

Collective modes of magnetized spin liquids

Oleg Starykh, Univ of Utah



Rapid Communication

Collective spinon spin wave in a magnetized $U(1)$ spin liquid

Leon Balents and Oleg A. Starykh

Phys. Rev. B **101**, 020401(R) – Published 6 January 2020

Anna Keselman, Leon Balents, Oleg Starykh, PRL **125**, 187201 (2020)

Ren-Bo Wang, Anna Keselman, Oleg Starykh, work in progress



KITP program "Correlated systems with multicomponent local Hilbert space",
October 29, 2020



Outline

- Quantum Spin Liquid, spinon Fermi surface
- Spin waves in magnetized conductors and U(1) spin liquids
- Spin-1/2 chain: d=1 spin liquid in magnetic field
- Conclusions

The big question(s)

What is quantum spin liquid?

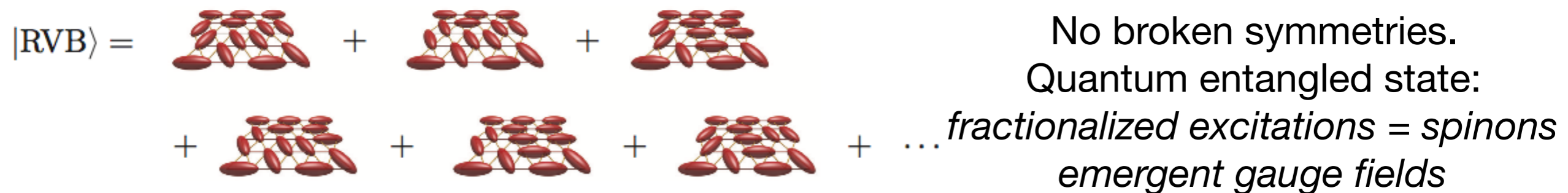


Figure 1. A 'resonating valence bond' (RVB) state. Ellipsoids indicate spin-zero singlet states of two $S = 1/2$ spins.

Savary, Balents 2017

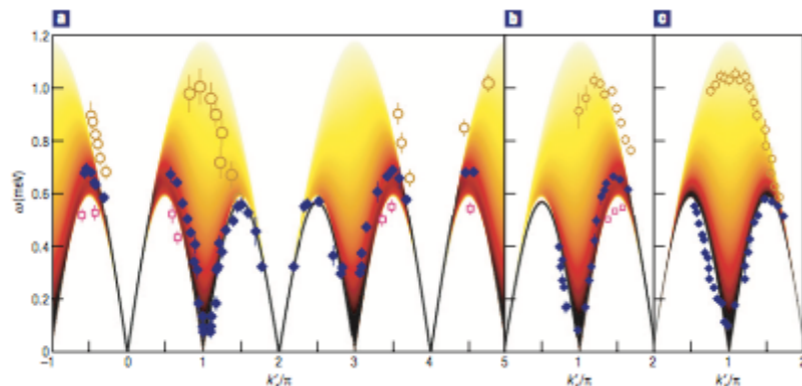
Which materials realize it?

Past candidates: Cs_2CuCl_4 , kagome volborthite...

Current candidates: kagome herbertsmithite, $\alpha\text{-RuCl}_3$, YbMgGaO_4 , organic Mott insulators

How to detect/observe it?

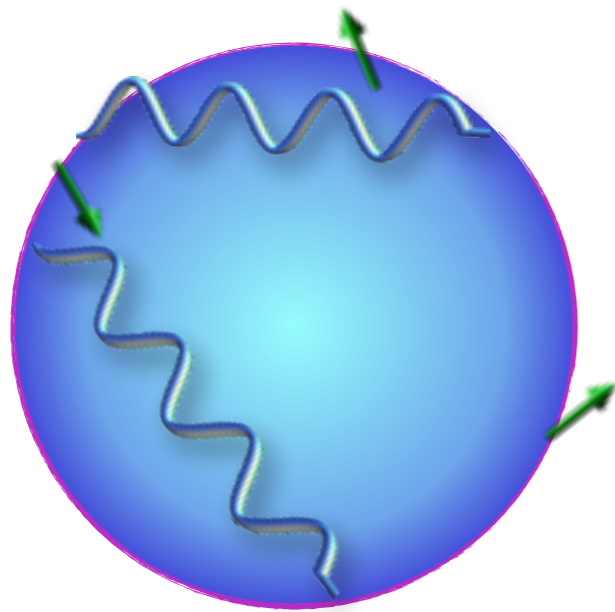
Neutrons, RIXS, NMR, thermal transport, terahertz optics, ESR



Dynamics in
Magnetic field!

Focus: Spinon Fermi surface in magnetic field

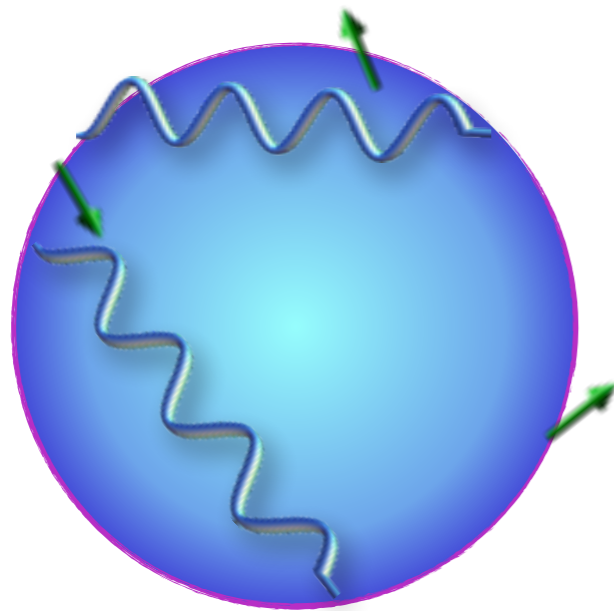
$$|\Psi\rangle = \prod_i \hat{n}_i (2 - \hat{n}_i) \prod_{k < k_F} c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger |0\rangle$$



- The most gapless/highly entangled QSL state
- Like a "metal" of neutral fermions with a U(1) gauge field
- Prototype "non-Fermi liquid" state of great theoretical interest

Many theoretical proposals, number of suggestive experiments...

Spin liquid with spinon Fermi surface



- The most gapless/highly entangled QSL state
- Like a “metal” of **neutral fermions** with a U(1) **gauge field**
- Prototype “non-Fermi liquid” state of great theoretical interest

Explore analogy with Fermi liquid!

Fermi liquid in (Zeeman) magnetic field: spin wave collective excitation

SOVIET PHYSICS JETP

VOLUME 6 (33), NUMBER 5

May, 1958

OSCILLATIONS OF A FERMI-LIQUID IN A MAGNETIC FIELD

V. P. SILIN

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Received by JETP editor May 6, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1227-1234 (November, 1957)

A study is made of the spin oscillations of a paramagnetic Fermi-liquid (He^3) placed in a constant magnetic field at low temperatures, where collisions can be ignored.

The present paper is devoted to a study of spin oscillations in a Fermi-liquid placed in a magnetic field. In formulating the relevant kinetic equation, Landau¹ ignored the presence of a magnetic field; consequently in the first section of this report we derive a kinetic equation which takes it into account.⁶ In Sec. 2 we investigate spin oscillations for the isotropic case and obtain the characteristic frequencies of these oscillations. These frequencies appear to be the limiting values of the spin-wave frequencies when the wavelength goes to infinity. Section 3 is devoted to a study of spin waves. Here it is shown, in contrast with the results of Landau quoted above, that it is possible for spin waves to be propagated in actual liquid He^3 in the presence of a constant magnetic field.

VOLUME 18, NUMBER 8

PHYSICAL REVIEW LETTERS

20 FEBRUARY 1967

OBSERVATION OF SPIN WAVES IN SODIUM AND POTASSIUM*

Sheldon Schultz and Gerald Dunifer

University of California, San Diego, La Jolla, California

(Received 12 December 1966)

We report the first observation of spin waves in sodium and potassium at low temperatures. Utilizing the theory of Platzman and Wolff, we are able to deduce the first two Legendre coefficients of the Landau correlation function for a Fermi liquid.

SPIN-WAVE EXCITATION IN NONFERROMAGNETIC METALS

P. M. Platzman and P. A. Wolff

Bell Telephone Laboratories, Murray Hill, New Jersey

(Received 12 December 1966)

Using the Fermi-liquid theory, we have calculated the wave-number-dependent rf spin susceptibility of an interacting electron gas. For large $\omega\tau$ the kernel exhibits a branch of singularities, spin waves, which show up as sidebands on the electron spin-resonance line in thin slabs of metals. The character of these spin waves and their influence on electron spin resonance is discussed in detail.

PHYSICAL REVIEW B

VOLUME 10, NUMBER 8

15 OCTOBER 1974

Experimental determination of the Landau Fermi-liquid-theory parameters: Spin waves in sodium and potassium*

Gerald L. Dunifer,[†] Daniel Pinkel, and Sheldon Schultz

University of California, San Diego, La Jolla, California 92037

(Received 18 March 1974)

Qualitative difference of
Fermi-liquid from Fermi-gas

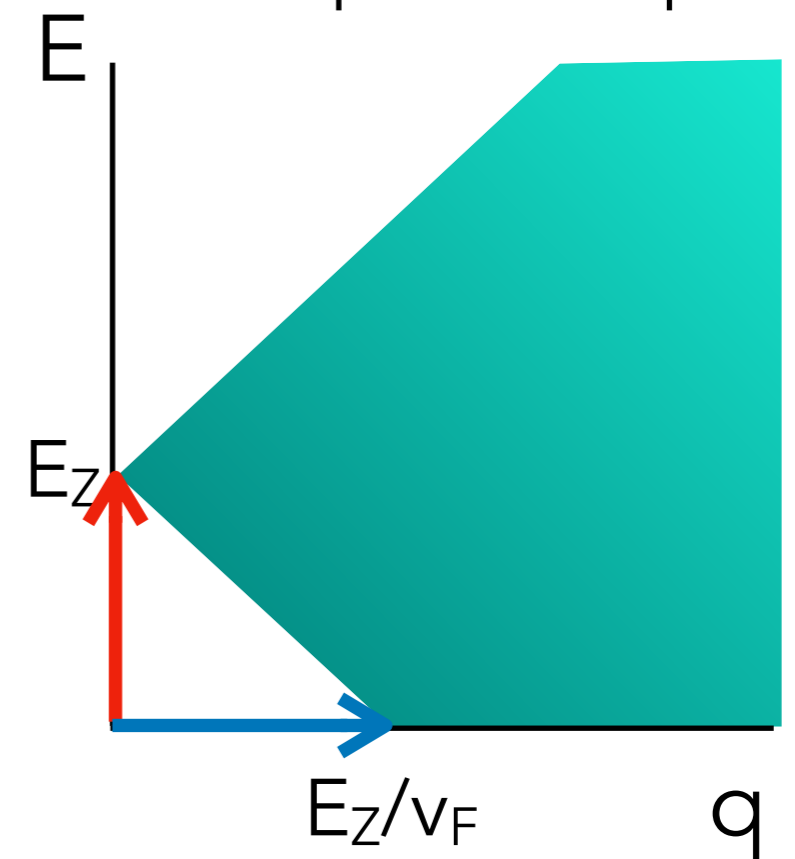
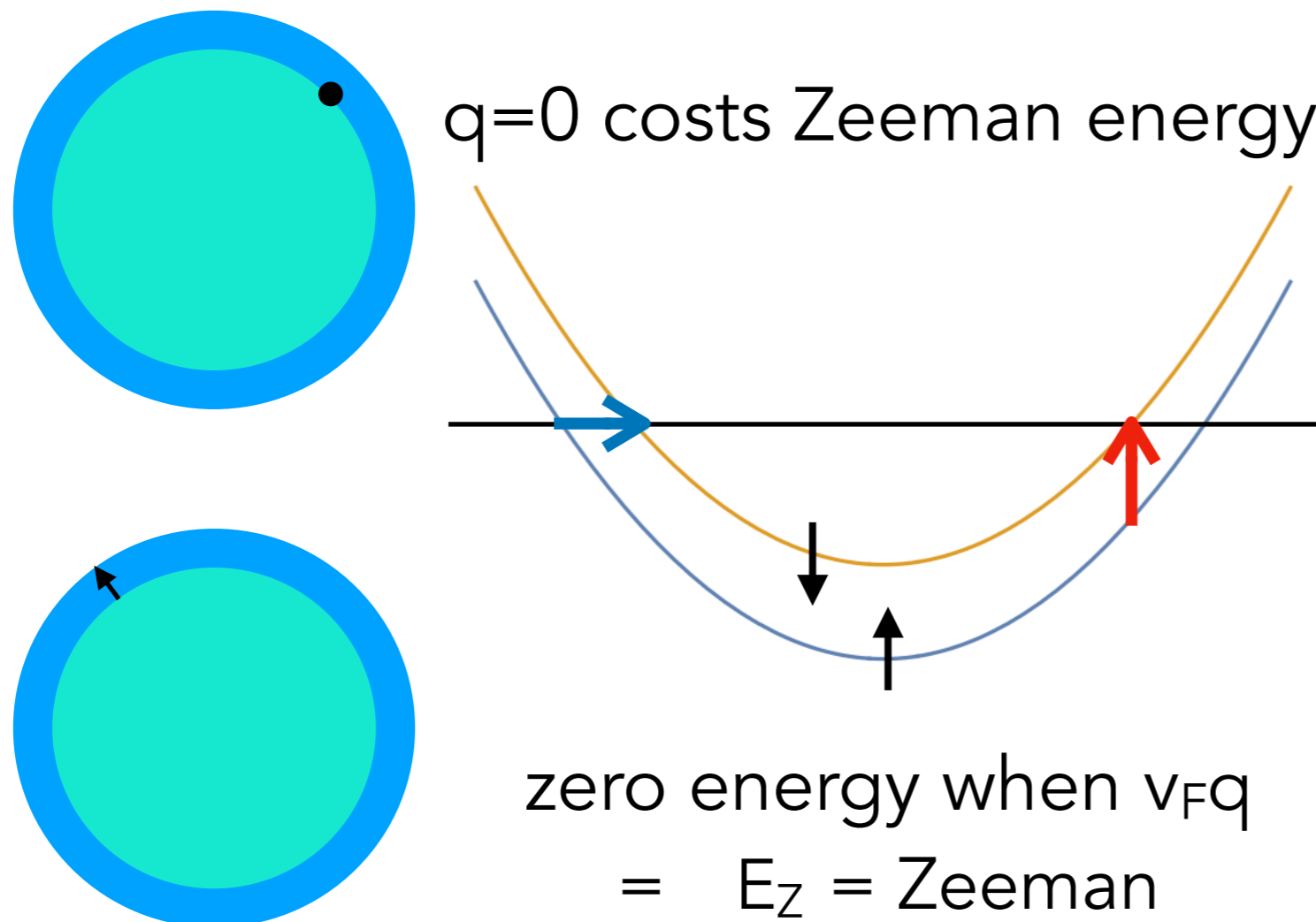
Particle-hole continuum

+ Zeeman field

$$-Bm = -\frac{1}{2}B(n_{\uparrow} - n_{\downarrow})$$

$$\chi_{\pm}(\mathbf{q}, \omega) = i \int_0^{\infty} dt \langle [S_{\mathbf{q}}^{\dagger}(t), S_{-\mathbf{q}}(0)] \rangle e^{i\omega t}$$

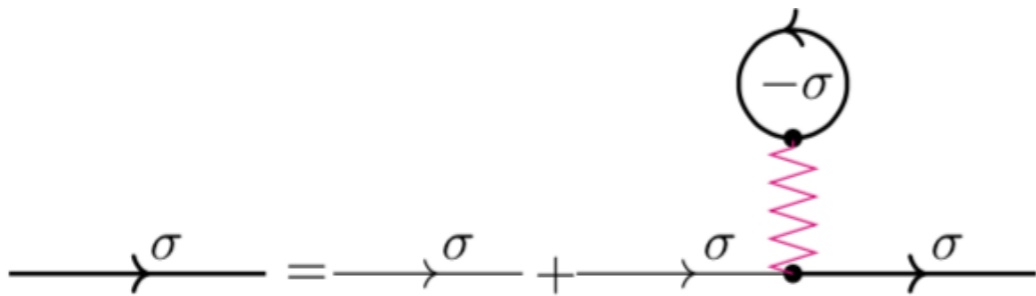
Transverse spin susceptibility



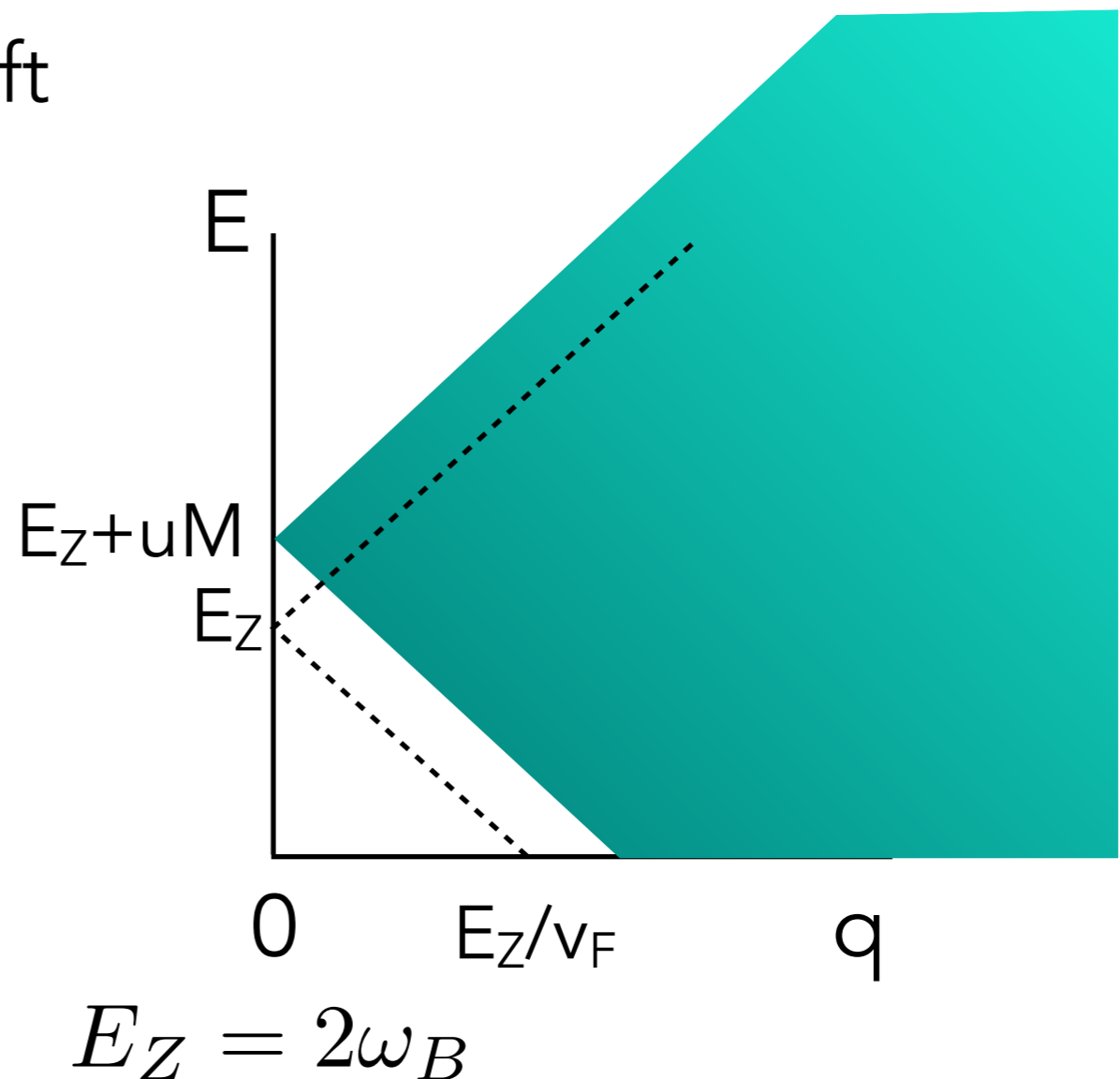
Short-range interaction + Zeeman magnetic field

$$u\psi_{\uparrow}^{\dagger}\psi_{\uparrow}\psi_{\downarrow}^{\dagger}\psi_{\downarrow} \longrightarrow = -\frac{1}{2}uM(\psi_{\uparrow}^{\dagger}\psi_{\uparrow} - \psi_{\downarrow}^{\dagger}\psi_{\downarrow})$$

