QCD at Colliders:
Status, Prospects and Open Issues

Thomas Gehrmann

Universität Zürich

Collider Physics Conference

KITP Santa Barbara

QCD

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- “testing QCD”-era is over for some time
- QCD today is becoming precision physics

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LEP precision physics:

Electroweak processes

Tevatron/LHC precision physics:

QCD processes

QCD

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- precise determination of
  - strong coupling constant
  - quark masses
  - electroweak parameters
  - parton distributions
  - LHC luminosity (!)

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  - quark masses
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  - parton distributions
  - LHC luminosity (!)
- precise predictions for
  - new physics effects
  - and their backgrounds

Theoretical challenges in QCD

- QCD describes quarks and gluons; experiments observe hadrons
  - describe parton $\longrightarrow$ hadron transition (fragmentation)
  - or define appropriate final state (jets)

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  - important higher order corrections

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  - frequent multiparticle final states
  - important higher order corrections
- observables involve different scales

$$m_Q, p_T, M_Z, m_T$$

$\longrightarrow$ large logarithms e.g. $\ln(p_T^2/M_Z^2)$

  - reorder (resum) perturbative series

Outline

- Heavy Quarks
- Jets and Multiparticle Production
- Photons
- Gauge and Higgs Bosons

Heavy Quarks

Heavy Quarks

Total cross sections

status: NLO + NLL soft gluon resummation

W. Beenakker et al., R. Bonciani, S. Catani, M. Mangano, P. Nason
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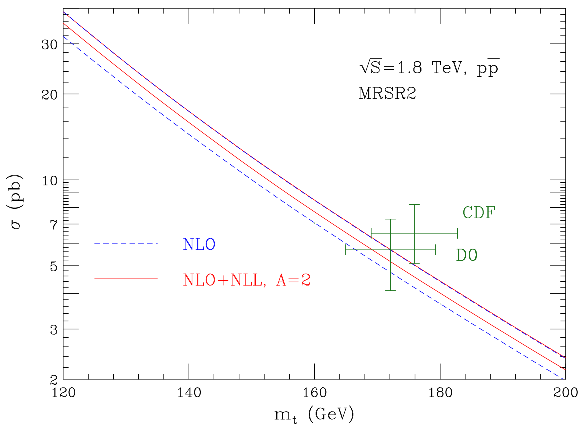

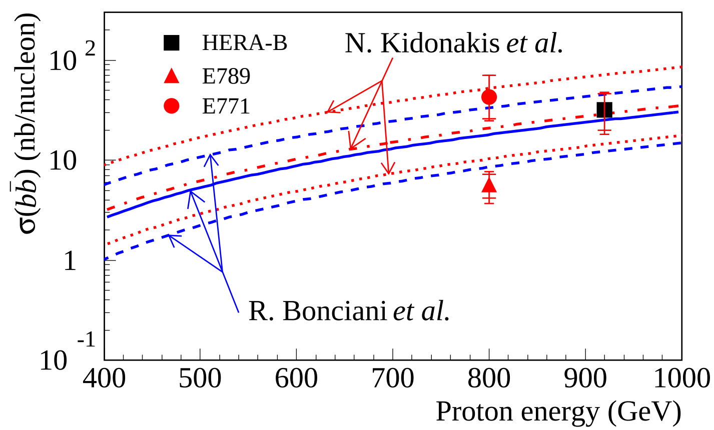

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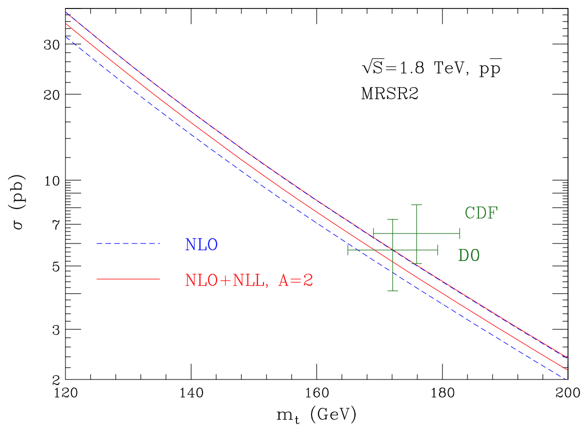

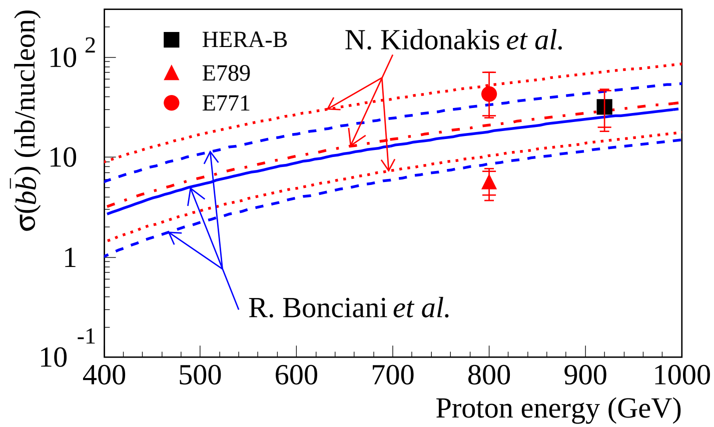

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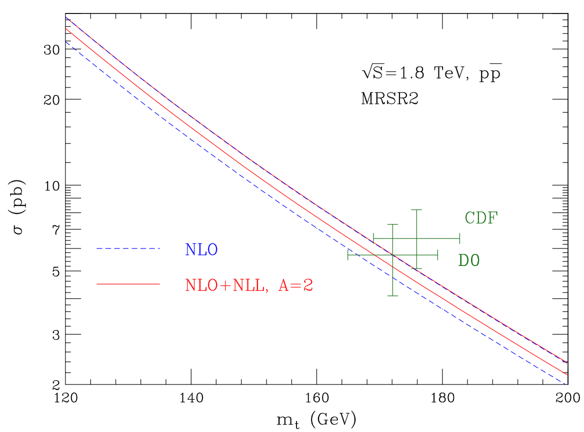

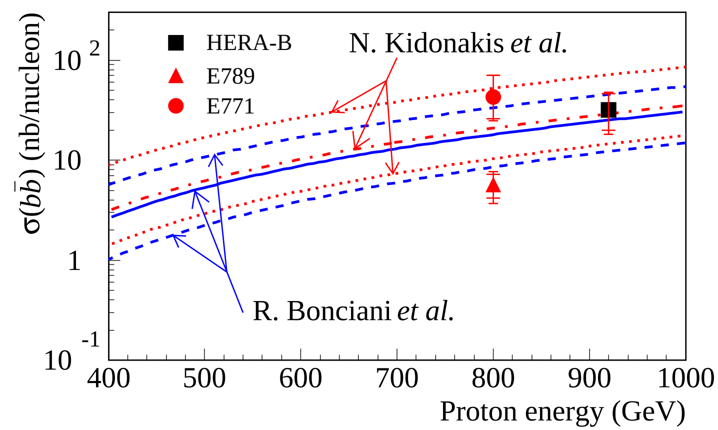

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- Theoretical uncertainty lower for $t\bar{t}$ than for $b\bar{b}$: $\alpha_s(m_b) \gg \alpha_s(m_t)$, $q\bar{q}$ dominance at Tevatron and gg dominance at HERA-B

