Topological SU(N) spin liquids

on the square lattice

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- Generalize spin-1/2 RVB spin liquids to SU(4) on the square lattice and investigate its relevance in a SU(4) NN spin model.
- Construct chiral SU(N) (N=2,3,…) spin liquids, analogs of the Fractional Quantum Hall states, and identify simple (local) quantum spin models hosting these CSL.

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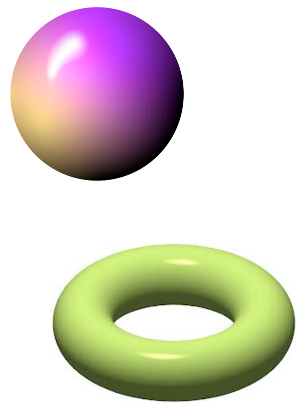
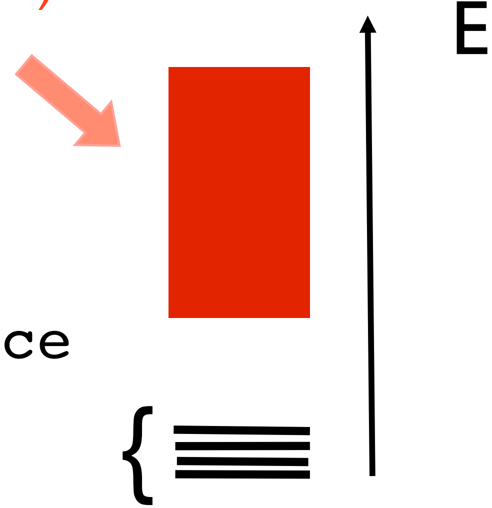
«Topological spin liquids»
beyond the
«order parameter» paradigm

* no spontaneous broken symmetry
* no local order but...
* Topological order

X. G. Wen International Journal of Modern Physics B4, pp. 239-271 (1990)



Excitations are fractional («anyons»)



GS degeneracy
depends on **topology** of space

TWO TYPES OF SPIN LIQUIDS:

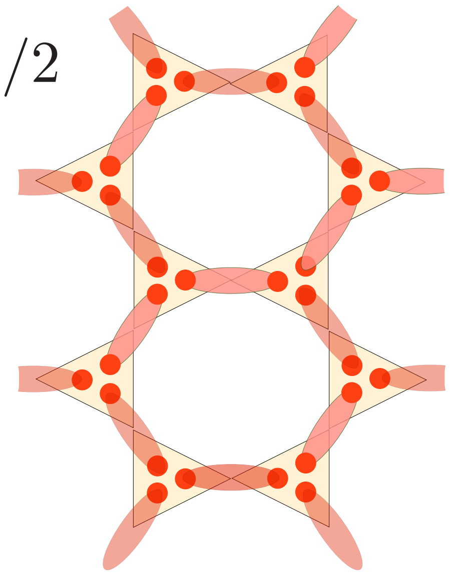
spin-S AKLT

$Bi_3Mn_4O_{12}(NO_3)$ material

J. Lavoie et al., Nat. Phys. 6, 850 (2010)

M. Matsuda et al., Phys. Rev. Lett. 105, 187201 (2010)

$S = z/2$



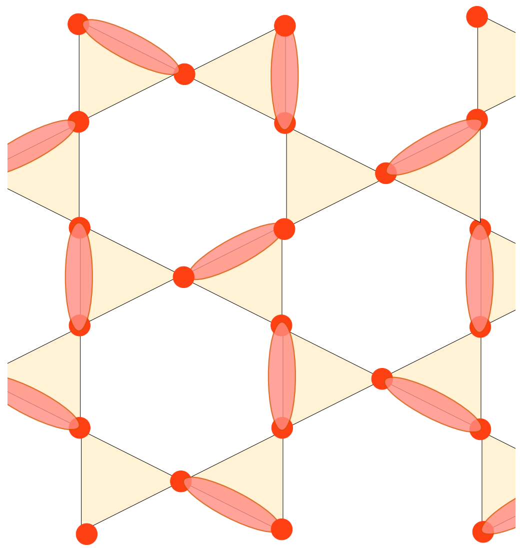
«Trivial» liquid

$S = 0$

spin-1/2 RVB

P. Fazekas and P.W. Anderson

Philosophical Magazine **30**, 423-440 (1974)



Equal-weight superposition
of NN singlet coverings

Topological liquid

Hasting-Oshikawa-LSM theorem

Tensor networks ansatze

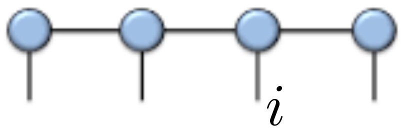
G. Vidal
I. Cirac
D. Perez-Garcia
F. Verstraete

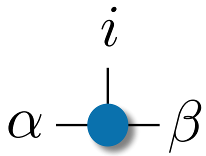
$$|\Psi\rangle = \sum C_{i_1,i_2,\ldots,i_N} |i_1, i_2, \ldots, i_N\rangle \qquad i_k = 1, \cdots, d$$

rank-N tensor with d^N elements

(d=2 for spin-1/2)

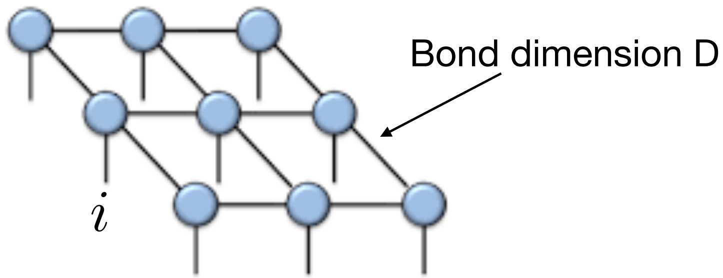
1D: Matrix product state (MPS)



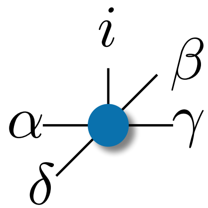


$A^{(i)}\ :\ D \times D$ matrix

(modern formulation of DMRG)



2D: Projected entangled pair state (PEPS)

