The Minimum Energy Principle
Linking protohalo shapes to anisotropic infall

Marcello Musso

University of Salamanca

(with Ravi Sheth)

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1. Peaks of the energy overdensity (as a model for DM haloes)

The way to energy peaks

- Geometrical radius: $R^3 = 3V/4\pi$
- Mean matter overdensity

$$\delta_R = \frac{1}{V}\int_V \mathrm{d}^3r\,\delta(\mathbf{r})$$

- Characteristic time ~ $(1/\delta_R)^{3/2}$
- Halos of mass M are peaks of $\delta_R(\mathbf{x})$

vs

- Inertial radius: $R_I^2 = \frac{5}{3}\int \frac{\mathrm{d}^3r}{V}|\mathbf{r}-\mathbf{r}_{\mathrm{cm}}|^2$
- Mean energy overdensity

$$\epsilon_R = 5\int_V \frac{\mathrm{d}^3r}{VR_I^2}\,\mathbf{r}\cdot(\nabla\phi - \nabla\phi_{\mathrm{cm}})$$

- Characteristic time ~ $(1/\epsilon_R)^{3/2}$
- Halos of mass M are peaks of $\epsilon_R(\mathbf{x})$

What is the advantage?

- No dynamical meaning in $\nabla\delta_R = 0$
- More small-scale power. In Fourier:

$$\delta_R = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\delta(\mathbf{k})\,\frac{3j_1(kR)}{kR}$$

- $\langle(\nabla^2\delta_R)^2\rangle$ diverges in ΛCDM.
- Usually resort to Gaussian filter.
  Blurred physical interpretation

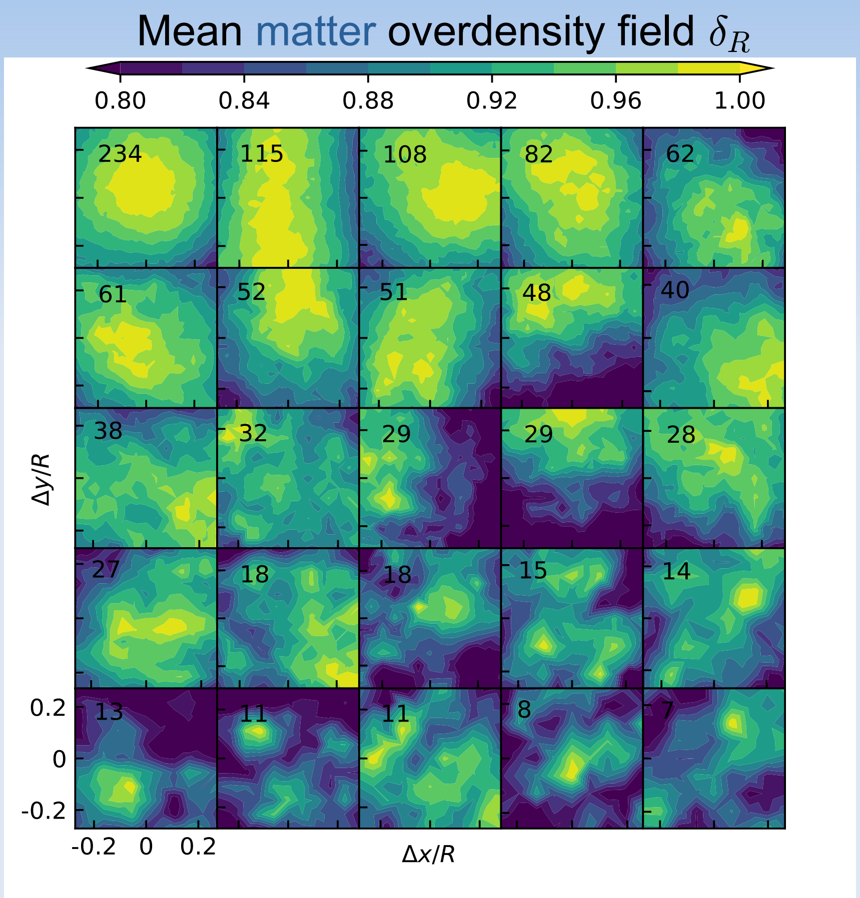
vs

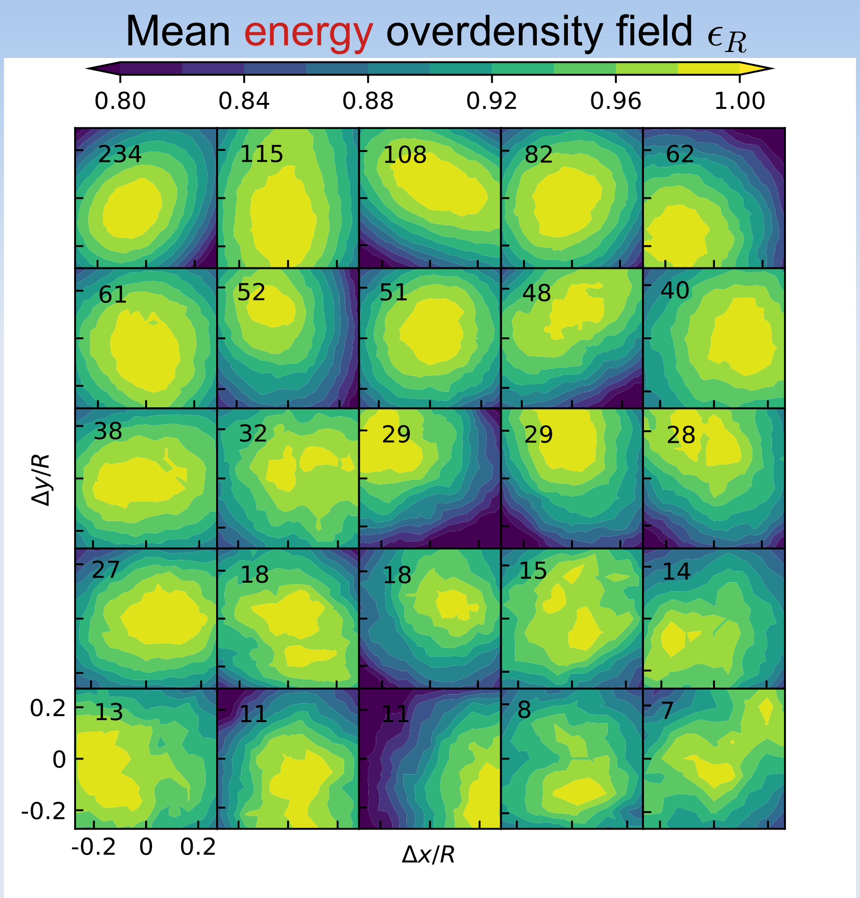
- $\nabla\epsilon_R$ ~ dipole moment.
  $\nabla\epsilon_R = 0$ implies radial infall
- Less small-scale power. In Fourier:

$$\epsilon_R = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\delta(\mathbf{k})\,\frac{15j_2(kR)}{(kR)^2}$$

- $\langle(\nabla^2\epsilon_R)^2\rangle$ remains finite.
- No need to “tweak” the filter.
  Clearly rooted in the EoM

