Statistical Mechanics of Money, Income, and Wealth

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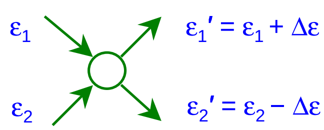
Publications

- European Physical Journal B **17**, 723 (2000), cond-mat/0001432
- European Physical Journal B **20**, 585 (2001), cond-mat/0008305
- Physica A **299**, 213 (2001), cond-mat/0103544
- *Modeling of Complex Systems: Seventh Granada Lectures*, AIP CP **661**, 180 (2003), cond-mat/0211175

Boltzmann-Gibbs probability distribution of energy

Collisions between atoms



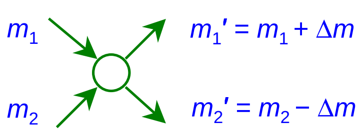
Conservation of energy:
$\varepsilon_1 + \varepsilon_2 = \varepsilon_1' + \varepsilon_2'$

Detailed balance:
$P(\varepsilon_1)\, P(\varepsilon_2) = P(\varepsilon_1')\, P(\varepsilon_2')$

Boltzmann-Gibbs probability distribution $P(\varepsilon) \propto \exp(-\varepsilon/T)$ of energy ε, where $T = \langle\varepsilon\rangle$ is temperature.

Boltzmann-Gibbs distribution maximizes entropy $S = -\Sigma_\varepsilon P(\varepsilon) \ln P(\varepsilon)$ under the constraint of conservation law $\Sigma_\varepsilon P(\varepsilon)\, \varepsilon = \text{const.}$

Economic transactions between agents

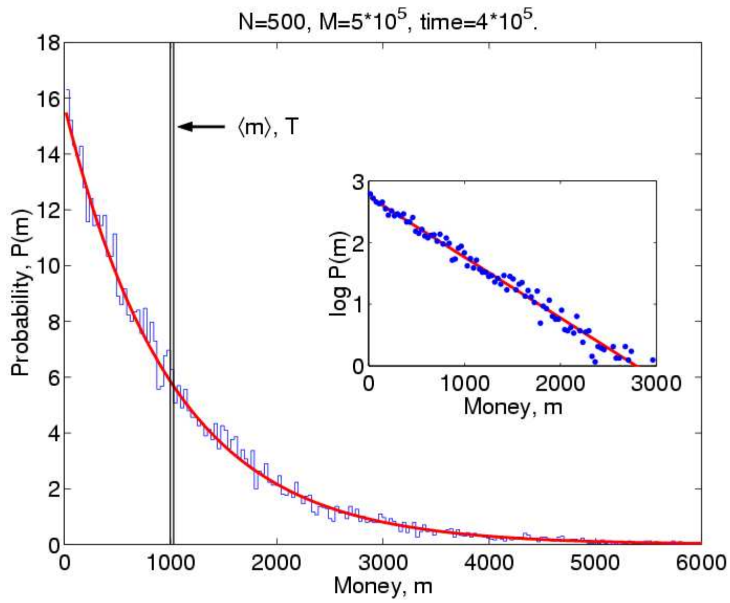


Conservation of money:
$m_1 + m_2 = m_1' + m_2'$

Detailed balance:
$P(m_1)\, P(m_2) = P(m_1')\, P(m_2')$

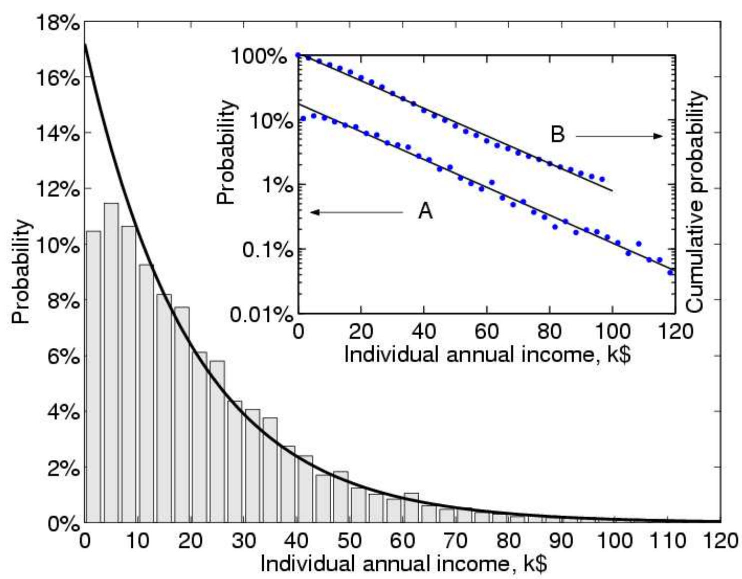
Boltzmann-Gibbs probability distribution $P(m) \propto \exp(-m/T)$ of money m, where $T = \langle m\rangle$ is the money temperature.