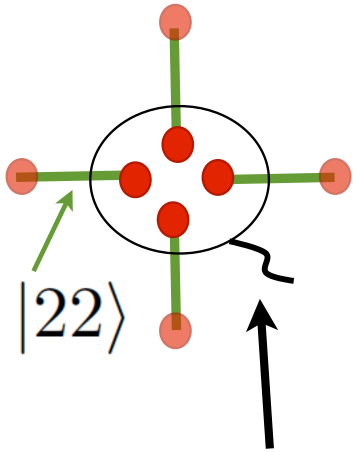


$D^{\otimes z}$ tensor

The spin-1/2 RVB can be written as a PEPS !

virtual states: $1/2 \oplus 0$

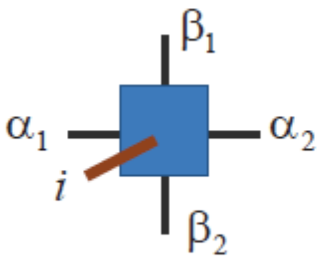
(D=3)



$$|\mathcal{S}\rangle = |01\rangle - |10\rangle + |22\rangle$$

Project onto physical subspace S=1/2 (d=2)

⇨

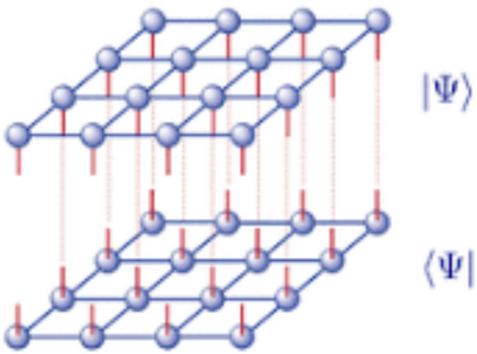


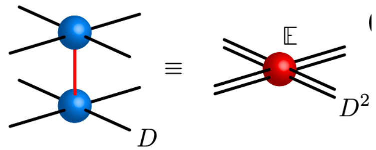
PEPS tensor

Z_2 gauge symmetry => topological order !

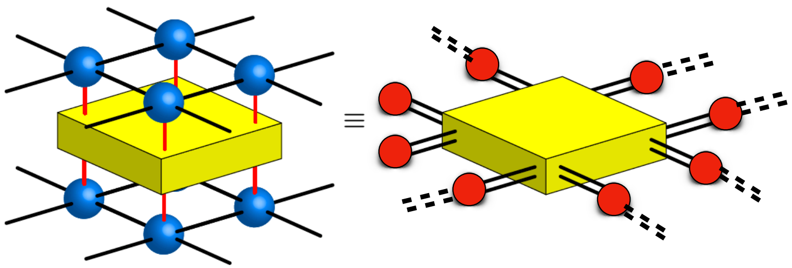
Computing observables

$$\langle \Psi | \Psi \rangle =$$





$$\langle \Psi | O_{1234} | \Psi \rangle =$$



How to deal with infinite double layer ?

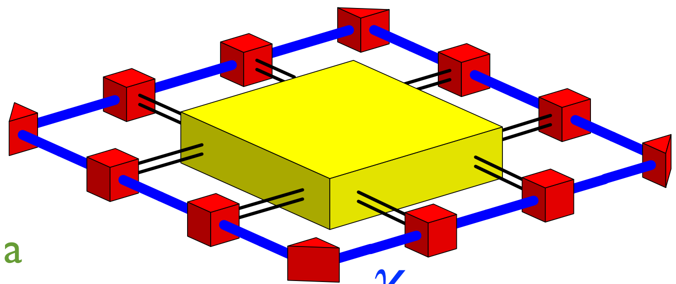
→ real space RG !

iPEPS method

(summary)

$$\langle \Psi_{\text{PEPS}} | H_{\square} | \Psi_{\text{PEPS}} \rangle$$

$$\|$$



- Environment constructed by renormalization of the corner transfer matrix (CTM)

T. Nishino & K. Okunichi, J. Phys. Soc. J. **65**, 891 (1996)
R. Orus & G. Vidal, Phys. Rev. B **80**, 094403 (2009)

- Variational optimisation scheme based on a conjugate gradient method

L. Vanderstraeten, J. Haegeman, P. Corboz, F. Verstraete, Phys. Rev. B **94**, 155123 (2016)

DP & M. Mambrini, Phys. Rev. B **96**, 014414 (2017)

DP, Phys. Rev. B **96**, 121118 (2017)

Enhancement of spin-space symmetry from SU(2) to SU(N) ?

Realized in ultracold alkaline-earth Fermi gases using nuclear spin degrees of freedom. But more challenging in electronic spin systems…

- Coupling between spin & orbital degrees of freedom —> trigger search for QSO liquids

K. I. Kugel and D. I. Khomskii, Sov. Phys. Usp.25, 231(1982)

- SU(4)-symmetric Hubbard model (1/4-filling) in $\alpha - ZrCl_3$

M. Yamada, M. Oshikawa, G. Jackeli, PRL 121, 097201 (2018)

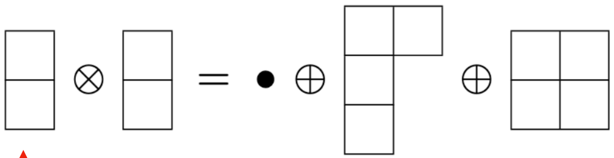
- SU(4)-symmetric spin models correspond to 1/2-filled insulator relevant to twisted bilayer graphene with valley degeneracy ?

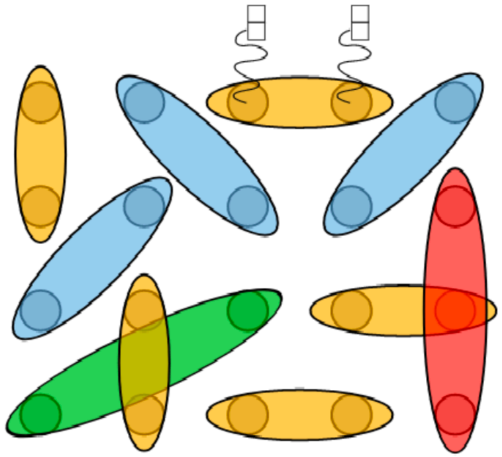
Y. Cao et al., Nature 556, 80 (2018)

