

Contrasting electronic nematicity in rigid lattices and moiré superlattices

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U.S. DEPARTMENT OF
ENERGY

Office of
Science

KITP, Oct 20th, 2020

Collaborators

Jörn Venderbos, Draxel University

Venderbos and RMF,
Phys. Rev. B **98**, 245103 (2018)

RMF and Venderbos,
Science Adv. **6**, eaba8834 (2020)

Rhine Samajdar, Harvard

Mathias Scheurer, Harvard

Samajdar, Scheurer, Venderbos,
and RMF, *in preparation*

Experimentalists

Pablo Jarillo-Herrero, MIT

Yuan Cao, MIT

Cao, ..., RMF, Fu, and Jarillo-Herrero,
arxiv:2004.04148

Abhay Pasupathy, Columbia

Carmen Verdú-Rubio, Columbia

Simon Turkel, Columbia

Verdú-Rubio, ..., RMF, Rubio, and
Pasupathy, arxiv:2009.11645

Outline

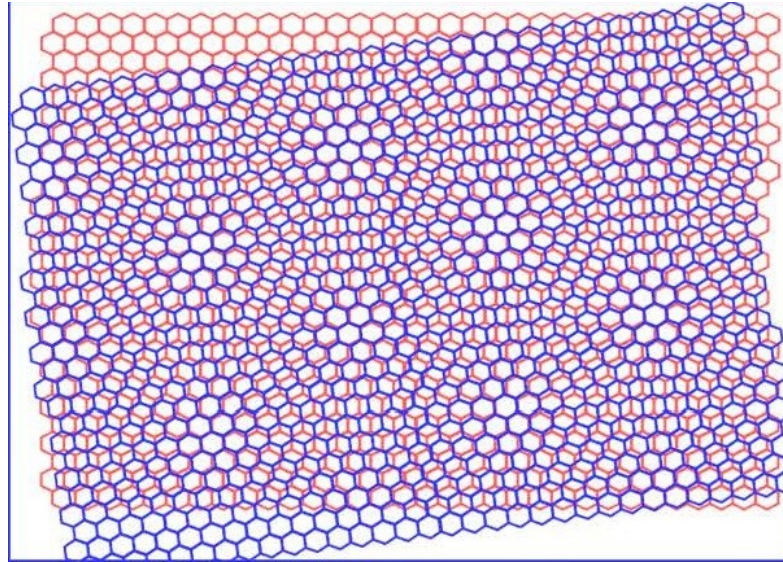
1. Brief overview of twisted moiré systems
2. Potts-nematicity in moiré superlattices: static strain
3. Potts-nematicity in moiré superlattices: fluctuating strain
4. Electric control of the nematic director

Outline

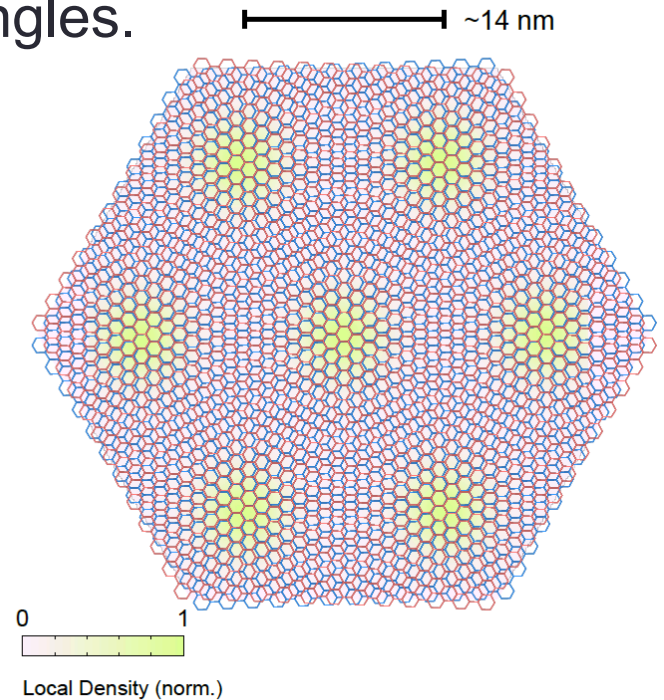
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Twisted bilayer graphene (TBG): moiré superlattice

- Two graphene layers twisted by small angles.



Eva Andrei website

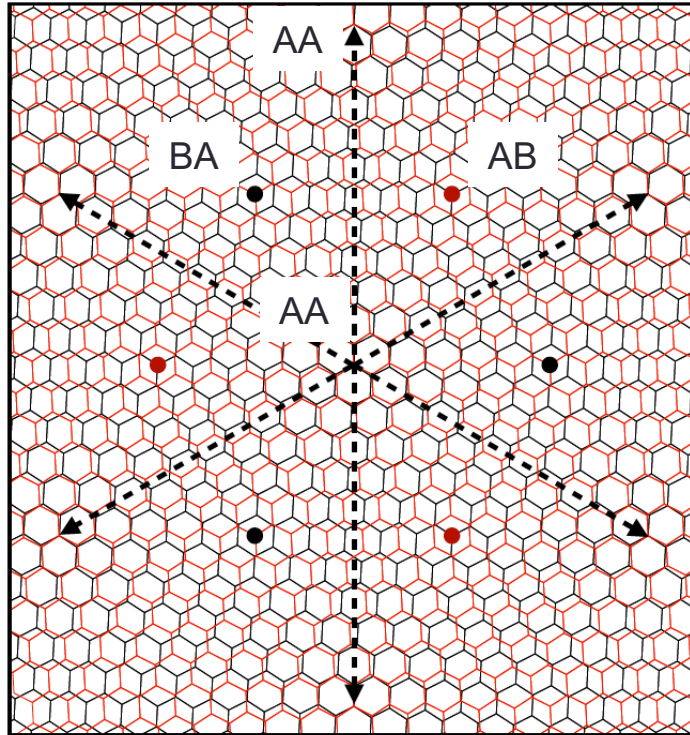
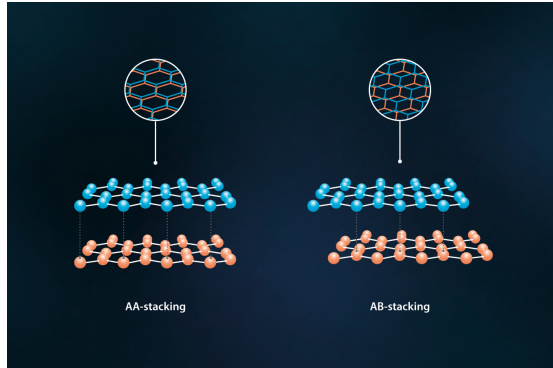


Emergent moiré lattice:
huge distance between lattice sites

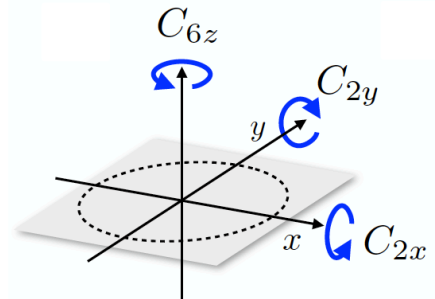
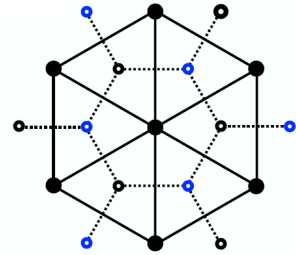
Twisted bilayer graphene (TBG): symmetries

- Emergence of a triangular moiré superlattice

Phys.org



- symmetries:
 C_{6z} , C_{2x} , C_{2y}



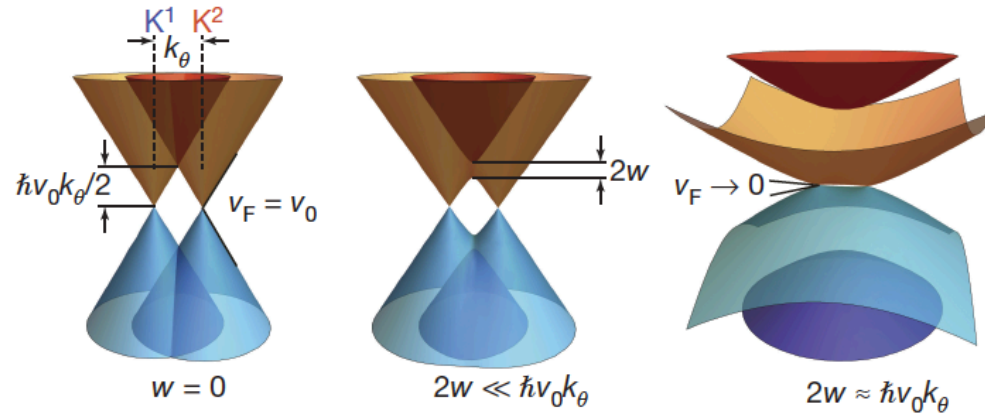
- eight electrons per moiré unit cell

Twisted bilayer graphene (TBG): “flat” bands

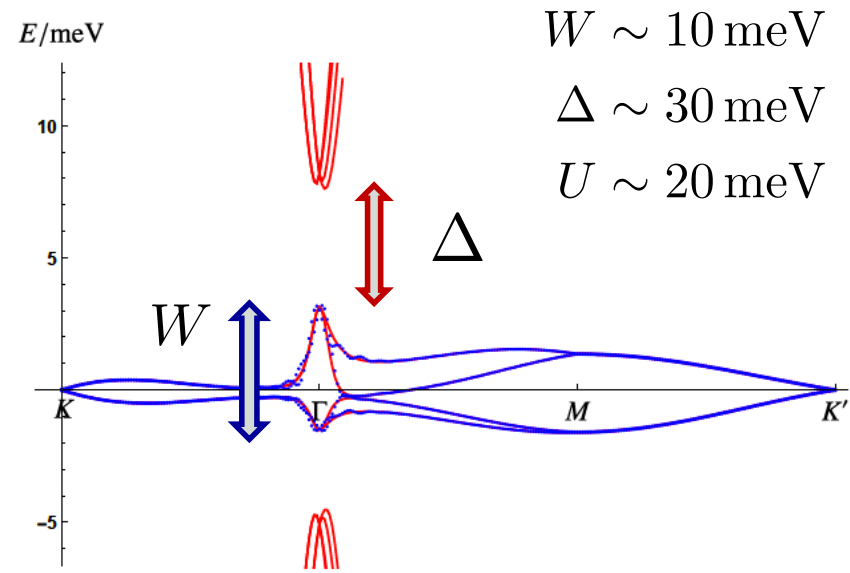
- Nearly-flat bands at “magic angle”

Lopes dos Santos et al, PRL (2007)

Bistritzer and MacDonald, PNAS (2011)



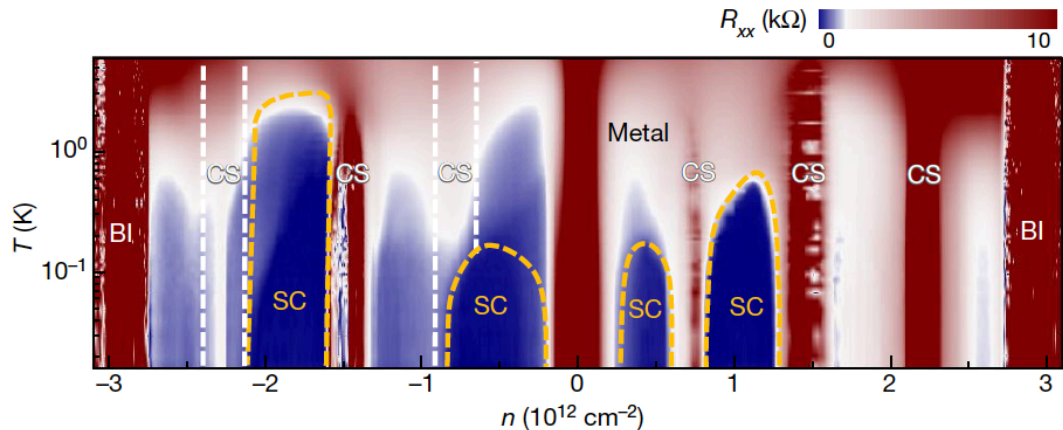
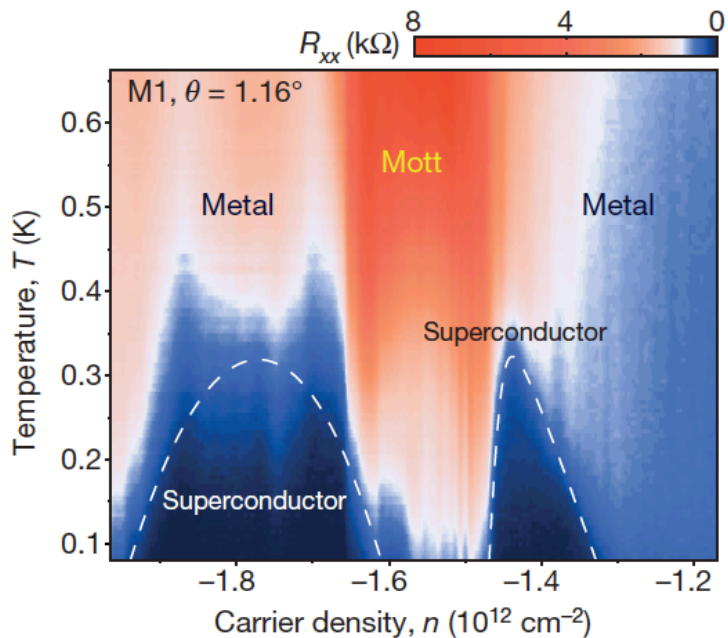
Cao et al, Nature (2018)