Computer simulation of money redistribution



The stationary distribution of money *m* is exponential: $P(m) \propto e^{-m/T}$

Probability distribution of individual income

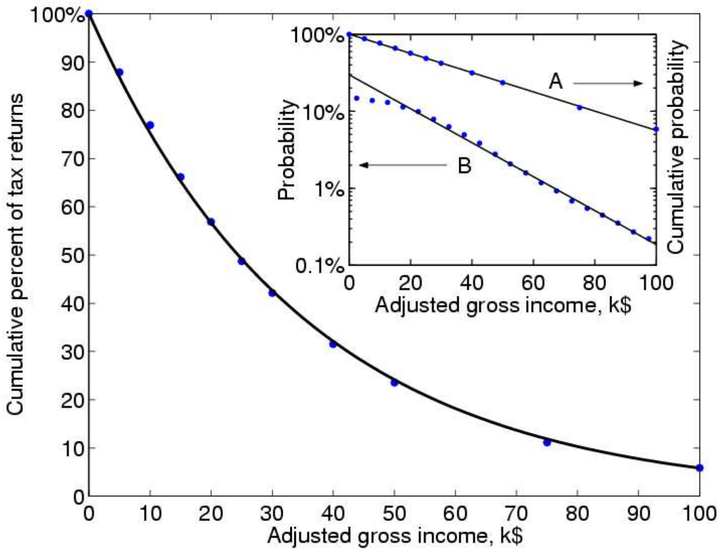


US Census data 1996 – histogram and points A

PSID: Panel Study of Income Dynamics, 1992 (U. Michigan) – points B

Distribution of income *r* is exponential: $P(r) \propto e^{-r/T}$

Probability distribution of individual income



IRS data 1997 – main panel and points A, 1993 – points B

Cumulative distribution of income r is exponential:

$$C(r) = \int_r^\infty dr' P(r') = \exp(-r/T)$$

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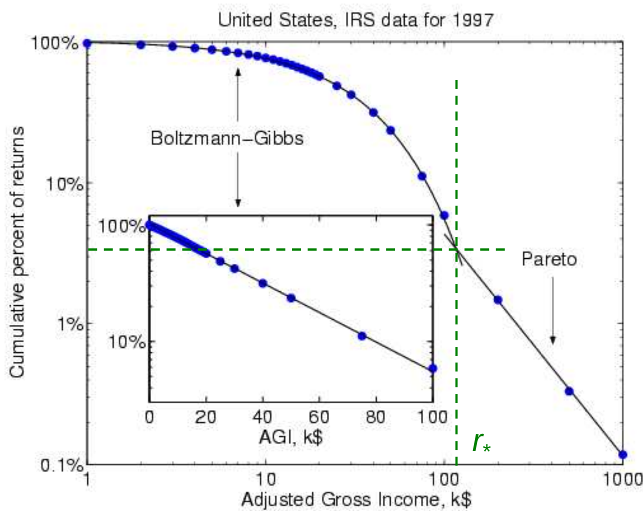
Income distribution in the USA, 1997



Two-class society

Upper Class
- Pareto power law
- 3% of population
- 16% of income
- Income > 120 k$: investments, capital

Lower Class
- Boltzmann-Gibbs exponential law
- 97% of population
- 84% of income
- Income < 120 k$: wages, salaries