A. Keselman, L. Savary & L. Balents, SciPost Phys. 8, 076 (2020)

SU(4) topological resonating valence bond spin liquid on the square lattice

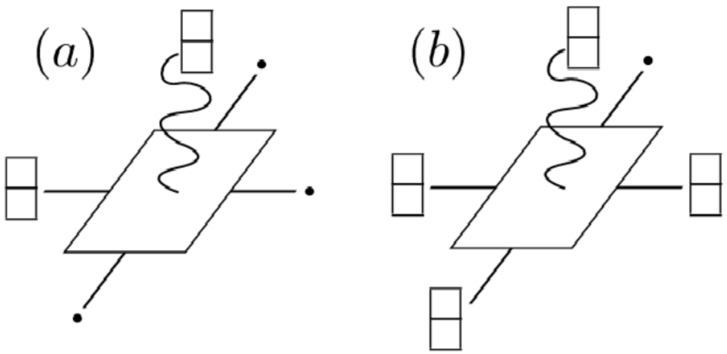
Olivier Gauthé, Sylvain Capponi, and Didier Poilblanc
Phys. Rev. B **99**, 241112(R) – Published 26 June 2019





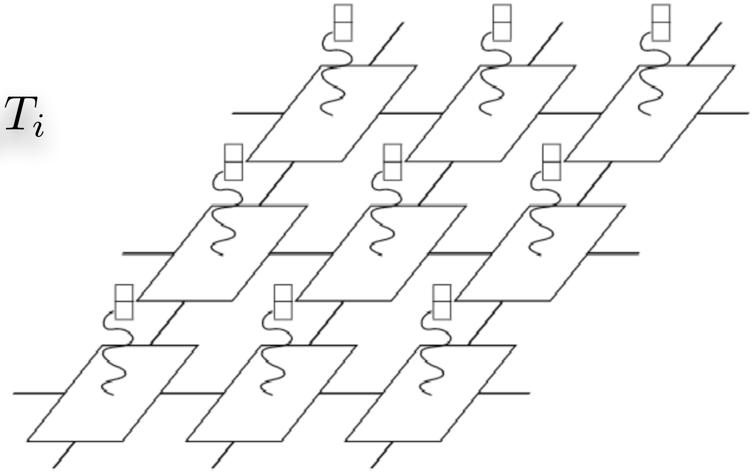
RVB

Two SU(4) fermions (6-dim antisymmetric irrep)

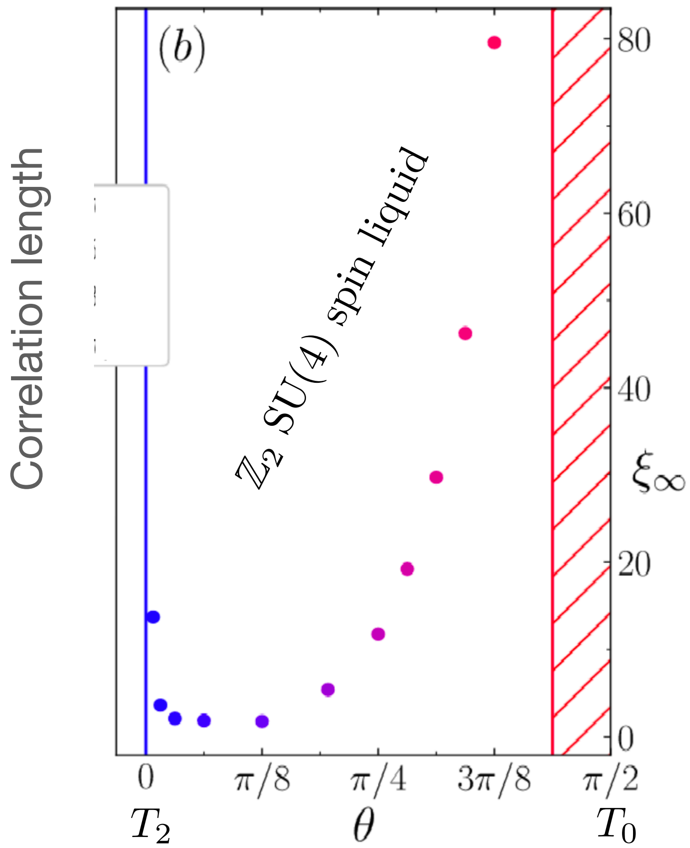


1 tensor T_0 2 tensors T_1, T_2

$$T = \sum \lambda_i T_i$$

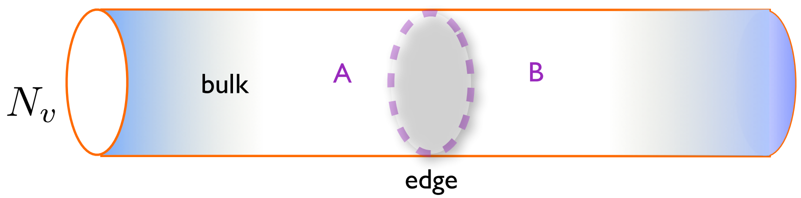


PEPS



Similar to SU(2) RVB

Ji-Yao Chen, DP, Phys. Rev. B 97, 161107 (2018)



$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$

PEPS bulk edge correspondence
(exact theorem) :