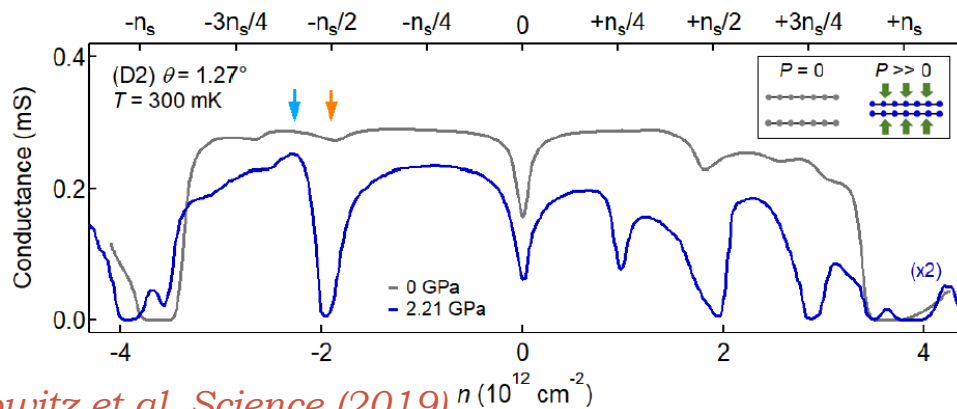
Kang & Vafek, PRX (2018)

- Interactions give rise to a rich phase diagram and superconductivity.

Cao, ..., Jarillo-Herrero, Nature (2018)



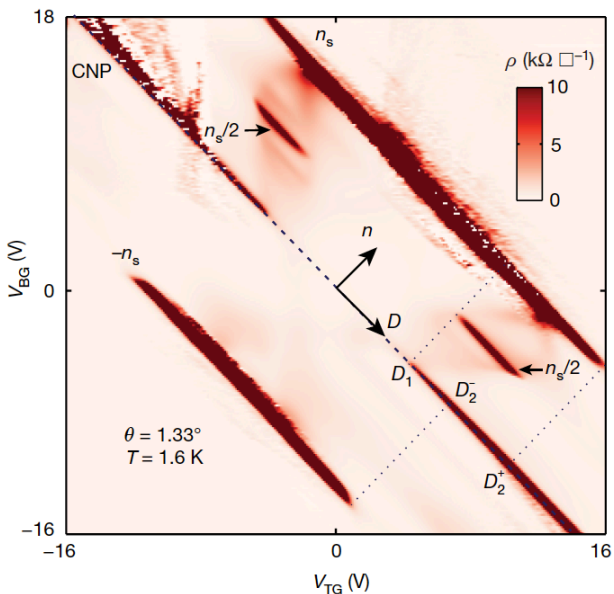
Lu et al, Nature (2019)



Yankowitz et al, Science (2019) n (10^{12} cm^{-2})

TBG and beyond: twisted moiré systems

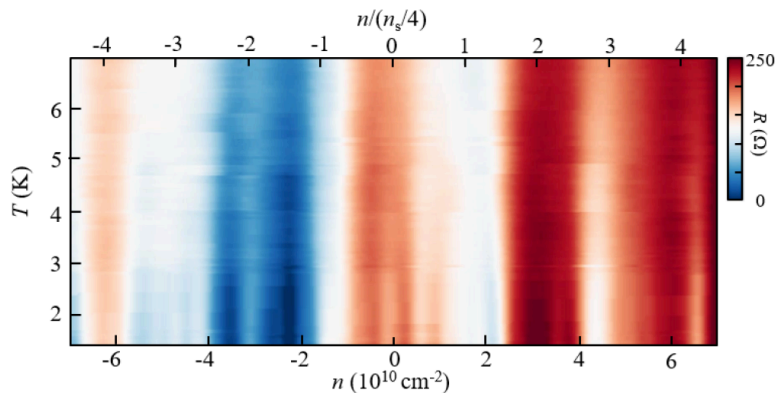
➤ twisted double bilayer graphene (TDBG)



Liu et al, Nature (2020)

Cao et al, Nature (2020)

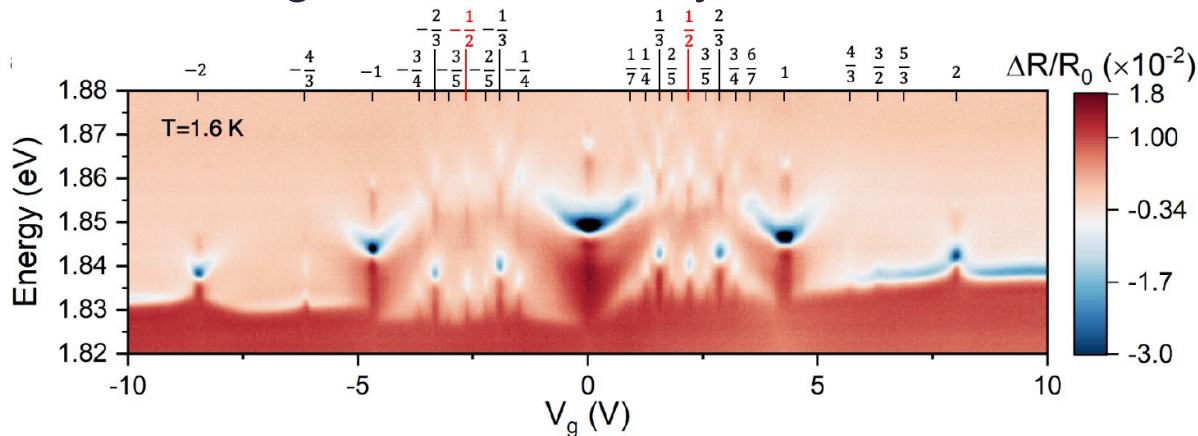
Chen et al, Nature Phys (2020)



➤ twisted trilayer graphene

Tsai et al, arxiv (2020)

➤ dichalcogenide heterobilayers



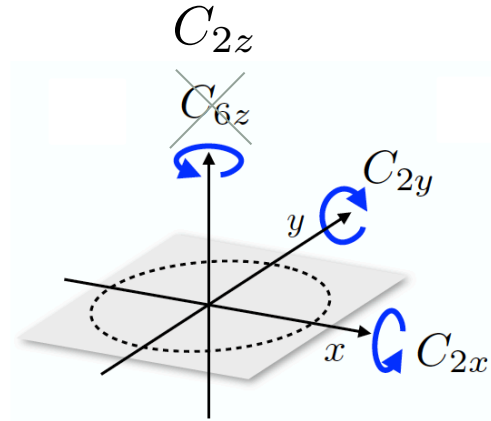
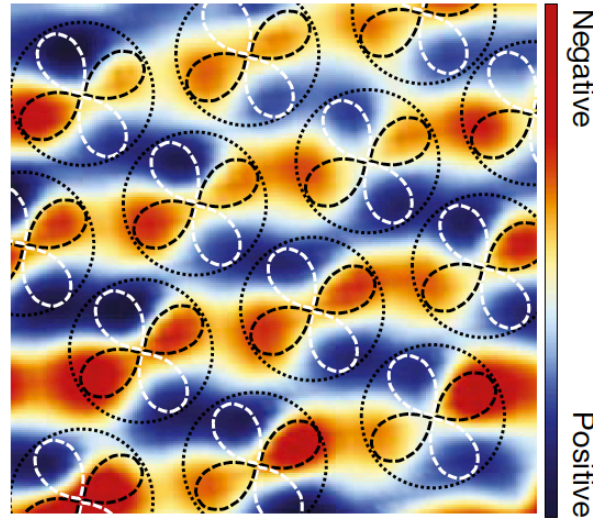
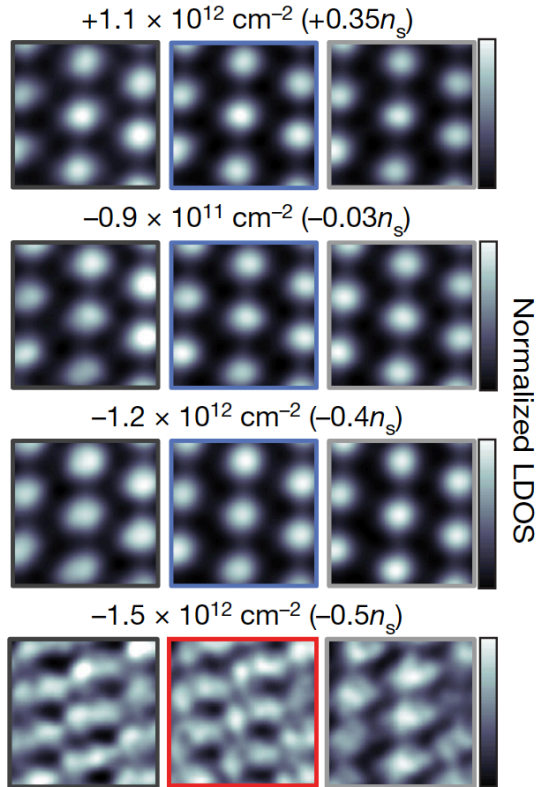
Xu et al, arxiv (2020); Regan et al, Nature Phys (2020)

Twisted moiré systems: common properties

- Are there “universal” features of the phase diagrams?
 - Correlated insulating phases? **Yes.**
 - Superconductivity? **Maybe.**
 - Symmetry-breaking phases? **Maybe.**

Nematicity in twisted bilayer graphene

- Nematic order = breaking of 3-fold rotational symmetry. STM data:



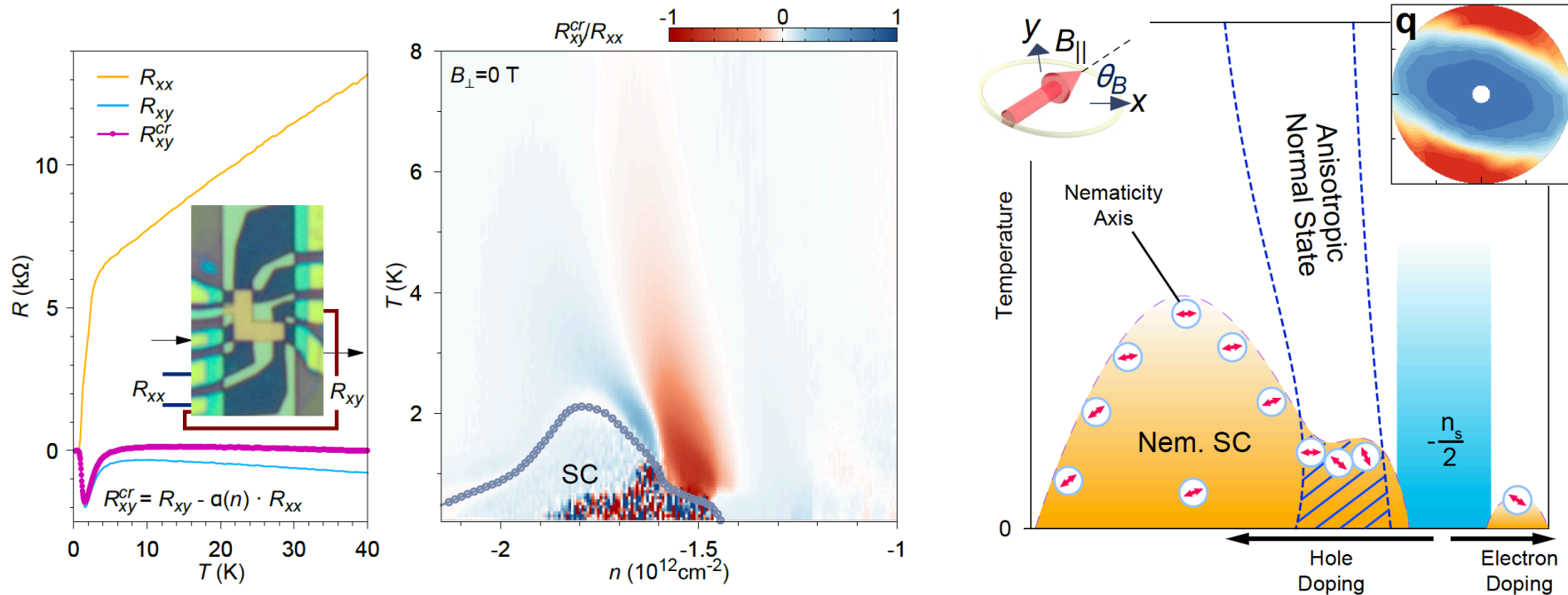
Jian et al, Nature (2019)

Kerelsky et al, Nature (2019)

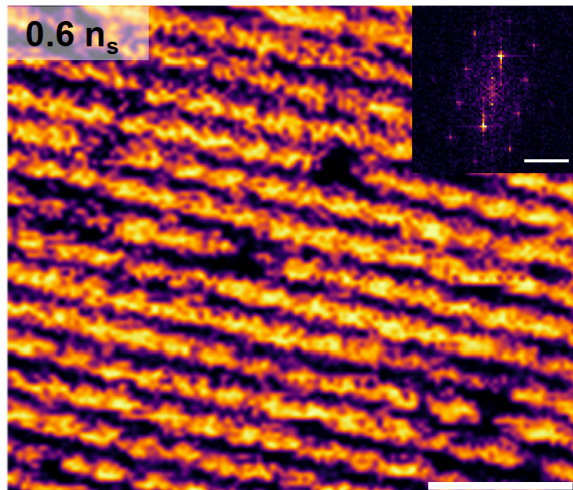
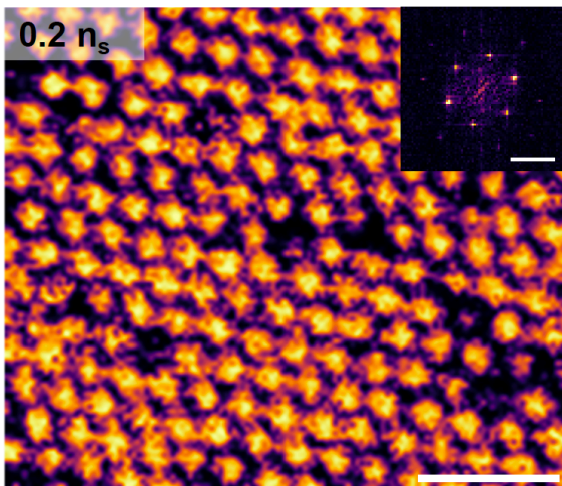
Choi et al, Nature Phys (2019)

Nematicity in twisted bilayer graphene

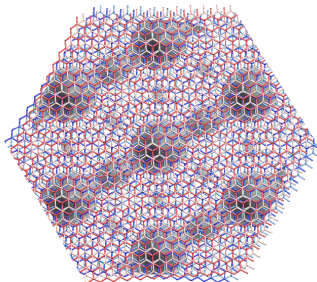
- Nematic order = breaking of 3-fold rotational symmetry. Signatures in transport measurements in the normal and superconducting states.



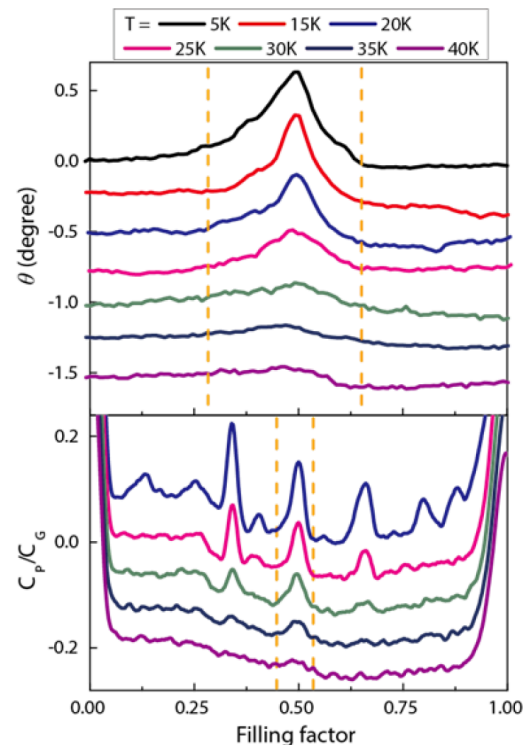
Nematicity in other twisted moiré systems



➤ twisted *double* bilayer graphene



Rubio-Verdú, ..., RMF, Rubio, Pasupathy arxiv (2020)



➤ WSe_2/WS_2 heterobilayer

Jin et al, arxiv (2020)

Nematicity in twisted moiré systems

- General low-energy model to study electronic nematicity in generic moiré superlattices.
- Take-home message: nematicity in moiré superlattices is fundamentally different than in rigid lattices. New effects emerge and unexpected tuning knobs can be used.

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Electronic nematicity: phenomenology

- Electronic nematicity: $\hat{Q}_{ij} = \psi^\dagger(\mathbf{r}) (2\partial_i\partial_j - \delta_{ij}\nabla^2) \psi(\mathbf{r})$

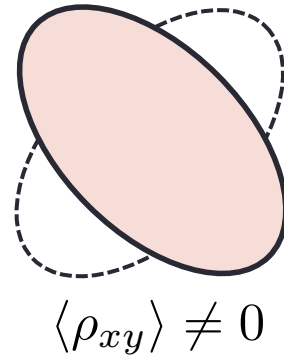
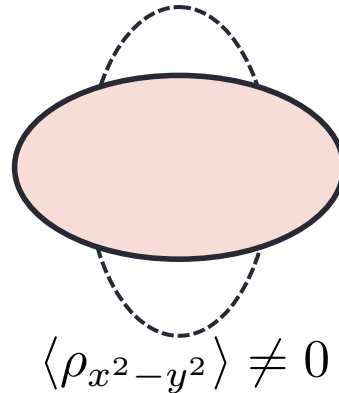
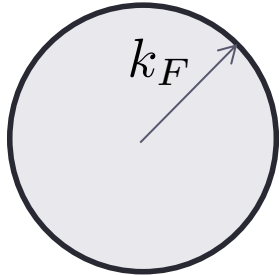
Kivelson, Fradkin, and Emery, Nature (1998)

Electronic nematicity: phenomenology

- Electronic nematicity: $\hat{Q}_{ij} = \psi^\dagger(\mathbf{r}) (2\partial_i\partial_j - \delta_{ij}\nabla^2) \psi(\mathbf{r})$

➤ order parameter can be expressed in terms of quadrupolar charge densities:

$$\langle \hat{Q} \rangle = \begin{pmatrix} \rho_{x^2-y^2} & \rho_{xy} \\ \rho_{xy} & -\rho_{x^2-y^2} \end{pmatrix} \begin{cases} \rho_{x^2-y^2} \equiv \langle (k_x^2 - k_y^2) \hat{\psi}^\dagger(\mathbf{k}) \hat{\psi}(\mathbf{k}) \rangle \\ \rho_{xy} \equiv \langle (2k_x k_y) \hat{\psi}^\dagger(\mathbf{k}) \hat{\psi}(\mathbf{k}) \rangle \end{cases}$$



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➤ XY-nematic order parameter Φ

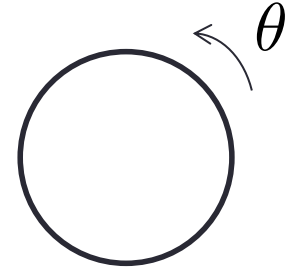
$$\langle \hat{Q} \rangle = \rho_{x^2-y^2} \sigma^z + \rho_{xy} \sigma^x \quad \Longrightarrow \quad \Phi = \begin{pmatrix} \rho_{x^2-y^2} \\ \rho_{xy} \end{pmatrix}$$

Electronic nematicity: phenomenology

- Electronic nematic free-energy: XY nematics

$$\Phi = \Phi \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix}$$

$$F_0 = \frac{a}{2}\Phi^2 + \frac{u}{4}\Phi^4$$



director space

- XY nematics has unique properties: Goldstone mode couples directly (i.e. not via the gradient) to the electronic density, promoting non-Fermi liquid behavior.

Oganesyan, Kivelson, and Fradkin, PRB (2001)

Watanabe and Vishwanath, PNAS (2014)

- Underlying crystal: introduces nematic-anisotropy terms.

$$F_{\text{nem}} = F_0 + F_{\text{cr}}$$

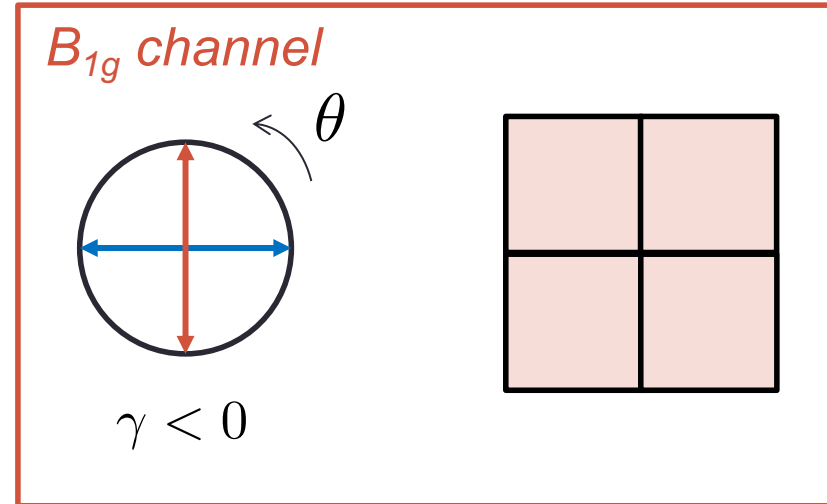
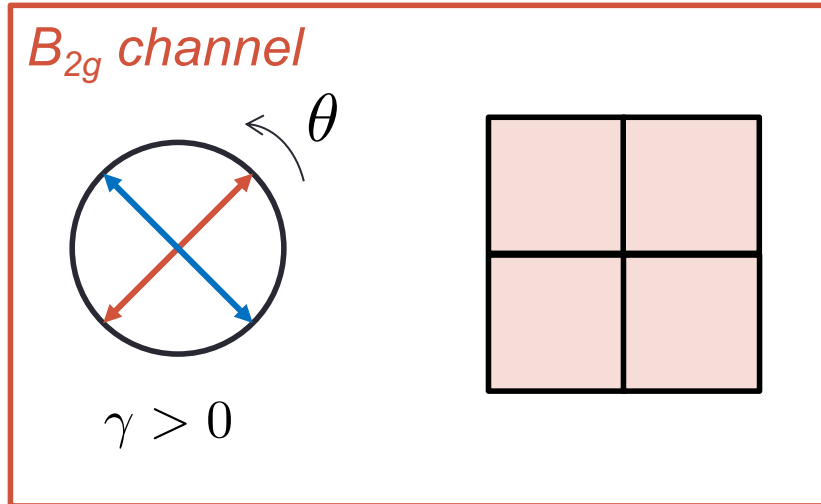
Electronic nematicity: impact of the lattice