"Thermal" bulk and "super-thermal" tail distribution

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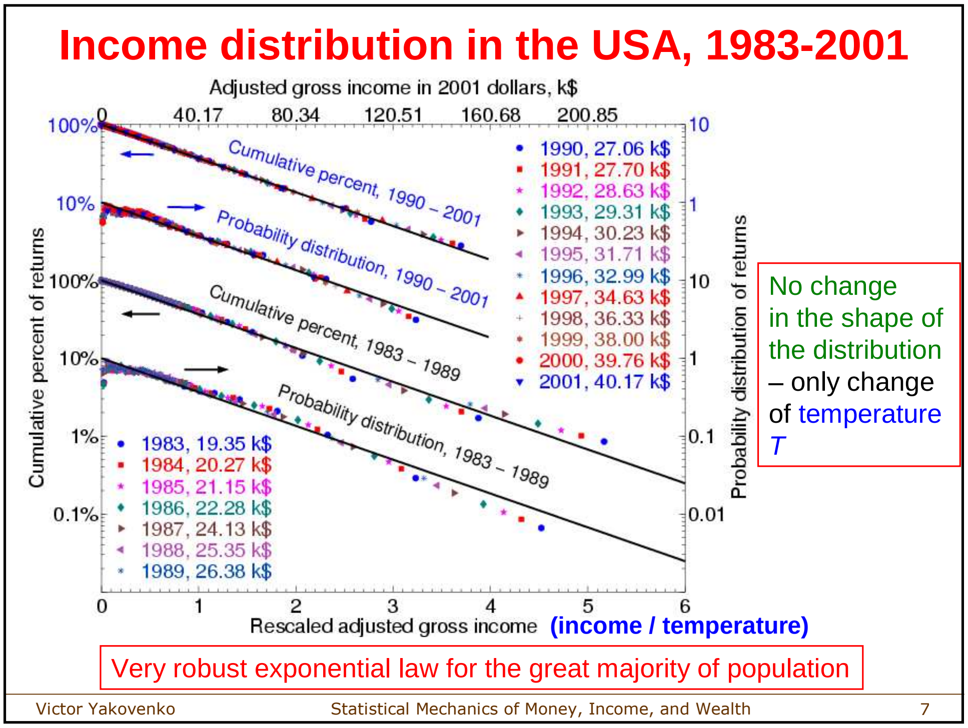
Income distribution in the USA, 1983-2001
Adjusted gross income in 2001 dollars, k$
0 40.17 80.34 120.51 160.68 200.85
Cumulative percent, 1990 – 2001
Probability distribution, 1990 – 2001
Cumulative percent, 1983 – 1989
Probability distribution, 1983 – 1989
1990, 27.06 k$
1991, 27.70 k$
1992, 28.63 k$
1993, 29.31 k$
1994, 30.23 k$
1995, 31.71 k$
1996, 32.99 k$
1997, 34.63 k$
1998, 36.33 k$
1999, 38.00 k$
2000, 39.76 k$
2001, 40.17 k$
1983, 19.35 k$
1984, 20.27 k$
1985, 21.15 k$
1986, 22.28 k$
1987, 24.13 k$
1988, 25.35 k$
1989, 26.38 k$
Cumulative percent of returns
Probability distribution of returns
Rescaled adjusted gross income (income / temperature)
No change in the shape of the distribution – only change of temperature T