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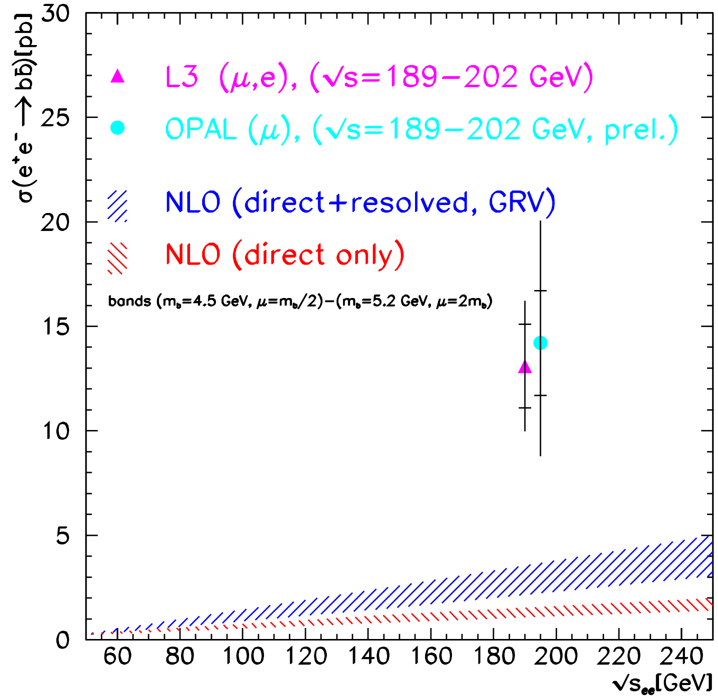
Total cross sections

agreement data–theory not always good

excess seen especially for $b\bar{b}$ in γp, $\gamma\gamma$

e.g. $\gamma\gamma \to b\bar{b}$ at LEP



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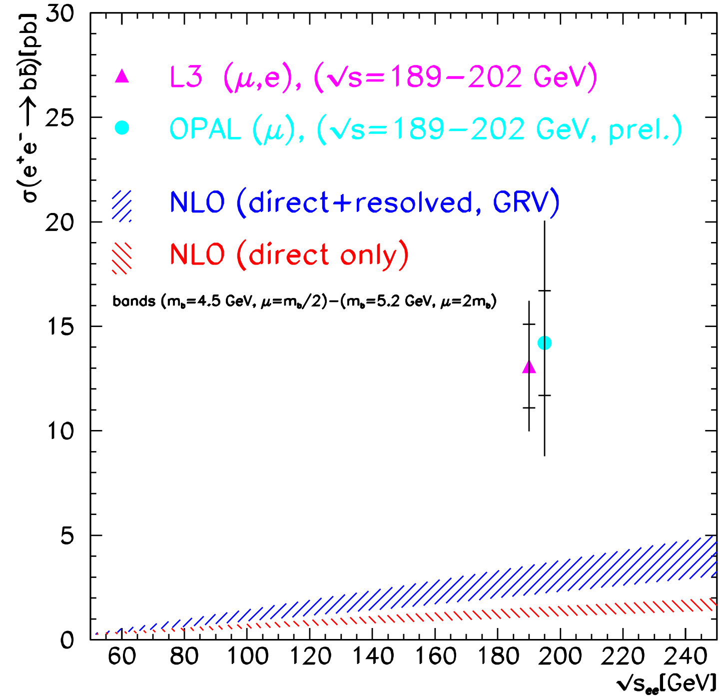

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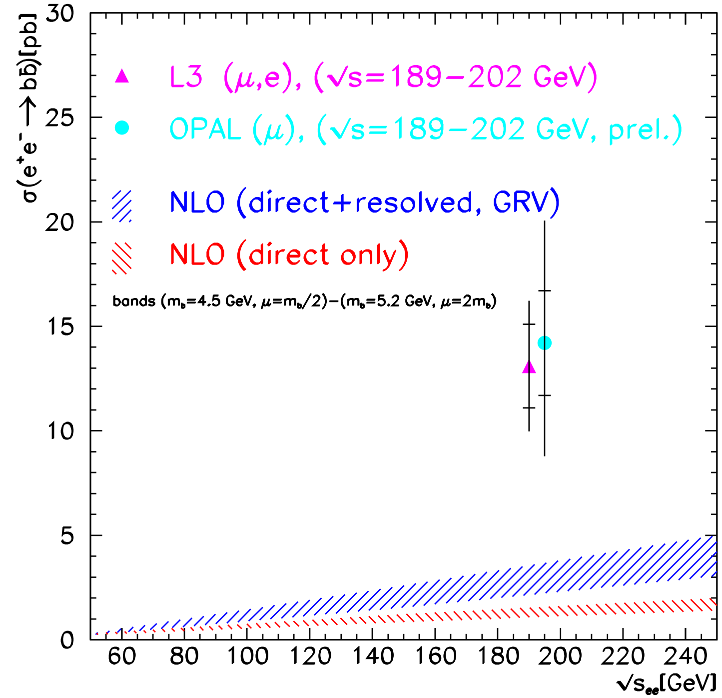

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- measurement relies on extrapolation of differential distributions into regions outside experimental coverage

Heavy Hadron Production

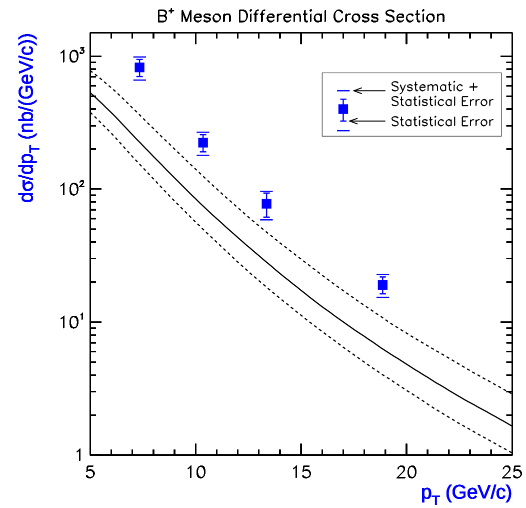
Differential distributions

long-standing excess of b-quark transverse momentum

distributions in $p\bar{p}$, γp, $\gamma\gamma$

measured final state: B hadrons, e.g. CDF: $B^{\pm}$

motivated careful reinvestigation of theory



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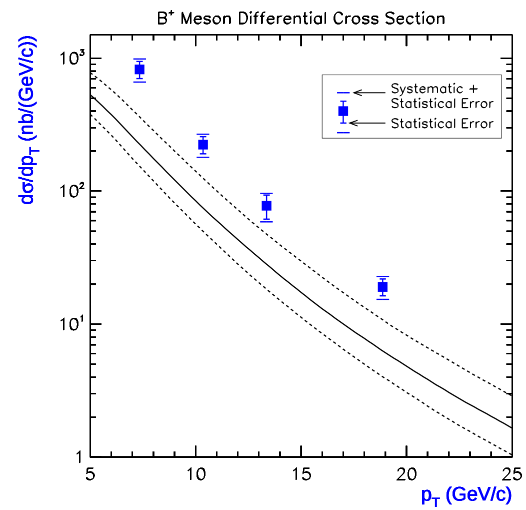

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Theoretical prediction:

$$\frac{d\sigma^{B^{\pm}}}{dp_T} = f_{a/p} \otimes f_{b/\bar{p}} \otimes \frac{d\sigma^{ab\to b\bar{b}}}{dp_T} \otimes D_{b\to B^{\pm}}$$

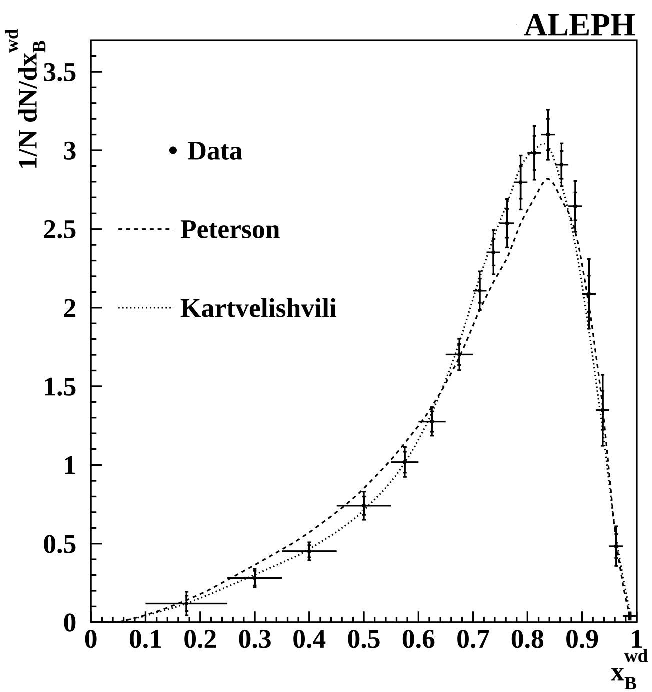
$f_{a/p}$: parton distribution function

$D_{b\to B^{\pm}}$: fragmentation function

Heavy Hadron Production

Fragmentation Functions

Extraction of $D_{b\to B^\pm}$ from LEP data on B meson spectra: many ambiguities