- Square lattice: **Ising-nematicity** (cuprates and pnictides)

Fradkin et al, Ann. Rev. Cond. Matter Phys (2010)

RMF, Chubukov, Schmalian, Nature Phys (2014)

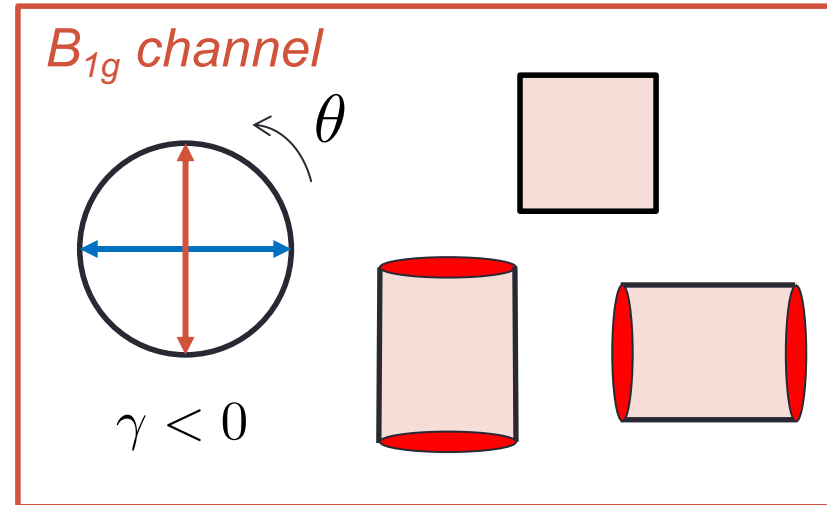
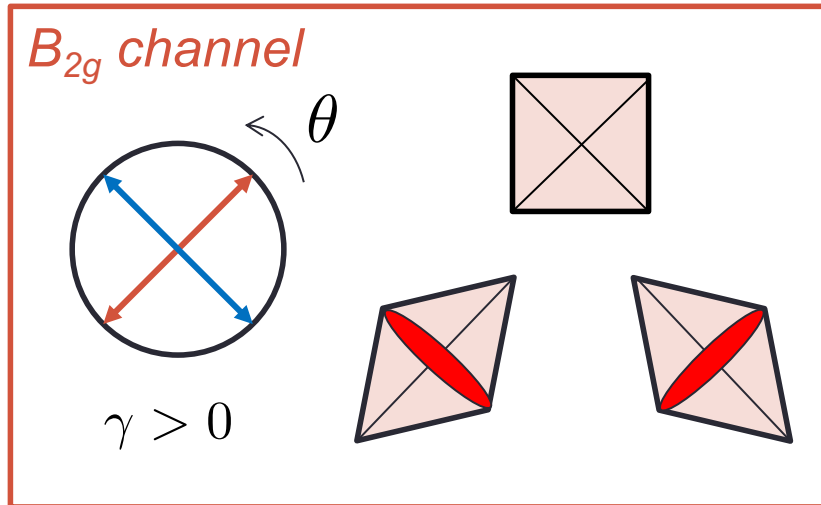
$$F_{\text{cr}} = \gamma (\Phi_1^2 - \Phi_2^2) = \gamma \Phi^2 \cos 4\theta$$



Electronic nematicity: impact of the lattice

- Square lattice: **Ising-nematicity** (cuprates and pnictides)
- Nematic order *always* triggers a structural distortion.

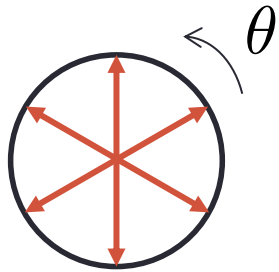
$$F_{\text{cr}} = \gamma (\Phi_1^2 - \Phi_2^2) = \gamma \Phi^2 \cos 4\theta$$



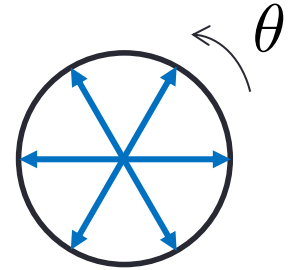
Electronic nematicity: impact of the lattice

- Triangular lattice: the two components of Φ transform as the same irreducible representation (E_{2g}). Cubic term is allowed:

$$F_{\text{cr}} = \frac{\gamma}{3} \Phi_1 (\Phi_1^2 - 3\Phi_2^2) = \frac{\gamma}{3} \Phi^3 \cos 6\theta$$



$$\gamma > 0$$



$$\gamma < 0$$

Hecker & Schmalian, npj QM (2018)

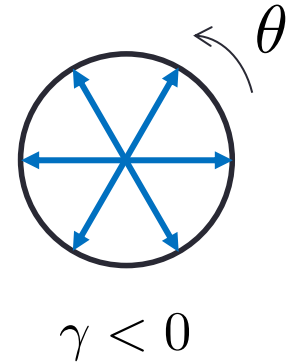
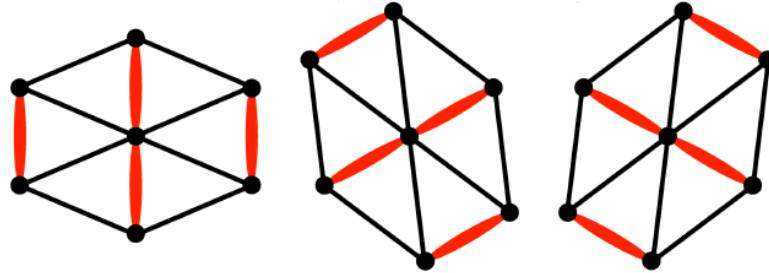
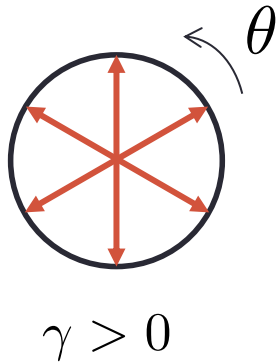
Venderbos & RMF, PRB (2018)

Little et al, Nature Materials (2020)

Electronic nematicity: impact of the lattice

- Triangular lattice: **3-state Potts nematicity** (twisted bilayer graphene, Bi_2Se_3 , $\text{Fe}_{1/3}\text{NbS}_2$)
- Nematic order *always* triggers a structural distortion.

$$F_{\text{cr}} = \frac{\gamma}{3} \Phi_1 (\Phi_1^2 - 3\Phi_2^2) = \frac{\gamma}{3} \Phi^3 \cos 6\theta$$



Hecker & Schmalian, npj QM (2018)

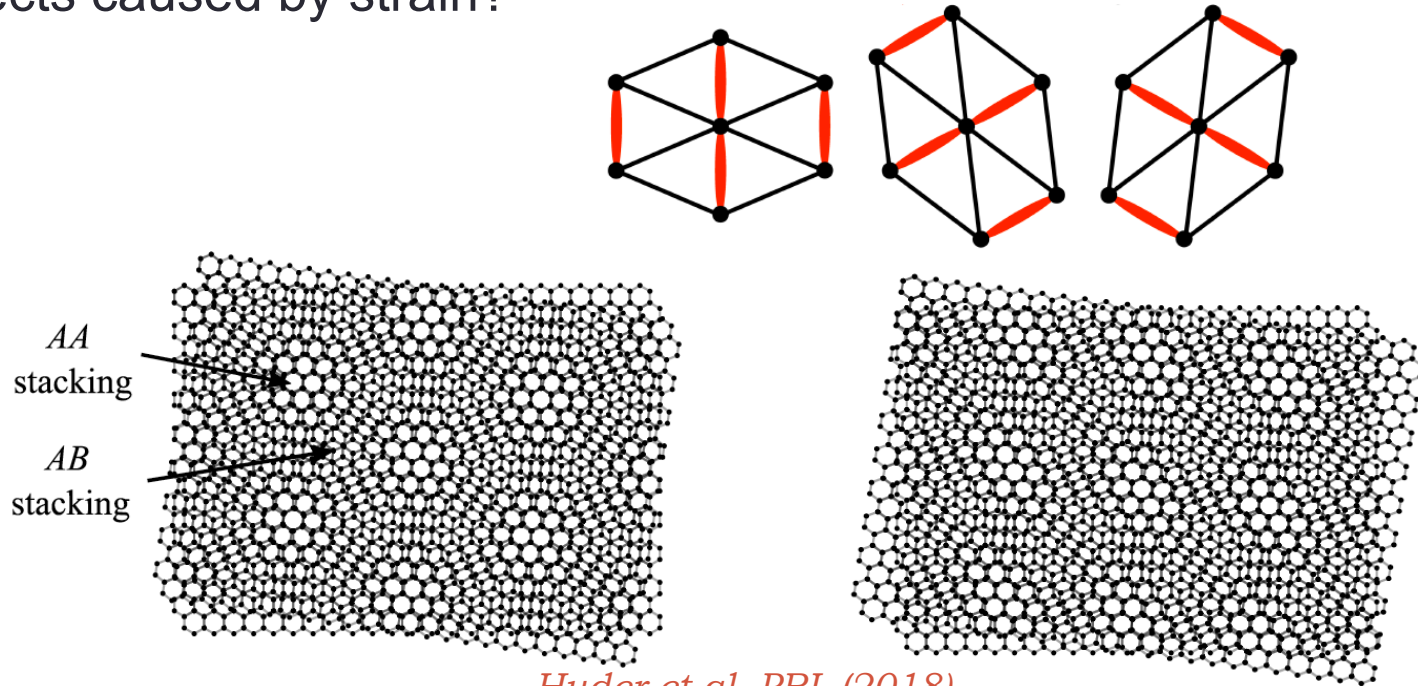
Venderbos & RMF, PRB (2018)

Little et al, Nature Materials (2020)

*twofold rotational
symmetries are preserved*

Nematicity in TBG: static strain

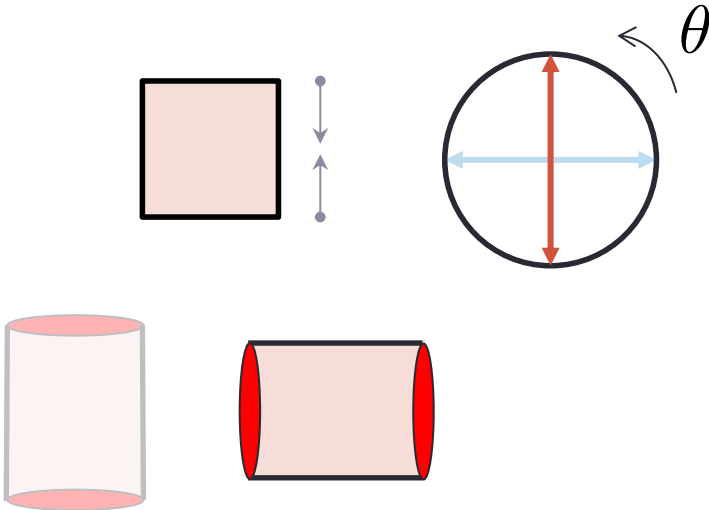
- Build-in strain is unavoidable in TBG. Strain induces structural distortions. How to distinguish effects caused by nematic order from effects caused by strain?



Nematicity in TBG: static strain

- Build-in strain is unavoidable in TBG. Can the nematic transition survive?