Very robust exponential law for the great majority of population
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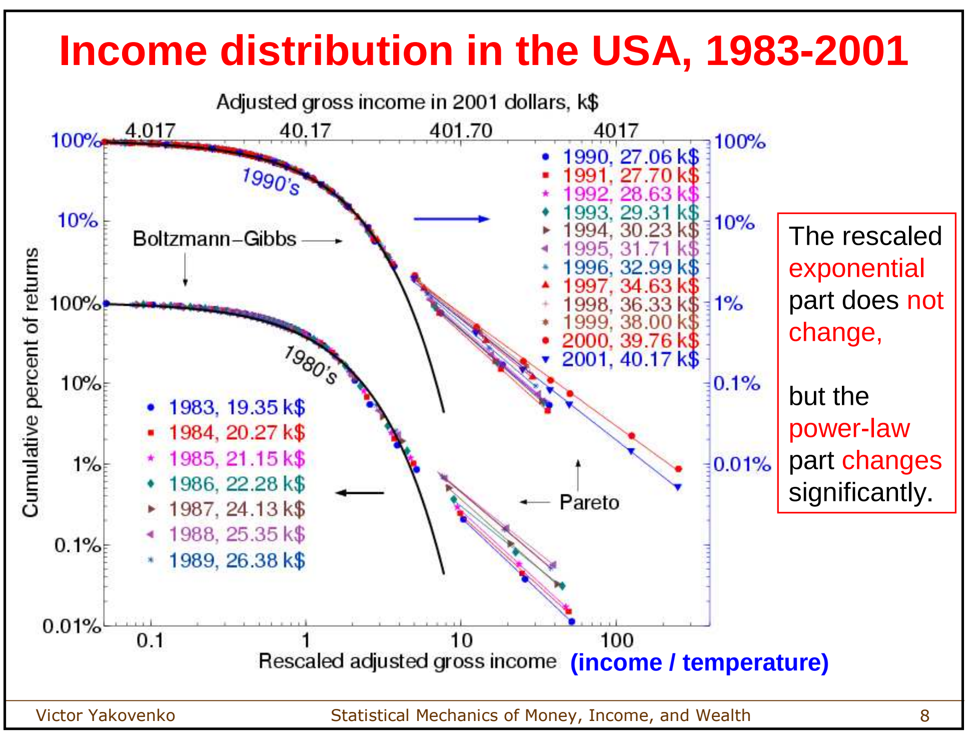
Income distribution in the USA, 1983-2001
Adjusted gross income in 2001 dollars, k$
4.017 40.17 401.70 4017
1990's
1980's
Boltzmann–Gibbs
Pareto
1990, 27.06 k$
1991, 27.70 k$
1992, 28.63 k$
1993, 29.31 k$
1994, 30.23 k$
1995, 31.71 k$
1996, 32.99 k$
1997, 34.63 k$
1998, 36.33 k$
1999, 38.00 k$
2000, 39.76 k$
2001, 40.17 k$
1983, 19.35 k$
1984, 20.27 k$
1985, 21.15 k$
1986, 22.28 k$
1987, 24.13 k$
1988, 25.35 k$
1989, 26.38 k$
Cumulative percent of returns
Rescaled adjusted gross income (income / temperature)
The rescaled exponential part does not change,
but the power-law part changes significantly.
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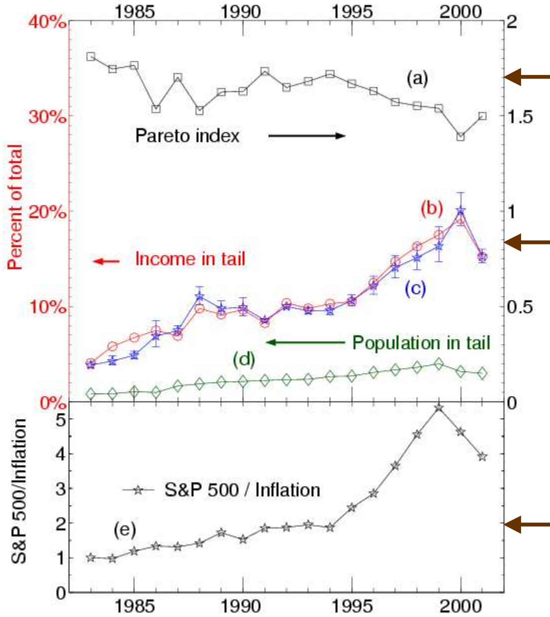
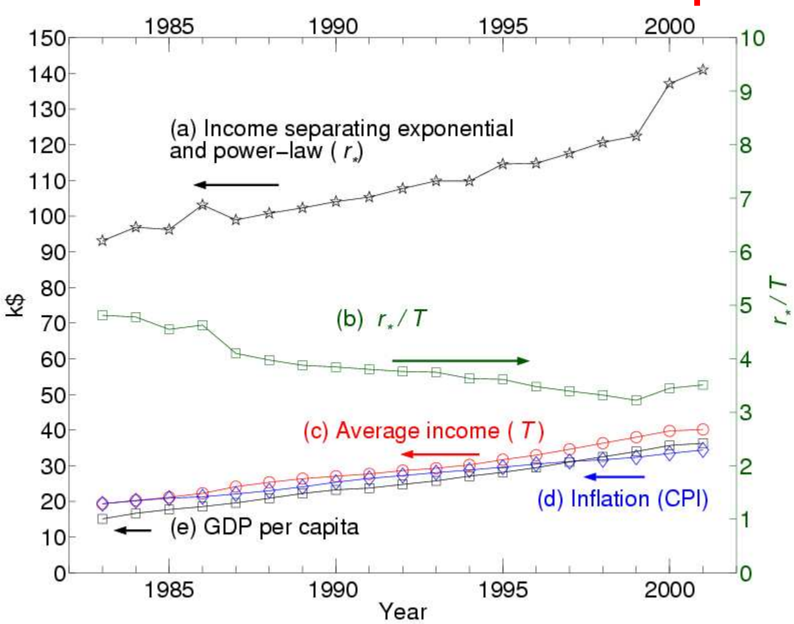
Time evolution of the tail parameters