Self-energy = mean field shift



PH continuum shifts
up in energy by **uM**



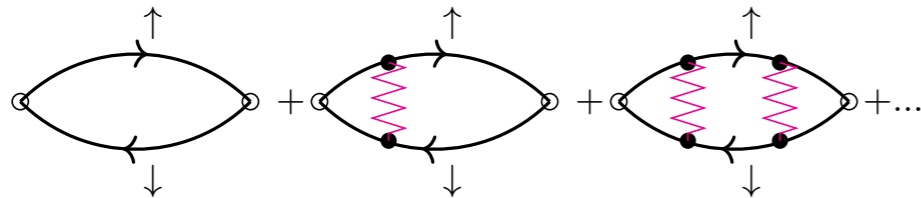
Silin spin wave

Larmor theorem: $\mathbf{q}=0$

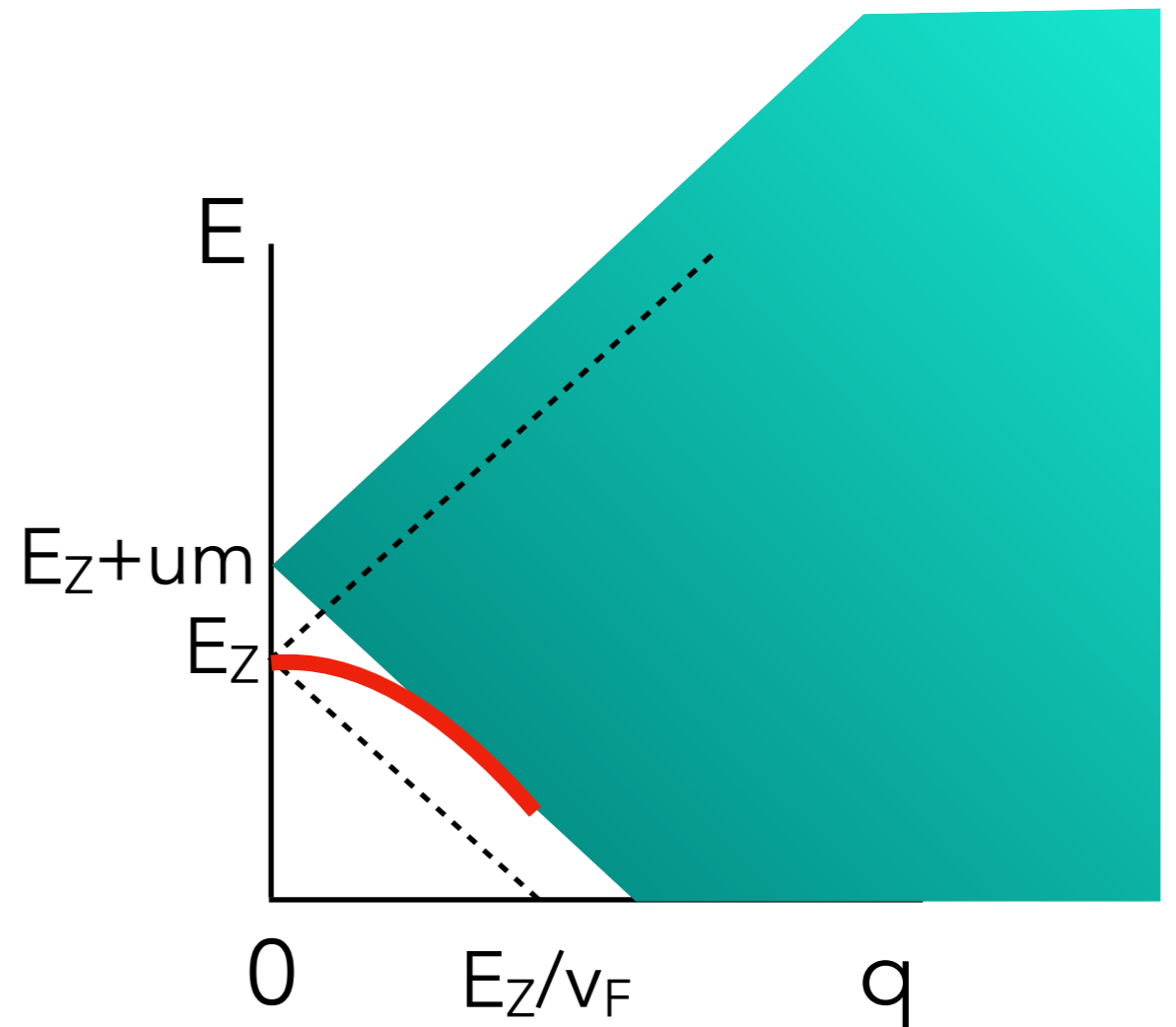
excitation *must* be at E_Z

$$\frac{dS_{\text{tot}}^+}{dt} = -iBS_{\text{tot}}^+$$

RPA: ladder series



$$\chi(\mathbf{q}, i\omega_n) = \frac{\chi^0(\mathbf{q}, i\omega_n)}{1 + u\chi^0(\mathbf{q}, i\omega_n)}$$



“Silin spin wave”

Theory: V.P. Silin, JETP 6, 945 (1958);

Platzman, Wolf, PRL 18, 280 (1967);

Exp: Schultz, Dunifer, PRL 18, 283 (1967).

pole: collective mode

$$\omega = E_Z + um - \sqrt{u^2m^2 + v_F^2q^2}$$

Collective spinon spin wave in a magnetized U(1) spin liquid

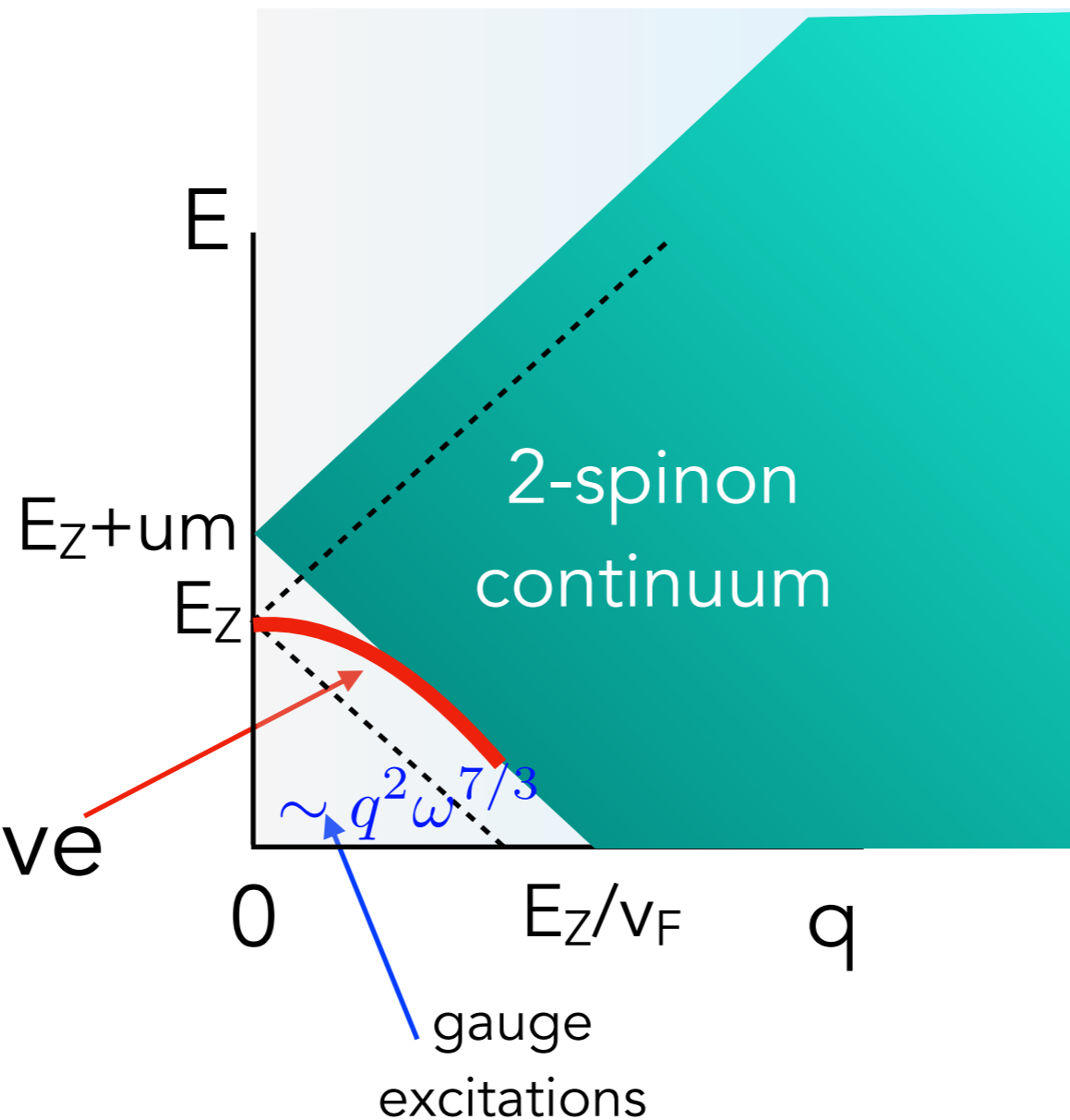
Leon Balents and Oleg A. Starykh

Phys. Rev. B **101**, 020401(R) – Published 6 January 2020

Distinct signature of spinons, interactions, and gauge fields

Transverse collective spin wave is dressed by gauge fluctuations, acquires finite lifetime.

spinon spin wave



$$\frac{E_Z}{v_F} \sim \sqrt{m E_Z} \sqrt{\frac{E_Z}{E_F}}$$

conductors $\frac{E_Z}{E_F} \rightarrow 0$

QSL $\frac{E_Z}{E_F} \sim 1$

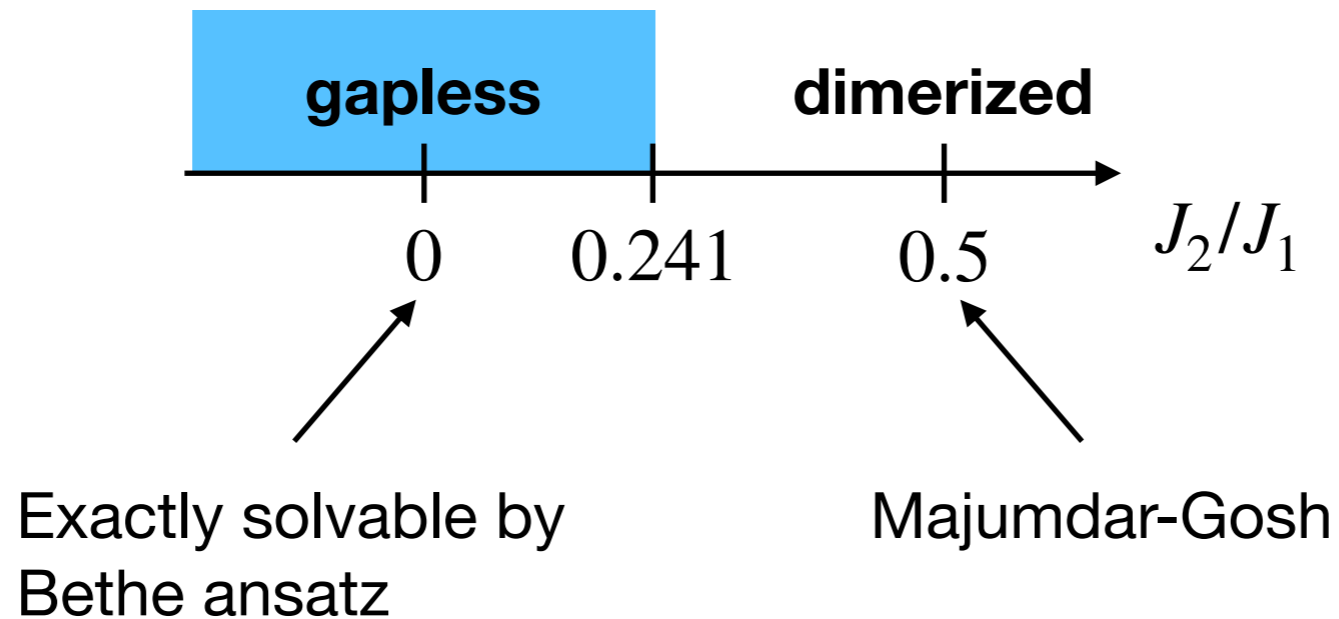
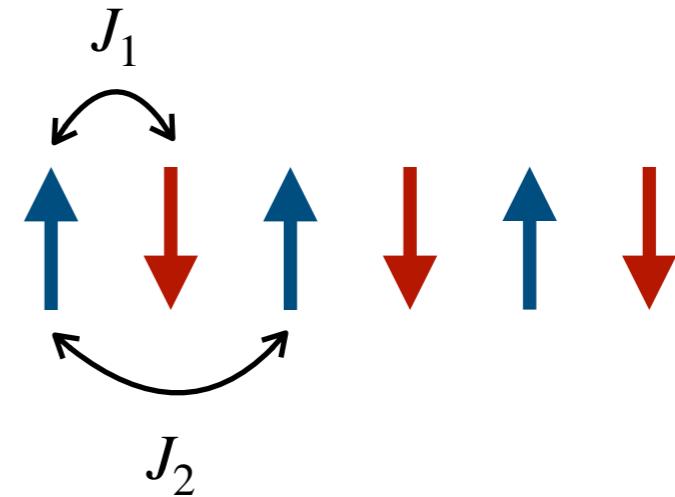
gauge excitations

Outline

- Quantum Spin Liquid, spinon Fermi surface
- Spin waves in magnetized conductors and U(1) spin liquids
- Spin-1/2 chain: $d=1$ spin liquid in magnetic field
- Conclusions

Spin-1/2 antiferromagnetic chain

$$H = \sum_i J_1 \vec{S}_i \cdot \vec{S}_{i+1} + J_2 \vec{S}_i \cdot \vec{S}_{i+2}$$



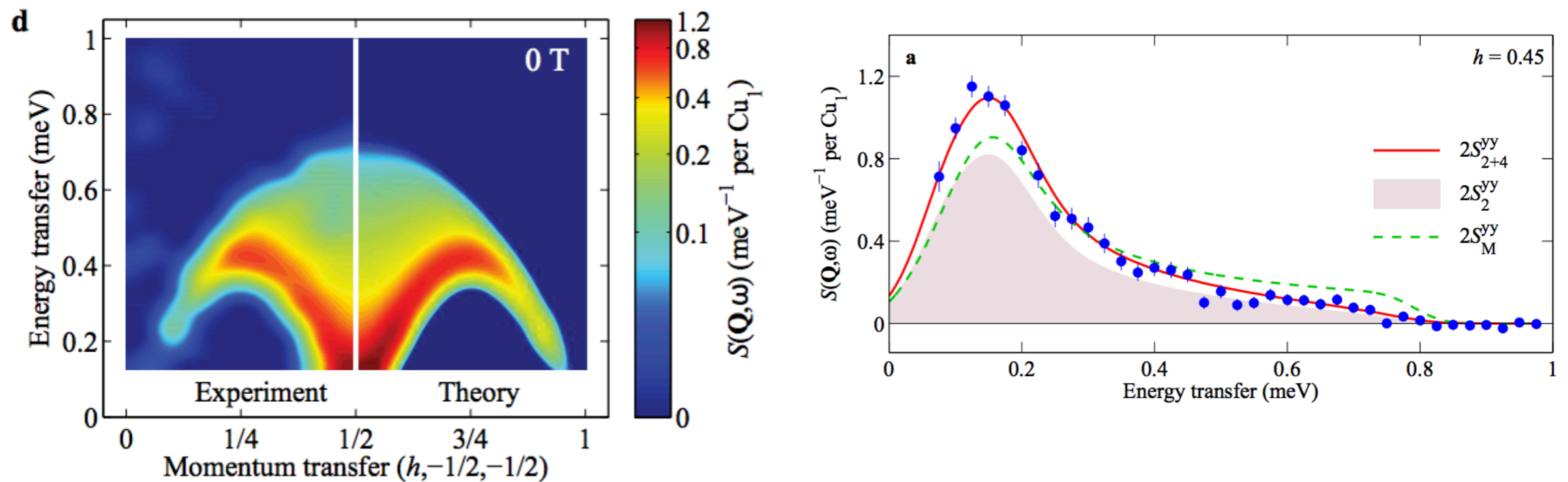
Very well understood and non-trivial many-body system