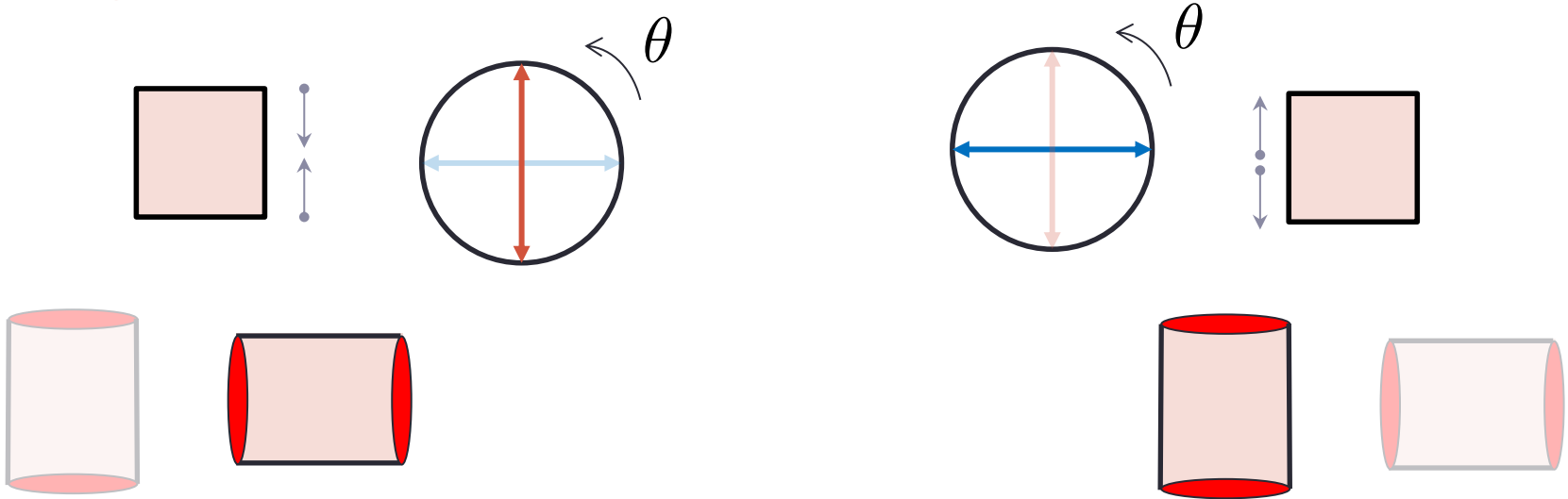
tetragonal lattice:



Nematicity in TBG: static strain

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tetragonal lattice:

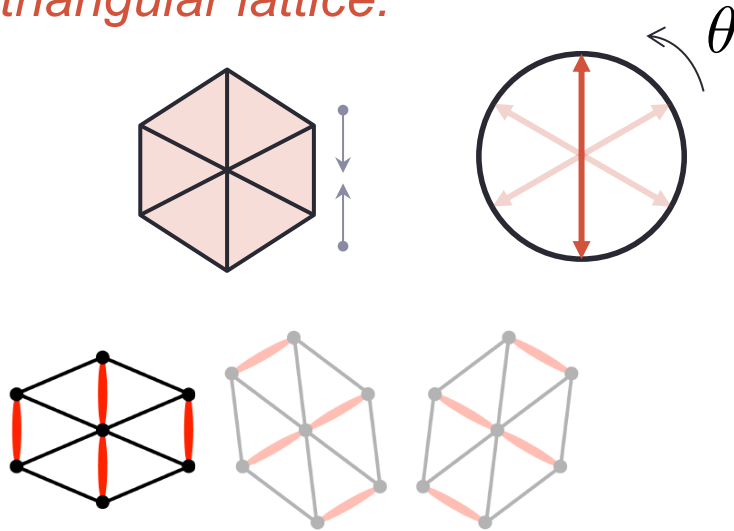


no Ising-nematic transition in the presence of strain

Nematicity in TBG: static strain

- Build-in strain is unavoidable in TBG. Can the nematic transition survive?

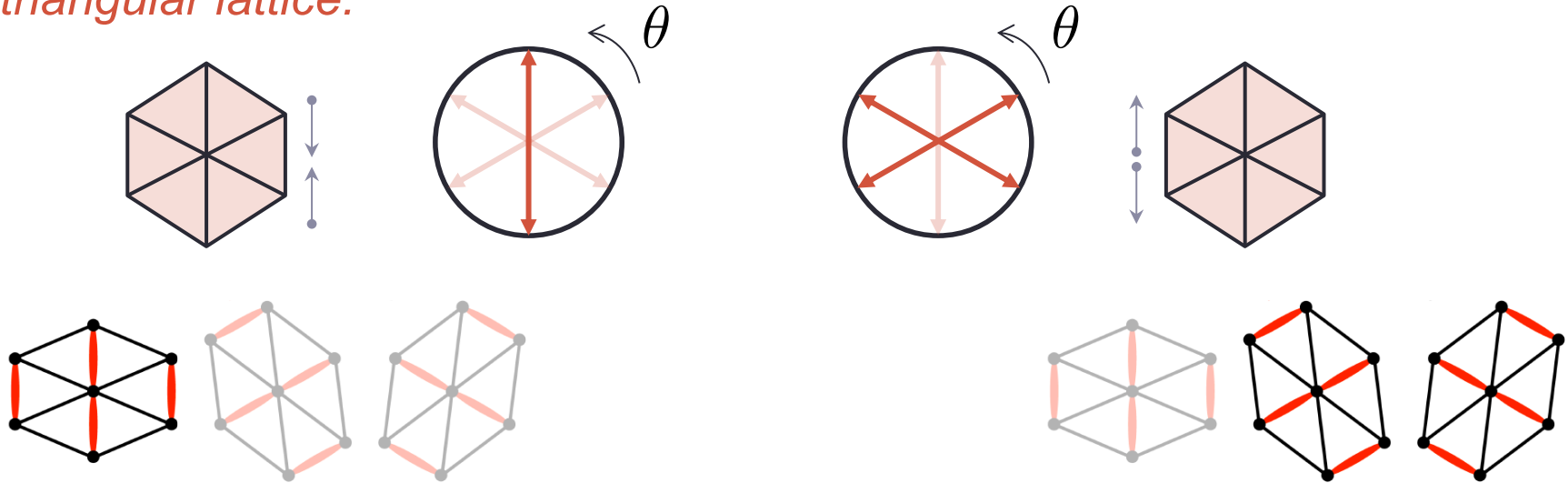
triangular lattice:



Nematicity in TBG: static strain

- Build-in strain is unavoidable in TBG. Can the nematic transition survive?

triangular lattice:



residual Ising symmetry related to in-plane rotations

Nematicity in TBG: static strain

- Nemato-elastic coupling: formalism $\Phi_{\pm} = \Phi_1 \pm i\Phi_2$

$$S_{\text{nem}}[\Phi] = S_0[\Phi] + \frac{\gamma}{6} \int_x (\Phi_+^3 + \Phi_-^3)$$

$$\varepsilon_{ij} = \frac{1}{2} (\partial_i u_j + \partial_j u_i) : \text{static strain}$$

\mathbf{u} : relative displacement
between the two layers

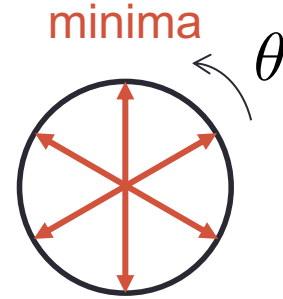
$$S'[\Phi, \hat{\varepsilon}] = -\lambda \int_x [(\varepsilon_{xx} - \varepsilon_{yy}) \Phi_1 + 2\varepsilon_{xy} \Phi_2]$$

uniaxial strain: frustration of the nematic director

Nematicity in TBG: static strain

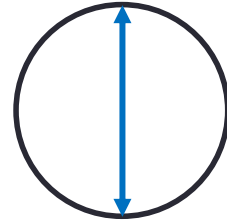
- Example: strain along y axis with $\lambda < 0$ and $\gamma > 0$.

$$S_{\text{nem}}[\Phi] = S_0[\Phi] + \underbrace{\frac{\gamma}{6} \int_x (\Phi_+^3 + \Phi_-^3)}_{+\Phi^3 \cos 6\theta}$$



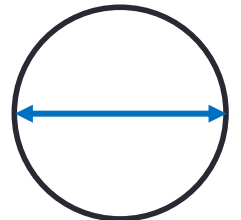
$$S'[\Phi, \hat{\varepsilon}] = \underbrace{-\lambda \int_x [(\varepsilon_{xx} - \varepsilon_{yy}) \Phi_1 + 2\varepsilon_{xy} \Phi_2]}_{-\varepsilon \Phi \cos 2\theta}$$

minimum
(compressive)



maximum
(tensile)

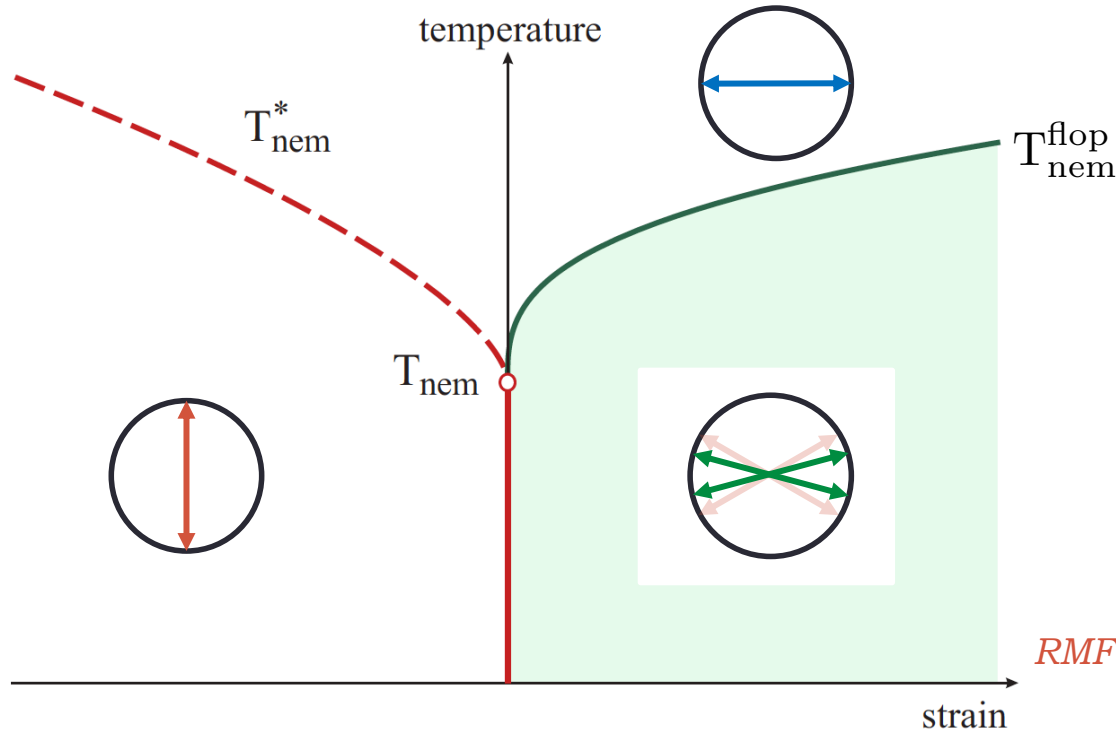
minimum
(tensile)



maximum
(compressive)