The Pareto index α in $C(r) \propto 1/r^{\alpha}$ is non-universal. It changed from 1.7 in 1983 to 1.3 in 2000.

- Pareto tail changes in time non-monotonously, in line with the stock market.
- The tail income swelled 5-fold from 4% in 1983 to 20% in 2000.
- It decreased in 2001 with the crash of the U.S. stock market.

Time evolution of income temperature



The nominal average income T doubled: 20 k$ 1983 40 k$ 2001, but it is mostly inflation.

Diffusion model for income kinetics

Suppose income changes by small amounts Δr over time Δt.
Then $P(r,t)$ satisfies the Fokker-Planck equation for $0<r<\infty$:

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial r}\left(AP + \frac{\partial}{\partial r}(BP)\right), \quad A = -\left\langle \frac{\Delta r}{\Delta t} \right\rangle, \quad B = \left\langle \frac{(\Delta r)^2}{2\Delta t} \right\rangle.$$

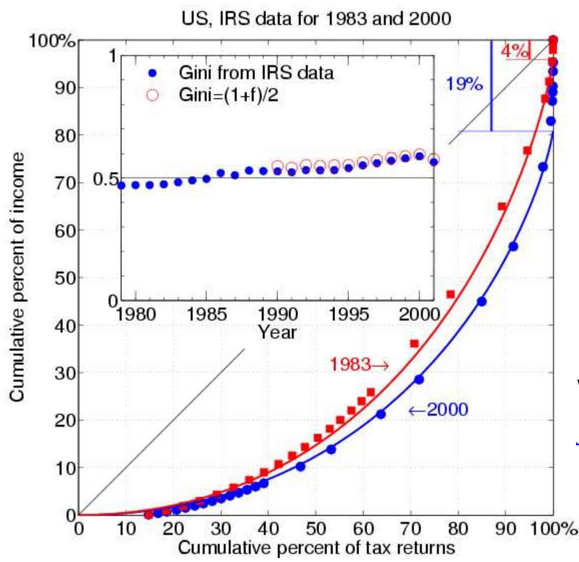
For a stationary distribution, $\partial_t P = 0$ and $\frac{\partial}{\partial r}(BP) = -AP.$

For the lower class, Δr are independent of r – additive diffusion, so A and B are constants. Then, $P(r) \propto \exp(-r/T)$, where $T = B/A$, – an exponential distribution.

For the upper class, $\Delta r \propto r$ – multiplicative diffusion, so $A = ar$ and $B = br^2$. Then, $P(r) \propto 1/r^{\alpha+1}$, where $\alpha = 1+a/b$, – a power-law distribution.

For the upper class, income does change in percentages, as shown by Fujiwara, Souma, Aoyama, Kaizoji, and Aoki (2003) for the tax data in Japan. For the lower class, the data is not known yet.

Lorenz curves and income inequality



Lorenz curve ($0<r<\infty$):

$$x(r) = \int_0^r P(r')\,dr'$$

$$y(r) = \int_0^r r'P(r')\,dr' \Big/ \langle r' \rangle$$

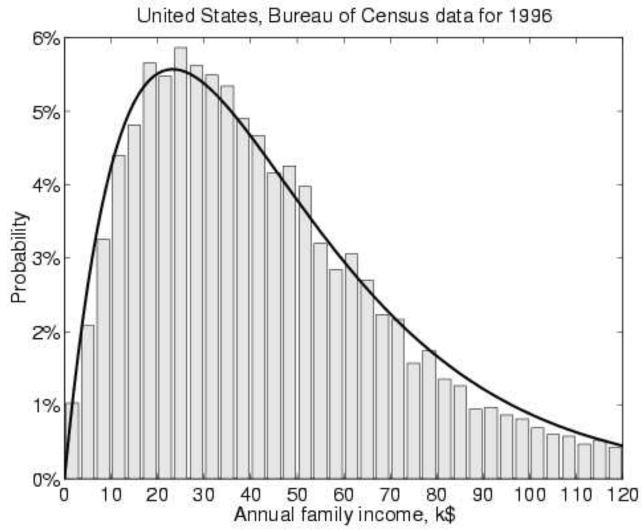
A measure of inequality, Gini coefficient is G = Area(diagonal line - Lorenz curve) / Area(Triangle beneath diagonal)

For exponential distribution with a tail, the Lorenz curve is

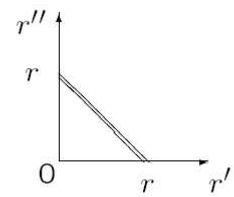
$$y = (1-f)[x + (1-x)\ln(1-x)] + f\delta(1-x),$$

where f is the tail income, and Gini coefficient is $G=(1+f)/2$.