Fractional spinon excitations in the quantum Heisenberg antiferromagnetic chain

Martin Mourigal , Mechthild Enderle, Axel Klöpperpieper, Jean-Sébastien Caux, Anne Stunault & Henrik M. Rønnow

Nature Physics **9**, 435–441(2013) | [Cite this article](#)

CuSO4 \cdot 5D2O



Quantitative description of 2- and 4-spinon continuum
(for $B=0$ and $J_2=0$)

Spin-1/2 antiferromagnetic chain

Low energy description $SU(2)_1$ WZW CFT

Fermionic representation

$$\vec{S}_i \sim \vec{J}_R(x_i) + \vec{J}_L(x_i) + (-1)^i N(x_i)$$

$$\vec{J}_{R/L} = \frac{1}{2} \psi_{R/L}^\dagger \vec{\sigma} \psi_{R/L}, \quad \psi_{R/L} = \begin{pmatrix} \psi_{R/L,\uparrow} \\ \psi_{R/L,\downarrow} \end{pmatrix}$$

Hamiltonian

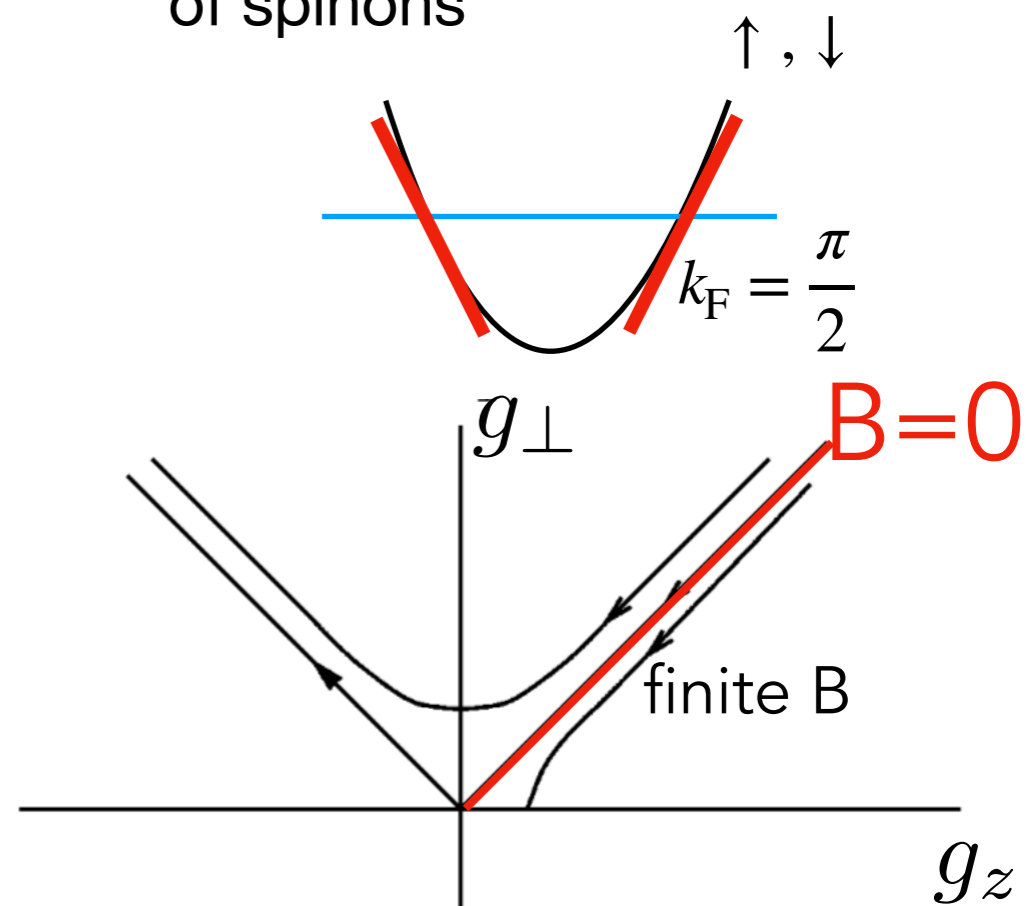
$$H = \underbrace{\int dx \left(\psi_R^\dagger (-iv_F \partial_x) \psi_R + \psi_L^\dagger (iv_F \partial_x) \psi_L \right)}_{H_0} - g \underbrace{\int dx \vec{J}_R \cdot \vec{J}_L}_V$$

Free fermions

$g > 0$: marginally irrelevant interaction of spin currents (spin backscattering)

$$g(\ell) = \frac{g(0)}{1 + g(0)\ell}, \quad \ell = \ln(J/E) \longrightarrow g(E \rightarrow 0) \rightarrow 1/\ln(J/E)$$

half-filled band of spinons



PHYSICAL REVIEW B

VOLUME 60, NUMBER 2

1 JULY 1999-II

Field-induced gap in Cu benzoate and other $S=1/2$ antiferromagnetic chains

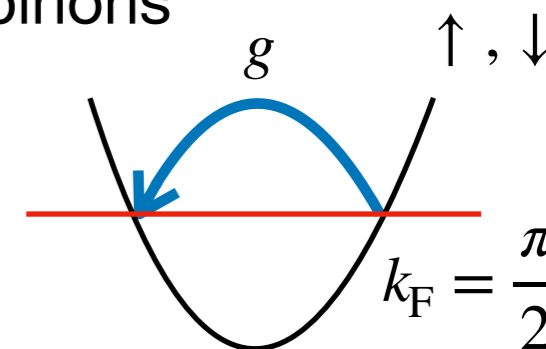
Ian Affleck
Department of Physics and Astronomy and Canadian Institute for Advanced Research, The University of British Columbia,
Vancouver, British Columbia, Canada V6T 1Z1

Masaki Oshikawa
Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551, Japan

Spin-1/2 antiferromagnetic chain

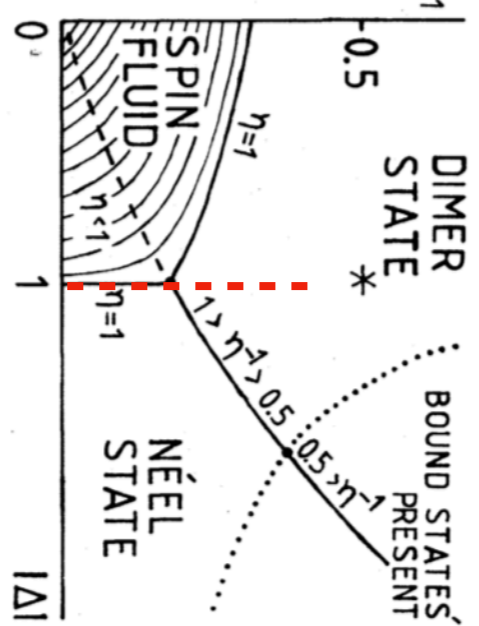
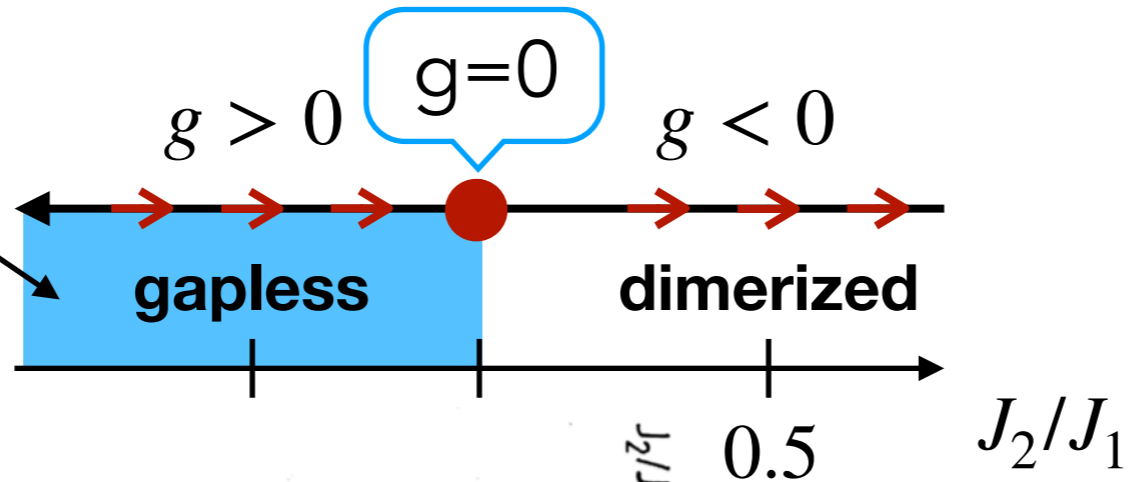
$$H = \int dx \left(\psi_R^\dagger (-iv_F \partial_x) \psi_R + \psi_L^\dagger (iv_F \partial_x) \psi_L \right) - g \int dx \vec{J}_R \cdot \vec{J}_L$$

half-filled band of spinons



$$\vec{J}_{R/L} = \frac{1}{2} \psi_{R/L}^\dagger \vec{\sigma} \psi_{R/L}$$

backscattering is marginally irrelevant!



Spontaneous dimerization in the $S = \frac{1}{2}$ Heisenberg antiferromagnetic chain with competing interactions

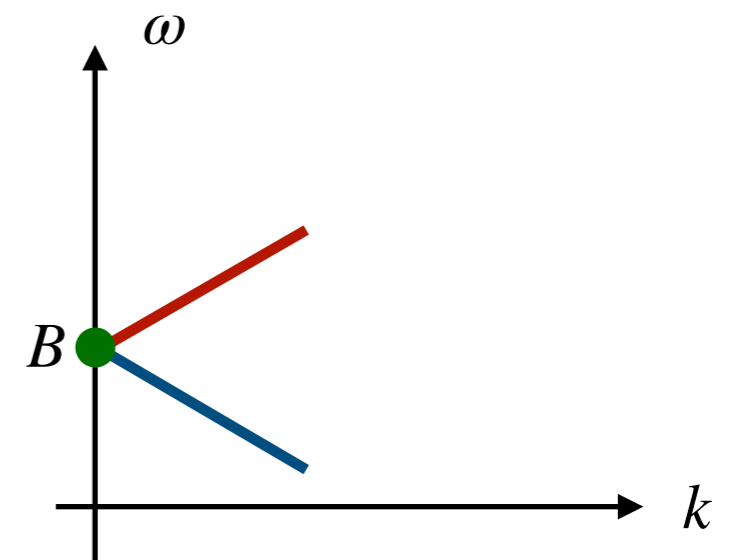
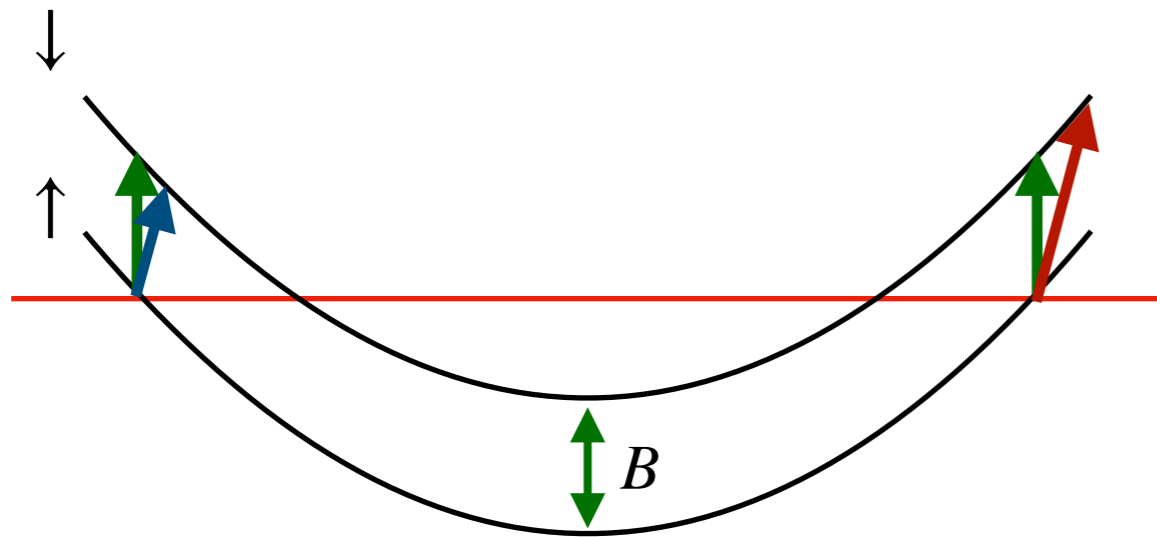
F. D. M. Haldane
 Department of Physics, University of Southern California, Los Angeles, California 90007
 (Received 5 February 1982)

Spontaneous dimerization is found in the $S = \frac{1}{2}$ isotropic Heisenberg antiferromagnetic chain with competing nearest- and next-nearest-neighbor exchange, $J_2/J_1 \geq \frac{1}{6}$, and results from the same umklapp processes that lead to the Néel state when easy-axis exchange anisotropy is present. Spontaneous and externally induced dimerizations are contrasted.

Spin-1/2 antiferromagnetic chain in magnetic field

Non-interacting limit ($g=0$) - small field splits the up/down bands

$$H = H_0 - B \int dx \left[\underbrace{J_R^z(x) + J_L^z(x)}_{\text{Magnetization}} \right]$$



Spin-1/2 Heisenberg chain in magnetic field

PHYSICAL REVIEW B, VOLUME 65, 134410

Electron spin resonance in $S = \frac{1}{2}$ antiferromagnetic chains

Masaki Oshikawa¹ and Ian Affleck^{2,*}

¹Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551, Japan

²Department of Physics, Boston University, Boston, Massachusetts 02215

(Received 13 August 2001; published 19 March 2002)

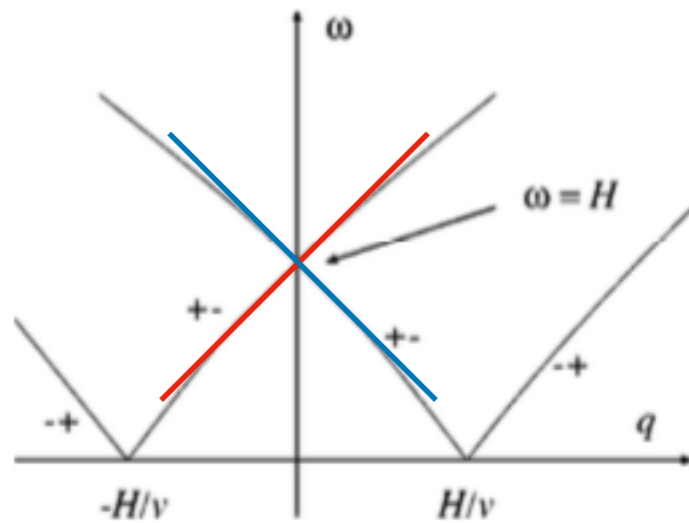


FIG. 2. The zero temperature transverse spin structure factor $S_{xx}(\omega, q) = S_{yy}(\omega, q)$ of the $S = 1/2$ Heisenberg antiferromagnetic chain under an applied field H , near $q = 0$. It is approximately proportional to $\omega[\delta(\omega - |q - H|) + \delta(\omega - |q + H|)]$, giving the resonance at $q = 0, \omega = H$. This consists of two branches coming from S_{+-} and S_{-+} , which are marked by $+-$ and $-+$ in the graph. In fact, there is a small spreading of the spectrum and the structure factor is generally not a perfect delta function. However, it is exactly the delta function $\delta(\omega - H)$ at $q = 0$, as explained in the text.

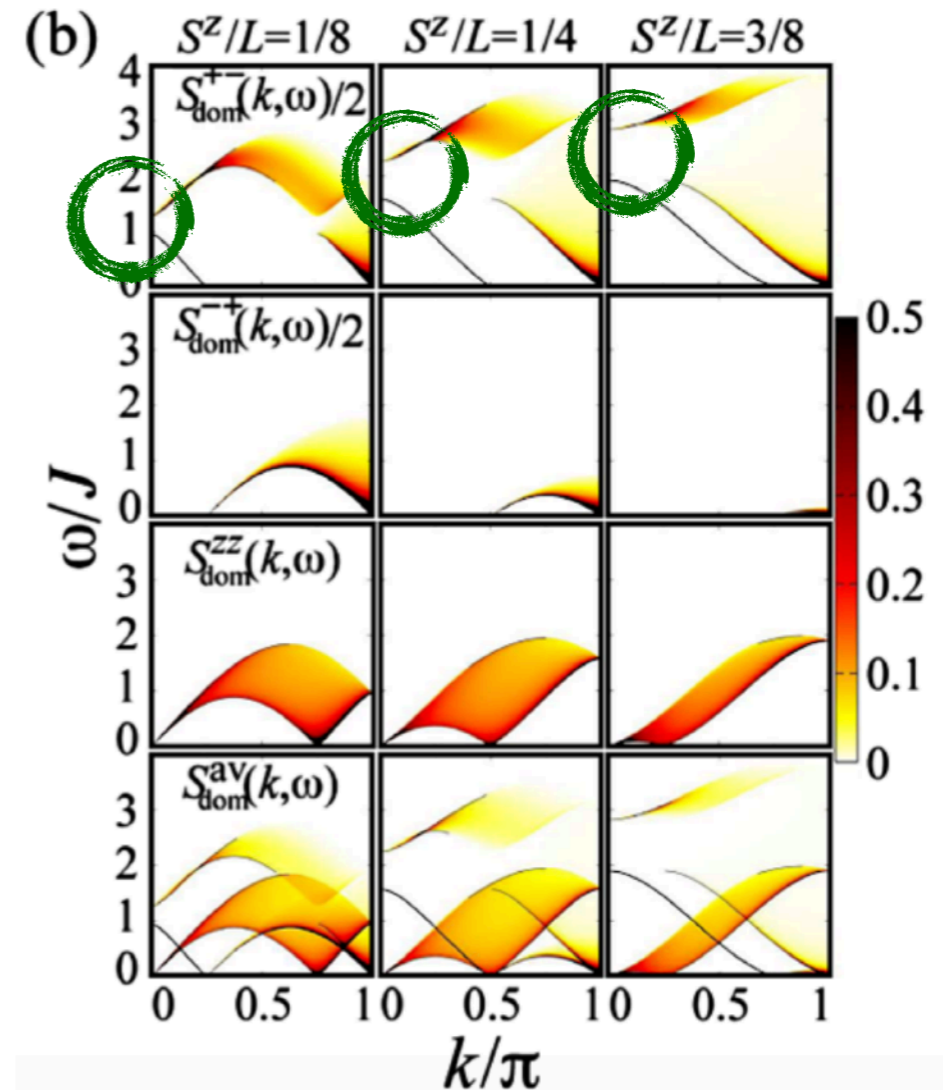
$$S_{xx}(\omega, q) = S_{yy}(\omega, q) \propto \omega[\delta(\omega - |q + H|) + \delta(\omega - |q - H|)].$$

BUT
???