Testing the energy peak ansatz

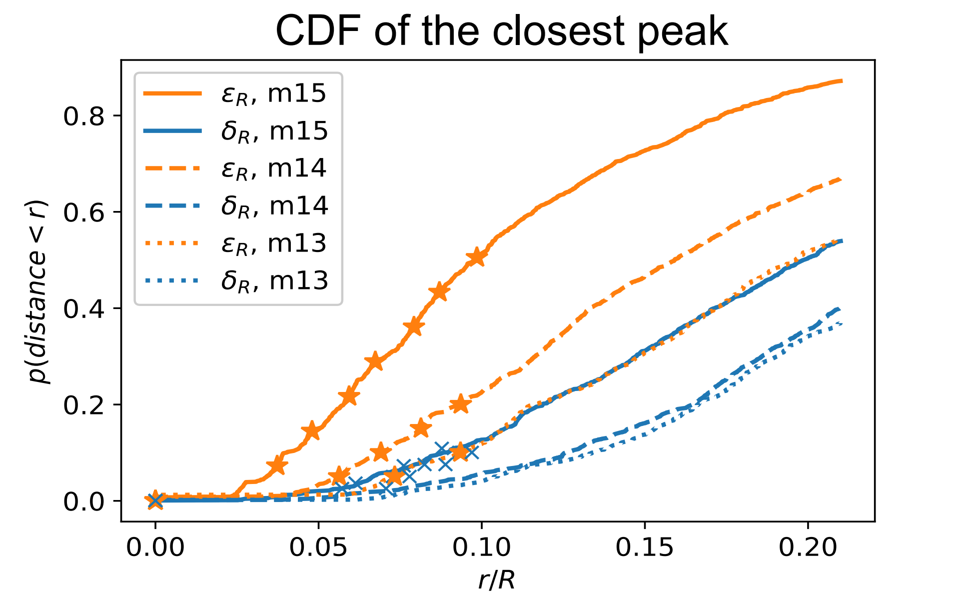


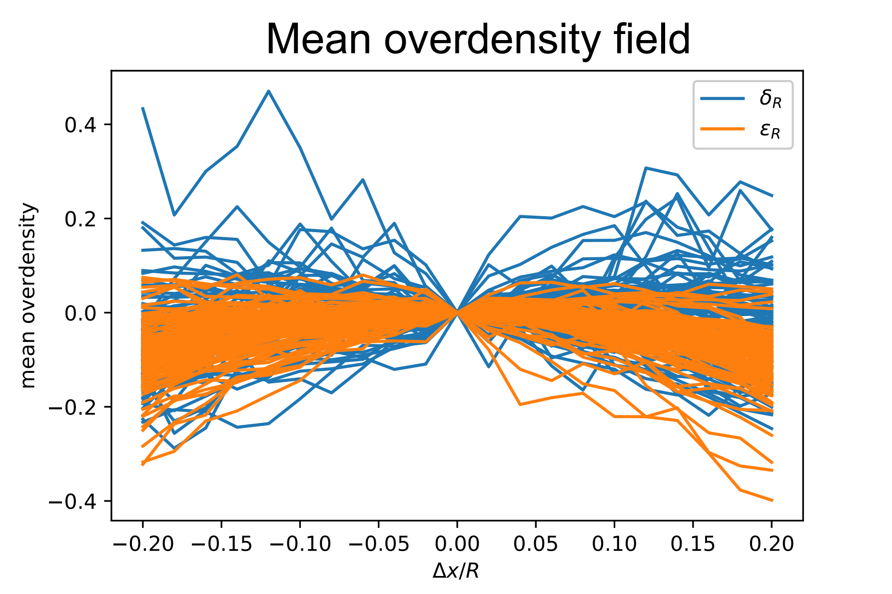


- Energy peaks are a better proxy for protohalo centers!

Testing the energy peak ansatz

- Regions around protohalo centers are more likely to be energy peaks than matter density ones:

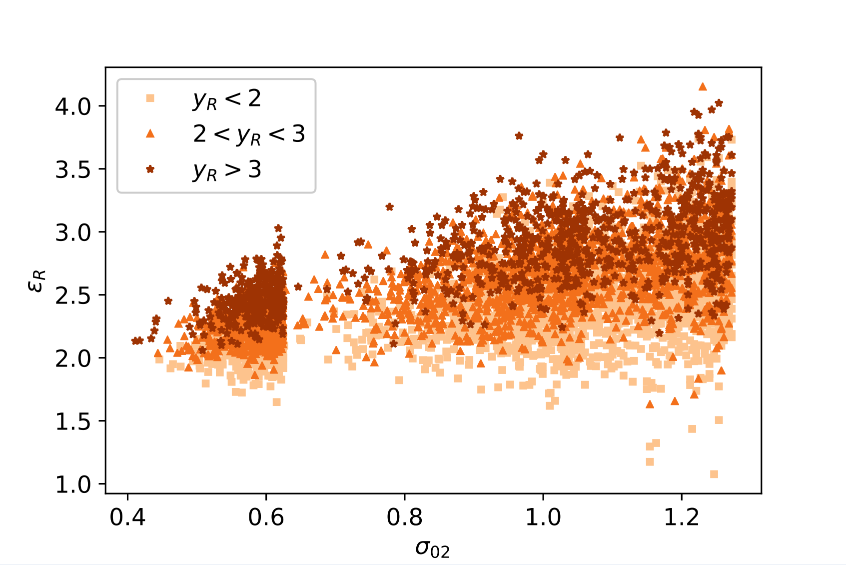


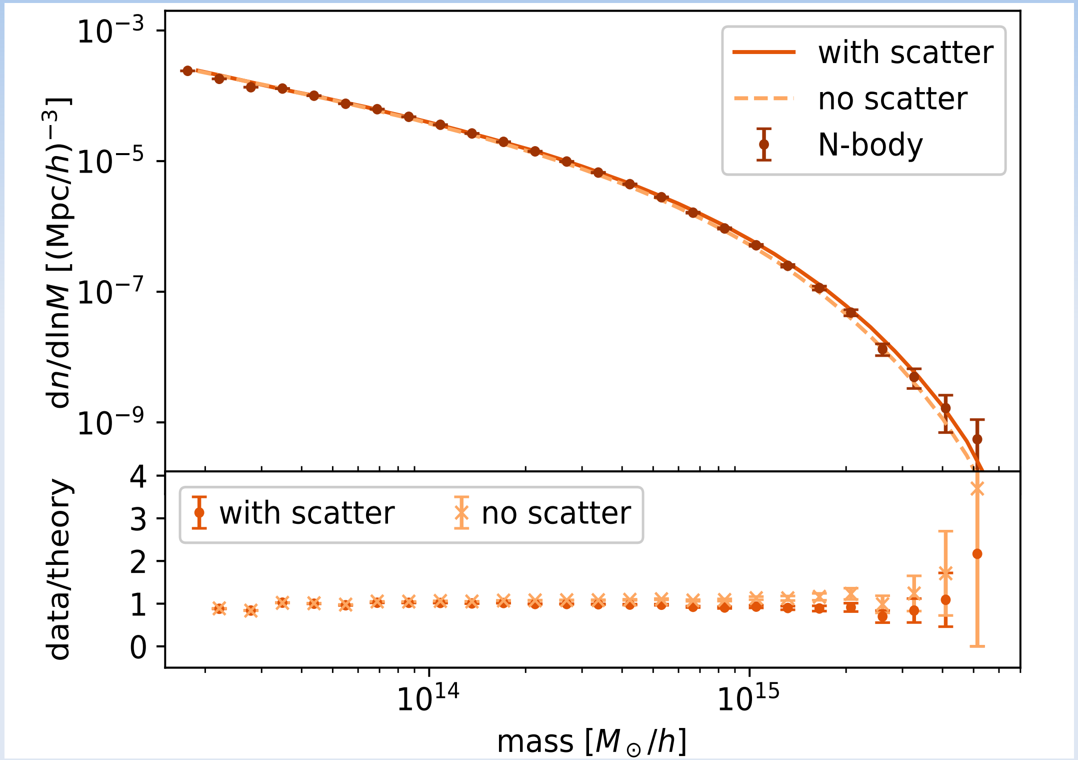


- Protohaloes are more likely to be close to energy peaks

Halo mass function

- Predicted, using a fit to ϵ_R





- Scatter can describe assembly bias.