Income distribution for two-earner families

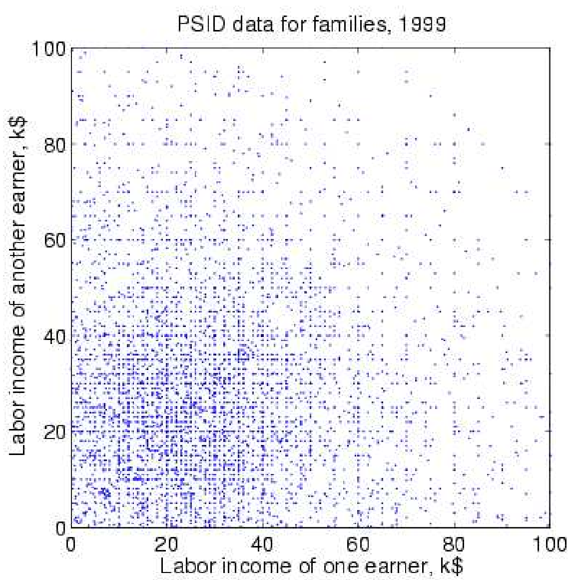


$$r = r' + r'', \quad P_2(r) = \int_0^r P_1(r') P_1(r - r') dr' \propto r \exp(-r/T)$$



The average family income is $2T$. The most probable family income is T.

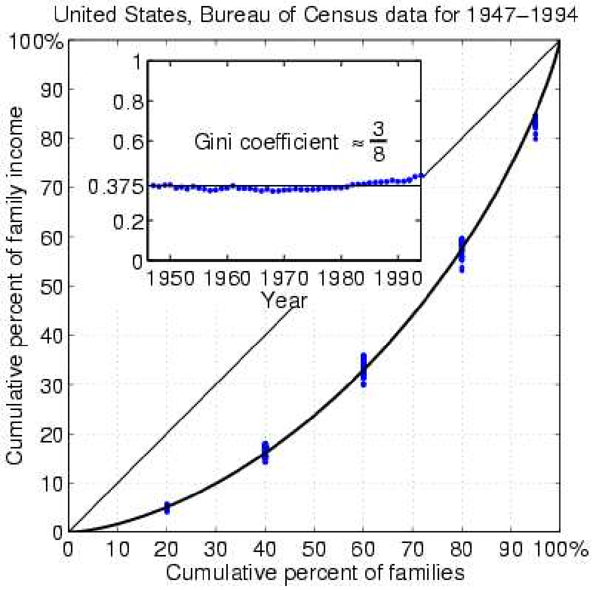
No correlation in the incomes of spouses



Every family is represented by two points (r_1, r_2) and (r_2, r_1).

The absence of significant clustering of points (along the diagonal) indicates that the incomes r_1 and r_2 are approximately uncorrelated.

Lorenz curve and Gini coefficient for families



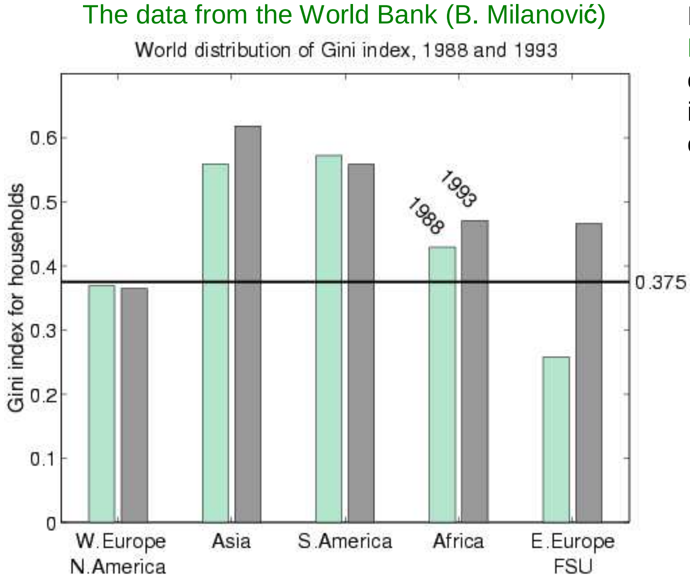
Lorenz curve is calculated for families $P_2(r) \propto r \exp(-r/T)$. The calculated Gini coefficient for families is $G=3/8=37.5\%$

No significant changes in Gini and Lorenz for the last 50 years. The exponential ("thermal") Boltzmann-Gibbs distribution is very stable, since it maximizes entropy.