Nematicity in TBG: static strain

- Ising-like *nematic-flop* transition in the presence of uniaxial strain

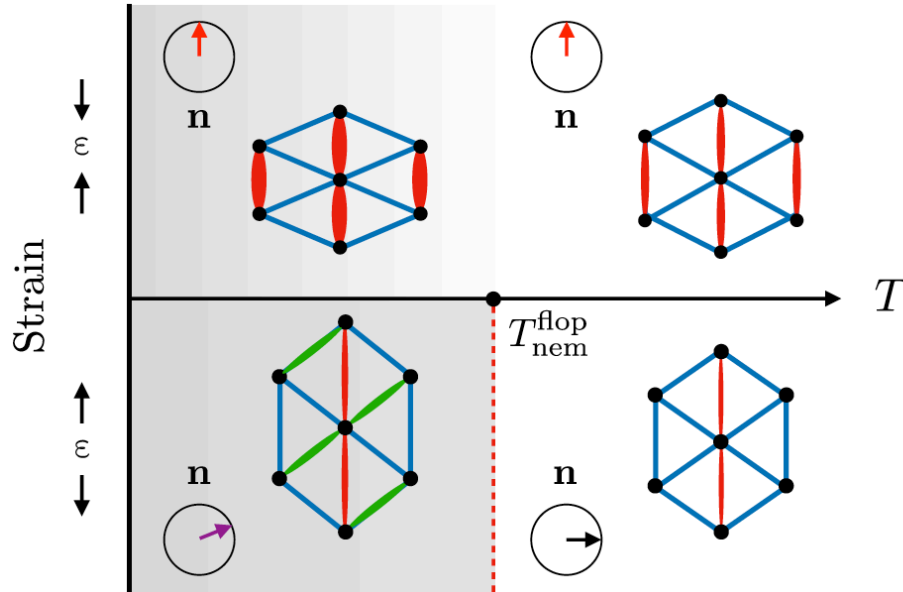


RMF & Venderbos, Sci. Adv. (2020)

can be used to establish long-range nematic order experimentally

Nematicity in TBG: static strain

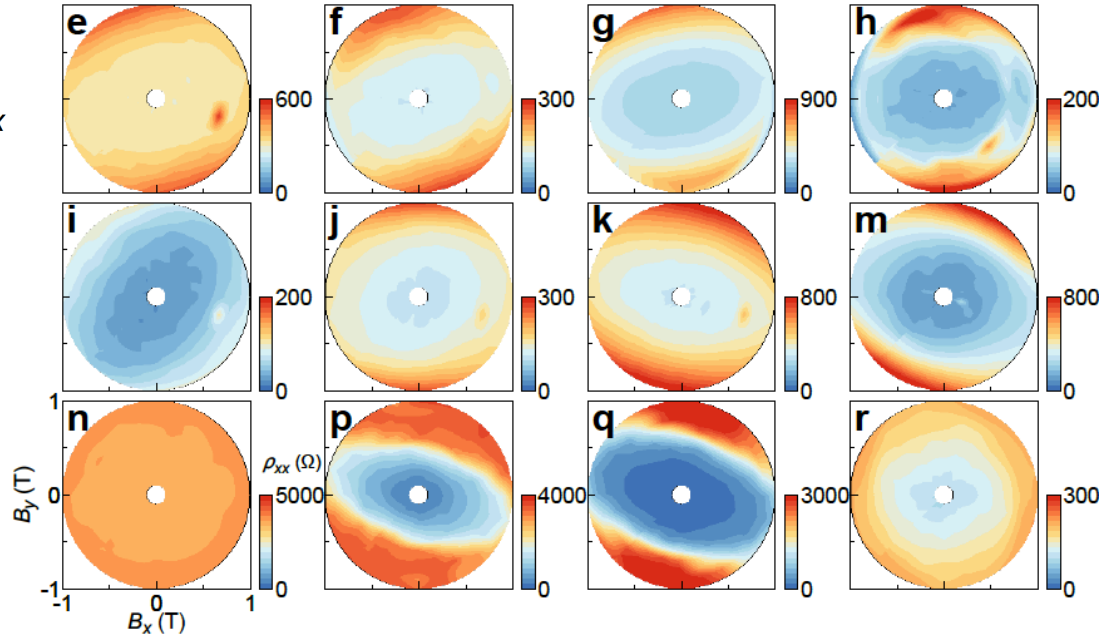
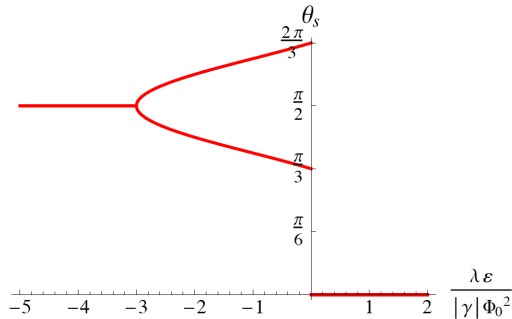
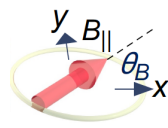
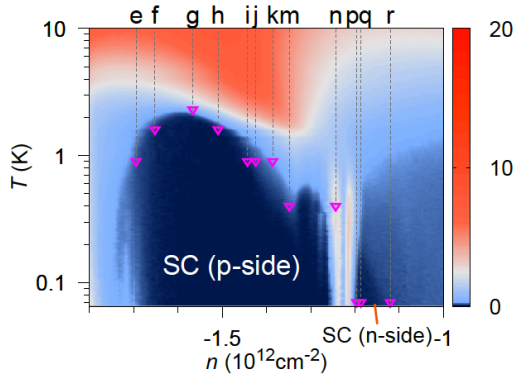
- Ising-like *nematic-flop* transition in the presence of static strain



bond-order pattern can establish long-range nematic order experimentally

Nematicity in the superconducting state of TBG

- Nematic director rotates as a function of doping inside the superconducting state: evidence for spontaneous nematic order.



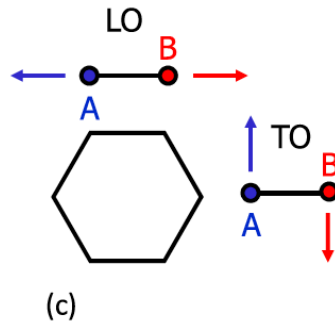
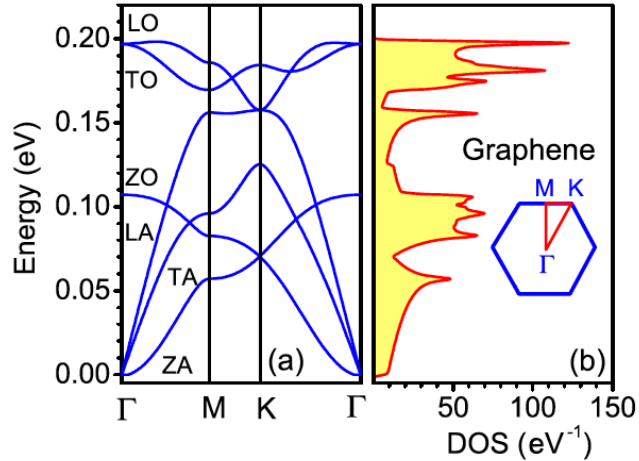
Cao, ..., RMF, Fu, and Jarillo-Herrero, arxiv (2020)

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Nematicity in TBG: fluctuating strain

- Finite-momentum strain fluctuations: acoustic phonons



$$S_{\text{el}} = \frac{1}{2} \sum_{\mu=L,T} \int_q \tilde{u}_{q,\mu} (\omega_n^2 + v_\mu^2 \mathbf{q}^2) \tilde{u}_{-q,\mu}$$

$$\hat{\mathbf{e}}_L = (\cos \zeta_{\mathbf{q}}, \sin \zeta_{\mathbf{q}})$$

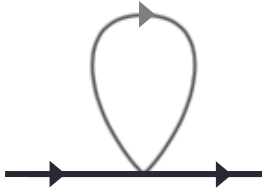
$$\hat{\mathbf{e}}_T = (-\sin \zeta_{\mathbf{q}}, \cos \zeta_{\mathbf{q}})$$

Sanders et al, J Phys: Cond Matt (2013)

triangular lattice: purely transversal and purely longitudinal modes

Nematicity in TBG: fluctuating strain

- Acoustic phonons mediate long-range anisotropic nematic interactions



$$\delta S = \int_{\mathbf{r}, \mathbf{r}'} \frac{\Phi_i(\mathbf{r}) A_{ij}(\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}') \Phi_j(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^2}$$

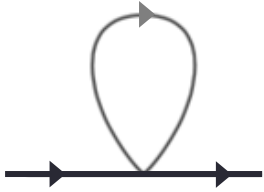
Cowley, PRB (1976)

Karahasonovic and Schmalian, PRB (2016)

Paul and Garst, PRL (2017)

Nematicity in TBG: fluctuating strain

- Acoustic phonons mediate long-range anisotropic nematic interactions



$$\delta S = \int_{\mathbf{r}, \mathbf{r}'} \frac{\Phi_i(\mathbf{r}) A_{ij}(\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}') \Phi_j(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^2}$$

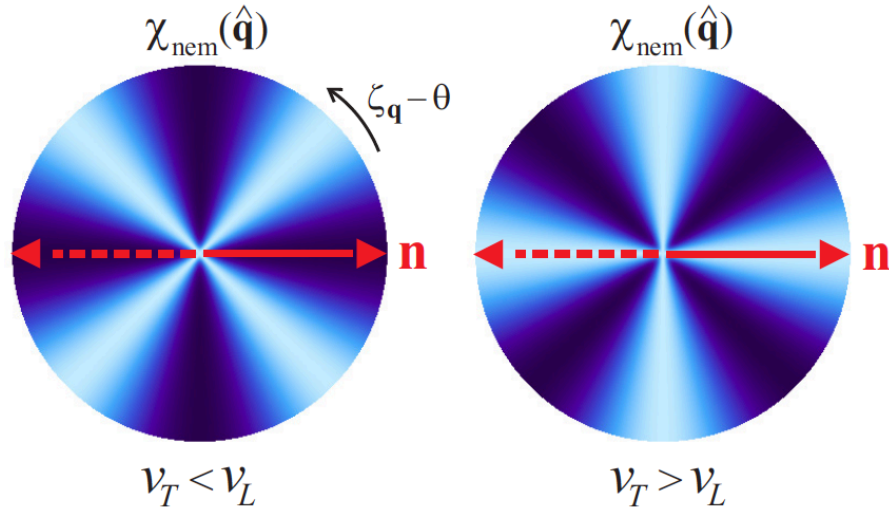
- In momentum-space, this corresponds to a nemato-orbital coupling

$$\delta S = \frac{\lambda^2}{v_T^2} \left(1 - \frac{v_T^2}{v_L^2}\right) \int_q (\boldsymbol{\Phi} \cdot \hat{\mathbf{D}})^2 \quad \hat{\mathbf{D}} = (\hat{q}_x^2 - \hat{q}_y^2, 2\hat{q}_x \hat{q}_y)$$

Nematicity in TBG: nemato-orbital coupling

- Nemato-orbital coupling ties the orientation of the nematic director to certain directions in momentum space.

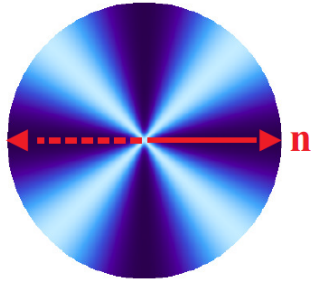
$$\chi_{\text{nem}}^{-1}(q, \zeta_{\mathbf{q}}) = r_0 + q^2 + \left(1 - \frac{v_T^2}{v_L^2}\right) \cos^2(2\theta - 2\zeta_{\mathbf{q}})$$



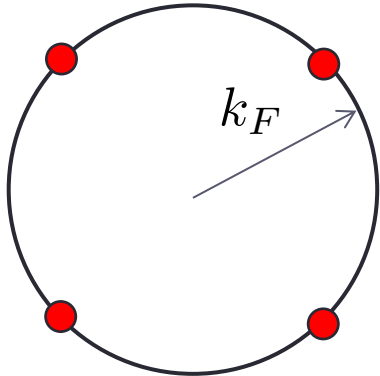
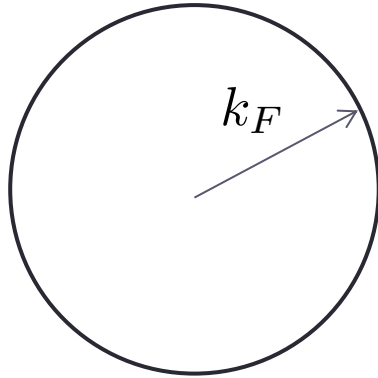
restriction of phase space is expected to render the transition mean-field and first-order ($d_c=3$)