I. Cirac et al., PRB 83, 245134 (2011)

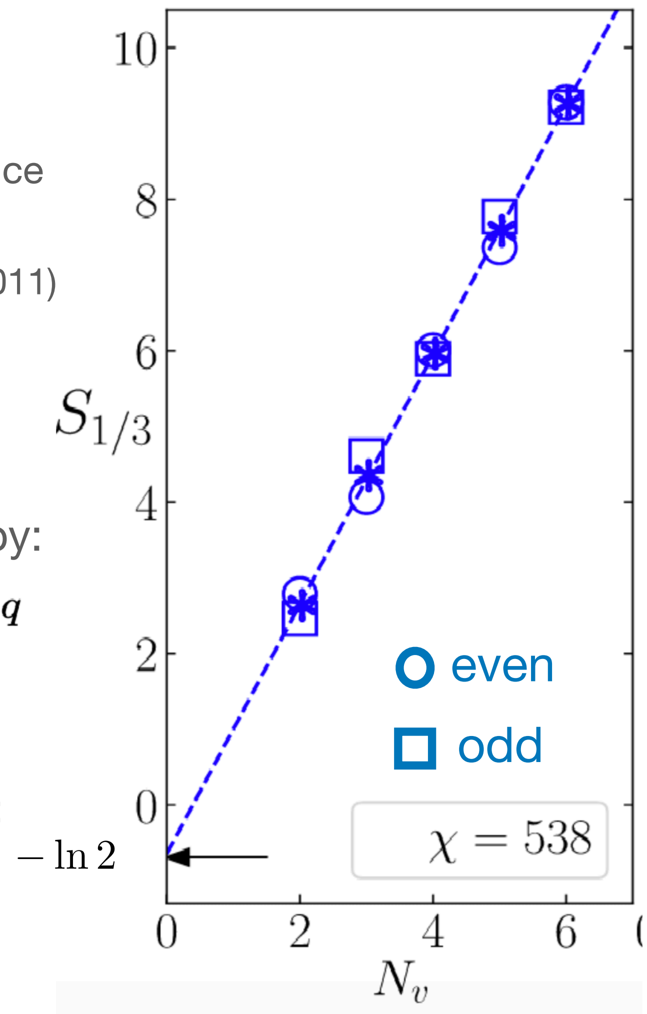
$$\rho_A = U \sigma_b^2 U^\dagger$$

Renyi entanglement entropy:

$$S_q = \frac{1}{1-q} \log \mathrm{Tr}(\rho_b)^q$$

Topological entanglement entropy:

Kitaev & Preskill, 2006
Levein & Wen, 2006

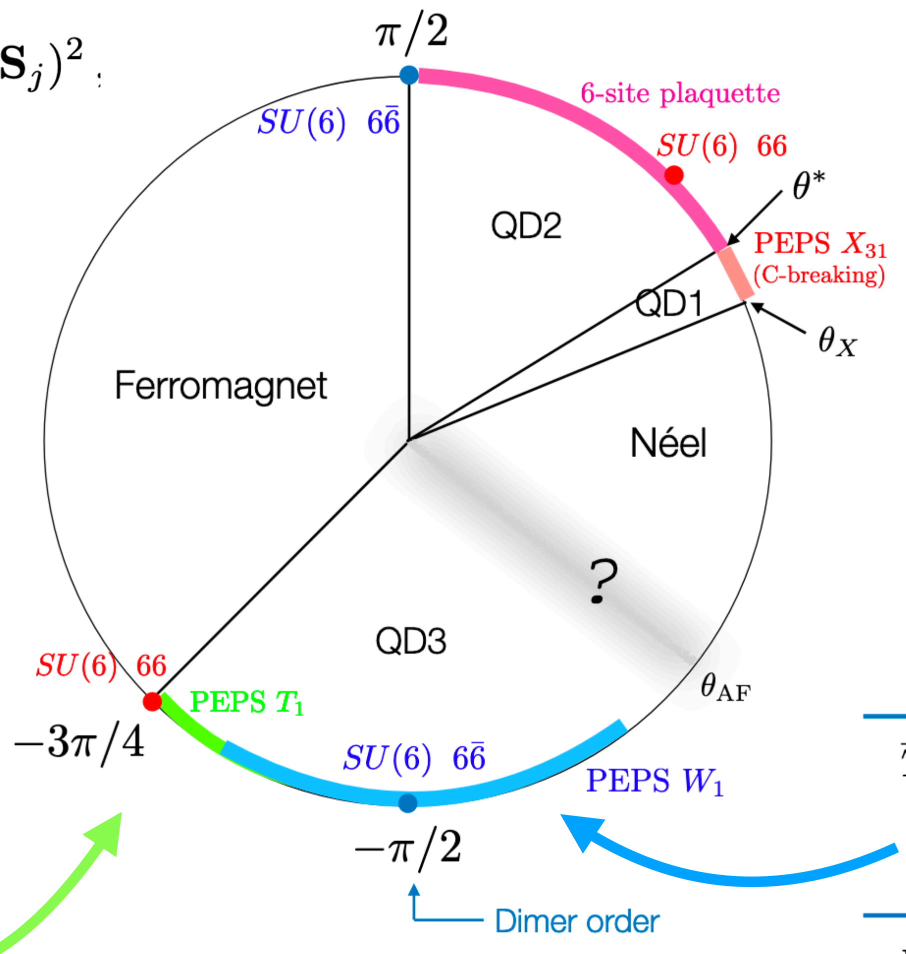


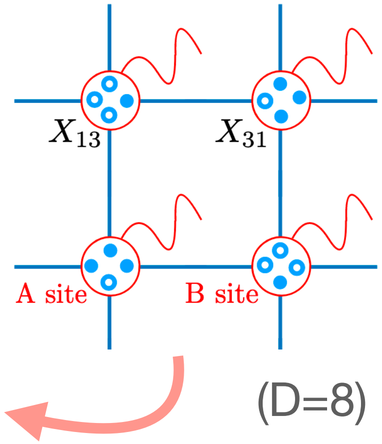
Bilinear-biquadratic model

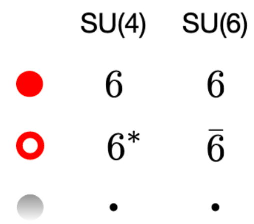
$$\mathcal{H}(\theta) = \cos\theta \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \frac{\sin\theta}{4} \sum_{\langle ij \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j)^2 ,$$

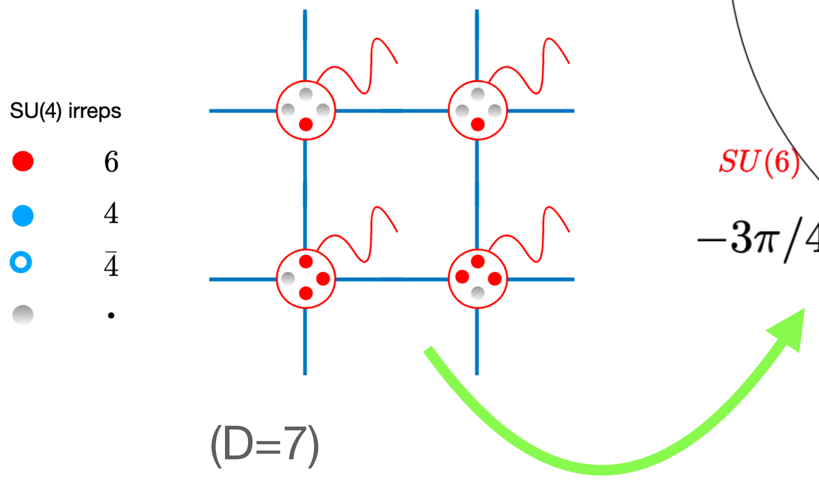
Improve phase diagram :