Maximum entropy (the 2nd law of thermodynamics) ⇒ **equilibrium inequality:**
G=1/2 for individuals,
G=3/8 for families.

World distribution of Gini coefficient

The data from the World Bank (B. Milanović)

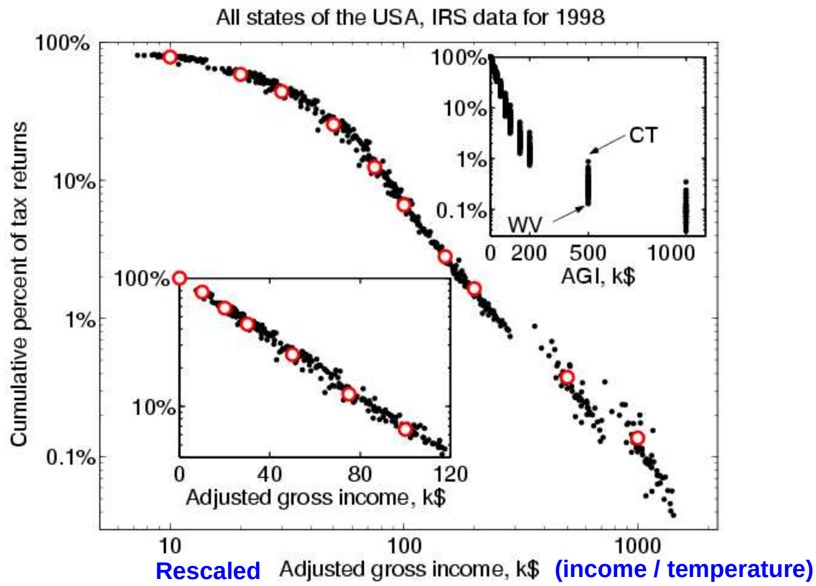


In W. Europe and N. America, G is close to 3/8=37.5%, in agreement with our theory.

Other regions have higher G, i.e. higher inequality.

A sharp increase of G is observed in E. Europe and former Soviet Union (FSU) after the collapse of communism – no equilibrium yet.

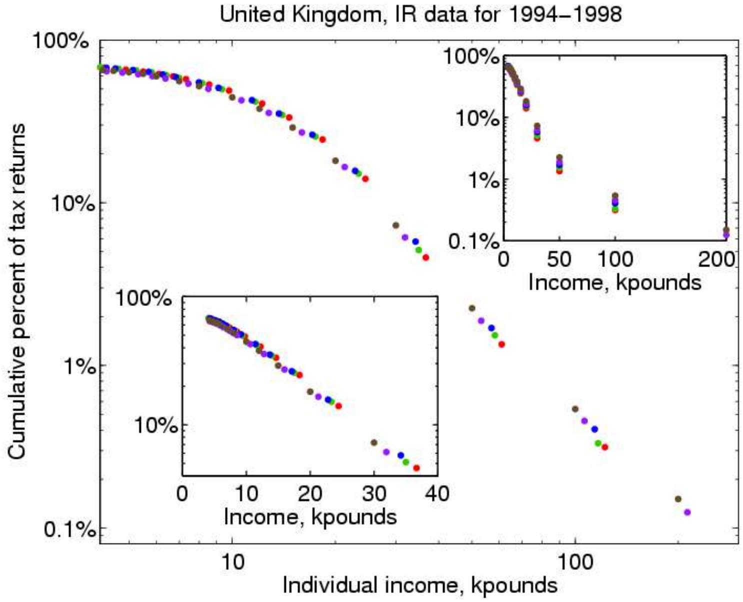
Income distribution in the states of USA, 1998



Deviations of the state income temperatures from the average US temperature

CT	NJ	MA	MD	VA	CA	NY	IL	CO
25%	24%	14%	14%	9%	9%	7%	6%	6%
NH	AK	DC	DE	MI	WA	MN	GA	
5%	5%	5%	4%	4%	2%	1%	0%	
TX	RI	AZ	PA	FL	KS	OR	HI	NV
-1%	-3%	-3%	-3%	-4%	-5%	-6%	-7%	-7%
NC	WI	IN	UT	MO	VT	TN	NE	
-7%	-8%	-8%	-9%	-9%	-9%	-11%	-12%	
OH	LA	AL	SC	IA	WY	NM	KY	ID
-12%	-13%	-13%	-13%	-14%	-14%	-14%	-14%	-15%
OK	ME	MT	AR	SD	ND	MS	WV	
-16%	-16%	-19%	-19%	-20%	-20%	-21%	-22%	

Income distribution in the United Kingdom



For UK in 1998, T = 12 k£ = 20 k$

Pareto index $\alpha = 2.1$

For USA in 1998, T = 36 k$

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Thermal machine in the world economy

In general, different countries have different temperatures T, which makes possible to construct a thermal machine:

Low T_1, developing countries	← Money (energy) ← $T_1 < T_2$ → Products →	High T_2, developed countries

Prices are commensurate with the income temperature T (the average income) in a country.