2. The Minimum Energy Principle

Shape of maximal ϵ

- The total initial energy (or curvature) of the patch is

$$E = -\frac{4\pi G\bar{\rho}}{3} R_I^2 \epsilon$$

- Once a spherical peak is found, one can further increase ϵ by deforming the sphere (at fixed volume)
- The inertial radius R_I of the deformed region collapses even faster
- The boundary of the region of maximal ϵ (minimal E) must be an isosurface of

$$\mathcal{V}(\mathbf{r}) \equiv (\mathbf{r} - \mathbf{r}_{\rm cm}) \cdot \left[\nabla\phi - \nabla\phi_{\rm cm} - \frac{\epsilon}{3}(\mathbf{r} - \mathbf{r}_{\rm cm})\right]$$

- Proxy for protohalo shape and boundary!

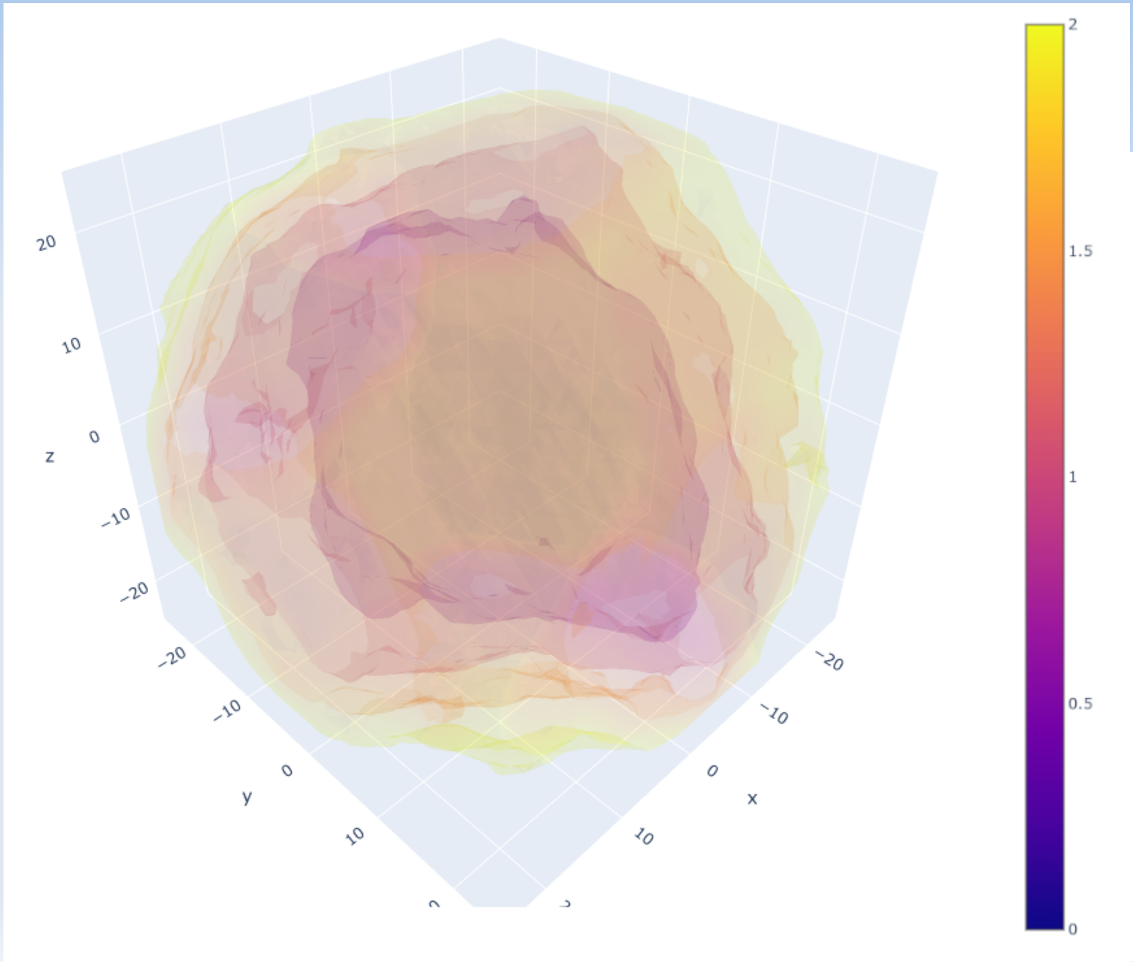
Shear-shape alignment

- The equipotential surface is non-spherical if $\nabla\phi$ is anisotropic.
- Deformation can be computed analytically. Starting from spherical peak ($\boldsymbol{r}_{\rm cm} = 0$):

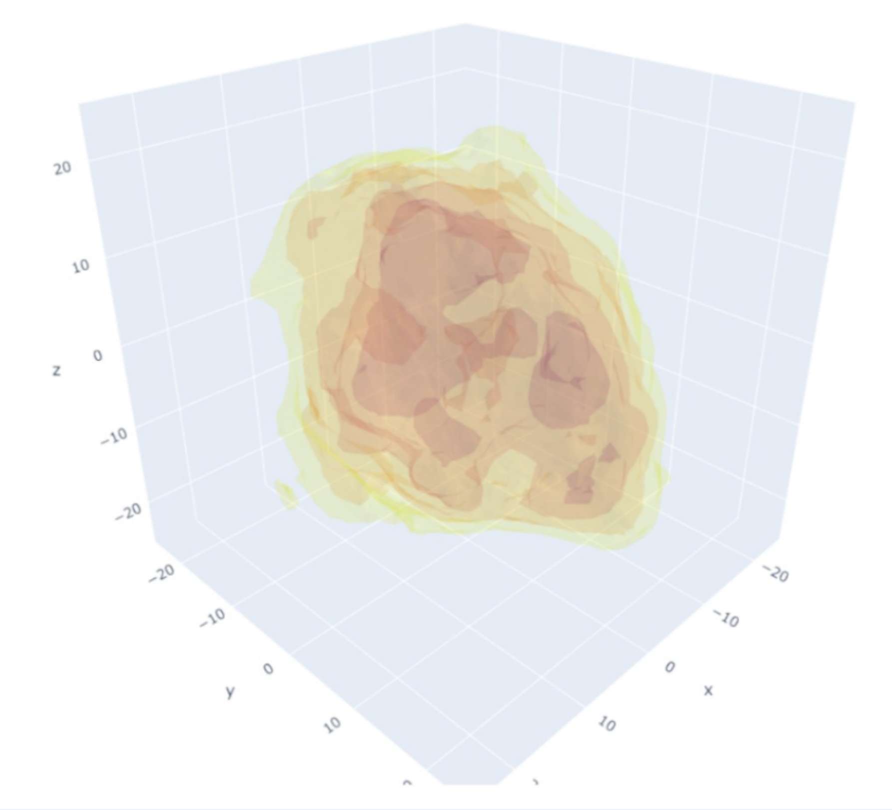
$$\Delta R(\theta, \phi) \simeq 3\frac{\hat{\mathbf{r}} \cdot \nabla\phi - R\delta_R/3}{R|\delta'_R + 2\epsilon'_R/5|}$$

- $\Delta R > 0$ if $\hat{\mathbf{r}} \cdot \nabla\phi$ is greater than its angular average $(\delta_R/3)R$.
- Longest axis in the direction of maximum compression (orthogonal to the filament)

