|\Xi\rangle$?
  $\Rightarrow$ don't know (part of bad news)

set the stage

We cannot diagonalize H_A: $\Rightarrow$ relate $|\Xi\rangle$ to $|\Phi\rangle$ perturbatively

$$\underbrace{\langle\Phi(\beta)|S_A|\Phi(\alpha)\rangle}_{\substack{\text{modified S operator} \\ \text{ordinary states}}} \equiv \underbrace{\langle\{\Phi(\beta)\}|S|\{\Phi(\alpha)\}\rangle}_{\substack{\text{ordinary S operator} \\ \text{modified (dressed) states}}}$$

Hamiltonian approach
+ split hard-soft at level of Hamiltonian
− (manifest) Lorentz invariance

effective theory approach
+ manifestly Lorentz invariant
− dressing depends on order of PT

previous work

- These are old ideas! Chung, Zwanziger
- QED with massive fermions (only soft singularities) Kulish, Fadeev
- Extension to soft singularities in a non-abelian theory; generalized coherent states, including multiple soft gluon emission
  Creco et.al; Butler, Nelson; Catani, Ciafaloni, Marchesini
- IR-finiteness of amplitudes Frenkel, Gatheral, Taylor; Giavarini, Marchesini
- Including collinear singularities $\Longrightarrow$ more complicated structure of H_A Havemann; Del Duca, Magnea, Sterman; Contopanagos, Einhorn

set the target

goal: **Complete evaluation of IR-finite amplitudes, order-by-order in perturbation theory**

why?

- a problem in field theory that is interesting in itself
- another approach to completely numerical evaluation of amplitudes
- merging of parton showers with fixed order calculations

why now?

- automatization is on the political agenda
- progress in numerical integration
- nobody has done it so far

Good News

formal construction

We follow closely:
Frenkel, Gatheral, Taylor; Catani, Ciafaloni, Marchesini; Contopanagos, Einhorn . . .

usual Møller operators

$$i\frac{d}{dt}\Omega^{(\pm)}(t) = e^{iH_0(t-t_0)}\left[\Omega_S^{(\pm)}, H_0\right] e^{-iH_0(t-t_0)} = H_I(t)\Omega^{(\pm)}(t)$$

$$\uparrow$$

$$\Omega_S^{(\pm)} H_0 = H_S\,\Omega_S^{(\pm)}$$

asymptotic Møller operator

$$i\frac{d}{dt}\Omega_A^{(\pm)}(t) = e^{iH_0(t-t_0)}\left[\Omega_{A,S}^{(\pm)}, H_0\right] e^{-iH_0(t-t_0)} = H_I(t)\Omega_A^{(\pm)}(t) - \Omega_A^{(\pm)}(t)H_\Delta(t)$$

$$\uparrow$$

$$\Omega_{A,S}^{(\pm)} H_{A,S} = H_S\,\Omega_{A,S}^{(\pm)}$$

solution: $\Omega_A^{(\pm)}(t) = \Omega^{(\pm)}(t)\,\Omega_\Delta^{(\pm)\dagger}(t)$

soft Moller operator

standard Moller operator

formal construction

Either compute $\langle f|S_A|i\rangle$ directly

$$
\begin{aligned}
S_A &= \Omega_A^{(-)\,\dagger}(t_0)\,\Omega_A^{(+)}(t_0) = 1+\sum_n S_A^{(n)} = 1+\sum_n \sum_{\{l|m|n\}} S_A^{(l|m|n)} \\
S_A^{(l|m|n)} &= \int_{t_2}^{t_0} dt_1 \ldots \int_{t_{l+1}}^{t_0} dt_l \times \int_{-\infty}^{\infty} dt_{l+1} \ldots \int_{-\infty}^{t_{m-1}} dt_m \times \int_{t_0}^{t_m} dt_{m+1} \ldots \int_{t_0}^{t_{n-1}} dt_n \\
&\times \underbrace{H_\Delta(t_1)\ldots H_\Delta(t_l)}_{\text{soft only}} \times \underbrace{H_h(t_{l+1})\ldots H_h(t_m)}_{\text{hard (and soft)}} \times \underbrace{H_\Delta(t_{m+1})\ldots H_\Delta(t_n)}_{\text{soft only}}
\end{aligned}
$$