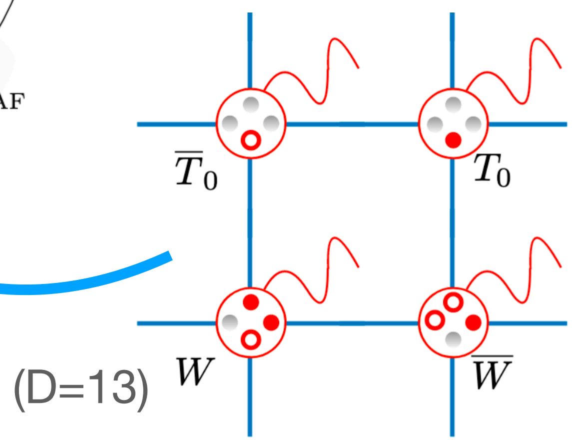
Paramekanti & Marston,
J. of Phys.: Cond. Matter 19, 125215 (2007).



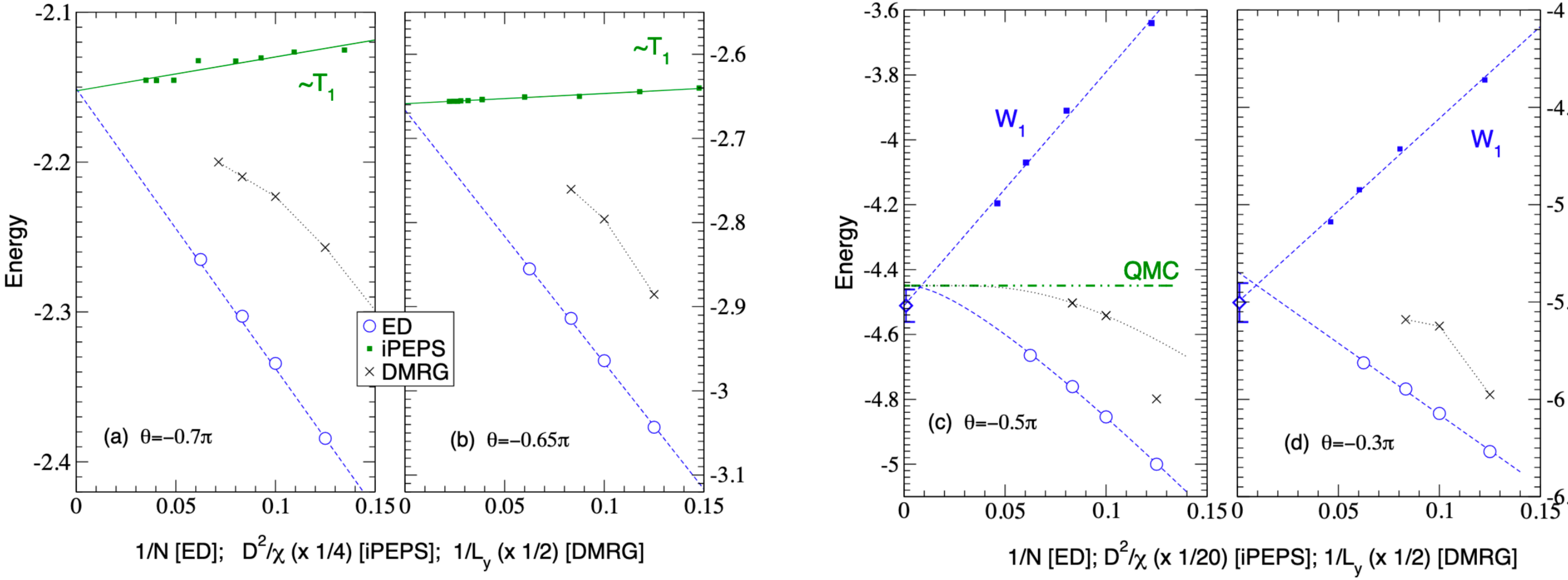








Energetics

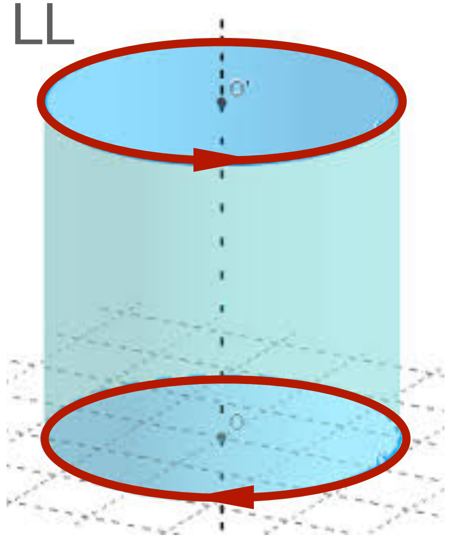


Energy
~T1
(a) θ=−0.7π
(b) θ=−0.65π
ED
iPEPS
DMRG
1/N [ED]; D²/χ (x 1/4) [iPEPS]; 1/Ly (x 1/2) [DMRG]
W1
QMC
(c) θ=−0.5π
(d) θ=−0.3π
1/N [ED]; D²/χ (x 1/20) [iPEPS]; 1/Ly (x 1/2) [DMRG]

if T & P are broken : chiral spin liquids
lattice analogs of FQH states

Low-energy physics described by 2+1 Chern-Simons theory

First example of CSL: $\nu = \frac{1}{2}$ FQHS on a lattice (Kalmeyer-Laughlin, 1987)



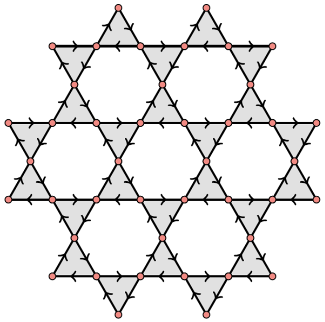
Protected edge modes described by $SU(2)_1$ CFT

"Long range entanglement"