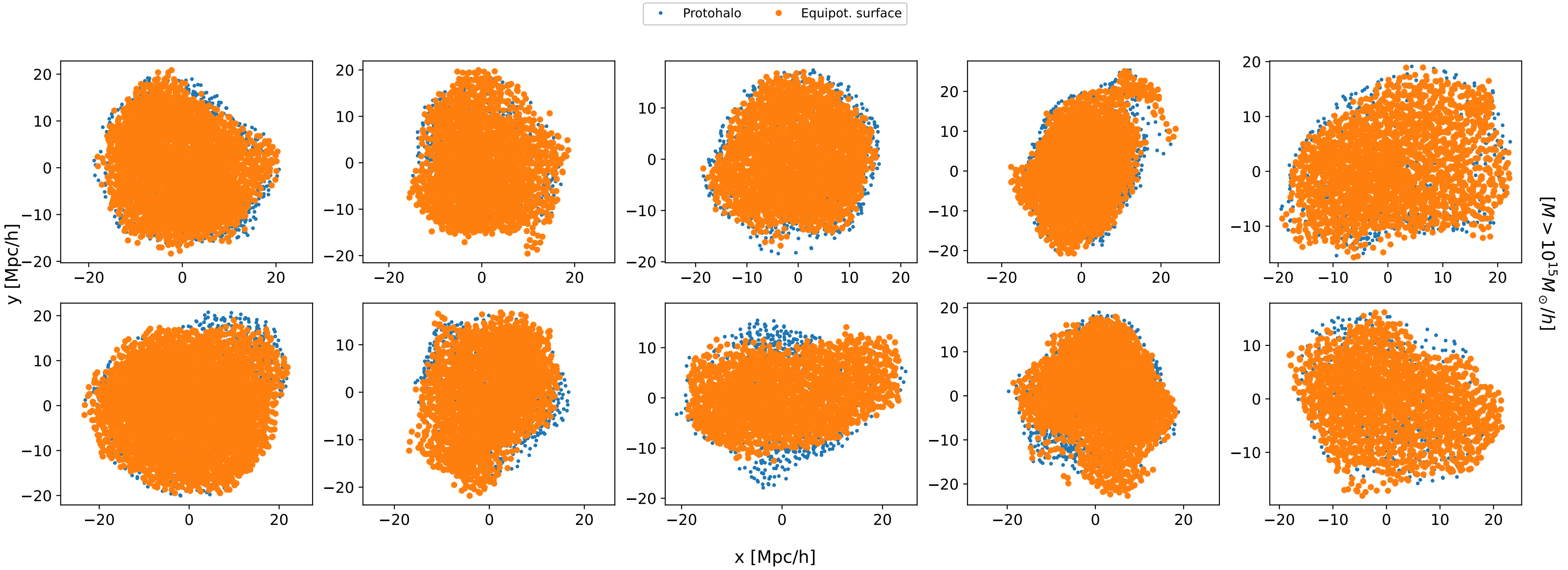
Equipotential surfaces



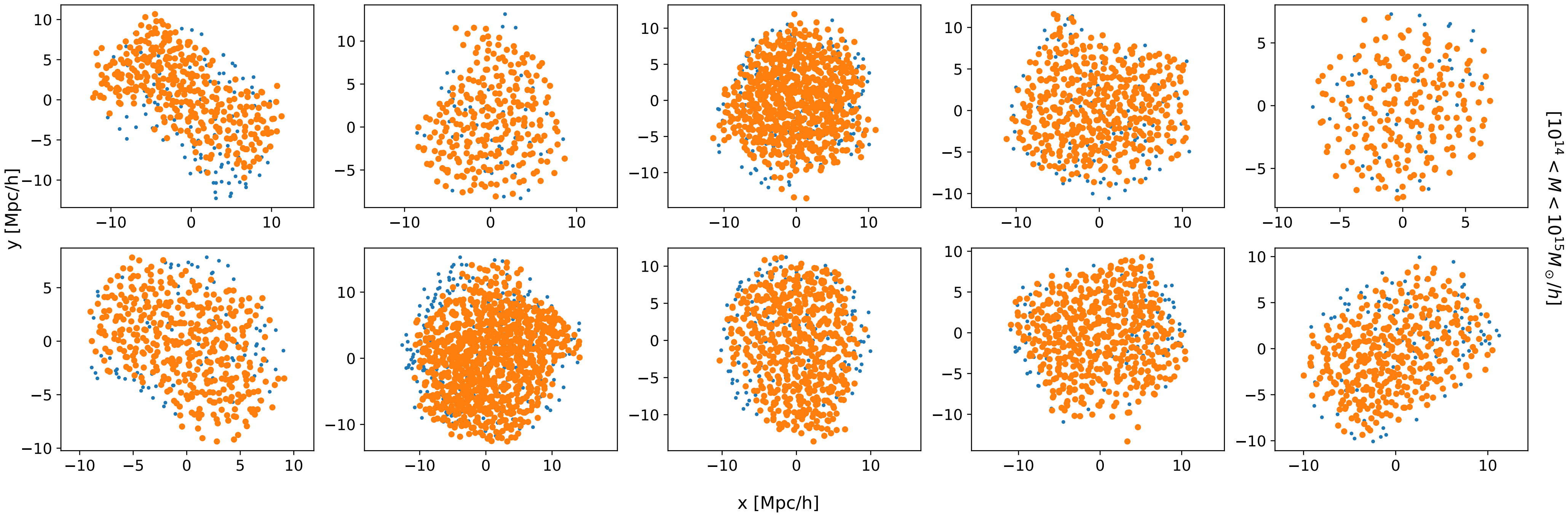
Two sets of surfaces of constant $\mathcal{V}$ around a protohalo center