Products can be manufactured in a low-temperature country at a low price T_1 and sold to a high-temperature country at a high price T_2.

The temperature difference $T_2 – T_1$ is the profit of an intermediary.

Money (energy) flows from high T_2 to low T_1 (the 2nd law of thermodynamics – entropy always increases) ⇔ **Trade deficit**

In full equilibrium, $T_2 = T_1$ ⇔ No profit ⇔ "Thermal death" of economy

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Conclusions

- An analogy in conservation laws between energy in physics and money in economics results in the exponential ("thermal") Boltzmann-Gibbs probability distribution of money and income $P(r) \propto \exp(-r/T)$ for individuals and $P(r) \propto r \exp(-r/T)$ for two-earner families.
- The tax and census data reveal a two-class structure of the income distribution in the USA: the exponential ("thermal") law for the great majority (97-99%) of population and the Pareto ("superthermal") power law for the top 1-3% of population.
- The exponential part of the distribution is very stable and does not change in time, except for slow increase of temperature T (the average income). The Pareto tail is not universal and was increasing significantly for the last 20 years with the stock market, until its crash in 2000.
- Stability of the exponential distribution is the consequence of entropy maximization. This results in the concept of **equilibrium inequality** in society: the Gini coefficient G=1/2 for individuals and G=3/8 for families. These numbers agree well with the data for developed capitalist countries.

Money, Wealth, and Income

Wealth = Money + Property (Material Wealth)

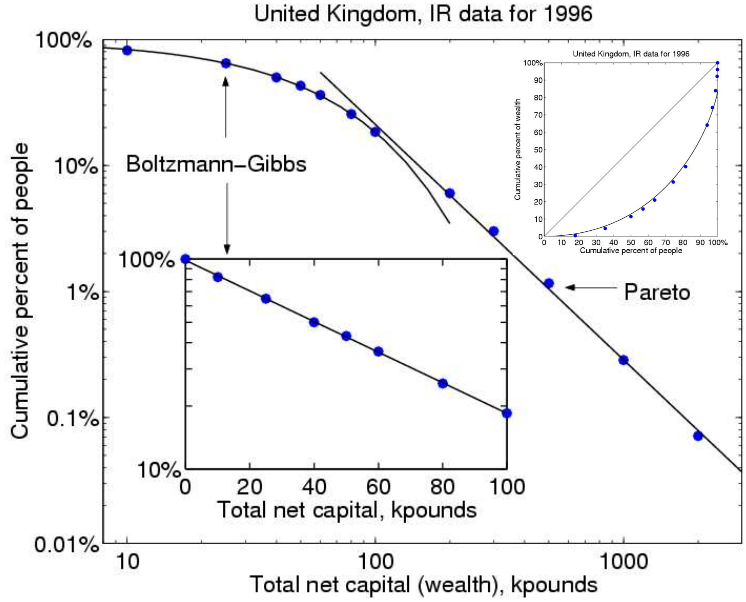
Material Wealth = Price x Goods

Money is conserved

Material Wealth is not conserved.

d(Money) / dt = Income - Spending

Wealth distribution in the United Kingdom



For UK in 1996, T = 60 k£

Pareto index $\alpha = 1.9$

Fraction of wealth in the tail f = 16%

Boltzmann-Gibbs versus Pareto distribution

Ludwig Boltzmann (1844-1906)

Vilfredo Pareto (1848-1923)

Boltzmann-Gibbs probability distribution $P(\varepsilon) \propto \exp(-\varepsilon/T)$, where ε is energy, and $T=\langle\varepsilon\rangle$ is temperature.

Pareto probability distribution $P(r) \propto 1/r^{(\alpha+1)}$ of income r.

Analogy: energy $\varepsilon \leftrightarrow$ money $m \Rightarrow P(m) \propto \exp(-m/T)$