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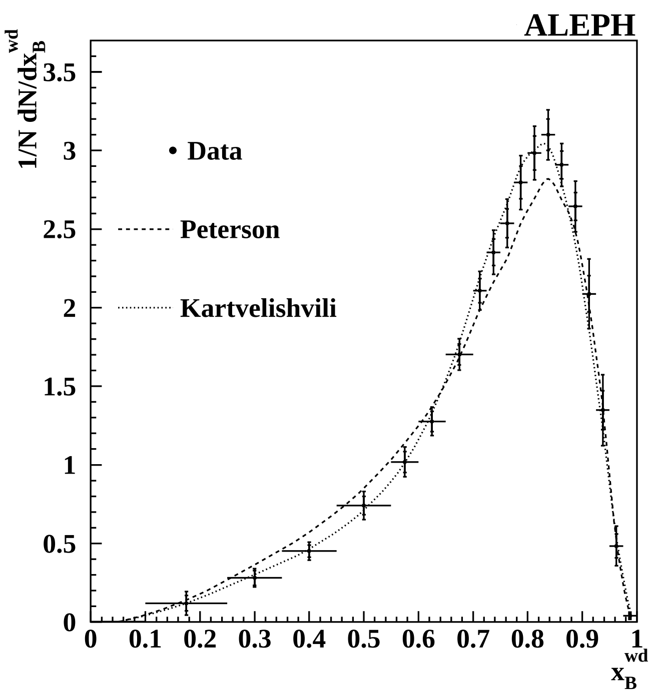

- order of perturbative calculation
- massless or massive hard matrix elements
- resummation of perturbative terms
- inclusion of power corrections $\mathcal{O}(\Lambda/m)$
  M. Cacciari, E. Gardi
- data corrected with parton showers ?
- parametric form of $D_{b\to B^\pm}$: e.g. Peterson:

$$D_{b\to B^\pm}(z,\mu_0) = N\frac{x(1-x)^2}{[(1-x)^2+\epsilon x]^2} \qquad \text{fit } \epsilon, N$$

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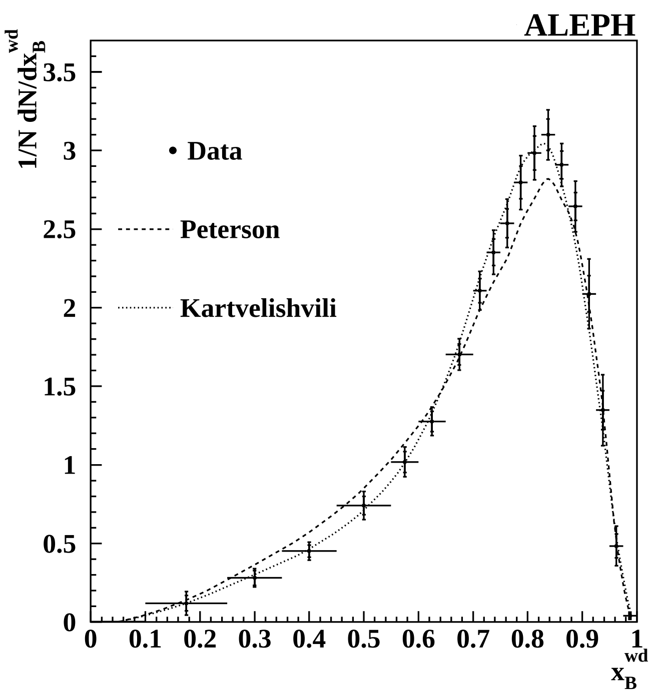

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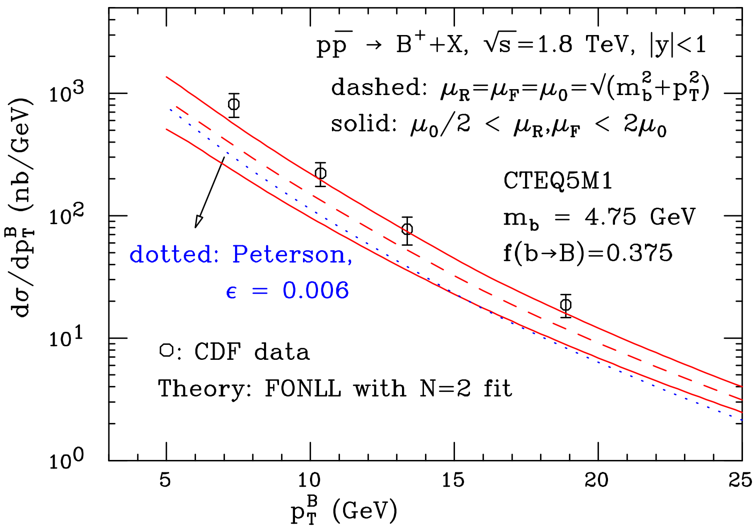
Prediction for $p\bar{p}$ must use same assumptions as made in the extraction from e^+e^-

Heavy Hadron Production

Resummation of $\ln(m_b^2/p_T^2)$ and $\ln(m_b^2/s)$

M. Cacciari, P. Nason



- must re-fit $D_{b\to B^\pm}$ from e^+e^- data
- done in moment space to expose information content relevant to $p\bar{p}$
- use of the resummed (FONLL) scheme enhances transverse momentum spectrum

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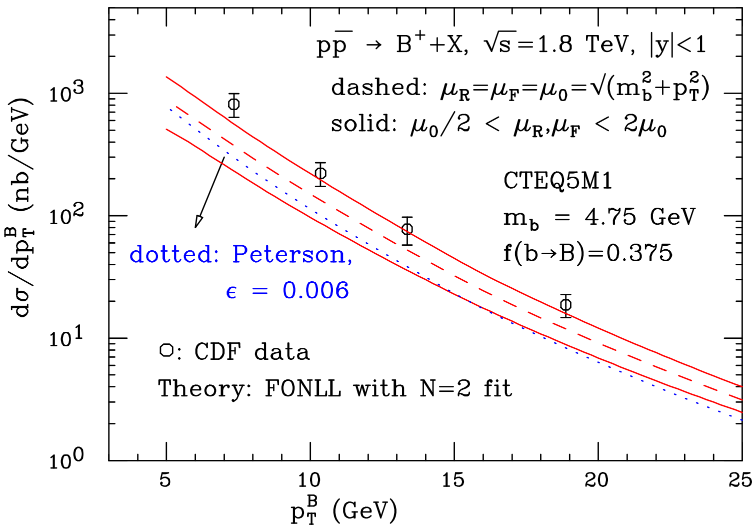
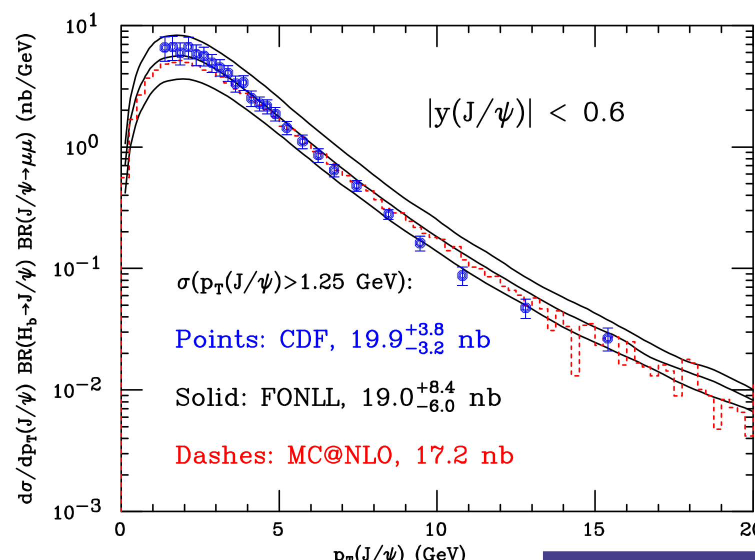


first CDF data at $\sqrt{s} = 1.96$ TeV do not confirm excess:

good agreement data–theory

M. Cacciari, S. Frixione, M. Mangano, P. Nason, G. Ridolfi



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- higher precision measurements (Tevatron-Run II, LHC) will require NNLO corrections

Jets and Multiparticle Production

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- sensitive probe of standard model parameters
  - measure α_s from $e^+e^- \to 3j$, $ep \to (2+1)j$, $pp \to j+X,\ 2j+X$
  - measure M_W, M_Z from $pp \to V+X,\ V+j+X$

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$\longrightarrow$ want QCD prediction as precise as possible

- multijet final states frequent: $pp \to nj \sim \alpha_s^n$
  - important background for searches

$\longrightarrow$ want robust QCD prediction as guidance

$\longrightarrow$ need flexible tools to predict any standard model process

$\longrightarrow$ also might want QCD to predict the full hadronic final state

Jets and Multiparticle Production

State-of-the-art: leading order $\longrightarrow$ M. Mangano

Jets and Multiparticle Production

State-of-the-art: leading order $\longrightarrow$ M. Mangano

- several efficient codes available for multiparton matrix element generation (from helicity amplitudes or fully numerically): $2 \rightarrow 8$ and beyond feasible on current computers
  - VECBOS W. Giele
  - COMPHEP E. Boos et al.
  - MADGRAPH T. Stelzer et al.
  - GRACE Minami–Tateya Group
  - HELAC C. Papadopoulos et al.
  - ALPHGEN M. Mangano et al.
  - AMEGIC++ F. Krauss et al.
- combined with automatic integration over multiparticle phase space
  - RAMBO R. Kleiss et al.
  - PHEGAS C. Papadopoulos
  - MADEVENT T. Stelzer et al.