Splitting
between
two
branches;
increases
with M

Dynamically Dominant Excitations of String Solutions in the Spin-1/2 Antiferromagnetic Heisenberg Chain in a Magnetic Field

Masanori Kohno
Phys. Rev. Lett. **102**, 037203 – Published 22 January 2009



Dynamical correlation functions of the $S=1/2$ nearest-neighbor and Haldane-Shastry Heisenberg antiferromagnetic chains in zero and applied fields

Kim Lefmann*

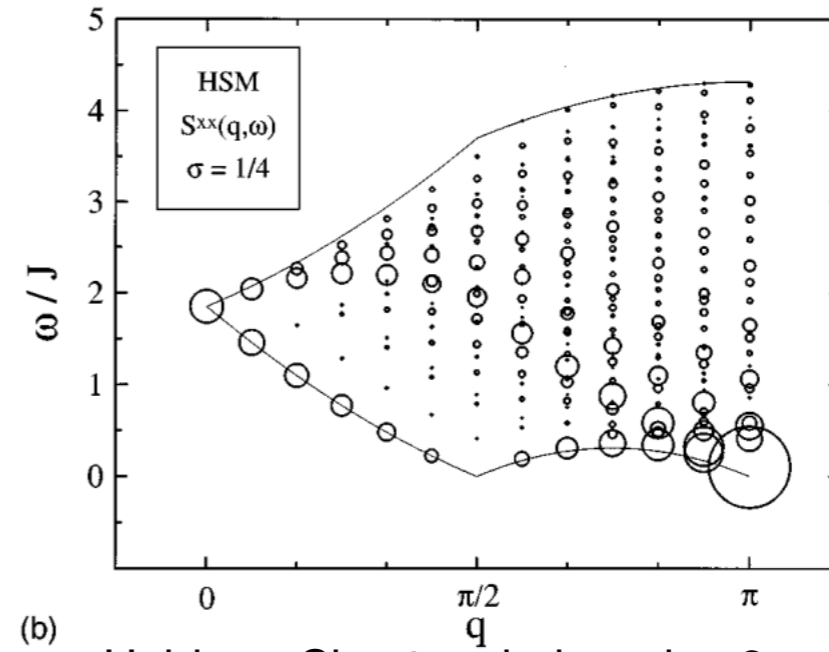
Department of Solid State Physics, Risø National Laboratory, DK-4000 Roskilde, Denmark

Christian Rischel†

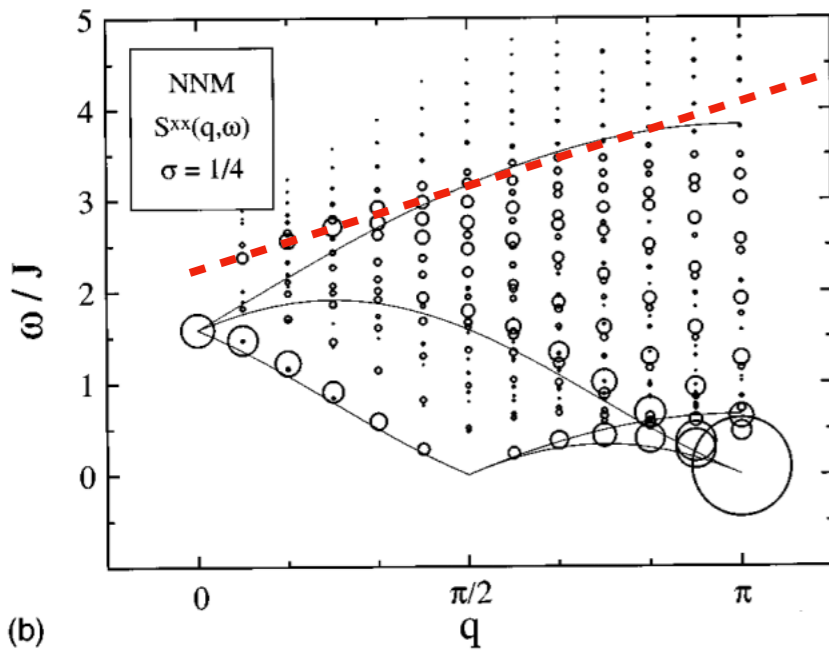
Ørsted Laboratory, Niels Bohr Institute, University of Copenhagen, DK-2100 København Ø, Denmark

(Received 12 February 1996)

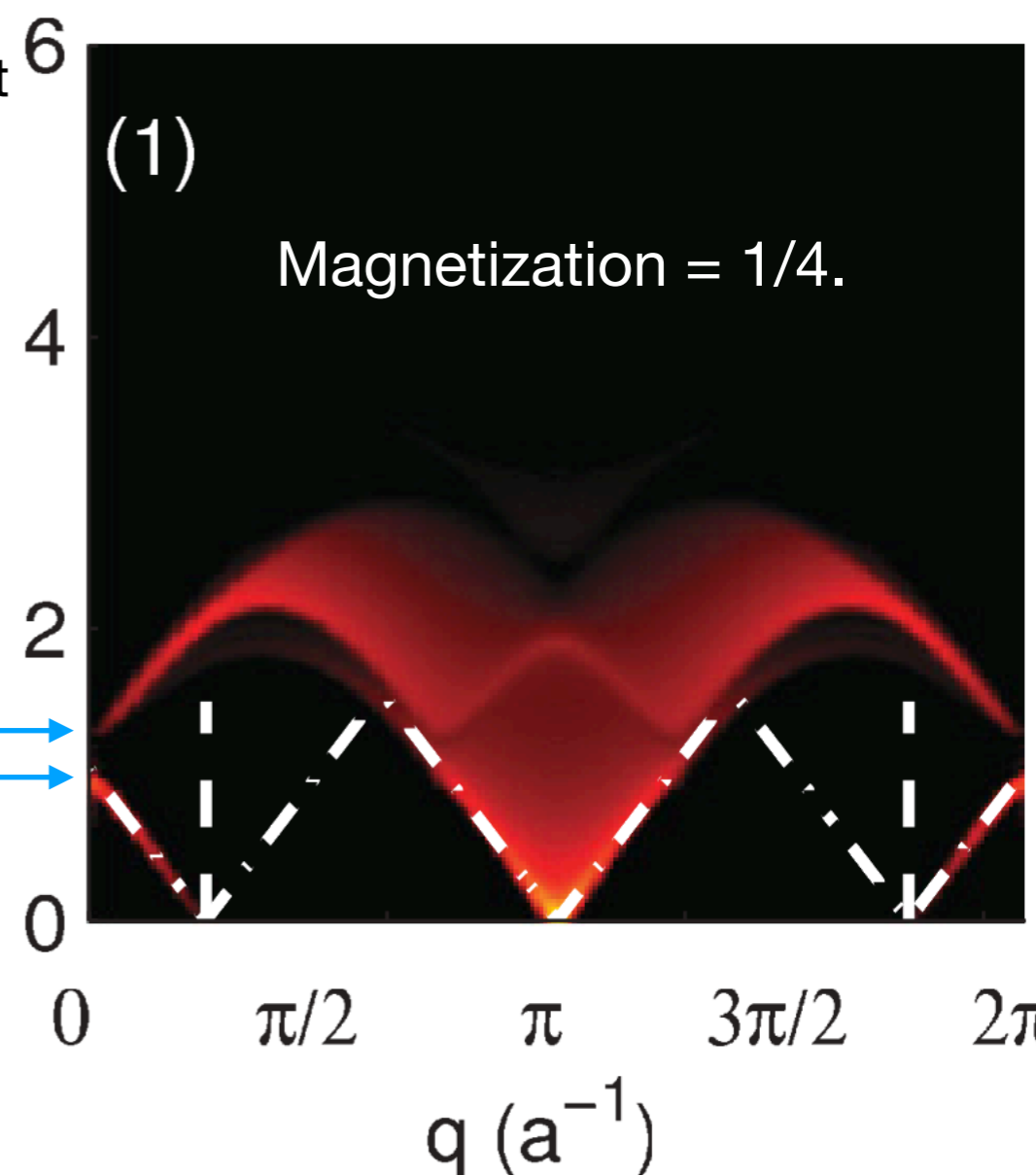
We present a numerical diagonalization study of two one-dimensional $S=1/2$ antiferromagnetic Heisenberg chains, having nearest-neighbor and Haldane-Shastry ($1/r^2$) interactions, respectively. We have obtained the $T=0$ dynamical correlation function, $S^{\alpha\alpha}(q, \omega)$, for chains of length $N=8-28$. We have studied $S^{zz}(q, \omega)$ for the Heisenberg chain in zero field, and from finite-size scaling we have obtained a limiting behavior that for large ω deviates from the conjecture proposed earlier by Müller *et al.* For both chains we describe the behavior of $S^{zz}(q, \omega)$ and $S^{xx}(q, \omega)$ for selected values of the applied field, and compare with previous work by Müller *et al.* and Talstra and Haldane. Suggestions for future finite-field neutron scattering experiments are made. [S0163-1829(96)00733-3]



(b) Haldane-Shastry chain - nice 2-spinon continuum



Heisenberg chain:
significant spectral weight
outside Muller continuum



(1)
Magnetization = 1/4.

Statics and dynamics of weakly coupled antiferromagnetic spin- $\frac{1}{2}$ ladders in a magnetic field

Pierre Bouillot, Corinna Kollath, Andreas M. Läuchli, Mikhail Zvonarev, Benedikt Thielemann, Christian Rüegg, Edmond Orignac, Roberta Citro, Martin Klanjšek, Claude Berthier, Mladen Horvatić, and Thierry Giamarchi
Phys. Rev. B **83**, 054407 – Published 9 February 2011

Spin backscattering remains present down to energy $E = B$

$$H = H_0 - g \int dx \left[J_R^z J_L^z + \frac{1}{2} J_R^+ J_L^- + \frac{1}{2} J_R^- J_L^+ \right] - B \int dx [J_R^z(x) + J_L^z(x)]$$

Transverse interaction
must restore Larmor Th

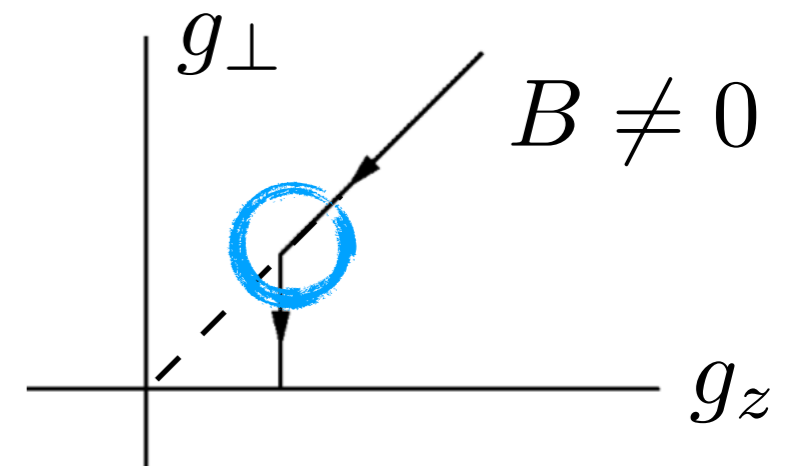
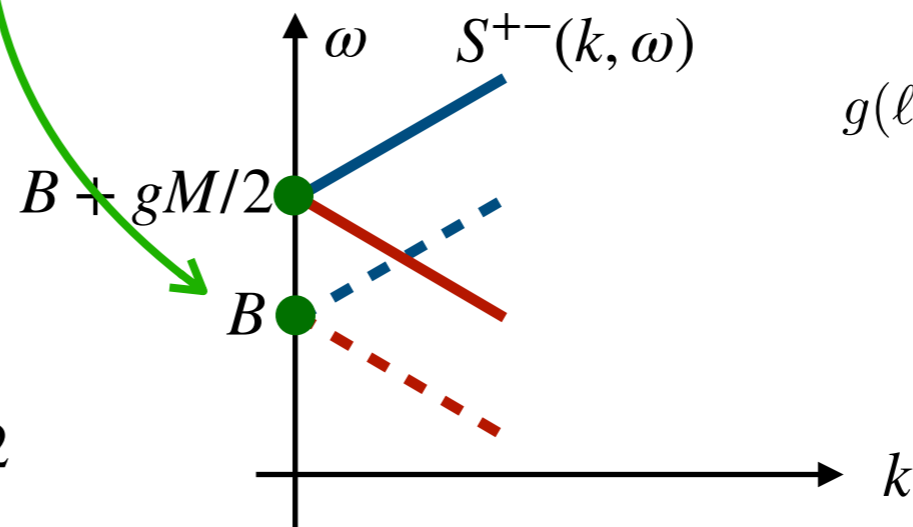
Self energy

$$-g \int dx \left[\langle J_L^z \rangle J_R^z + J_L^z \langle J_R^z \rangle \right]$$

$$\langle J_{L,R}^z \rangle = M/2$$

(magnetization)

shifts the mode at $k=0$: $B \rightarrow B + gM/2$



$$g(\ell) = \frac{g(0)}{1 + g(0)\ell}, \quad \ell = \ln(J/E)$$

$$g \rightarrow g(E = B)$$

- The essence — RPA-like treatment — Hubbard-Stratonovich decouple spin backscattering, integrate fermions out, expand fermion determinant about saddle point with finite magnetization, evaluate spin susceptibility.

Backscattering interaction

RPA-like treatment:

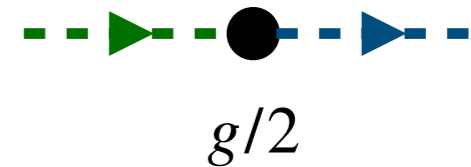
$$G_{\mu\nu}(x, \tau) = - \left\langle \hat{T}_\tau J_\mu^+(x, \tau) J_\nu^-(0, 0) \right\rangle$$

$$G_{RR}^0 \quad \text{---} \blacktriangleright \text{---}$$

$$G_{LL}^0 \quad \text{---} \blacktriangleright \text{---}$$

$$G_{RL}^0 = G_{LR}^0 = 0$$

$$V_{int, \perp} = -\frac{g}{2} \int dx [J_R^+ J_L^- + J_R^- J_L^+]$$



$$G_{RR} \quad \text{---} \blacktriangleright \text{---} = \overset{R}{\text{---} \blacktriangleright \text{---}} \overset{R}{\text{---} \blacktriangleright \text{---}} + \overset{R}{\text{---} \blacktriangleright \text{---}} \overset{R}{\text{---} \blacktriangleright \text{---}} \overset{L}{\bullet} \overset{L}{\text{---} \blacktriangleright \text{---}} \overset{L}{\bullet} \overset{R}{\text{---} \blacktriangleright \text{---}} \overset{R}{\text{---} \blacktriangleright \text{---}}$$

$$G_{RR} = \frac{G_{RR}^0}{1 - \frac{g^2}{4} G_{RR}^0 G_{LL}^0}$$

$$G(k, \omega_n) = G_{RR} + G_{LL} + G_{LR} + G_{RL} = \frac{G_{RR}^0 + G_{LL}^0 - g G_{RR}^0 G_{LL}^0}{1 - \frac{g^2}{4} G_{RR}^0 G_{LL}^0} \rightarrow \chi^\pm(k, \omega) = G(k, \omega + i0)$$

The result: Dynamical susceptibility of interacting spinon liquid

$$\chi^\pm(k, \omega) = M \left(\frac{A_+(k)}{\omega - \omega_+(k)} + \frac{A_-(k)}{\omega - \omega_-(k)} \right)$$

$$\tilde{v} = v \sqrt{1 - g^2 \chi_0^2 / 4}$$

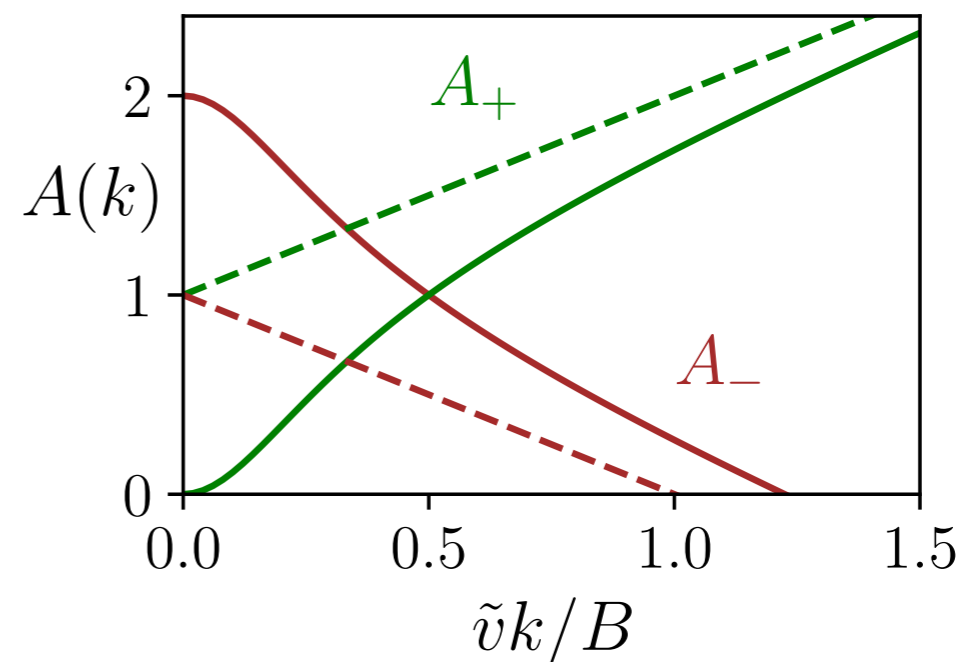
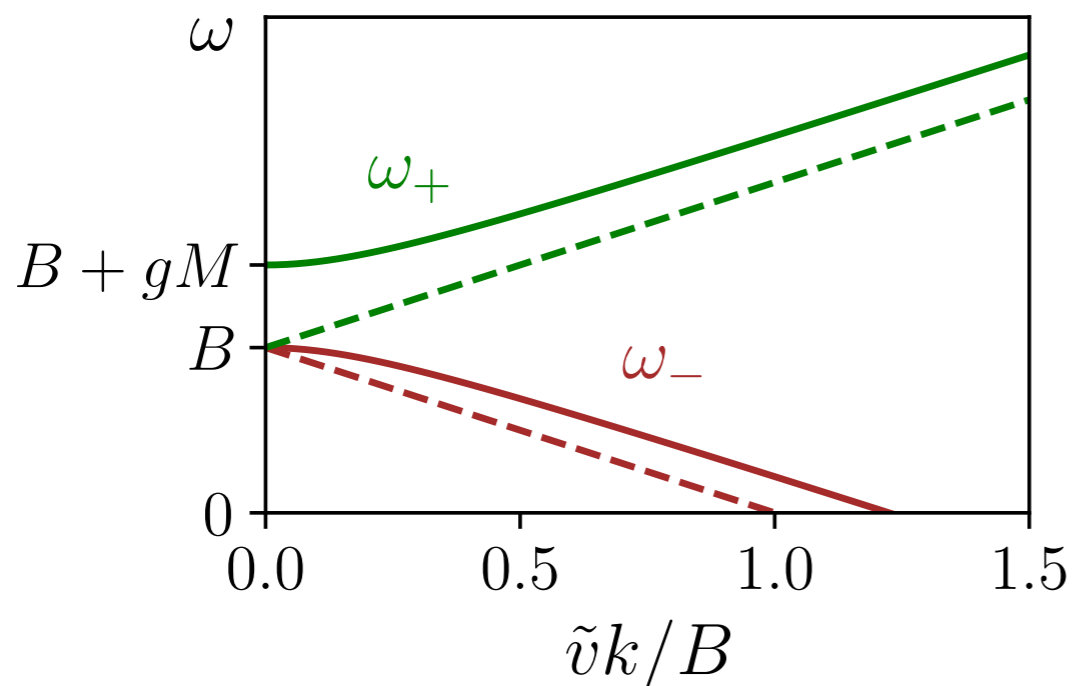
$$\chi = M/B = \frac{\chi_0}{1 - g\chi_0/2}$$