or make a connection to conventional amplitudes

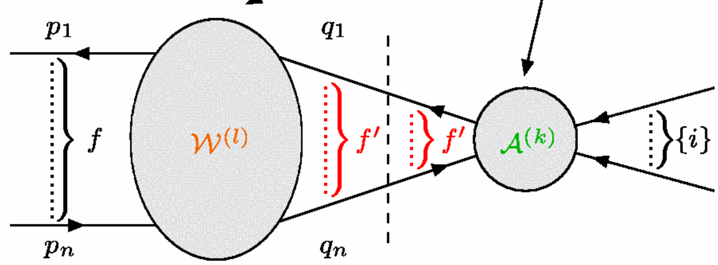
$$
\langle f|S_A|i\rangle \equiv \langle f|\Omega_\Delta^{(-)}\,S\,\Omega_\Delta^{(+)\,\dagger}|i\rangle = \sum_{i',f'} \underbrace{\langle f|\Omega_\Delta^{(-)}|f'\rangle}_{\substack{\text{dressing}\\ \text{factor}}} \;\underbrace{\langle f'|S|i'\rangle}_{\substack{\text{ordinary}\\ \text{amplitude}}} \;\underbrace{\langle i'|\Omega_\Delta^{(+)\,\dagger}|i\rangle}_{\substack{\text{dressing}\\ \text{factor}}}
$$

IR finiteness

- $t \to \pm\infty$ only for $H_h(t)$
- if split $H_I = H_\Delta + H_h$ such that H_h does not give IR-singularities S_A is finite
- H_Δ has to contain all interaction terms giving rise to IR singularities
- think of dressed states $|\{i\}\rangle \equiv \Omega_\Delta^{(+)\,\dagger}|i\rangle$ and $\langle\{f\}| \equiv \langle f|\Omega_\Delta^{(-)}$ as “proto-jets”
- if the dressed states are “too fat” we cannot use them to describe very exclusive final states, ideally choose H_Δ such that dressed states are less inclusive than experimental resolution.
- we cannot compute observable that are not IR safe
- setting $H_h = H_I \cdot \Theta(|\sum(\pm)_j \omega(\vec{k}_j)| - \Delta)$ removes all soft and collinear singularities, but severely limits our chances for a covariant treatment
- remaining UV singularities have to be subtracted

final-state dressing

$$\langle f|\Omega_\Delta^{(-)}\, S\, \Omega_\Delta^{(+)\,\dagger}|i\rangle = \langle\{f\}|S|\{i\}\rangle = \sum_{f'} \underbrace{\langle f|\Omega_\Delta^{(-)}|f'\rangle}\underbrace{\langle f'|S|i'\rangle}\langle i'|\Omega_\Delta^{(+)\,\dagger}|i\rangle$$



- computation similar to TOPT
- all particles on-shell
- breaks Lorentz invariance
- no UV singularities

- computed in the standard way
- needs IR regulator
- contains UV divergences

example $e^+e^- \to$ 2 jets

In the conventional approach at NLO we get contributions from $\mathcal{A}(q(p_1),\bar q(p_2);\gamma)$ and $\mathcal{A}(q(p_1),\bar q(p_2),g(p_3);\gamma)$, both of which are IR divergent.

For the total cross section at NLO we have

$$\begin{aligned}
\sigma_{q\bar q} &\sim |\mathcal{A}(q(p_1),\bar q(p_2);\gamma)|^2 = \sigma_0\, C_F \frac{\alpha_s}{\pi} c_\Gamma \left(\frac{2}{\epsilon^2}+\frac{3}{2\epsilon}+\frac{19}{4}-\frac{\pi^2}{2}\right)\\
\sigma_{q\bar q g} &\sim |\mathcal{A}(q(p_1),\bar q(p_2),g(p_3);\gamma)|^2 = \sigma_0\, C_F \frac{\alpha_s}{\pi} c_\Gamma \left(-\frac{2}{\epsilon^2}-\frac{3}{2\epsilon}-4+\frac{\pi^2}{2}\right)\\
\sigma_1 &= \sigma_{q\bar q}+\sigma_{q\bar q g} = \sigma_0\left(1 + C_F\frac{3}{4}\frac{\alpha_s}{\pi}\right)
\end{aligned}$$

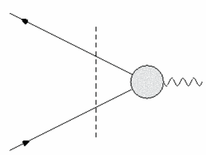
In our approach at NLO we get contributions from $\mathcal{A}(\{q(p_1),\bar q(p_2)\};\gamma)$ and $\mathcal{A}(\{q(p_1),\bar q(p_2),g(p_3)\};\gamma)$, both of which are IR finite.