Jets and Multiparticle Production

State-of-the-art: leading order

- generic procedure to interface partonic final state with parton shower:
  modified matrix element plus vetoed parton shower
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Leading order QCD is good tool to estimate relative magnitudes of processes and to design searches. Once precision is required (e.g. to identify a discovery with a particular model), it is not sufficient.

Jets and Multiparticle Production

State-of-the-art: next-to-leading order (NLO)
$\longrightarrow$ J. Campbell

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- nature of calculations: parton level Monte Carlo integrator

Jets and Multiparticle Production

State-of-the-art: NLO $2 \to 2$

programs available for $\simeq 10$ years, e.g.

- $pp \to 2j$
  S.D. Ellis, D. Soper, Z. Kunszt
  W. Giele, N. Glover, D. Kosower
- $pp \to V + j$
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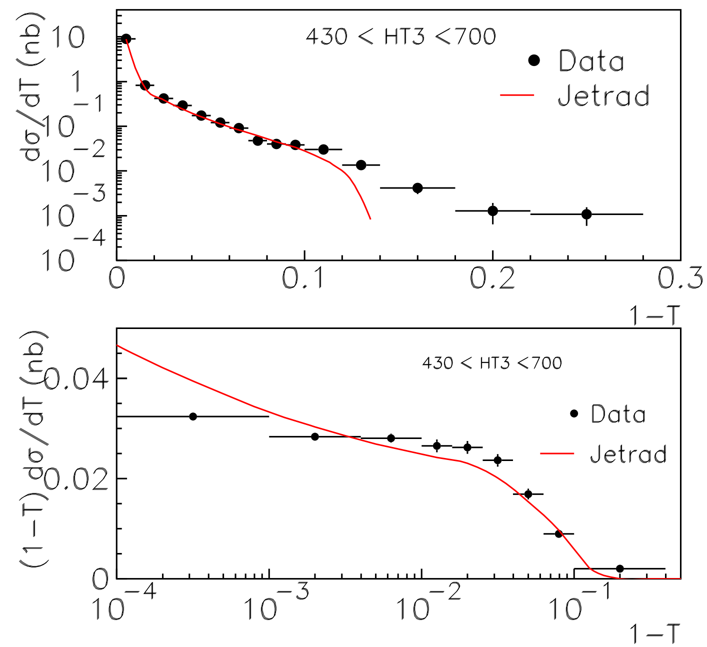
comparison data–theory

- generally good agreement on total rates
- fully differential data become available only now
- might need resummation of large logartihms in certain regions
  A. Banfi, G. Salam, G. Zanderighi



D0 collaboration

Jets and Multiparticle Production

State-of-the-art: NLO $2 \to 3$

recent results

- $pp \to V + 2j$
  J.M. Campbell, R.K. Ellis
- $ep \to (3+1)j$
  Z. Nagy, Z. Trocsanyi
- $pp \to 3j$
  Z. Nagy
- $pp \to \gamma\gamma j$
  V. Del Duca, F. Maltoni, Z. Nagy, Z. Trocsanyi
- $pp \to t\bar{t}H$
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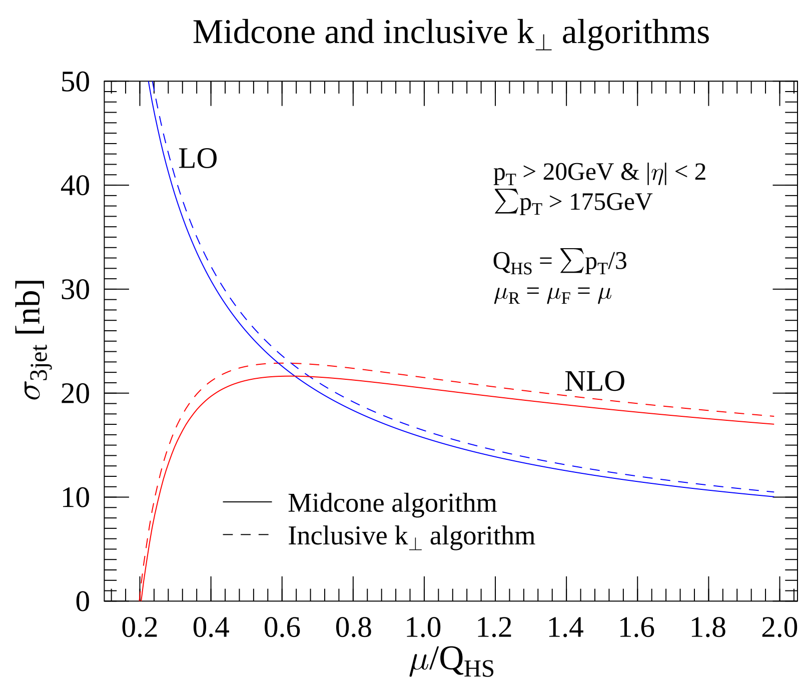
  B. Plümper, M. Spira, P. Zerwas
  S. Dawson, L. Orr, L. Reina, D. Wackeroth
- $pp \to H + 2j$
  C. Oleari, D. Zeppenfeld



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- ingredients to NLO calculation for n parton final state
  - virtual one-loop matrix element n partons
  - real matrix element $n+1$ partons
  - procedure to extract infrared singularities from both and to combine them
- last two points solved for arbitrary n some time ago
  phase space slicing, subtraction, dipole formalism

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  phase space slicing, subtraction, dipole formalism
- no generic procedure for automatic computation of one-loop integrals
- but many ideas
  - subtraction formalism for virtual corrections Z. Nagy, D. Soper
  - analytic reduction of hexagon integrals T. Binoth, J.P. Guillet, G. Heinrich
  - numerical evaluation of hexagon amplitudes T. Binoth, N. Kauer
  - soft/collinear separation S. Dittmaier
  - infrared rearrangement D.A. Forde, A. Signer

Jets and Multiparticle Production

Interfacing NLO calculations with parton showers

MC@NLO approach $\longrightarrow$ S. Frixione

S. Frixione, B. Webber

- introduces modified NLO subtraction method
- modified real and virtual contributions become initial conditions for parton shower

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- modified real and virtual contributions become initial conditions for parton shower
- hard radiation from NLO matrix element
- avoids double counting
- NLO result explicitly recovered from expansion

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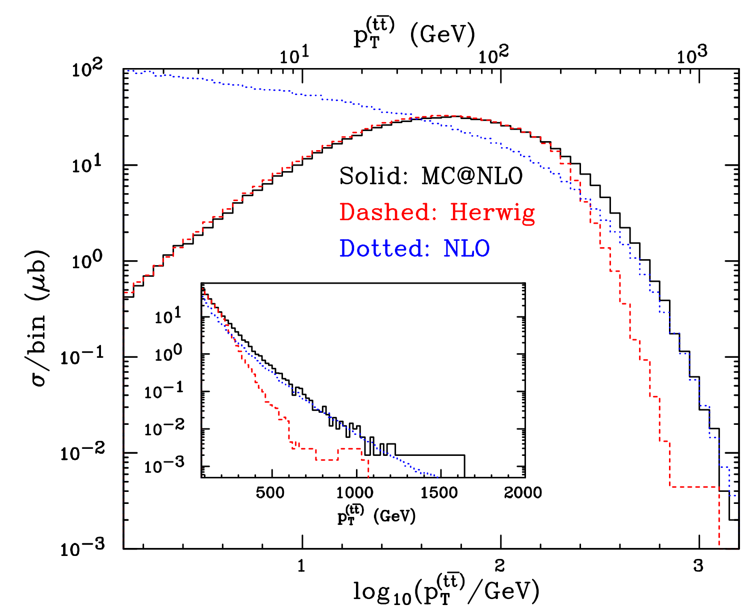
MC@NLO approach $\longrightarrow$ S. Frixione