Nematicity in TBG: coupling to electrons

- **Hot spots:** electrons efficiently exchange soft nematic fluctuations

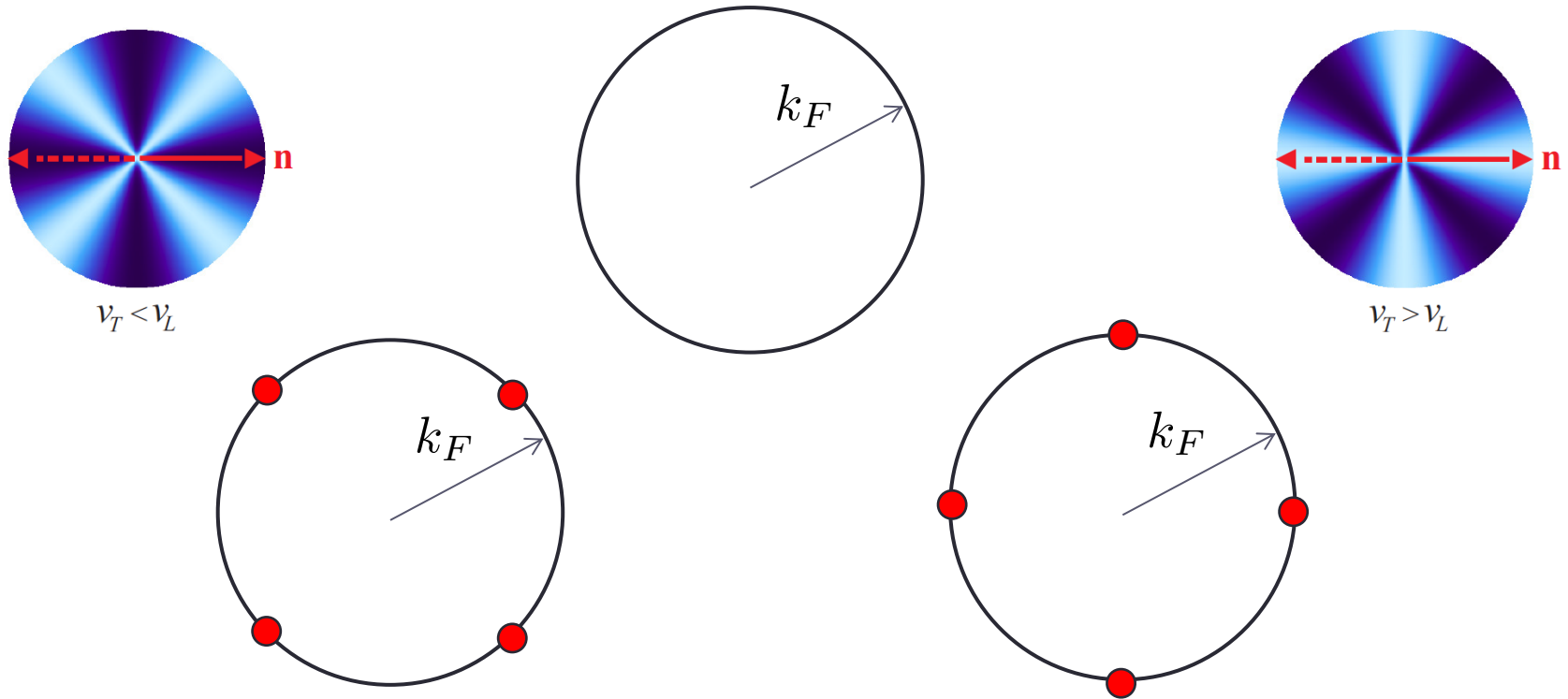


$$v_T < v_L$$



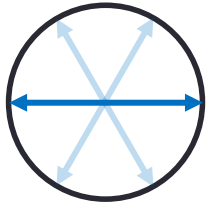
Nematicity in TBG: coupling to electrons

- **Hot spots:** electrons efficiently exchange soft nematic fluctuations

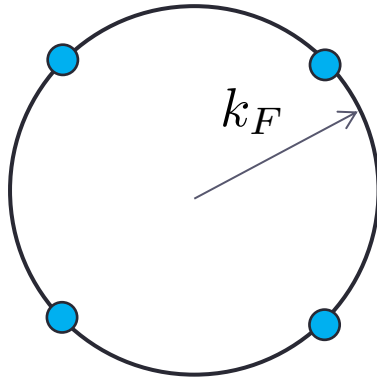


Nematicity in TBG: coupling to electrons

- **Cold spots:** vanishing of the nematic form factor.



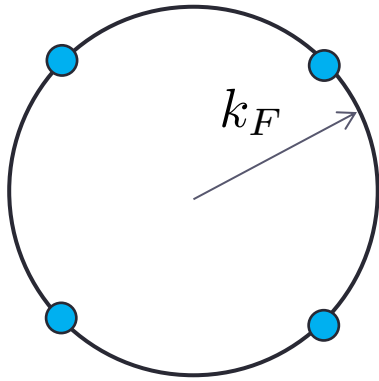
$$H = \sum_{\mathbf{k}, \mathbf{q}} \cos(2\theta - 2\theta_{\mathbf{k}}) \Phi_{\mathbf{q}} \hat{\psi}_{\mathbf{k}-\mathbf{q}/2}^{\dagger} \hat{\psi}_{\mathbf{k}-\mathbf{q}/2}$$



Nematicity in TBG: coupling to electrons

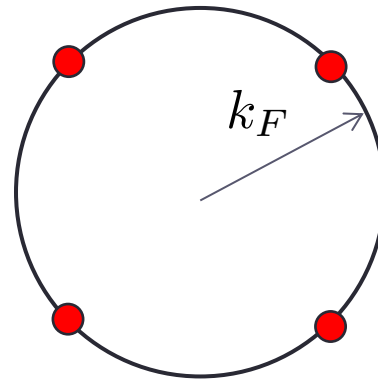
- When the transverse sound velocity is *smaller* than the longitudinal one, hot spots overlap with cold spots: **decoupling between low-energy nematic fluctuations and the electrons.**

*similar to Ising-nematic case:
Paul & Garst, PRL (2017)*



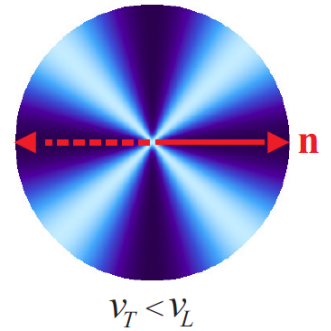
cold spots

vanishing of nematic form factor



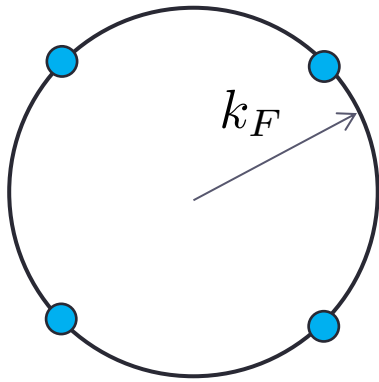
hot spots

exchange of nematic fluctuations



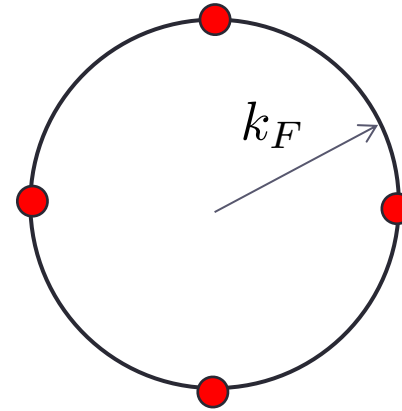
Nematicity in TBG: coupling to electrons

- When the transverse sound velocity is *larger* than the longitudinal one, hot spots do not overlap with cold spots: **maximum coupling between low-energy nematic fluctuations and the electrons.**



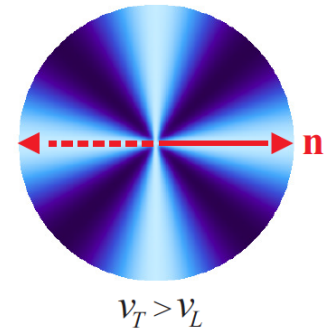
cold spots

vanishing of nematic form factor



hot spots

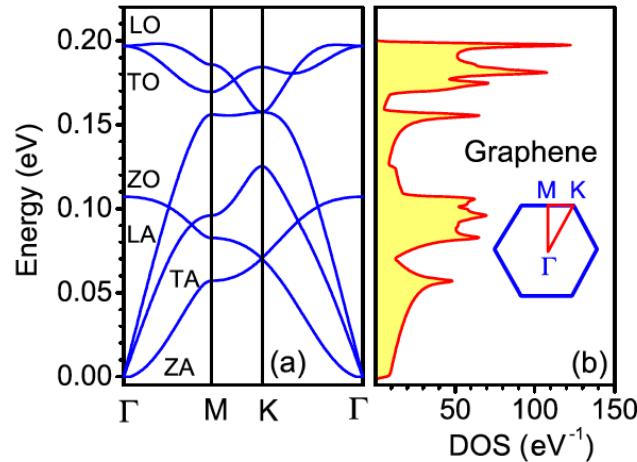
exchange of nematic fluctuations



Nematicity in TBG: coupling to electrons

- Rigid lattice: lattice stability requires $v_T < v_L$

$$F_s(\mathbf{u}) = \frac{C_{11}}{2} (\varepsilon_{xx}^2 + \varepsilon_{yy}^2) + C_{12}\varepsilon_{xx}\varepsilon_{yy} + (C_{11} - C_{12})\varepsilon_{xy}^2$$

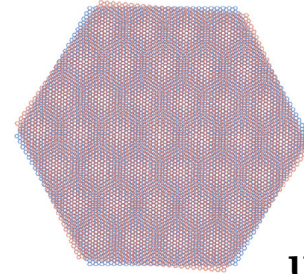


$$\left\{ \begin{aligned} v_L &= \sqrt{\frac{C_{11}}{\rho}} \\ v_T &= \sqrt{\frac{C_{11} - C_{12}}{2\rho}} \end{aligned} \right.$$
$$|C_{12}| < C_{11}$$

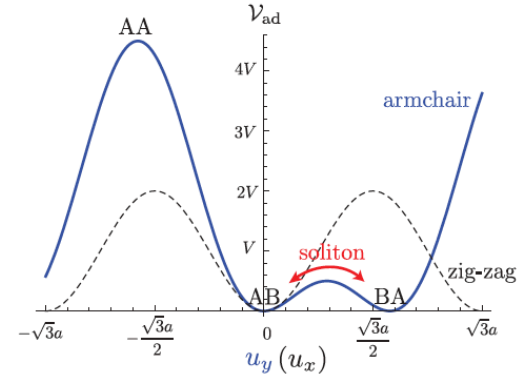
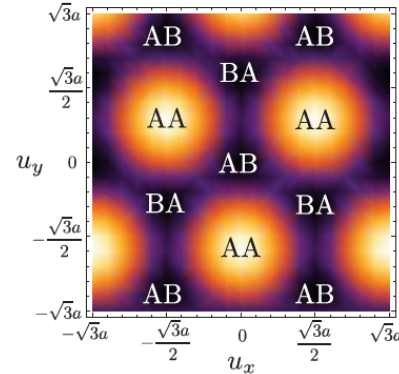
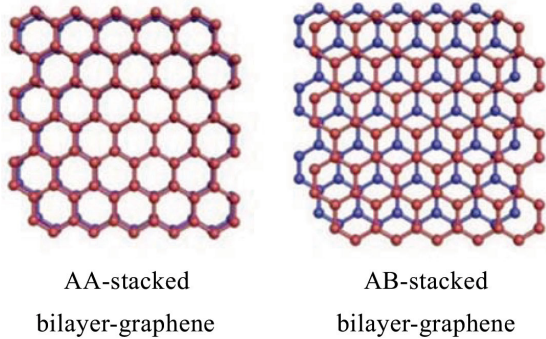
- *in the rigid lattice, electrons are nearly decoupled from low-energy nematic fluctuations*

Nematicity in TBG: coupling to electrons

- Rigid lattice: lattice stability requires $v_T < v_L$
- But the moiré superlattice is not a rigid lattice.
 - adhesion potential favors AB stacking



$$\mathbf{u} = \mathbf{u}_{\text{top}} - \mathbf{u}_{\text{bot}}$$



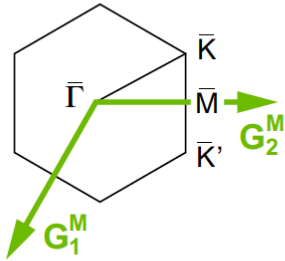
Huang et al,

Current Graphene Science (2018)