- simplifying feature: no initial state radiation
- non-abelian nature does not yet show up

$\mathcal{A}(\{q(p_1), \bar{q}(p_2)\}; \gamma)$

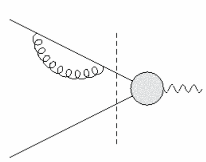
Decomposition of $\mathcal{A}(\{q(p_1), \bar{q}(p_2)\}; \gamma)$ at $\mathcal{O}(g_s^2)$:

$$
\begin{aligned}
\mathcal{A}^{(2)}(\{q_1, \bar{q}_2\}; \gamma) &\equiv \langle \{q(p_1, r_1) \bar{q}(p_2, r_2)\} | S | 0 \rangle \Big|_{g^2} \\
&= \mathcal{W}^{(0)}(q_1(p_1), \bar{q}_2(p_2); q_1(q_1), \bar{q}_2(q_2)) \otimes \mathcal{A}^{(2)}(q_1(q_1), \bar{q}_2(q_1); \gamma) \\
&+ \mathcal{W}^{(1)}(q_1(p_1), \bar{q}_2(p_2); q_1(q_1), \bar{q}_2(q_2), g_3(q_3)) \otimes \mathcal{A}^{(1)}(q_1(q_1), \bar{q}_2(q_2), g_3(q_3); \gamma) \\
&+ \mathcal{W}^{(2)}(q_1(p_1), \bar{q}_2(p_2); q_1(q_1), \bar{q}_2(q_2)) \otimes \mathcal{A}^{(0)}(q_1(q_1), \bar{q}_2(q_2); \gamma)
\end{aligned}
$$

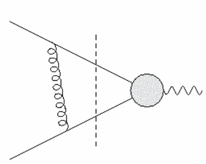


$$
\begin{aligned}
&= \mathcal{A}^{(2)}(q(p_1), \bar{q}(p_2); \gamma(P)) \\
&= C_F \left(\frac{\alpha_s}{2\pi}\right) \left(\frac{\mu^2}{s}\right)^{\epsilon} \left(-\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} - 4 + \frac{\pi^2}{12}\right) \\
&\quad \times \mathcal{A}^{(0)}(q(p_1), \bar{q}(p_2); \gamma(P))
\end{aligned}
$$

$\mathcal{W}^{(2)} \otimes \mathcal{A}^{(0)}$

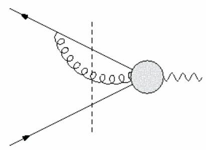


$$
\begin{aligned}
&= C_F \left(\frac{\alpha_s}{2\pi}\right) \left(\frac{\mu^2}{s}\right)^{\epsilon} \left(-\frac{1}{\epsilon^2} - \frac{5}{2\epsilon} + g_1(\Delta) \right. \\
&\quad \left. + \int \mathrm{d}\widetilde{q}_3\, f_1(p_1, p_2, q_3)\right) \mathcal{A}^{(0)}(q(p_1), \bar{q}(p_2); \gamma(P))
\end{aligned}
$$



$$
\begin{aligned}
&= C_F \left(\frac{\alpha_s}{2\pi}\right) \left(\frac{\mu^2}{s}\right)^{\epsilon} \left(\left(\frac{1}{\epsilon} + g_2(\Delta)\right) \mathcal{A}^{(0)}(q, \bar{q}; \gamma) \right. \\
&- (-ie)\, \delta_{ij}\, \langle p_1 | \gamma^{\alpha} | p_2 \rangle (2\pi)^{(3)} \delta^{(D-1)}(\vec{P} - \vec{p}_1 - \vec{p}_2) \\
&\quad \left. \int \mathrm{d}\widetilde{q}_3\, f_2(p_1, p_2, q_3)\right)
\end{aligned}
$$

$\mathcal{W}^{(1)} \otimes \mathcal{A}^{(1)}$



$$= C_F \left(\frac{\alpha_s}{2\pi}\right) \left(\frac{\mu^2}{s}\right)^{\epsilon} \Bigg(\left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + g_3(\Delta)\right) \mathcal{A}^{(0)}(q(p_1), \bar{q}(p_2); \gamma(P))$$

$$+ \quad (-ie)\,\delta_{ij}\,\langle p_1|\gamma^\alpha|p_2\rangle (2\pi)^{(D-1)} \delta^{(D-1)}(\vec{P} - \vec{p}_1 - \vec{p}_2) \int \mathrm{d}\tilde{q}_3\; f_3(p_1, p_2, q_3) \Bigg)$$

- g_1, g_2 and g_3 are finite
- f_1, f_2 and f_3 can be integrated (numerically) without producing any singularities

$\mathcal{A}(\{q(p_1), \bar{q}(p_2)\}; \gamma)$

On combining all these terms the IR divergences cancel

$$\mathcal{A}(\{q(p_1), \bar{q}(p_2)\}; \gamma) = \left(1 + C_F \left(\frac{\alpha_s}{2\pi}\right) \left(g_1(\Delta) + g_2(\Delta) + g_3(\Delta) - 4 + \frac{\pi^2}{12}\right)\right) \mathcal{A}^{(0)}(q(p_1), \bar{q}(p_2); \gamma(P))$$