Tensor networks formalism well suited

Abelian chiral SL in chiral AFM on non-bipartite lattices

Kagome lattice



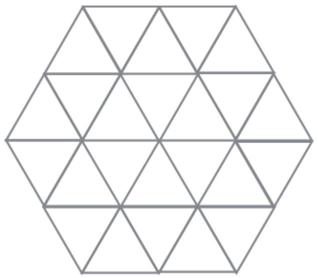
B. Bauer, L. Cincio, B. P. Keller, M. Dolfi, G. Vidal, S. Trebst, A. W. W. Ludwig
Nature Communications 5, 5137 (2014)

SU(2) — S=1/2

$$H = J_{\rm HB} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

Edge modes : chiral c=1 CFT ($SU(2)_1$ WZW)

Triangular lattice



P. Nataf, M. Lajko, A. Wietek, K. Penc, F. Mila & A. Laeuchli
Phys. Rev. Lett. 117, 167220 (2016)

SU(N) — 1 particle/site

$$H = J \sum_{\langle i,j \rangle} P_{ij} + K_3 \sum_{(i,j,k)} (iP_{ijk} + h.c.)$$

→ $SU(N)_1$ chiral SL

Chen, Capponi, Wietek, Mambrini, Schuch, DP
PRL **125,** 017201 (2020)

SU(N) chiral (frustrated) antiferromagnet on square lattice

$$H = J_1 \sum_{\langle i,j \rangle} P_{ij} + J_2 \sum_{\langle\langle k,l \rangle\rangle} P_{kl} + J_\mathrm{R} \sum_{\triangle ijk} (P_{ijk} + P_{ijk}^{-1}) + iJ_\mathrm{I} \sum_{\triangle ijk} (P_{ijk} - P_{ijk}^{-1})$$

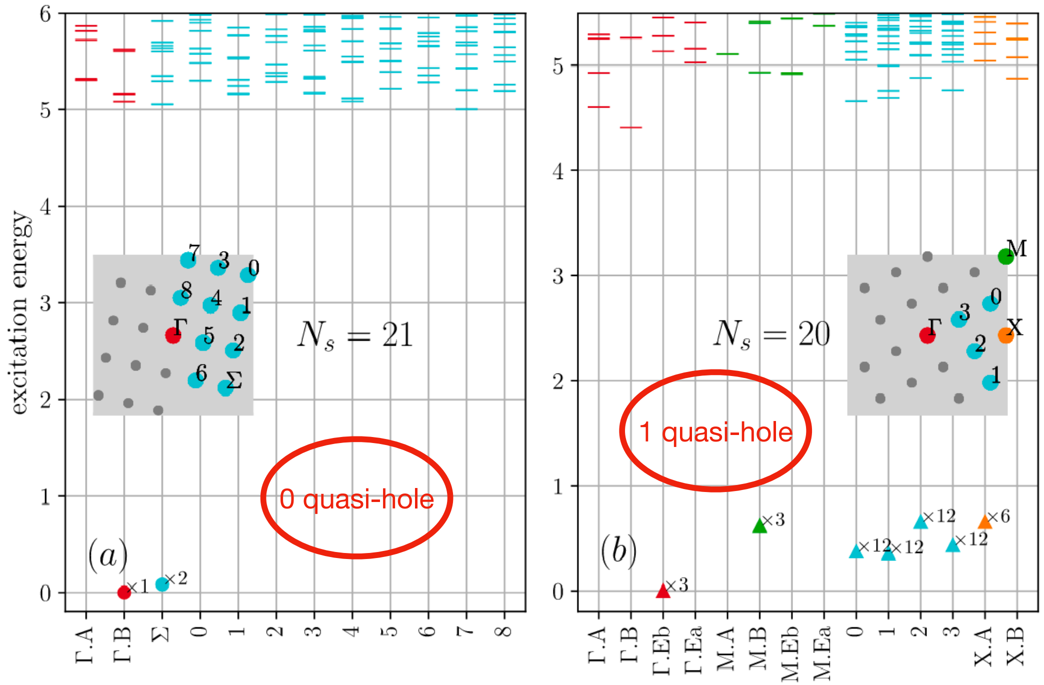
- N-dimensional physical spins (ie fundamental rep. of SU(N))
- Generic 3-site interaction

Example: N=3 (3 “quarks” / per site)

$$J_1 = 2/3 \;\; J_2 = 1/3$$
$$J_R = 1/2 \;\; J_I = 1/\sqrt{2}$$

ED spectra on periodic tori

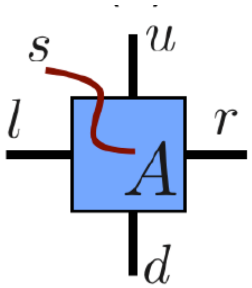
GS topological degeneracy



Same counting rule as 221 Halperin FQH state

Chiral spin liquid with PEPS

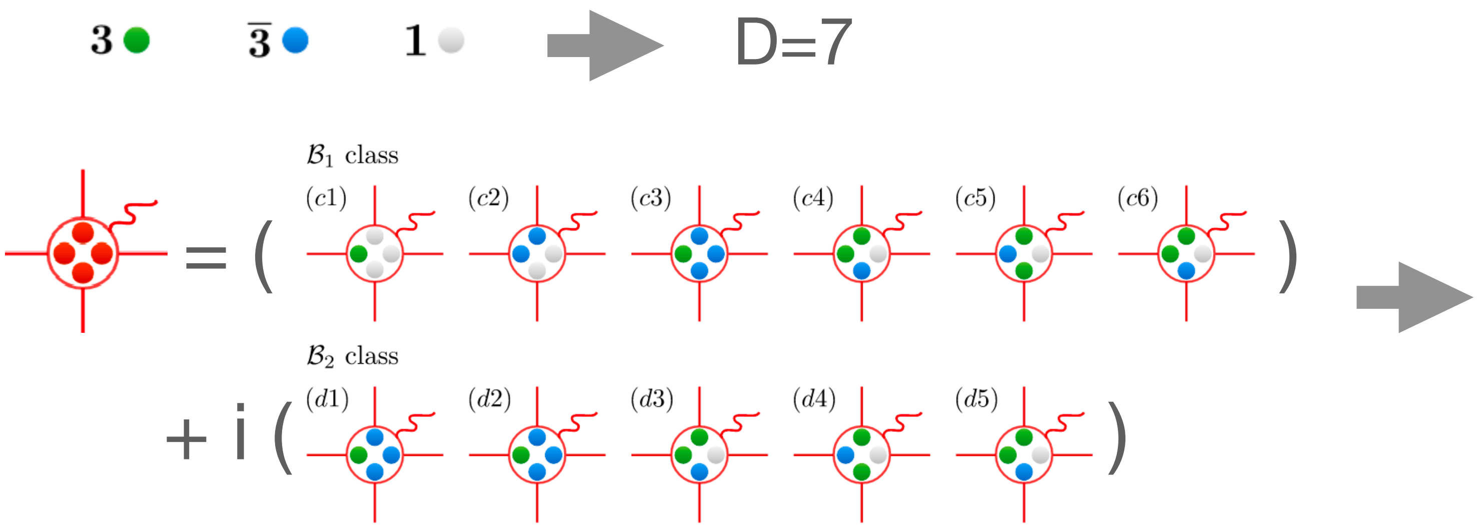
Generalize a classification of SU(2)-invariant PEPS