Dispersion

$$\omega_\pm(k) = B + \Delta \pm \sqrt{\Delta^2 + \tilde{v}^2 k^2}$$

Spectral weight

$$A_\pm(k) = 1 \pm \frac{\tilde{v}^2 k^2 - B\Delta}{B\sqrt{\Delta^2 + \tilde{v}^2 k^2}}$$



Dashed lines: free spinon gas ($g=0$)

Dynamical susceptibility - numerics



$$T = 0$$

$$S^{+-}(\vec{k}, \omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \int d\vec{r} e^{-i\vec{k} \cdot \vec{r}} \langle 0 | S_r^+(t) S_0^-(0) | 0 \rangle$$

The ground state
can be obtained
using DMRG

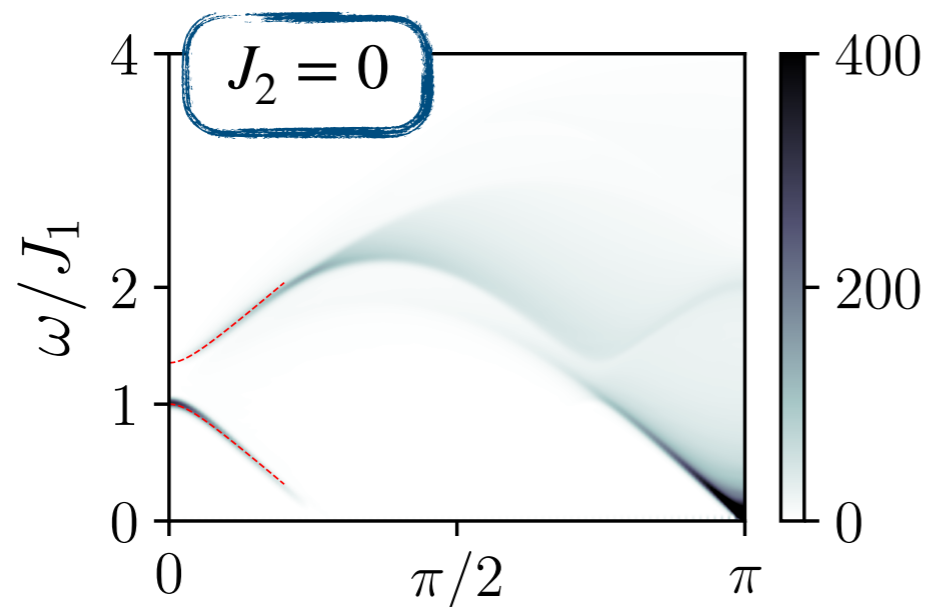
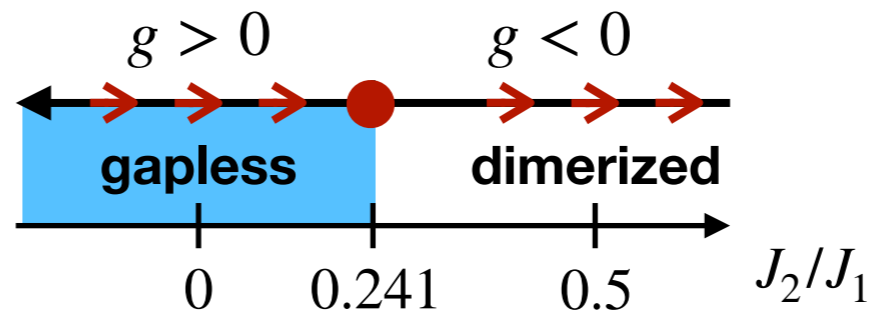
$$\langle 0 | e^{iHt} S_r^+ e^{-iHt} S_0^- | 0 \rangle = e^{iE_0 t} \langle 0 | S_r^+ e^{-iHt} S_0^- | 0 \rangle$$

Time evolution of a quenched state -
numerous MPS-based techniques:

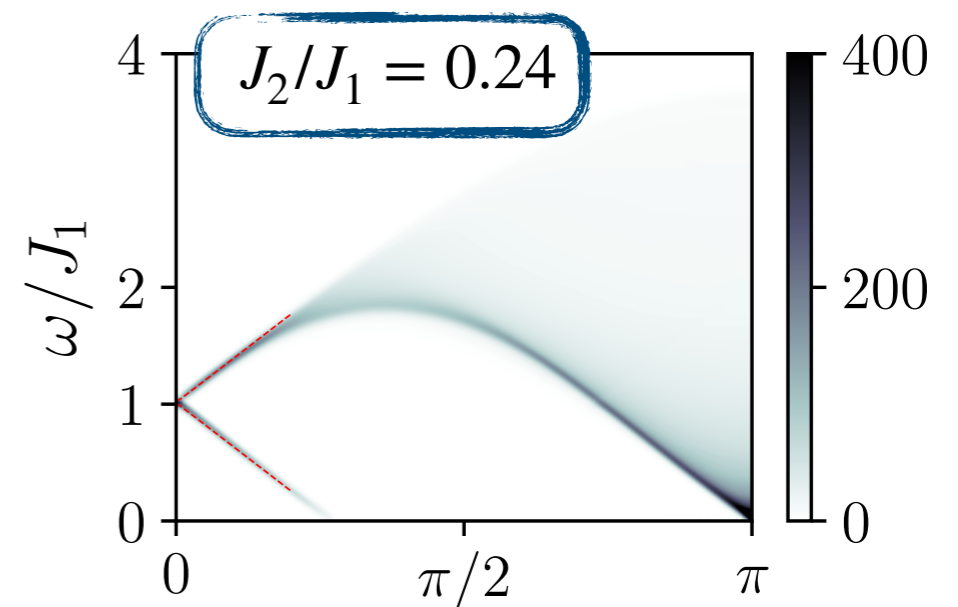
TEBD (in 1D), tDMRG, TDVP,
MPO representation of the evolution
operator (also for long range or 2D!)

Numerical results

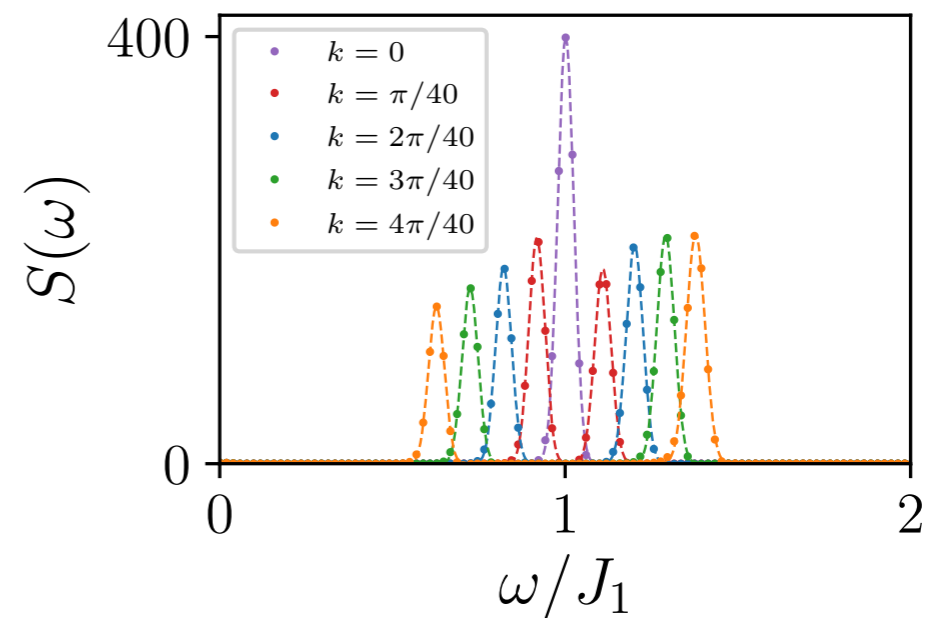
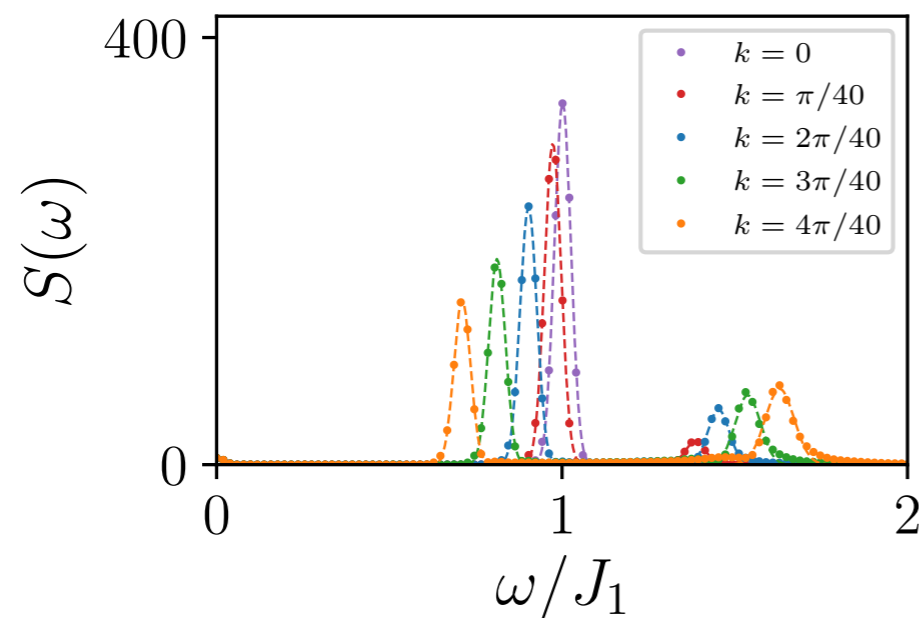
$$B/J_1 = 1$$



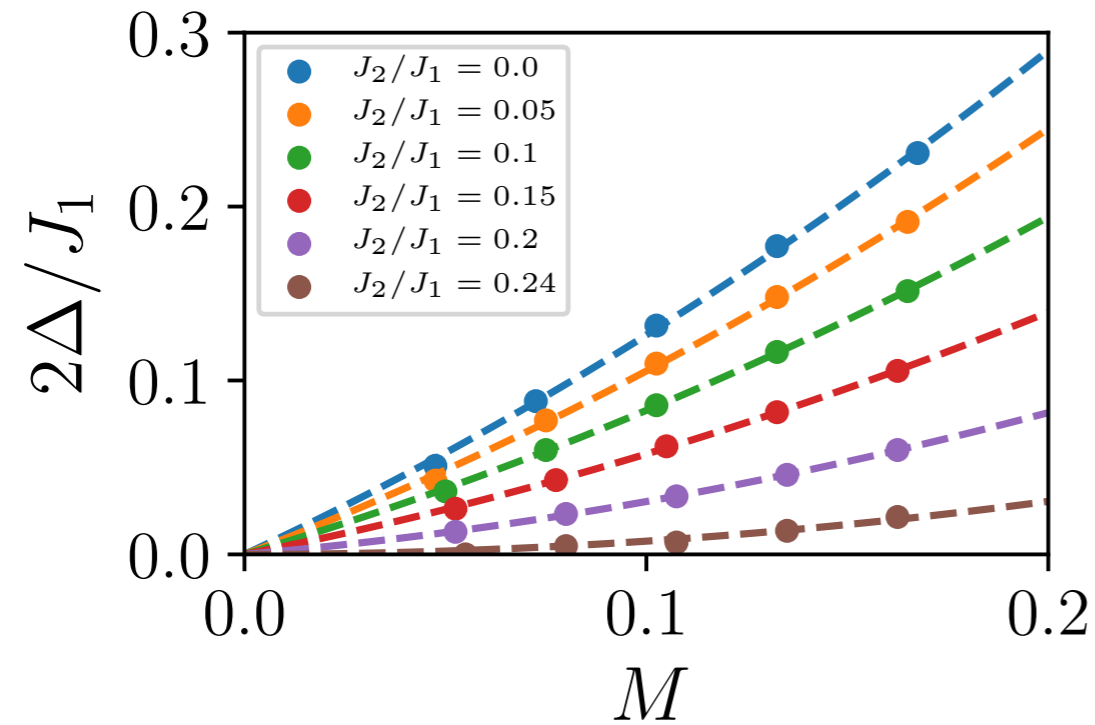
interacting spinon liquid k



spinon gas k

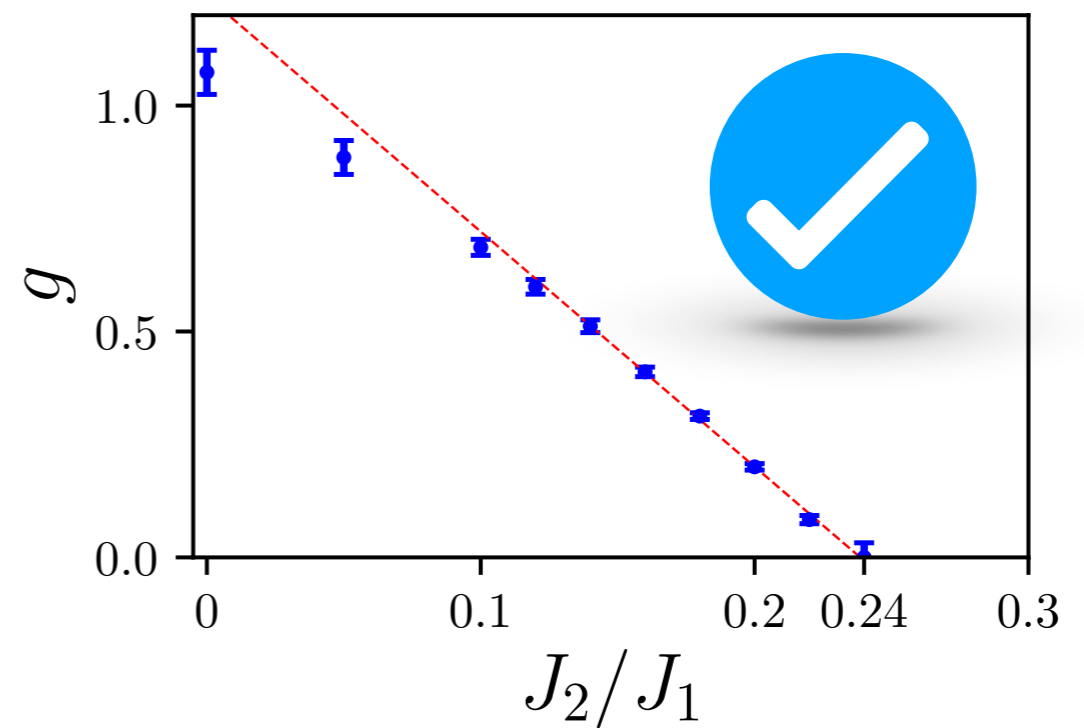
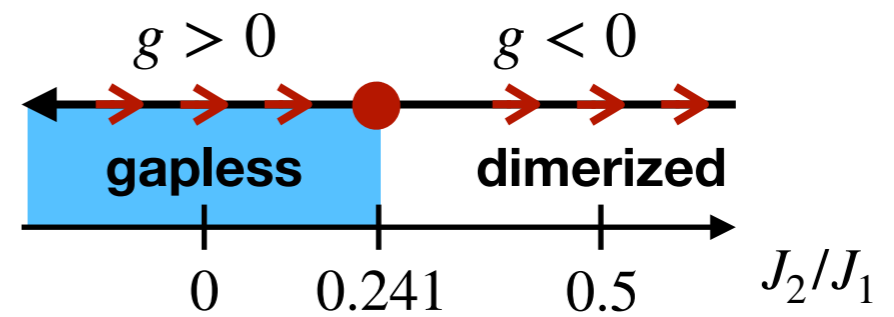