S. Frixione, B. Webber

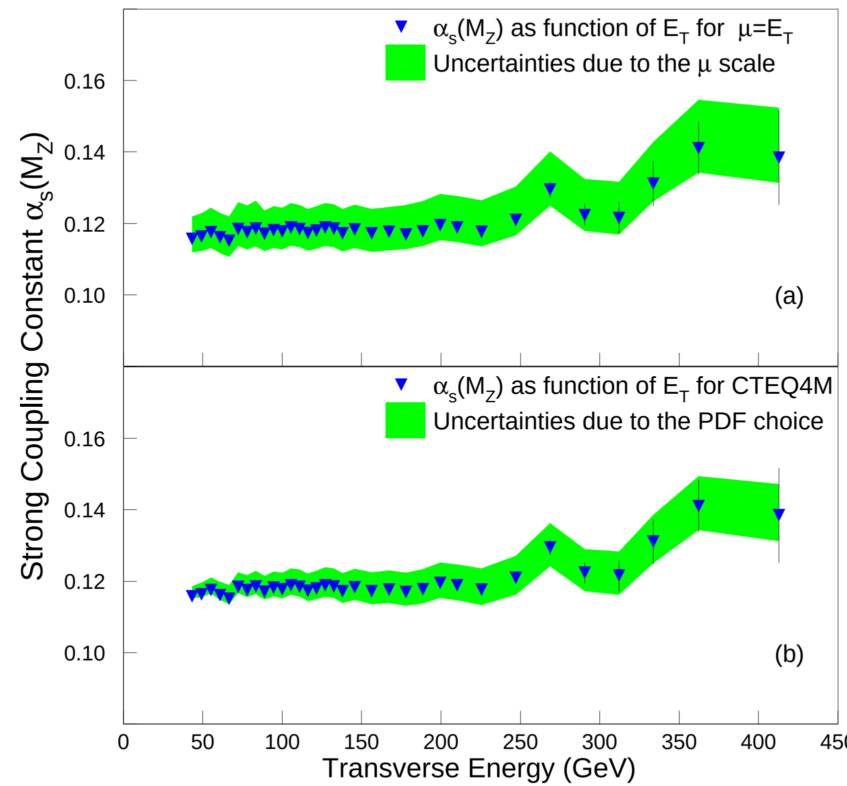
- introduces modified NLO subtraction method
- modified real and virtual contributions become initial conditions for parton shower
- hard radiation from NLO matrix element
- avoids double counting
- NLO result explicitly recovered from expansion
- so far applied to V, H, VV, $b\bar{b}$, $t\bar{t}$
  S. Frixione, P. Nason, B. Webber



NLO jet physics precise enough ?

Measurement of strong coupling constant α_s from single jet inclusive cross section

CDF collaboration



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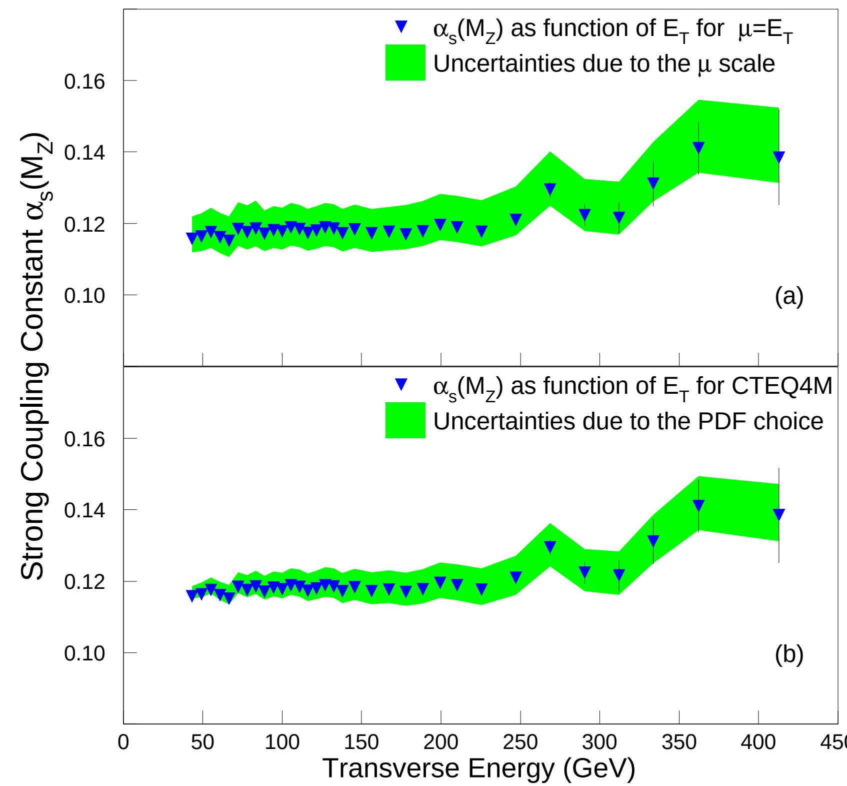

$$\alpha_s(M_Z) = 0.1178 \pm 0.0001(\text{stat})^{+0.0081}_{-0.0095}(\text{sys})$$
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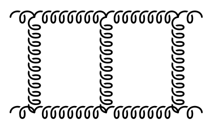
- modified power corrections as higher perturbative powers $1/\ln(Q^2/\Lambda^2)$ can mimic genuine power corrections Q/Λ
- allow full NNLO global fits to parton distributions $\longrightarrow$ lower error on benchmark processes at LHC and Tevatron

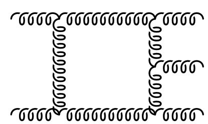
Towards NNLO jet physics

Ingredients to NNLO n-jet: $\longrightarrow$ D. Kosower

- Two-loop matrix elements

  $|\mathcal{M}|^2_{\text{2-loop},n \text{ partons}}$ 

- One-loop matrix elements

  $|\mathcal{M}|^2_{\text{1-loop},n+1 \text{ partons}}$ 

- One-loop one-particle subtraction terms

  $\int |\mathcal{M}^{R,1}|^2_{\text{1-loop},n+1 \text{ partons}} \, d\Phi_1$

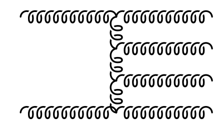
  D. Kosower, P. Uwer

  Z. Bern et al.

  S. Weinzierl

  D. Kosower

- Tree level matrix elements

  $|\mathcal{M}|^2_{\text{tree},n+2 \text{ partons}}$ 

- Tree-level one-particle subtraction terms

  $\int |\mathcal{M}^{R,1}|^2_{\text{tree},n+2 \text{ partons}} \mathrm{d}\Phi_1$

  W. Giele, N. Glover

  S. Catani, M. Seymour

- Tree-level two-particle subtraction terms

  $\int |\mathcal{M}^{R,2}|^2_{\text{tree},n+2 \text{ partons}} \mathrm{d}\Phi_2$

  remain to be calculated

Towards NNLO jet physics

Virtual two-loop corrections to jet observables have seen enormous progress in the past years

Technical breakthroughs:

- algorithms to reduce the ~ 10000's of integrals to a few $(10-30)$ master integrals
  - Integration-by-parts (IBP)
    K. Chetyrkin, F. Tkachov
  - Lorentz Invariance (LI)
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  - and their implementation in computer algebra
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- new methods to compute master integrals
  - Mellin-Barnes Transformation V. Smirnov, O. Veretin; B. Tausk
  - Differential Equations E. Remiddi, TG
  - Sector Decomposition (numerically) T. Binoth, G. Heinrich
  - Nested Sums S. Moch, P. Uwer, S. Weinzierl

Towards NNLO jet physics

Virtual two-loop matrix elements have recently been computed for:

- Bhabha-Scattering: $e^+e^- \to e^+e^-$
  Z. Bern, L. Dixon, A. Ghinculov
- Hadron-Hadron 2-Jet production: $qq' \to qq'$, $q\bar{q} \to q\bar{q}$, $q\bar{q} \to gg$, $gg \to gg$
  C. Anastasiou, N. Glover, C. Oleari, M. Yeomans-Tejeda
  Z. Bern, A. De Freitas, L. Dixon [SUSY-YM]
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- Three-jet production: $e^+e^- \to \gamma^* \to q\bar{q}g$
  L. Garland, N. Glover, A.Koukoutsakis, E. Remiddi, TG
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- DIS (2+1) jet production: $\gamma^* g \to q\bar{q}$, Hadronic (V+1) jet production: $qg \to Vq$
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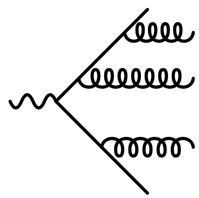
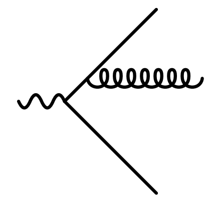
Ongoing:

- Matrix elements with internal masses: $\gamma^* \to Q\bar{Q}$, $Q\bar{Q} \to Q\bar{Q}$
  R. Bonciani, P. Mastrolia, E. Remiddi; U. Aglietti, R. Bonciani

Towards NNLO jet physics

Double real radiation

 $\mathrm{d}\Phi_2 \longrightarrow C$ 

- Singular configuartions:
  - triple collinear
  - double single collinear
  - soft/collinear
  - double soft

Towards NNLO jet physics

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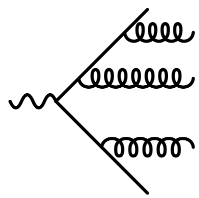
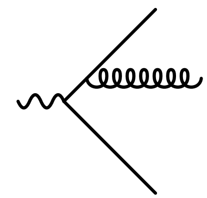


$$\mathrm{d}\Phi_2 \longrightarrow C$$



- Singular configuartions:
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  - double single collinear
  - soft/collinear
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- Issue: find subtraction functions which
  - approxmiate full $n+2$ matrix element in all singular limits
  - are sufficiently simple to be integrated analytically

Towards NNLO jet physics

Double real radiation

results for specific processes:

- $\gamma^* \to \gamma + j$ at $\mathcal{O}(\alpha\alpha_s)$
  triple collinear, double single collinear, soft/collinear
  A. Gehrmann-De Ridder, N. Glover

Towards NNLO jet physics

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- NLO evolution kernels for semi-inclusive deep inelastic scattering
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  A. Daleo, R. Sassot

Towards NNLO jet physics

Double real radiation

recent progress:

- smooth mapping $\mathrm{d}\phi_{n+2} \to \mathrm{d}P \cdot \mathrm{d}\phi_n$
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Towards NNLO jet physics

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    A. Gehrmann-De Ridder, G. Heinrich, TG
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    G. Heinrich; A. Gehrmann-De Ridder, G. Heinrich, TG
    C. Anastasiou, K. Melnikov, F. Petriello

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available at present:

- subtraction terms not integrated up to now
- class of integrals not matching structure of subtraction terms yet
- promising purely numerical method

N^3LO jet physics: first steps

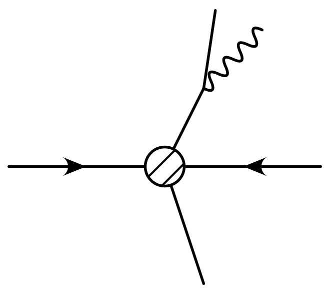
Exploring the calculational structure:

- virtual three-loop four-point functions
  V. Smirnov; T. Binoth, G. Heinrich
- simple unresolved limits of two-loop amplitudes
  C. Anastasiou, Z. Bern, L. Dixon, D. Kosower
- double unresolved limits of one-loop amplitudes
  S. Catani, D. de Florian, G. Rodrigo
- triple unresolved limits of tree amplitudes
  V. Del Duca, A. Frizzo, F. Maltoni

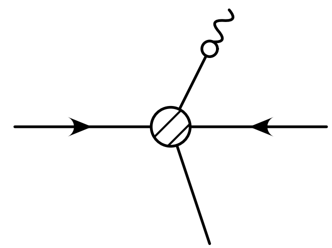
Photons

Photons

Two production processes for photons



direct



fragmentation

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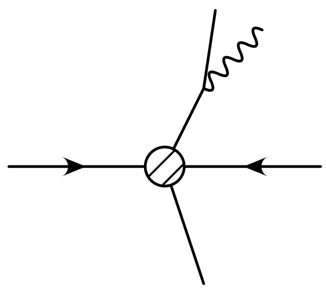

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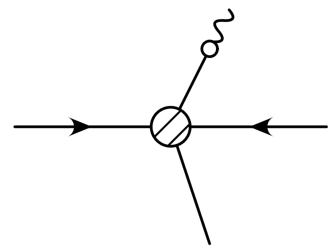

fragmentation

Photons never fully isolated from hadrons

- isolation cone: $E^{\text{had}} < E^{\text{isol}}$ for $R < R^{\text{isol}}$
- cluster photon into jet: $E^{\gamma} > z_{\text{cut}} E^{\text{jet}}$ N. Glover, A. Morgan

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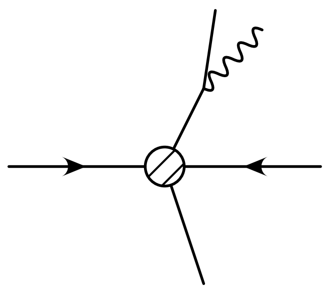

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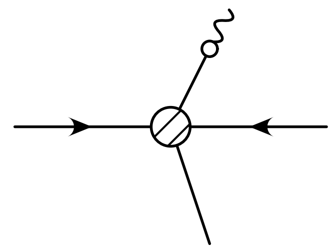

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Both isolation criteria

- are infrared safe
- induce contribution from non-perturbative quark-to-photon fragmentation function

Cone-based isolation fails for small cones:
S. Catani, M. Fontannaz, J.P. Guillet, E. Pilon

$$\sigma^{\rm isol} > \sigma^{\rm incl} \quad \text{for } R \leq 0.1 \quad (\alpha_s \ln R^{-2} \sim 1)$$

Photon Pairs

$pp \to \gamma\gamma X$: background to Higgs search in $H \to \gamma\gamma$ decay mode

- Subprocesses: $q\bar{q} : \mathcal{O}(\alpha_s^0), \quad qg : \mathcal{O}(\alpha_s^1), \quad gg : \mathcal{O}(\alpha_s^2)$
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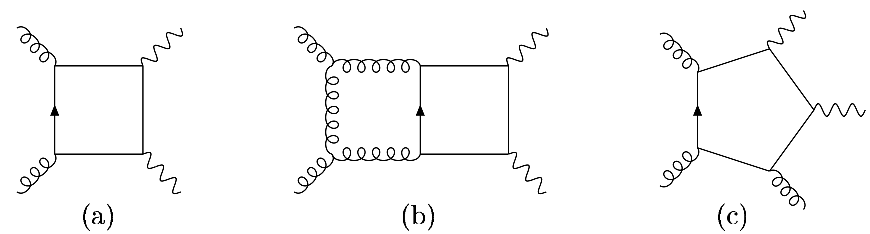

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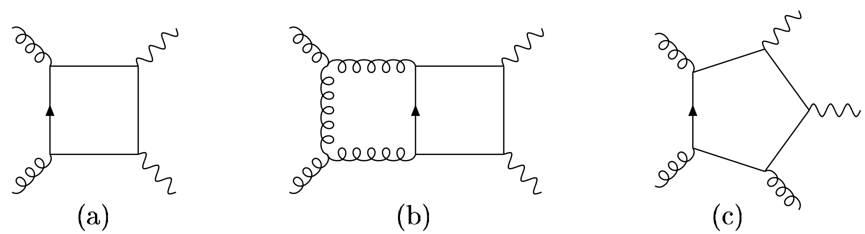
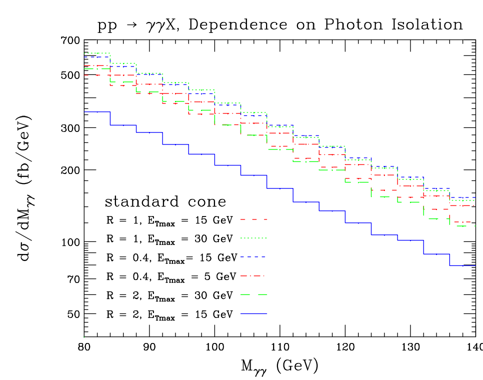