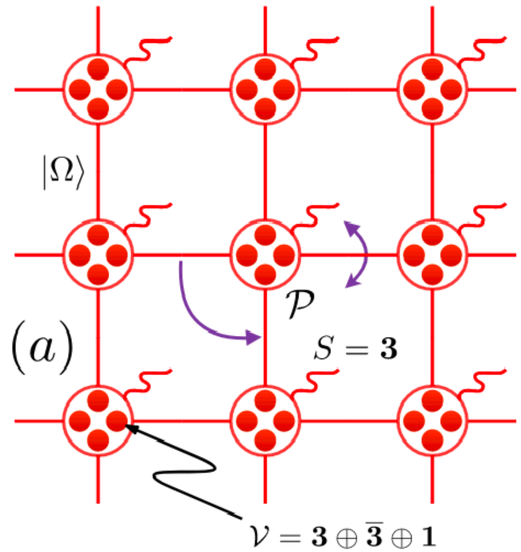


M. Mambrini, R. Orus & DP, Phys. Rev. B 94, 205124 (2016)

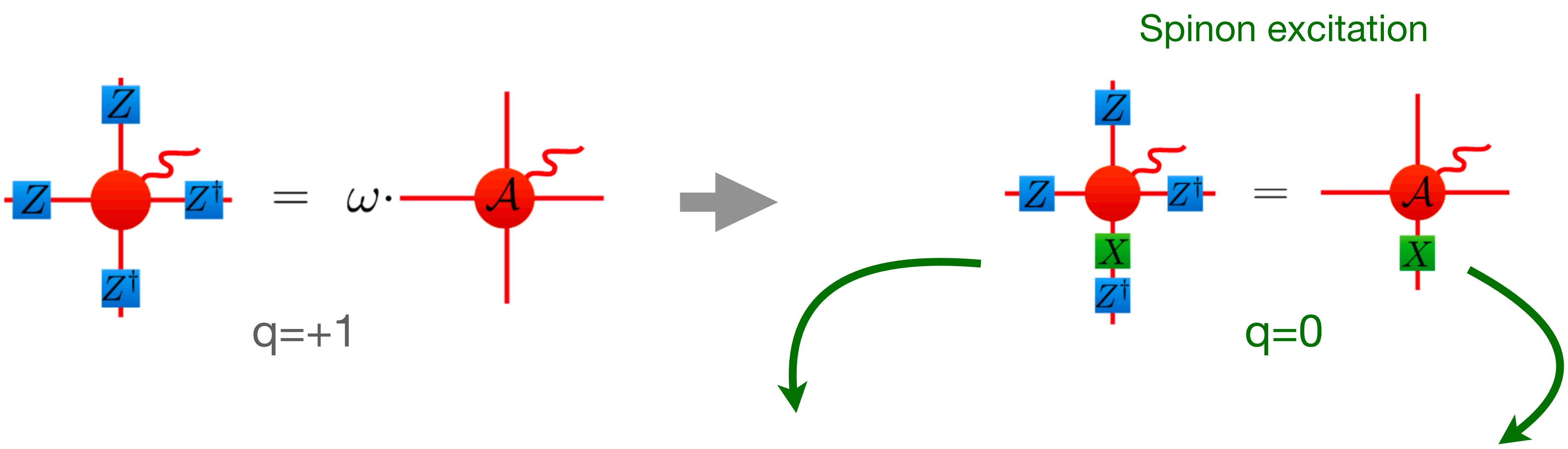
DP, PRB 96, 121118 (2017)

* virtual space : $V = S_1 \oplus S_2 \oplus \cdots S_p$
* Irreps of point group
  (C4v for square lattice)