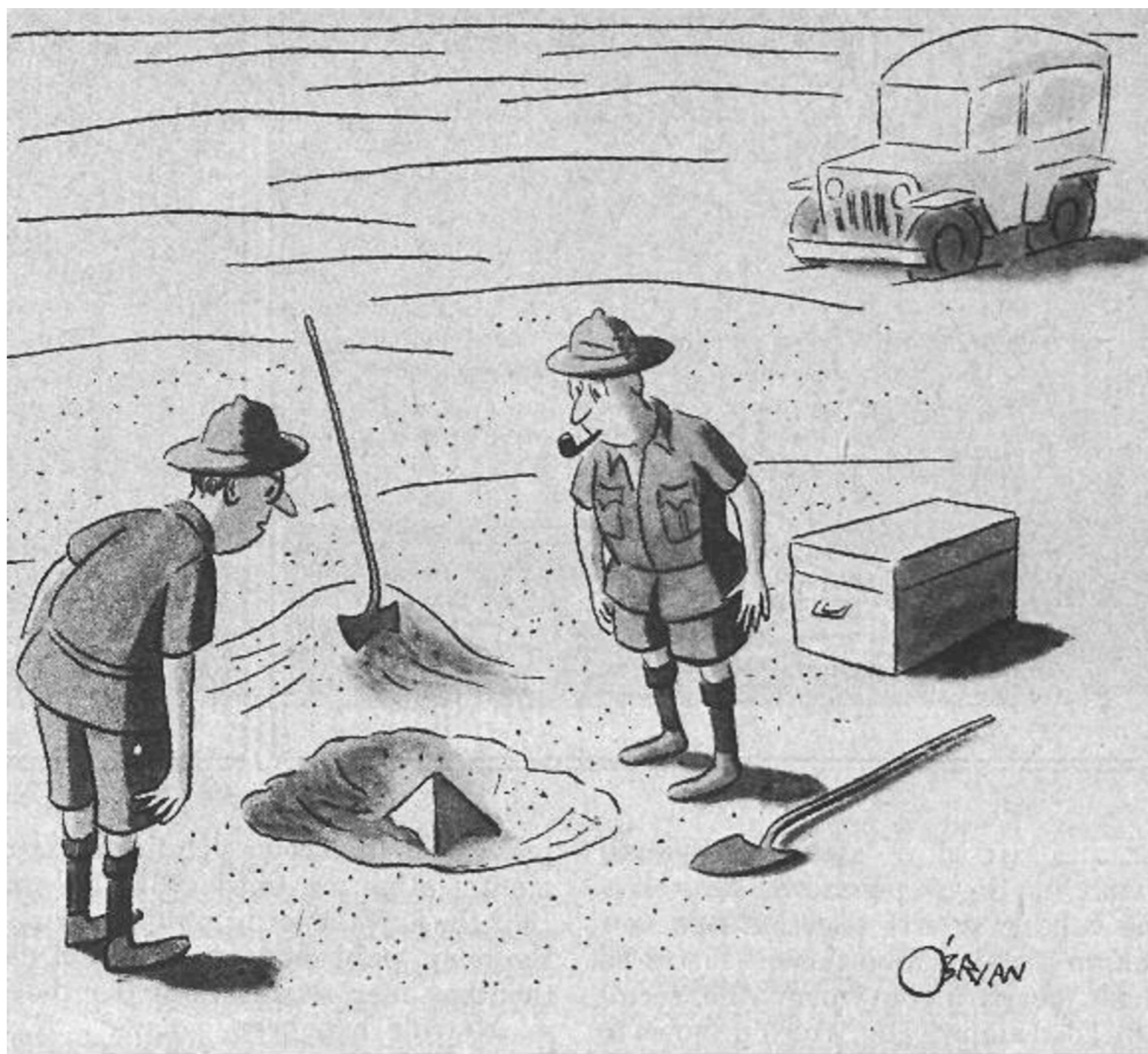
Numerical results



$$2\Delta/J_1 = g(J_2/J_1)M + O(M^2)$$

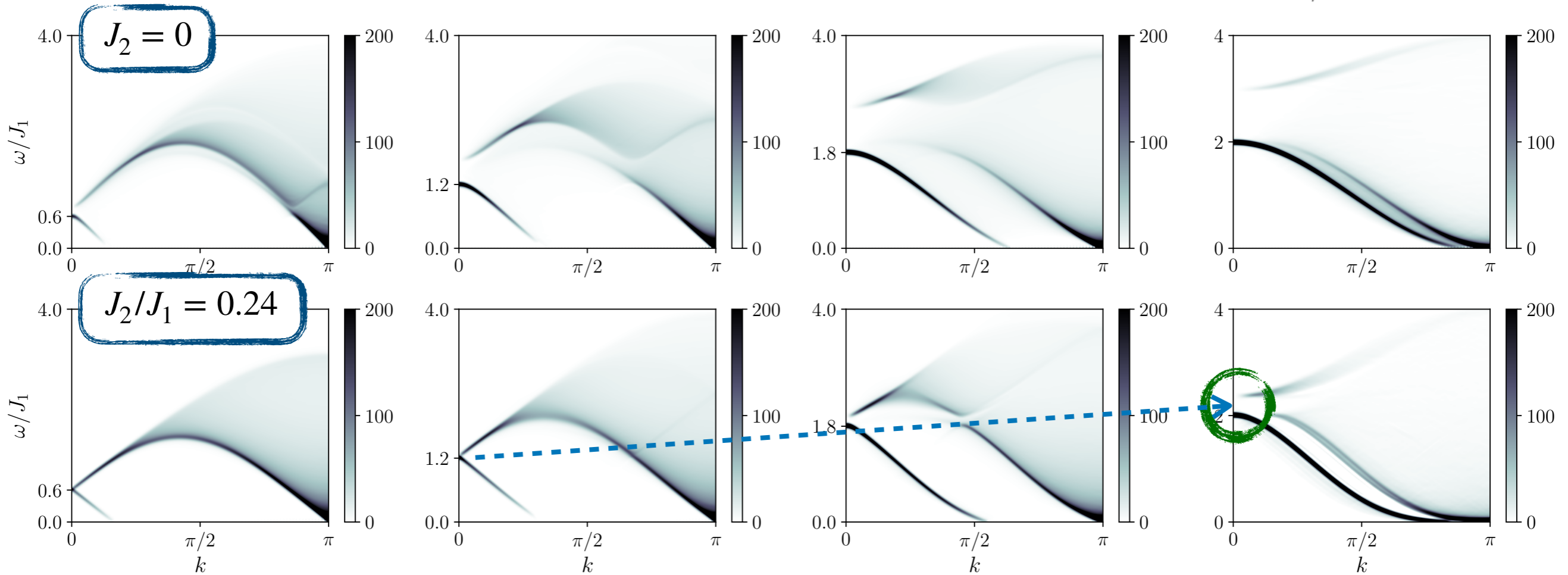
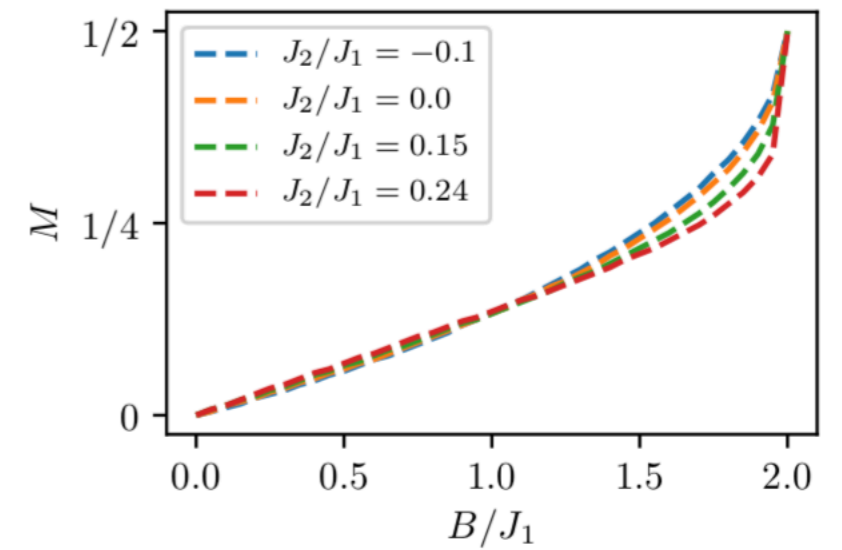
It works!



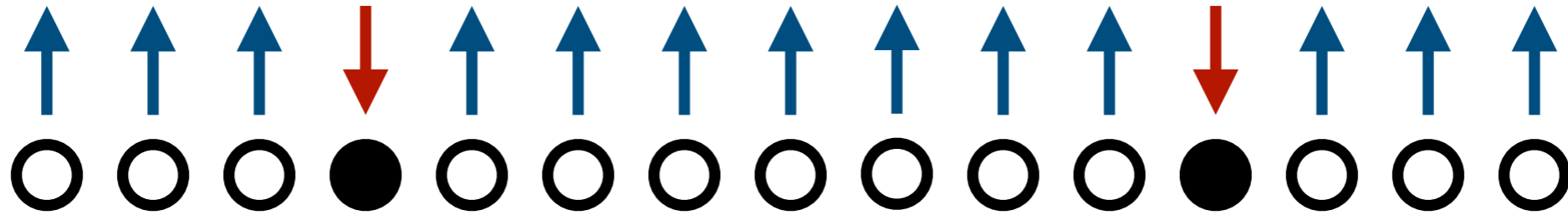


"This could be the discovery of the century. Depending, of course, on how far down it goes."

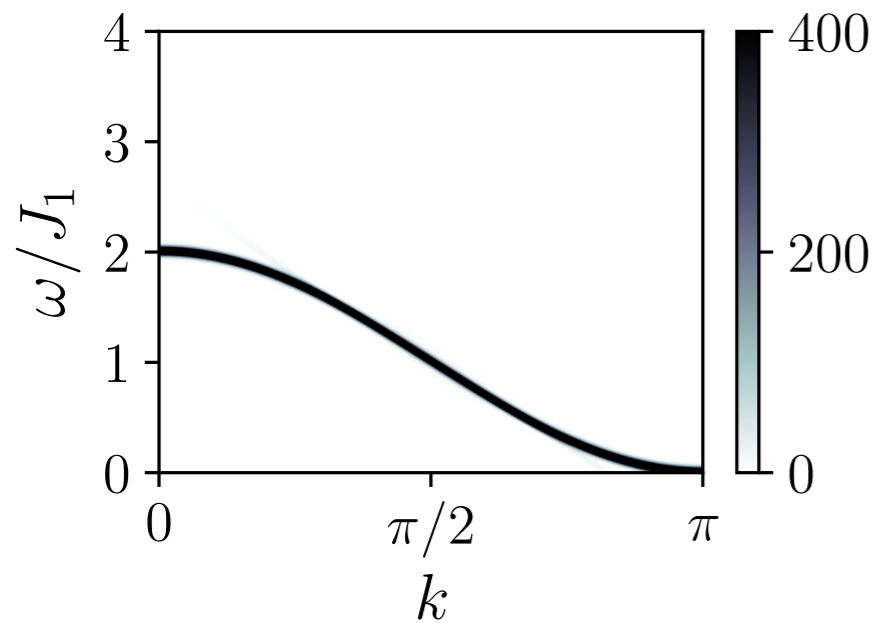
Increasing the magnetic field



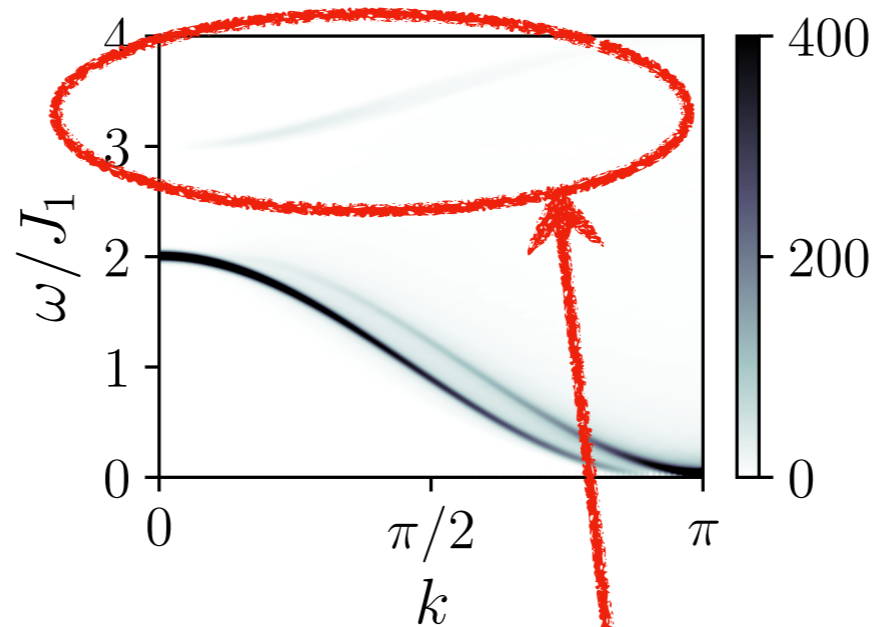
Dynamical susceptibility in the large magnetization limit: The limit of interacting **magnons**!



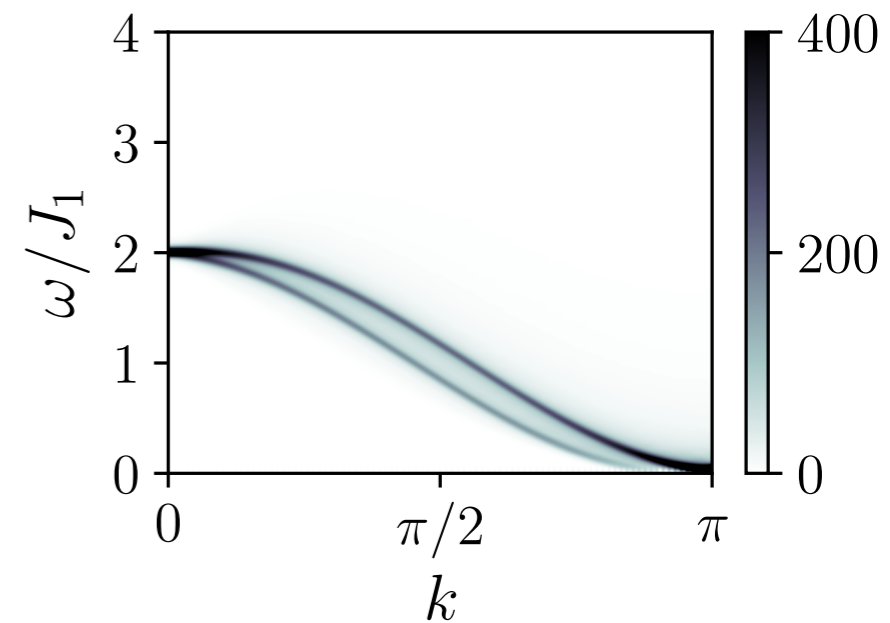
$$J^z = J, M = M_{sat}$$



$$J^z = J, M = 0.9M_{sat}$$

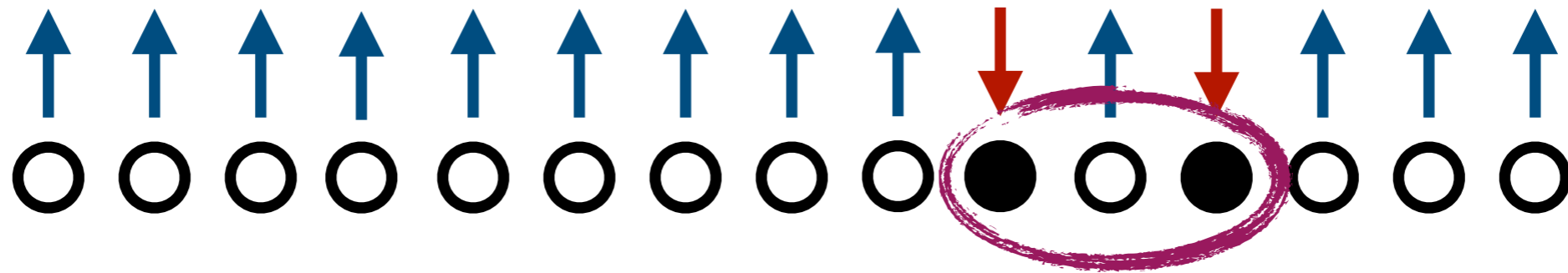


$$J^z = 0, M = 0.9M_{sat}$$



Interactions effect!