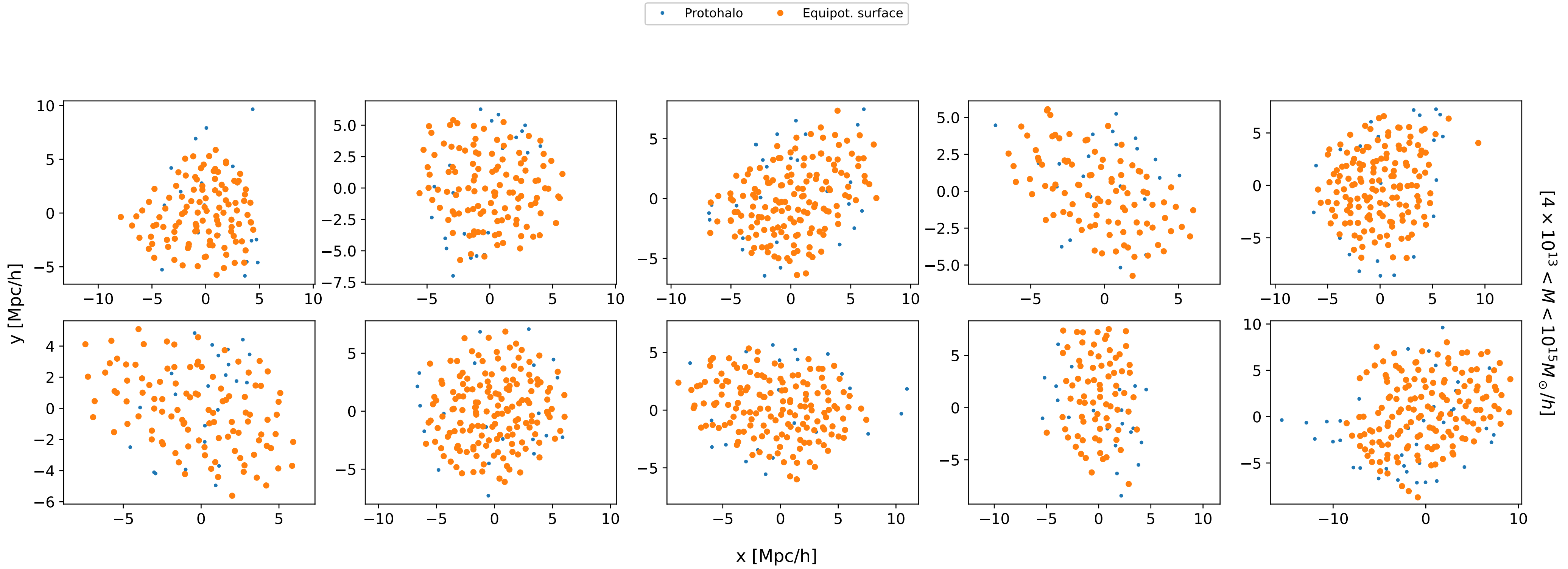
Protohaloes vs equipotential surfaces



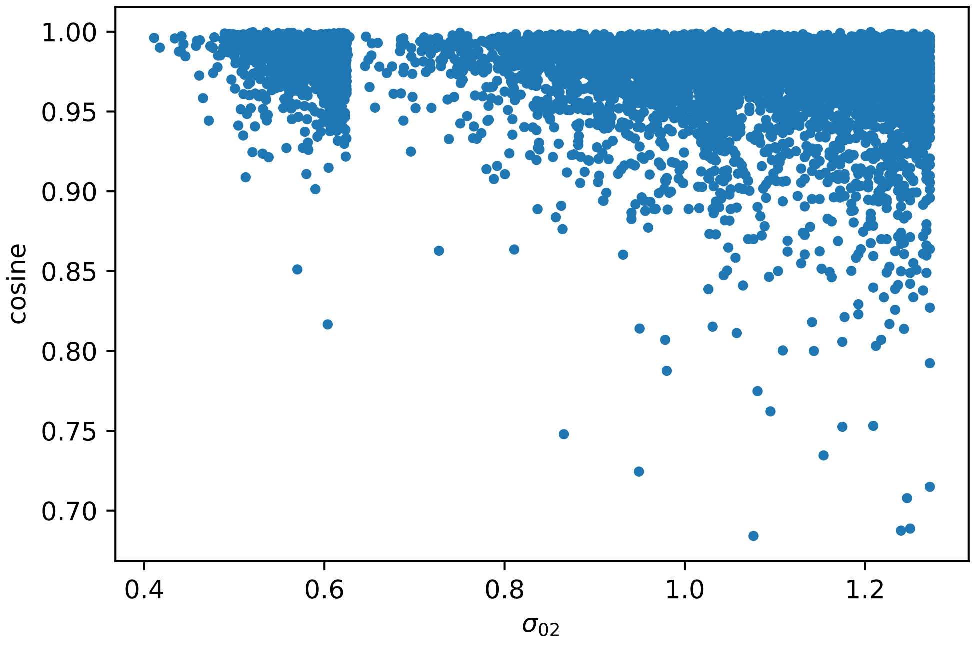
Protohaloes vs equipotential surfaces



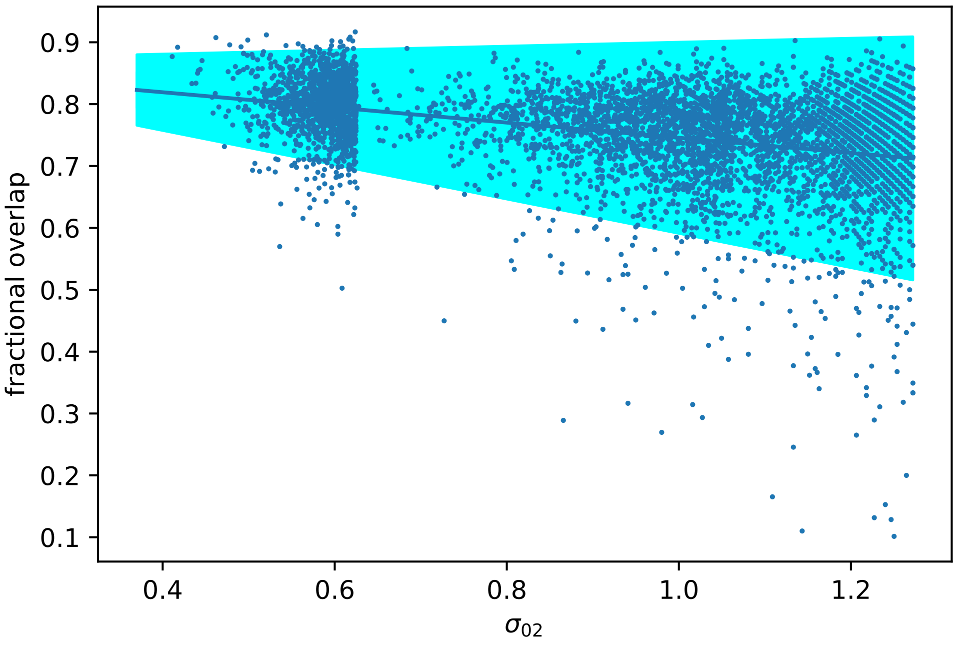
Protohaloes vs equipotential surfaces



Alignments

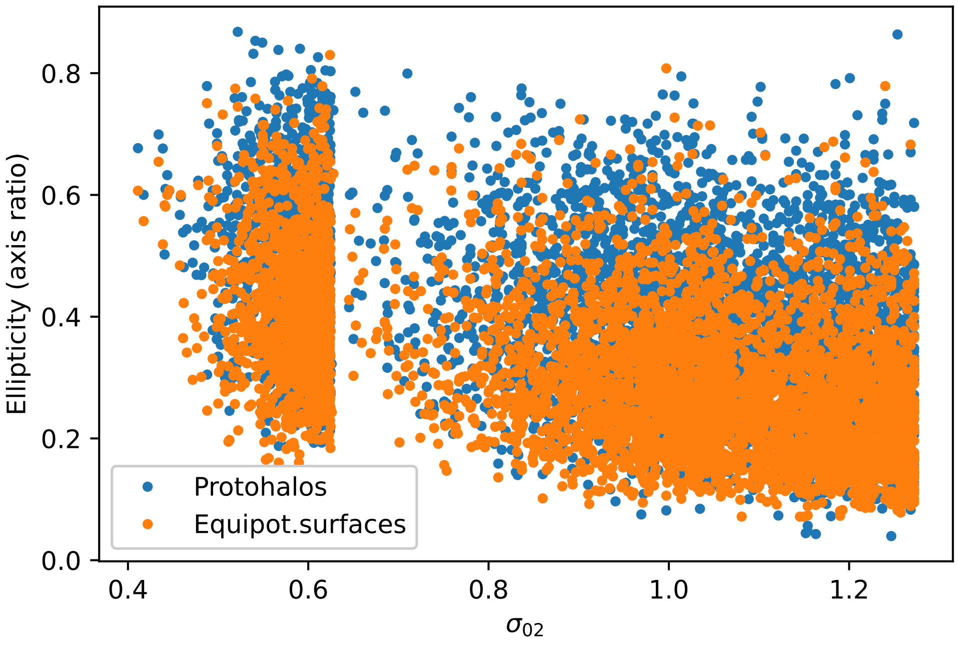


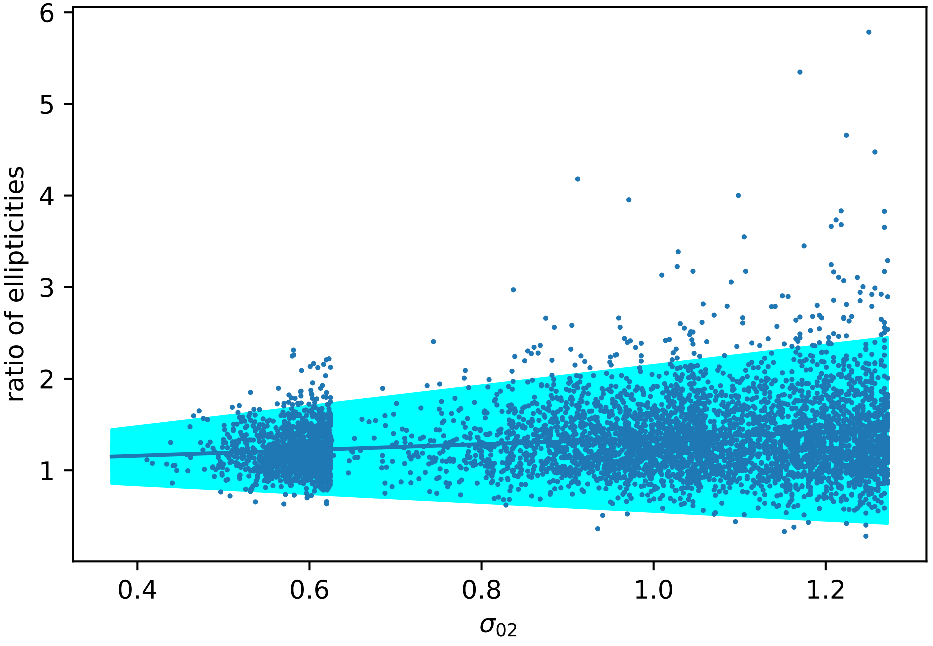
1.00
0.95
0.90
0.85
0.80
0.75
0.70
cosine
0.4
0.6
0.8
1.0
1.2
σ_{02}