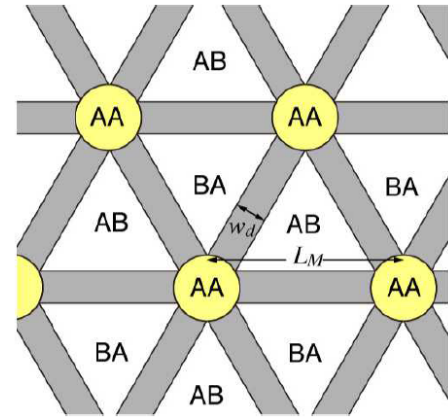
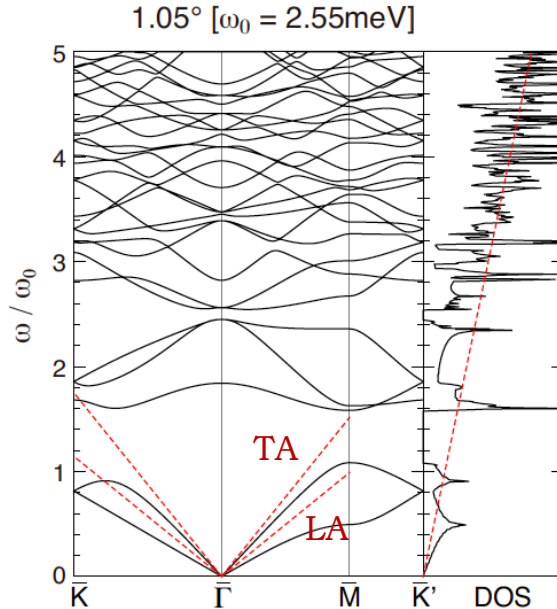
Ochoa, PRB (2019)

Nematicity in TBG: coupling to electrons

- Rigid lattice: lattice stability requires $v_T < v_L$
- But the moiré superlattice is not a rigid lattice: $v_T > v_L$



Koshino, PRB (2019)
Ochoa, PRB (2019)

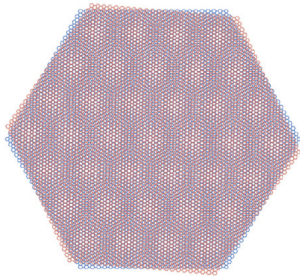


➤ *in the moiré superlattice, electrons are maximally coupled to low-energy nematic fluctuations*

Nematicity in TBG: elastic degrees of freedom

- Adhesion potential favors sharp domain walls between AB/BA stacking regions.
- In contrast to a rigid crystal, rotations of the moiré superlattice cost energy.

$$F_s(\mathbf{u}) = \frac{C_{11}}{2} (\varepsilon_{xx}^2 + \varepsilon_{yy}^2) + C_{12}\varepsilon_{xx}\varepsilon_{yy} + (C_{11} - C_{12}) \varepsilon_{xy}^2 + \frac{K}{2}\omega_{xy}^2$$



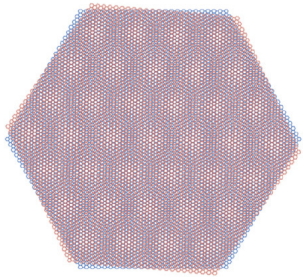
$$\mathbf{u} = \mathbf{u}_{\text{top}} - \mathbf{u}_{\text{bot}}$$

$$\begin{cases} \varepsilon_{ij} = \frac{1}{2} (\partial_i u_j + \partial_j u_i) \\ \omega_{ij} = \frac{1}{2} (\partial_i u_j - \partial_j u_i) \end{cases}$$

Nematicity in TBG: elastic degrees of freedom

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$$F_s(\mathbf{u}) = \frac{C_{11}}{2} (\varepsilon_{xx}^2 + \varepsilon_{yy}^2) + C_{12}\varepsilon_{xx}\varepsilon_{yy} + (C_{11} - C_{12}) \varepsilon_{xy}^2 + \frac{K}{2}\omega_{xy}^2$$



$$\mathbf{u} = \mathbf{u}_{\text{top}} - \mathbf{u}_{\text{bot}}$$

$$\left\{ \begin{array}{l} v_L = \sqrt{\frac{C_{11}}{\rho}} \\ v_T = \sqrt{\frac{C_{11} - C_{12} + \frac{K}{2}}{2\rho}} \end{array} \right. \quad \begin{array}{l} \hat{\mathbf{e}}_L = (\cos \zeta_{\mathbf{q}}, \sin \zeta_{\mathbf{q}}) \\ \hat{\mathbf{e}}_T = (-\sin \zeta_{\mathbf{q}}, \cos \zeta_{\mathbf{q}}) \end{array}$$

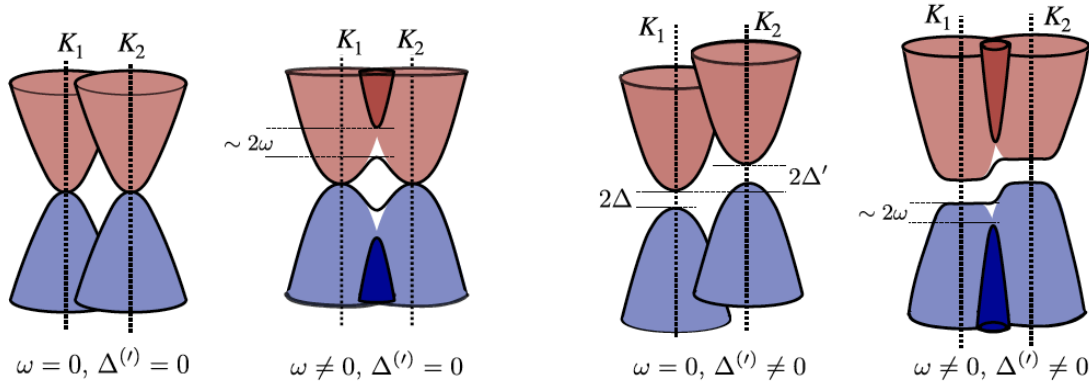
➤ *rotation term contributes only to the transverse phonon velocity*

Outline

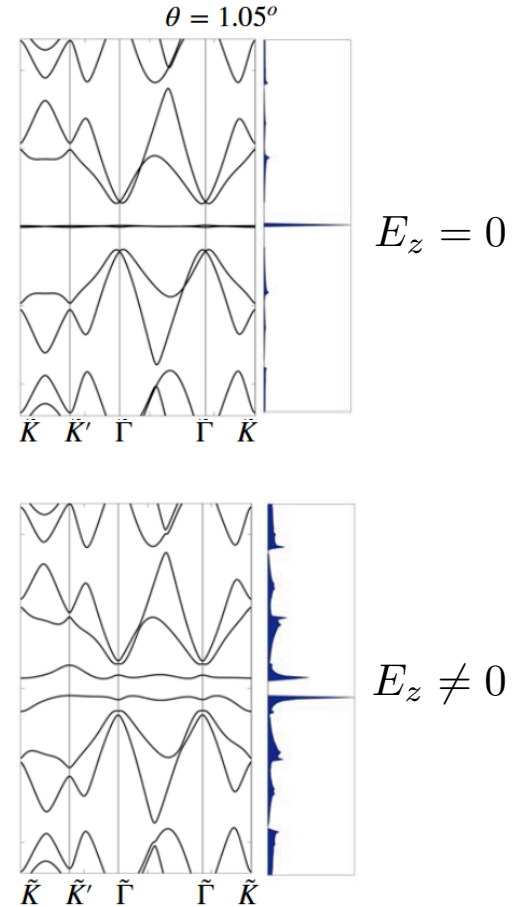
1. Brief overview of twisted moiré systems
2. Potts-nematicity in moiré superlattices: static strain
3. Potts-nematicity in moiré superlattices: fluctuating strain
4. **Electric control of the nematic director**

Nematicity in TDBG: electric field control

- Band structure of twisted double-bilayer graphene can be efficiently tuned by a perpendicular electric field.



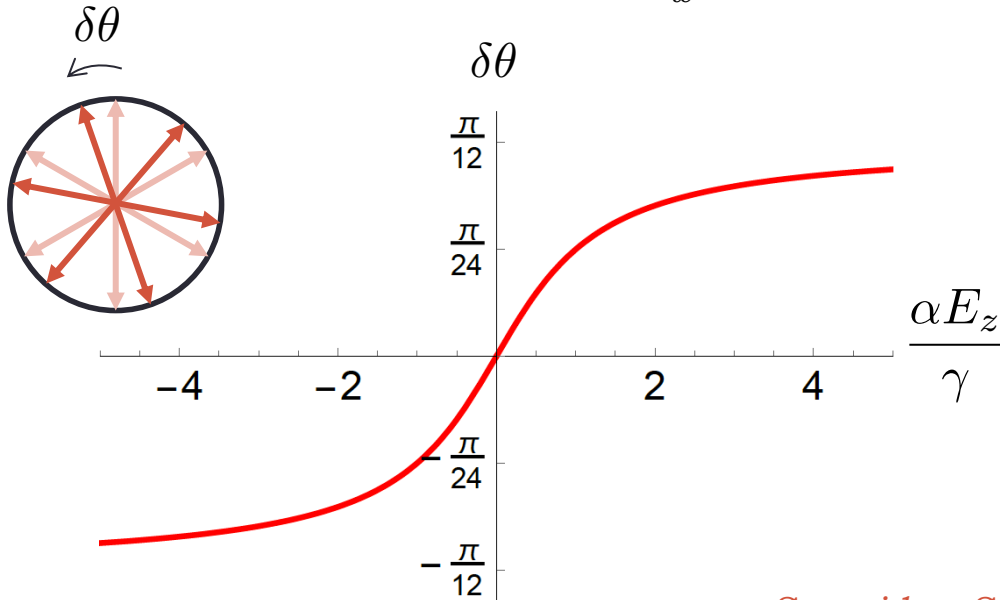
Chebrolu et al, PRB (2019)



Nematicity in TDBG: electric field control

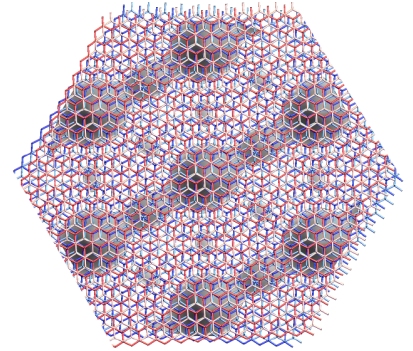
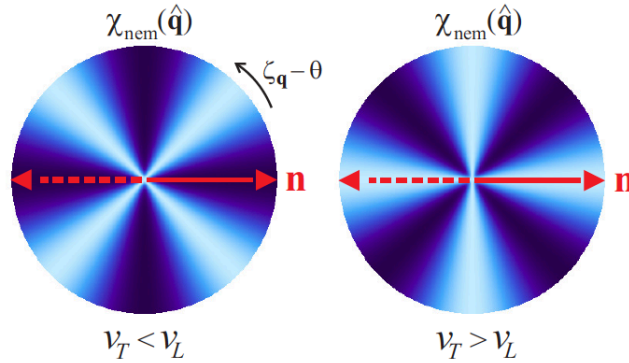
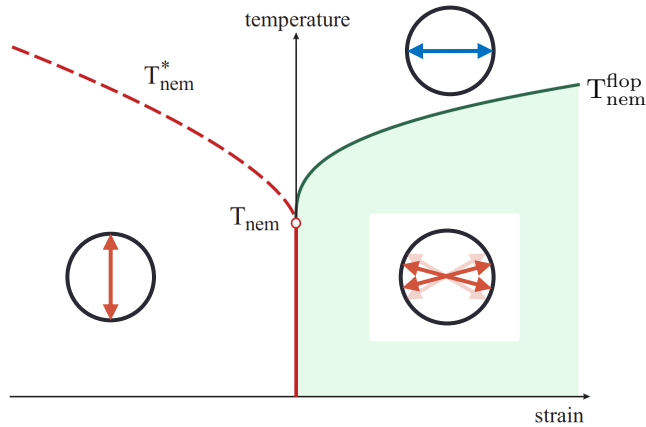
- Perpendicular electric field lowers the symmetry from D_3 to C_3 .

$$S_{\text{nem}}[\Phi] = S_0[\Phi] + \frac{\gamma}{6} \int_x (\Phi_+^3 + \Phi_-^3) + \frac{\alpha E_z}{6i} \int_x (\Phi_+^3 - \Phi_-^3)$$



- nematic director is no longer pinned to high-symmetry directions*
- nematic director (and related thermodynamic quantities) can be rotated by the electric field*

Conclusions



- Nematic order in twisted moiré systems belongs to the 3-state Potts-model universality class.
- Nematic transition can survive in the presence of static strain, becoming an Ising-like nematic-flop transition.
- Impact of the nematic-acoustic phonon coupling on the electronic properties is fundamentally different in moiré superlattices as compared to rigid lattices.
- Electric control of the nematic director may be possible in twisted double-bilayer graphene