$B > B_{\text{sat}}$: 2-magnon (anti-)bound states



Schrodinger equation
for 2-magnon states

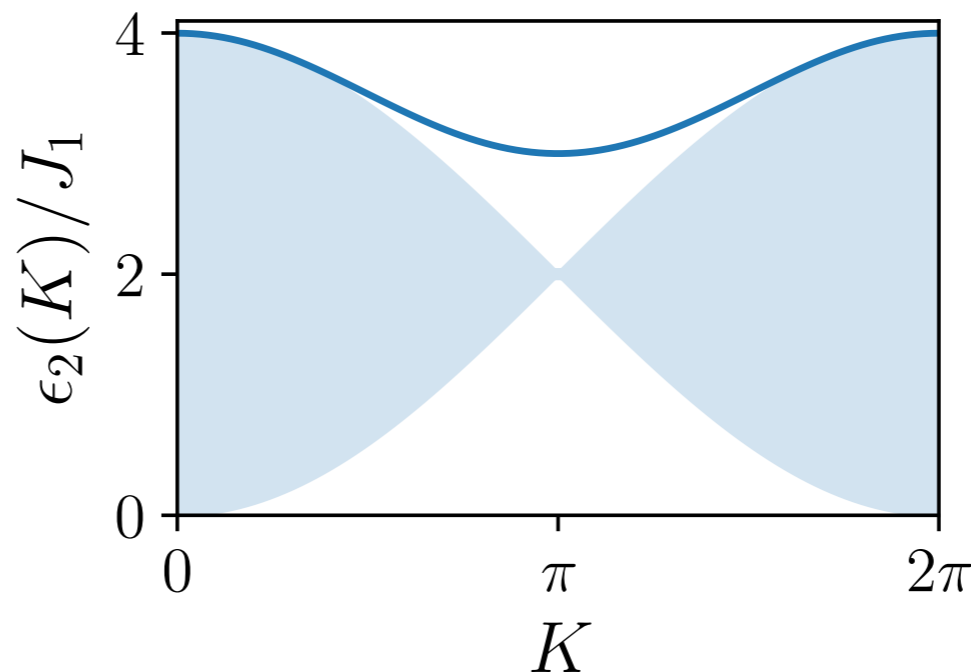
$$|2\rangle = \sum_{n,m} \Psi_{n,m} S_n^- S_m^- |0\rangle$$

fully polarized state

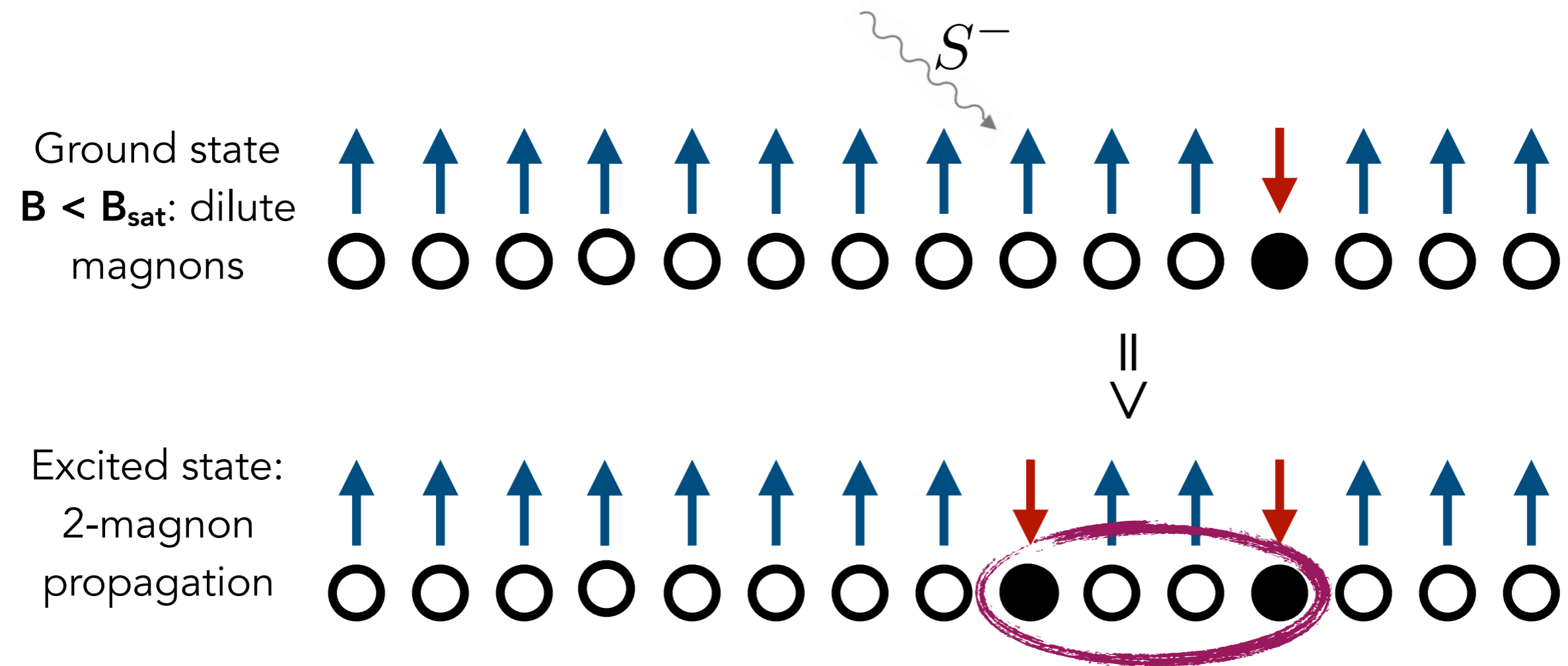
$$\Psi_{n,m} = e^{iK(n+m)/2} f(|n-m|)$$

center of mass momentum

can show there is a bound state above the 2-magnon continuum!



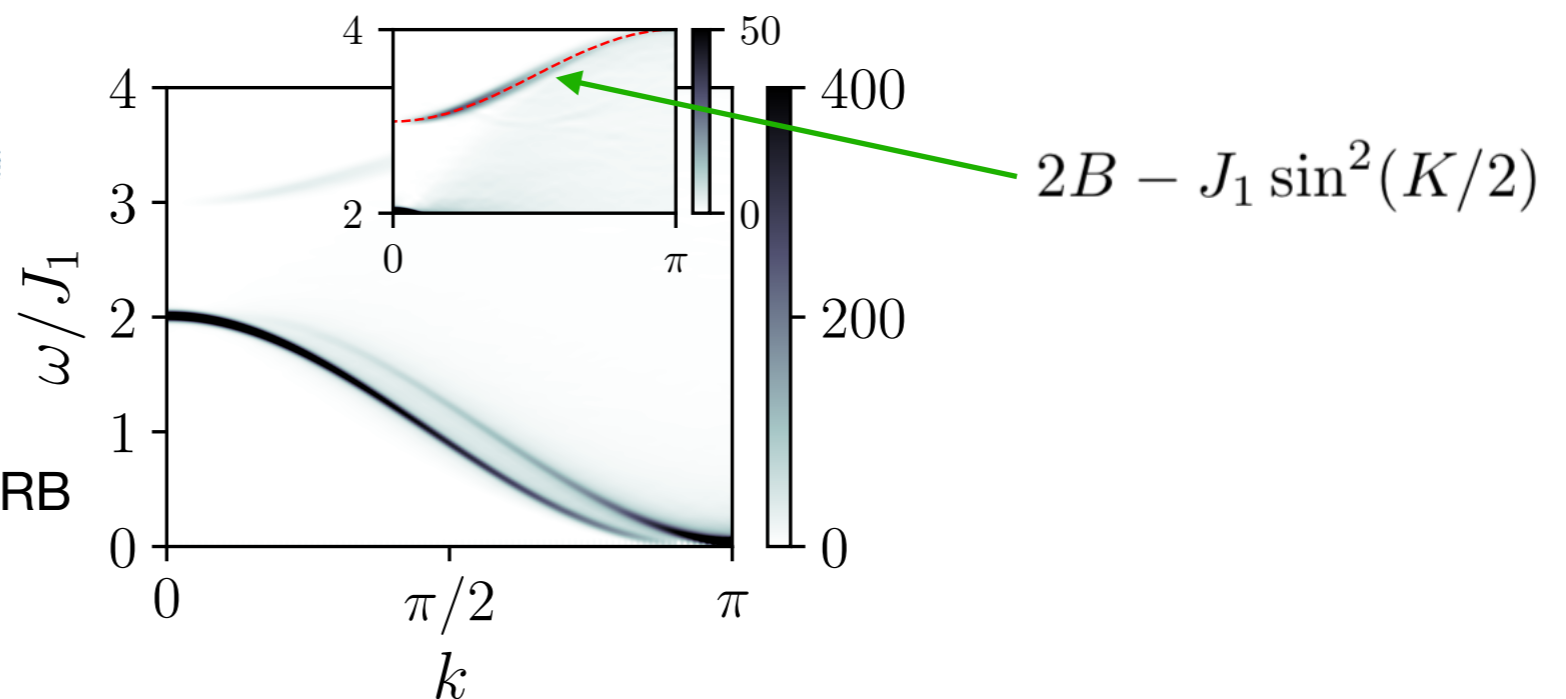
2-magnon (anti-)bound states: *probe magnon binds* with one of the magnons in the ground state



$$J_2 = 0$$

This is 2-string solution of the Bethe ansatz!

see also Kohno PRL 2009, Yang et al. PRB



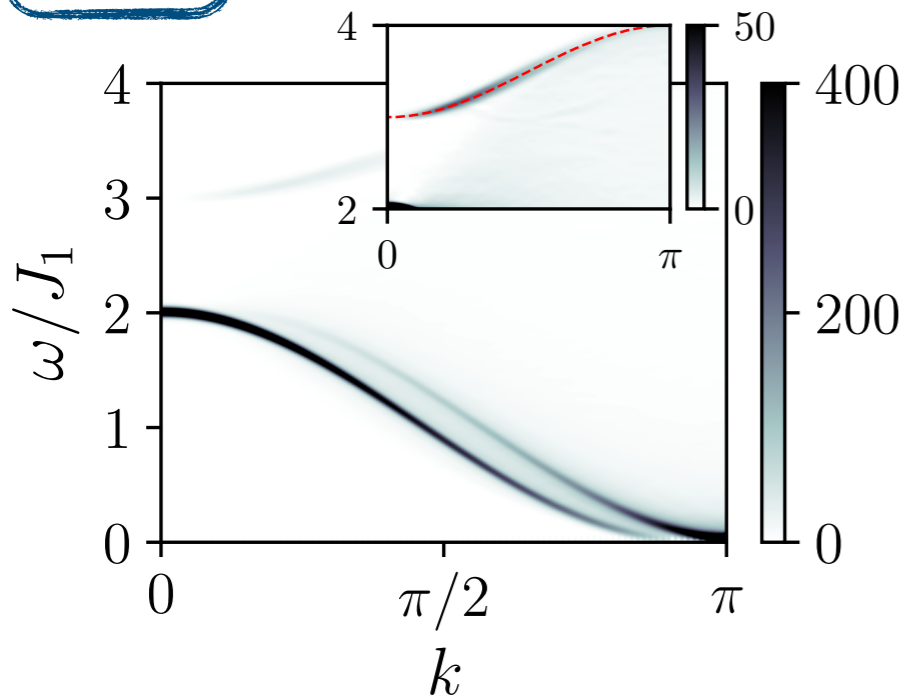
How does the 2-magnon bound state show up in the dynamical correlations?

Assume a single magnon (at $k = \pi$) in the ground state, i.e. $|0\rangle = |1_\pi\rangle$

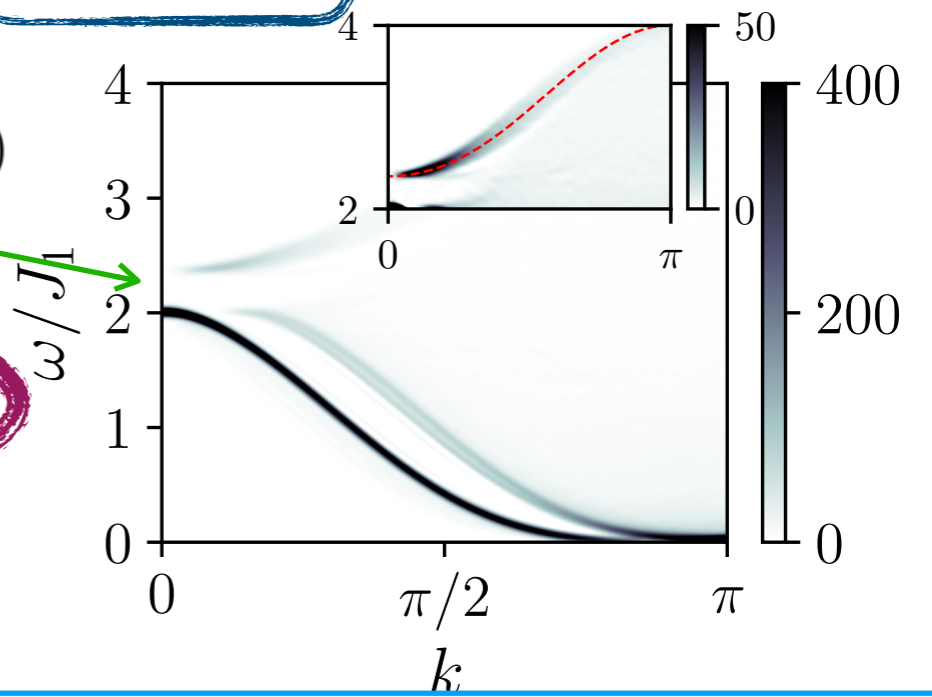
$$S^{+-}(k, \omega) = \sum_m \left| \langle m | S_k^- | 0 \rangle \right|^2 \delta(\omega - E_m) = \left| \langle 2_{\pi+k} | S_k^- | 1_\pi \rangle \right|^2 \delta(\omega - \epsilon_2(\pi + k)) + \dots$$

Scattering and 2-magnon states

$J_2 = 0$



$J_2/J_1 = 0.24$



$$J_1 - 3J_2 + J_2^2 / (J_1 - J_2)$$

* also valid for $J_2 \neq 0$!

Works in higher dimensions too!
(you can ask me about this)

Independent arguments for spin ladder:
Magnetic-Field-Induced Bound States in Spin- $\frac{1}{2}$ Ladders
Mithilesh Nayak, Dominic Blosser, Andrey Zheludev, and Frédéric Mila
Phys. Rev. Lett. **124**, 087203 – Published 26 February 2020

Need experiments!

Extended Quantum Critical Phase in a Magnetized Spin- $\frac{1}{2}$ Antiferromagnetic Chain

M. B. Stone,^{1,*} D. H. Reich,¹ C. Broholm,^{1,2} K. Lefmann,³ C. Rischel,⁴ C. P. Landee,⁵ and M. M. Turnbull⁵

¹Department of Physics and Astronomy, Johns Hopkins University, Baltimore, Maryland 21218, USA

²National Institute of Standards and Technology, Gaithersburg, Maryland 20899, USA

³Materials Research Department, Risø National Laboratory, DK-4000 Roskilde, Denmark

⁴Ørsted Laboratory, Niels Bohr Institute, University of Copenhagen, DK-2100, København Ø, Denmark

⁵Carlson School of Chemistry and Department of Physics, Clark University, Worcester, Massachusetts 01610, USA

(Received 18 March 2003; published 17 July 2003)

Measurements are reported of the magnetic field dependence of excitations in the quantum critical state of the spin $S = 1/2$ linear chain Heisenberg antiferromagnet copper pyrazine dinitrate (CuPzN). The complete spectrum was measured at $k_B T/J \leq 0.025$ for $H = 0$ and $H = 8.7$ T, where the system is $\sim 30\%$ magnetized. At $H = 0$, the results are in agreement with exact calculations of the dynamic spin correlation function for a two-spinon continuum. At $H = 8.7$ T, there are multiple overlapping continua with incommensurate soft modes. The boundaries of these continua confirm long-standing predictions, and the intensities are consistent with exact diagonalization and Bethe ansatz calculations.

DOI: 10.1103/PhysRevLett.91.037205

PACS numbers: 75.10.Jm, 75.40.Gb, 75.50.Ec

Looks like an upper branch!?

Finally, we note that Fig. 4(a) shows some evidence of weak scattering intensity for $\hbar\omega > 2$ meV. This could be due to the presence of short chains resulting from impurities, or to higher-order processes not included in the spinon/pson picture. However, we note that our error bars are much larger here than at lower energy due to shorter counting times (see Fig. 2), and so a definitive statement on the existence of excitations in this energy range cannot be made at this time.

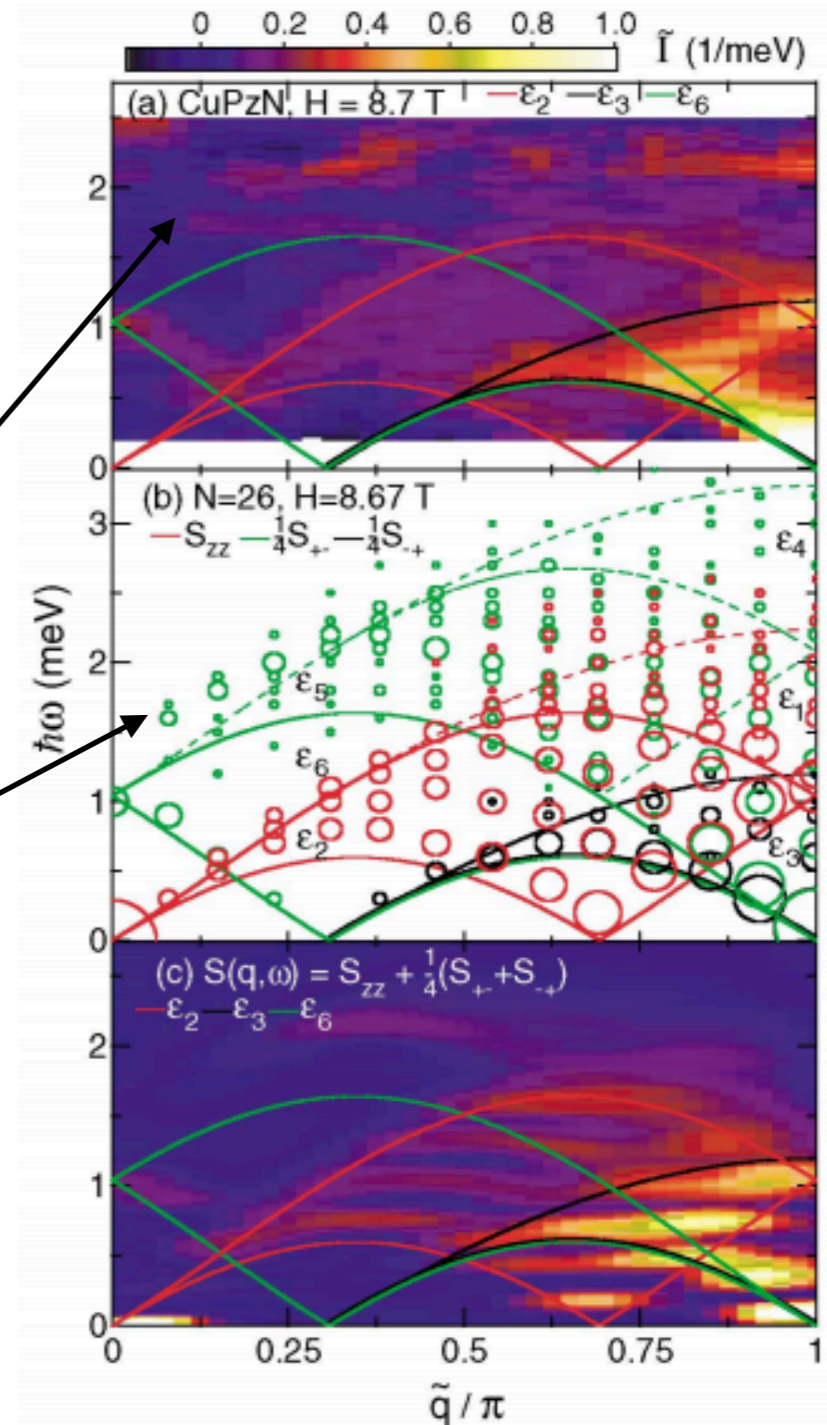


FIG. 4 (color). (a) Inelastic neutron scattering intensity $\tilde{I}_m(\vec{q}, \omega)$ for CuPzN at $T = 0.25$ K and $H = 8.7$ T. (b) Calculations of the different components of $S(\vec{q}, \omega)$ for $N = 26$ spins and $m = 2/13$. The area of each circle is proportional to $S(\vec{q}, \omega)$. (c) $\tilde{I}_m(\vec{q}, \omega)$ calculated for ensemble of chains with $N = 24, 26,$ and 28 . The curves in (a)–(c) show the bounds of the excitation continua ϵ_1 – ϵ_6 . Solid lines: Continua predicted to predominate as $N \rightarrow \infty$. In (b), ϵ_2 (upper) = ϵ_1 (lower).