- cross section depends on photon isolation



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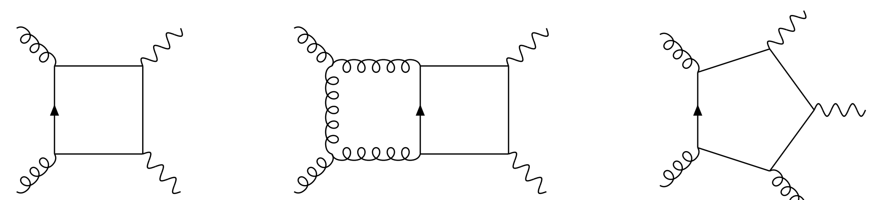
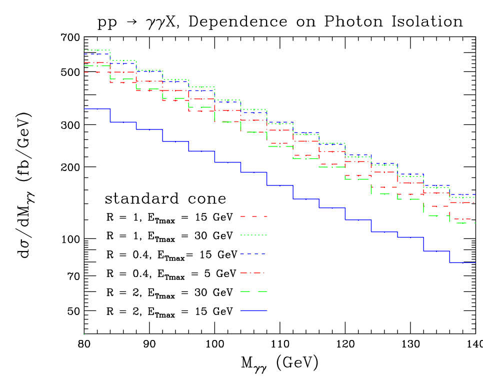


(a) (b) (c)

- cross section depends on photon isolation
- sensitive on photon fragmentation function $D_{q\to\gamma}$ at large momentum transfer
  (only experimental constraint: LEP)



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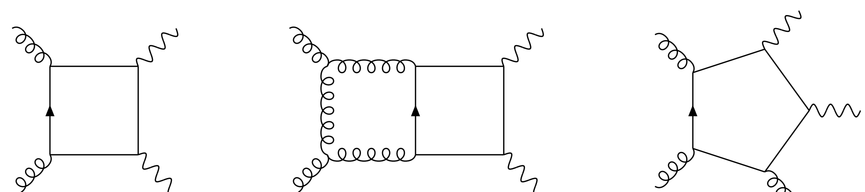

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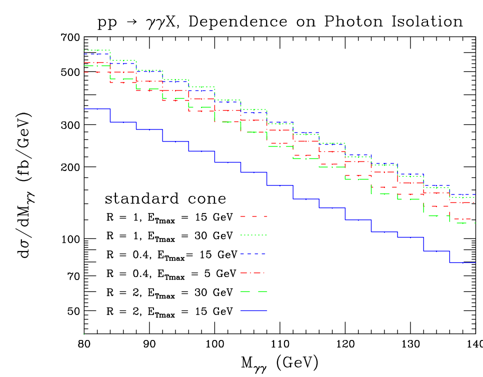
- NLO avaliable also for $pp \to \gamma\gamma j$
  V. Del Duca, F. Maltoni, Z. Nagy, Z. Trocsanyi



Gauge and Higgs Boson Production

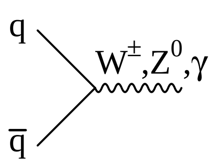
Gauge and Higgs Boson Production

$\longrightarrow$ B. Kilgore

Inclusive Production: NNLO corrections

Drell–Yan process

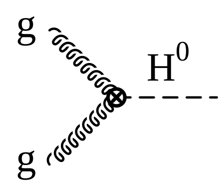
$q\bar{q} \to W^{\pm}, Z^0, \gamma^*$



T. Matsuura, R. Hamberg,
W. van Neerven
R. Harlander, W. Kilgore

Higgs production

$gg \to H^0$



R. Harlander, W. Kilgore
C. Anastasiou, K. Melnikov
V. Ravindran, J. Smith, W. van Neerven

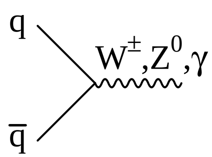
Gauge and Higgs Boson Production

$\longrightarrow$ B. Kilgore

Inclusive Production: NNLO corrections

Drell–Yan process

$q\bar{q} \to W^{\pm}, Z^0, \gamma^*$



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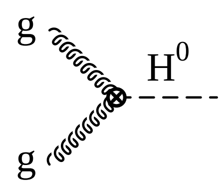

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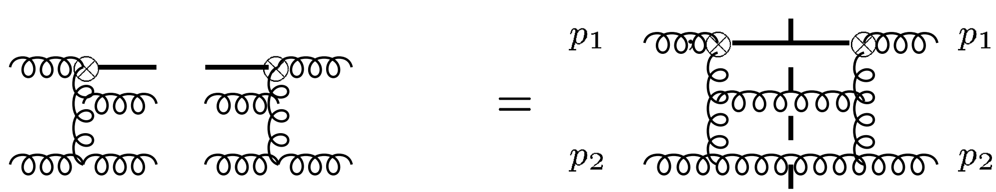
New calculational techniques:

- expansion around the soft limit
  R. Harlander, W. Kilgore
- extension of multi-loop techniques (IBP and LI, differential equations) to phase space integrals



C. Anastasiou, K. Melnikov

Inclusive Higgs Boson Production

Recent NNLO results

- Standard model Higgs from gluon fusion
  $gg \to H + X$
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- including NNLL soft gluon resummation
  S. Catani, D. de Florian, M. Grazzini, P. Nason
- Pseudoscalar Higgs from gluon fusion
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- Higgs from bottom quark fusion $b\bar{b} \to H + X$
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  $q\bar{q} \to W^{\pm}H + X$, $q\bar{q} \to Z^0 H + X$
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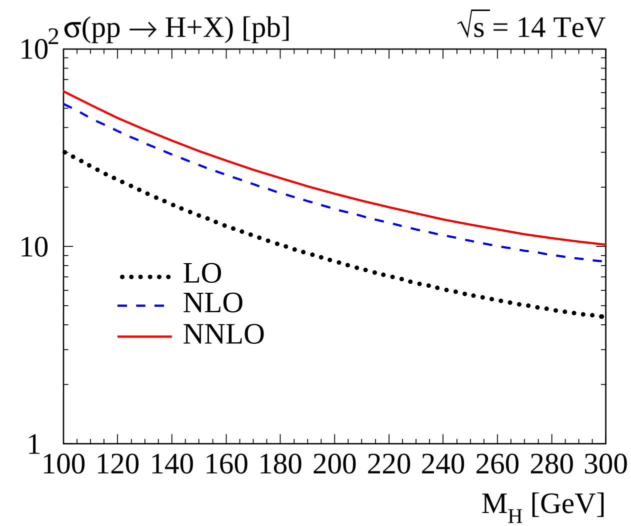
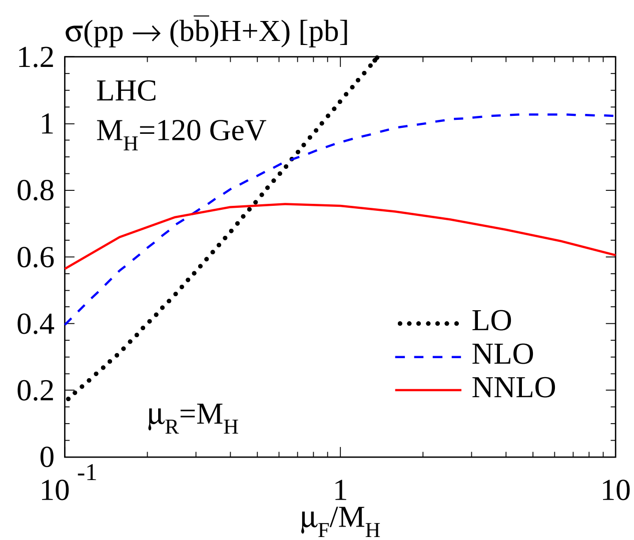
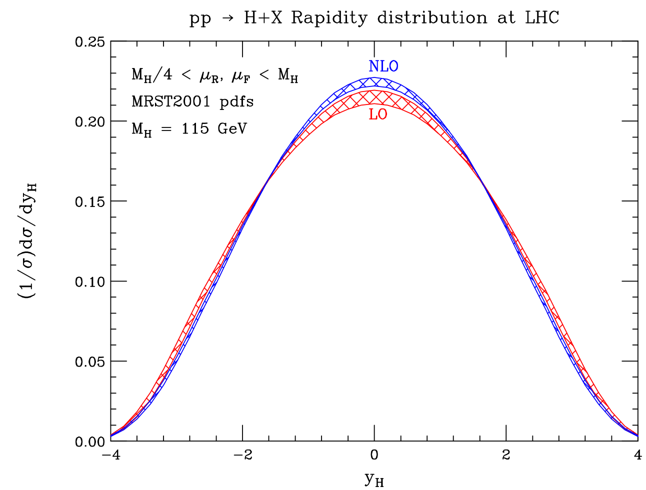