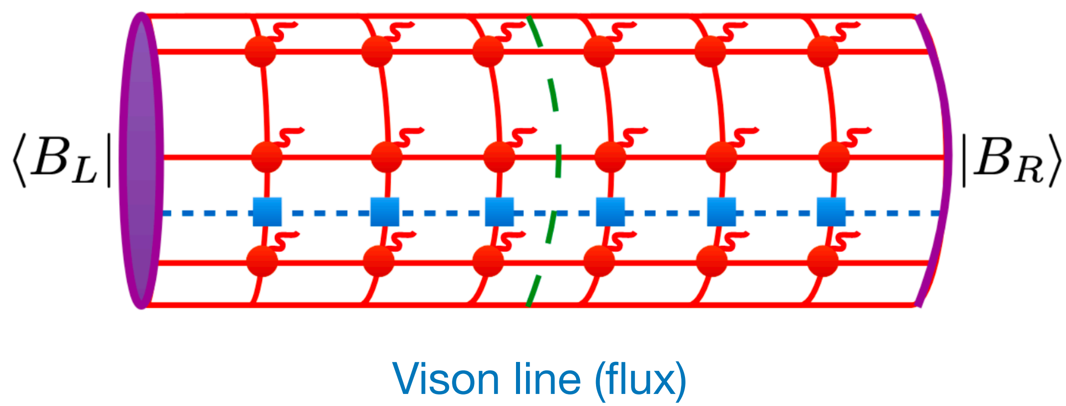


Z_3 gauge (virtual) symmetry



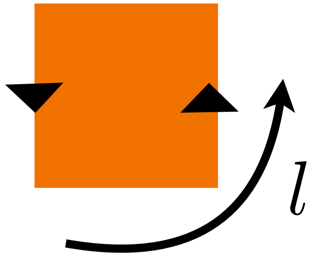
$$Q = 0, \pm 1$$

N=3 charge sectors on the cylinder

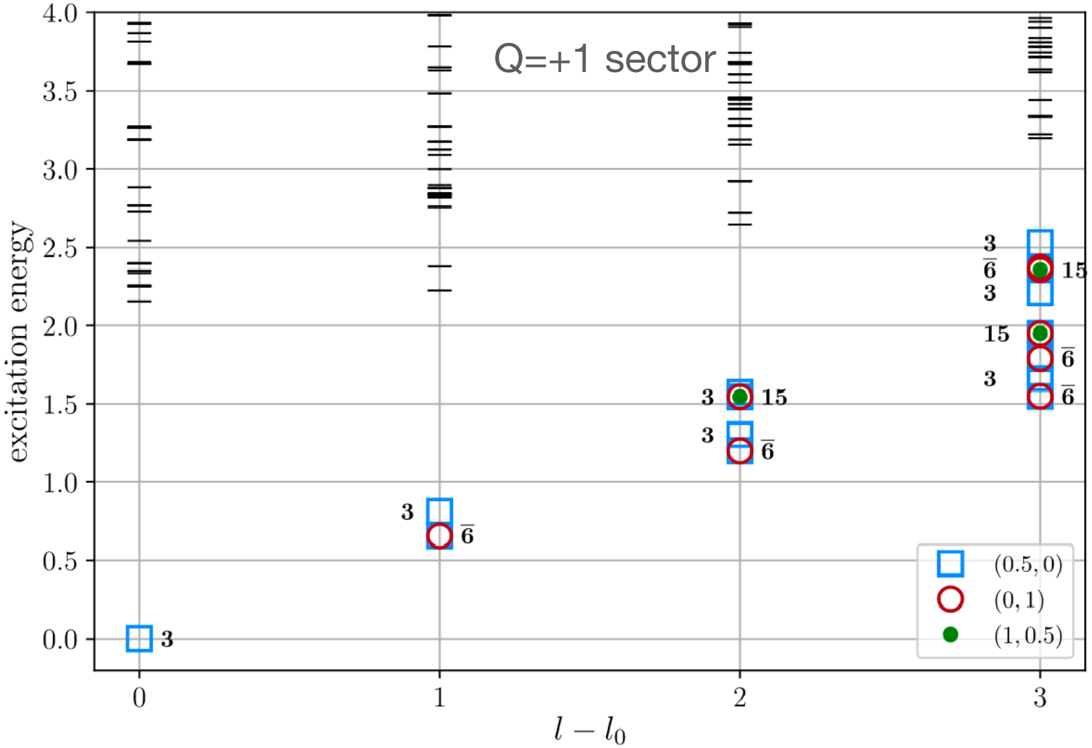


Chiral edge mode described by SU(3)$_1$ WZW CFT

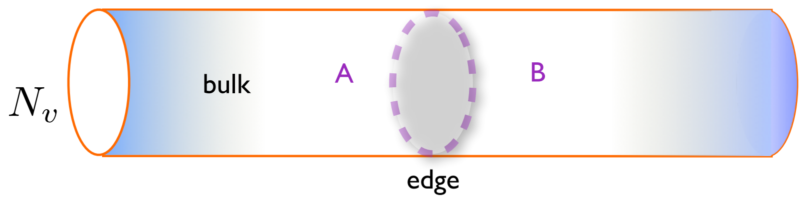
Numerical observation of "conformal towers"



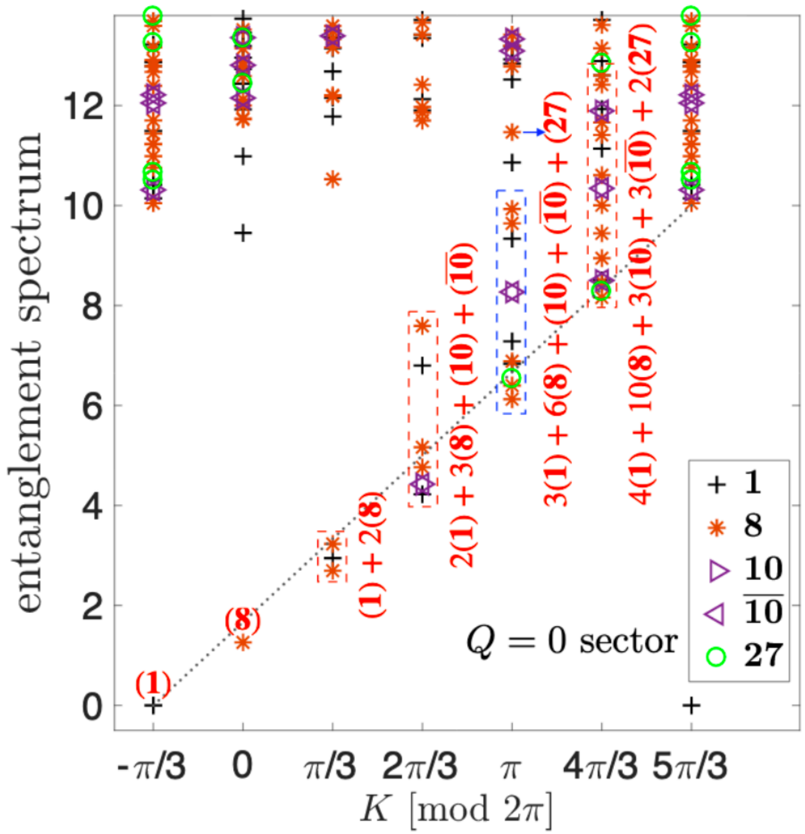
ED spectra
on 4x4 open system



Nv=6 infinite PEPS cylinder



cf. Li & Haldane



Conclusion & outlook

- Existence of SU(4) spin liquids needs to be substantiated - may be QMC ? Also could be investigated on other lattices, triangular, etc…

- Simple SU(2) / SU(3) spin models hosting chiral topological spin liquids

- Can be extended to SU(N), N>3 (in progress, results up to N=10)

- More exotic non-Abelian CSL with $SU(2)_2$, $SU(2)_3$, $SU(3)_2$, $SU(4)_2$, etc… edges physics

Ising anyons   Fibonacci anyons

S=1

JY Chen, L Vanderstraeten, S Capponi, DP
Physical Review B 98 (18), 184409 (2018)

→ Higher irrep for physical spins