String solutions

Dispersions of Many-Body Bethe strings

Anup Kumar Bera^{1,2*}, Jianda Wu^{3*}, Wang Yang^{4*}, Zhe Wang⁵, Robert Bewley,⁶ Martin Boehm,⁷ Maciej Bartkowiak,¹ Oleksandr Prokhnenko,¹ Bastian Klemke,¹ A. T. M. Nazmul Islam,¹ Joseph Mathew Law,⁸ Bella Lake^{1,9,*}

$\text{SrCo}_2\text{V}_2\text{O}_2$
Neutron scattering

Experimental observation of Bethe strings

Zhe Wang , Jianda Wu, Wang Yang, Anup Kumar Bera, Dmytro Kamenskyi, A. T. M. Nazmul Islam, Shenglong Xu, Joseph Matthew Law, Bella Lake, Congjun Wu & Alois Loidl

THz spectroscopy

Nature **554**, 219–223(2018) | [Cite this article](#)

Cold atoms

Observation of Complex Bound States in the Spin-1/2 Heisenberg XXZ Chain Using Local Quantum Quenches

Martin Ganahl, Elias Rabel, Fabian H. L. Essler, and H. G. Evertz
Phys. Rev. Lett. **108**, 077206 – Published 17 February 2012

Microscopic observation of magnon bound states and their dynamics

Takeshi Fukuhara , Peter Schauß, Manuel Endres, Sebastian Hild, Marc Cheneau, Immanuel Bloch & Christian Gross

Nature **502**, 76–79(2013) | [Cite this article](#)

ESR: Spinon magnetic resonance

PRL 107, 037204 (2011)

PHYSICAL REVIEW LETTERS

week ending
15 JULY 2011

Modes of Magnetic Resonance in the Spin-Liquid Phase of Cs_2CuCl_4

K. Yu. Povarov,^{1,*} A. I. Smirnov,^{1,2} O. A. Starykh,^{3,4} S. V. Petrov,¹ and A. Ya. Shapiro⁵

¹*P. L. Kapitza Institute for Physical Problems, RAS, 119334 Moscow, Russia*

²*Moscow Institute for Physics and Technology, 141700, Dolgoprudny, Russia*

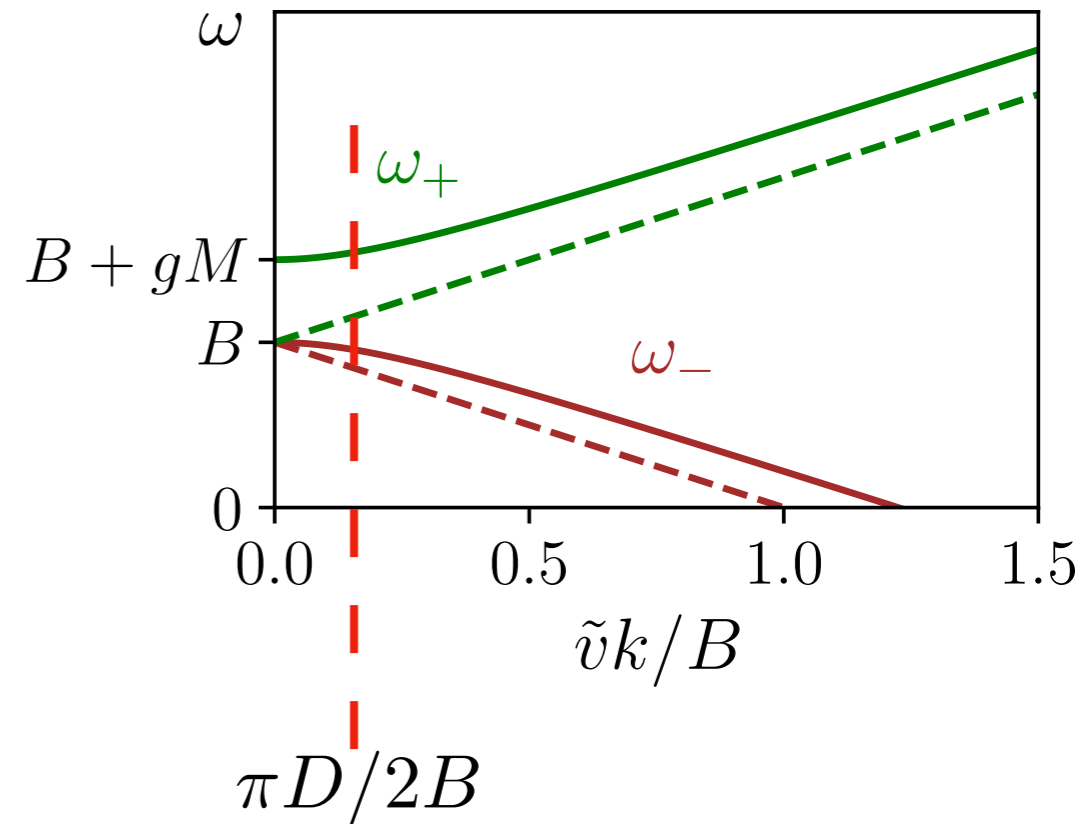
³*Department of Physics and Astronomy, University of Utah, Salt Lake City, Utah 84112, USA*

⁴*Max-Planck-Institut für Physik Komplexer Systeme, D-01187 Dresden, Germany*

⁵*A. V. Shubnikov Institute of Crystallography, RAS, 119333 Moscow, Russia*

(Received 7 February 2011; revised manuscript received 23 May 2011; published 14 July 2011)

We report the observation of a frequency shift and splitting of the electron spin resonance (ESR) mode of the low-dimensional $S = 1/2$ frustrated antiferromagnet Cs_2CuCl_4 in the spin-correlated state above the ordering temperature 0.62 K. The shift and splitting exhibit strong anisotropy with respect to the direction of the applied magnetic field and do not vanish in a zero field. The low-temperature evolution of the ESR is a result of the modification of the one-dimensional spinon continuum by the uniform Dzyaloshinskii-Moriya interaction within the spin chains.



Heisenberg chain with **uniform DM**

$$\mathcal{H} = \sum_n J \vec{S}_n \cdot \vec{S}_{n+1} - \vec{D} \cdot \vec{S}_n \times \vec{S}_{n+1} - BS_n^z$$

for $B \parallel D$ maps onto $\left[S_n^+ = \tilde{S}_n^+ e^{iQn}, Q = \tan^{-1}(D/J), S_n^z = \tilde{S}_n^z \right]$

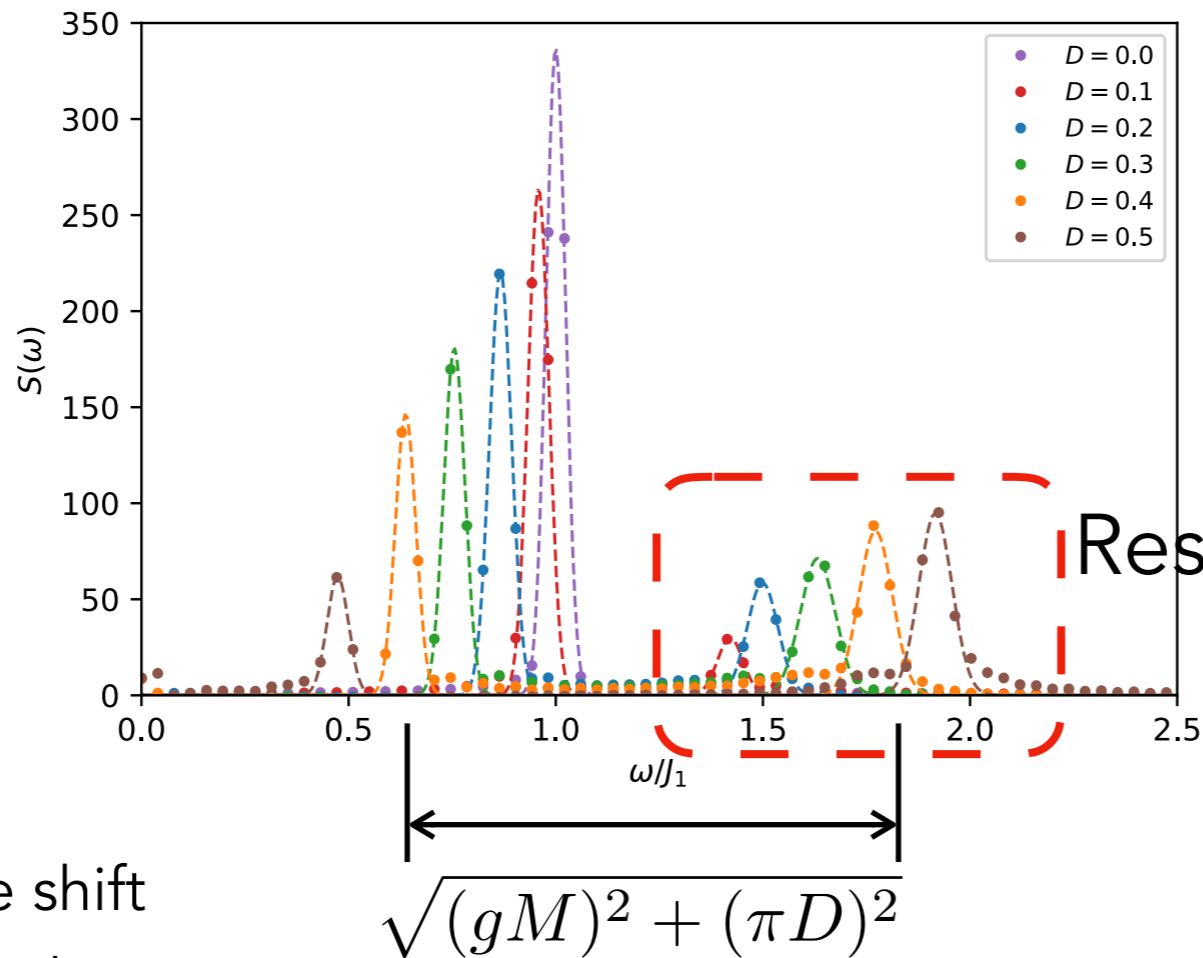
$$\tilde{\mathcal{H}} = \sum_n \sqrt{J^2 + D^2} (\tilde{S}_n^x \tilde{S}_{n+1}^x + \tilde{S}_n^y \tilde{S}_{n+1}^y) + J \tilde{S}_n^z \tilde{S}_{n+1}^z - B \tilde{S}_n^z \approx \sum_n J \tilde{S}_n^a \tilde{S}_{n+1}^a - B \tilde{S}_n^z$$

$$\text{Structure factor } \mathcal{S}(q = 0, \omega)|_{\text{DM}} = \tilde{\mathcal{S}}(D/J, \omega)$$

DM allows ESR to probe upper (forbidden) branch at $Q = D/J$.

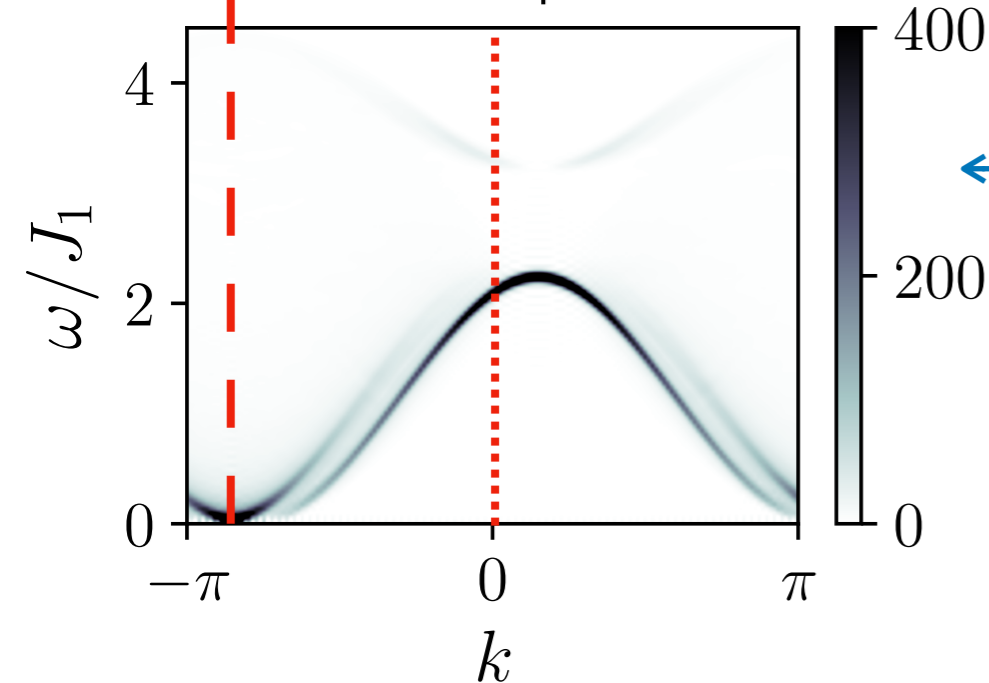
Spinon/2-magnon magnetic resonance B || D

Small M
(B=J)

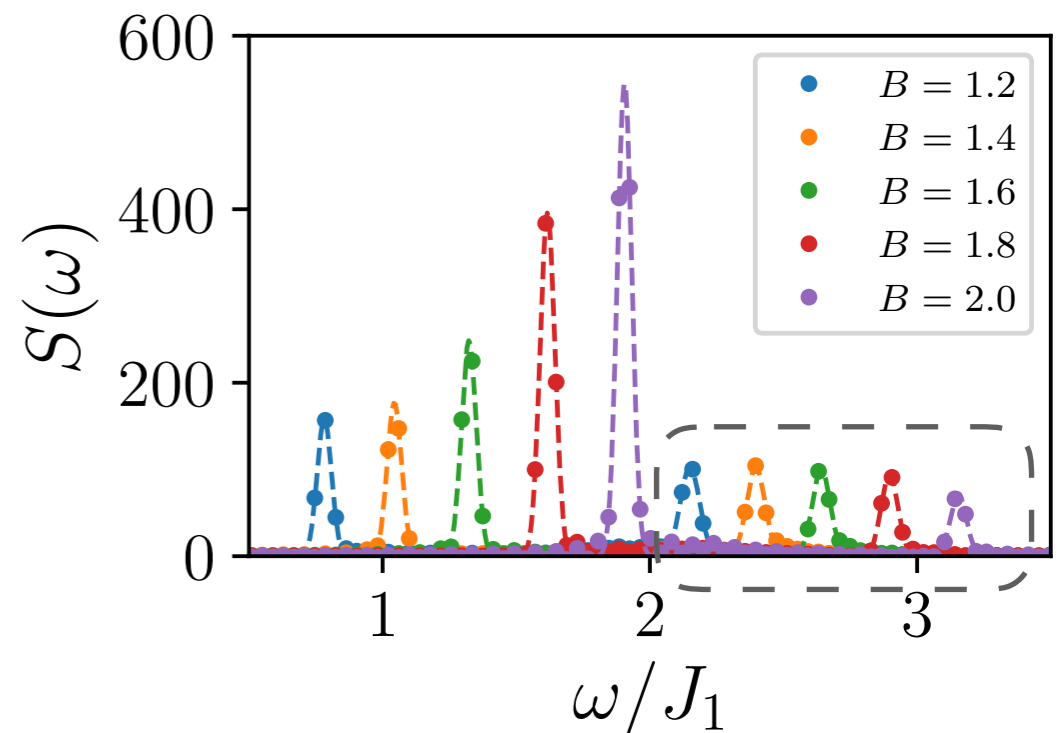


$\pi D / 2B$

Note the shift
of the spectrum



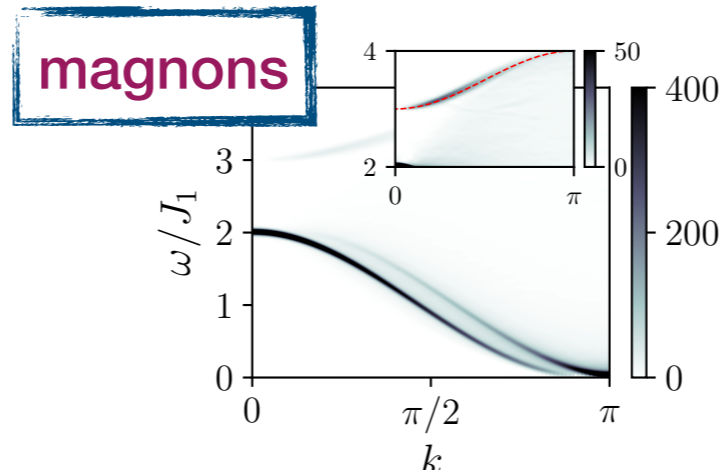
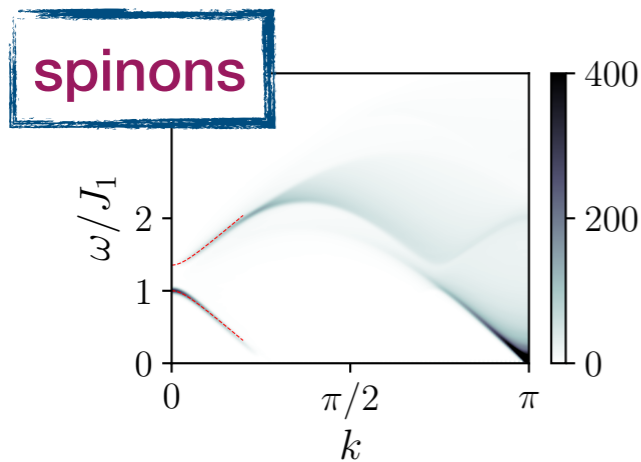
Large M:
2-magnon
mode is probed
by DM as well.



Summary and outlook

• 1D

- Interaction between quasiparticles qualitatively changes transverse dynamical susceptibility
- Finite energy gap between two branches of spin-1 excitations near $k=0$
- Appearance of 2 magnon anti-bound states near saturation



Dynamical Signatures of Quasiparticle Interactions in Quantum Spin Chains

Anna Keselman, Leon Balents, and Oleg A. Starykh
 Phys. Rev. Lett. **125**, 187201 – Published 27 October 2020

• 2D and higher: Applies to higher dimensional U(1) spin liquids.

- Collective transverse spin-1 mode below spinon continuum