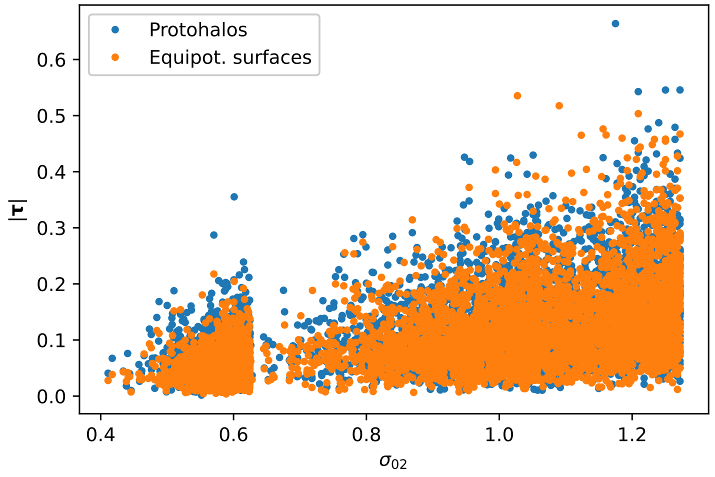
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
fractional overlap
0.4
0.6
0.8
1.0
1.2
σ_{02}

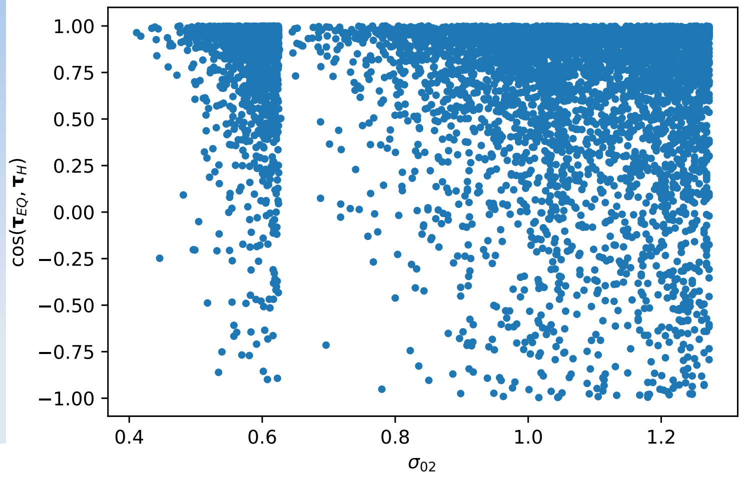
Ellipticities





Torques

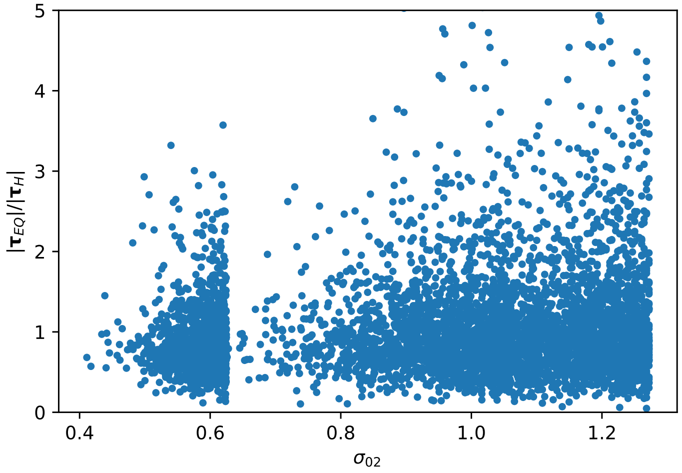




$$\tau_i = -\frac{MR_I^2}{5}\varepsilon_{ijk}\epsilon_{jk}$$

$\epsilon_{ij} =$ energy ovedensity tensor.

No approx here!



Conclusions

- Protohaloes are peaks of the initial energy overdensity field. Not densest but most energetically bound initial regions, having fastest collapse times.
- Peaks in ϵ_R create convergent matter flows. Initial evolution matches perturbation theory. Final high mean density results dynamically, not put in “by hand”.
- Using ϵ_R instead of δ_R simply means changing the filter (to a more convergent one)
- Energy density peaks are better behaved, and better proxies for protohalo centers

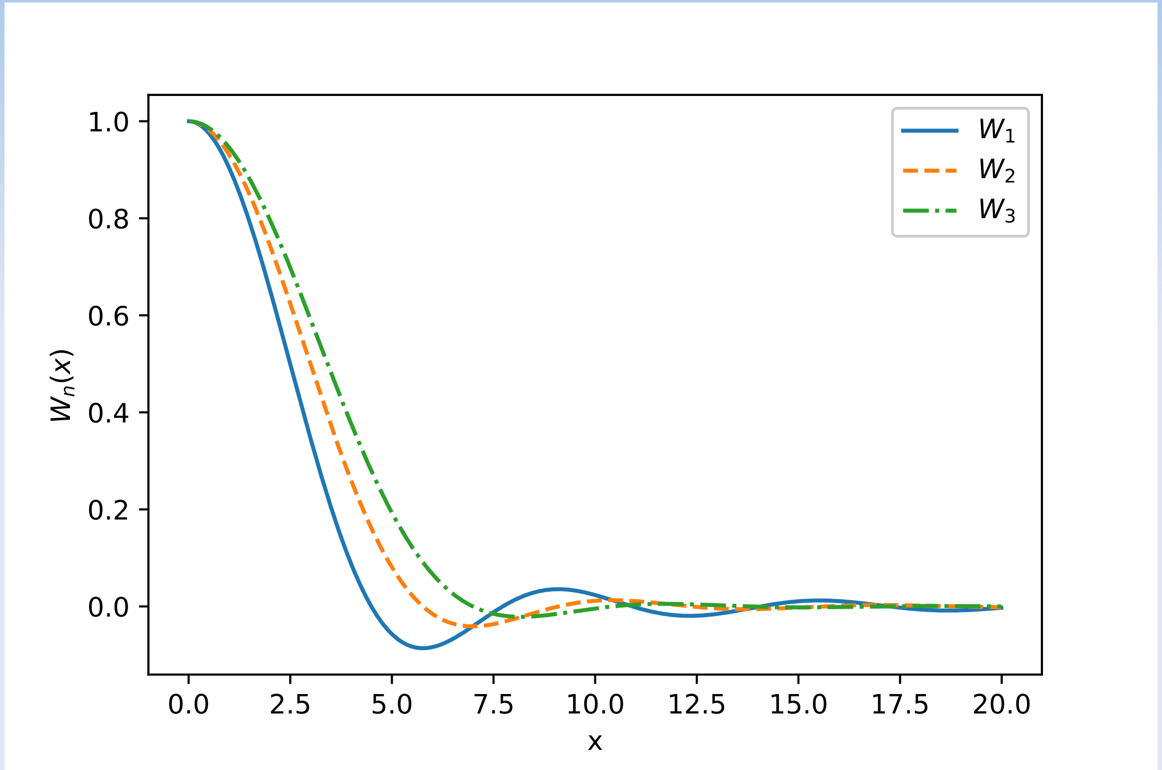
- Non-spherical shapes of maximal ϵ_R are equipotential surfaces
- Excellent description of protohalo shapes and shear-shape alignments
- Handle on assembly history and secondary halo properties (e.g. accretion rate, torque...)