Higgs Boson Spectra

Rapidity distribution

- experiments cover only limited range in rapidity
- recently computed to NLO $\mathcal{O}(\alpha_s)$
  C. Anastasiou, L. Dixon, K. Melnikov



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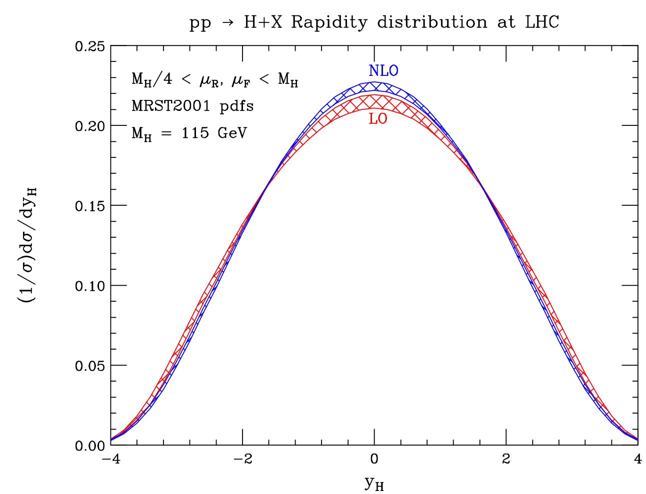
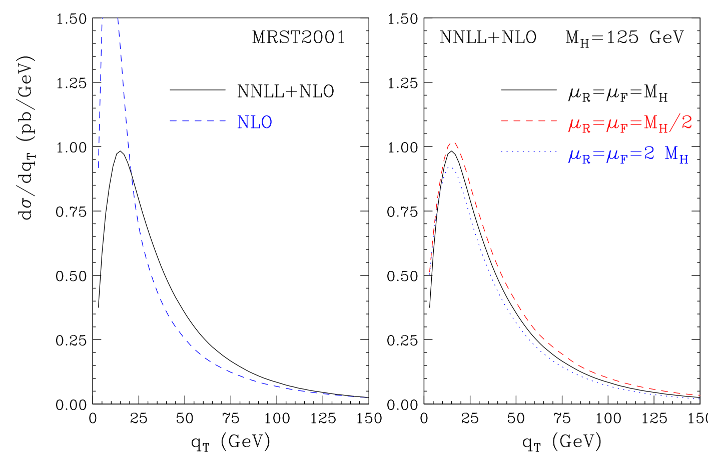


Transverse momentum distribution $\longrightarrow$ G. Sterman

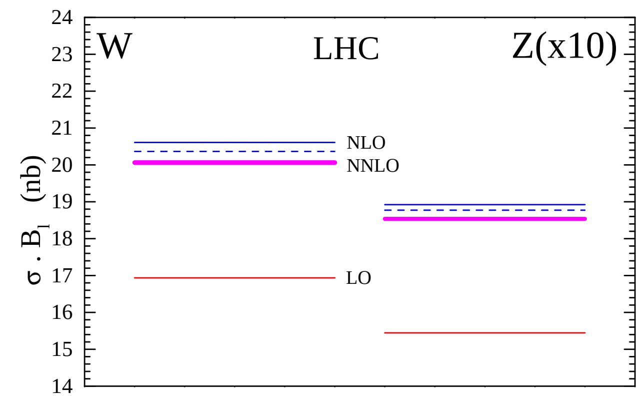
- fixed order NLO $\mathcal{O}(\alpha_s^2)$ reliable only for $q_T \geq M_H$
  D. de Florian, M. Grazzini, Z. Kunszt
  C. Glosser, C. Schmidt
  V. Ravindran, J. Smith, W. van Neerven
- small $q_T \ll M_H$ require soft gluon resummation (NNLL) of $\ln(q_T/M_H)$
  G. Bozzi, S. Catani, D. de Florian, M. Grazzini



Vector Boson Production

Inclusive cross section

- can be measured precisely
- are theoretically well understood
  - NNLO corrections known
  - relevant partons well constrained
- benchmark reaction for LHC
  (luminosity monitor)



A. Martin, R. Roberts, J. Stirling, R. Thorne

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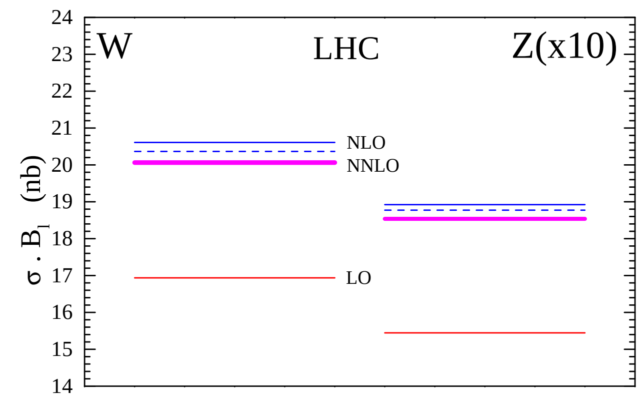

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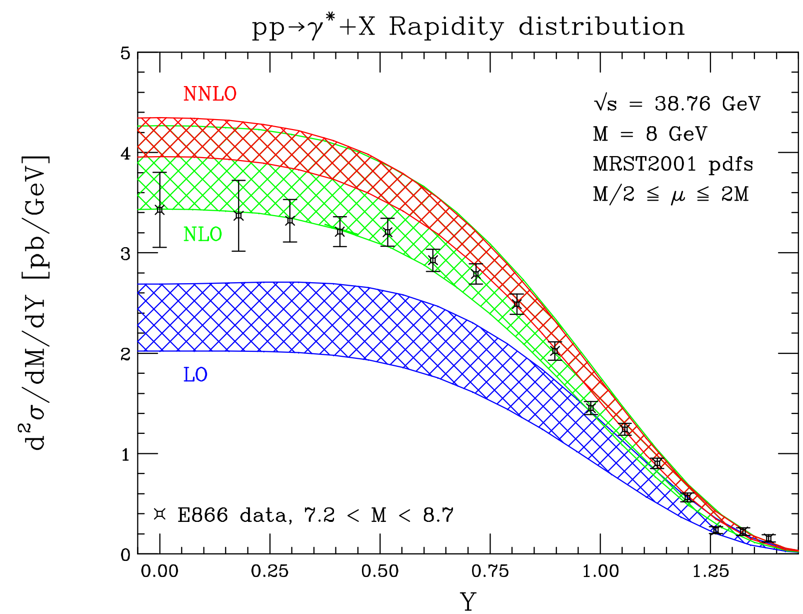
Rapidity distribution

- accounts for limited experimental coverage
  M. Dittmar, F. Pauss, D. Zürcher
- recently computed to NNLO
  (further extension of multi-loop tools)
  C. Anastasiou, L. Dixon, K. Melnikov,
  F. Petriello



Parton Distributions at NNLO

Parton distributions from global fit $\longrightarrow$ J. Stirling

analyze data from different observables to determine all partonic distribution functions

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- need control of error propagation from data to distributions
- NLO theory error already comparable to data errors
- need partonic cross sections to NNLO
  - deep inelastic scattering: neutral and charged current (OK)
  - Drell-Yan process (OK)
  - jet production
  - (direct photon production)

Parton Distributions at NNLO

Splitting kernels to NNLO $\longrightarrow$ S. Moch

- technique: evaluate $\gamma^* q \to \gamma^* q$ and $\phi g \to \phi g$ at three loops (in moment space)
- some fixed moments known
  S. Larin, P. Nogueira, T. van Ritbergen, J. Vermaseren; A. Retey
- allow approximate reconstruction
  W. van Neerven, A. Vogt
- non-singlet n_f piece known
  S. Moch, J. Vermaseren, A. Vogt
- remaining pieces under way

Summary and Conclusions

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    - many generic tools available
    - interface to parton shower and hadronization
  - NLO important to refine searches and to identify signals
    - current frontier $2 \to 3$; $2 \to 4$ requires new tools
    - interface to parton showers
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- resummation often important
- many predictions require input from LEP data (e.g. fragmentation functions)