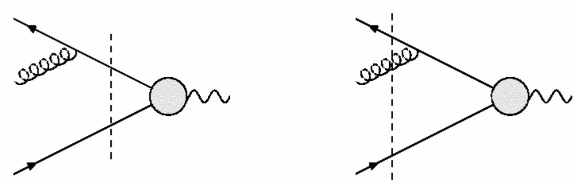
$$+(-ie)\,\delta_{ij}\,\langle p_1|\gamma^\alpha|p_2\rangle (2\pi)^{(D-1)} \delta^{(D-1)}(\vec{P} - \vec{p}_1 - \vec{p}_2) \int \mathrm{d}\tilde{q}_3 \left(f_1(p_1, p_2, q_3)\delta(\sqrt{S} - \omega(\vec{p}_1) - \omega(\vec{p}_2)) + f_2(p_1, p_2, q_3) + f_3(p_1, p_2, q_3)\right)$$

Only Δ dependent finite pieces and the (numerically) integrated finite terms remain

$\mathcal{A}(\{q(p_1),\bar{q}(p_2),g(p_3)\};\gamma)$

Decomposition of $\mathcal{A}(\{q(p_1),\bar{q}(p_2),g(p_3)\};\gamma)$ at $\mathcal{O}(g_s)$:

$$\begin{aligned}
\mathcal{A}^{(2)}(\{q_1,\bar{q}_2,g(p_3)\};\gamma) &\equiv \langle\{q(p_1,r_1)\bar{q}(p_2,r_2)g(p_3,r_3)\}|S|0\rangle\Big|_{g^2} \\
&= \mathcal{W}^{(0)}(q_1(p_1),\bar{q}_2(p_2),g_3(p_3);q_1(q_1),\bar{q}_2(q_2),g_3(q_3)) \otimes \\
&\qquad\qquad \mathcal{A}^{(1)}(q_1(q_1),\bar{q}_2(q_2),g_3(q_3);\gamma) \\
&+ \mathcal{W}^{(1)}(q_1(p_1),\bar{q}_2(p_2),g_3(p_3);q_1(q_1),\bar{q}_2(q_2)) \otimes \mathcal{A}^{(0)}(q_1(q_1),\bar{q}_2(q_2);\gamma)
\end{aligned}$$



Total Cross Section

- The total cross section is given by, $\sigma = \sigma_{\{q\bar{q}\}} + \sigma_{\{q\bar{q}g\}}$.
- The individual cross sections are (omitting additional terms that vanish for $\Delta \to 0$)

$$\begin{aligned}
\sigma_{\{q\bar{q}\}} &= \int |\mathcal{A}(\{q(p_1),\bar{q}(p_2)\};\gamma)|^2 \\
&= \left(1 + C_F\,\frac{\alpha_s}{\pi}\left(-\frac{1}{2} + \log 4 - \frac{3}{2}\log\left(\frac{\Delta}{2}\right) - \log^2\left(\frac{\Delta}{2}\right)\right)\right), \\
\sigma_{\{q\bar{q}g\}} &= \int |\mathcal{A}(\{q(p_1),\bar{q}(p_2),g(p_3)\};\gamma)|^2 \\
&= C_F\left(\frac{\alpha_s}{\pi}\right)\left(\frac{5}{4} - \log 4 + \frac{3}{2}\log\left(\frac{\Delta}{2}\right) + \log^2\left(\frac{\Delta}{2}\right)\right)
\end{aligned}$$

- Adding the two cross sections then gives the result for the total cross section

$$\frac{1}{\sigma_0}\sigma = \left(1 + C_F\frac{3}{4}\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right).$$

Bad News

structure of amplitudes

- IR-finite amplitudes are more tedious to compute
- IR-finite amplitudes have a more complicated structure
- standard amplitudes: $\mathcal{A} = (2\pi)^4 \delta(\sum p_i)\, a(p_i)$ (should use wave packets)
- standard cross sections: $\sigma \sim (2\pi)^4 \delta(\sum p_i)\, |a(p_i)|^2$ (we can get away with non-normalizable states)
- even if we start with a non-normalizable state $|i\rangle$, the dressing distorts the sharp momentum/energy (by an amount controlled by Δ)
- do we get away with with non-normalizable states?
- can we take the $\Delta \to 0$ limit in a meaningfull way?
- can we take a more covariant approach?

conclusions

a lot has to be understood before such a method becomes competitive to the standard cross-section approach

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if the progress is twice as fast as for the cross-section method, this method should be ready to use in ∼20 years from now, just in time for the L(H?)C