Open questions and outlook

- Can we predict critical value ϵ_c ? Must model virialization (in progress)
- Relation with halo finder? Ellipsoidal? FOF? Energy-based?
- Angular momentum? (in progress)
- How to improve even more? Account for non-conservation of energy?
- Final shear/shape alignments?
- (Assembly) bias? Voids? Skeleton/cosmic web?
- Primordial BHs?
- …

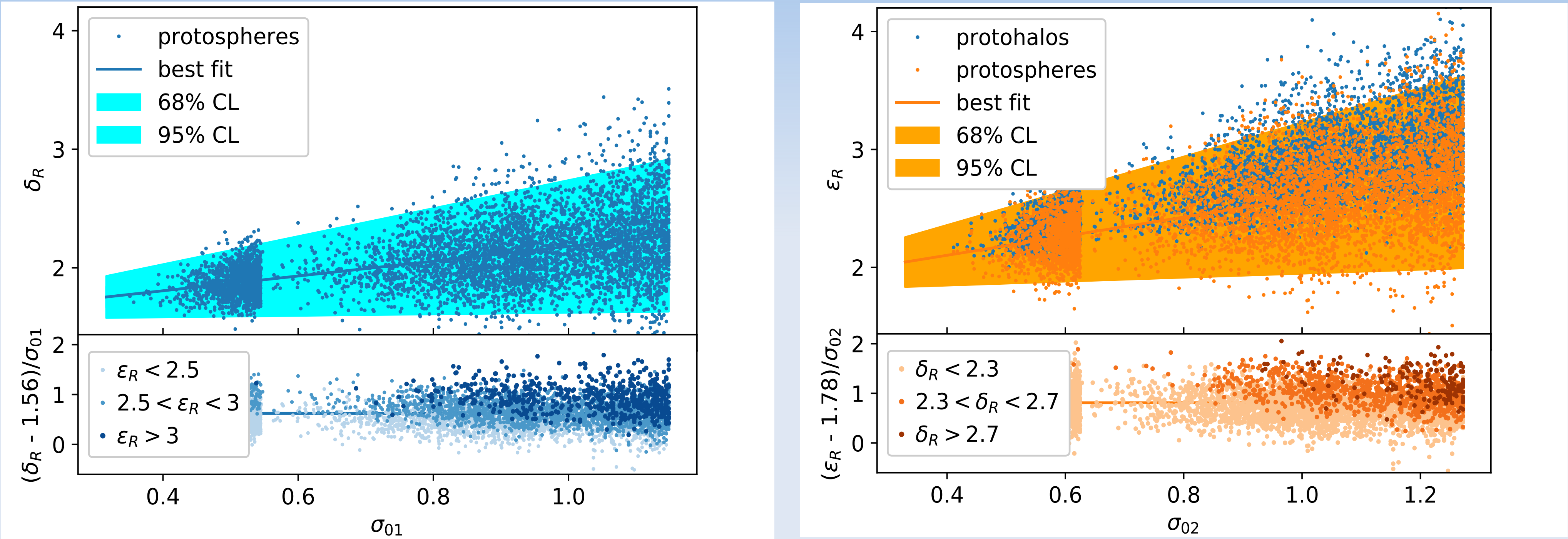
Thank you!!

The filters



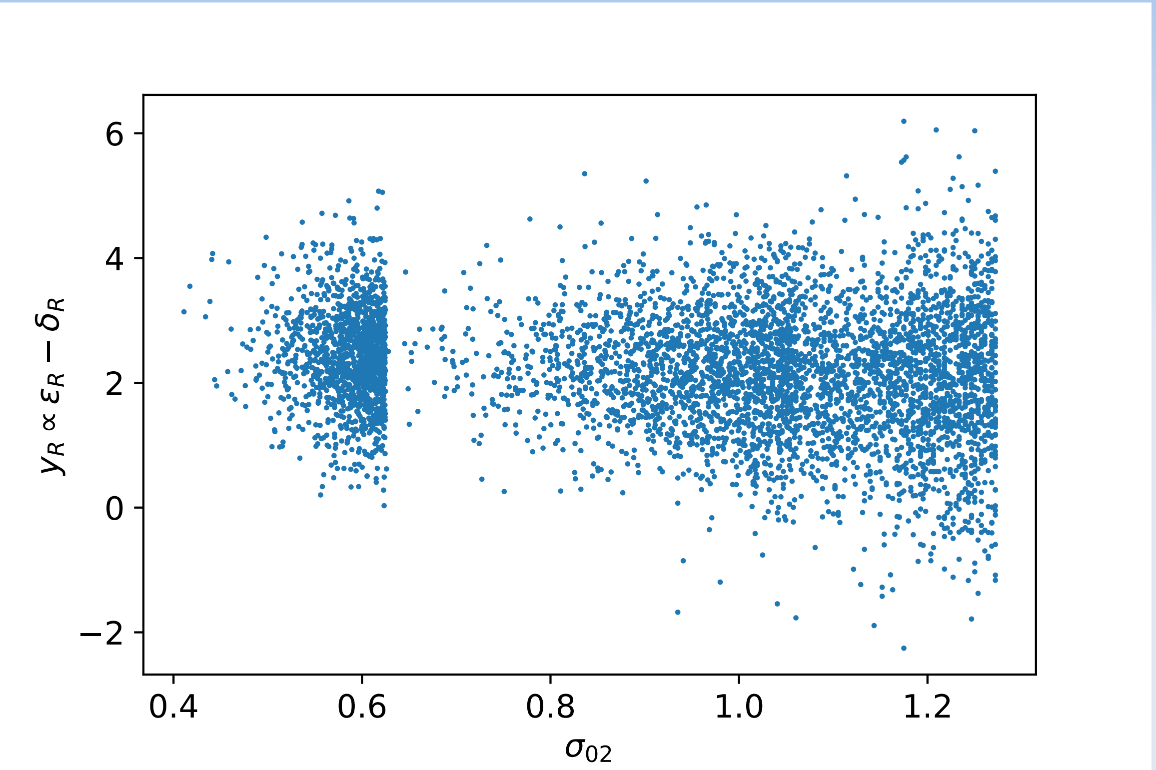
- The W_1 filter converges more slowly and has more pronounced wiggles

Peak heights



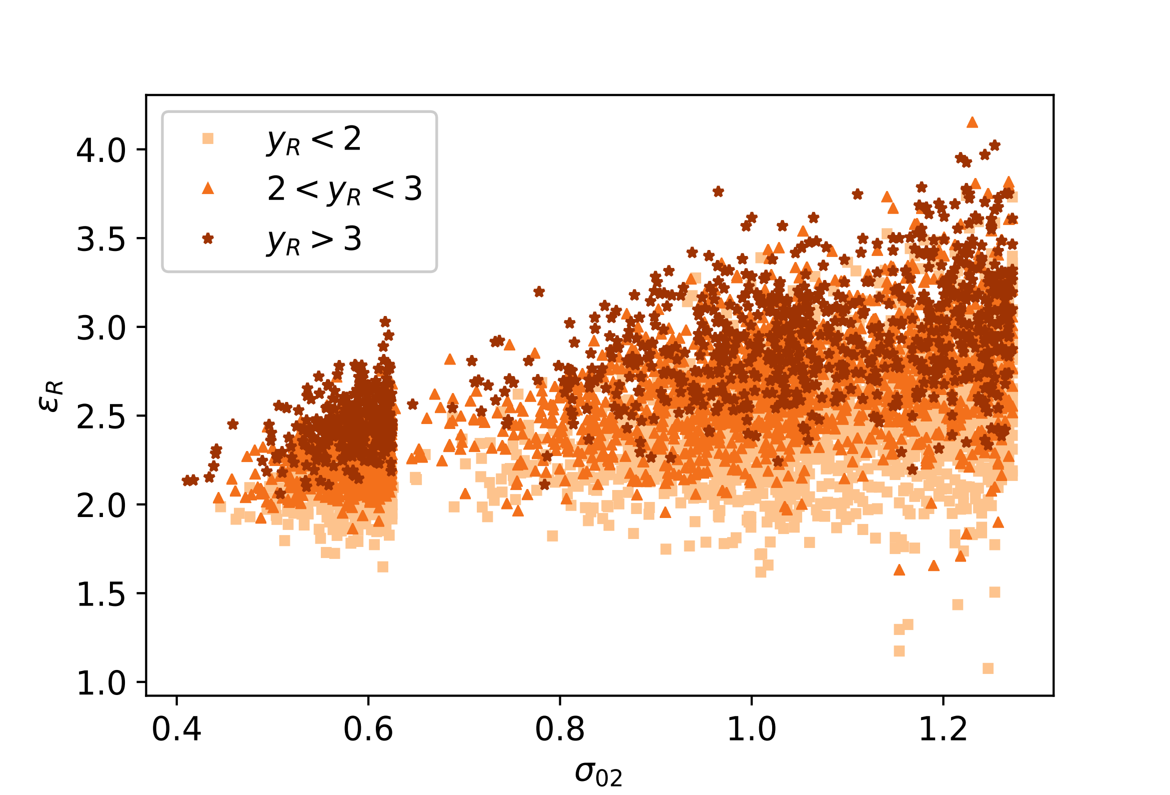
- The measured value of δ_R is not really $\delta_c = 1.686$ and it correlates with ϵ_R
- But ϵ_R is not a constant either...

Excursion set slopes



- The “slope of the excursion set” -$\mathrm{d}\epsilon_R/\mathrm{d}R$ at the center is always positive. Consistent with the peak ansatz.

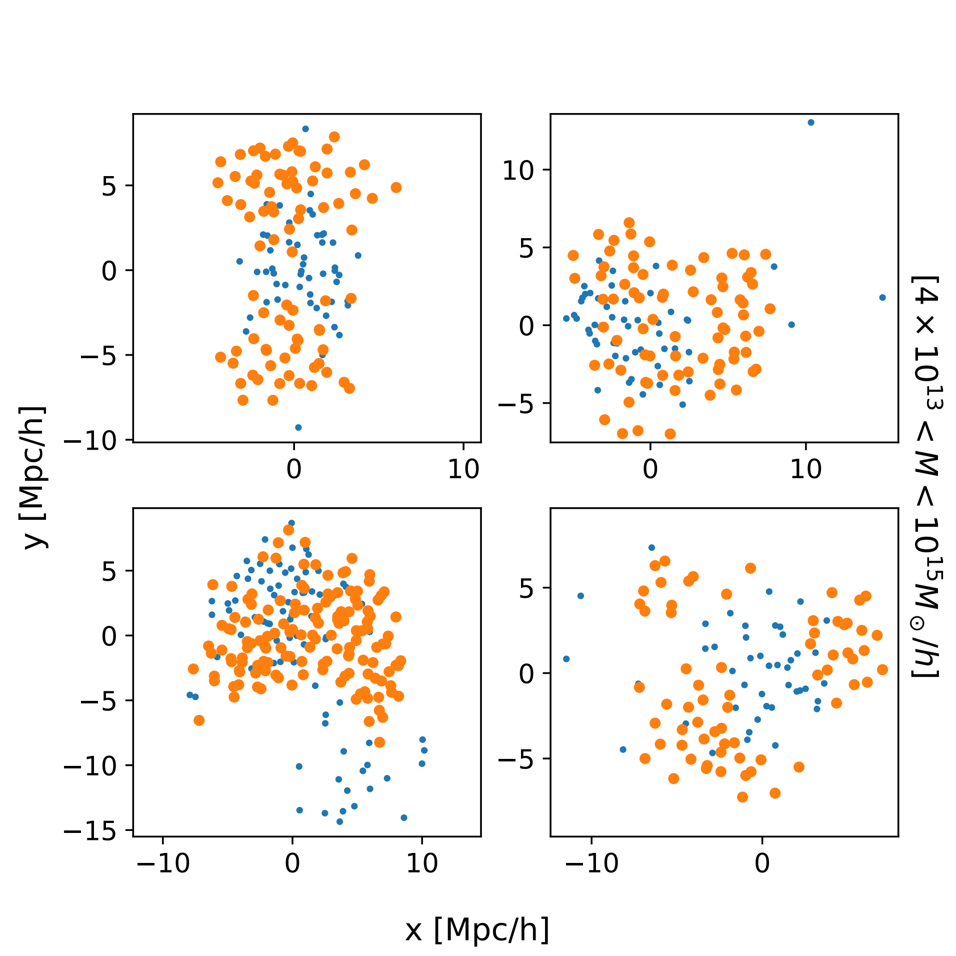
Excursion set slopes



- Peak height and excursion set slope correlate.
- What is the slope for the final halo? Accretion rate?

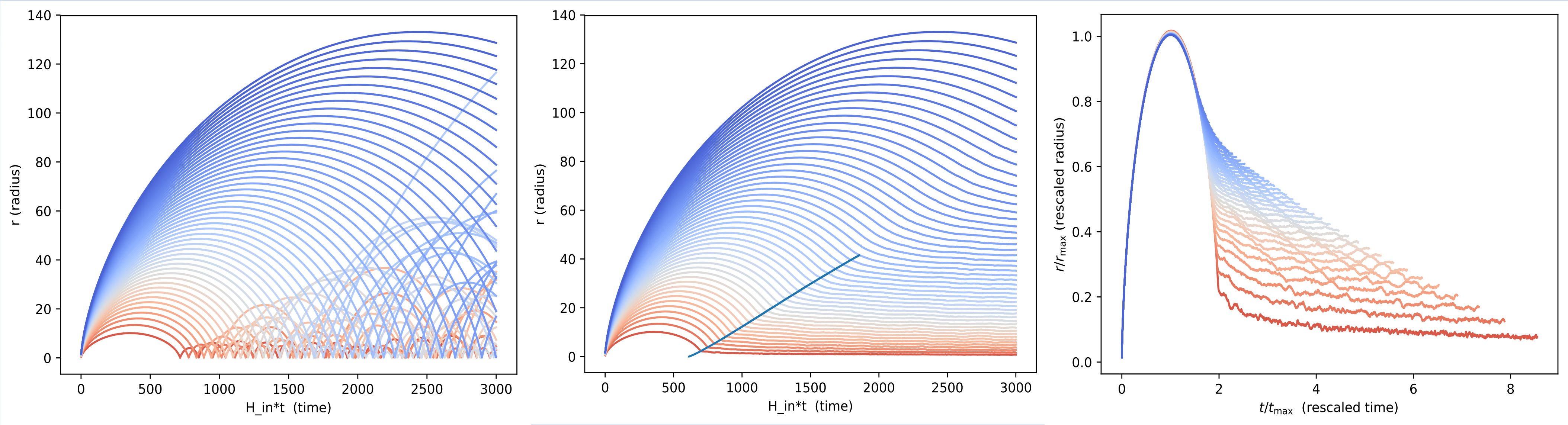
It does not always work...

- At low mass, sometimes the prediction fails
(here, < 40% of protohalo particles identified correctly)



Spherical model of virialization

- Shells cross the center at different times, and then start crossing each other. Mass and energy withing each shell are NOT conserved
- The radius of mass-conserving spheres freezes (null mean velocity)



- The virialization radius of each shell is NOT half of the turnaround radius

MM (in prep)